Update on radiative capture reactions with Halo/cluster EFT

Renato Higa

Instituto de Física
Universidade de São Paulo

Workshop on Multineutron Clusters in Nuclei and Stars
ICTP-SAIFR, São Paulo, June 02-06, 2025

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- Collaborators: Gautam Rupak, Lakma Fernando, Akshay Vaghani, Pradeepa Premarathna (MSSU)

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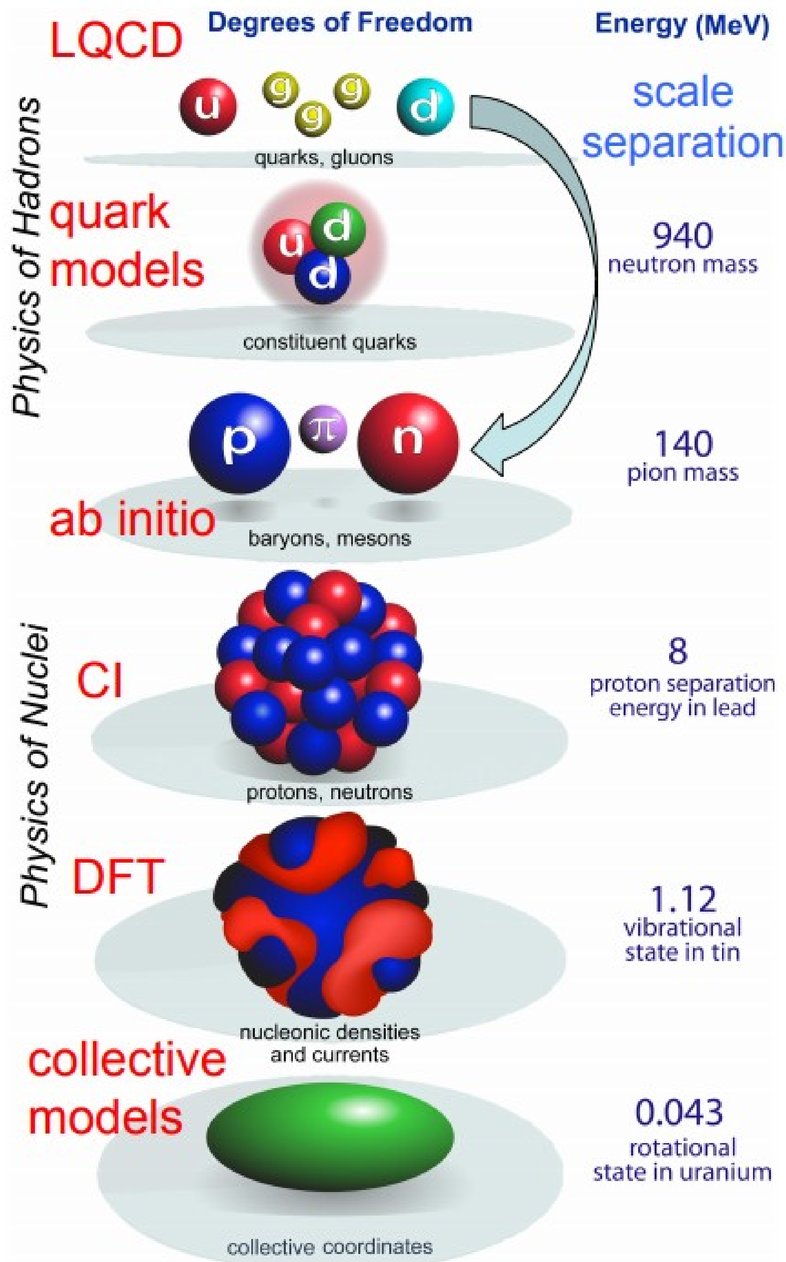
How are nuclei made?

Origin of elements, isotopes



Resolution

Hot and dense quark-gluon matter

Hadron structure

Hadron-Nuclear interface

Effective Field Theory

Nuclear structure
Nuclear reactions
New standard model

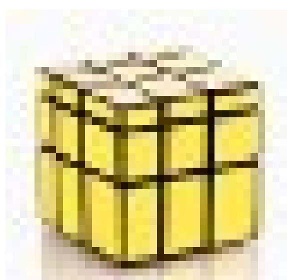
Applications of nuclear science

To explain, predict, use…

(Slide from W. Nazarewicz)

EFT: resolution scale

EFT: resolution scale



EFT: resolution scale

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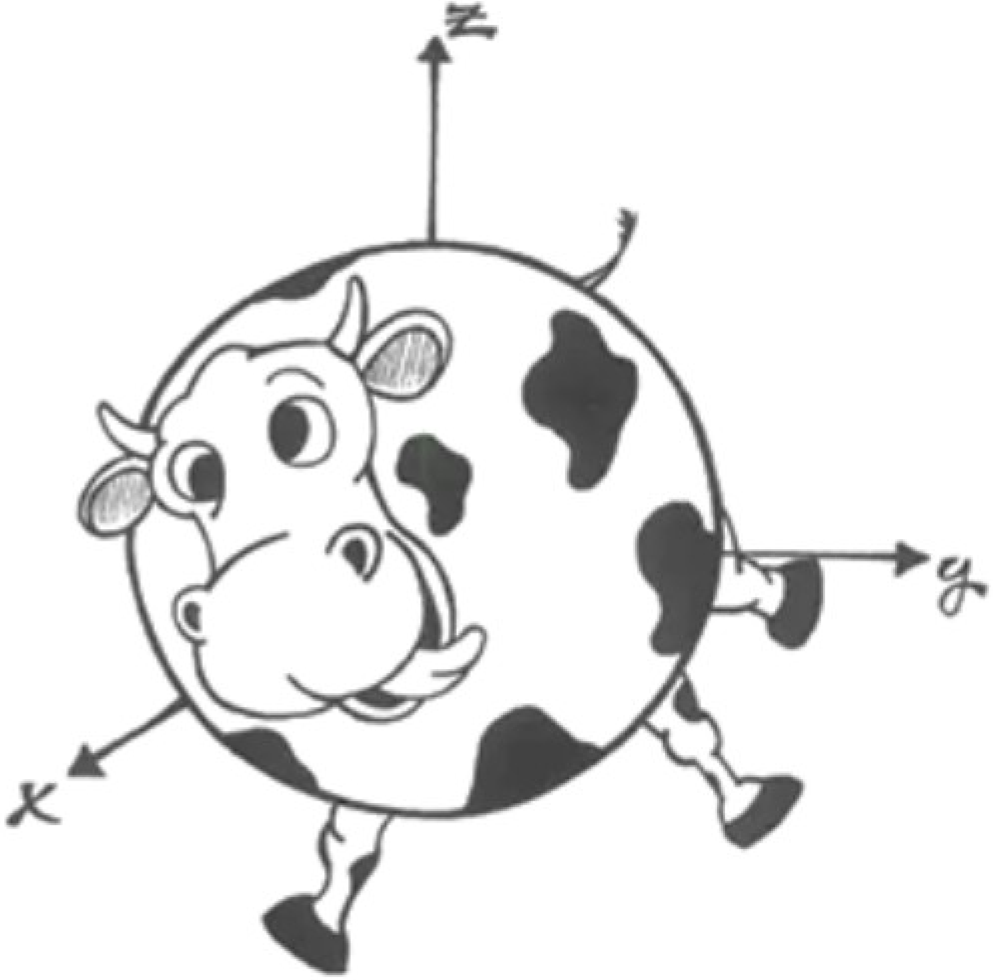
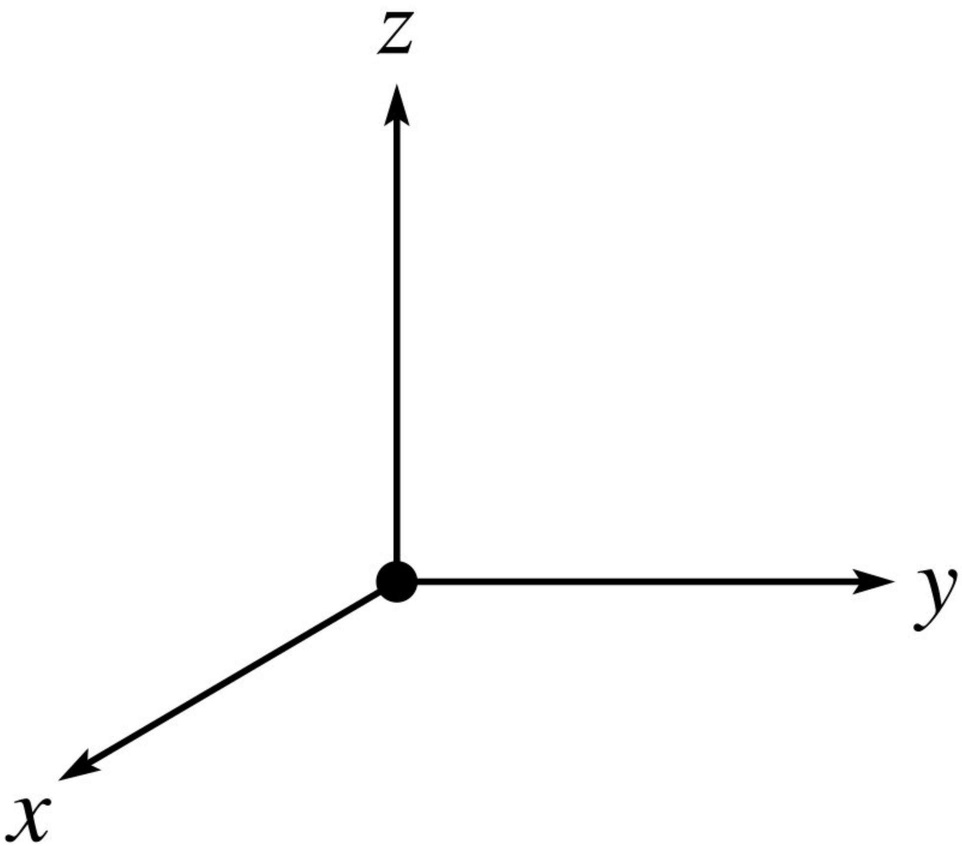


EFT: resolution scale



EFT with SRI

$k \sim 1/a \sim M_{lo}$, $1/R \sim M_{hi}$,

2-body: shallow bound state ($E_2 = \hbar^2/ma_2^2 + \ldots$), scaling limit at LO

RG flow towards a non-trivial fixed point (Birse et al., ...)

$|a| \rightarrow 0 \ \Rightarrow$ unitary limit $\Rightarrow$ no scales (NR-conf. inv.)

3-body: renormalization demands a 3-body interaction at LO

RG flow towards a limit cicle

- V. Efimov, PLB33, 563 (1970)
- Amado and Nobel, Adhikari et al., Fonseca et al., Minlos and Faddeev, Frederico et al., Fedorov et al., Bedaque et al., Greene et al., ...

