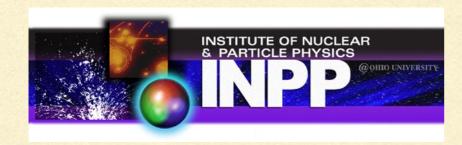
Testing universality in breakup reactions on halo nuclei

Daniel Phillips with Matthias Göbel, Hans-Werner Hammer, Bijaya Acharya, Tom Aumann, Carlos Bertulani, Tobias Frederico, Raúl Briceño, Caroline Costa

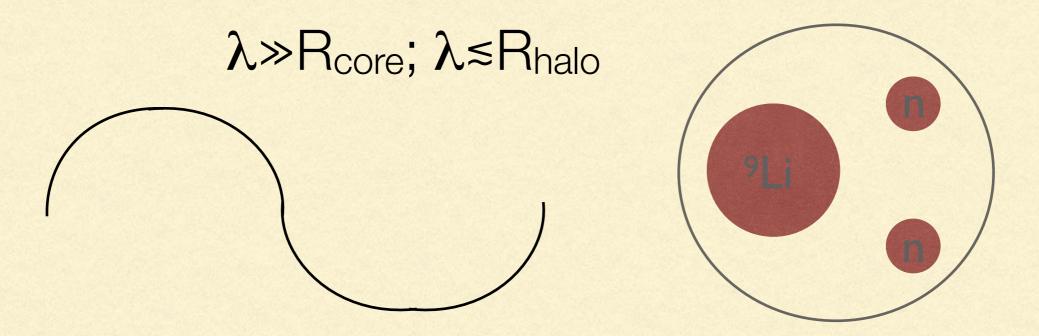




SUPPORTED BY THE US DEPARTMENT OF ENERGY

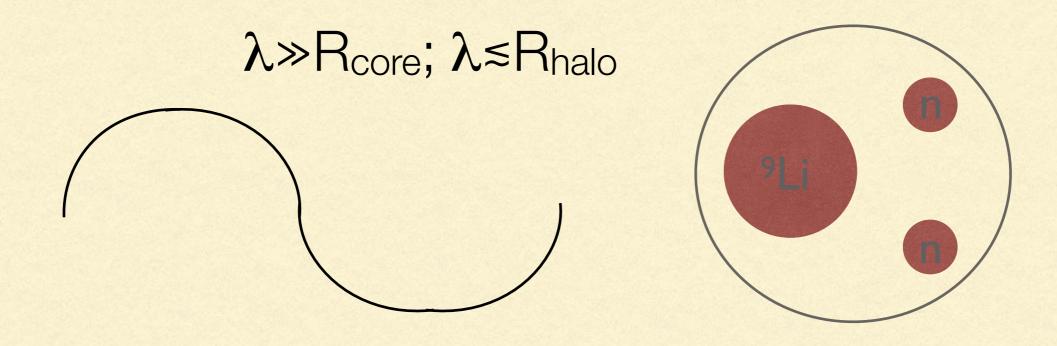
Halo EFT

Bertulani, Hammer, van Kolck, NPA (2003); Bedaque, Hammer, van Kolck, PLB (2003); Review: Hammer, Ji, DP, J. Phys. G 44, 103002 (2017)



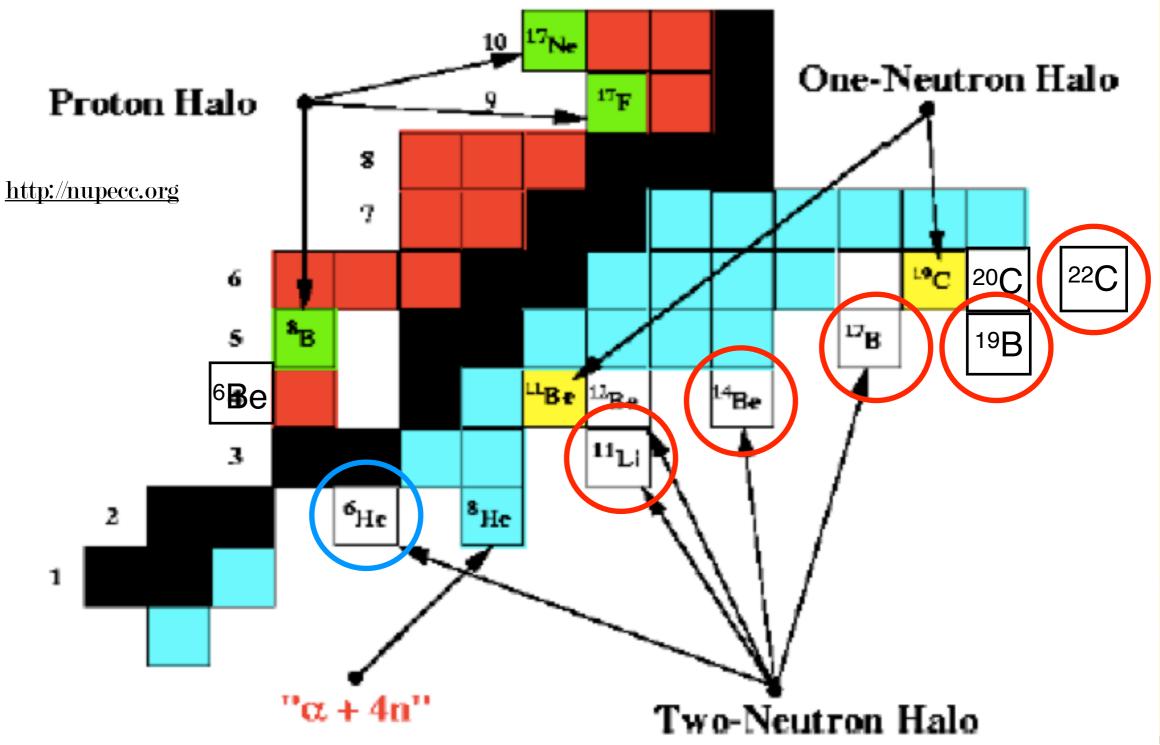
Halo EFT

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- Define $R_{halo} = \langle r^2 \rangle^{1/2}$. Seek EFT expansion in R_{core}/R_{halo} . Valid for $\lambda \leq R_{halo}$
- Typically R=R_{core}~2 fm. Since <r²> is related to the neutron separation energy we seek systems with neutron separation energies of order I MeV
- ²²C, ¹¹Li, ¹²Be, ¹⁹B, ⁶²Ca (hypothesized), and ³H: all s-wave 2n halos

Halo nuclei: examples



- What is Halo EFT and what does it do for us?
- Halo EFT for Borromean s-wave 2n halos
- Coulomb-induced breakup of ¹¹Li
- Measuring the nn relative-momentum distribution in ⁶He using fast breakup
- The unitarity limit in momentum distributions of 2n halos

An EFT approach to computing quasi-free core knockout

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- The unitarity limit in momentum distributions of 2n halos We can measure the bound-state nn momentum distribution in the halo!
- An EFT approach to computing quasi-free core knockout All other reaction mechanisms can be filtered out!

Two-body scattering amplitude in Halo EFT

$$t_0^{2B}(E) = -\frac{2\pi}{m_R} \frac{1}{k \cot \delta(E) - ik}; \quad k = \sqrt{2m_R E}$$

$$k \cot \delta(E) = -\frac{1}{a} + \frac{1}{2}rk^2 + O(k^4 R^3)$$

- Effective-range expansion, valid for kR < I
- Typical situation $|r| \sim R$. Here we assume $|r| \ll |a|$
- LO in an expansion in powers of r/a: reproduce a, or equivalently SIn
- NLO in the expansion: reproduce r and a, or equivalently S_{In} and ANC
- Errors for scattering are then $O(r^3/a^3)$ and $O(k^3r^3)$

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Elastic scattering: this is effective-range theory with built-in UQ

But it's more than just s-wave nn & nc scattering

So not just two-body scattering: also EM processes

Chen, Rupak, Savage (1999); Hammer, DP (2011)

- And other partial waves Bertulani, Hammer, van Kolck (2003); Bedaque, Hammer, van Kolck (2003); Brown & Hale (2005); Braun et al. (2018); Ando (2016-present)
- Extension to pp, p-core, and cluster-cluster scattering

Kong & Ravndal (1999); Higa, Hammer, van Kolck (2008); Ryberg, Forssén, Hammer, Platter (2014, 2016)

Expansion around limit of a bound or unbound state near threshold.
 Include higher-order effects in ERE in proportion to their importance.
 Expansion in kR_{core} , where R_{core} is scale of unresolved core physics

Extends to three-body states at cost of an additional parameter (S_{2n})

Bedaque, Hammer, van Kolck (1999); Hammer & Mehen (2001); Bedaque et al. (2002); Ji, Platter, DP (2009)

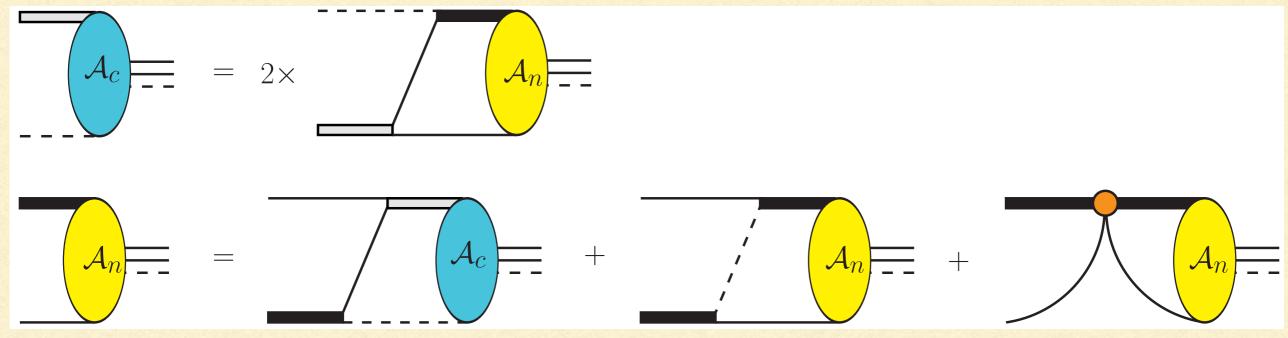
 Then predictive for four-body states (bosons or distinguishable particles) at LO accuracy

Platter, Hammer, Meißner (2005); Bazak, Kirscher, König, Pavon Valderrama, Barnea, van Kolck (2018)

Canham, Hammer (2008)

Canham, Hammer (2008)

