Fusion reactions in collisions of weakly bound systems*

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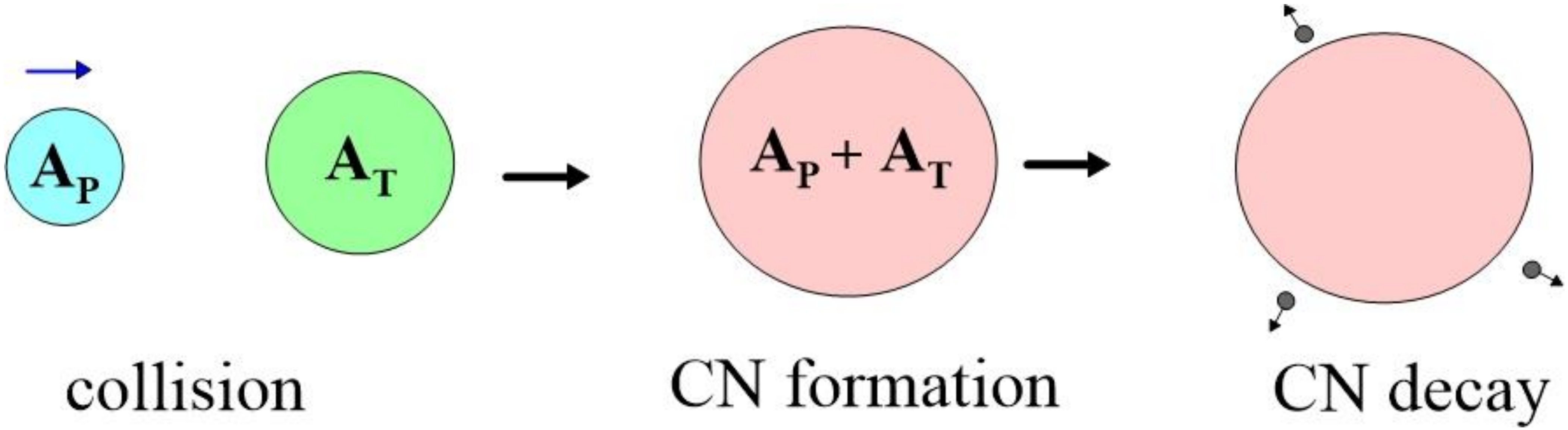
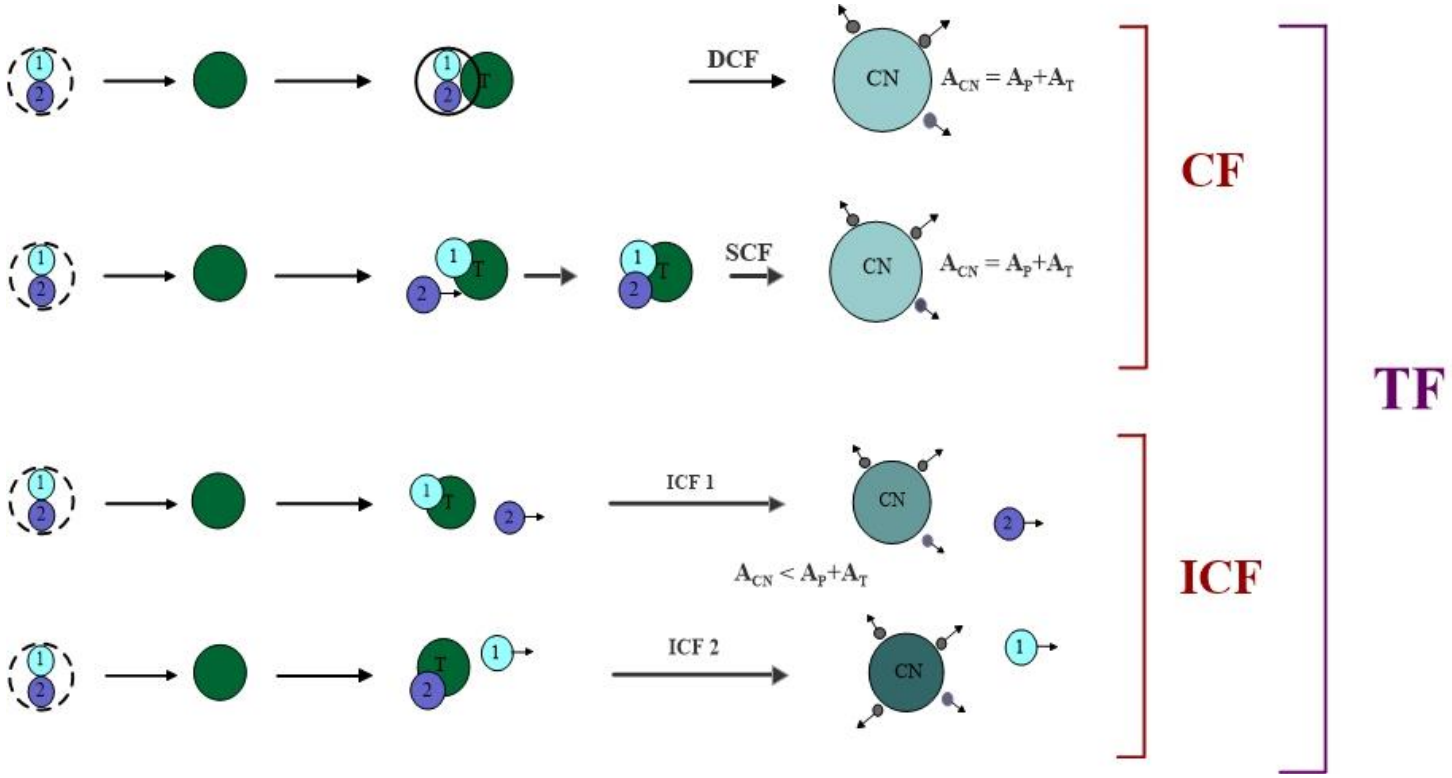
Fusion of tightly bound nuclei



