האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

The Relative Abundance of Correlated Spin-zero Nucleon Pairs

Raz Yankovich, Ehoud Pazy, Nir Barnea

The Hebrew University, Jerusalem, Israel

Workshop on Multineutron Clusters in Nuclei and in Stars
ICTP-SAIFR, Sao Paulo, June 2–6, 2025

Modern times ...

AI Overview

Neutron-neutron short-range correlations (SRCs) are a quantum mechanical phenomenon where two neutrons in a nucleus are closely correlated due to the strong nuclear force. These correlations lead to high-momentum nucleons and influence nuclear structure and interactions.

Key aspects of neutron-neutron SRCs:

Induced by the strong nuclear force:

The short-range and tensor components of the nucleon-nucleon interaction are responsible for these correlations.

High-momentum nucleons:

SRCs cause nucleons to occupy high-momentum states above the Fermi momentum.

Impact on nuclear structure:

SRCs play a role in the structure of nuclei, including their size and shape.

Influence on neutron stars:

SRCs can affect the thermal evolution and properties of neutron stars, influencing the proton fraction and cooling rates.

Experimental verification:

SRCs have been observed through experiments like electron and proton scattering.

Generalized Contact Formalism:

This theory helps describe many aspects of electron scattering measurements, particularly those related to SRCs.

Impact on nuclear pasta:

Modern times ...

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Impact on nuclear pasta:

… and in stars

PHYSICAL REVIEW LETTERS **133,** 171401 (2024)

Short-Range Correlations and Urca Process in Neutron Stars

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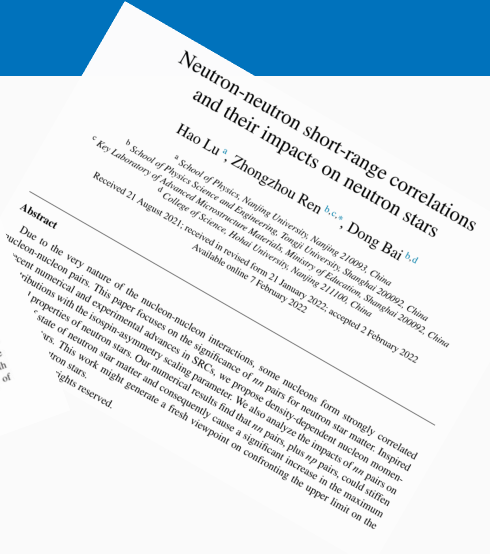
Frankfurt Institute for Advanced Studies, D-60438 Frankfurt am Main, Germany
and Institute of Theoretical Physics, University of Wroclaw, 50-204 Wroclaw, Poland

(Received 26 June 2024; revised 24 August 2024; accepted 18 September 2024; published 21 October 2024)

Recent measurements of high-momentum correlated neutron-proton pairs at JLab suggest that the dense nucleonic component of the compact stars contains a fraction of high-momentum neutron-proton pairs that is not accounted for in the familiar Fermi-liquid theory of the neutron-proton fluid mixture. We compute the rate of the Urca process in compact stars taking into account the non-Fermi liquid contributions to the proton's spectral widths induced by short-range correlations. The Urca rate differs strongly from the Fermi-liquid prediction at low temperatures; in particular, the high threshold on the proton fraction precluding the Urca process in neutron stars is replaced by a smooth increase with the proton fraction. This observation may have a profound impact on the theories of cooling of compact stars.

DOI: 10.1103/PhysRevLett.133.171401



Neutron-neutron short-range correlations and their impacts on neutron stars

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Available online 7 February 2022

Abstract

Due to the very nature of the nucleon-nucleon interactions, some nucleons form strongly correlated
ucleon-nucleon pairs. This paper focuses on the significance of *nn* pairs for neutron star matter. Inspired
ecent numerical and experimental advances in SRCs, we propose density-dependent nucleon momen-
ributions with the isospin-asymmetry scaling parameter. We also analyze the impacts of *nn* pairs on
l properties of neutron stars. Our numerical results find that *nn* pairs, plus *np* pairs, could stiffen
state of neutron star matter and consequently cause a significant increase in the maximum
ars. This work might generate a fresh viewpoint on confronting the upper limit on the
tron stars.