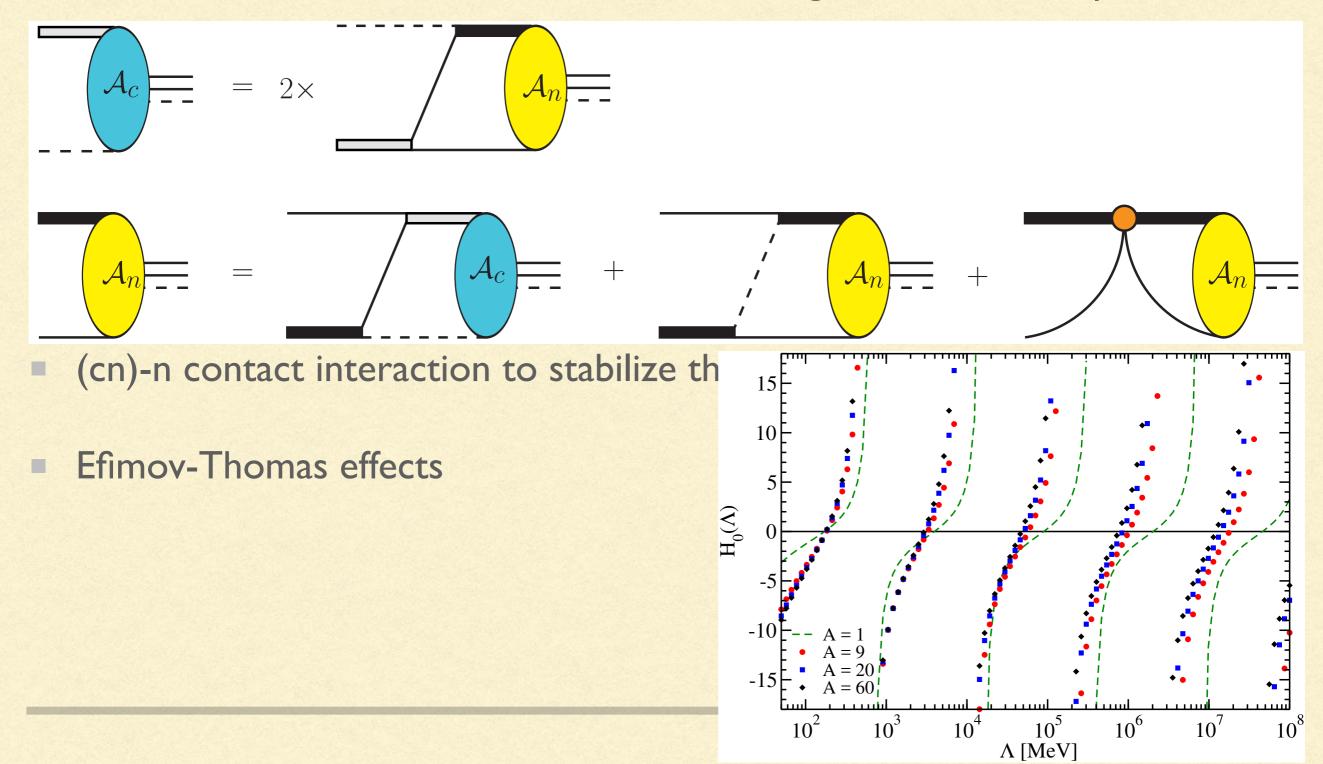
Core-n and n-n contact interactions at leading order: solve 3B problem



(cn)-n contact interaction to stabilize three-body system

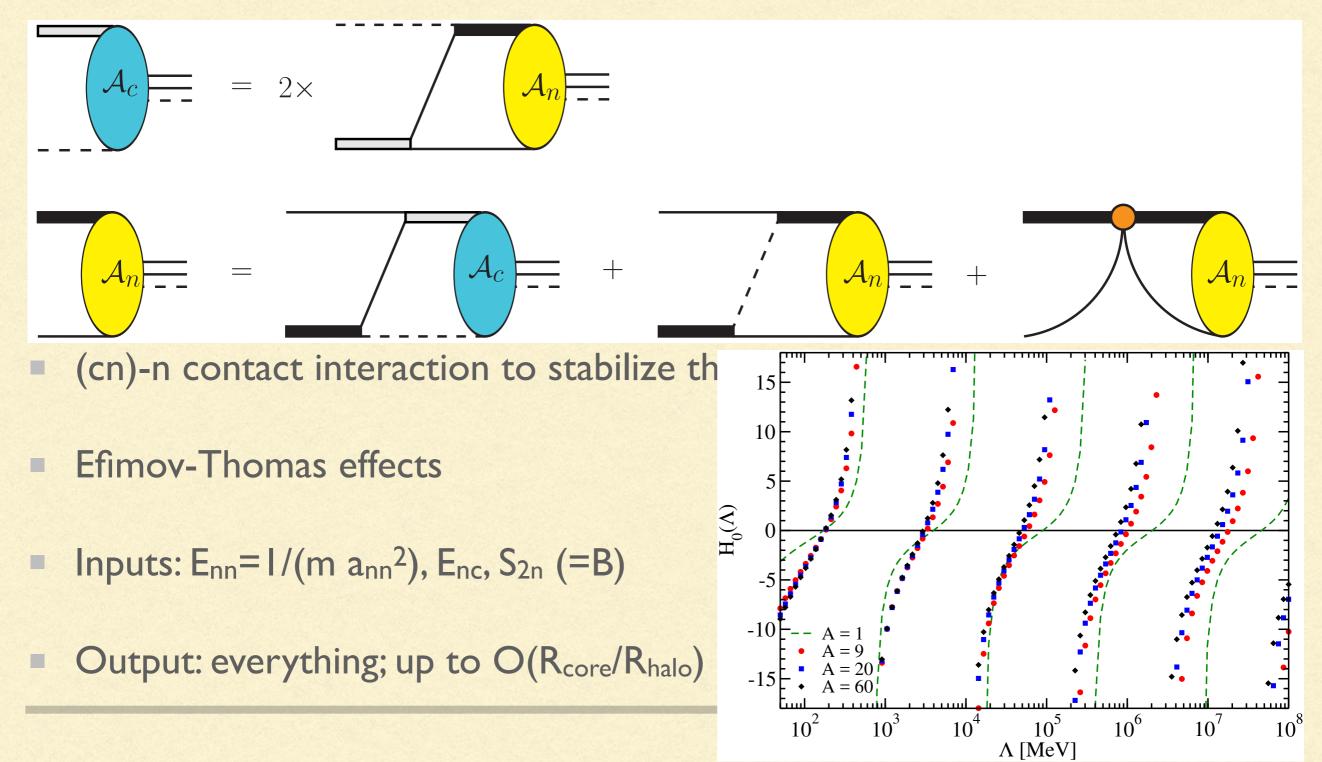
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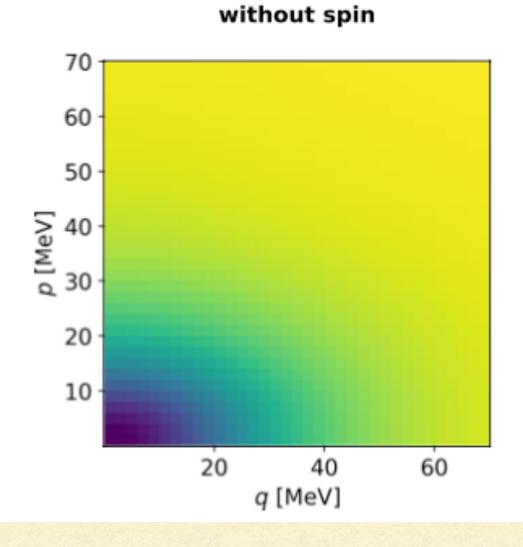


¹¹Li as a 2n halo

- ann=-18.7 fm, Enc=0.026 MeV
- S_{2n}=369 keV
- Calculations done with a cutoff of 470 MeV, but results checked for a cutoff of 700 MeV
- Here results with a spin-0 core, but we also examined case of spin-3/2 core
- Results identical if spin-1 and spin-2 nc interactions have equal strength

Göbel, Acharya, Hammer, DP, PRC (2023)

¹¹Li wave function



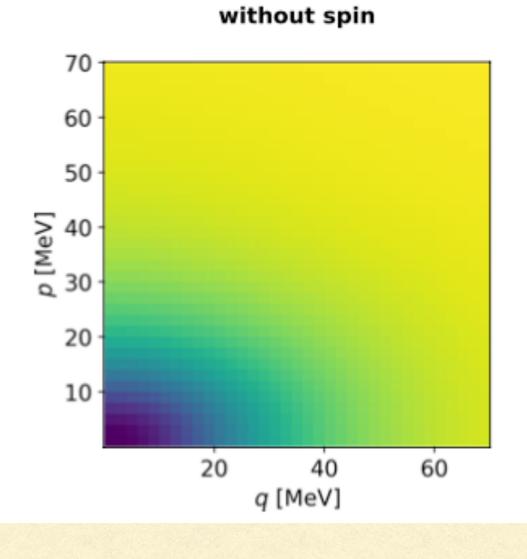
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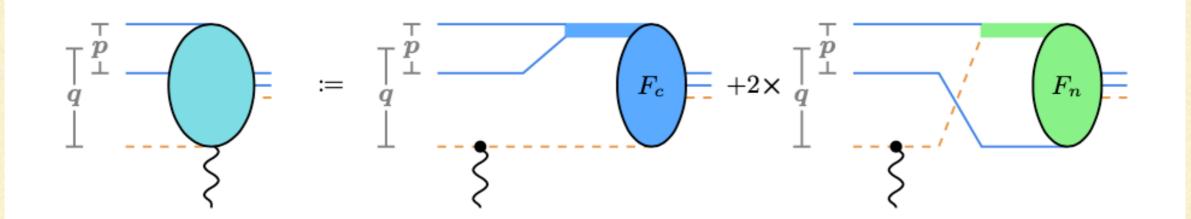
Can we test this prediction for ground-state momentum distribution?

Göbel, Acharya, Hammer, DP, PRC (2023)

¹¹Li wave function



PWIA



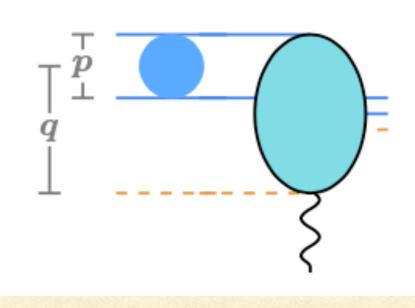
PWIA

$\frac{dB(E1)}{dE} = \sum_{\mu} \int dp \, p^2 \int dq \, q^2 |_c \langle p, q, \Omega_c^{(1,\mu)} | \mathcal{M}(E1,\mu) | \Psi \rangle |^2 \delta(E_f - E)$

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tnn FSI



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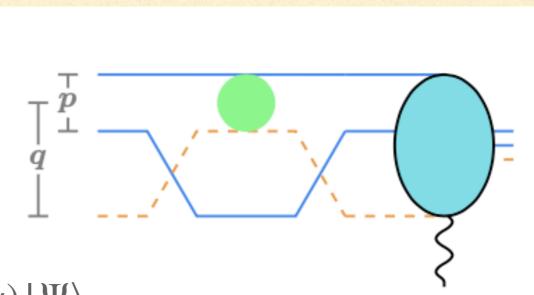
 ${}_{c}\langle p,q,\Omega_{c}^{(1,\mu)}|(1+t_{nn}(E_{p})G_{0}^{(nn)}(E_{p}))\mathcal{M}(E1,\mu)|\Psi\rangle$

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tnn FSI

tnc FSI

 ${}_{n}\langle p,q,\Omega_{n}^{(0,\xi)}|(1+t_{nc}(E_{p})G_{0}^{(nc)}(E_{p}))\mathcal{M}(E1,\mu)|\Psi\rangle$