Detection

Collisions of weakly nuclei (particular case of 2 clusters)



$$\sigma_{\mathrm{CF}} = \sigma_{\mathrm{DCF}} + \sigma_{\mathrm{DCF}}$$

$$\sigma_{\mathrm{TF}} = \sigma_{\mathrm{CF}} + \sigma_{\mathrm{ICF}}$$

$$\sigma_{\mathrm{ICF}} = \sigma_{\mathrm{ICF}_1} + \sigma_{\mathrm{ICF}_2}$$

Finding CF and ICF cross section is a great challenge

(both for experimentalists and theorists)

Experiment:

- Individual σ_{CF} and/or σ_{ICF} have been measured for some particular stable P-T combinations:

Some examples: (^{6}Li: B = 1.47 MeV, ^{7}Li: B = 2.45 MeV, ^{9}Be: B = 1.65 MeV

^{6,7}Li + ^{209}Bi, ^{6,7}Li + ^{159}Tb, ^{6,7}Li + ^{197}Au, ^{9}Be + ^{208}Pb

Theoretical models

Classical & Semiclassical:

Hagino et al., NPA 738, 475 (2004); Diaz-Torres et al., PRL 98, 152701 (2007);
Marta et al., PRC 89, 034625 (2014)

CC (CDCC)-based models:

Hagino Hagino et al., PRC 61, 037602 (2000); Diaz-Torres and Thompson, PRC 65, 024606 (2002);
Diaz-Torres et al., PRC 68, 044607 (2003);

The subject of my talk

Recently developed CDCC-based model of Rangel et al. (PLB 803, 135337 (2020))

Potential scattering approach (the simplest QM approach):

$$H = T + V \longrightarrow H = T + V - iW \ ; \qquad H\,|\Psi\rangle = E\,|\Psi\rangle$$

Fusion cross section (from the violated continuity equation)

$$\sigma_{\mathrm{F}} \equiv \sigma_{\mathrm{abs}} = \frac{k}{E}\ \langle\Psi|\,W\,|\Psi\rangle \ ; \ \sigma_{\mathrm{F}} = \frac{\pi}{k^2}\sum_l (2l+1)\ P_{\mathrm{F}}(l)$$

$$P_{\mathrm{F}} = 1 - |S_l|^2 \equiv \frac{4k}{E}\int dr\ |u_l(r)|^2\ W_{\mathrm{F}}(r)$$

Nuclear structure effects on σ_F

The CC method: (neglecting spins, for simplicity)

The full Schrödinger equation

$$\left[E - \mathbb{H}\right] \Psi(\mathbf{R}, \zeta) = 0 \; ;$$

Channel expansion:

$$\Psi(\mathbf{R}, \zeta) = \sum_{\alpha} \psi_\alpha(\mathbf{R}) \, \varphi_\alpha(\zeta)$$

