Connecting electromagnetic observables to astrophysics

FRANCESCA BONAITI, FRIB&ORNL

WORKSHOP ON «MULTINEUTRON CLUSTERS IN NUCLEI AND STARS», SÃO PAULO, BRAZIL

JUNE 3, 2025

Extreme matter in neutron stars

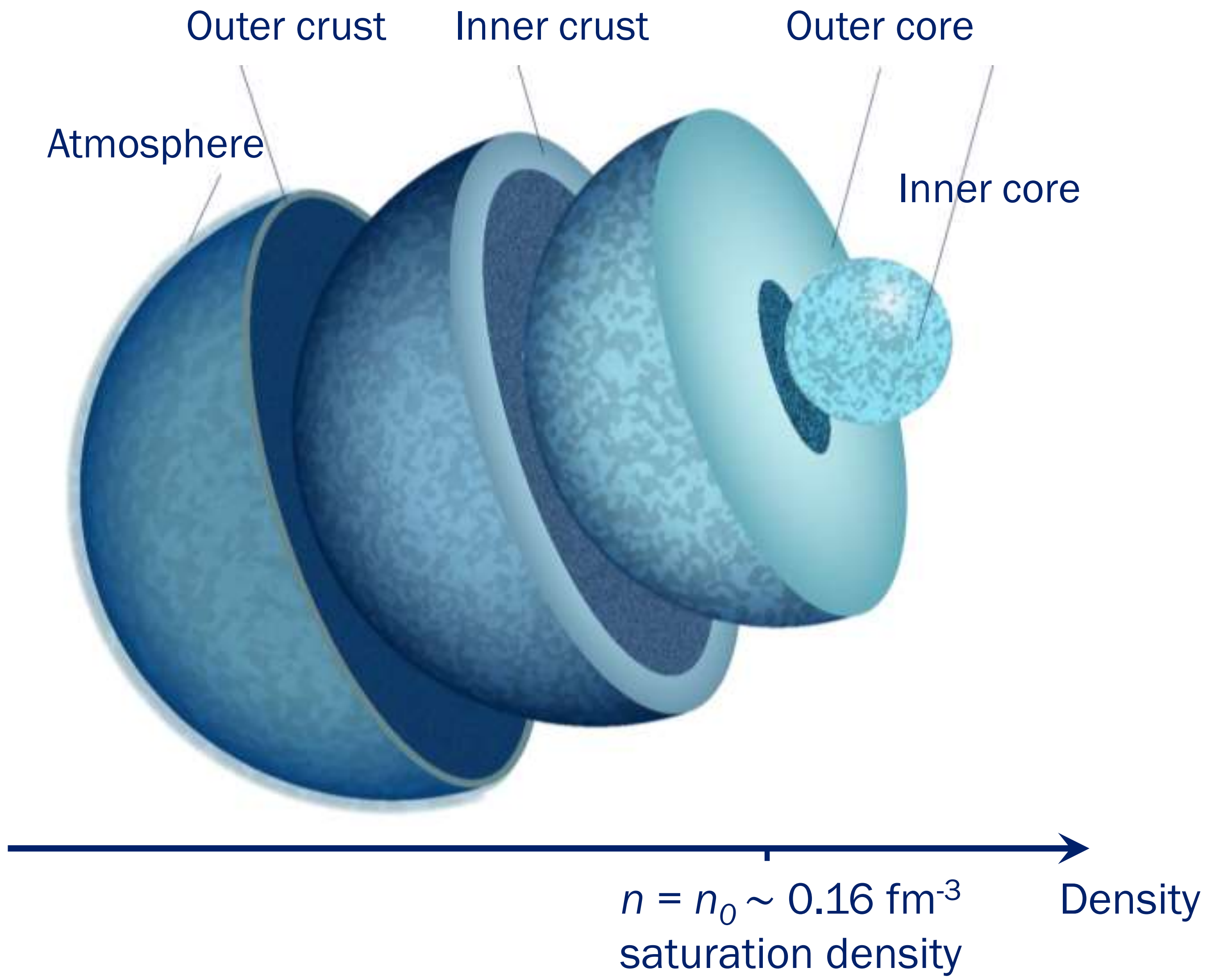


Image credits:
MUSES Collaboration

Nuclear matter equation of state (EOS)

$$\frac{E}{A}(n,\alpha) = \frac{E}{A}(n,0) + S(n)\alpha^2 + \mathcal{O}[\alpha^4]$$

symmetry energy

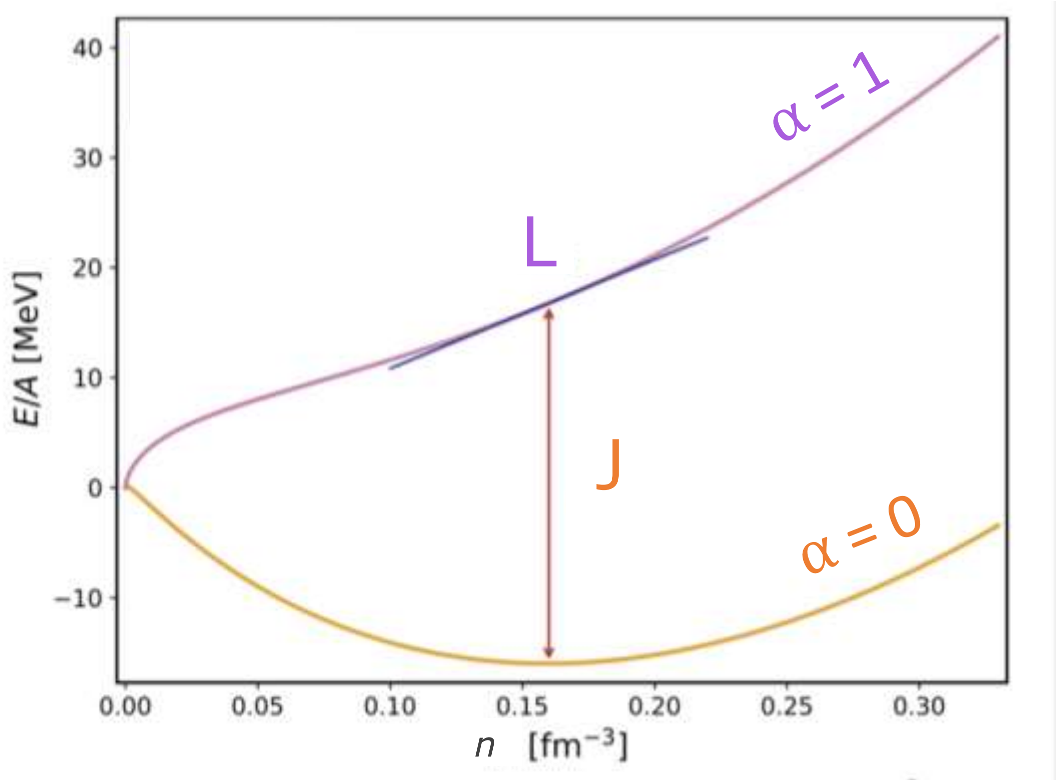
$$n = n_p + n_n$$

$$\alpha = \frac{n_n - n_p}{n}$$

$$S(n) = J + L\frac{n - n_0}{3n_0} + \dots$$

symmetry energy
at saturation density

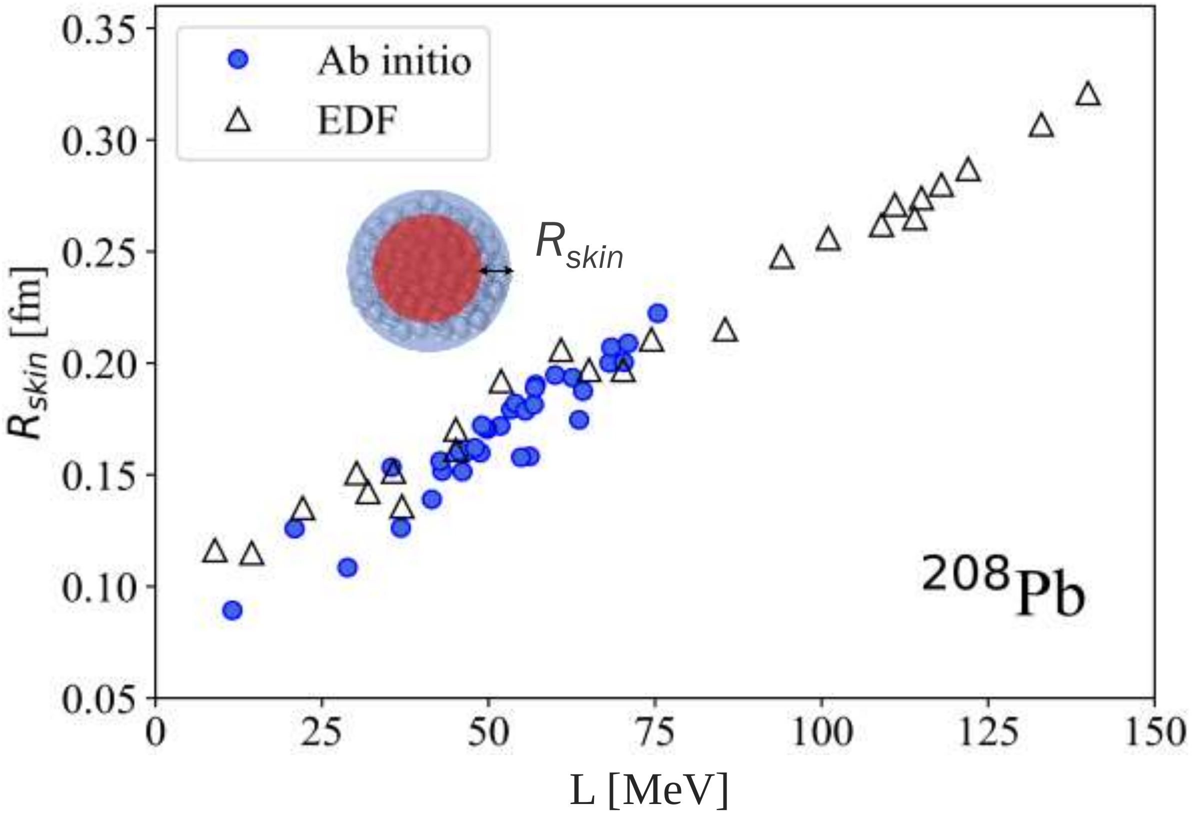
slope parameter,
related to pressure of pure neutron matter
at saturation density



How to constrain the symmetry energy?

Neutron-skin thickness

$$R_{skin} = R_n - R_p$$



Data from M. Centelles et al, PRC 82, 054314 (2010), X. Roca-Maza et al, PRC 88, 024316 (2013),
Hu et al, Nat Phys. 18, 1196–1200 (2022).

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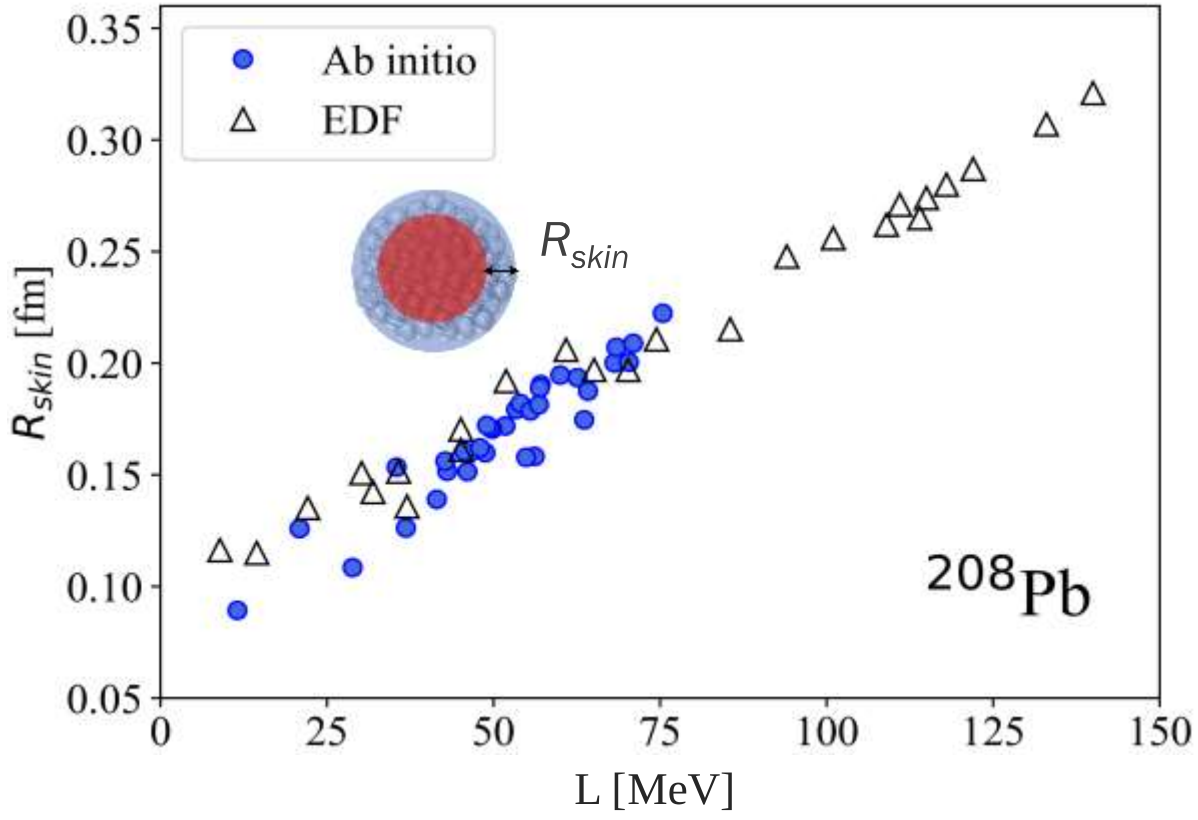
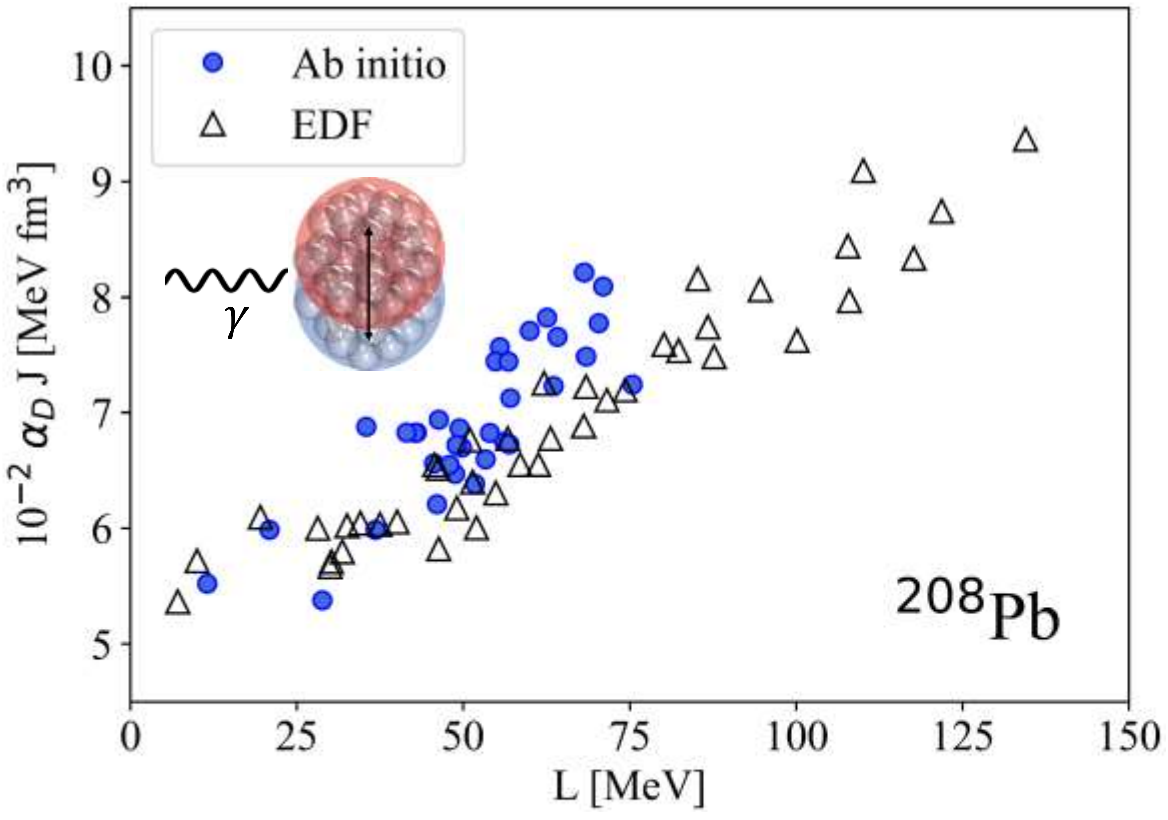
Electric dipole polarizability