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tnn FSI

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Modifications to matrix element due to one final-state interaction/ wave-function distortion encoded in Møller operators

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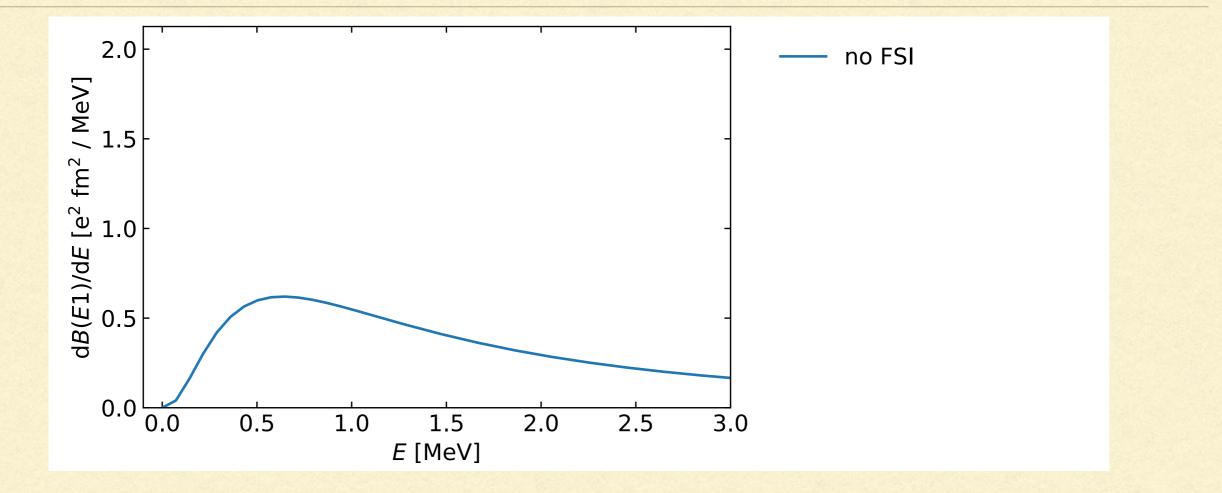
tnc FSI

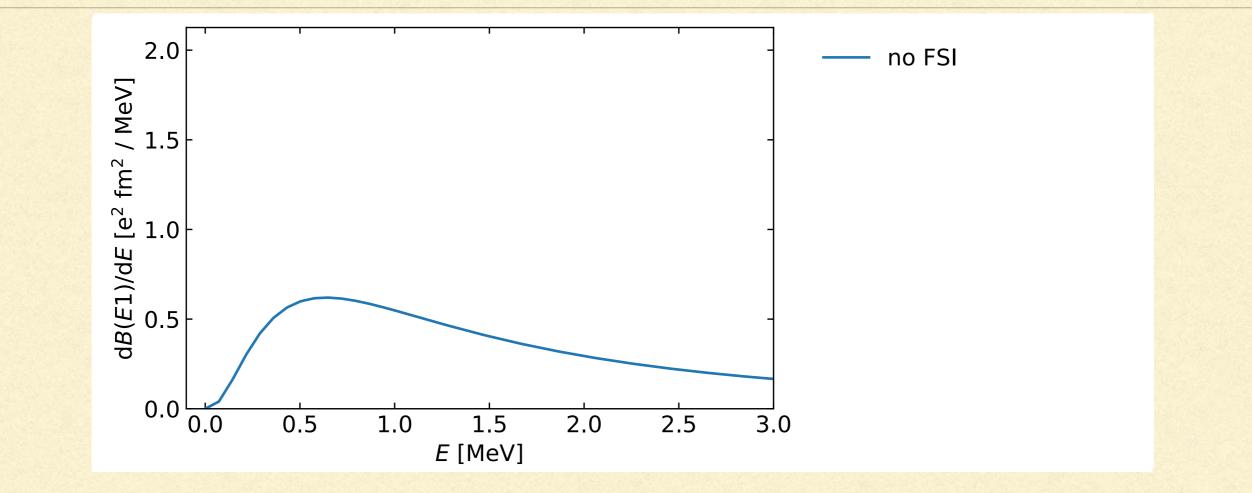
Modifications to matrix element due to one final-state interaction/ wave-function distortion encoded in Møller operators

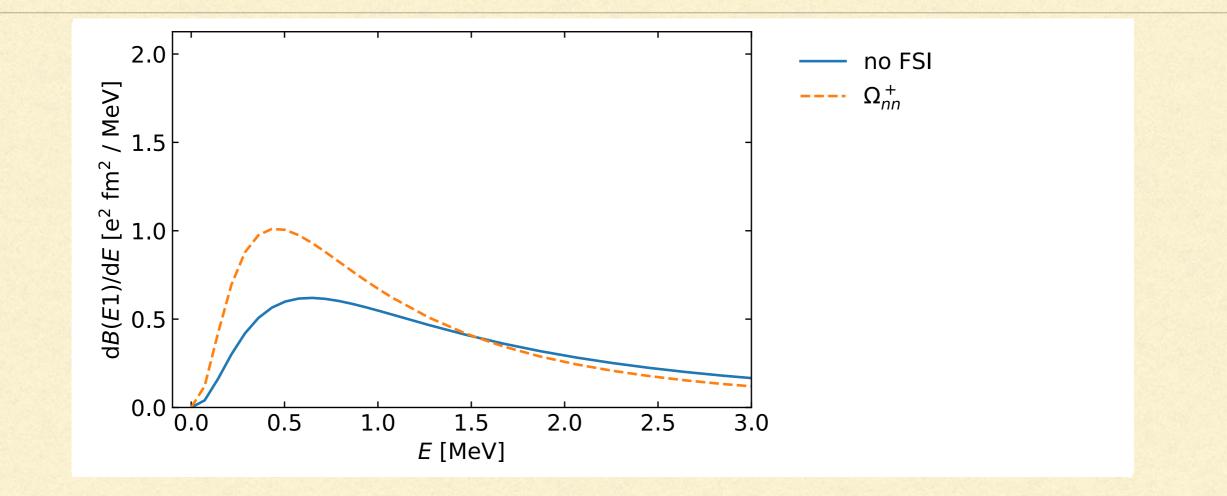
then can multiply Møllers for two t's in final-state, three t's in final-state, etc.

 ${}_{n}\langle p,q,\Omega_{n}^{(0,\xi)}|(1+t_{nc}(E_{p})G_{0}^{(nc)}(E_{p}))\mathcal{M}(E1,\mu)|\Psi\rangle$

 $_{c}\langle p,q,\Omega_{c}^{(1,\mu)}|(1+t_{nn}(E_{p})G_{0}^{(nn)}(E_{p}))\mathcal{M}(E1,\mu)|\Psi\rangle$

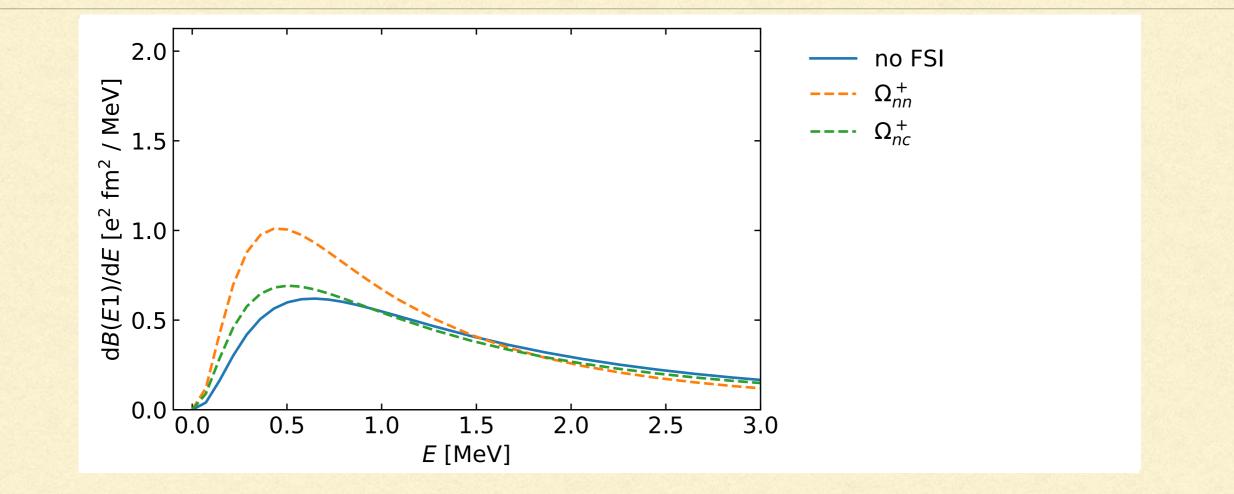




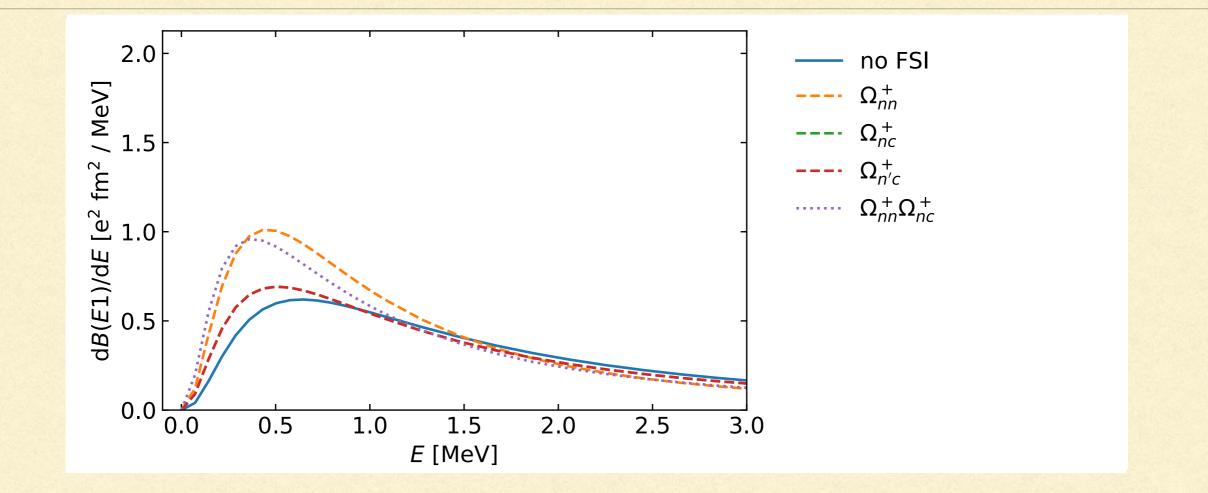


• $\mathcal{M}(E1,\mu) = eZ_c r_c Y_{1,\mu}(\hat{r}_c)$ so PWIA images \mathbf{r}_c times probability distribution