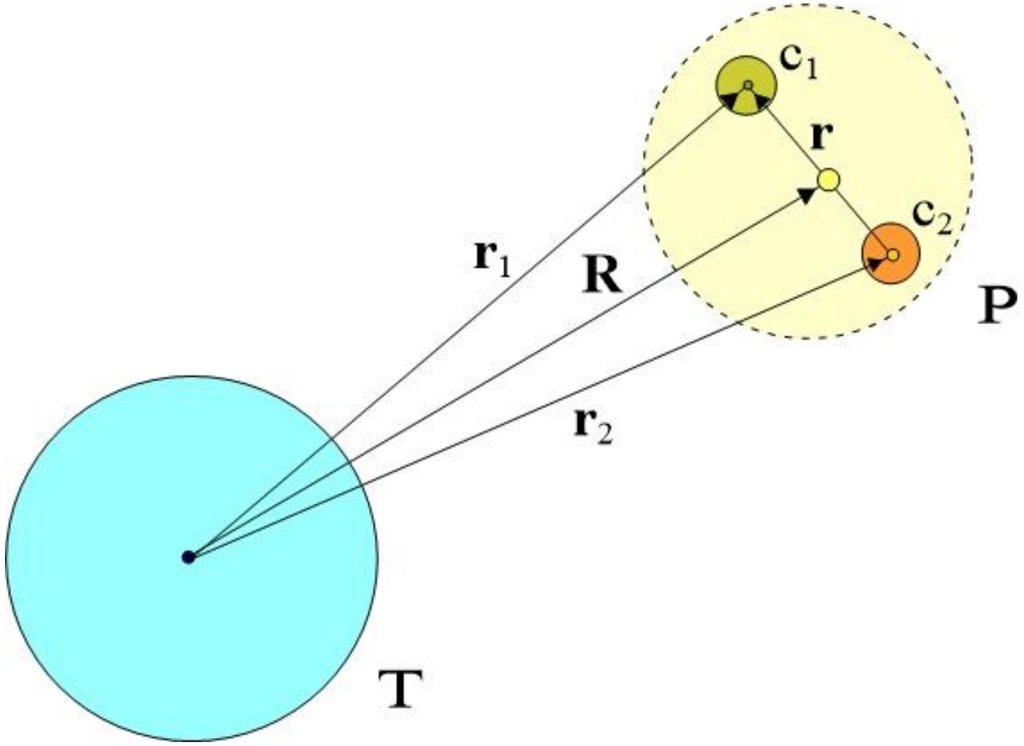
Coupled equations

$$\times \left(\varphi_\alpha \right| \quad \Longrightarrow \quad \left[E_\alpha - H_\alpha\right] \psi_\alpha^{(+)} = \sum_{\alpha' \neq \alpha} V_{\alpha\alpha'} \, \psi_{\alpha'}^{(+)}, \quad \alpha, \alpha' = 1, N$$

$$H_\alpha = \langle \varphi_\alpha \left| \mathbb{H} \right| \varphi_\alpha \rangle \, , \; V_{\alpha,\alpha'} = \langle \varphi_\alpha \left| \mathbb{V} \right| \varphi_{\alpha'} \rangle$$

Projectiles of 2 clusters

The nucleus-nucleus potential



$$\mathbb{U}(\mathbf{r},\mathbf{R}) = \mathbb{U}^{(1)}(r_1) + \mathbb{U}^{(2)}(r_2)$$

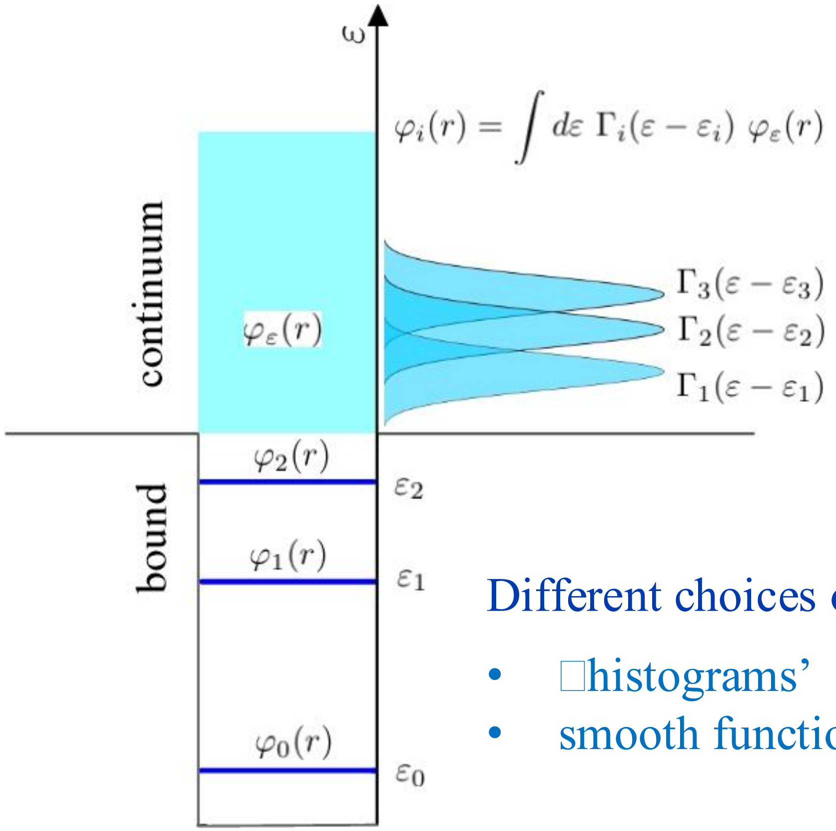
$$\mathbb{U}^{(j)}(r_j) = \mathbb{V}^{(j)}(r_j) - i\,\mathbb{W}^{(j)}(r_j), \quad j = 1, 2$$