Short Range Correlations in a many-body systems

Kenya (2016).

Short Range Correlations in many-body systems

Kenya (2016).

Ideal Gas

Getting closer to home

Length scales:

The interparticle distance:

$$d \propto n^{1/3}$$

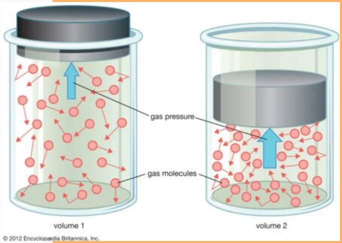
Interaction range:

$$\sigma \approx \pi R^2$$

$V(r) \to 0$ at typical distance $R \ll d$

I. Particles move freely most of the time

II. Equilibrium resluts from **short range** collisions



Factorization and Universality in QM

The Short Range Wave Function

The 2-body system

$$\Psi(\boldsymbol{r}) \xrightarrow[r \to 0]{} \varphi_0(\boldsymbol{r})$$

The N-body system

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A) \xrightarrow[r_{12} \to 0]{} \varphi_0(\boldsymbol{r}_{12}) A(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_N)$$

Short Range Observables

The Contact - Tan, Braaten & Platter,...

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$$\langle \Psi | \sum O_{ij} | \Psi \rangle \approx \underbrace{\frac{N(N-1)}{2} \langle A | A \rangle}_{C_N} \langle \varphi_0 | O_{12} | \varphi_0 \rangle$$

Short Range Observables - Momentum space

The Contact - Tan, Braaten & Platter,...

The 2-body system

$$\tilde{\psi}(\boldsymbol{k}) \xrightarrow[k \to \infty]{} \tilde{\varphi}_0(\boldsymbol{k})$$

The N-body system

$$\tilde{\Psi}(\boldsymbol{k}_1, \boldsymbol{k}_2, \ldots, \boldsymbol{k}_N) \xrightarrow[k_{12} \to \infty]{} \tilde{\varphi}_0(\boldsymbol{k}_{12})\tilde{A}(\boldsymbol{K}_{12}, \boldsymbol{k}_3, \ldots, \boldsymbol{k}_N)$$

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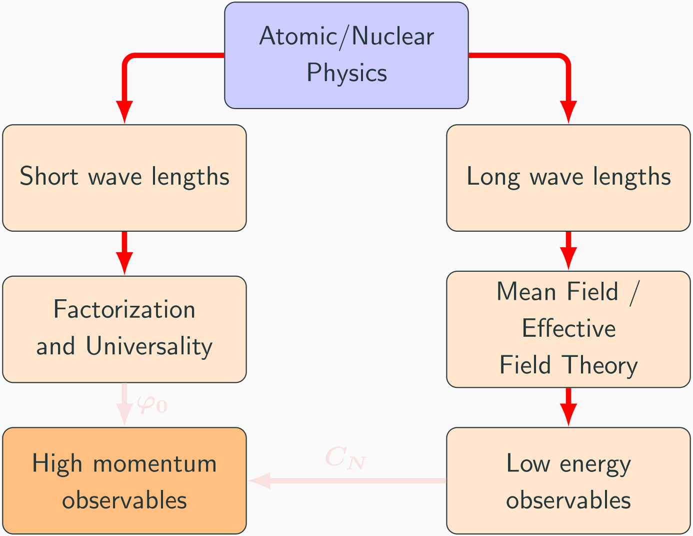
Short and Long