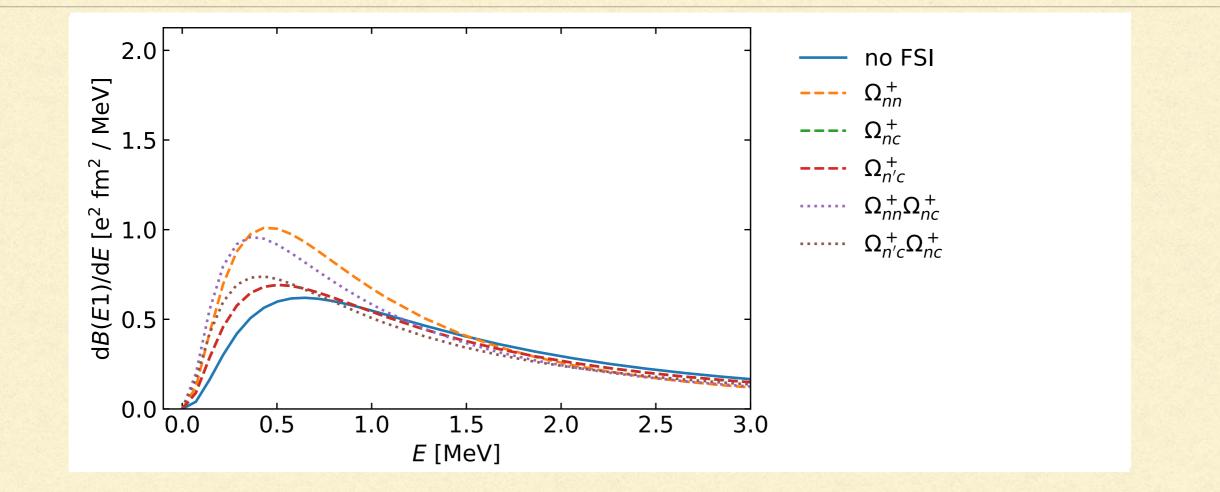
• $\Omega_{nn}^{\dagger} = (1 + t_{nn}G_0)$: nn FSI effect is large



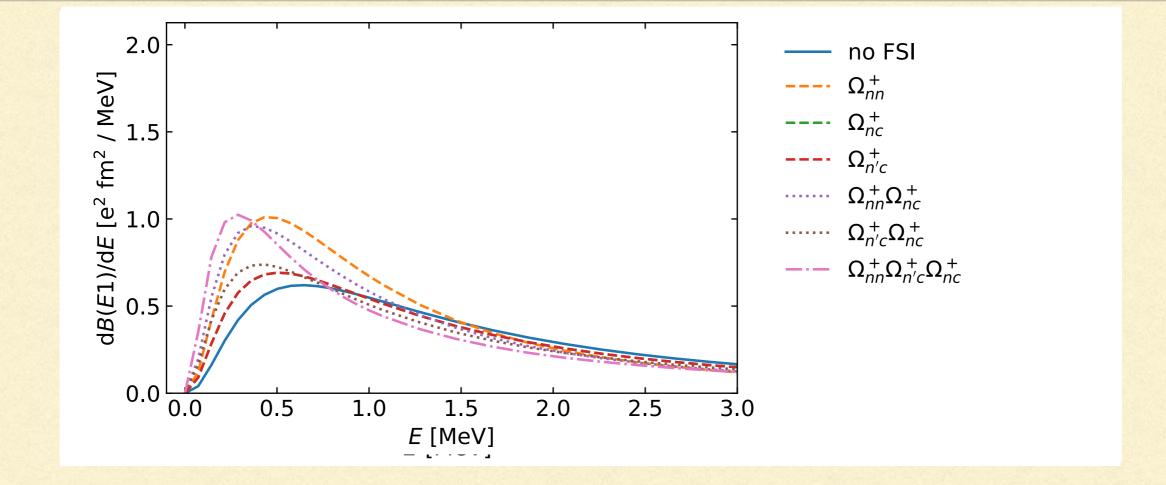
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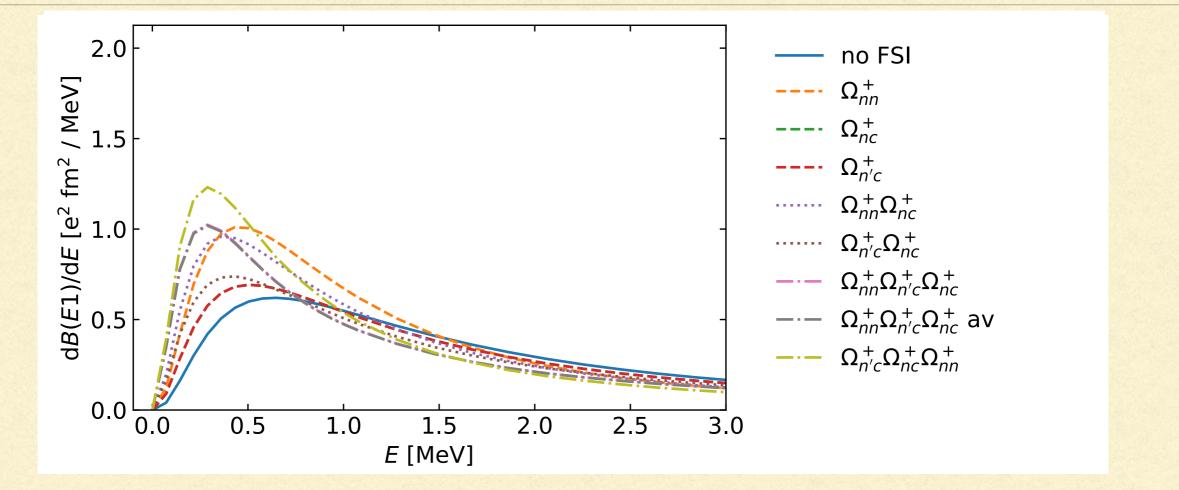


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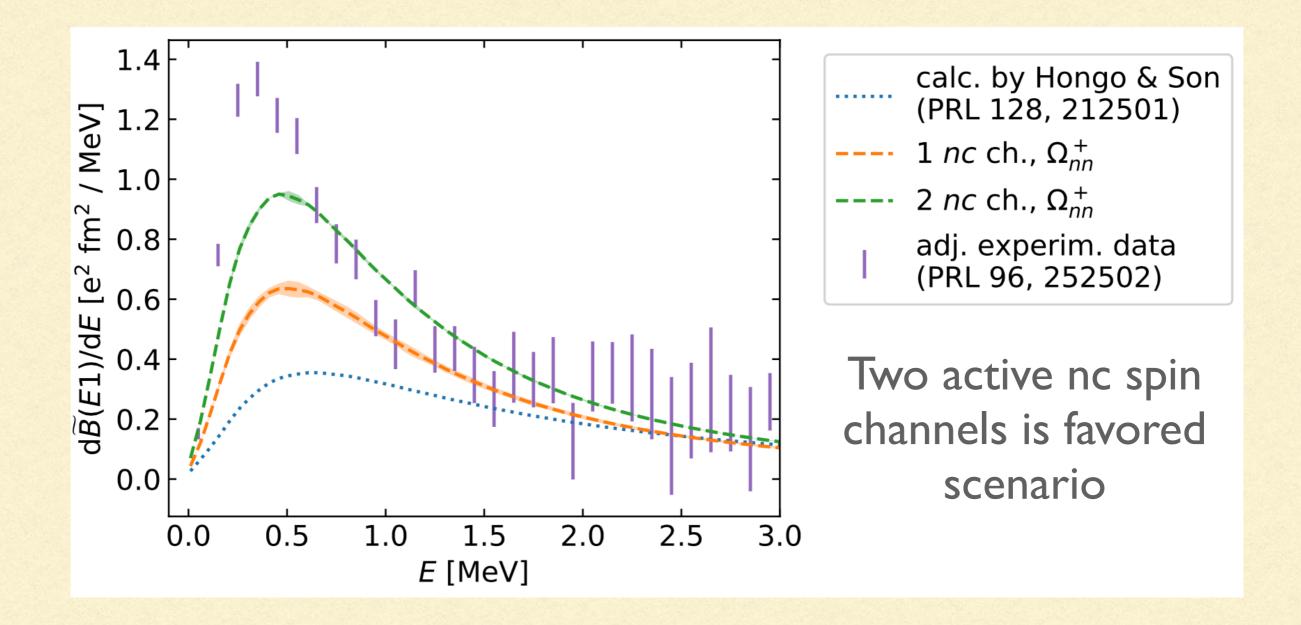
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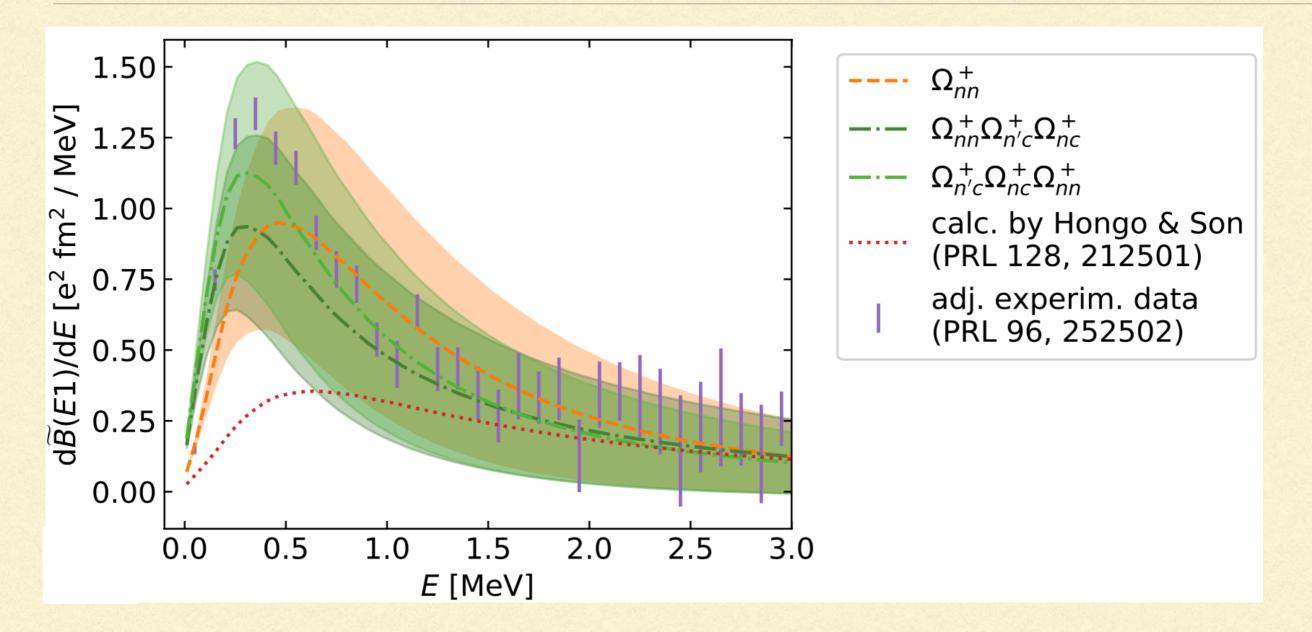


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- Order in which Møller operators are applied affects peak height somewhat