Difficulty: unbound states of the projectile

Infinite norms

Continuous energy label $\Longrightarrow$ Infinite set of equations

Solution: discretize the continuum



CDCC method with bins

$$\{\varphi_\varepsilon\} \Longrightarrow \{\varphi_i\}$$

Reduces to a standard CC problem,
(finite number of coupled equations)

Different choices of Γ

- □histograms’
- smooth functions

Channel space (dimension N) split into 2 sub-spaces

- Bound (B) chanels, dimension N_B
- Continuum discretized channels (*bins*), dimension N_C
- $N_B + N_C = N$

$$|\Psi\rangle = |\Psi_B\rangle + |\Psi_C\rangle$$

The CDCC calculation can only give:

- Direct CF (no breakup)

$$\sigma_{\mathrm{DCF}} = \frac{k}{E} \; \langle \Psi_{\mathrm{B}} | \, \mathbb{W}^{(1)} + \mathbb{W}^{(2)} \, | \Psi_{\mathrm{B}} \rangle$$

- Inclusive fusion of the each cluster

(capture of one cluster, independently of what happens to the other)

$$\sigma_{\mathrm{F}}^{(1)} = \frac{k}{E} \; \langle \Psi_{\mathrm{C}} | \, \mathbb{W}^{(1)} \, | \Psi_{\mathrm{C}} \rangle \;\; ; \qquad \sigma_{\mathrm{F}}^{(2)} = \frac{k}{E} \; \langle \Psi_{\mathrm{C}} | \, \mathbb{W}^{(2)} \, | \Psi_{\mathrm{C}} \rangle$$

However, these are not the measure cross sections

Experiments give:

$$\sigma_{CF} = \sigma_{DCF} + \sigma_{SCF}; \qquad \sigma_{ICF1}; \quad \sigma_{ICF2}$$

Thus, to extract these cross sections from the available ones,

$$\sigma_F^{(1)}, \quad \sigma_F^{(2)} \quad \text{and} \quad \sigma_{DCF}$$

we need further assumptions

The method of Rangel *et al.* (PL B803 (2020) 135337; PRC 102 (2020) 064628)

Carrying out angular momentum expansions of $\sigma_{\rm F}^{(1)}$ and $\sigma_{\rm F}^{(2)}$ one gets inclusive fusion probabilities $P^{(1)}(J)$ and $P^{(2)}(J)$

Then, use classical probability theory:

$$P^{\rm ICF1}(J) = P^{(1)}(J) \times \left[1 - P^{(1)}(2)(J)\right]$$

$$P^{\rm ICF2}(J) = P^{(2)}(J) \times \left[1 - P^{(1)}(1)(J)\right]$$

$$P^{\rm SCF}(J) = P^{(2)}(J) \times P^{(1)}(1)(J)$$

Then, carrying out the sum over J, one gets

$\sigma^{\mathrm{ICF1}};\ \sigma^{\mathrm{ICF2}}$, and σ^{SCF}

and the CF cross section is

$$\sigma^{\mathrm{CF}} = \sigma^{\mathrm{DCF}} + \sigma^{\mathrm{SCF}}$$

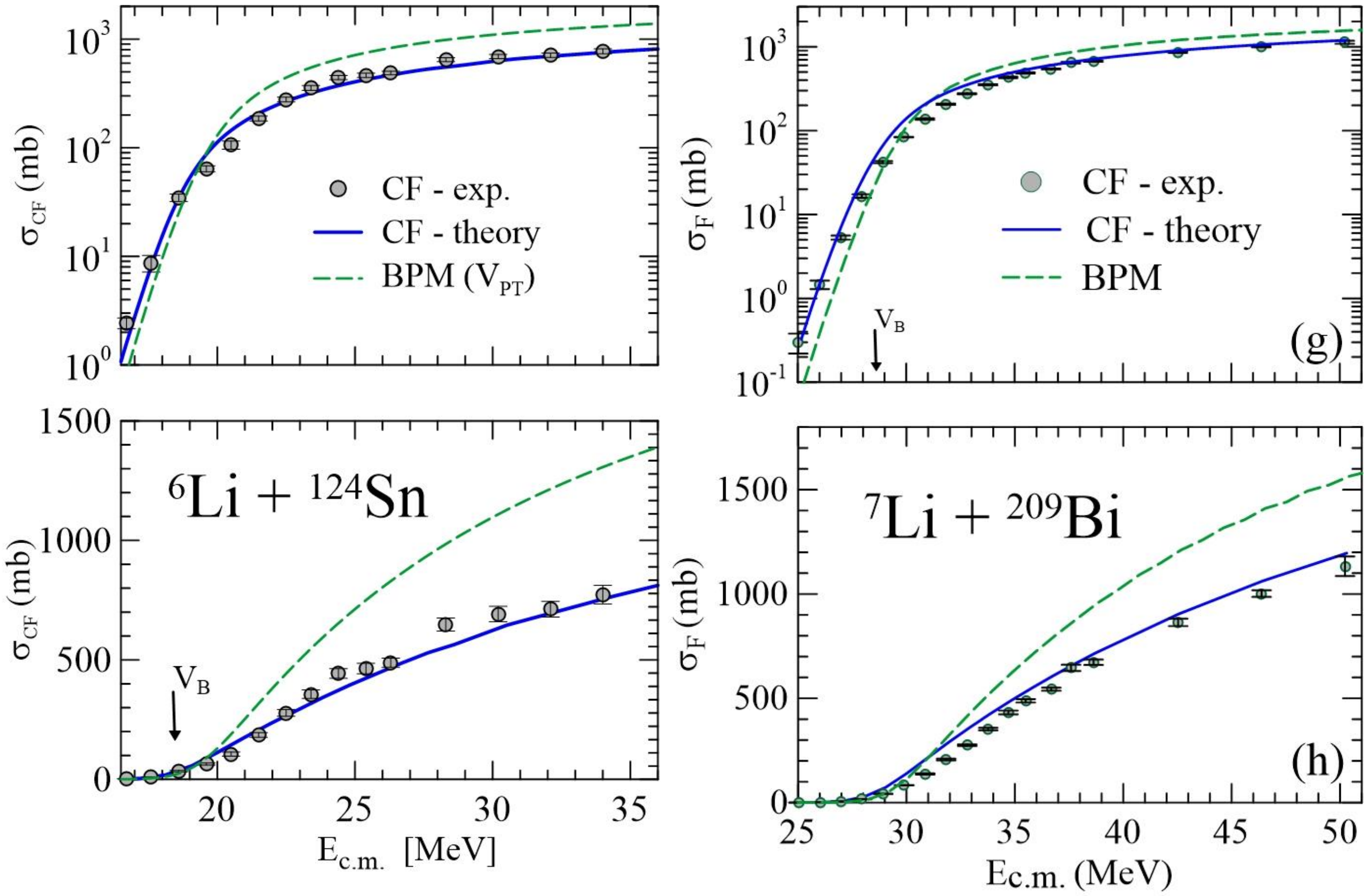
The method was applied to several systems with

$\mathbb{V}^{(i)}(r_i)$: São Paulo potential between cluster c_i and the target

$\mathbb{W}^{(i)}(r_i)$: Short-range absorption of cluster c_i

Good overall agreement between theory and experiment

A few examples



- Good agreement
- Enhancement below V_B,
- Suppression above V_B. This is the most interesting effect

- ^{209}Bi: data from Dasgupta *et al*., PRC **70**, 024606 (2004)
  ^{124}Sn: data from Parkar *et al*., PRC **98**, 014601 (2018)

Systematic comparison theory vs. experiment

To compare different systems it is necessary to reduce the cross section.
The reduction procedure eliminates the influence of the Coulomb barrier

The classical function method* (works above V_B)

Based on the improved classical cross section

*Canto et al., PRC **109**, 054609 (2024)

$$\sigma_{\mathrm{F}}\,(E > V_{\mathrm{B}}) \simeq \pi \overline{R}^{2} \left[1 - \frac{V_{\mathrm{B}}}{E}\right] \; ;$$

$\overline{R} \to$ average barrier radius

$$\overline{R}^{2} \simeq R_{\mathrm{B}}^{2}\,\left[1 - 0.14\,y - 0.14\,y^{2}\right]$$