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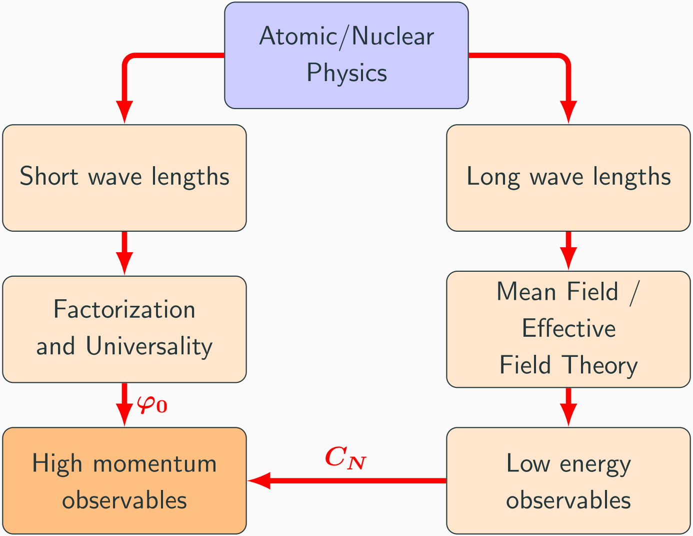

Universality and Factorization

Theoretical develpoments in nuclear physics

(with personal bias)

- **Levinger - Photoabsorption**

  J. S. Levinger, Phys. Rev. 84, 43 (1951).

- **Amado, Woloshyn - Momentum Distribution**

  R. D. Amado, Phys. Rev. C 14, 1264 (1976).

  R. D. Amado and R. M. Woloshyn, Phys. Lett. B 62, 253 (1976).

- **Zabolitsky - Coupled Cluster**

  J. G. Zabolitsky and W. Ey, Phys. Lett. B 76, 527 (1978).

- **Frankfurt, Strikman - Factorization**

  L. Frankfurt, and M. Strikman, Phys. Rep. **160**, 235 (1988).

- **Bogner, Roscher - Factorization**

  S. K. Bogner and D. Roscher, Phys. Rev. C **86**, 064304 (2012).

- **Ciofi degli Atti - Electron scattering**

  C. Ciofi degli Atti, Phys. Rep. 590, 1 (2015).

Tan's Relations

Universal Fermi gas - an ideal system

Neutral, Cold, Dilute, Two components ↑,↓

Tan relations connects the contact C with:

- **Tail of momentum distribution** $|a|^{-1} \ll k \ll R^{-1}$

$$n_\sigma(\boldsymbol{k}) \longrightarrow \frac{C}{k^4}$$

- The energy relation

$$T + U = \sum_\sigma \int \frac{d\boldsymbol{k}}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} \left(n_\sigma(\boldsymbol{k}) - \frac{C}{k^4} \right) + \frac{\hbar^2}{4\pi m a} C$$

- Adiabatic relation

$$\frac{dE}{da^{-1}} = -\frac{\hbar^2}{4\pi m} C$$

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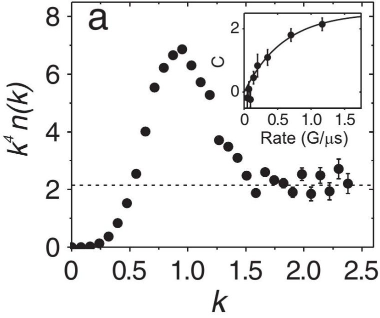
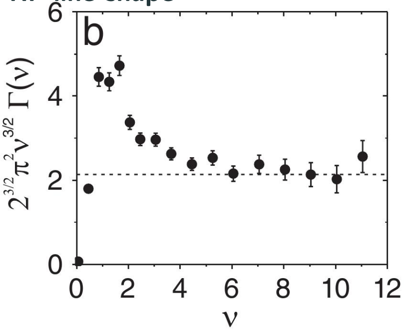
The Contact - Experimental Results

Momentum Distribution



RF line shape



Verification of Universal Relations in a Strongly Interacting Fermi Gas
J. T. Stewart, J. P. Gaebler, T. E. Drake, and D. S. Jin

Phys. Rev. Lett. 104, 235301 (2010)