EFT with SRI

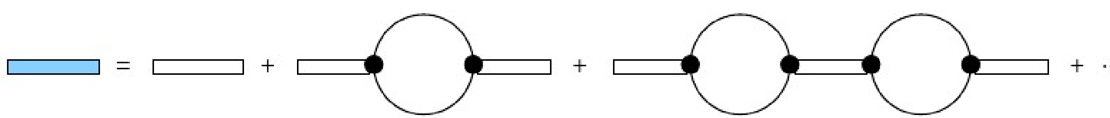
$k \sim 1/a \sim M_{lo}$, $1/R \sim M_{hi}$,

2-body: shallow bound state ($E_2 = \hbar^2/ma_2^2 + \ldots$), scaling limit at LO

RG flow towards a non-trivial fixed point (Birse et al., ...)

$|a| \to 0 \Rightarrow$ unitary limit $\Rightarrow$ no scales (NR-conf. inv.)

$$\mathcal{L} = N^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{2M} \right] N + \phi_i^{(s)\dagger} \left[\underbrace{i\partial_0 + \frac{\vec{\nabla}^2}{4M}}_{\text{NLO}} + \underbrace{\Delta}_{\text{LO}} \right] \phi_i^{(s)} + g_0 \left[\phi_i^{(s)\dagger} N^T \tilde{P}_i^{(s)} N + \text{H.c.} \right] + \cdots ,$$



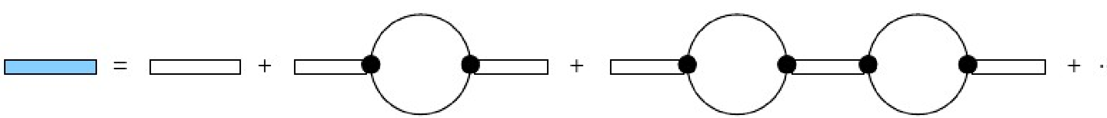
$$T_0 = -\frac{4\pi}{M} \frac{1}{-\frac{4\pi\Delta}{Mg_0^2} + \frac{8\pi}{M^2 g_0^2}\frac{k^2}{2} + \cdots - ik}$$

EFT with SRI

$$k \sim 1/a \sim M_{lo}, \qquad 1/R \sim M_{hi},$$

Bethe's ERE: $$T_0 = -\frac{4\pi}{M}\frac{1}{-\frac{1}{a_0} + r_0\frac{k^2}{2} + \ldots - ik}$$

$$\mathcal{L} = N^\dagger\left[i\partial_0 + \frac{\vec{\nabla}^2}{2M}\right]N + \phi_i^{(s)\dagger}\left[\underbrace{i\partial_0 + \frac{\vec{\nabla}^2}{4M}}_{\text{NLO}} + \underbrace{\Delta}_{\text{LO}}\right]\phi_i^{(s)} + g_0\left[\phi_i^{(s)\dagger}N^T\tilde{P}_i^{(s)}N + \text{H.c.}\right] + \cdots ,$$



$$T_0 = -\frac{4\pi}{M}\frac{1}{-\frac{4\pi\Delta}{Mg_0^2} + \frac{8\pi}{M^2g_0^2}\frac{k^2}{2} + \cdots - ik}$$

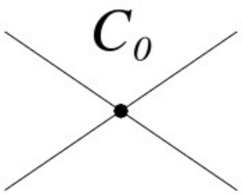
Halo/cluster EFT

$k \sim 1/a \sim M_{lo}, \quad 1/R \sim M_{hi},$

- excitation of each cluster: $\sqrt{m_c E_c^*} \sim M_{hi} \sim m_\pi$
- binding of the valence nucleons (clusters): $\sqrt{2\mu E_{rel}} \sim M_{lo} \ll M_{hi}$
- extension of the core – treated in *perturbation theory*
- power-counting: essential to assure convergence and reliable theoretical uncertainties

Bertulani, Hammer, van Kolck, NPA 172, 37 (2000)
Bedaque, Hammer, van Kolck, PLB 569, 159 (2003)

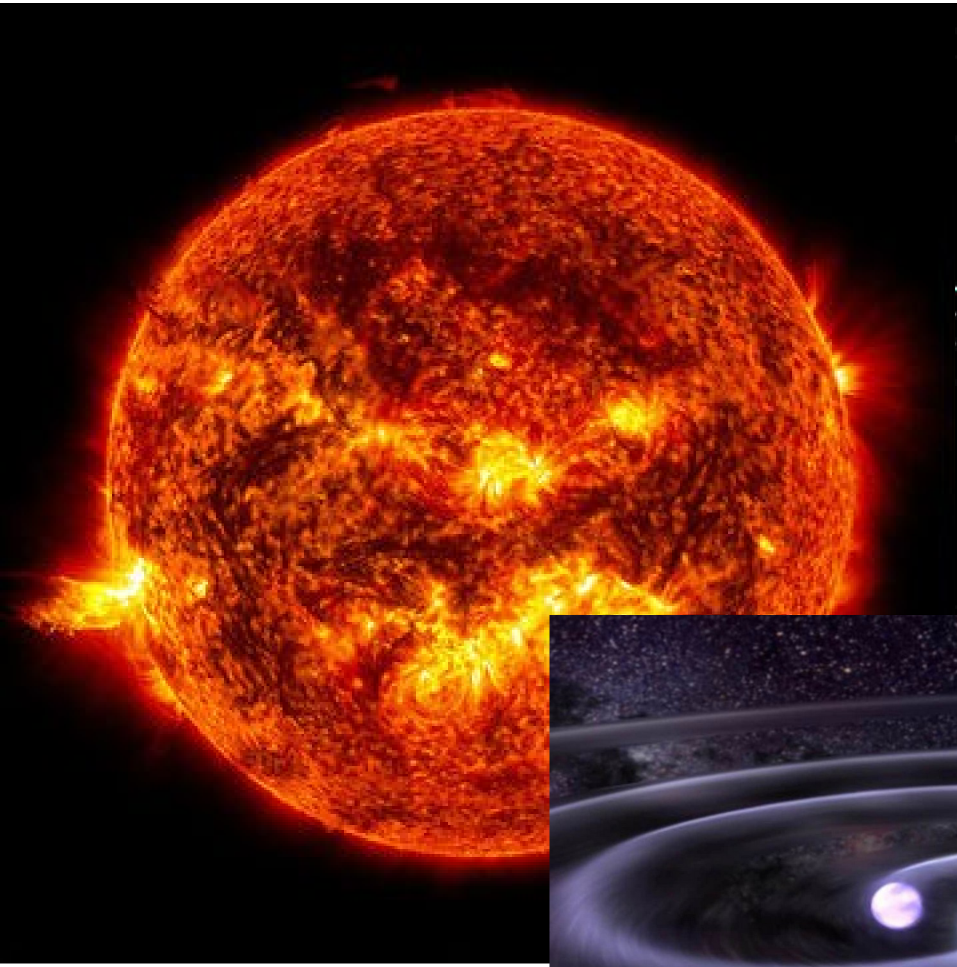
Potential models vs EFT

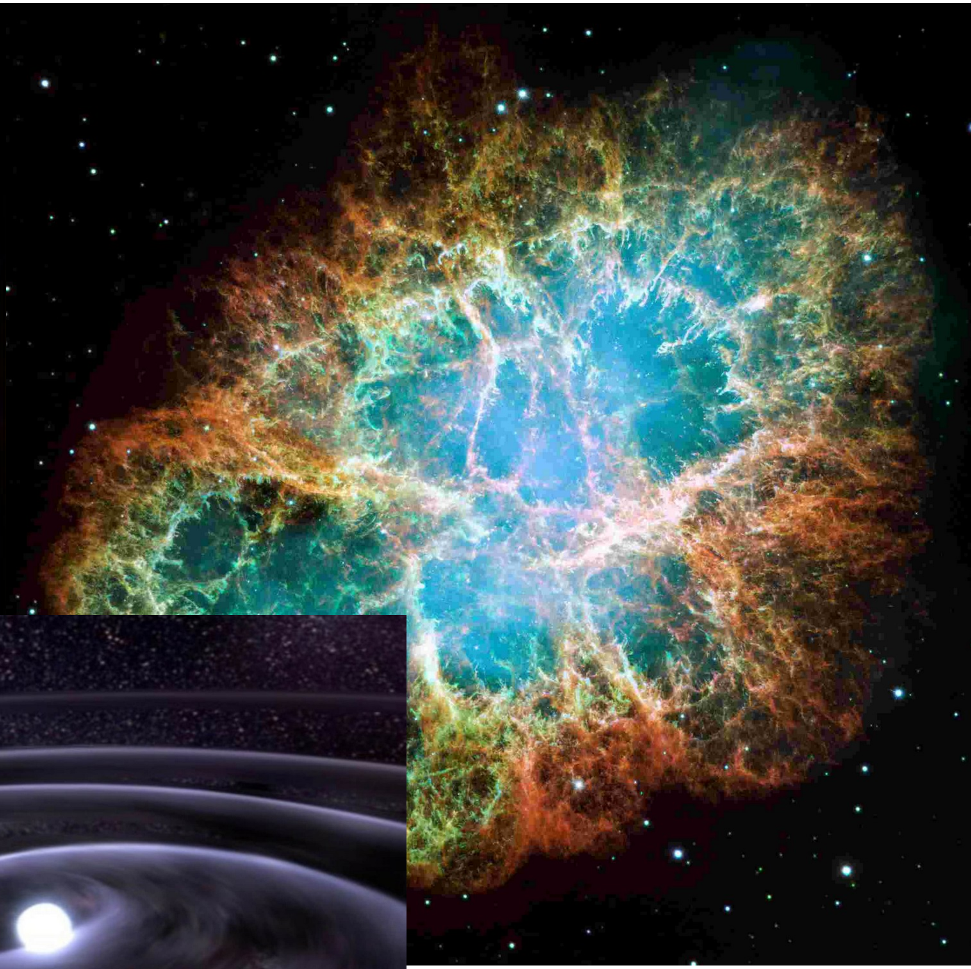
	V_{WS}	EFT
	$V_{WS}(r) = \frac{-V_0}{1+\exp\left(\frac{r-R}{d}\right)}$	
bound state	Sch. Eq. for V_0^B, SF/ANC	Feynman graphs, resum., $\mathcal{Z}$
scatt. states	Sch. Eq. for $V_0^{S,\nu}$	Feynman graphs (resum.), a, r
EM	$\mathcal{O}_{E1} = Z_C \frac{\mu}{M_C} e\, r\, Y_{1m}(\hat{r})$	QED