$$\alpha_D = 2\alpha \int d\omega \, \frac{R(\omega)}{\omega}$$





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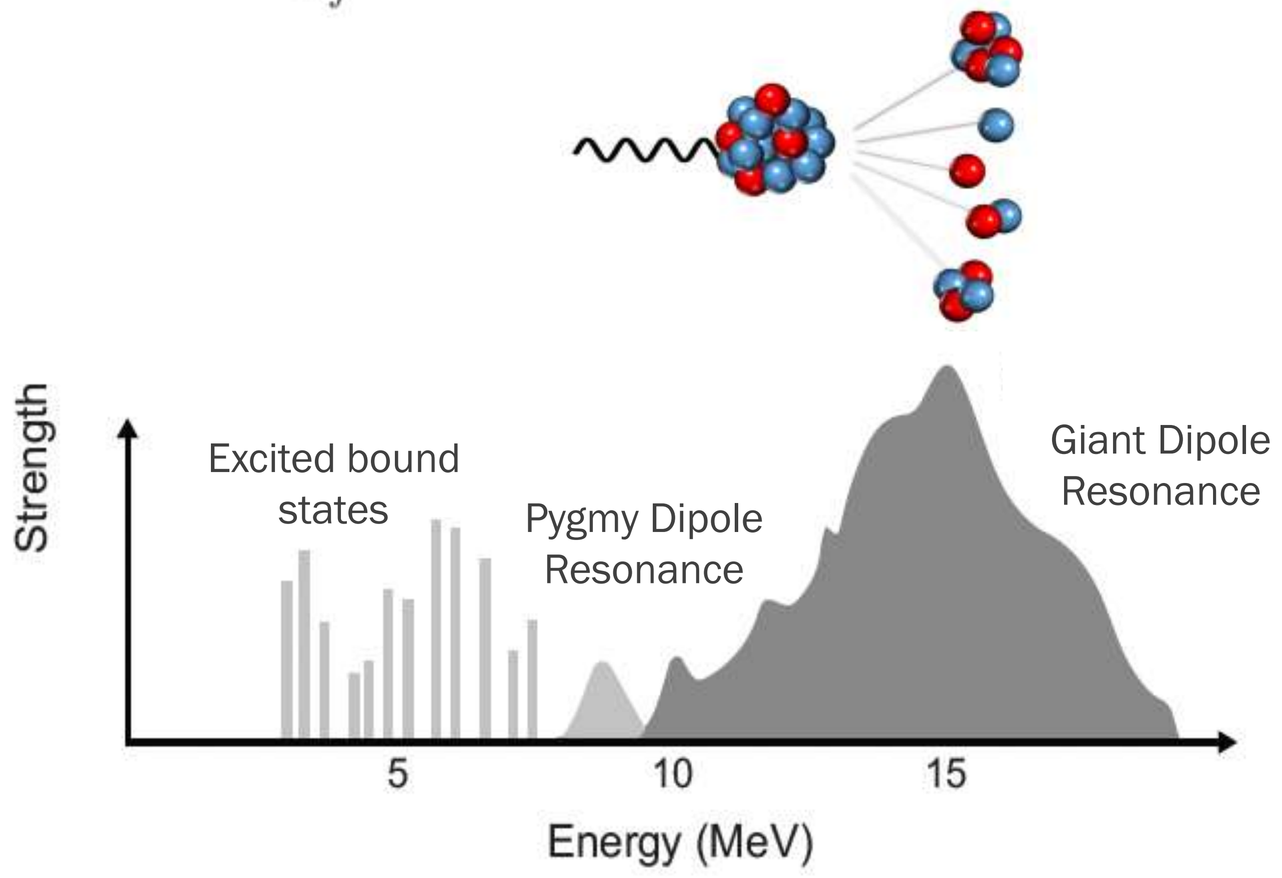
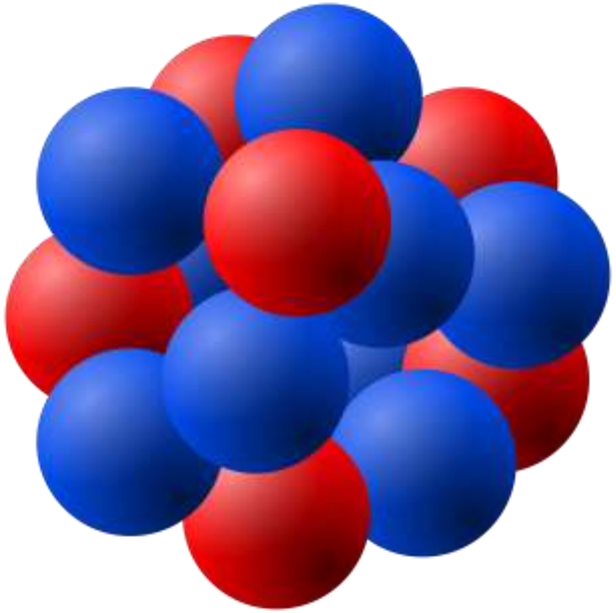
Nuclear response functions

$$R(\omega) = \sum\!\!\!\!\!\!\!\int_f |\langle \Psi_f | \Theta | \Psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$



Ab initio nuclear theory



- Building blocks: protons and neutrons.
- Solve quantum many-body problem

$$H\,|\psi\rangle = E\,|\psi\rangle$$
$$H = T + V_{NN} + V_{3N}$$

with controlled approximations.

- 2 ingredients: nuclear interactions and many-body solver.

Coupled-cluster theory

- Starting point: Hartree-Fock reference state $|\Phi_0\rangle$
- Add correlations via:

$$|\Psi_0\rangle = e^T |\Phi_0\rangle$$

with

$$T = \sum t_i^a a_a^\dagger a_i + \sum t_{ij}^{ab} a_a^\dagger a_b^\dagger a_j a_i + \sum t_{ijk}^{abc} a_a^\dagger a_b^\dagger a_c^\dagger a_k a_j a_i + \ldots$$

G. Hagen, T. Papenbrock, M. Hjorth-Jensen, D. J. Dean, RPP 77, 096302 (2014).

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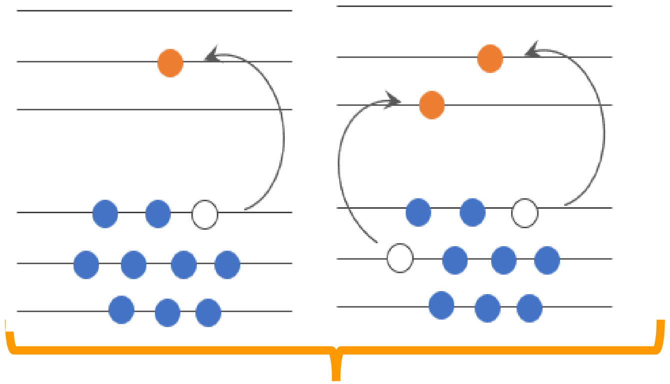
singles and doubles (CCSD)



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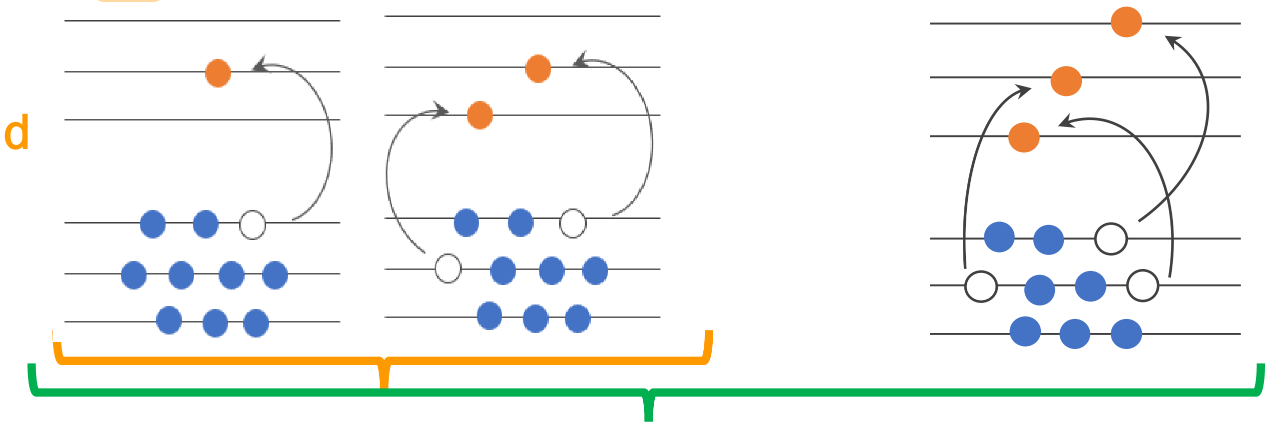

→ coefficients from coupled-cluster equations

+ triples (CCSDT-1)

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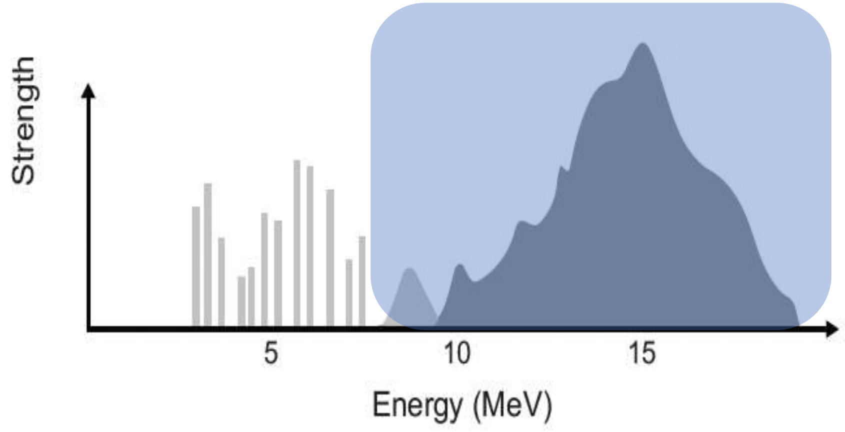
From bound to dipole-excited states

$$R(\omega)$$



Continuum problem

S. Bacca, N. Barnea, G. Hagen, G. Orlandini, T. Papenbrock, PRL **111**, 122502 (2013).

From bound to dipole-excited states

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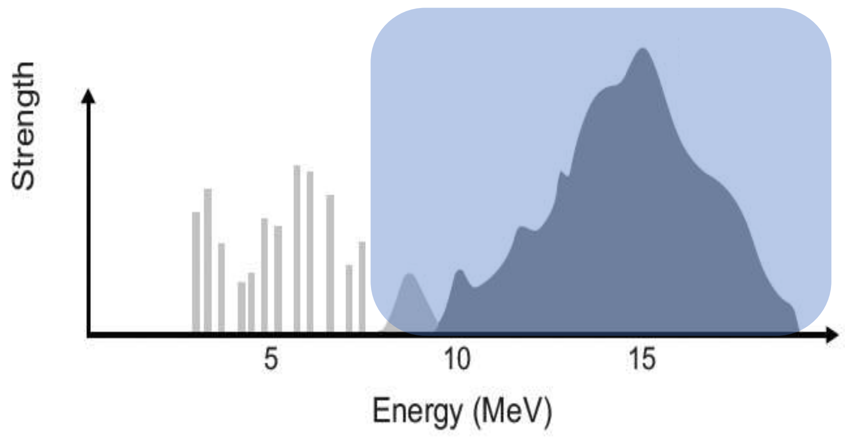
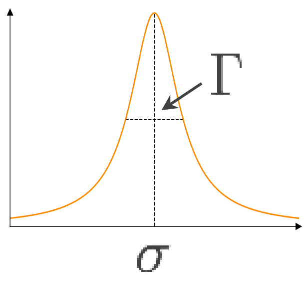
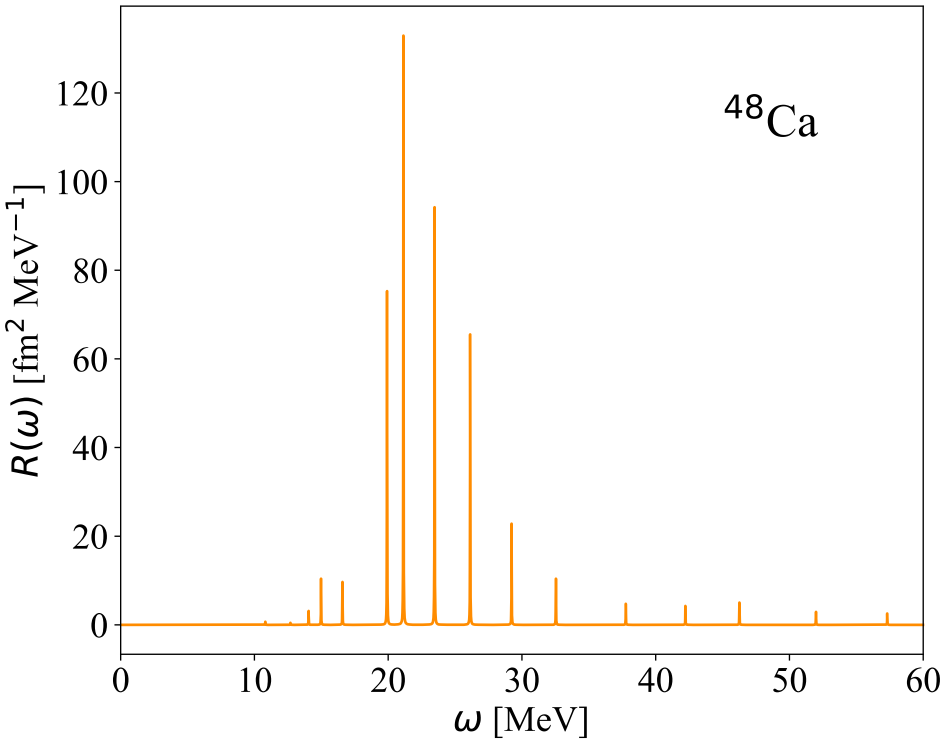




Lorentz Integral Transform (LIT)



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Bound-state like problem

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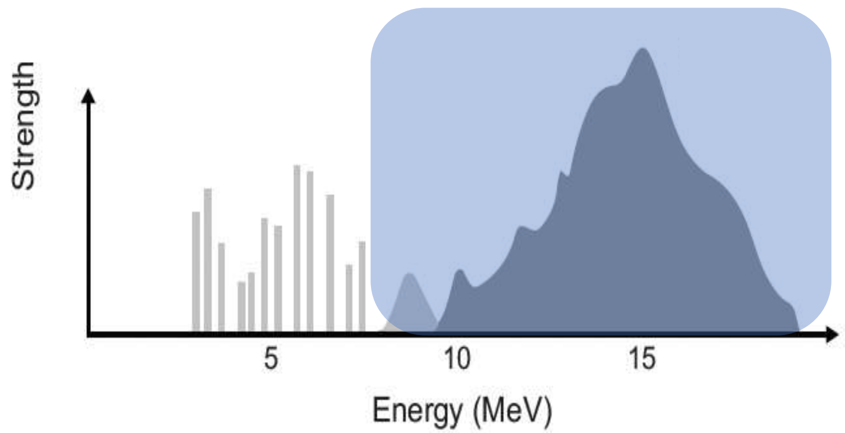
From bound to dipole-excited states

For more details on LIT → see **Miriam El Batchi**'s poster

$R(\omega)$



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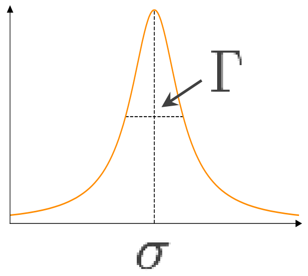

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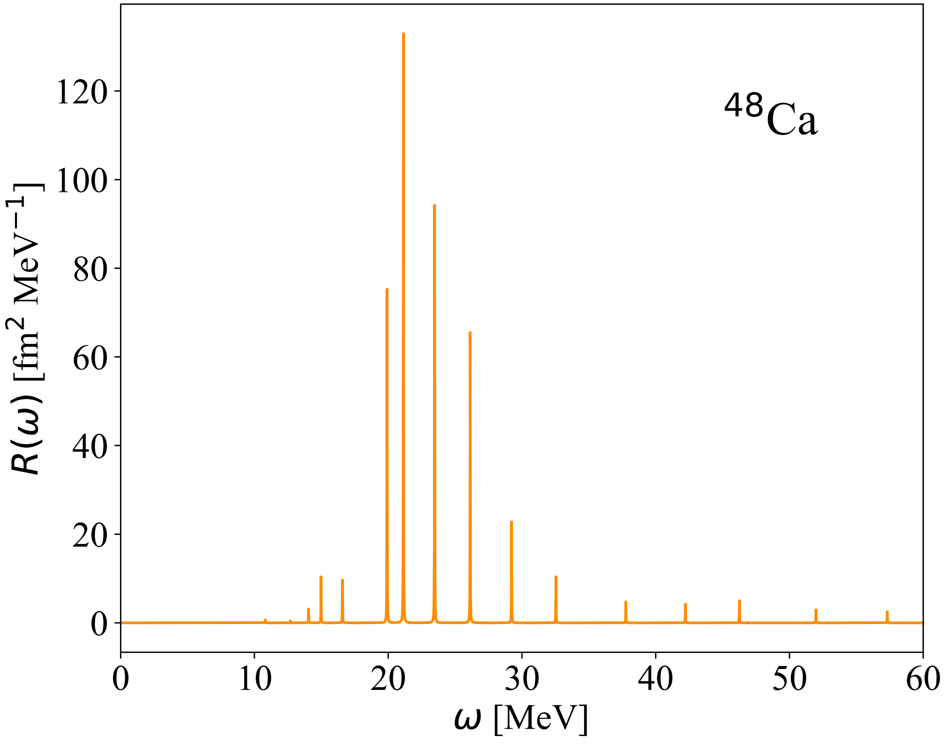