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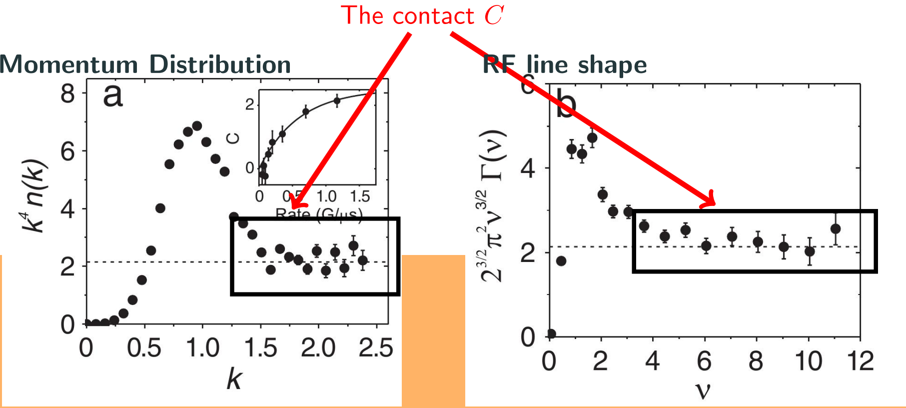

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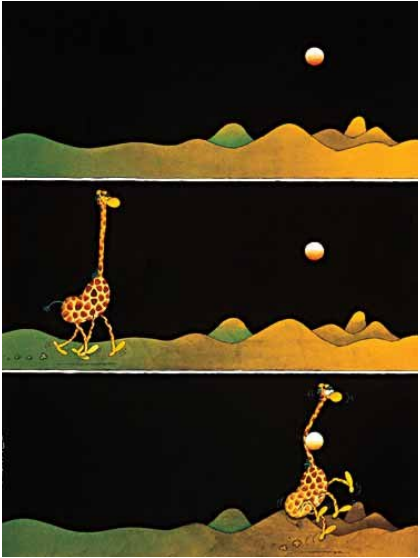
Nuclear Physics

The Generalized Contact Formalism

Factorization
and
Universality
in
Nuclear-Physics



Generalized Contact Formalism (GCF)

R. Weiss, B. Bazak, N. Barnea, PRC (2015)

Nuclear 2-body Short Range Correlations (SRCs) are successfully described by the **GCF**

The GCF is based on the factorization ansatz

$$\lim_{\boldsymbol{r}_{ij}\to 0} \Psi = \sum_\alpha \varphi_{ij}^\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha\left(\boldsymbol{R}_{ij}^{\text{C.M.}}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right) \qquad ij \in pp,\ np,\ nn$$

- For each pair there are different channels, $\alpha = (s,\ell)jm$
- φ is a universal **zero-energy** two-body wave-function, $\hat{H}\varphi = 0$
- A is the residual part, the "wave-function" of the spectator subsystem

The Nuclear Contact(s)

- For each pair we define the contact matrix

$$C_{ij}^{\alpha\beta} \equiv N_{ij} \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

using normalization $\int_{k_F}^{\infty} \frac{d\boldsymbol{k}}{(2\pi)^3} |\tilde{\varphi}_{\alpha}(\boldsymbol{k})|^2 = 1$

- **The contact $C_{ij}^{\alpha\alpha}$ counts the number of SRC pairs in channel α**
- For $\ell = 0$ we need consider **4** contacts

$$\{C_{pp}^{S=0}, C_{nn}^{S=0}, C_{np}^{S=0}, C_{np}^{S=1}\}$$

- If **isospin symmetry holds** the number of contacts is **2**,

$$C_{np}^{S=1} \qquad C_{np}^{S=0} = C_{pp}^{S=0} = C_{nn}^{S=0}$$

- It doesn't ... (for large nuclei)

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The nuclear contact relations/applications

- Photoabsorption cross-section
- Momentum distributions
- The nuclear spectral function
- Electron scattering
- Symmetry energy
- Two-body density
- Double beta-decay

and more ...