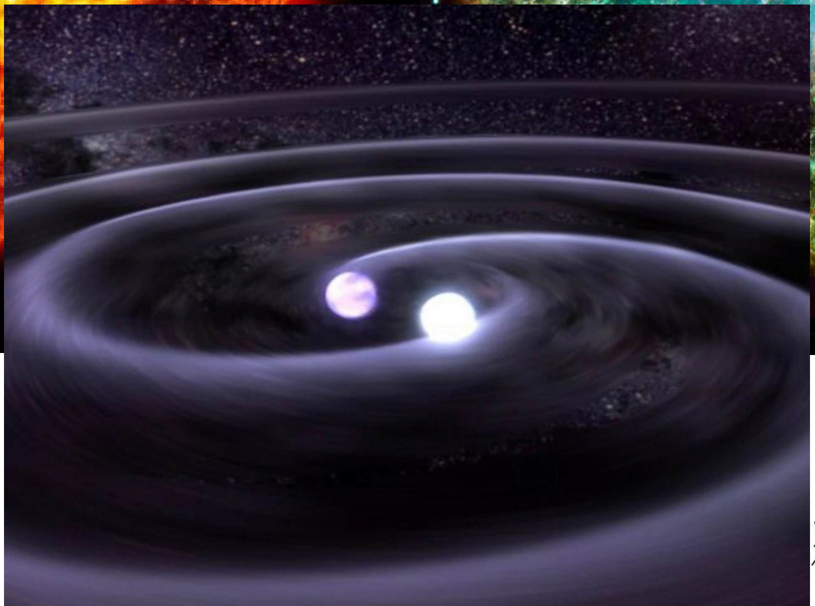
Halo/Cluster EFT:

$$k \sim 1/a_0 \sim M_{lo} \ll m_\pi, \sqrt{m_c E_c^*} \sim 1/r_0 \sim M_{hi}$$

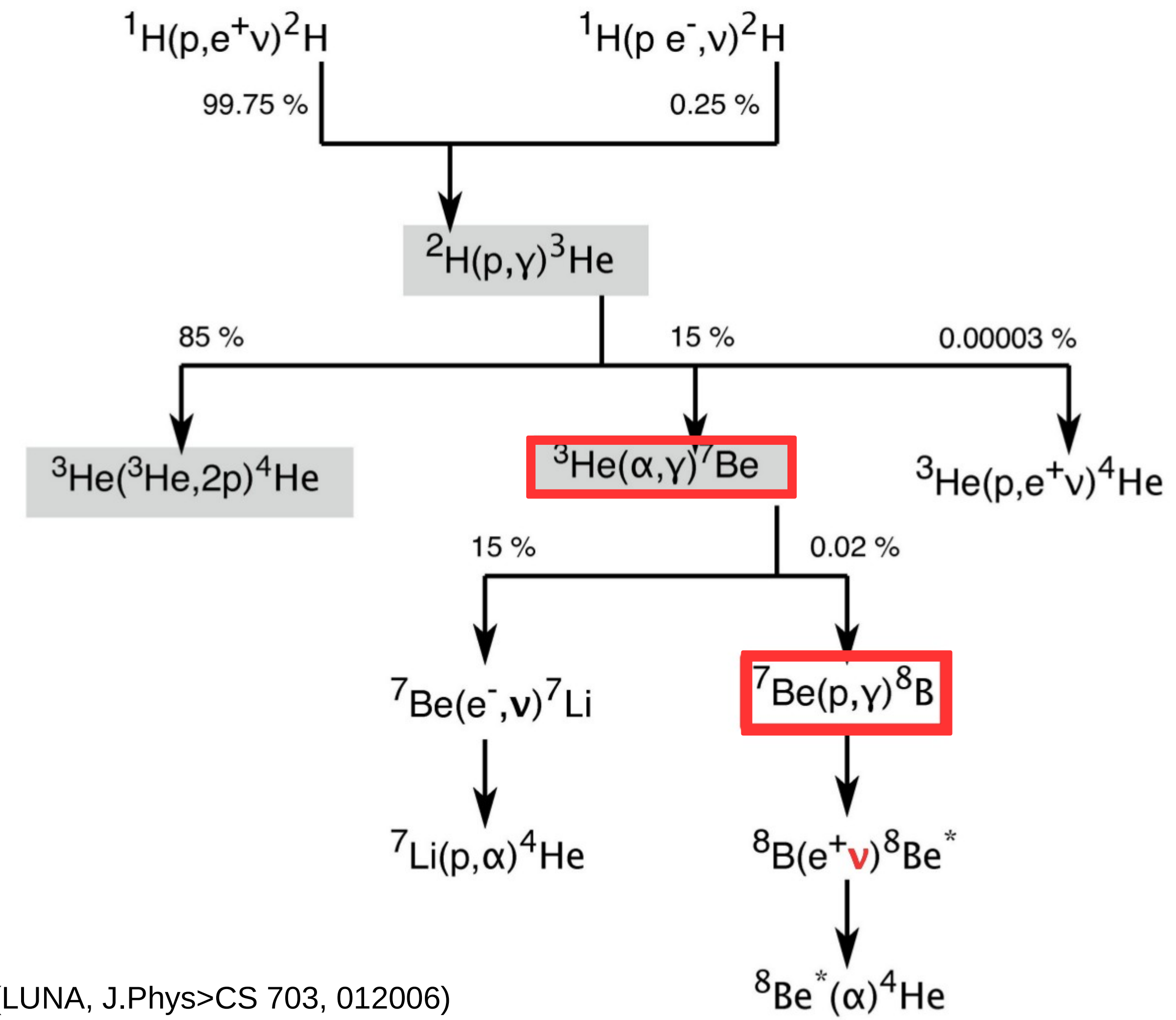
Nuclear Physics and Astrophysics







Nuclear Physics and Astrophysics

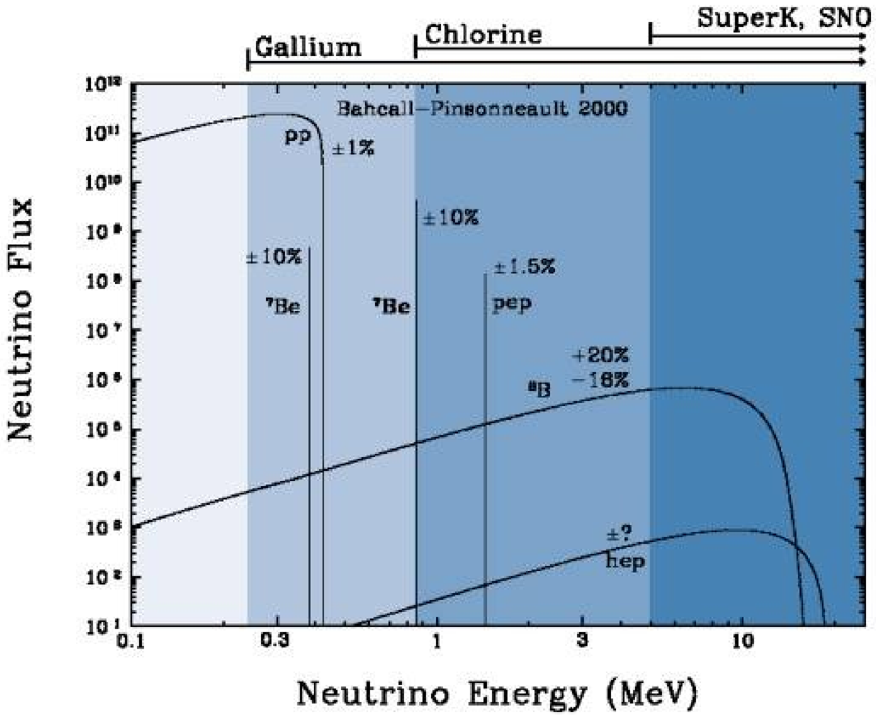


(LUNA, J.Phys>CS 703, 012006)

Radiative capture reactions with Halo/Cluster EFT

- *n*-^{7}Li:
  Rupak, RH, PRL106, 222501 (2011)
  Fernando, RH, Rupak, EPJA48, 24 (2012)
  Zhang, Nollett, Phillips, PRC89, 024613 (2014)
  RH, Premarathna, Rupak, EPJA57, 269 (2021)
- *n*-^{14}C:
  Rupak, Fernando, Vaghani, PRC86, 044608 (2012)
- *p*-^{16}O:
  Ryberg, Forssén, Hammer, Platter, PRC89, 014325 (2014)
  Ann.Phys.367, 13 (2016)
- *n*-^{14}C:
  Braun, Hammer, Platter, EPJA54, 196 (2018)
- α-^{3}He:
  RH, Rupak, Vaghani, EPJA54, 89 (2018)
  Zhang, Nollett, Phillips, JphysG47, 054002 (2020)
- *p*-^{7}Be:
  Zhang, Nollett, Phillips, PRC89, 051602 (2014), PLB751, 535 (2015), PRC98, 034616 (2018)
  Ryberg, Forssén, Hammer, Platter, EPJA50, 170 (2014)
  RH, Premarathna, Rupak, PRC106, 014601 (2022)
- α-^{12}C:
  Ando, EPJA52, 130, PRC97, 014604, PRC100, 015817, EPJA57, 17 (2021)

$^{7}Li(n,\gamma)^{8}Li$ and $^{7}Be(p,\gamma)^{8}B$ reactions



$p + {}^{7}Be \rightarrow {}^{8}B + \gamma$

${}^{8}B \rightarrow {}^{8}Be + e^{+} + \nu_e$

$\Rightarrow$ uncertainty on energetic ν_e flux

$\Rightarrow$ $S_{17}(0)$: low-energy extrapolation

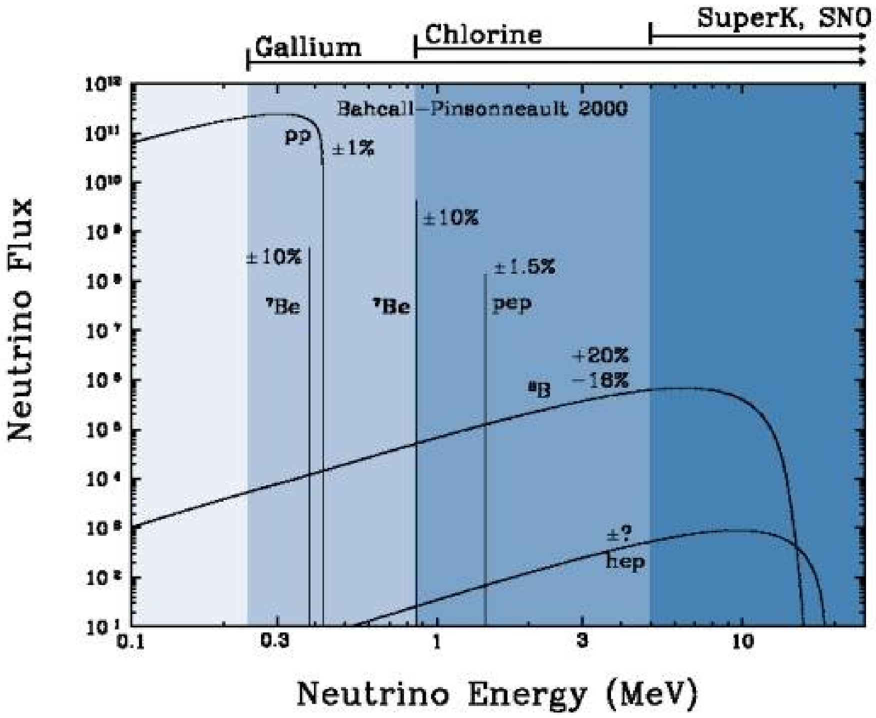
$\Rightarrow$ matter/vacuum oscillations

