





Correlations in multinucleon transfer reactions.

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Outline

Nuclear spectroscopy via transfer reactions between heavy ions ➤ The (¹⁸O,¹⁶O) reaction (two neutron transfer correlations)

> CRC, two-step CCBA and DWBA calculation

Experimental results about ${}^{12,13}C({}^{18}O,{}^{16}O){}^{14,15}C,$ ${}^{16}O({}^{18}O,{}^{16}O){}^{18}O, {}^{64}Ni({}^{18}O,{}^{16}O){}^{66}Ni$ and ${}^{28}Si({}^{18}O,{}^{16}O){}^{30}Si$ reactions @ 84 MeV incident energy.

> Correlations in 2p transfer react.: ⁴⁰Ca(¹⁸O,²⁰Ne)³⁸Ar

Correlations in np transfer react.: ¹²C(⁶Li,⁴He)¹⁴N ¹⁷F(⁶Li,⁴He)²¹Ne

Outline

³H and ³He cluster transfer in the ⁶Li + ⁸⁹Y reaction

Alpha clusters in nuclei. Transfer reactions. Fusion cross section

Conclusions and perspectives

Brief introduction. (t,p) reactions

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SPECTROSCOPY OF ¹⁶C*

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The ¹⁴C(t, p)¹⁶C reaction locates five new states in ¹⁶C, at excitation energies of 3020 ± 15 , 3983 ± 10 , 4038 ± 10 , 4136 ± 10 and 6109 ± 15 keV, in addition to the g.s. and 1.76 MeV states. The 3.02 and 3.98 MeV states appear to be the second 0⁺ and 2⁺ 2p-2h states, respectively. The 4.14 MeV state has $J^{\pi} = 4^+$ and the 6.11 MeV state has $J^{\pi} = 2^+$, 3^- , or 4^+ .

E _x (Ι ^π)	Ν
0.0 (0+)	300
1.766 (2+)	400
3.020 (0+)	300
3.983 (2+)	400
4.136 (4+)	360



Main reasons for these discrepancies:

- the use of oversimplified triton wave functions
- the use of the zero-range approximation
- the use of only simultaneous transfer.
- Numerical simplifications to solve six-dimension integrals to determine transition amplitudes???

The production of triton beam was prohibited for safety reasons!

Exact finite range CRC and Two-step CCBA or DWBA calculations

- Prior representation
- Complex remnant
- Non-orthogonality corrections

- São Paulo Potential (SPP) used as optical potentials L.C. Chamon. et al., PRL 79 (1997) 5218
- Wood-Saxon form factors were used to generate single-particle and cluster wave functions. Depth were varied to fit the exp. separation energies
- > **Deformation parameters** for collective excitations from systematics
- Spectroscopic Amplitudes by shell-model, IBM2, IBFM, QRPA, semimicroscopic algebraic cluster model.

The CRC equations are in many cases of the form

$$\begin{aligned} \left[E_{\kappa pt} - T_{\kappa L}(R_{\kappa}) - U_{\kappa}(R_{\kappa})\right] f_{\alpha}(R_{\kappa}) &= \sum_{\alpha', \Gamma > 0} i^{L'-L} V_{\alpha:\alpha'}^{\Gamma}(R_{\kappa'}) f_{\alpha'}(R_{\kappa'}) \\ &+ \sum_{\alpha', \kappa' \neq \kappa} i^{L'-L} \int_{0}^{R_{m}} V_{\alpha:\alpha'}((R_{\kappa}), R_{\kappa'}) f_{\alpha'}(R_{\kappa'}) dR_{\kappa'} \end{aligned}$$