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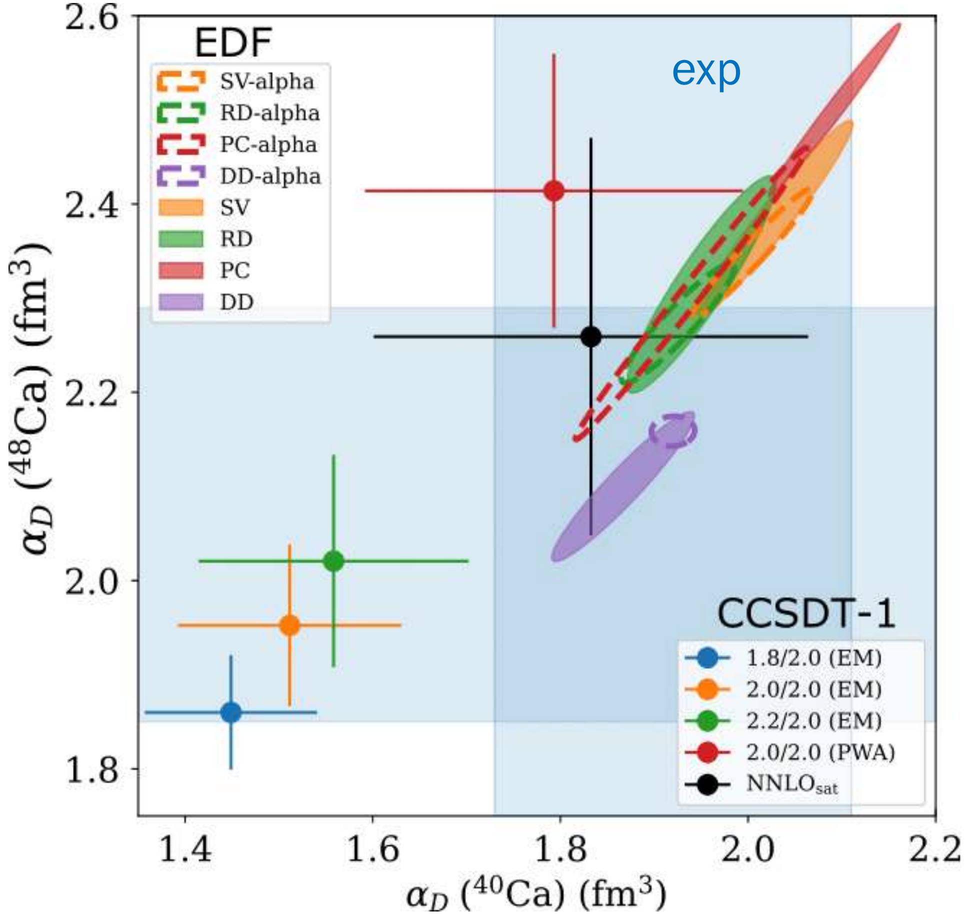
The case of ^{40,48}Ca



R. Fearick, P. von Neumann-Cosel, S. Bacca, **FB** et al, Phys. Rev. Research **5**, L022044 (2023).

Constraints on symmetry energy

$$S(n) = J + L\frac{n - n_0}{3n_0} + \dots$$

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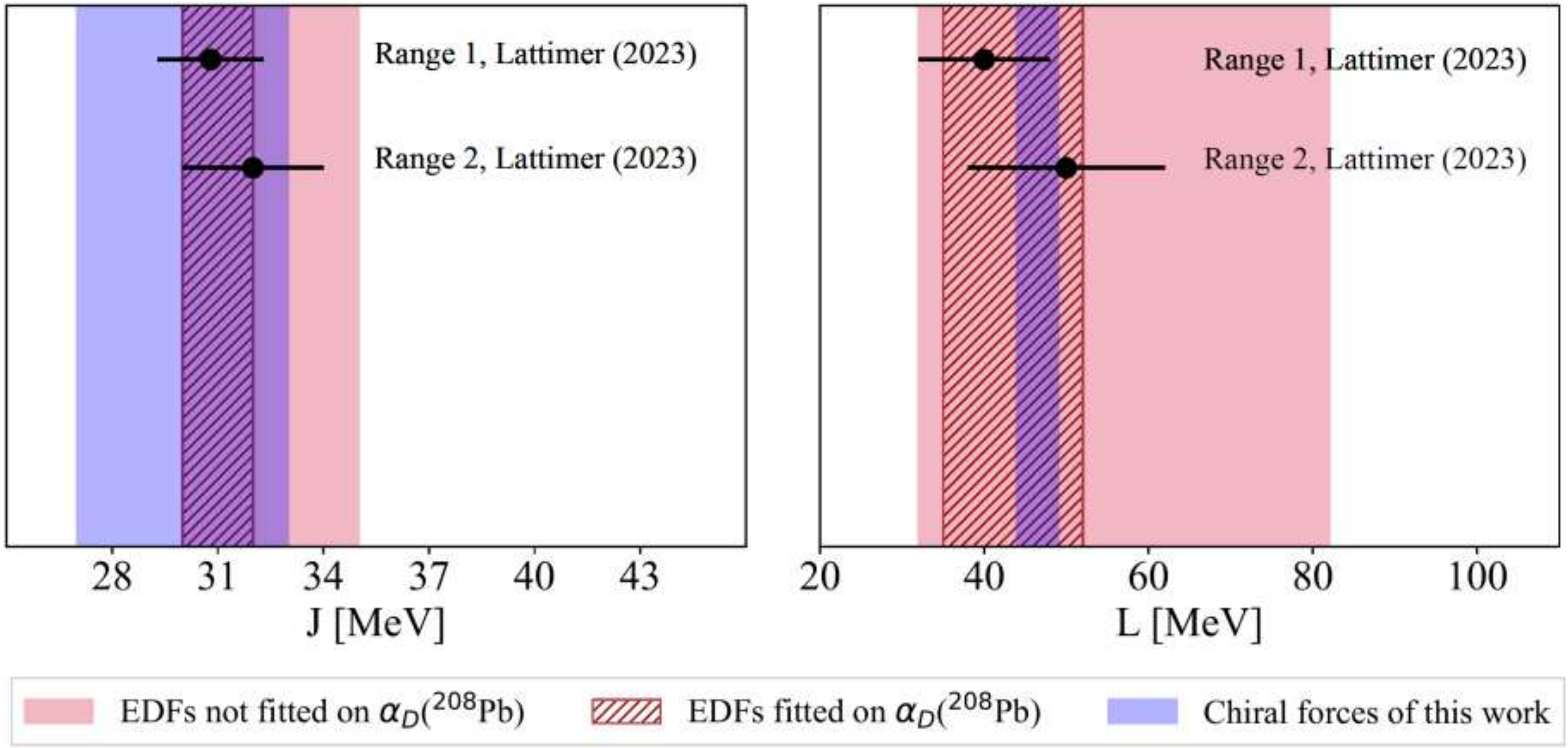

FB, PhD Thesis, JGU Mainz (2024).

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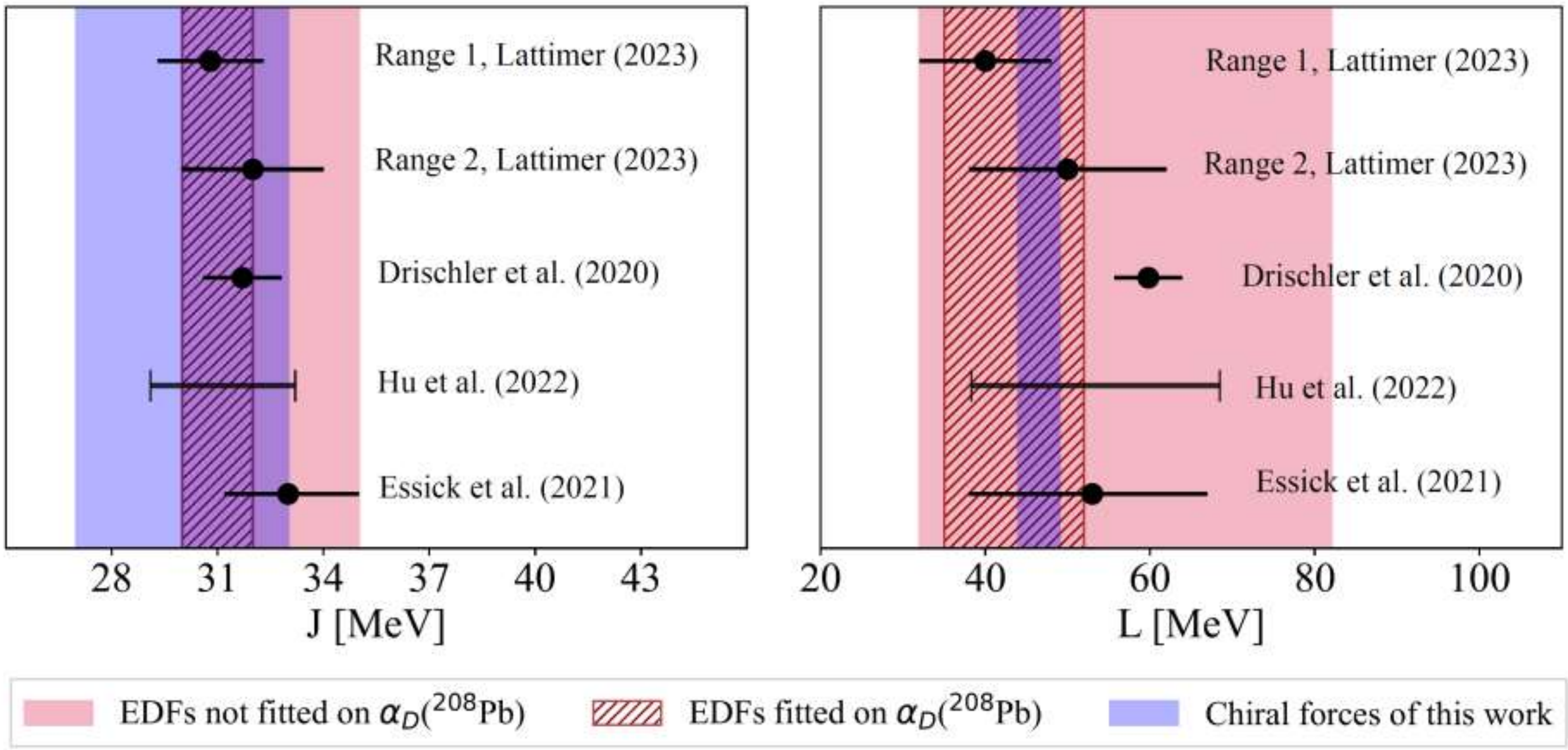

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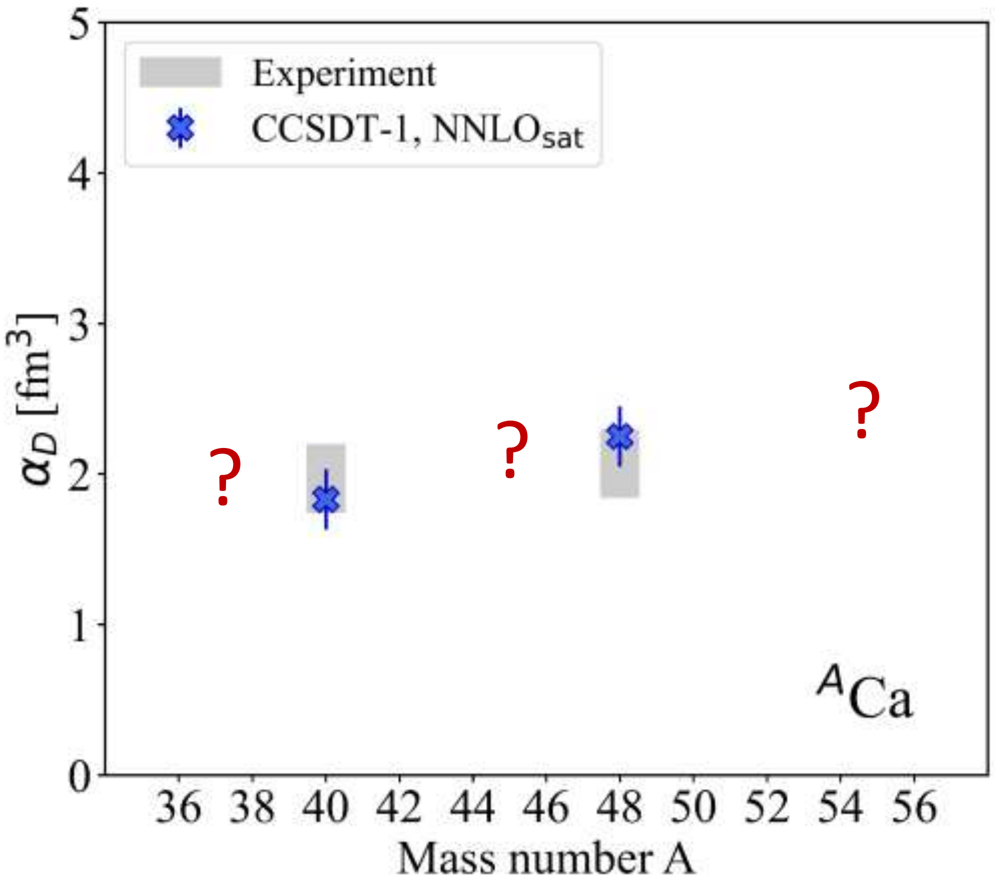
Happy ending for ^{40,48}Ca… but what's next?



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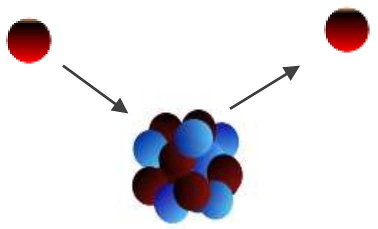
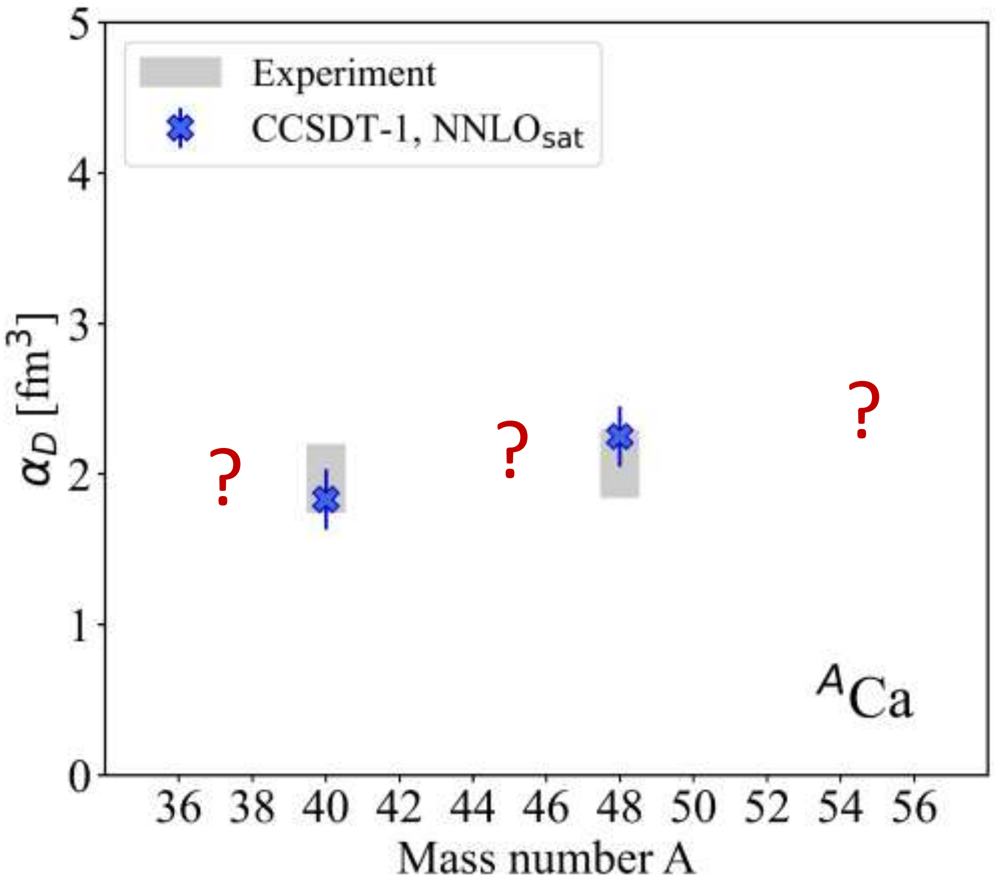