$\Rightarrow$ direct/inverse hierarchy

mirror symmetry: $n + {}^{7}Li \rightarrow {}^{8}Li + \gamma$

$\Rightarrow$ non-homogeneous BBN: bridge the $A = 8$ gap

$^3He(\alpha,\gamma)^7Be$ reaction



$\alpha + {}^3He \rightarrow {}^7Be + \gamma$

${}^7Be + e^- \rightarrow {}^7Li + \nu_e$

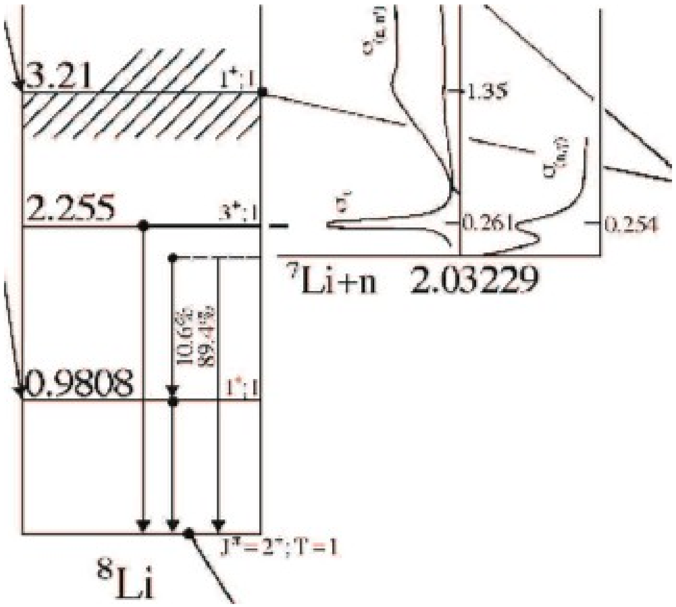
$\Rightarrow$ $S_{34}(0)$: low-energy extrapolation

$\Rightarrow$ matter/vacuum oscillations

$\Rightarrow$ direct/inverse hierarchy

Lithium abundance in the universe

The n-^{7}Li system



⇒ Bound states:

$2^+(-2.03\ \text{MeV}): ({}^5P_2 + {}^3P_2)/\sqrt{2}\ \ (p_{3/2})$

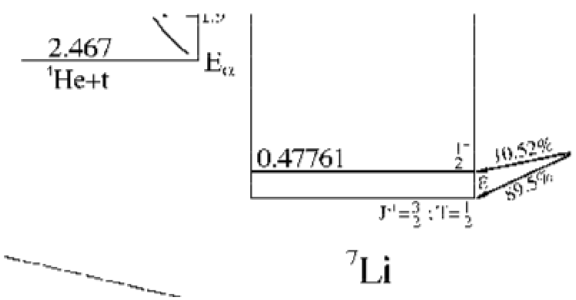
$1^+(-1.05\ \text{MeV}): (\sqrt{5}\,{}^5P_2 - {}^3P_2)/\sqrt{6}\ \ (p_{1/2})$

⇒ Scattering states:

${}^5S_2:\ a_0^{(2)} = -3.63 \pm 0.05\ \text{fm}$

${}^3S_1:\ a_0^{(1)} = 0.87 \pm 0.07\ \text{fm}$

${}^5P_3:\ E_R = 0.222\ \text{MeV},\ \Gamma_R = 0.031\ \text{MeV}$

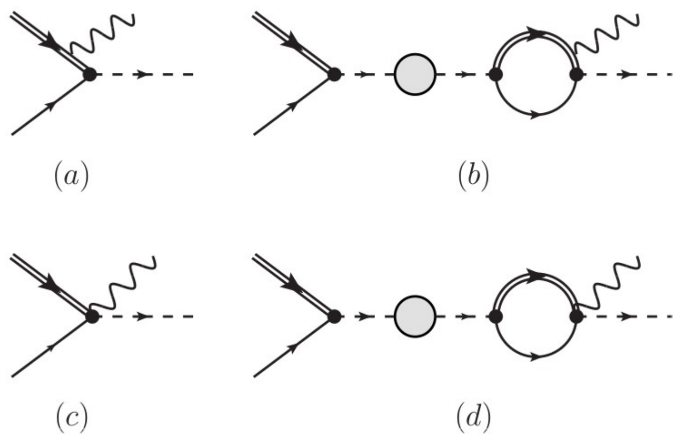


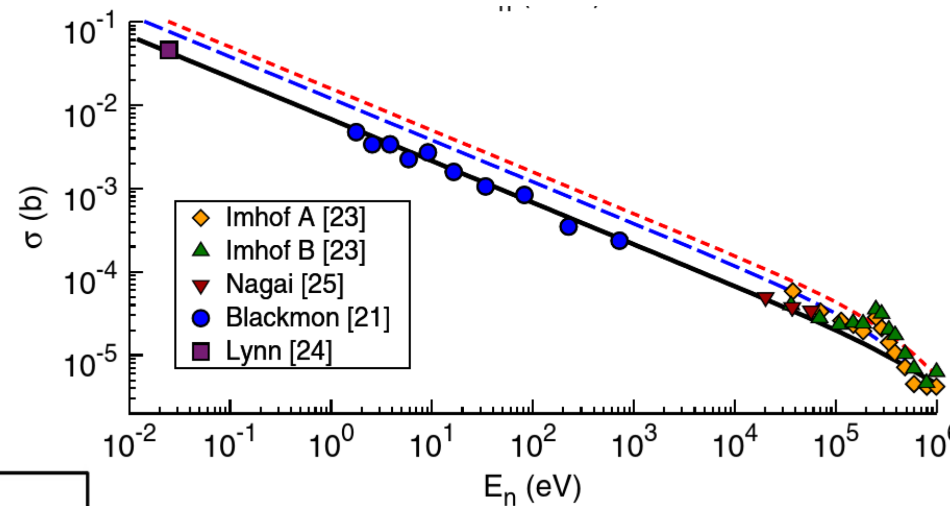
⇒ Radiative capture:

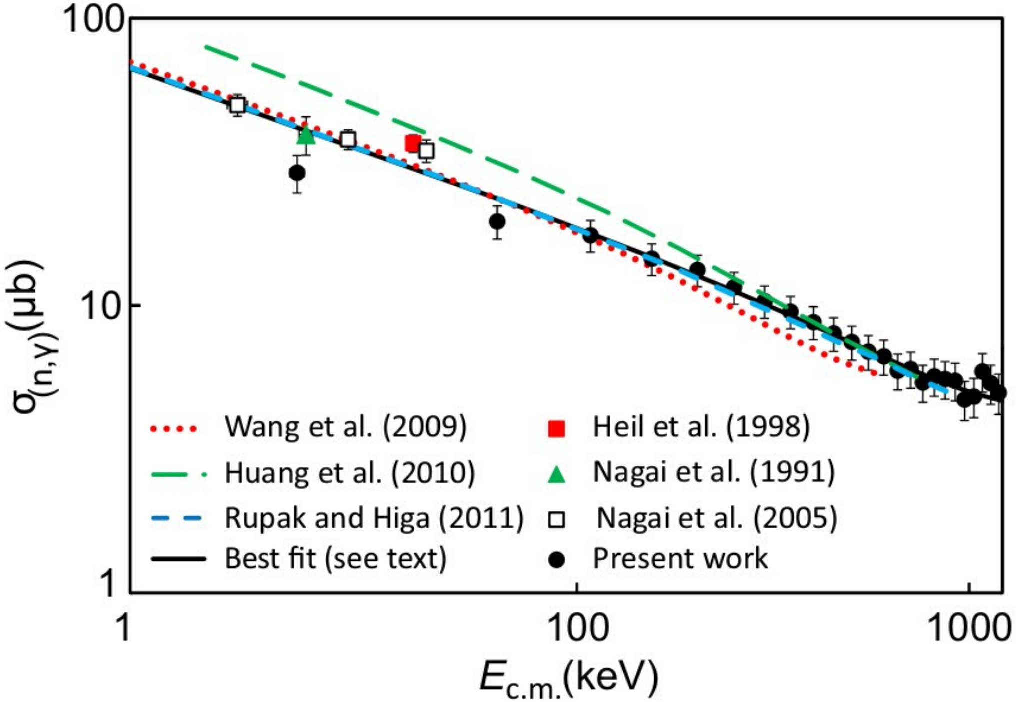
${}^5S_{2,}\,{}^3S_1 \rightarrow 2^+$ (E1, 89.4%)

${}^5S_{2,}\,{}^3S_1 \rightarrow 1^+$ (E1, 10.6%)

^{7}Li(n,γ)^{8}Li reaction

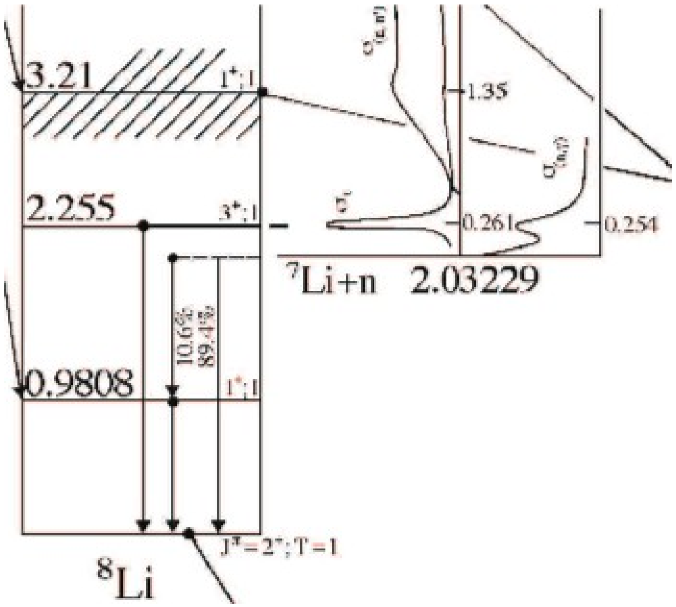






- Rupak, RH, PRL106, 222501 (2011)
- Fernando, RH, Rupak, EPJA48, 24 (2012)
- Izsák et al., PRC88, 065808 (2013)

Core excitation: a coupled-channel appr.



⇒ Bound states: $^5P_{2,}\ {}^3P_2 - {}^3P_2^*;\ {}^5P_{1,}\ {}^3P_1 - {}^3P_1^*$

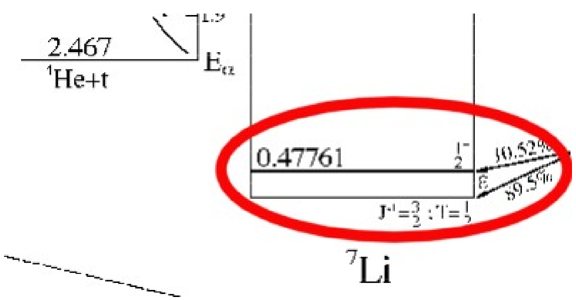
⇒ Scattering states: $^5S_{2,}\ {}^3S_1 - {}^3S_1^*$

⇒ Power counting:

LO: direct + ISI $S=2$

NLO: direct $S=1$, + $r_0^{(2)}$ corr. $S=2$

HO: ISI $S=1$, + direct d-wave, $P_0^{(2)}$ $S=2$



- RH, Premarathna, Rupak, EPJA57, 269 (2021)
- Cohen, Gelman, van Kolck, PLB588, 57 (2004)
- Lensky, Birse, EPJA47, 142 (2011)

Core excitation: a coupled-channel appr.