Numerical Verifications

Momentum distributions

1-body neutron and proton momentum distributions

$$\rho_n(\boldsymbol{k}), \;\; \rho_p(\boldsymbol{k})$$

2-body nn, np, pp momentum distributions

$$\rho_{nn}(\boldsymbol{k}), \;\; \rho_{pn}(\boldsymbol{k}), \;\; \rho_{pp}(\boldsymbol{k})$$

Momentum distributions - asymptotic relations

Using the factorization ansatz

$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_\alpha \varphi_\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

We can derive **asymptotic** relations between the 1-body and 2-body momentum distributions

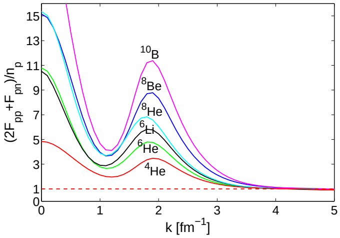
$$\boxed{\rho_p(\boldsymbol{k}) \xrightarrow[k\to\infty]{} 2\rho_{pp}(\boldsymbol{k}) + \rho_{pn}(\boldsymbol{k})}$$

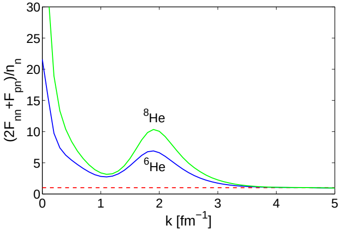
$$\boxed{\rho_n(\boldsymbol{k}) \xrightarrow[k\to\infty]{} 2\rho_{nn}(\boldsymbol{k}) + \rho_{pn}(\boldsymbol{k})}$$

These are **model independent** relations, that hold regardless of the specific form of φ_α and without any assumptions on $\{\alpha\}$

Numerical verification of the momentum relations

Weiss, Bazak, Barnea





VMC calculations of light nuclei

R. B. Wiringa, *et al.*, PRC **89**, 024305 (2014)

- 1-body, 2-body momentum distributions .
- Data available for $2 \leq A \leq 10$ and $A = 12, 16, 40$
  https://www.phy.anl.gov/theory/research/momenta/
- Potential - AV18+UX.

The momentum relations holds for $4\ \mathrm{fm}^{-1} \leq k \leq 5\ \mathrm{fm}^{-1}$

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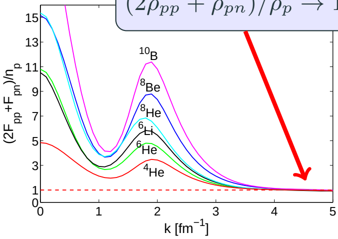
$(2\rho_{pp} + \rho_{pn})/\rho_p \to 1$



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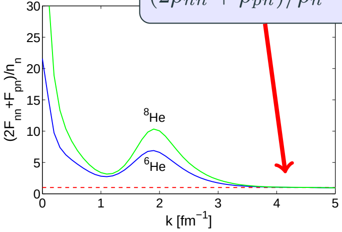

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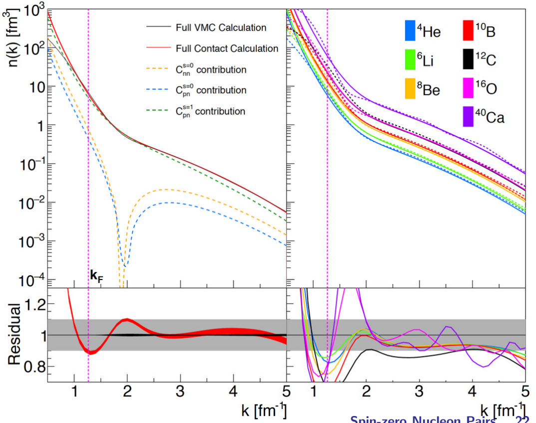
Further numerical verifications

Weiss et al

The resulting **asymptotic** 1-body momentum distribution

$$\rho_n(\boldsymbol{k}) \longrightarrow C_{np}^{s=0}|\tilde{\varphi}_{np}^{s=0}(\boldsymbol{k})|^2 + C_{np}^{s=1}|\tilde{\varphi}_{np}^{s=1}(\boldsymbol{k})|^2 + 2C_{nn}^{s=0}|\tilde{\varphi}_{nn}^{s=0}(\boldsymbol{k})|^2$$

Comparing with the VMC data:



Surprisingly, the agreement holds for $k_F \leq k \leq 6 \text{ fm}^{-1}$

2-body densities

The **asymptotic** 2-body density of a NN pair in quantum state α

$$\rho^{\alpha}_{pp}(\boldsymbol{r}) \xrightarrow[r\to 0]{} C^{\alpha}_{pp}|\varphi^{\alpha}_{pp}(\boldsymbol{r})|^2$$