New **(p,p') experiments**
in open-shell Ca, Ni isotopes,
as e.g. ^{42}Ca, ^{58}Ni…



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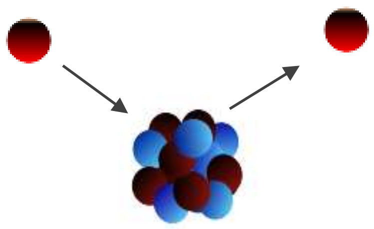

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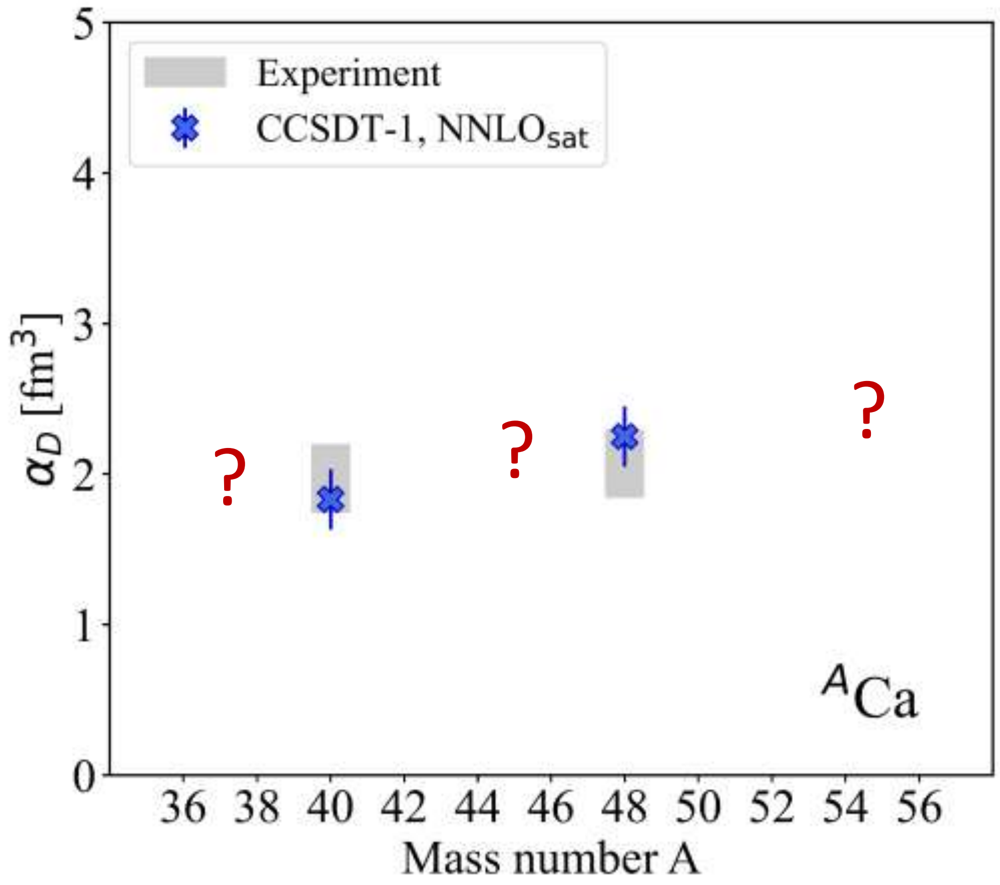
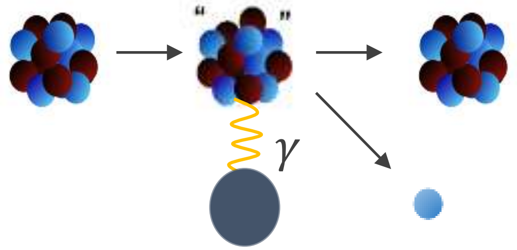
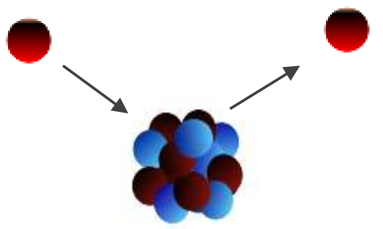




... and with future upgrades, **Coulomb excitation possible** for **very neutron-rich nuclei**.

Happy ending for ^{40,48}Ca… but what's next?



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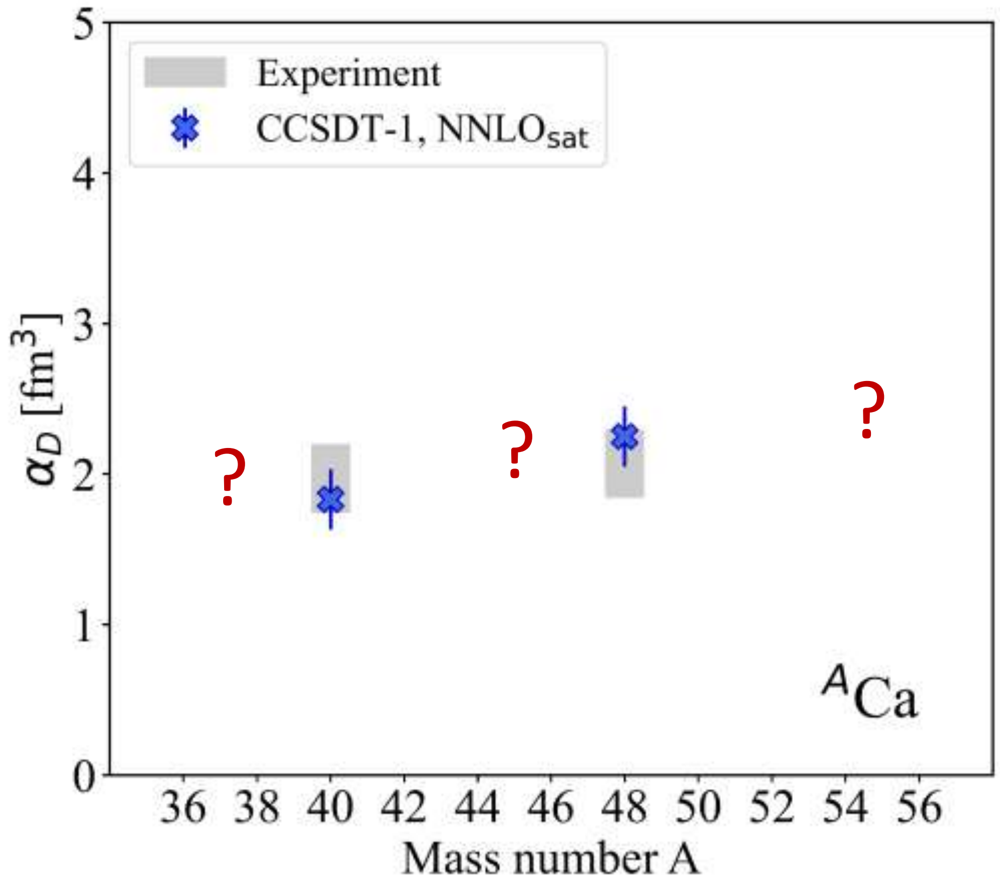
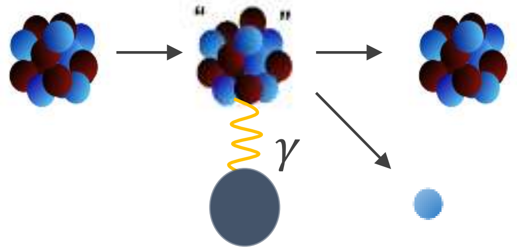




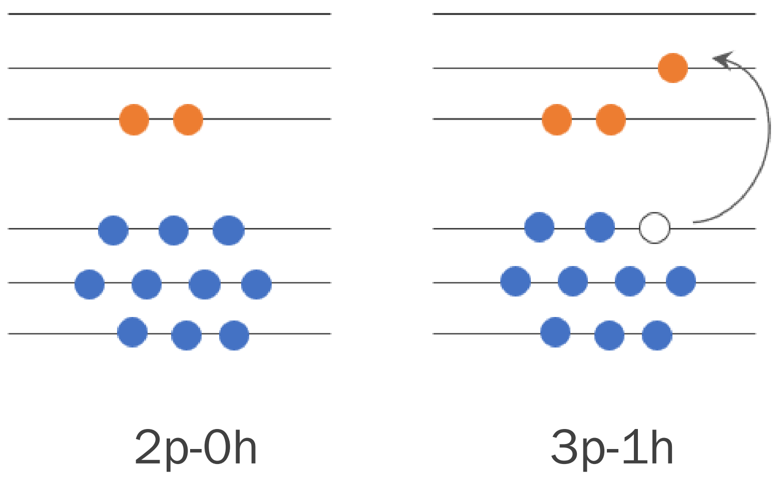
… and with future upgrades, **Coulomb excitation possible** for **very neutron-rich nuclei**.

We need to extend our method beyond closed-shell nuclei!

Open-shell nuclei: two-particle-attached systems (2PA)

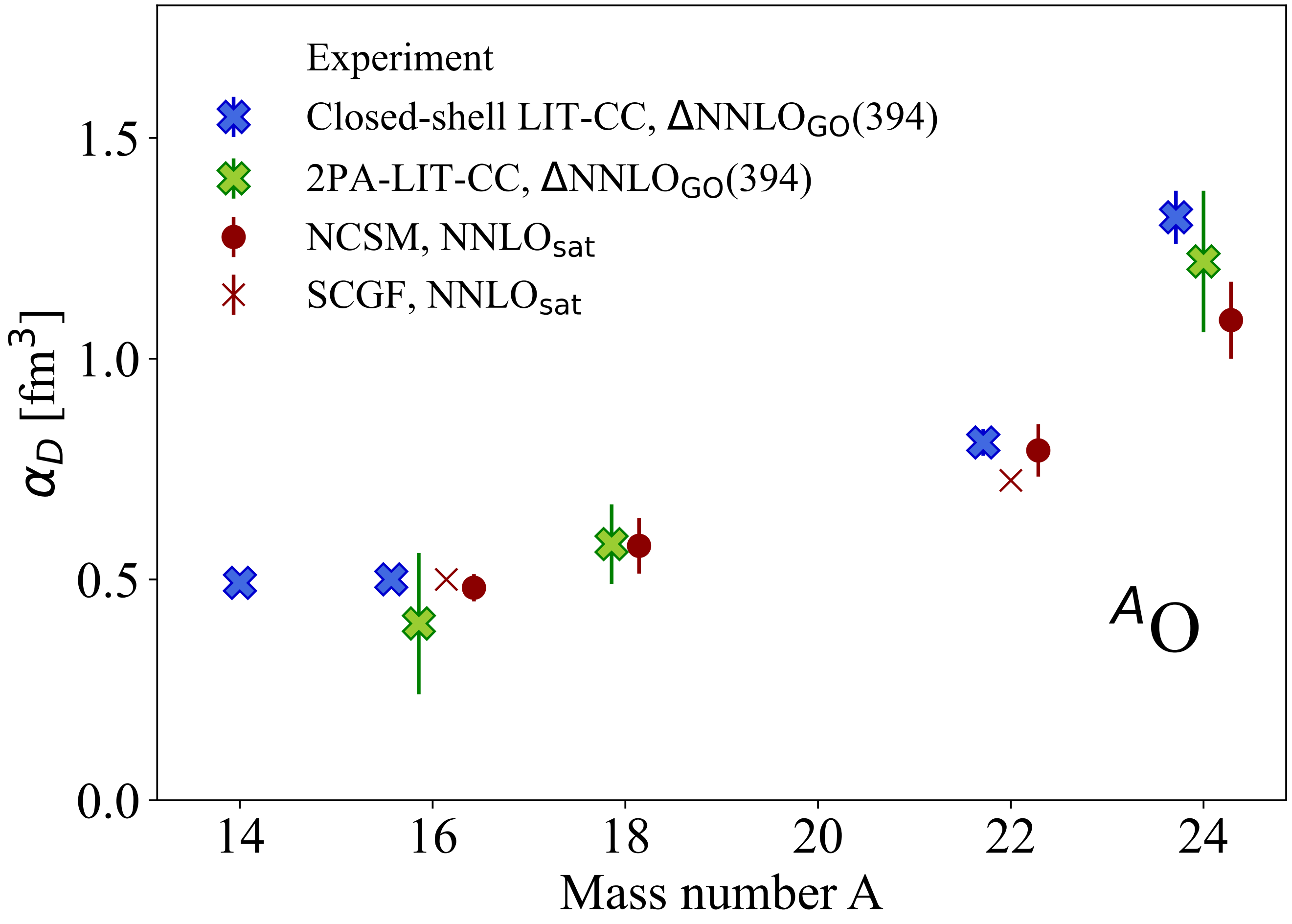
$$\mathcal{R} = \frac{1}{2}\sum r^{ab} a_a^\dagger a_b^\dagger + \frac{1}{6}\sum r_i^{abc} a_a^\dagger a_b^\dagger a_c^\dagger a_i + \ldots$$

$$|\Psi_{\mathrm{2PA}}\rangle = \mathcal{R}|\Psi_{\mathrm{closed-shell}}\rangle$$



FB et al., PRC 110, 044306 (2024).

α_D along the oxygen chain



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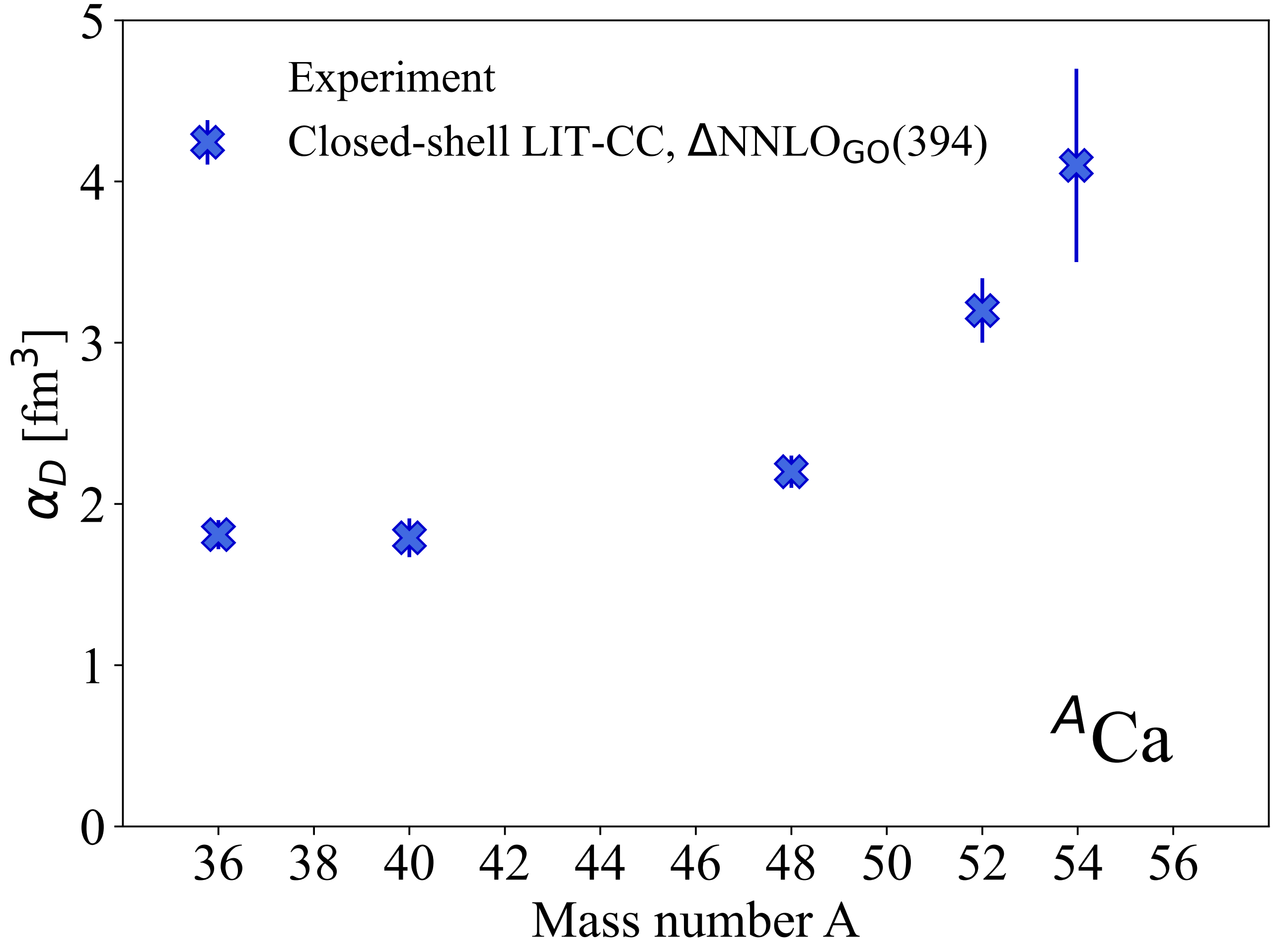
α_D along the calcium chain