Single nucleon states are given by

$$\phi_{JM}(\xi_c, \mathbf{r}) = \sum_{\ell j I} A_{\ell s j}^{jIJ} \left[\phi_I(\xi_c) \varphi_{\ell s j}(\mathbf{r}) \right]_{JM}$$
$$= \sum_{\ell j I, m \mu m_s m_\ell} A_{\ell s j}^{jIJ} \langle j m I \mu | JM \rangle \phi_{I\mu}(\xi_c) \langle \ell m_\ell s m_s | j m \rangle Y_\ell^{m_\ell}(\hat{\mathbf{r}}) \phi_s^{m_s} \frac{1}{r} u_{\ell s j I}(r)$$

and are the solution of

$$[T_{\ell}(r) + V(r) + \epsilon_I - E] u_{\ell s j I}(r) + \sum_{\ell' j' I', \Gamma > 0} V_{\ell s j I : \ell' s j' I'}^{\Gamma}(r) u_{\ell' s j' I'}(r) = 0$$

Independent coordinate model

$$\varphi_{12}(\mathbf{r}_{1}, \mathbf{r}_{2}) = \sum_{i} c_{i} |(\ell_{1}(i), s_{1})j_{1}(i), (\ell_{2}(i), s_{2})j_{2}(i); J_{12}T\rangle \rightarrow \sum_{u} c_{i} \sum_{L \ell S j} |L, (\ell, (s_{1}s_{2})S)j; J_{12}T\rangle \phi_{L(\ell S)j}^{J_{12}T, i}(r, \rho)$$

$$\phi_{L(\ell S)j}^{J_{12}T,i}(r,\rho) = \langle L, (\ell, (s_1s_2)S)j; J_{12}T | (\ell_1(i), s_1)j_1(i), (\ell_2(i), s_2)j_2(i); J_{12}T \rangle \\
\times \langle [Y_L(\hat{\mathbf{r}})Y_\ell(\hat{\rho})]_\lambda | [\varphi_{\ell_1s_1j_1}(\mathbf{r}_1)\varphi_{\ell_2s_2j_2}(\mathbf{r}_2)]_{J_{12}T} \rangle$$

and the radial integral overlaps are derived from using Moshinsky harmonic oscillator expansion



The São Paulo potential (SPP)

$$V_F = \int \rho_1(\mathbf{r}_1) \mathcal{V}(\mathbf{R} - \mathbf{r}_1 + \mathbf{r}_2) \rho_2(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2$$

 $\mathcal{V}(\mathbf{R} - \mathbf{r}_1 + \mathbf{r}_2)$ is the known nucleon-nucleon M3Y interaction

 $V_{\rm LE}^{\rm SPP}(\mathbf{\bar{R}}, E) = V_{\rm F}(\mathbf{\bar{R}})e^{4v^2/c^2}$

SPP systematics: Two parameters fermi-Dirac distribution

$$\rho(r) = \frac{\rho_0}{1 + exp\left(\frac{r - R_0}{a}\right)} \qquad \qquad R_0 = (1.31A^{1/3} - 0.84) \text{ fm}$$
$$a = 0.56 \text{ fm}$$

Optical potentials:

 $U=(1+0.78i)V^{SPP}$ in DWBA and CRC or CCBA in intermediate and final partitions $U=(1+0.60i)V^{SPP}$ in CRC and CCBA initial partitions (D. Pereira et al. PLB 670, 330 (2009)

Does the SPP systematic always work?

- > SPP systematic mostly works, but not always
- ▶ a = 0.61 fm, for ¹⁸O
- > a = 0.62 fm, for ¹⁷O
- \succ ^AAr, ^AK ?



See more details in V. Singh et al. PRC 104, L041601 (2021) BUT: SPP systematic is always the starting point!! Especially for nuclei around the stability line

Nuclear spectroscopy via (180,160) reaction

The (180,160) reactions are good candidates to show the role of correlations thanks to

The presence of a correlated pair of neutrons in the ¹⁸O_{g.s.} wave function

The very low polarizability of the ¹⁶O core

¹⁴C is a good benchmark for considerations on the reaction mechanism, ⁶⁴Ni and ²⁸Si are good benchmark for studying collective vs two particle correlations