⇒ Bound states: ${}^5P_{2},\ {}^3P_2 - {}^3P_2^*;\ {}^5P_{1},\ {}^3P_1 - {}^3P_1^*$

⇒ Scattering states: ${}^5S_{2},\ {}^3S_1 - {}^3S_1^*$

⇒ Power counting:

LO: direct + ISI $S=2$

NLO: direct $S=1$, + $r_0^{(2)}$ corr. $S=2$

HO: ISI $S=1$, + direct d-wave, $P_0^{(2)}$ $S=2$

$$\frac{\sigma^{({}^5P_2)}}{\sigma^{({}^5P_2)}+\sigma^{({}^3P_2)}}=\frac{(3-2a_0^{(2)}\gamma_0)^2 Z^{({}^5P_2)}}{(3-2a_0^{(2)}\gamma_0)^2 Z^{({}^5P_2)}+(3-2a_0^{(1)}\gamma_0)^2 Z^{({}^3P_2)}}\approx 0.8$$

Core excitation: a coupled-channel appr.

- Comparison with Zhang, Nollett, Phillips, PRC89, 024613 (2014)

$$T_{ab} \sim (i\, g_a) \frac{1}{g_a^2 F(a,r,k)} (i\, g_a) \sim \frac{1}{F(a,r,k)} \quad \Rightarrow \quad \text{RG-indep.}$$

$$T_{aa} \sim (i\, g_b) \frac{1}{g_a^2 F(a,r,k)} (i\, ga) \sim \frac{g_b}{g_a} \quad \Rightarrow \quad \text{not RG-indep!!!}$$

${}^5P_2, {}^3P_2, {}^3P_2^*$: same dimer field

$\Rightarrow$ same effective momentum r

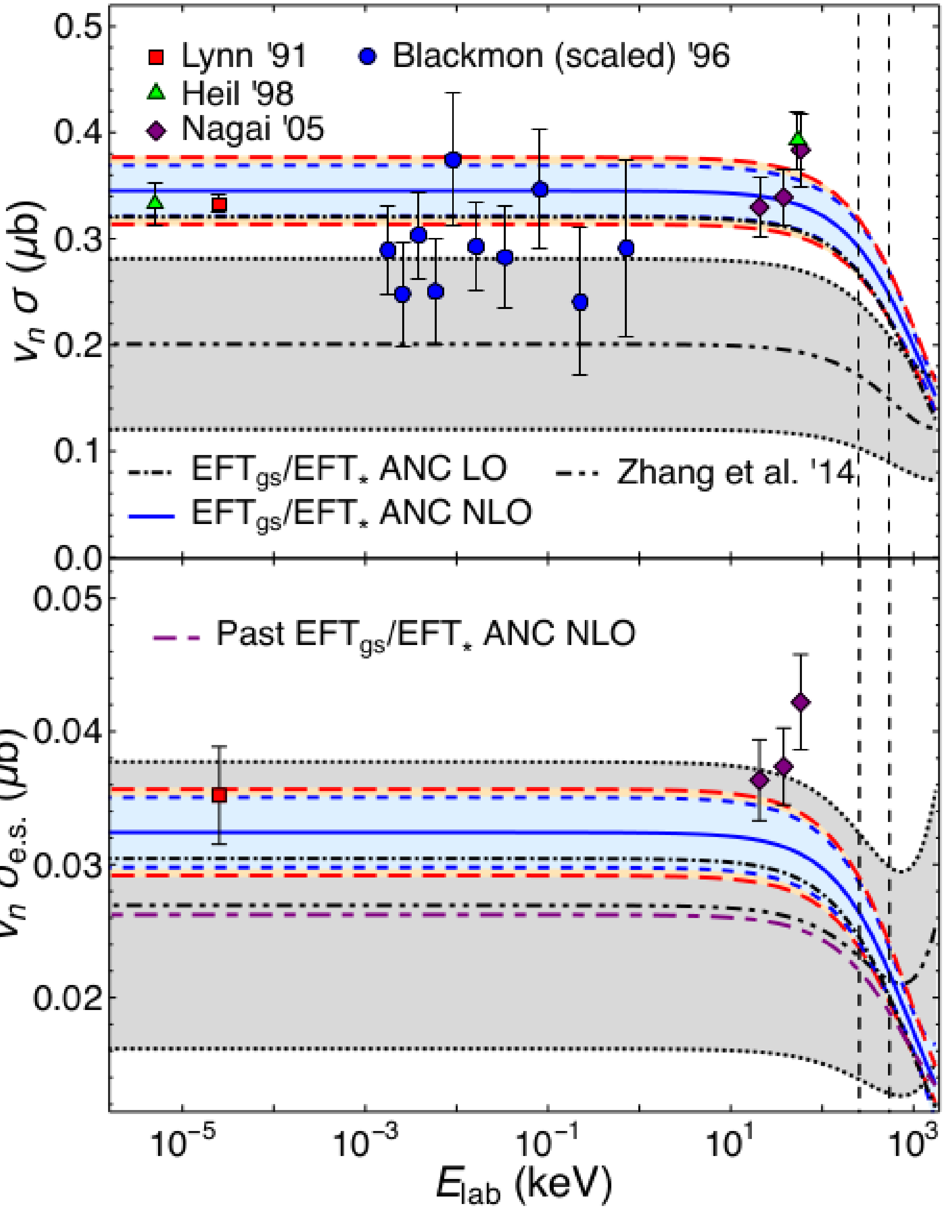
$\Rightarrow$ unknown related symmetry

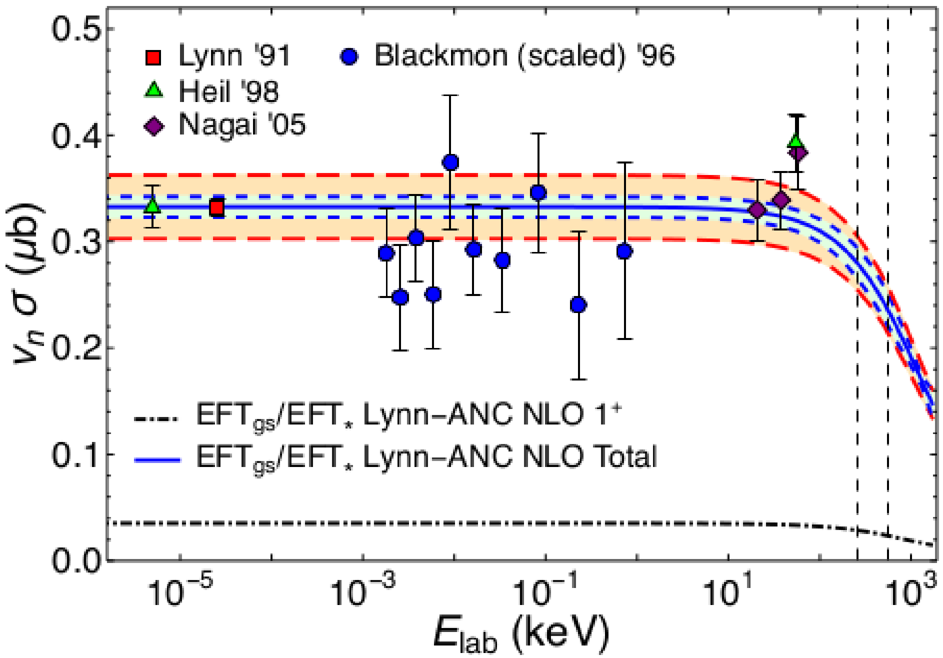
Our ${}^3S_1$-${}^3S_1^*$ coupled channels: $r_0^{(1)} \sim 1/Q$

[Cohen, Gelman, van Kolck, PLB 588, 57 (2004)]

Zhang, Nollet, Phillips: unknown [probably $r_0^{(1)} \sim 1/\Lambda$]

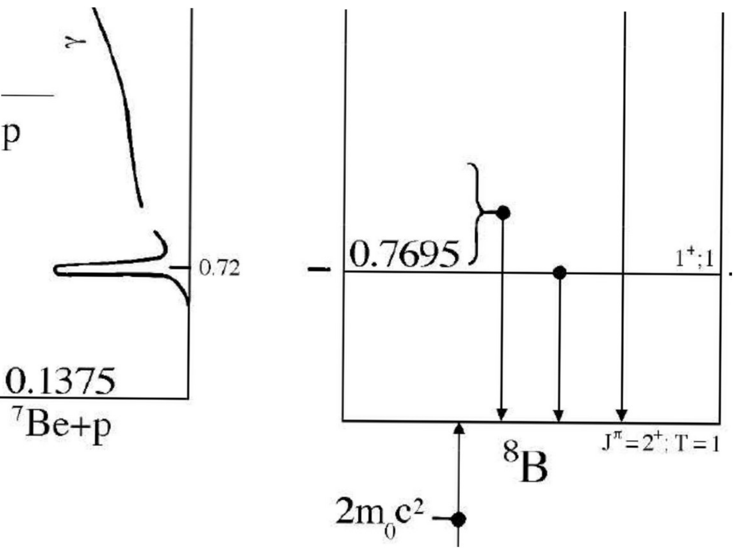
Core excitation: a coupled-channel appr.

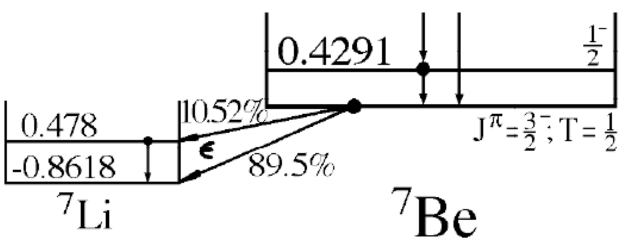




$p_R = 19.1$ MeV, $\gamma_\Delta = 28.0$ MeV

The p-^{7}Be system





⇒ Bound state:

$2^+(-0.1364\ \text{MeV})$: $^5P_2, {}^3P_2\text{-}{}^3P_2^*$

⇒ Scattering states:

$^5S_2$: $a_0^{(2)} = -3.18 \pm 0.05\ \text{fm}$

$^3S_1\text{-}{}^3S_1^*$: $a_0^{(1)} = 17.34 \pm 0.07\ \text{fm}$

$^5P_1$: $E_R = 0.634\ \text{MeV},\ \Gamma_R = 0.031\ \text{MeV}$

⇒ Power counting:

LO: direct $S=2$ s-wave

NLO: direct $S=1$ s-wave + $S=2$ d-wave

NNLO: ISI $S=1,2$; direct $S=1$ d-wave

- RH, Premarathna, Rupak, PRC106, 014601 (2022)