Comparison with data



Comparison with data



Agreement with data is good, given that this is only a leading-order calculation

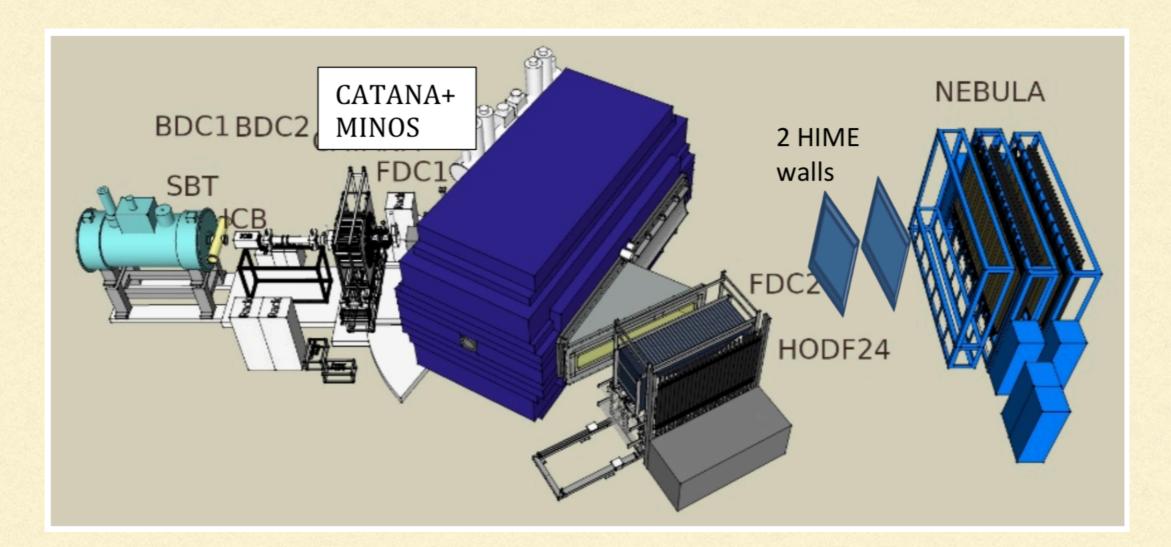
What we learn from ¹¹Li

¹¹Li Coulomb dissociation work shows

- Momentum distribution of s-wave 2n halos dominated by lowmomentum part
- In principle momentum c-nn momentum distribution "imaged" in Coulomb dissociation
- But FSI is large, especially due to nn interactions
- An order-by-order treatment of FSI using Møller operators seems to converge, but reasons for that are not entirely clear

Seek a more selective probe of the nn momentum distribution

RIKEN experiment with 6He beam



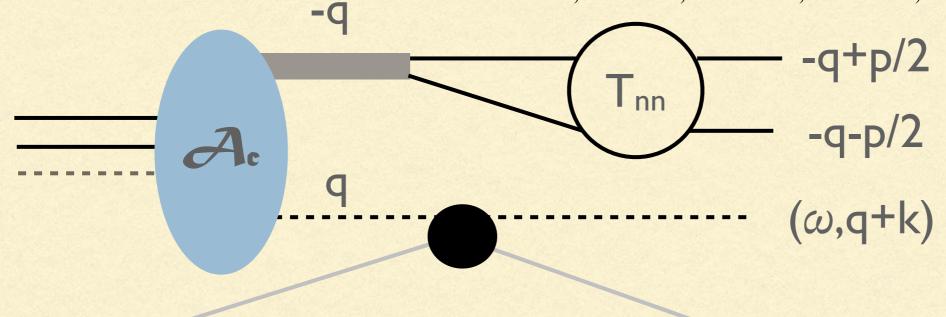
Tom Aumann spokesperson

Detect proton and alpha in TPC

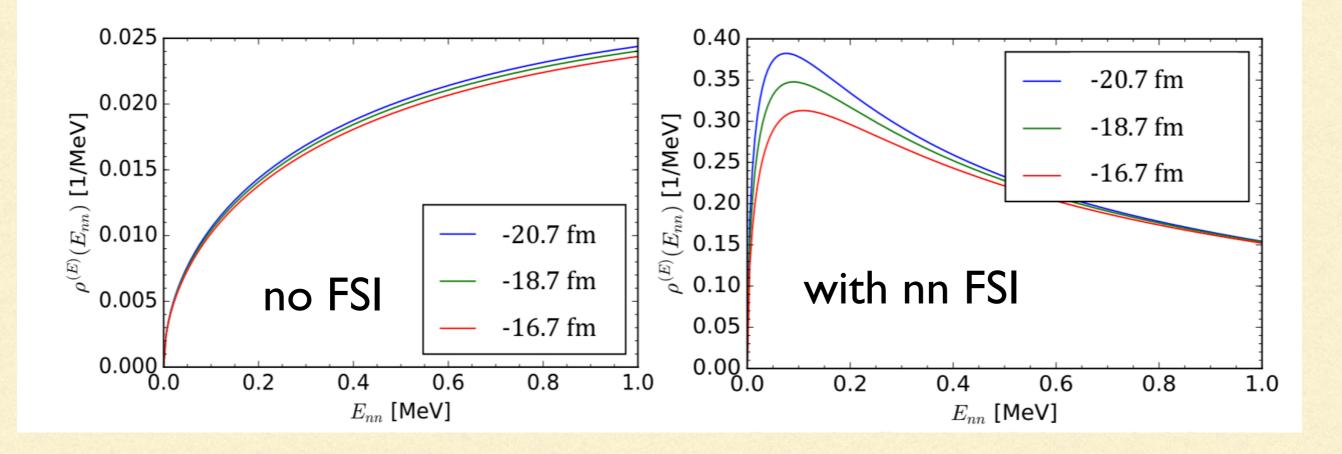
Detect neutrons in HIME + NEBULA: excellent energy resolution

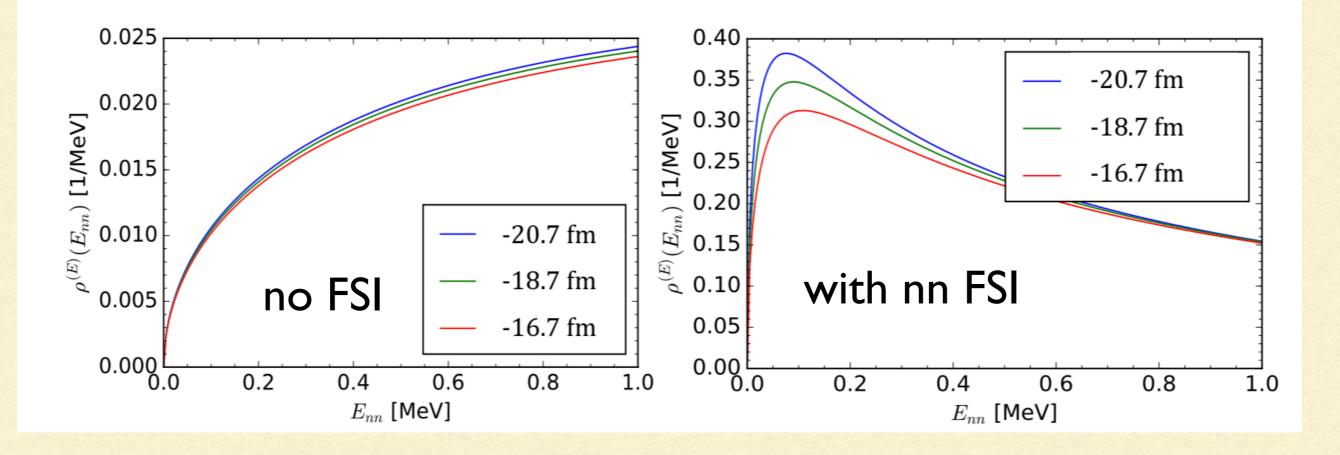
⁶He(p,p' α) and the nn scattering length



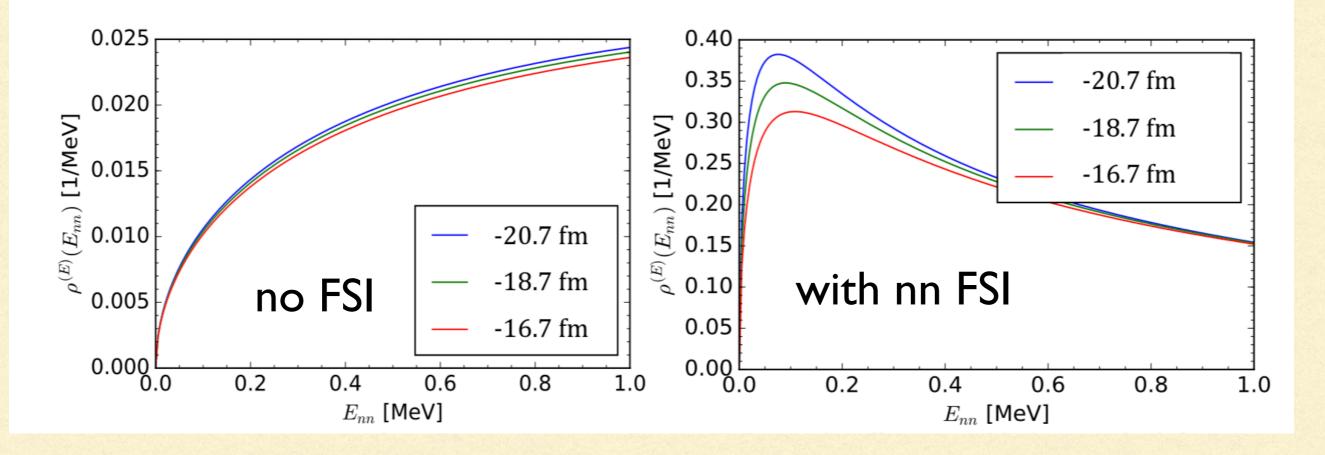


- Quasi-free alpha-particle knockout can leave nn pair almost at rest
- Final-state interaction then generates significant dependence of neutron relative-energy spectrum f(p²/m_n) on a_{nn}
- ⁶He acts as a "holder" for low-momentum neutrons
- Neutrons actually move fast in lab. frame: inverse kinematics



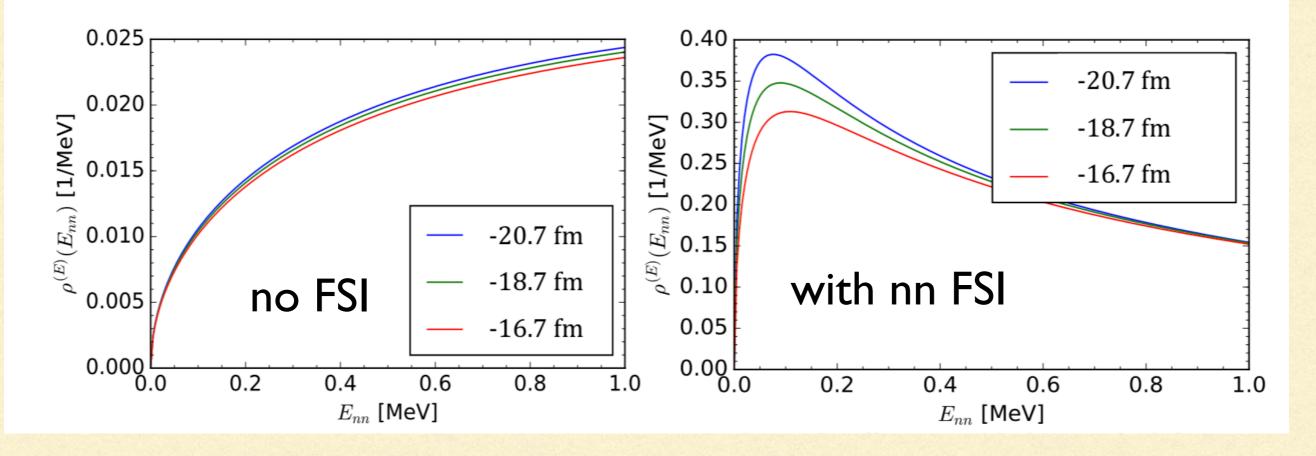


Note that since this is not an absolute measurement we need to decide how to normalize the spectra