Comparison with (t,p) results upon availability

Studies on both ¹³C(¹⁸O,¹⁷O)¹⁴C **1n transfer** and ¹²C(¹⁸O,¹⁶O)¹⁴C **2n transfer**



M. Cavallaro et al., PRC 88 (2013) 054601

Theoretical results for ¹²C(¹⁸O,¹⁶O)¹⁴C



ZBM model space (valence orbits: $1p_{1/2}$, $2s_{1/2}$, and $1d_{5/2}$)

Theoretical results for ¹²C(¹⁸O,¹⁶O)¹⁴C



Presence of two-neutron correlations in ¹⁴C state, especially the g.s.

No arbitrary scaling

S.A. from shell-model calculations using the ZBM model space M. Cavallaro et al., PRC 88 (2013) 054601

What happens if we add a neutron to the ¹⁴C system?

Study of the ¹³C(¹⁸O,¹⁶O)¹⁵C reaction at 84 MeV incident energy

D. Carbone et al., PRC 95, 034603 (2017)

¹⁵C energy spectrum



CRC and CCBA calculations



Extreme cluster model

- Relative motion of the 2n frozen and separated by the c.m.
- Only the term with the 2n coupled to S = 0 participates to the transfer
- S.A. = 1 for all configurations

Independent coordinate model

The transfer is described taking into account spectroscopic information obtained by shell model calculations



Sequential transfer (DWBA)

Introducing the ¹⁷O + ¹⁴C intermediate partition



CRC and DWBA calculations



Extreme cluster model overestimate the cross section (S.A. = 1) **Independent coordinate** model describes quite well the cross section **Sequential transfer (CCBA)** underestimate the cross section

S.A. from shell-model calculations using the ZBM model space

More recent results









Microscopic results: g.s.: IC results are better, specially in the bell-shaped region. Same order: one and two step. 2⁺ : Collective correl. dominates over pairing

J. Lubian MNC in N&S, 2-6 Jun, SP 2025

Results of theoretical calculations



Microscopic results: g.s.: IC results are better, specially in the bell-shaped region. 2⁺ : Collective correlations dominate over the pairing



B. Paes et al PRC 96.044612 (2017)

IBM2 for ^{64,66}Ni and IBFM for ⁶⁵Ni

Study of the ¹⁸O(²⁸Si,³⁰Si)¹⁶O reaction at 84 MeV incident energy

Мо	del space (⁴ He	core)	val	ence orbital	s (similar to	o Ni)
Pro	tons		1p ₃	_{/2} , 1p _{1/2} , 1d ₅	_{/2} , 2s _{1/2} , 1d ₃	3/2
neu	itrons		1p ₃	_{/2} , 1p _{1/2} , 1d ₅	_{/2} , 2s _{1/2} , 1d ₃	/2
(a)	Ε(MeV); J ^π	Ε (MeV); J ^π	(b)	$E(MeV); J^{\pi}$	E(MeV); J [*] 3.843 (5/2 ⁻)	E(MeV); J ^π
Projectile Overlaps	$ \begin{array}{c} 1.982 (2^{+}) \\ \hline 0.0 (0^{+}) \\ \hline \end{array} \begin{array}{c} 180 \end{array} $	6.130 (3 ⁻) 0.0 (0 ⁺) ¹⁶ O	Projectile Overlaps	1.982 (2 ⁺) 0.0 (0 ⁺)	3.055 (1/2 ⁻) 0.871 (1/2 ⁺) 0.0 (5/2 ⁺) ¹⁷ O	6.130 (3 ⁻) 0.0 (0 ⁺) ¹⁶ O
Target Overlaps	4.617 (4 ⁺) 1.779 (2 ⁺) 0.0 (0 ⁺) ²⁸ Si	3.498 (2*) 2.235 (2*) 0.0 (0*) ³⁰ Si	Target Overlaps	4.617 (4*) 1.779 (2*) 0.0 (0*) ²⁹ Si	3.067 (5/2 ⁺) 2.425 (3/2 ⁺) 2.028 (5/2 ⁺) 1.273 (3/2 ⁺) 0.0 (1/2 ⁺) ²⁹ Si	3.498 (2 ⁺) 2.235 (2 ⁺) 0.0 (0 ⁺) ³⁰ Si