Reduction procedure:

$$E \;\to\; y = 1 - \frac{V_{\mathrm{B}}}{E}; \qquad \sigma_{\mathrm{F}} \;\to\; G(y) = \frac{\sigma_{\mathrm{F}}}{\pi \overline{R}^{2}}$$

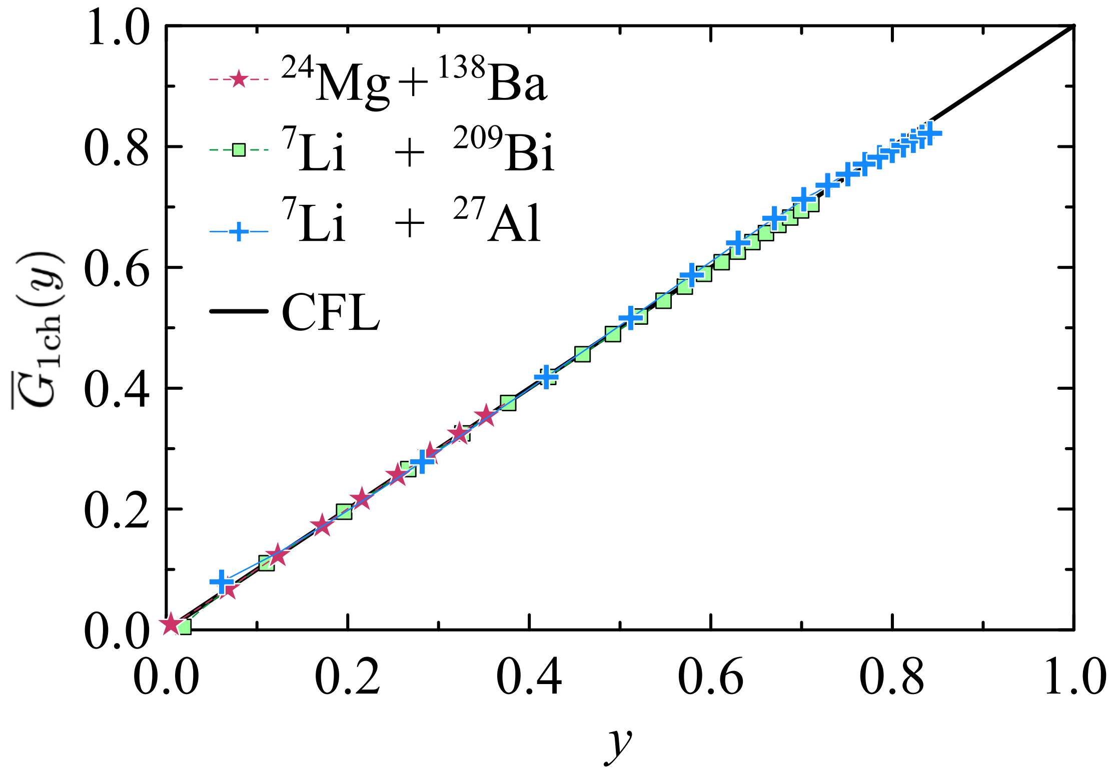
In the absense of coupling effects, one gets the universal function:

$$G_0(y) = y \qquad \text{(classical fusion line – CFL)}$$

Deviation from the CFL measure the influence of breakup

Checking the reduction method

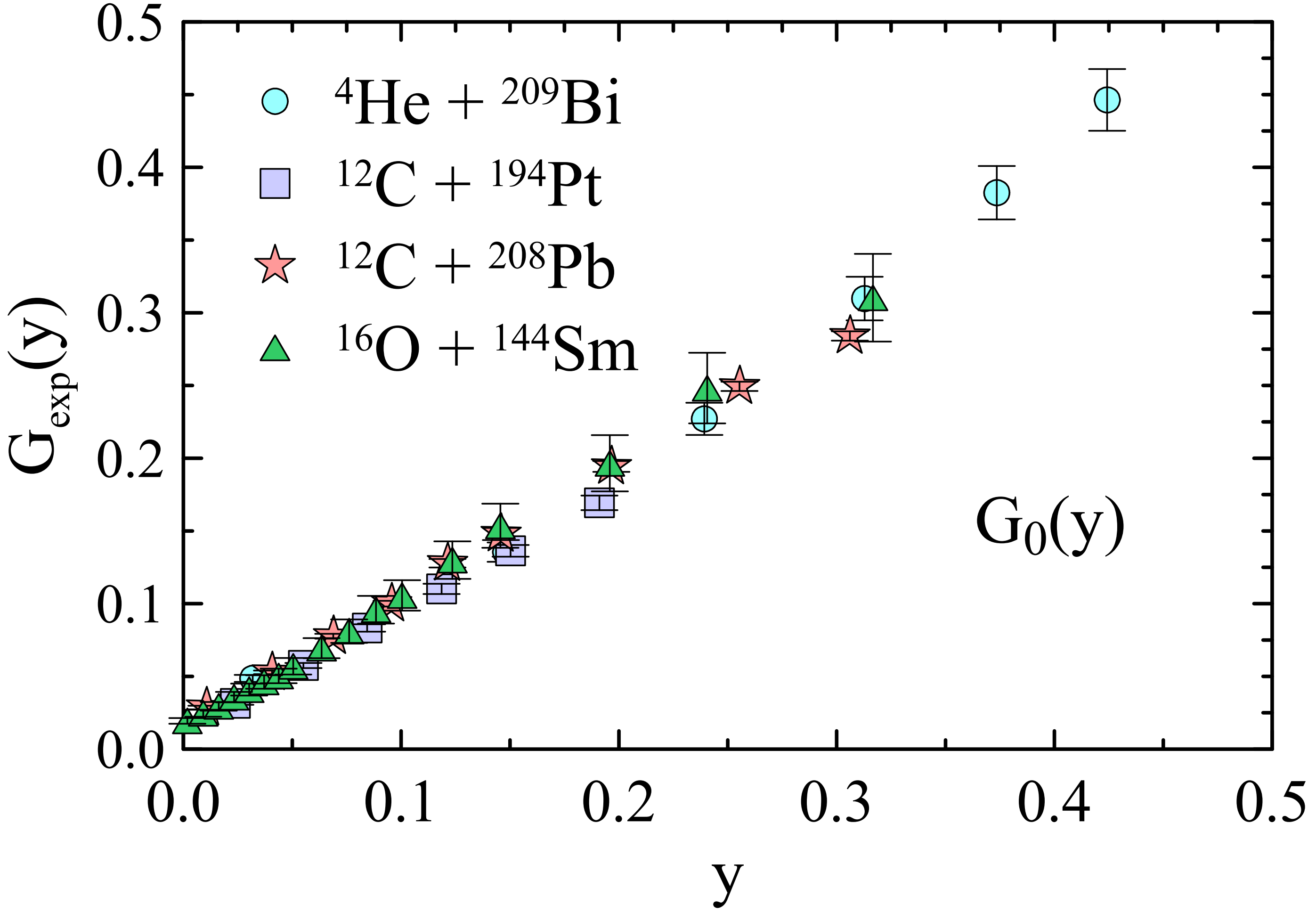
Apply it to σ_F of one-channel calculations with the SPP and short-rangel absorption, for several systems



Reduced cross sections close to the Classical Fusion Line

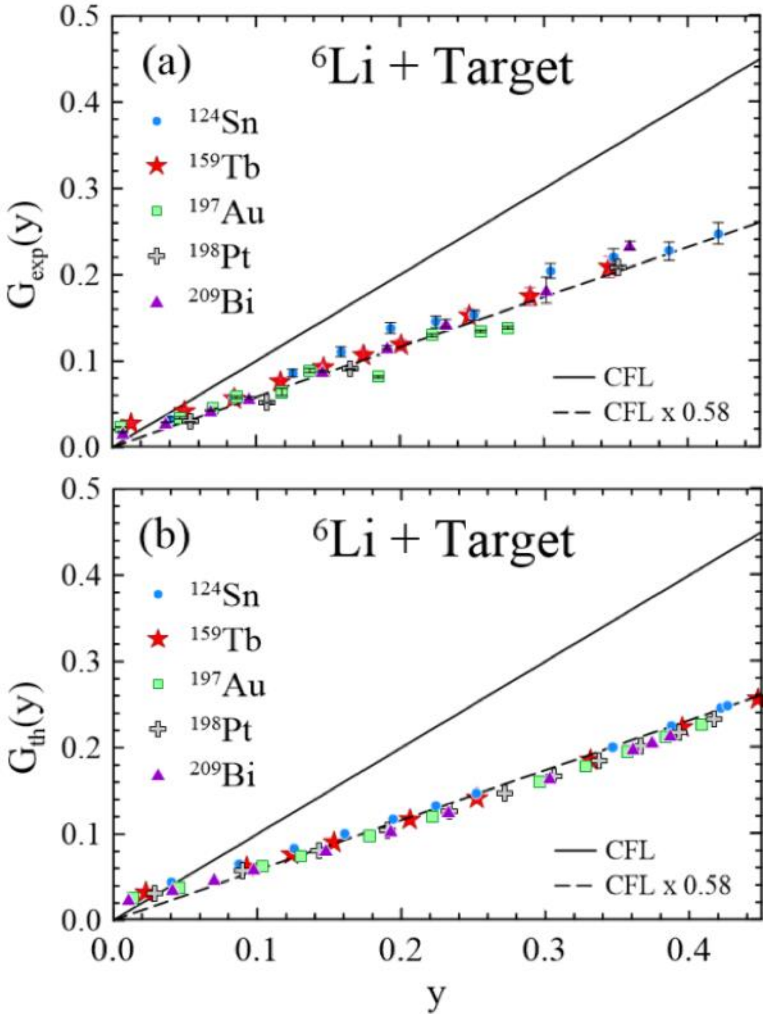
Does it work for experimental data of tightly bound systems