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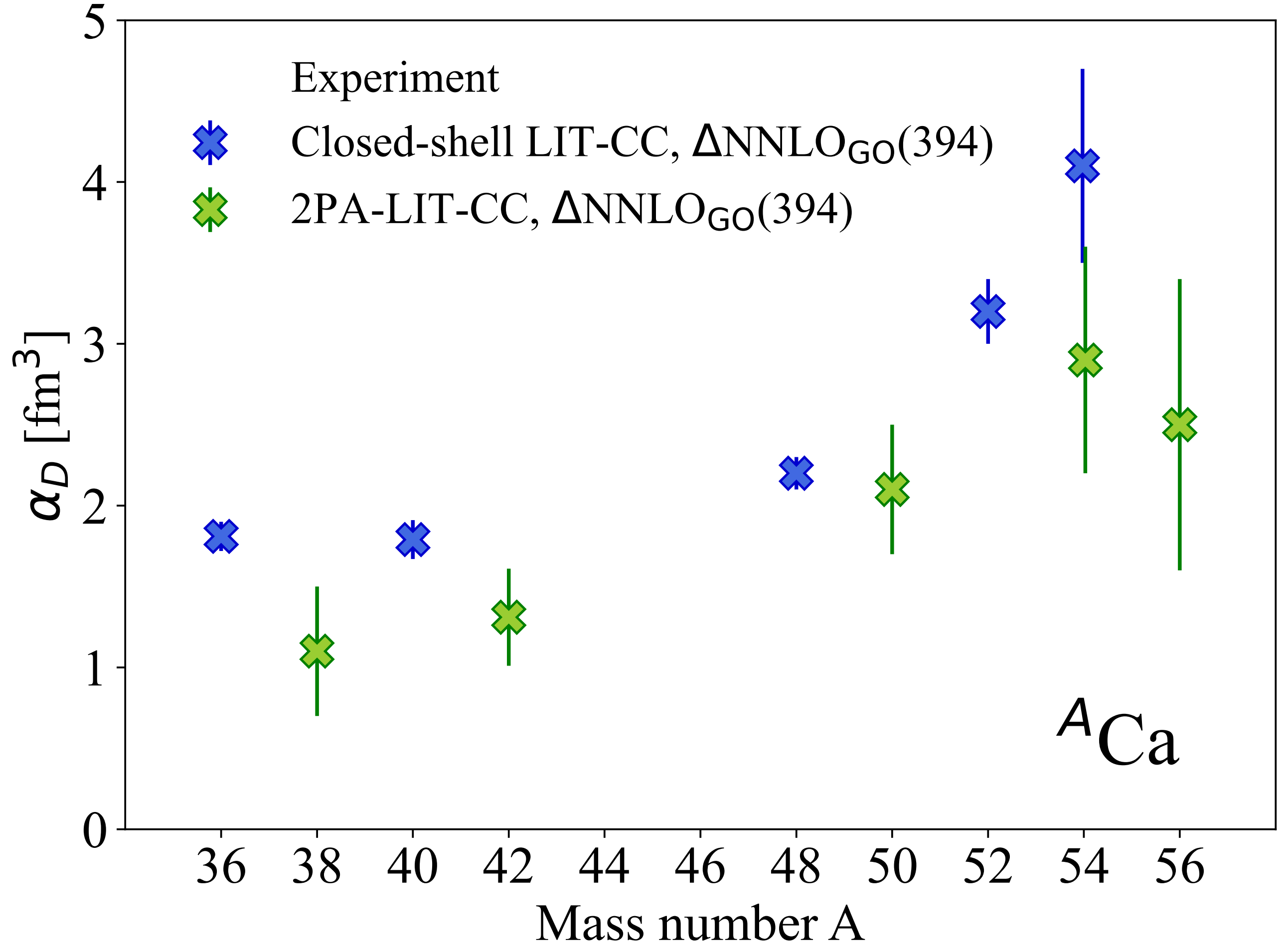
α_D along the calcium chain



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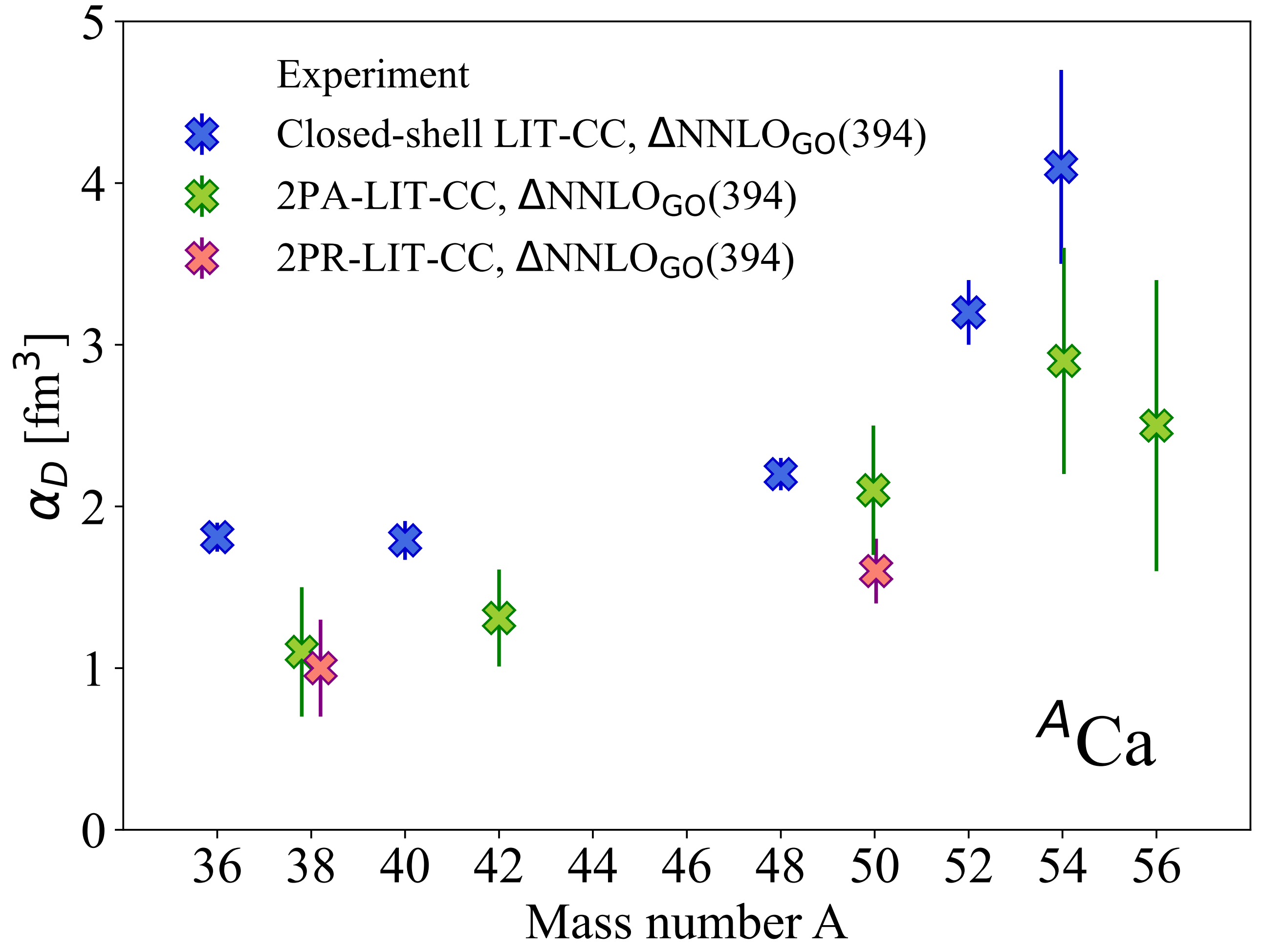
Adding two-particle removed nuclei



F. Marino, **FB** et al., arXiv:2504.11012 [nucl-th].

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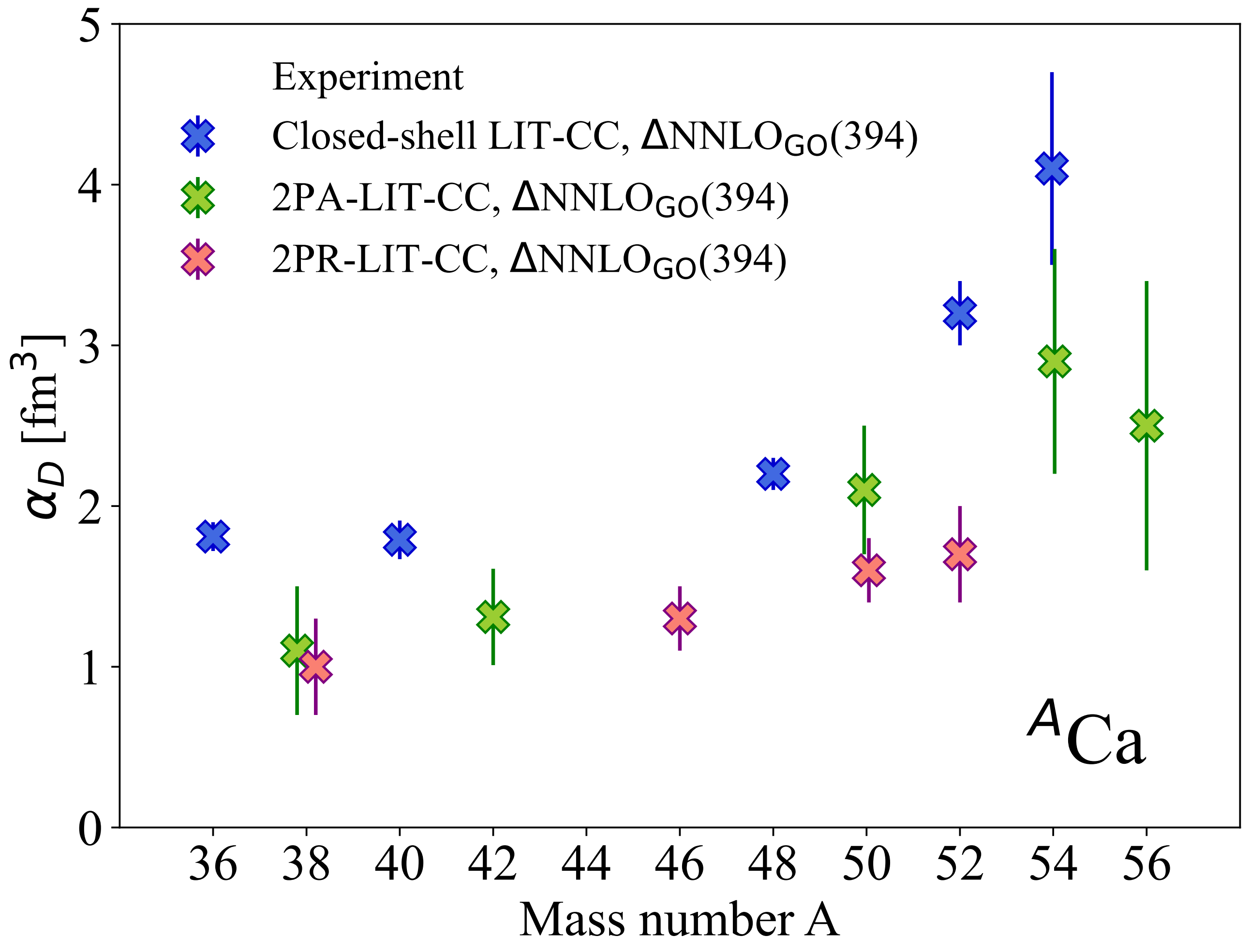

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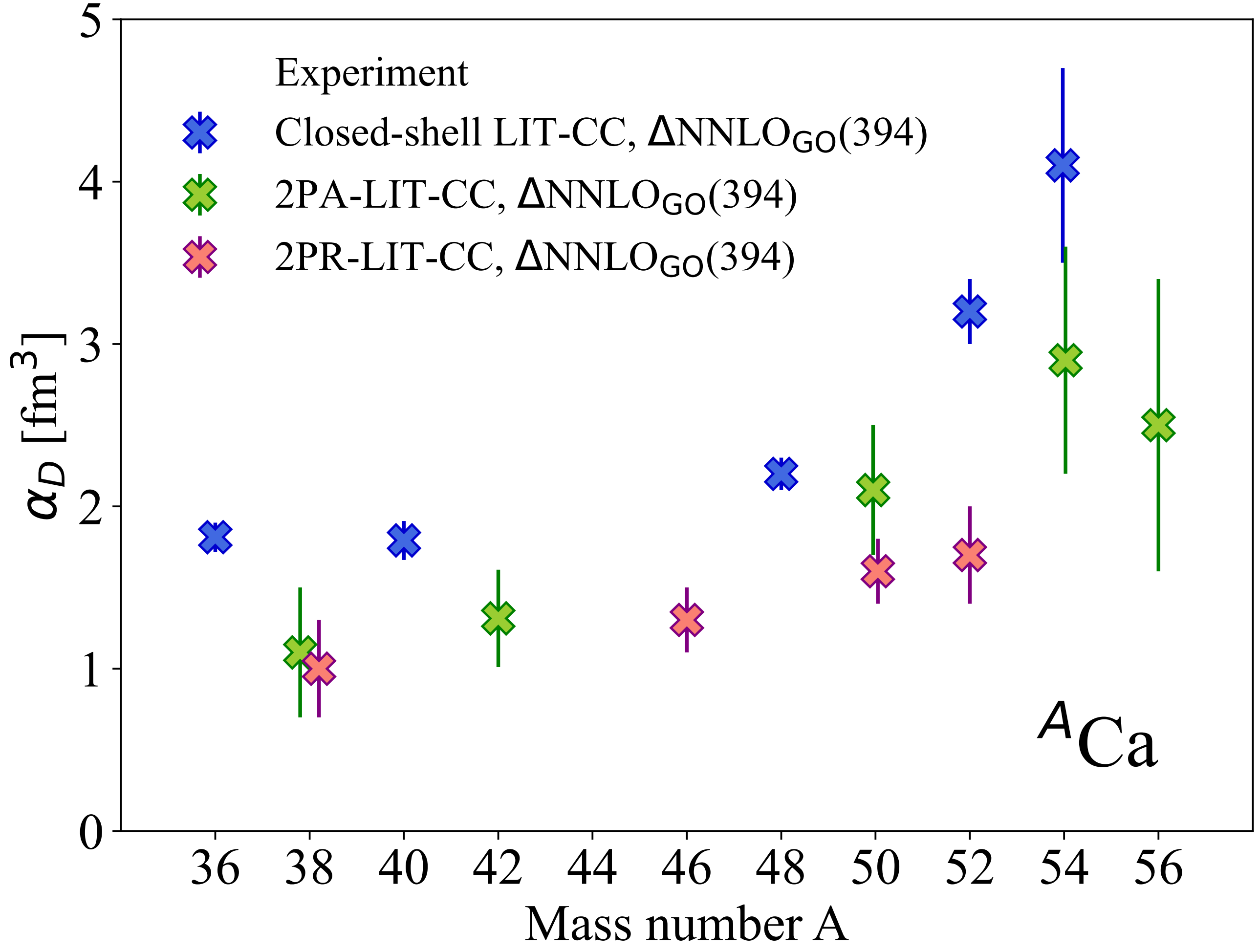
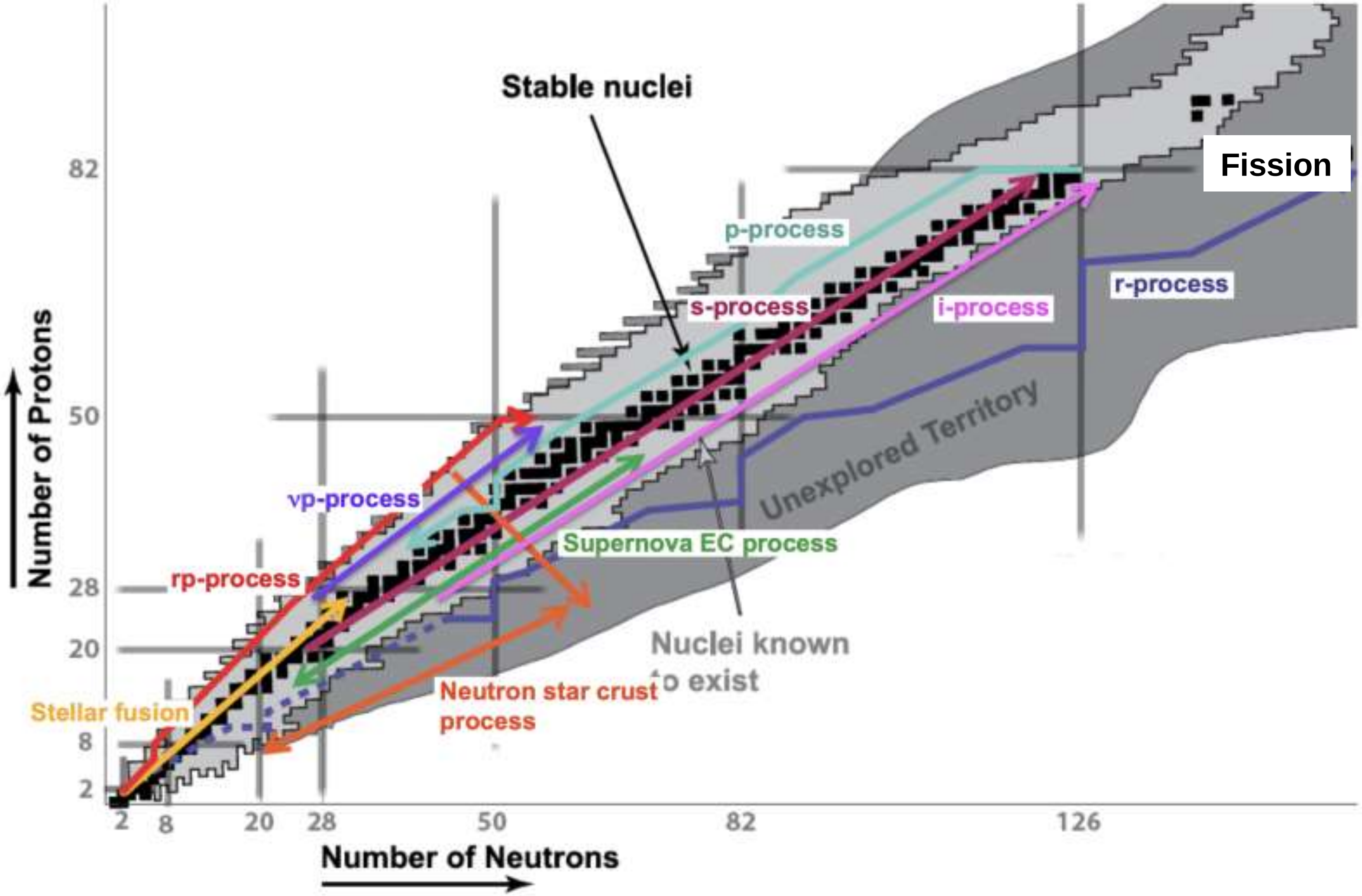