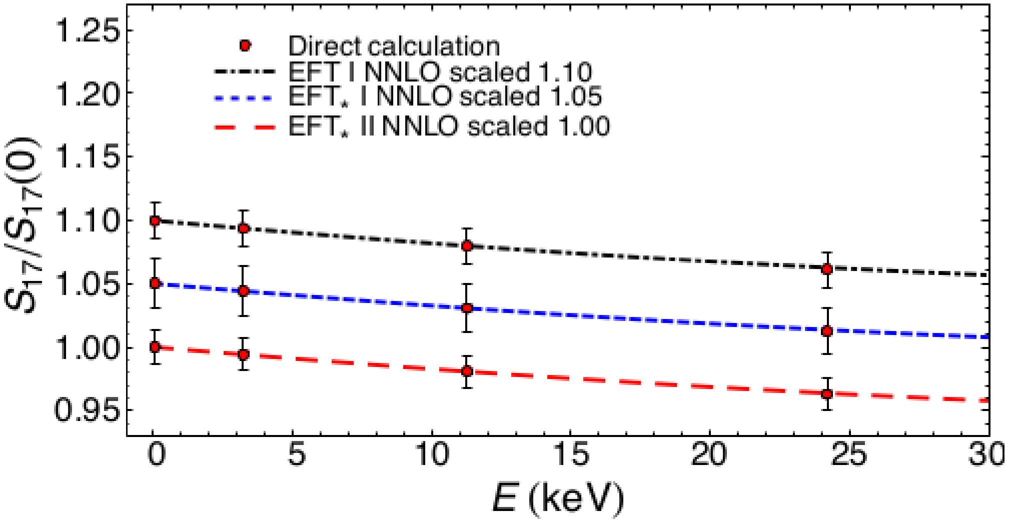
Halo EFT for ^{7}Be$(p,\gamma)^8$B

⇒ Power counting:

LO: direct $S=2$ s-wave

NLO: direct $S=1$ s-wave
+ $S=2$ d-wave

NNLO: ISI $S=1,2$ s-wave
+ direct $S=1$ d-wave

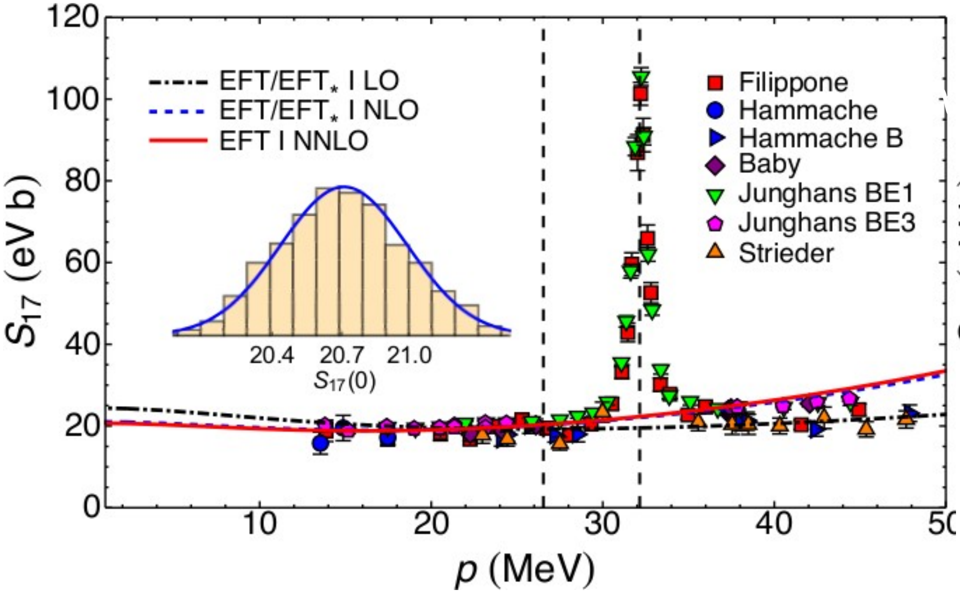


- Mostly peripheral, inspired by Daniel Baye:

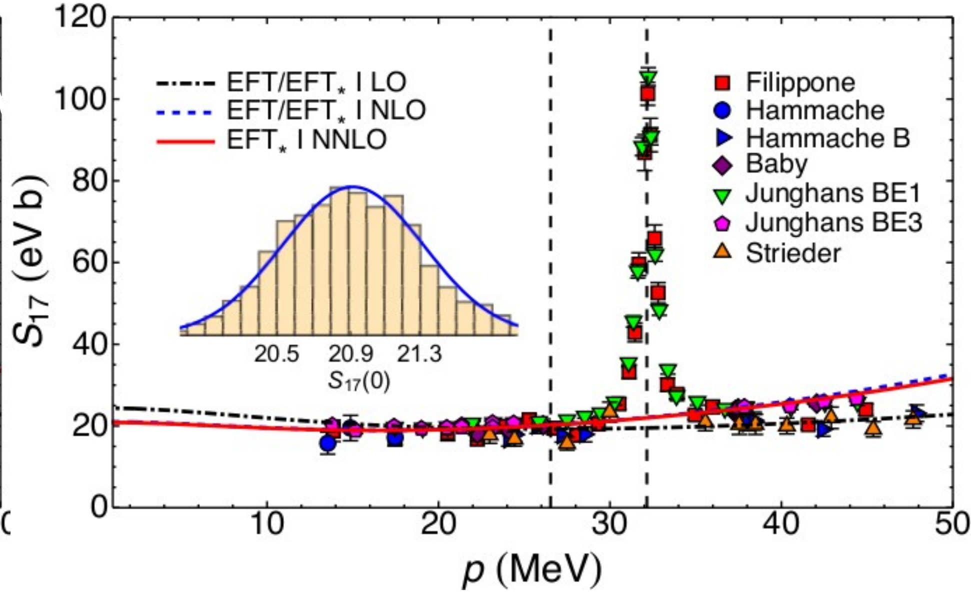
$$S_{17}/C^2_{1,\zeta} \approx 35.6\left(1 - a_0\, 0.00266\ \text{fm}^{-1} + 0.0657 + \ldots\right) \text{eV b fm}$$

- RH, Premarathna, Rupak, PRC106, 014601 (2022)
- Baye, PRC62, 065803 (2000)
- Zhang, Nollett, Phillips, PRC98, 034616 (2018)

Bayesian fits below 500 keV
($p \leq 29$ MeV, region I)



EFT: without 7Be* core
1+1+0 fit params.

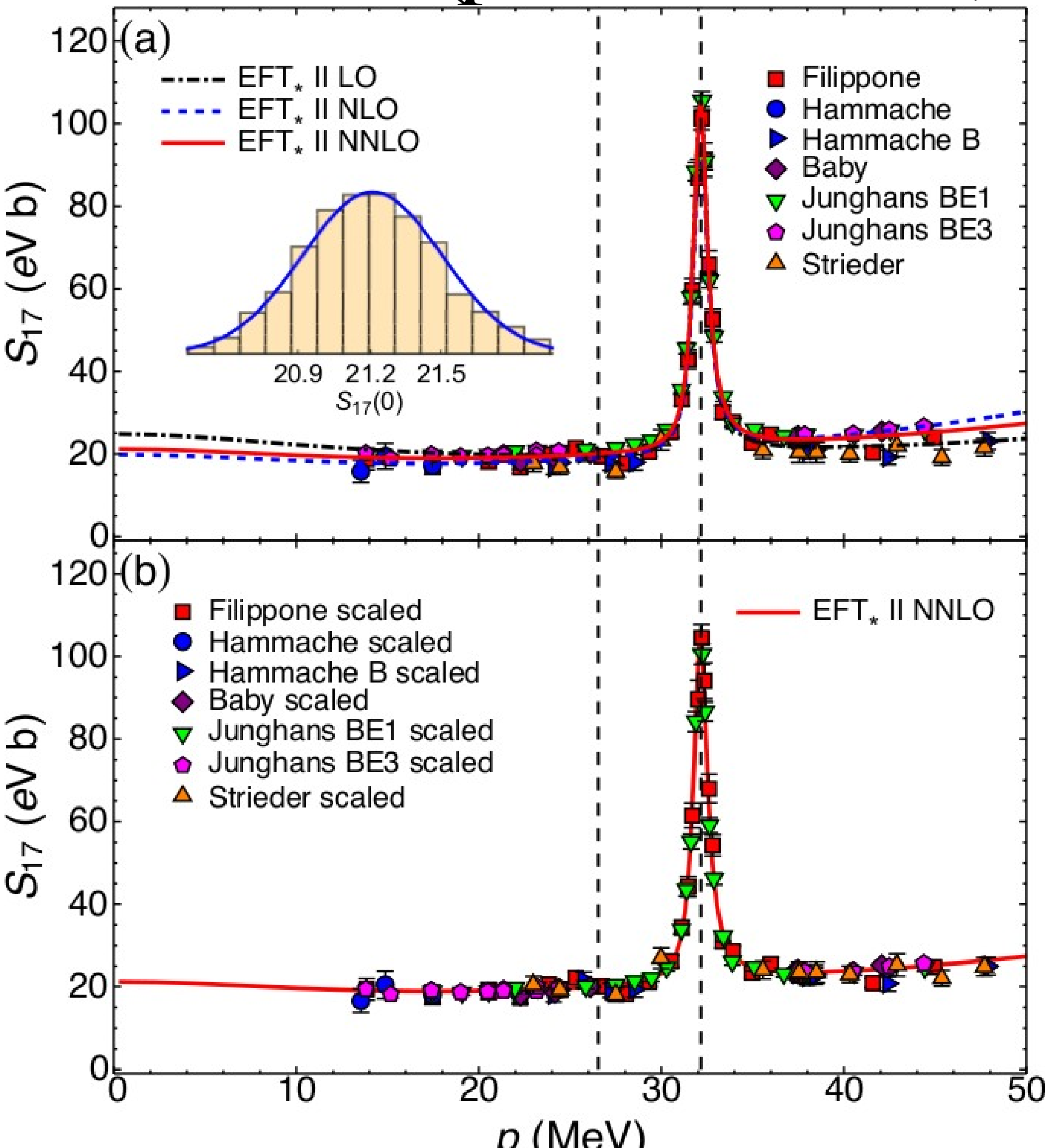


EFT: with 7Be* core
1+1+3 fit params.

$$\gamma_{\Delta}=26.5\ \text{MeV},\ p_R=32.2\ \text{MeV}$$

- previous Halo EFT studies:
- Zhang, Nollett, Phillips, PRC89, 051602 (2014), PLB751, 535 (2015), PRC98, 034616 (2018) – 7 fit params. @NLO
- Ryberg et al., EPJA50, 170 (2014)

Bayesian fits below 1000 keV ($p \leq 40$ MeV, region II)