⁶He structure at low momentum not significantly affected by cutoff or a_{nn} (or r_{nn})

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But measured neutron spectrum is affected by ann (and not by rnn)

Note that since this is not an absolute measurement we need to decide how to normalize the spectra

What we learn from ⁶He

So 6He relative-momentum distribution work shows:

- Very little sensitivity to ann in "structure part"
- NLO corrections to structure part should be small (not this talk)
- Even less sensitivity to rnn
- Strong ann dependence of final spectrum from FSI
- This modification can be well described by an enhancement factor

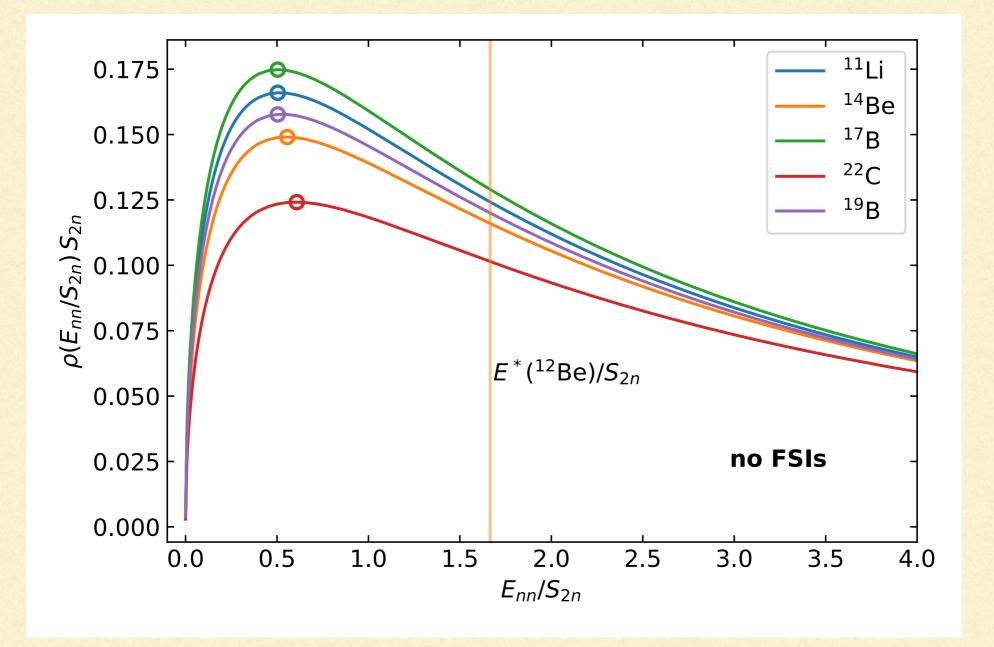
$$\rho^{\text{full}}(E_{nn}) \approx G(E_{nn}, a_{nn}, r_{nn})\rho^{g.s.}(E_{nn})$$

Same conclusions hold for ³H

Göbel, Kirchner, Hammer, PRC (2025)

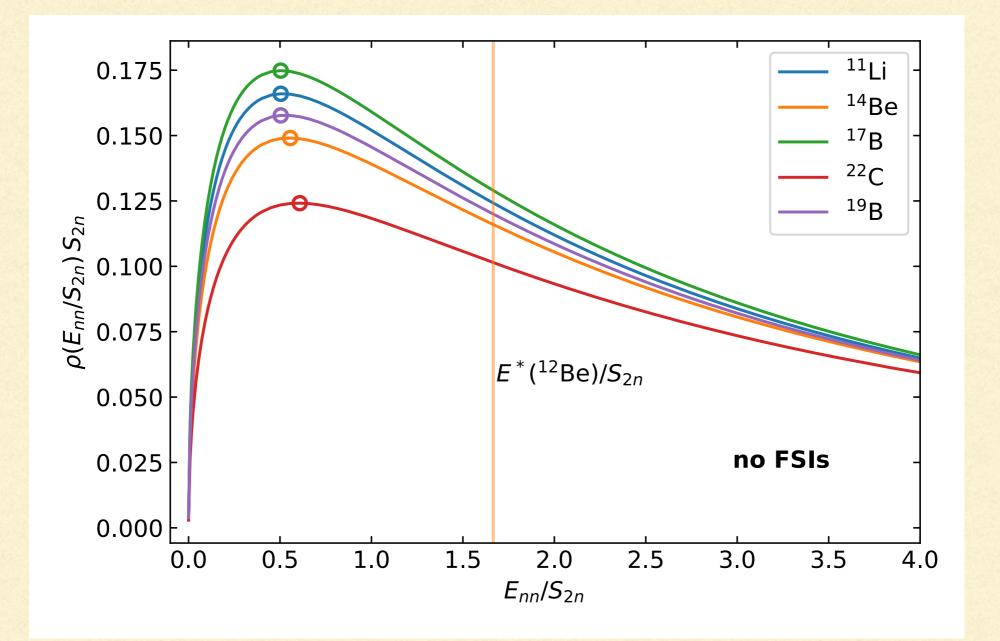
nn momentum distributions for s-wave 2n halos

Göbel, Hammer, DP, PRC (2024)



nn momentum distributions for s-wave 2n halos

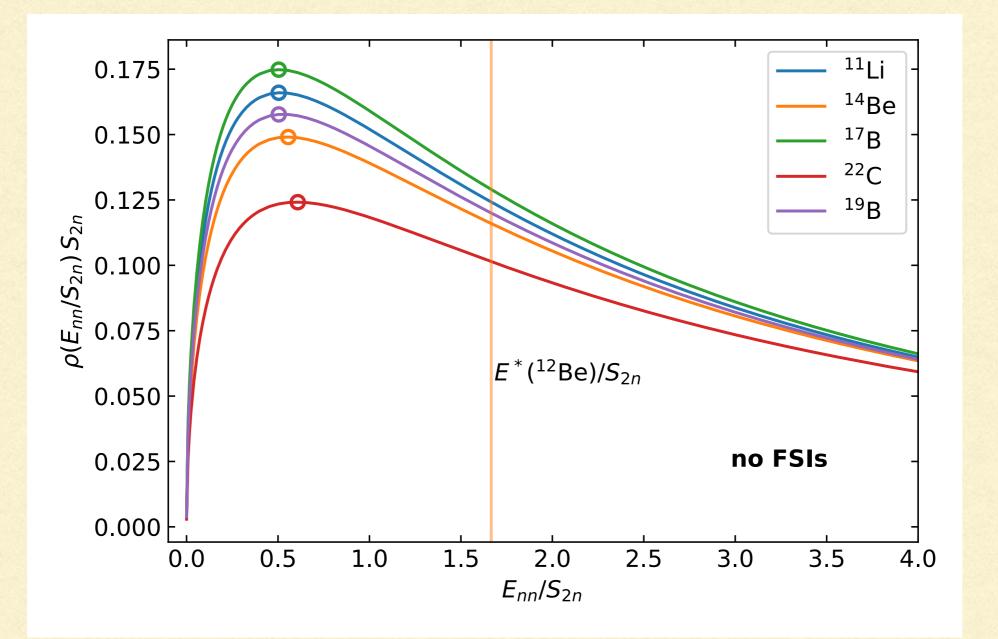
Göbel, Hammer, DP, PRC (2024)



Plot in dimensionless units so all 2n halos fall on same plot

nn momentum distributions for s-wave 2n halos

Göbel, Hammer, DP, PRC (2024)



- Plot in dimensionless units so all 2n halos fall on same plot
- Entirely within EFT's domain of validity for all but ¹⁴Be

Going to the unitarity limit

- The "unitarity limit" is another limit on top of LO Halo EFT: $|a| \rightarrow \infty$
- The 2B state is then right at threshold. No scales left: $r \rightarrow 0$, $|a| \rightarrow \infty$.

• 2B amplitude: $t^{2B}(E = k^2/m_R) \sim \frac{1}{ik}$, 2B problem has conformal invariance

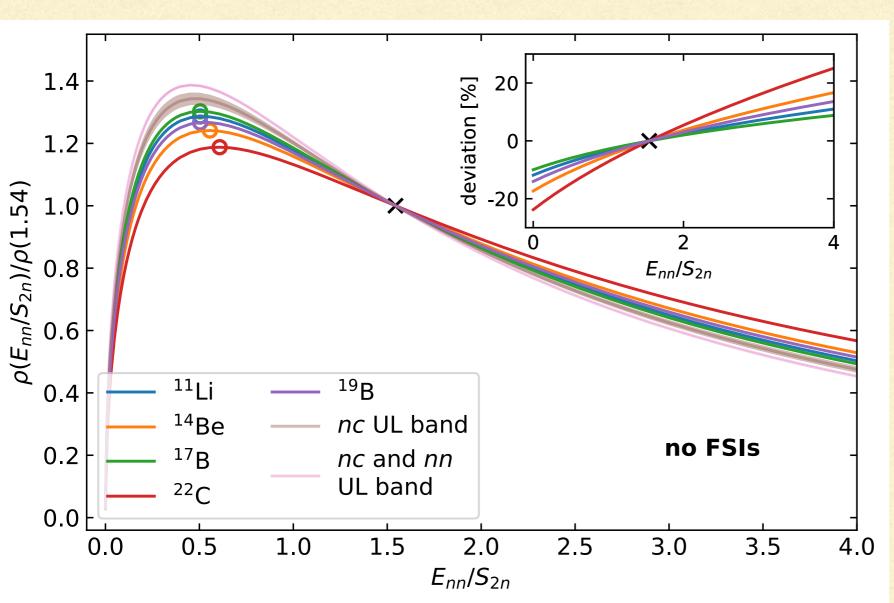
Efimov effect in 3B system: infinite tower of bound states $\frac{E^{(n)}}{E^{(n-1)}} = 515$

- Ratio of 4B and 3B binding energies $E^{4B,n}/E^{3B,n} = 4.6$ + excited tetramer Platter & Hammer (2007); Deltuva (2012)
- Scaling dimension of multi-neutron momentum distributions calculable Son & Hammer (2022); Chowdry, Mishra, Son (2023)
- What about momentum distribution of nn relative-momentum distributions in Borromean s-wave 2n halos?