What can we do while we wait for new data?
→ see Tim Egert's poster

F. Marino, **FB** et al., arXiv:2504.11012 [nucl-th].

Many other nuclear properties impact astrophysics



Hendrik Schatz, *J. Phys. G: Nucl. Part. Phys.* **43** 064001 (2016)

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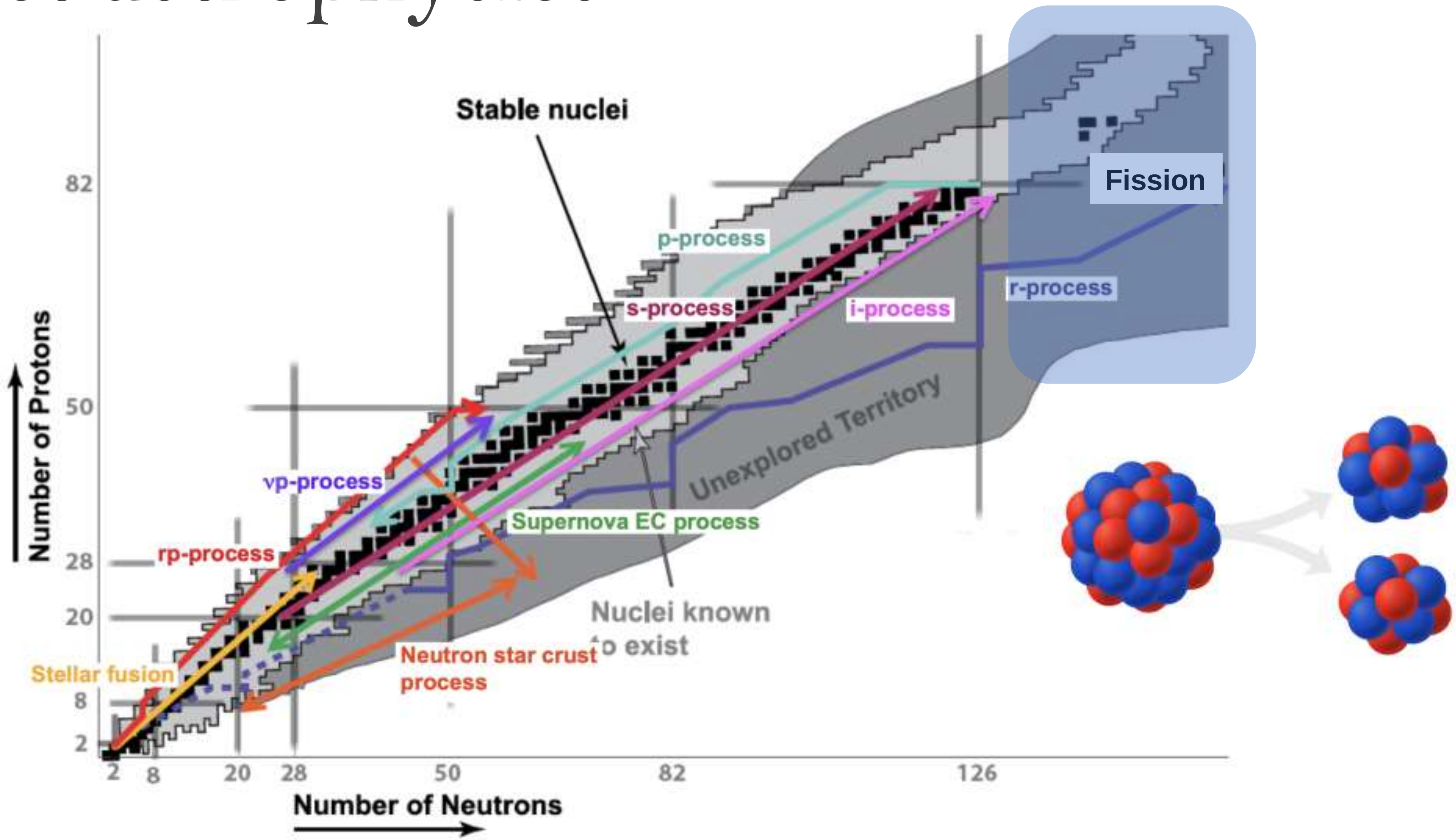

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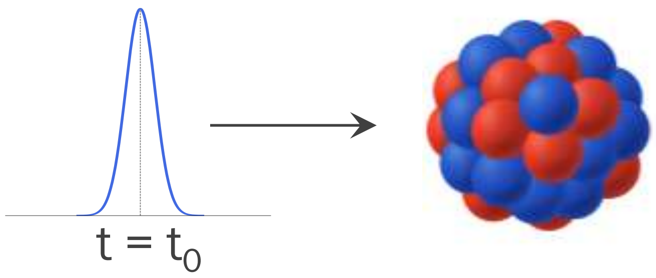
Responses in a time-dependent approach

Goal: solving

$$i\hbar \frac{d}{dt} |\Psi(t)\rangle = \hat{H}(t) |\Psi(t)\rangle$$

with

$$\hat{H}(t) = \hat{H}_0 + \epsilon f(t) \hat{D}$$



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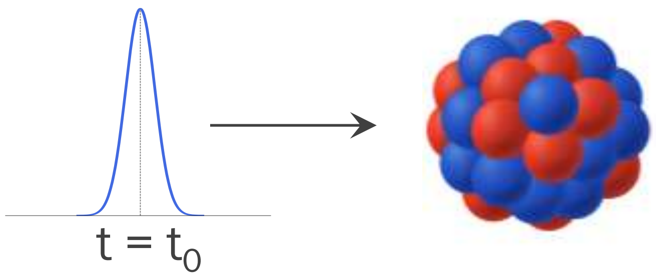

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For small ε, first-order time-dependent perturbation theory yields:

$$D(t) = \langle \Psi(t) | \hat{D} | \Psi(t) \rangle$$

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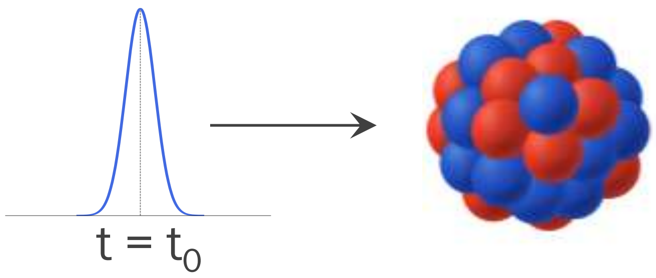

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Fourier transform

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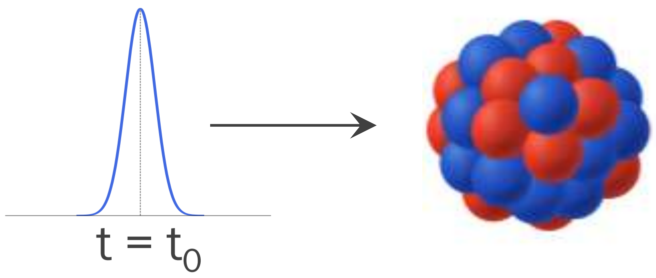

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For small ε, first-order time-dependent perturbation theory yields:

$$D(t) = \langle \Psi(t) | \hat{D} | \Psi(t) \rangle \longrightarrow \tilde{D}(\omega) \longrightarrow R(\omega) = \mathrm{Im}\left(\frac{\tilde{D}(\omega)}{\epsilon \tilde{f}(\omega)} \right)$$

Fourier transform

Time-dependent coupled-cluster equations

Time-dependent coupled-cluster (TDCC) ansatz:

$$|\Psi(t)\rangle = e^{T(t)}|\Phi_0\rangle$$

where

$$T(t) = t_0(t) + \sum_{ia} t_i^a(t) a_a^\dagger a_i + \sum_{ijab} t_{ij}^{ab}(t) a_a^\dagger a_b^\dagger a_j a_i$$

D.A. Pigg, G. Hagen, H. Nam, T. Papenbrock, PRC 86, 014308 (2012).

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Cluster amplitudes evolve in time according to:

$$i\hbar\dot{t}_0(t) = \langle\Phi_0|\overline{H}|\Phi_0\rangle$$

$$i\hbar\dot{t}_i^a(t) = \langle\Phi_i^a|\overline{H}|\Phi_0\rangle \qquad \overline{H} = e^{-T(t)}H(t)e^{T(t)}$$

$$i\hbar\dot{t}_{ij}^{ab}(t) = \langle\Phi_{ij}^{ab}|\overline{H}|\Phi_0\rangle$$

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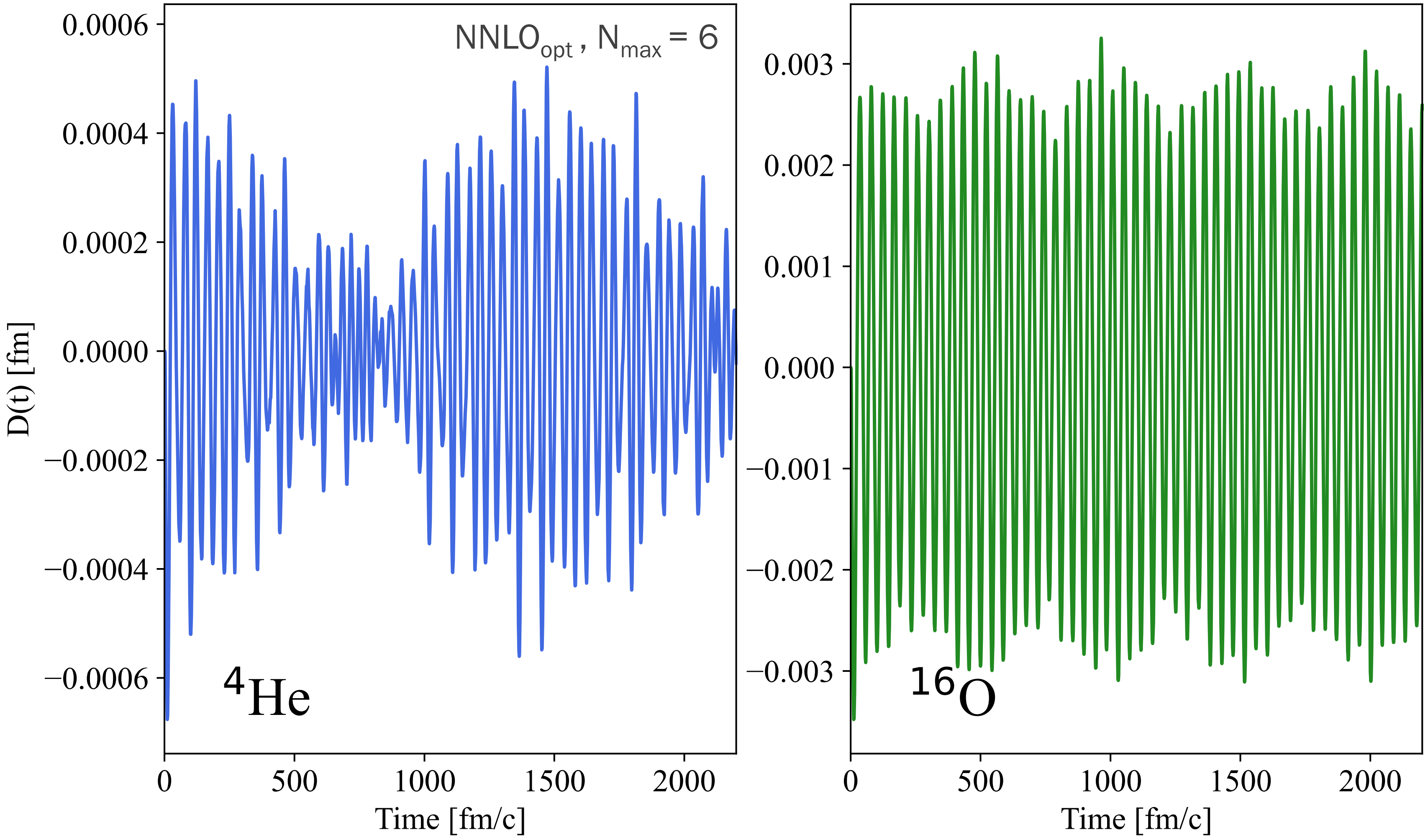
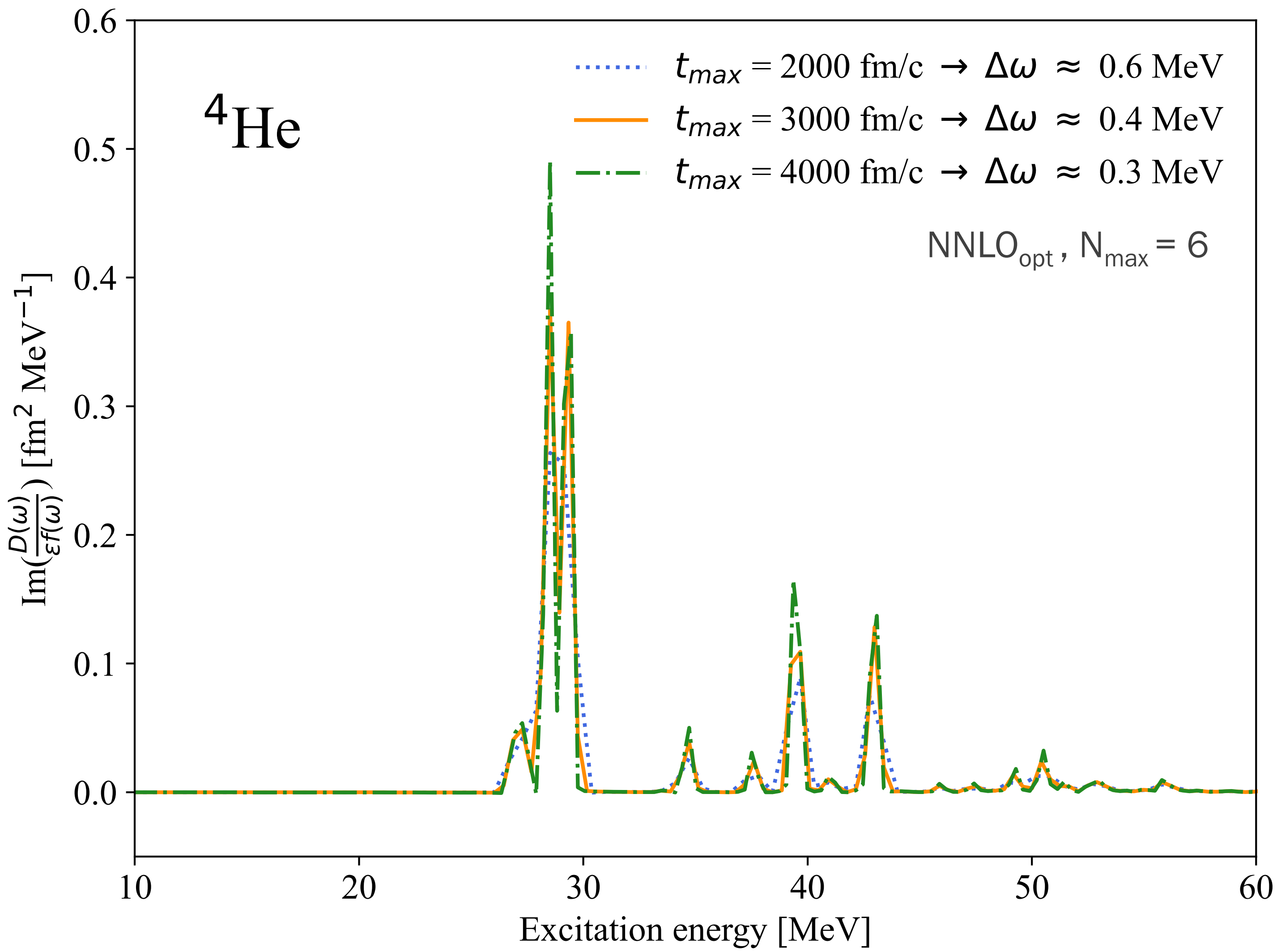
Time-dependent dipole moment