Results of theoretical calculations



Microscopic results: g.s.: Two-step DWBA results are better. Same order: one and two step. 2⁺ : Collective correlations dominate over the two-particle Si*: the same results as the 2⁺ state

Nucleus	B(E2);
	$0^+ \to 2^+ \ (e^2 b^2)$
$^{14}\mathrm{C}$	0.0018
180	0.0045
^{28}Mg	0.035
³⁰ Si	0.022
⁶⁶ Ni	0.060
⁷⁶ Ge	0.270

Small for ¹⁴C ¹⁸C Big for ²⁸Mg ³⁰Si ⁶⁶Ni ⁷⁶Ge

Results of theoretical calculations



Does our theoretical calculations describe other observables?

- Elastic scattering
- Inelastic scattering

Conclusions and outlooks for (180,160) reactions

^{12,13}C(¹⁸O,¹⁶O)¹⁵C, ¹⁶O(¹⁸O,¹⁶O)¹⁸O, ⁶⁴Ni(¹⁸O,¹⁶O)⁶⁶Ni,
 ²⁸Si(¹⁸O,¹⁶O)³⁰Si, at 84 MeV incident energy
 Four models were used to calculate the cross section:

✓ Extreme cluster

- ✓ Independent coordinate
- ✓ CCBA
- ✓ Microscopic cluster (only for 13 C)

> no need for any "unhappiness" factor to reproduce the absolute cross sections

Two –neutron correlations in ^{14,15}C, ¹⁸O
 In ¹³C importance of a two-neutron correlation in the nuclear wave function, the extra neutron does not destroy the correlations observed in the ¹⁴C case
 Dominance of collective correlations for the excited 2⁺ state of ⁶⁶Ni over the two neutron correlations. The opposite for the g.s.
 Dominance of collective correlations in all states of ³⁰Si.

2p-stripping reaction in the ¹⁸O+⁴⁰Ca collision at 270 MeV

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d-transfer stripping reactions in the ⁶Li+¹²C and ⁶Li+¹⁹F collitions at 20 MeV

Spectroscopic Amplitudes for the target overlap

- Shell Model Calculations NusheLLX
 - \checkmark Model Space: 1p_{1/2}, 1d_{5/2} and 2s_{1/2}
 - Effective Interaction: Rewile

Spectroscopic Amplitudes for the proj. overlap

- Taken from Phys. Rv. C, 8, 1-8, (1973)
- SF = 0.69 for L=0 component
- SF = 0.04 for L=2 component
- Direct two-particle transfer is the main process in both reaction.
- Sequential pn or np mechanism are almost three order of magnitudes lower than the direct one.



J. Zamora et al. 2022, PRC 106, 014603. Data measured at Tandar Lab. at Buenos Aires

d-transfer stripping reactions in the ⁶Li+¹²C and ⁶Li+¹⁹F collitions at 20 MeV



Direct x Sequential mechanisms

J. Zamora et al. 2022, PRC 106, 014603. Data meassured at Tandar Lab. at Buenos Aires

α -transfer reaction (¹²C+¹⁶O) at 20 MeV



Initial State	NLSJ	Final State	SA	SA_SACM
⁸ Be _{gs} (0 ⁺)	3000	¹² C _{g5} (0 ⁺)	0,265	0,294
120 (0+)	3000	160 (0+)	0,194	0,302
	4000	$- O_{gs}(0^{\circ})$	0,095	
160 (0+)	4000	20No (0+)	-0,092	
O ^g (0 ⁻)	5000	$1^{-\circ Ne_{gs}(U^{\circ})}$	-0,146	- 0,104
²⁰ Ne _{gs} (0 ⁺)	4000	248.4 (0+)	-0,103	-
	5000		-0,137	- 0,228