The unitarity limit can be seen in 2n halos

 $\rho^{g.s.}(E_{nn}/S_{2n}; V_{nn}, V_{nc}, S_{2n}, A) \approx \rho^{g.s.}(E_{nn}/S_{2n})$

Cf. for ¹⁹B: Hiayma, Lazauskas, Marqués, Carbonell (2019); Hiyama, Lazauskas, Carbonell, Frederico (2023)



i.e., ρ^{g.s.} is the same function for all halos to better than 20%

- Works because halos are sufficiently bound that precise values of ann and anc do not matter.
- A dependence also goes away

But can it be measured?

Results for other 2n halos after FSI modification

Use Møller operator to include nn FSI:

$$\psi_c^{(\text{wFSI})}(p,q) = \langle p,q;\zeta_c,\xi_c | (1+t_{nn}(E_p)G_0(E_p)) | \Psi \rangle$$

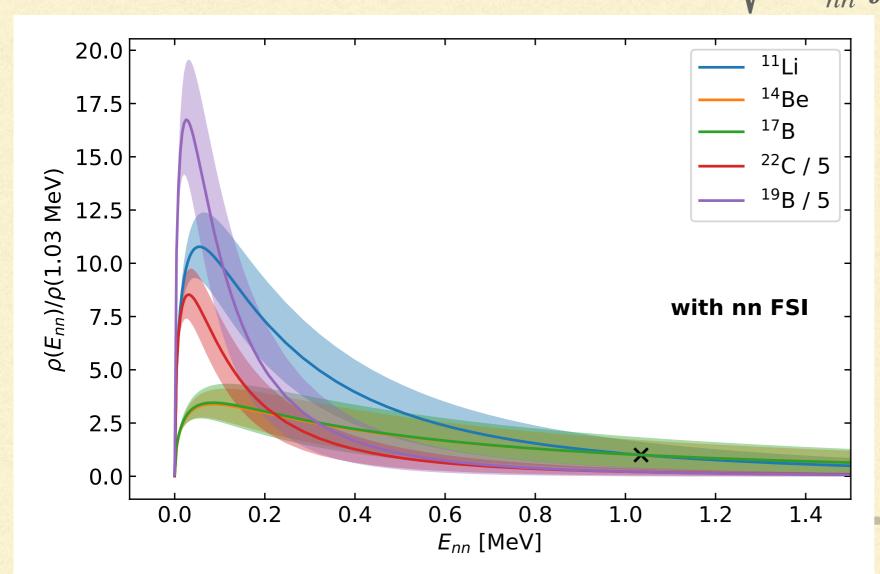
Relative energy distribution: $\rho(E_{nn}) = \sqrt{\frac{m_n}{4E_{nn}}} \int_0^\Lambda dq \, q^2 |\Psi_c(p_{nn},q)|^2 p_{nn}^2$

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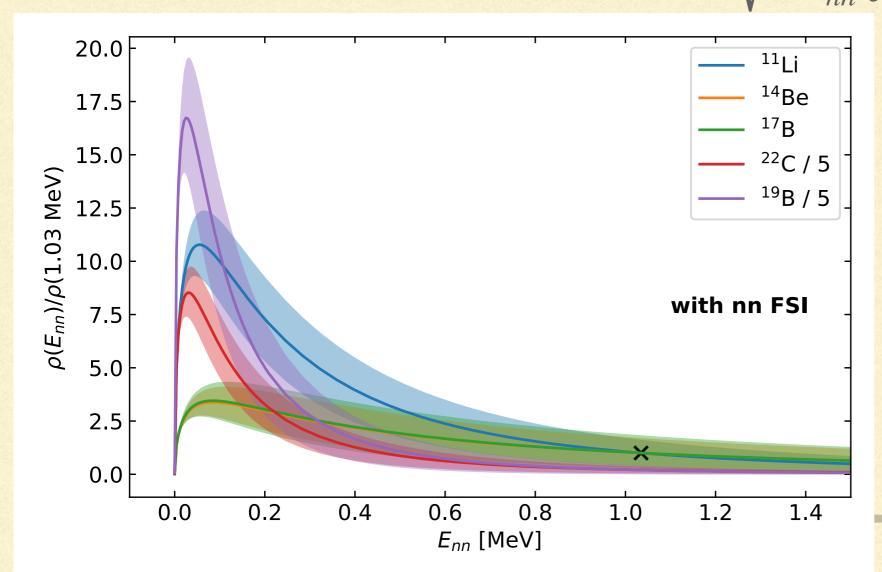


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nn interaction produces variation on scale $1/(m_n a_{nn}^2)$

Ground-state distribution varies on scale S_{2n}

Divide out by FSI factor

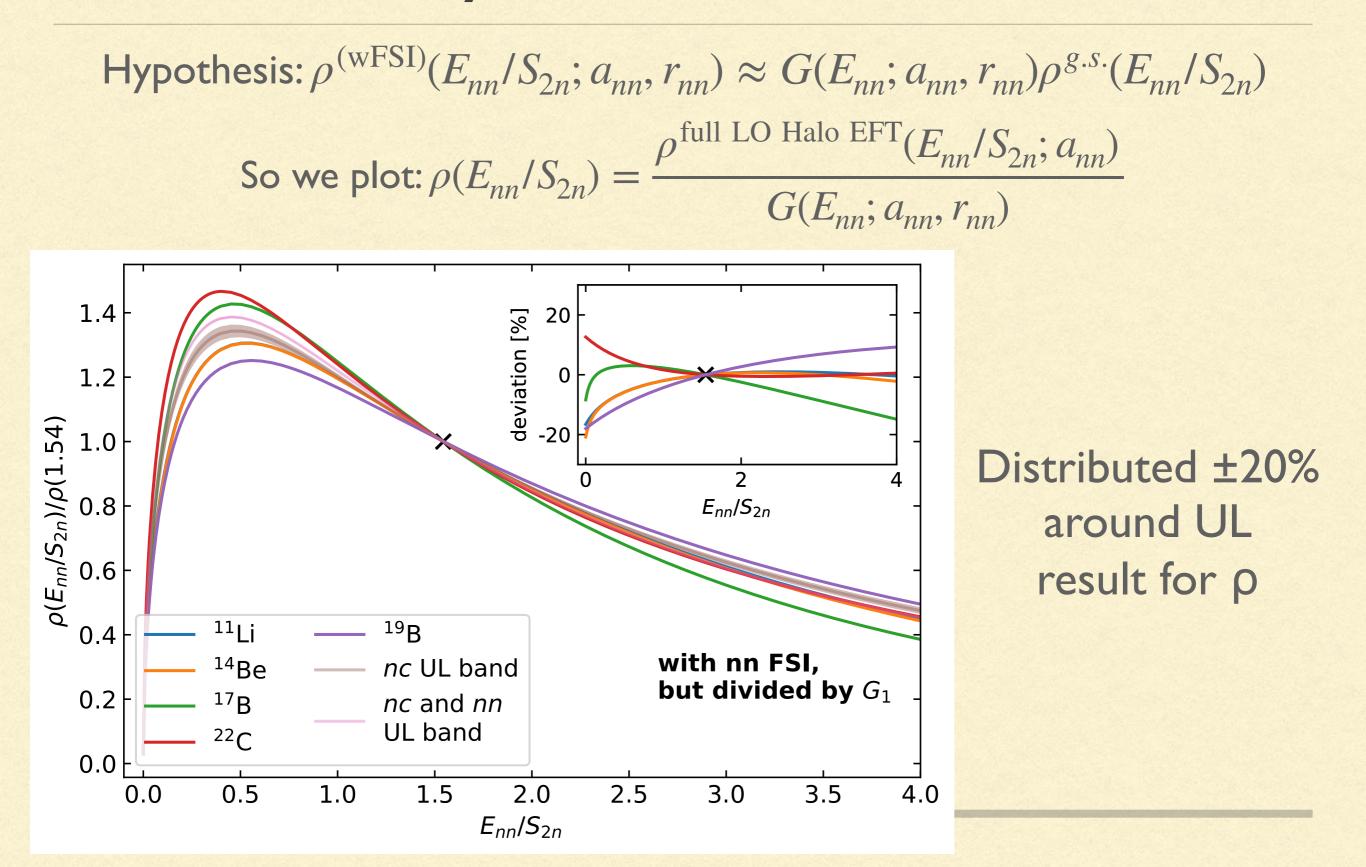
Hypothesis: $\rho^{(\text{wFSI})}(E_{nn}/S_{2n}; a_{nn}, r_{nn}) \approx G(E_{nn}; a_{nn}, r_{nn})\rho^{g.s.}(E_{nn}/S_{2n})$

Divide out by FSI factor

Hypothesis:
$$\rho^{(\text{wFSI})}(E_{nn}/S_{2n}; a_{nn}, r_{nn}) \approx G(E_{nn}; a_{nn}, r_{nn})\rho^{g.s.}(E_{nn}/S_{2n})$$

So we plot: $\rho(E_{nn}/S_{2n}) = \frac{\rho^{\text{full LO Halo EFT}}(E_{nn}/S_{2n}; a_{nn})}{G(E_{nn}; a_{nn}, r_{nn})}$

Divide out by FSI factor



Outline

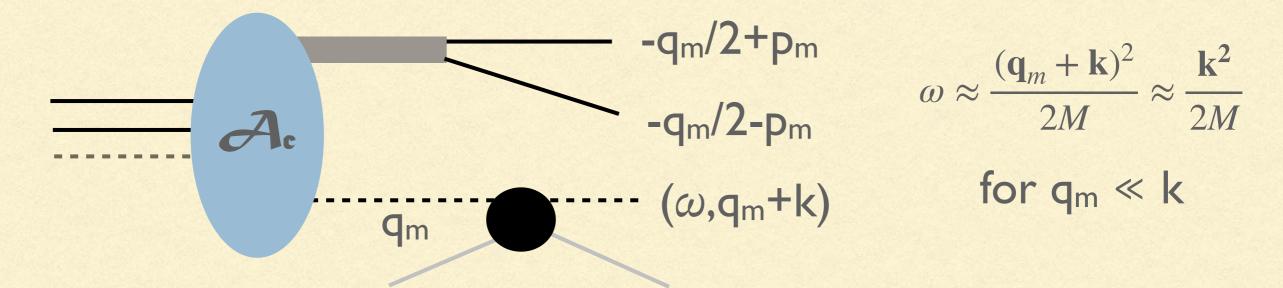
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The unitarity limit in momentum distributions of 2n halos We can measure the bound-state nn momentum distribution in the halo!

An EFT approach to computing quasi-free core knockout All other reaction mechanisms can be filtered out!