Simulation time and resolution

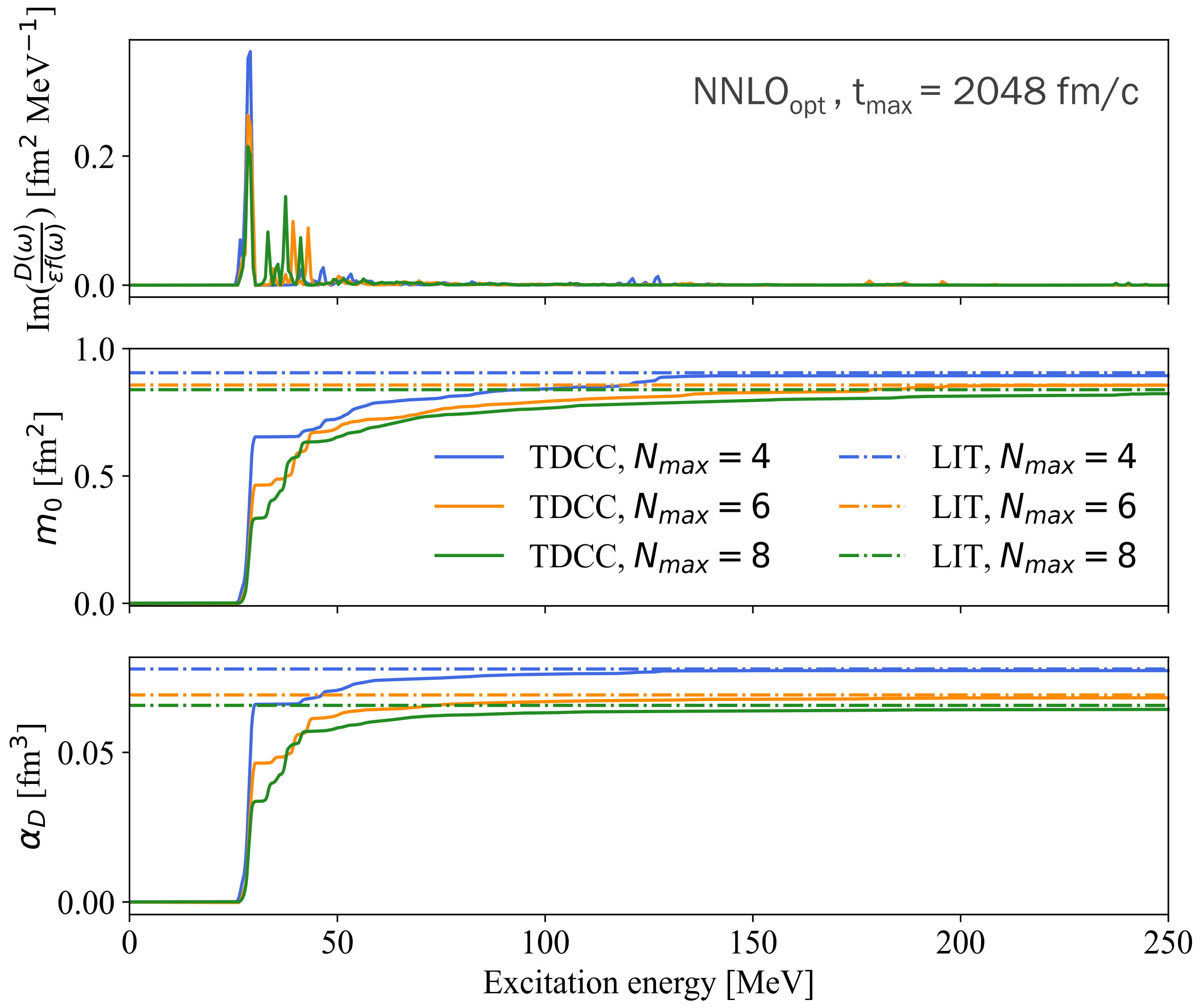


Resolution

$$\Delta\omega = \frac{2\pi\hbar c}{t_{max}}$$

Maximum simulation time

Static LIT-CC vs time-dependent CC: ^{4}He



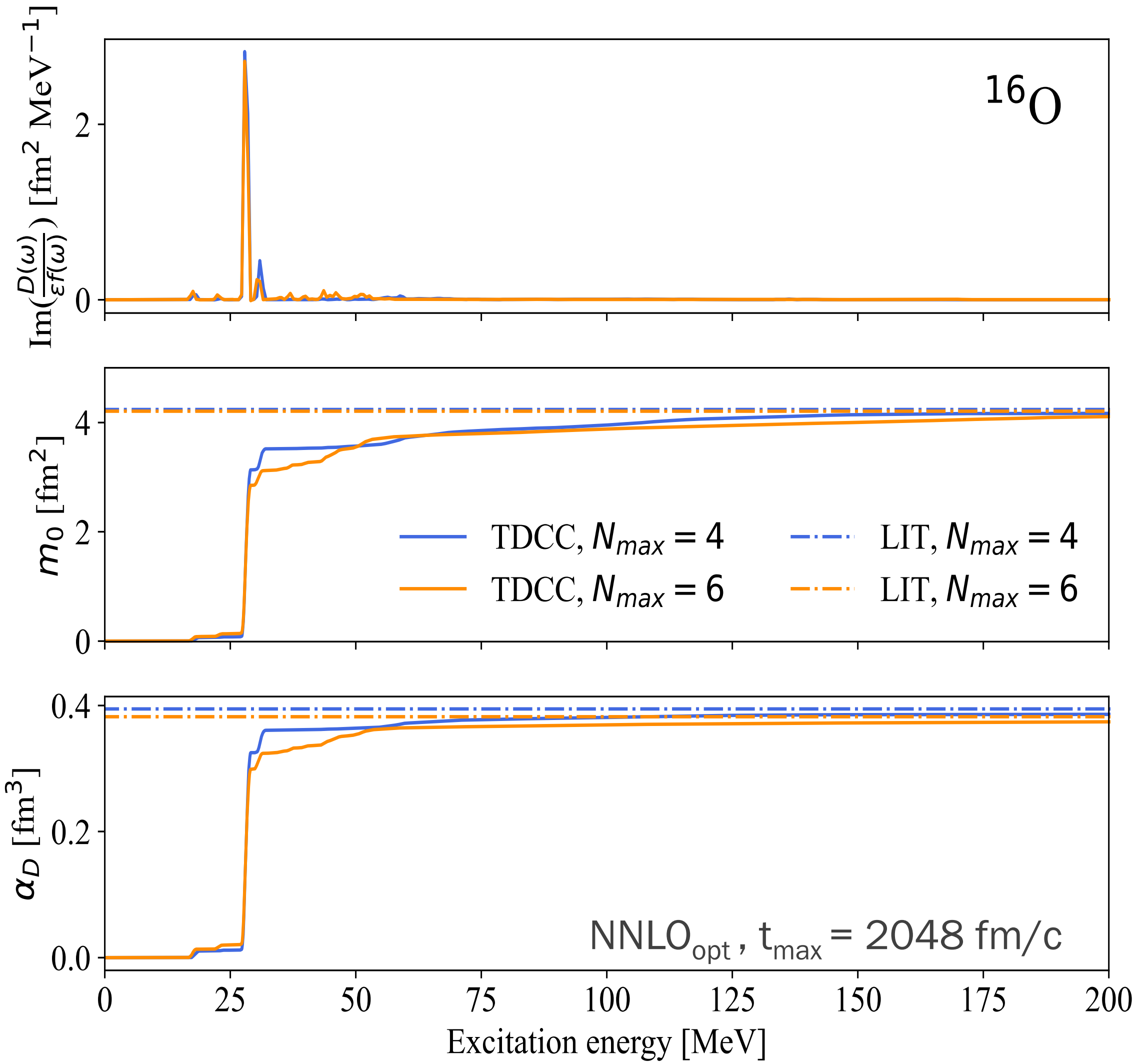
$$R(\omega) = \mathrm{Im}\left(\frac{\tilde{D}(\omega)}{\epsilon \tilde{f}(\omega)}\right)$$

$$m_0 = \int d\omega R(\omega)$$

$$\alpha_D = 2\alpha \int d\omega\, \omega^{-1} R(\omega)$$

Deviations of less than 1-2% between the two complementary approaches!

Static LIT-CC vs time-dependent CC: ^{16}O



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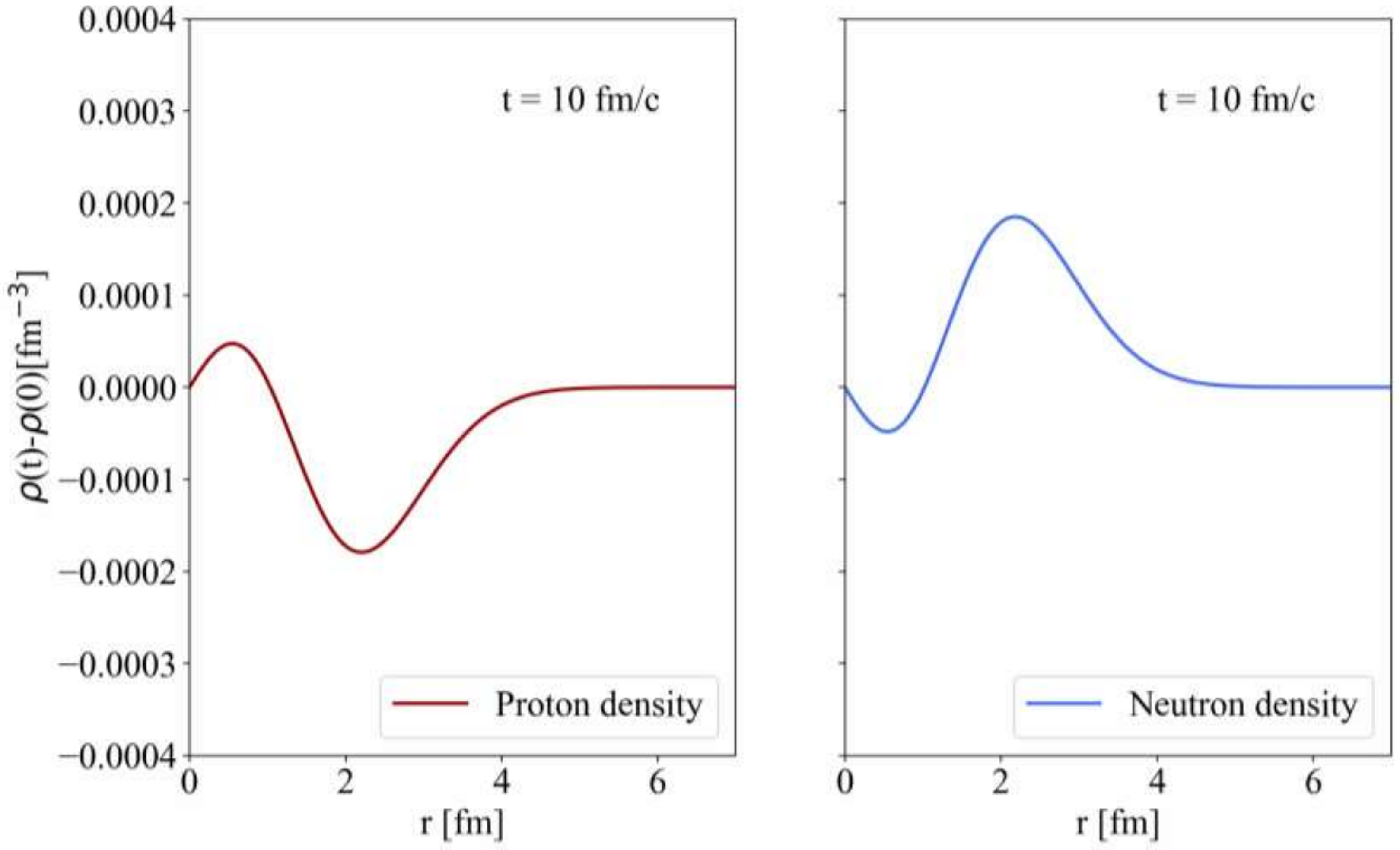
Very good agreement also for ^{16}O!

Collective oscillations in real time

^{16}O



What happens when we increase ε?

$$\hat{H}(t) = \hat{H}_0 + \epsilon f(t)\hat{D}$$

- Up to now, ε = 0.1 fm/MeV, where we are still in the linear regime.
- Non-linearities emerge when the perturbation becomes comparable to typical scale of H_0.
- For ^{16}O, $B(E1)^{1/2} \sim 10^{-2}$ e fm [TUNL database], so we need ε = 100 MeV/fm to get a perturbation ~ MeV.

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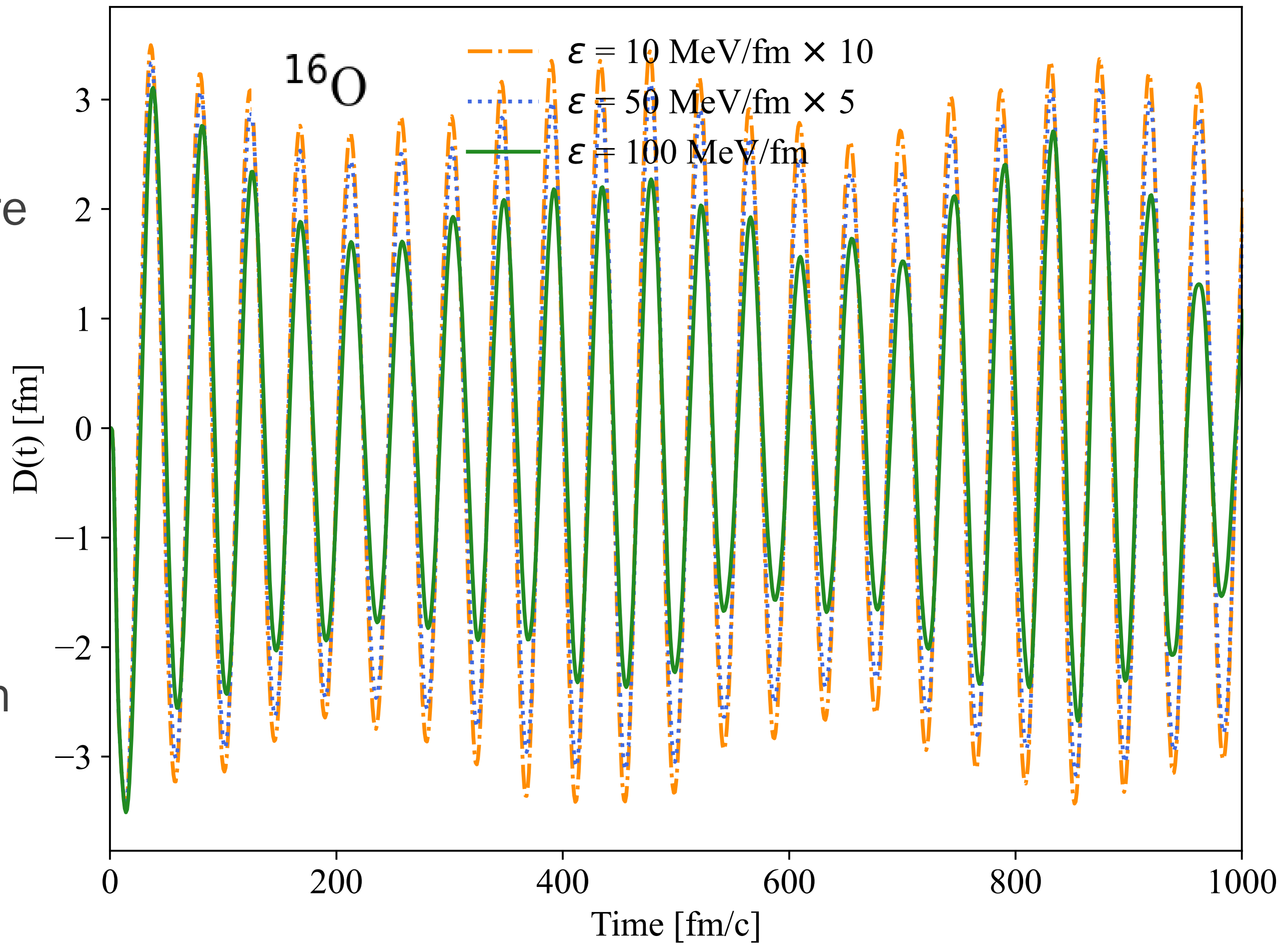
$NNLO_{opt}$, $N_{max} = 4$



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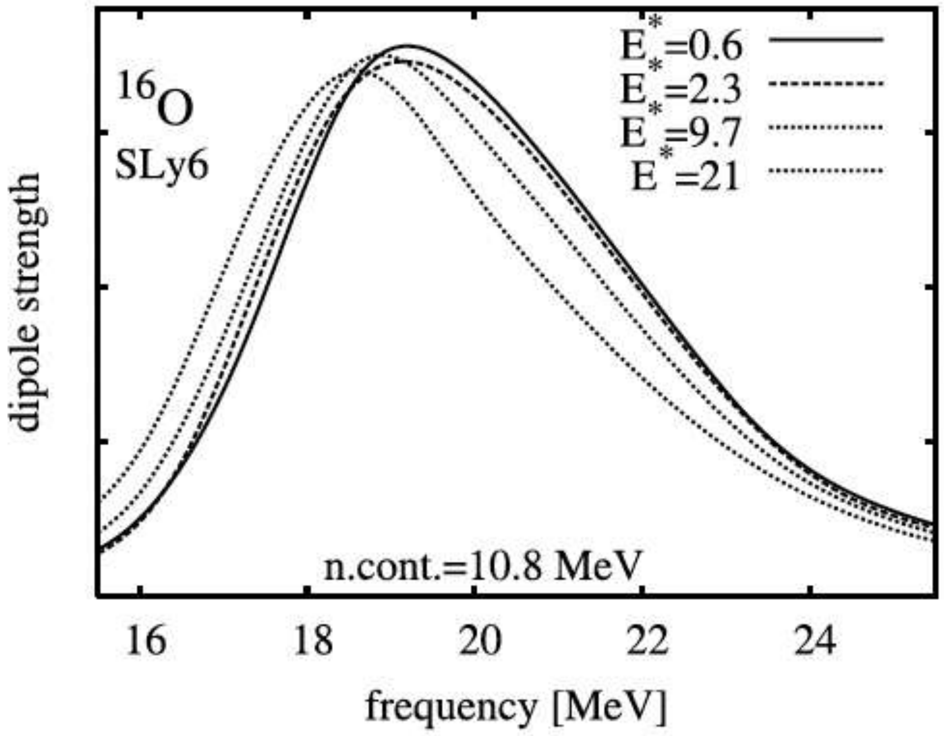

P.-G. Reinhard et al, Eur. Phys. J. A 32, 19–23 (2007).