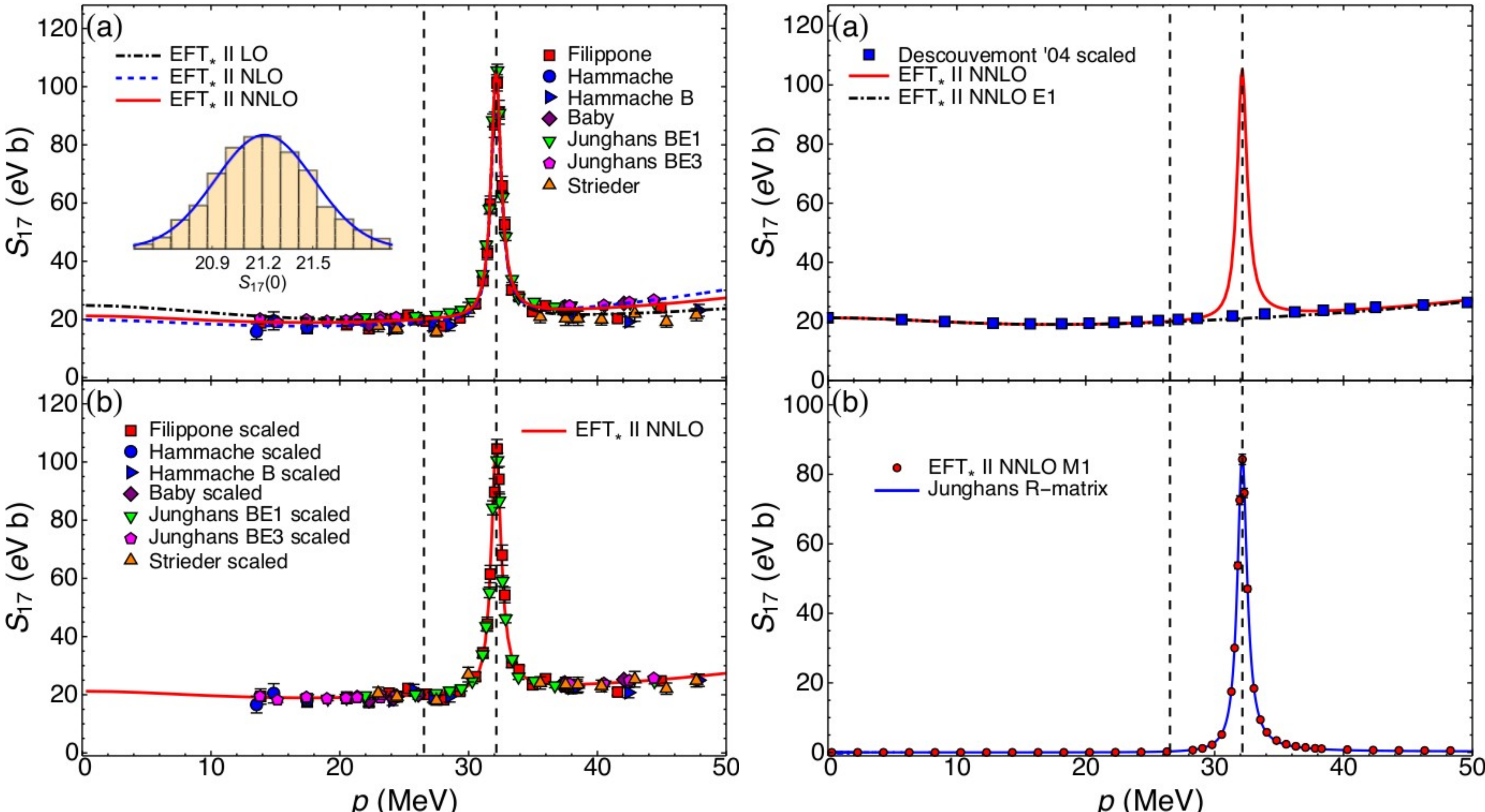


EFT$_*$ with 1(+1)+1+3 fit params.

→ "Artist's" illustration

$\gamma_\Delta = 26.5$ MeV, $p_R = 32.2$ MeV

Bayesian fits below 1000 keV ($p \leq 40$ MeV, region II)



Bayesian fits below 1000 keV ($p \leq 40$ MeV, region II)

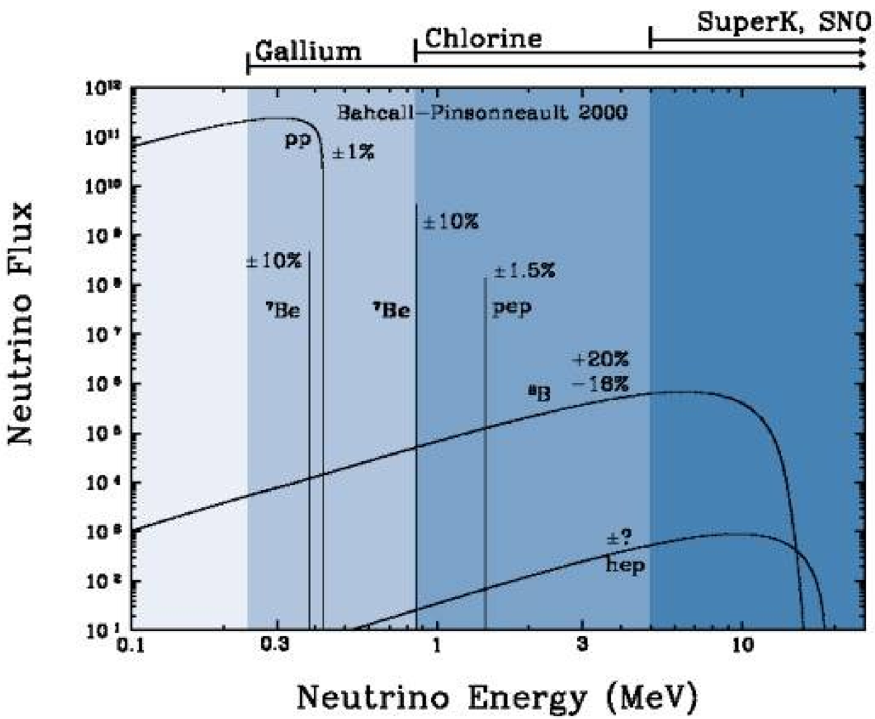
TABLE III. S_{17} and its first two energy derivatives at $E_0 = 50 \times 10^{-3}$ keV. The first set of errors are from the fits. The second set is the estimated LO 30%, NLO 10%, and NNLO 3% EFT errors, respectively, from higher-order corrections.

Theory	S_{17} (eV b)	S'_{17}/S_{17} (MeV^{-1})	S''_{17}/S_{17} (MeV^{-2})
EFT$_{gs}$/EFT$_\star$ I LO	24.4(3)(73)	−2.44(5)(73)	35.8(7)(108)
EFT$_{gs}$/EFT$_\star$ I NLO	21.1(3)(21)	−1.87(4)(19)	32.4(6)(32)
EFT$_{gs}$ I NNLO	20.7(3)(6)	−1.79(4)(5)	31.9(6)(10)
EFT$_\star$ I NNLO	20.9(4)(6)	−1.82(8)(5)	31.9(8)(10)
EFT$_\star$ II LO	24.8(3)(74)	−2.44(4)(73)	35.8(6)(108)
EFT$_\star$ II NLO	19.8(2)(20)	−1.91(3)(19)	32.7(5)(33)
EFT$_\star$ II NNLO	21.2(3)(6)	−1.89(4)(6)	31.9(6)(10)

$$S_{17}(0) = 20.5 \pm 0.7 \text{ eV b}$$

- RH, Premarathna, Rupak, PRC106, 014601 (2022)
- **Solar Fusion III recommended value**