S.A. from semimicroscopic algebraic cluster model. (by P. Hess)







Ratio of cross sections of ⁹²Zr excited to the first excited state (2+)

M.L. Wang, B. Pinheiro et al. 2024 (to be published)

	¦Ĥ) ⁹² Nb				ECM	DMCM
	annel	Outgoing ch	annel	Q value(keV)	Integrated cross section(mb)	Integrated cross section(mb)
an teacharta an teac An teacharta an teac	¹⁰ ⁸⁹ Y	$^{3}\mathrm{H}$	$^{92}\mathrm{Nb}$	-2120.8		
	$1/2^{-}$ 0.0 (g.s.)	$1/2^{+} 0.0 {\rm ~keV}$	7^+ 0.0 keV	T	5.6107	4.3966E-06
• 1790 gate	$9/2^+$ 909		$2^+ \ 136.0$		2.3474	1.4146E-06
\uparrow \downarrow	$3/2^- \ 1507$		$3^+ 285.7$		2.6697	1.0806E-06
Ŷͺϧϔͺϙʹʹͺϙϔͺͺͺͺͺʹϙϙ _{ϙϧ} Ϙ	$5/2^- \ 1745$		$5^+ 357.4$		7.9131	5.1893E-06
12 14 16 18 20 22 24 26			4^+ 480.5		7.4949	3.8354 E-06
roy of tritium (MeV)			$6^+ 501.0$		8.0594	4.5101E-06
igy of unuum (wie v)			$2^+ 1345.5$		5.2533	3.9683E-08
			$9^- \ 2087.5$		7.2290	5.3522 E-07
			$9^+ 2287.2$		4.2783	7.6842E-08

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Ratio of cross sections of ⁹²Nb excited to the first excited state (2+. 501 keV)

Direct versus sequencial (d+n)

⁸⁹ Y(⁶ Li, ³ He) ⁹² Zr.	3H (direct)		d+n (sequential)
	Integrated cross section(mb)		Integrated cross section(mb)
	2.3021E-04		9.3951E-11
	2.3814 E-05		3.4651E-09
	3.8838E-05		1.0888E-09
	2.6415 E-05	>>	4.3316E-11
	3.5430 E-06		3.4245 E-09
	1.1700E-05		3.8676E-07

Projectile overlap



Direct cluster tranfer of ³H dominates

Direct versus sequencial (d+p)

⁸⁹Y(⁶Li,³H)⁹²Nb

3He (direct)	d+p (sequential)
Integrated cross section(mb)	Integrated cross section(mb)
4.3966E-06	1.5926E-05
1.4146E-06	1.8663E-06
1.0806E-06	2.2839 E-07
5.1893E-06	1.7532 E-06
3.8354E-06	2.9726E-06
4.5101E-06	
3.9683E-08	8.4902 E-09
$5.3522 ext{E-07}$	
7.6842 E-08	

Projectile overlap



Direct cluster and sequential transfer are of the same order

Evidence of α – clustering in ⁴⁰Ca by studing the fusion cross section of the ⁴⁰Ca + ⁹⁰Zr



Using the cluster model the fusion cross section can be described very well independently of the Number of states considered

At the same elastic and inelastic Scattering ang. dist. Are very well Described.



F. R. Vasconcelos et al. 2025 (to be published)

Spectroscopic investigation ¹³C+⁹Be @ 55 MeV



⁹Be(¹³C,⁷Li)¹⁵N => 6Li transfer or ⁴He + d or d + ⁴He? Data taken at Tandar, BA, Argentina by V. Guimarães (unpublished)

Summary

- We have shown the relevance on the 2n, 2p, and pn correlations in two-particle transfer reacitons.
- The nucleon correlations are responsable for unusual cluster configurations in some light nuclei.
- Alpha cluster structure may very important in describing the reaciton mechanism of N x alpha nuclei

Working group

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