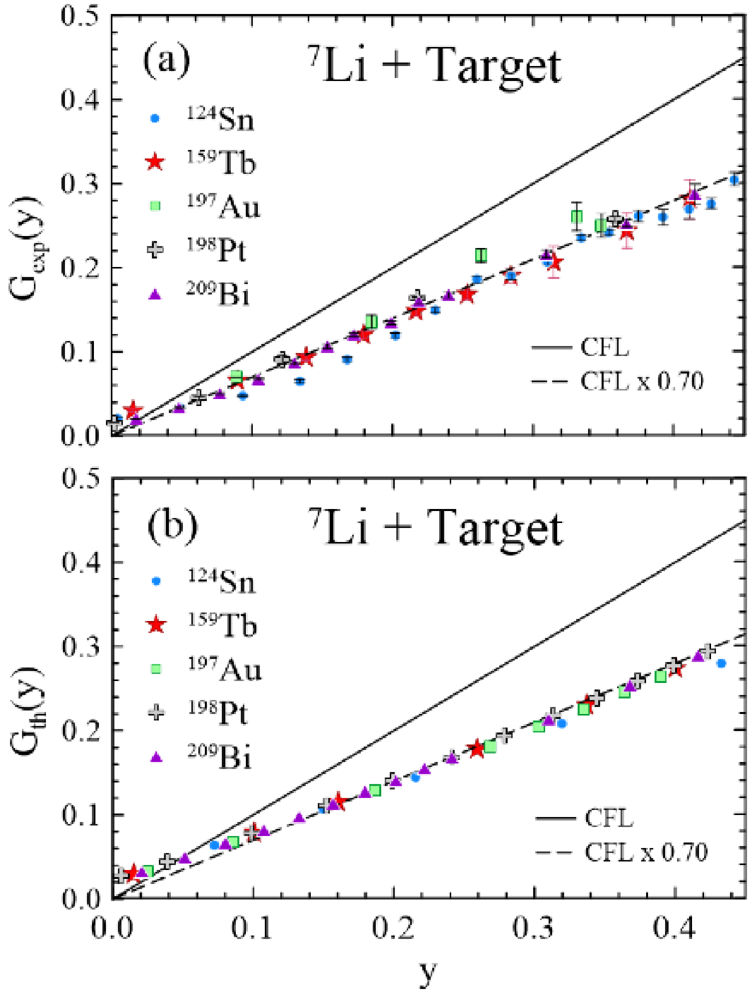
Yes, it does



What about σ_{CF} of weakly bound systems?

$G_{exp}(y)$ x $G_{th}(y)$ for ^{6,7}Li on several targets





- Excellent agreement theory - experiment
- Suppression increases as breakup threshold (B) decreases:
  ^{6}Li (B = 1.46 MeV) = 42%
  ^{7}Li(B = 2.47 MeV) = 30%
- Suppression is independent of target (Z_T varies from 50 to 83)
- Indicates that the suppression is dominated by nuclear couplings

Conclusions

- CDCC-based theory to evaluate CF and ICF cross sections. Calculations for several systems used SPP and short-range absorption. The only free parameter is the spectroscopic amplitude, with the same value for all targets

- The method describes the data quite well. Both theory and experiment find suppressions of 43% for ^{6}Li and 30% for ^{7}Li

- The suppression is independent of the target. Results for $Z_T = 50$ are qualitatively the same as for $Z_T = 83$

- Nuclear breakup seems to be the dominant reaction mechanism in CF suppression

Future plans

- Include effects of target excitation (important to deformed targets for $E < V_B$)
- Extend the method to projectiles that breaks up into 3 fragments (4-body CDCC)
- Study collisions with lighter targets (scant data available)