$^{3}He(\alpha,\gamma)^{7}Be$ reaction



$\alpha + {}^{3}He \rightarrow {}^{7}Be + \gamma$

${}^{7}Be \rightarrow {}^{7}Li + e^{+} + \nu_e$

⇒ $S_{34}(0)$: low-energy extrapolation

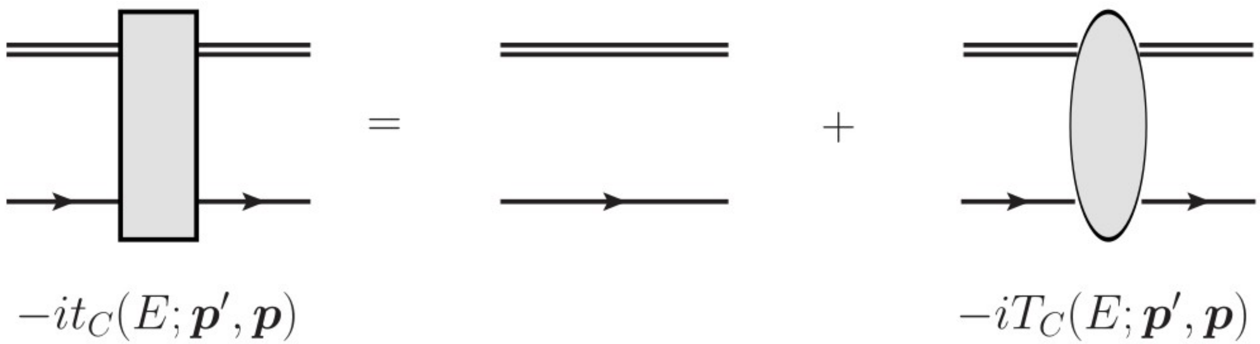
⇒ matter/vacuum oscillations

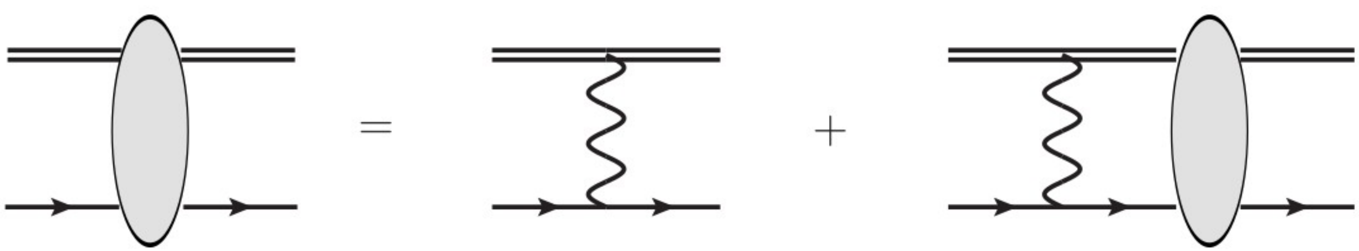
⇒ direct/inverse hierarchy

Lithium abundance in the universe

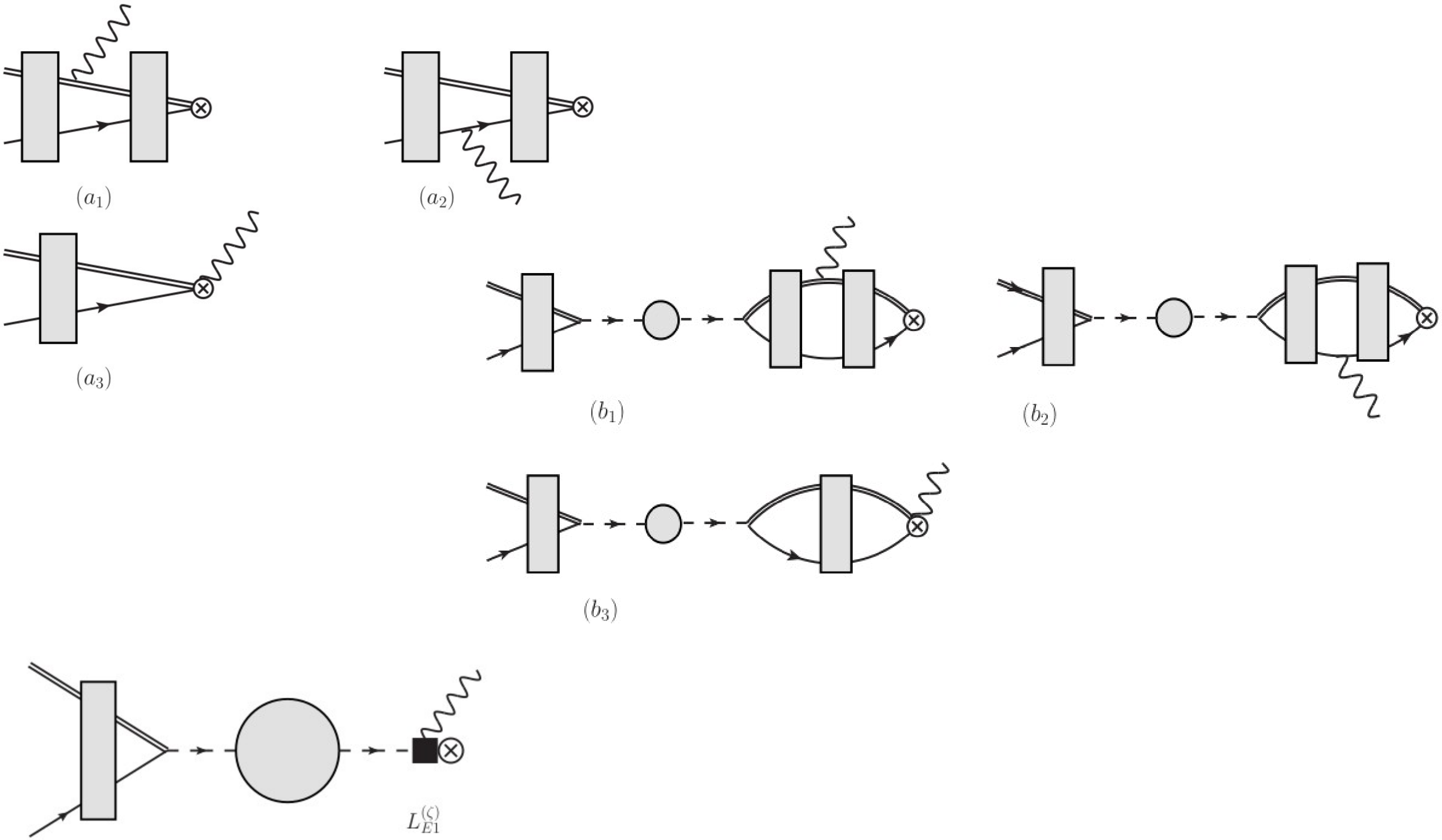
$^3He(\alpha,\gamma)^7Be$ radiative capture

- ^7Be: predominant ^3He-α cluster
- $B_0(^2P_{3/2}) \sim 1.6$ MeV, $B_1(^2P_{1/2}) \sim 1.2$ MeV $\ll S_N$ (~ 5.5 MeV), E^*_α (~ 20 MeV)

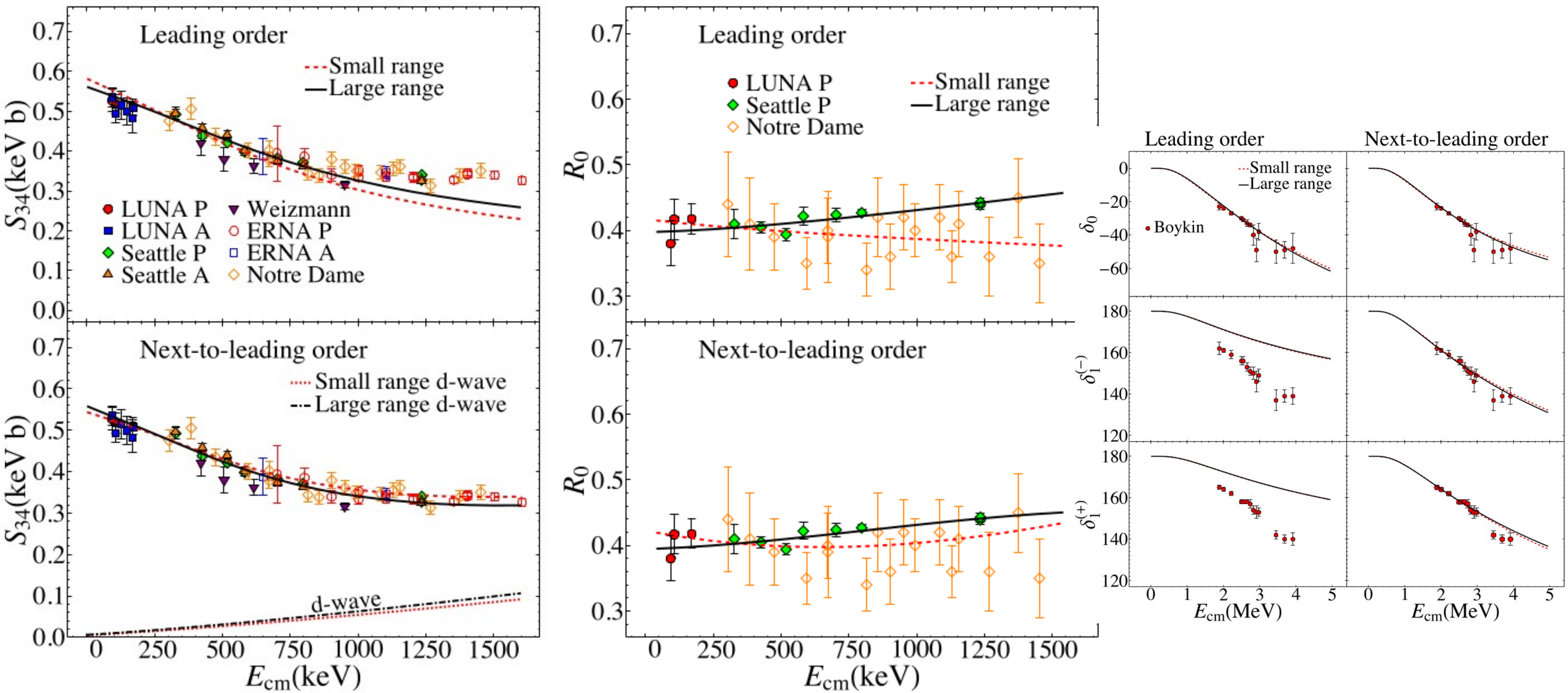




^{3}He$(\alpha,\gamma)^7$Be radiative capture



$^3He(\alpha,\gamma)^7Be$ radiative capture



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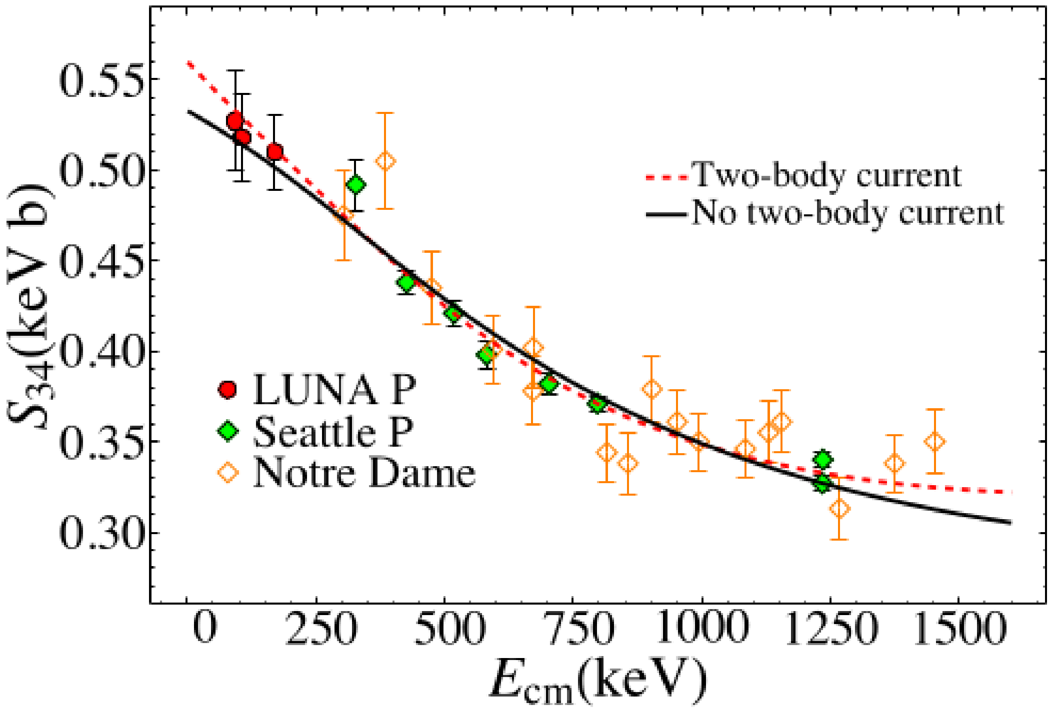
“Small range”: capture to 500 keV, S-wave scatt to 2.5 MeV
“Large range”: capture to 1000 keV, S-wave scatt to 3.0 MeV

$^{3}He(\alpha,\gamma)^{7}Be$ radiative capture



- RH, G. Rupak, A. Vaghani, EPJA 54, 89

Strong correlation between two-body currents and renormalization constants (ANCs)

$^3He(\alpha,\gamma)^7Be$ radiative capture

Fit	$S_{34}(0)$ (keV b)
Small range LO	0.582 ± 0.011 (fit) ± 0.194 (EFT)
Large range LO	0.561 ± 0.007 (fit) ± 0.187 (EFT)
Small range NLO	0.544 ± 0.012 (fit) ± 0.054 (EFT)
Large range NLO	0.558 ± 0.008 (fit) ± 0.056 (EFT)

$$S_{34}(0)=0.561\pm0.018_{\mathrm{exp}}\pm0.022_{\mathrm{theor}}\ \mathrm{keV\ b}$$

$$\frac{S'_{34}(0)}{S_{34}(0)}=-0.54\pm0.07\ \mathrm{MeV}^{-1}\qquad (E=0\text{-}500\ \mathrm{keV})$$

- RH, G. Rupak, A. Vaghani, EPJA 54, 89
- **Solar Fusion III recommended value**

Summary

- Halo/Cluster EFT: systematic framework, few-body correlations, EM/W currents

- ^{7}Li(n,γ)^{8}Li and ^{7}Be(p,γ)^{8}B:
  - coupled-channel approach → more convergent power-counting
  - S=2 channel dominance due to a, r (^{8}Li) / ANC (^{8}B) values
  - previous works: problems with RG-invariance, microscopic interpretation of parameters, low-energy correlations
  - ^{7}Li excited core contrib noticeable only at NNLO but required for $p \sim p_R$ ($\gamma_\Delta < p_R$)

- ^{3}He(α,γ)^{7}Be:
  - LO: scatt a, r S- and P-waves, 2-body currents L_{E1}
  - NLO: scatt S- and P-wave shape parameters
  - Large correlation between ANCs and 2-body currents

Solar Fusion III: $S_{34}(0)$ and $S_{17}(0)$ recommended values