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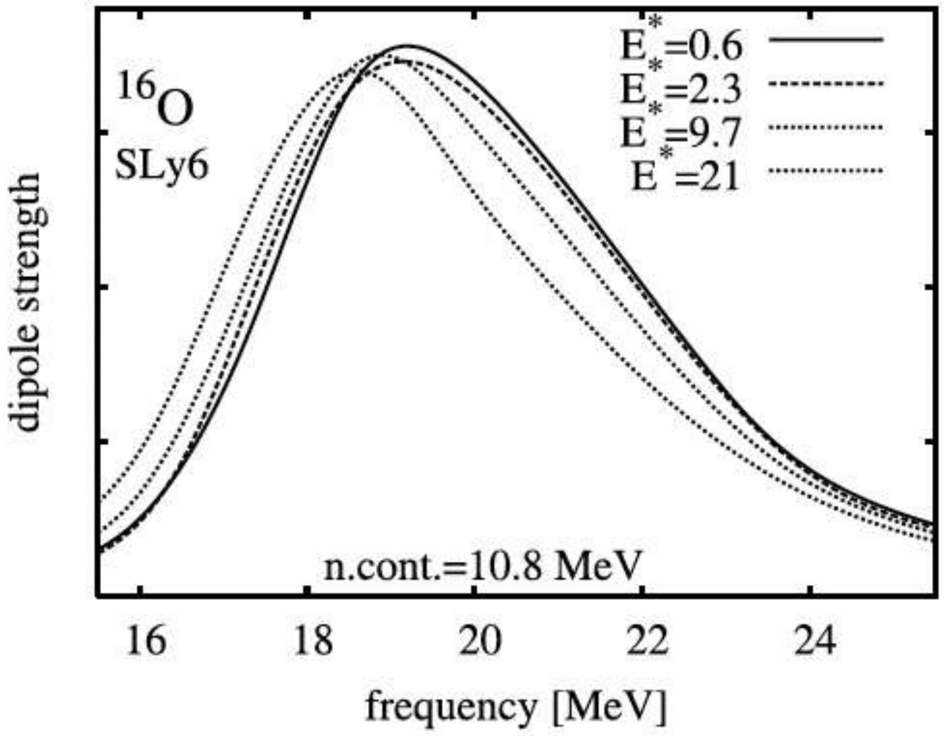
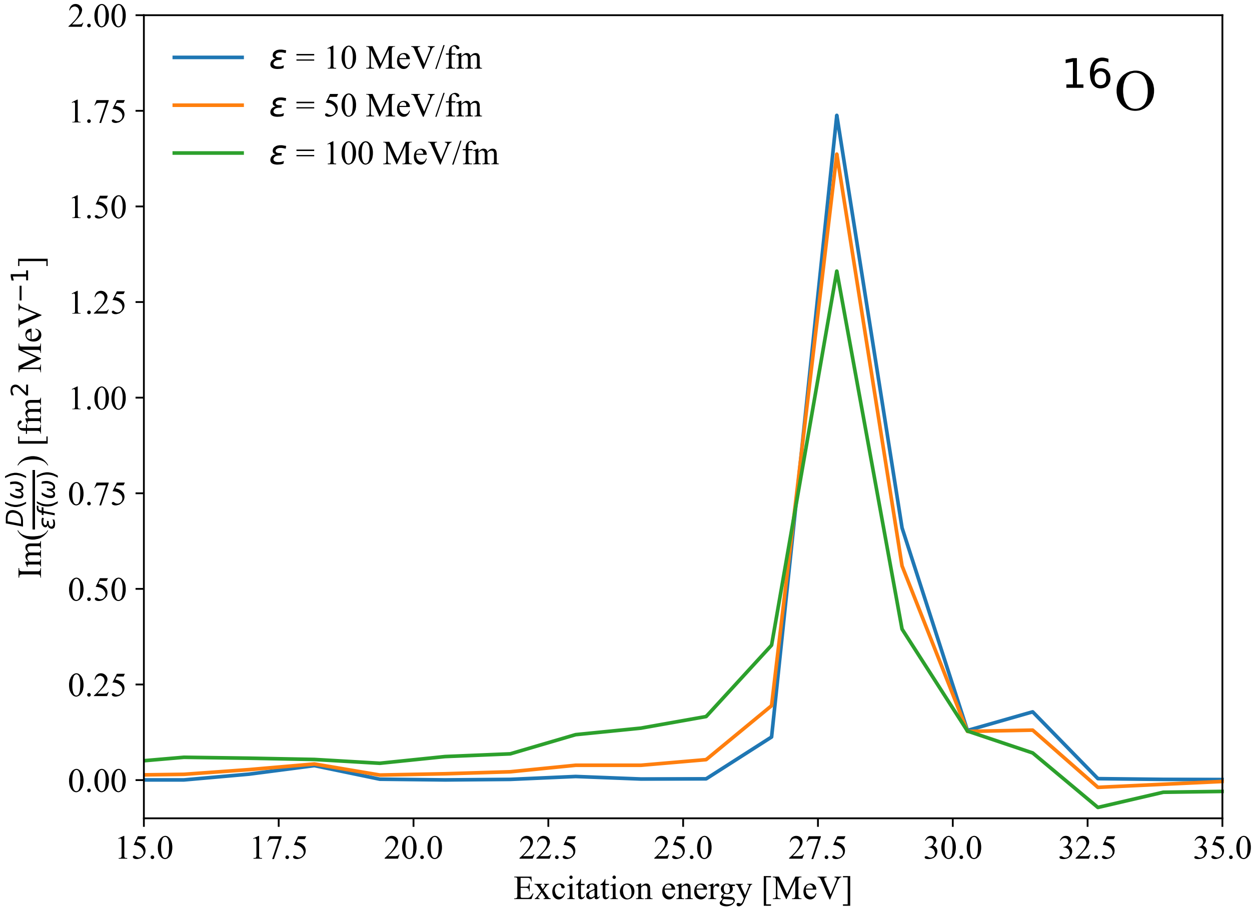


P.-G. Reinhard et al, Eur. Phys. J. A 32, 19–23 (2007).

$NNLO_{opt}$, N_{max} = 4, t_{max} = 1024 fm/c



Conclusions

- Electric dipole polarizabilities cast light on the collective excitations of the nucleus as well as constraining the symmetry energy.

- We extended ab initio reach of this observable to nuclei in the vicinity of closed shells.

- We started working on a time-dependent description of nuclear responses and working on different strategies to optimize it (natural orbital basis + adapting solver to GPUs + emulators…) for applications to non-linear problems and reactions in the long term.

Thanks to my collaborators:

@ORNL/UTK: Gaute Hagen, Gustav R. Jansen, Thomas Papenbrock
@FRIB/MSU: Kyle Godbey
@JGU Mainz: Sonia Bacca, Tim Egert, Weiguang Jiang, Francesco Marino, Joanna Sobczyk
@LLNL: Cody Balos, Carol Woodward
@TU Darmstadt: Andrea Porro, Alex Tichai, Achim Schwenk (theory), Isabelle Brandherm, Peter von Neumann-Cosel (exp)

and to you for your attention!

Work supported by:

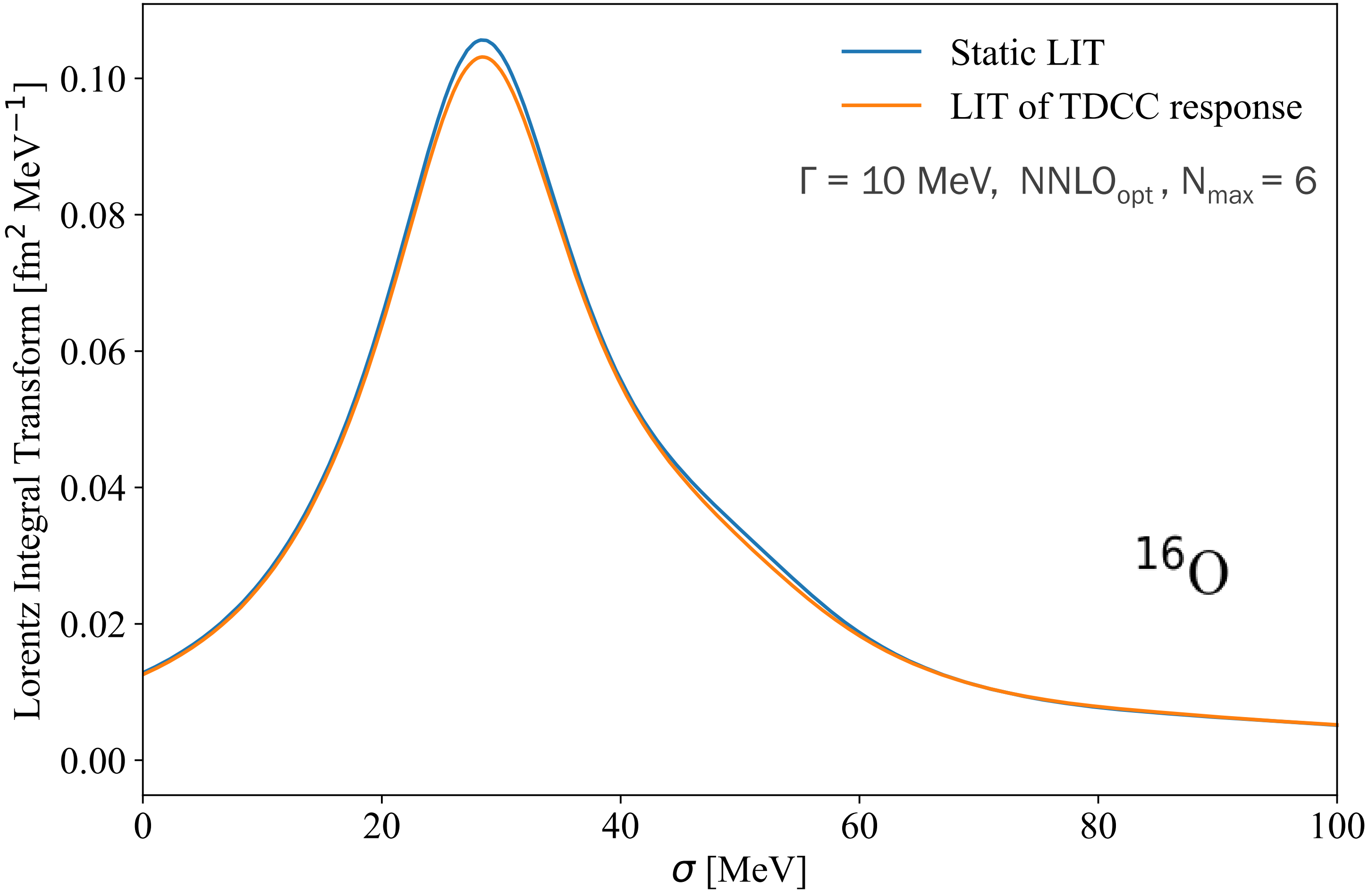
SFB 1245

U.S. DEPARTMENT OF ENERGY

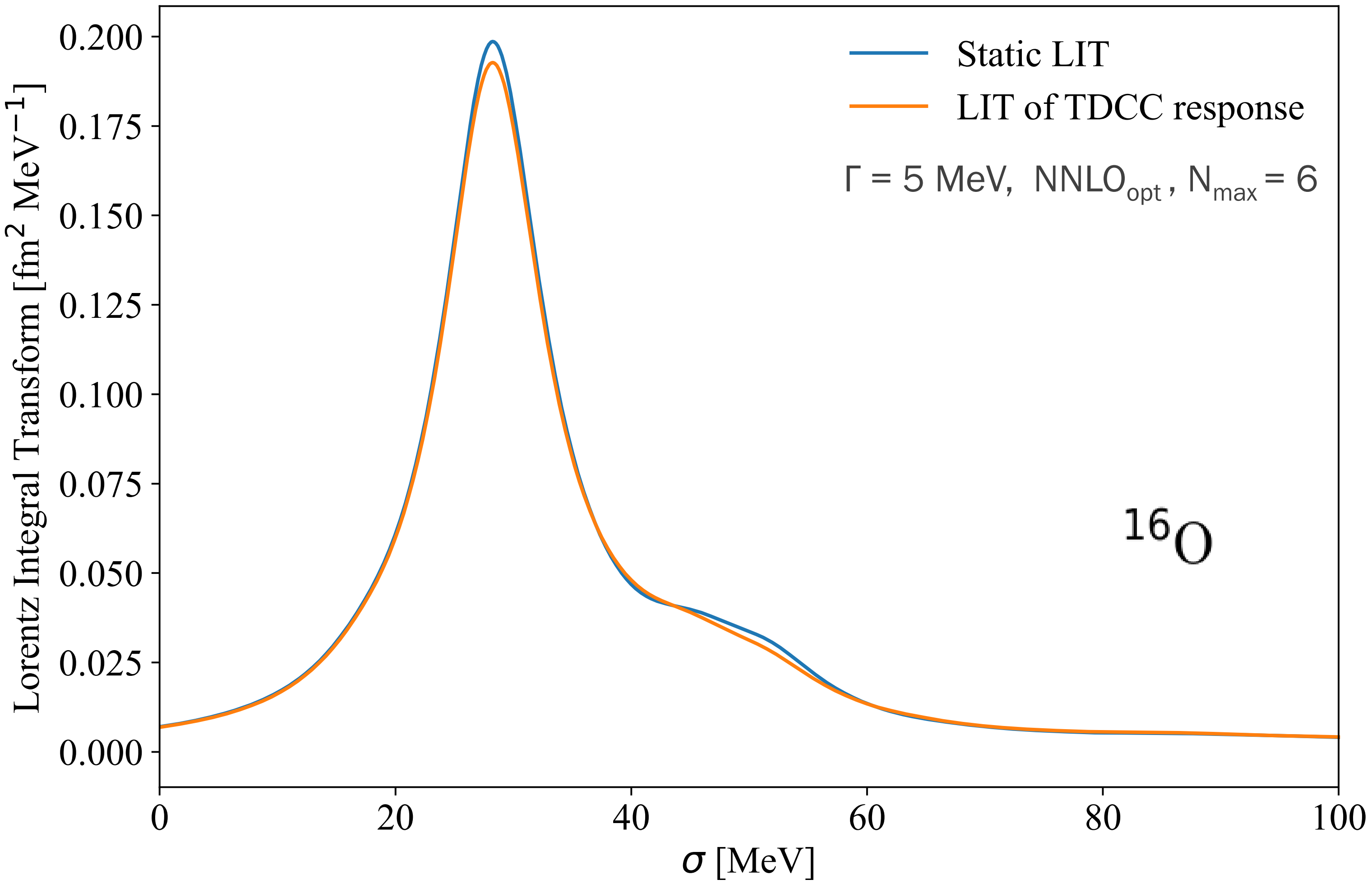
FRIB Theory Alliance

NUCLEI Nuclear Computational Low-Energy Initiative

Static and time-dependent LITs



Static and time-dependent LITs



Static and time-dependent LITs