An EFT treatment of QF knockout

Briceño, Costa, Hammer, DP, in preparation



- This impulse approximation diagram has a $q_m \rightarrow 0$ pole if the 2n halo has zero binding energy
- Momenta \sim k in wave function suppressed by $(\gamma/k)^4$ in Halo EFT
- Consider behavior of diagrams with different FSIs in terms of their lowmomentum scaling: count powers of p and q in each diagram
- Only one-body operator, as only it has this on-shell pole (aka QF peak)

Assumptions

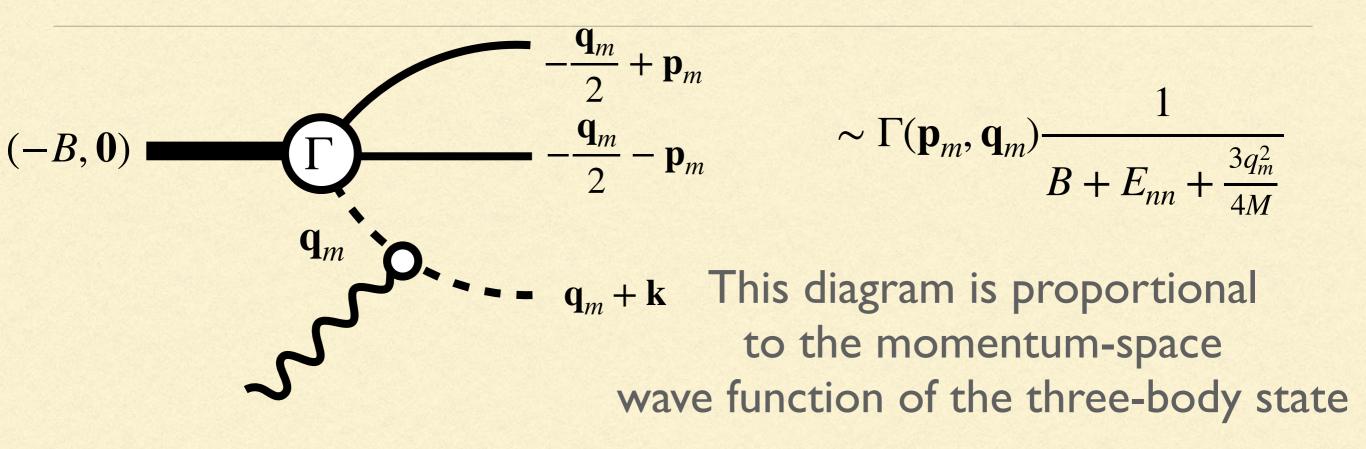
Experiment

- Events are selected in which core particle is in quasi-free kinematics
- Events are plotted vs qm and anything that is constant as a function of qm is removed as background

EFT

- Only low-momentum part of loops generate the rapid dependence on qm that we are looking for
- Scaling of low-momentum part of loops can be computed by replacing scattering amplitude by its on-shell value
- Work with nn amplitude at unitarity and zero binding of halo
- Two-body currents generate at best weak qm dependence

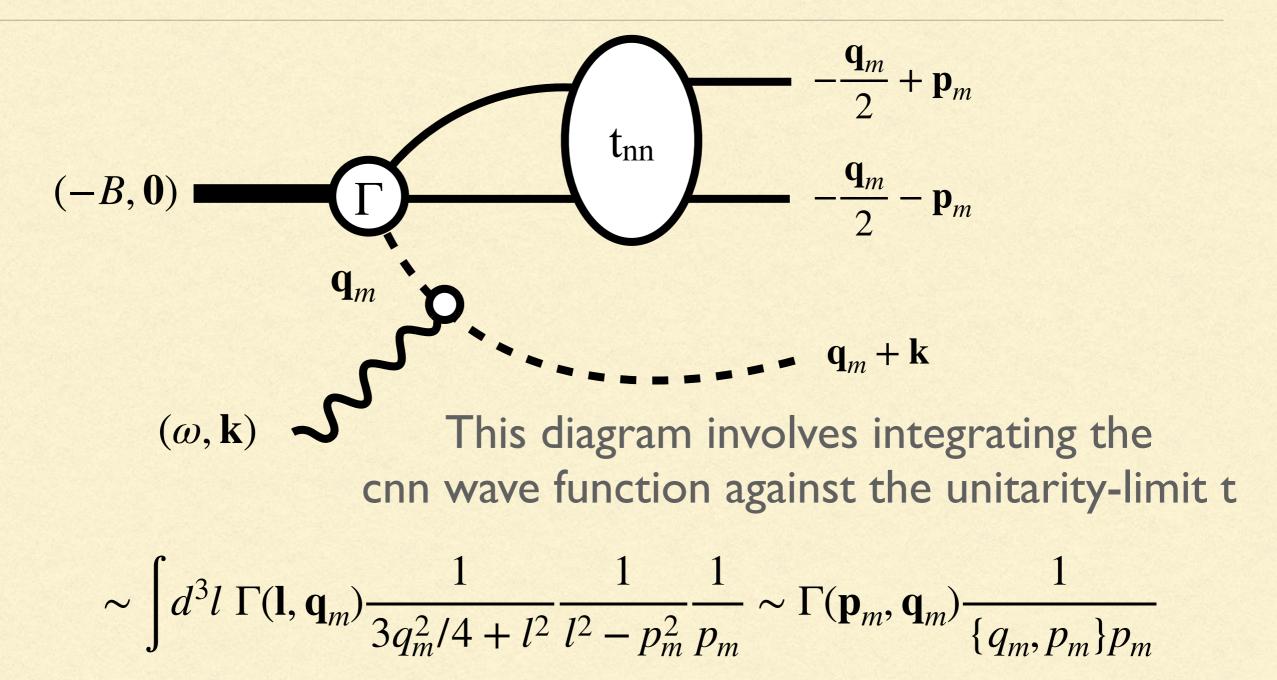
Start with the simplest case: PWIA



Diverges as $B \rightarrow 0$ and $E_{nn} \rightarrow 0$ and $q_m \rightarrow 0$

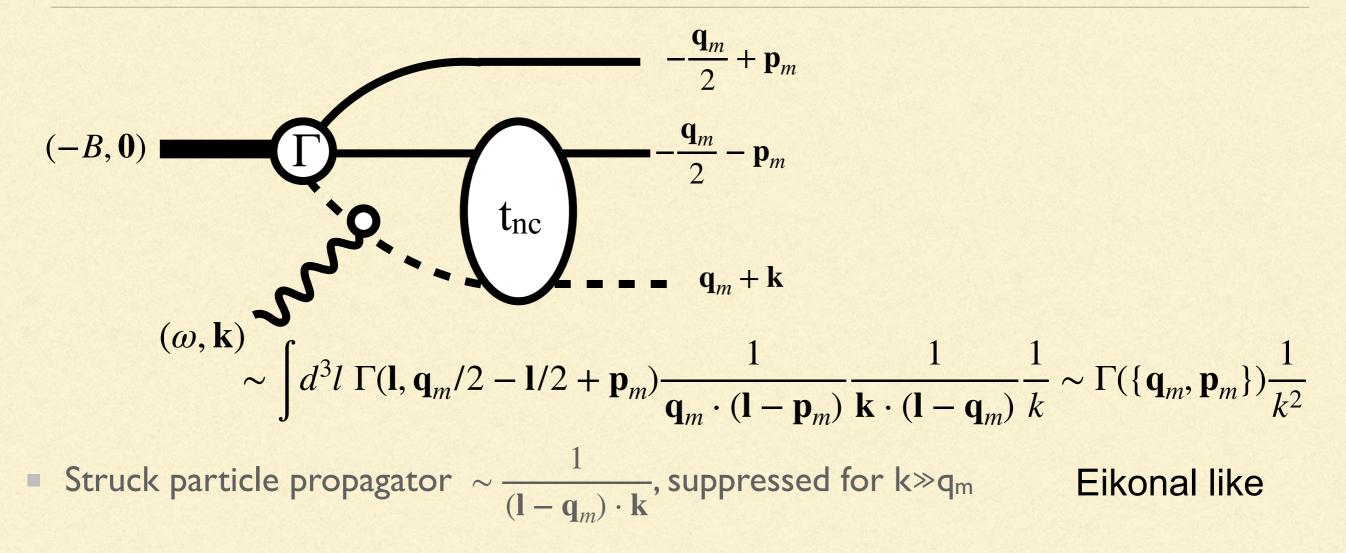
Infra-red pole for zero energy bound-state $\sim \Gamma(\mathbf{p}_m, \mathbf{q}_m) \frac{1}{\mathbf{p}_m^2 + 3\mathbf{q}_m^2/4}$

Adding the nn FSI



Same order (or even larger in certain regions of phase space) as diagrams where ground-state momentum distribution imaged directly

Key observation for nc FSI diagrams



- And scattering amplitudes at relative momenta of order k carry at least an additional power of 1/k.
- Core-neutron FSI is suppressed by $\{q_m, p_m\}^2/k^2$ (no pole as halo nucleus' binding $\rightarrow 0$)
- And this will hold for all diagrams, no matter how complicated, involving such FSI

Summary and outlook

- Claim part I: Fast core removal on 2n halos offers the opportunity to measure the neutron-neutron scattering length via final-state interactions
 ⁶He, ³H
- Claim part 2: S-wave 2n halos all have the same ground-state nn momentum distribution, within the size of higher-order corrections $1/(a_0 p) \sim r_0 p$ 11Li. 14Be. 17B. 22C
- Claim part 3: It is approximately the unitary limit momentum distribution: nothing about the nn and nc interactions matters except that they're strong
- Test: measure the nn relative energy distribution on several halos and divide out FSI effects
- EFT argument based on the infra-red poles of the quasi-free knockout amplitude implies that these conclusions receive "reaction-mechanism" corrections $\sim q_m^2/k^2$
- **To do I**: assess impact of NLO corrections to both structure and reaction mechanisms
- To do 2: Complete treatment of FSI in Coulomb dissociation
- To do 3: Extend QF knock-out argument to more neutrons left behind: tri-neutrons, tetra-neutrons, etc.

Lagrangian: shallow S- and P-states

$$\mathcal{L} = c^{\dagger} \left(i\partial_{t} + \frac{\nabla^{2}}{2M} \right) c + n^{\dagger} \left(i\partial_{t} + \frac{\nabla^{2}}{2m} \right) n$$

+ $\sigma^{\dagger} \left[\eta_{0} \left(i\partial_{t} + \frac{\nabla^{2}}{2M_{nc}} \right) + \Delta_{0} \right] \sigma + \pi^{\dagger}_{j} \left[\eta_{1} \left(i\partial_{t} + \frac{\nabla^{2}}{2M_{nc}} \right) + \Delta_{1} \right] \pi_{j}$
- $g_{0} \left[\sigma n^{\dagger} c^{\dagger} + \sigma^{\dagger} nc \right] - \frac{g_{1}}{2} \left[\pi^{\dagger}_{j} (n \ i \overleftrightarrow{\nabla}_{j} \ c) + (c^{\dagger} \ i \overleftrightarrow{\nabla}_{j} \ n^{\dagger}) \pi_{j} \right]$
- $\frac{g_{1}}{2} \frac{M - m}{M_{nc}} \left[\pi^{\dagger}_{j} \ i \overrightarrow{\nabla}_{j} \ (nc) - i \overleftrightarrow{\nabla}_{j} \ (n^{\dagger} c^{\dagger}) \pi_{j} \right] + \dots,$

- c, n: "core", "neutron" fields. c: boson, n: fermion.
- σ, π_j: S-wave and P-wave fields
- Minimal substitution generates leading EM couplings
- Additional EM couplings at sub-leading order