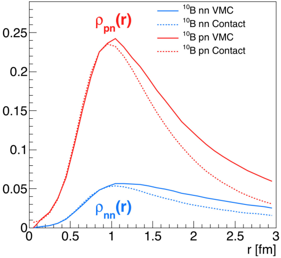
$$\rho^{\alpha}_{np}(\boldsymbol{r}) \xrightarrow[r\to 0]{} C^{\alpha}_{np}|\varphi^{\alpha}_{np}(\boldsymbol{r})|^2$$

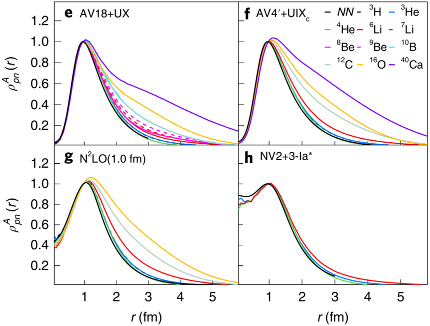
$$\rho^{\alpha}_{nn}(\boldsymbol{r}) \xrightarrow[r\to 0]{} C^{\alpha}_{nn}|\varphi^{\alpha}_{nn}(\boldsymbol{r})|^2$$

2-body densities

Compare 4 realistic $NN + 3N$ interactions:

- Phenomen. - **AV18 + UX**
- Phenomen. - **AV4' + UIX$_c$**
- χEFT - **N^{2}LO cutoff 1.0 fm**
- χEFT - **N^{2}LO cutoff 1.2 fm**





Cruz-Torres, Lonardoni, et al., Nature Phys. 17, 306 (2021)

Contacts from charge distribution

Weiss et al. PLB (2019)

Continuity, Phenomenology

Uncorrelated pp density **(blue)**

$$\rho^{UC}_{pp}(\boldsymbol{r}) = \int d\boldsymbol{R}\, \rho_p(\boldsymbol{R}+\boldsymbol{r}/2)\rho_p(\boldsymbol{R}-\boldsymbol{r}/2),$$

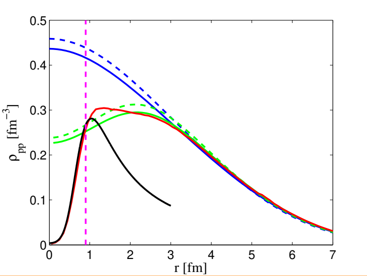
Fermi correlations **(green)**

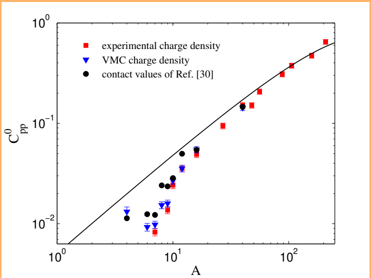
$$\rho^{F}_{pp}(r) = \mathcal{N}\rho^{UC}_{pp}(r)\left[1-\frac{1}{2}\left(\frac{3j_1(k^p_F r)}{k^p_F r}\right)^2\right]$$

Short range correlations **(black)**

$$\rho_{pp}(\boldsymbol{r}) = C^0_{pp}|\varphi^0_{pp}(\boldsymbol{r})|^2$$

For comparison - VMC (red)





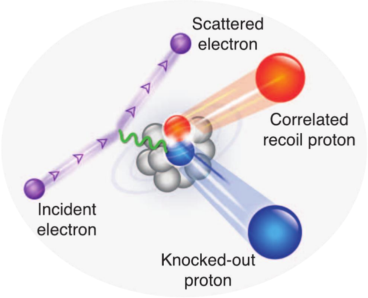
Experimental Verifications

Electron scattering

The Bjorken scaling parameter - x_B

$$x_B = \frac{Q^2}{2m\omega}$$

Where the virtual photon carries $(\boldsymbol{q}, \omega)$ and $Q^2 = -q^2$



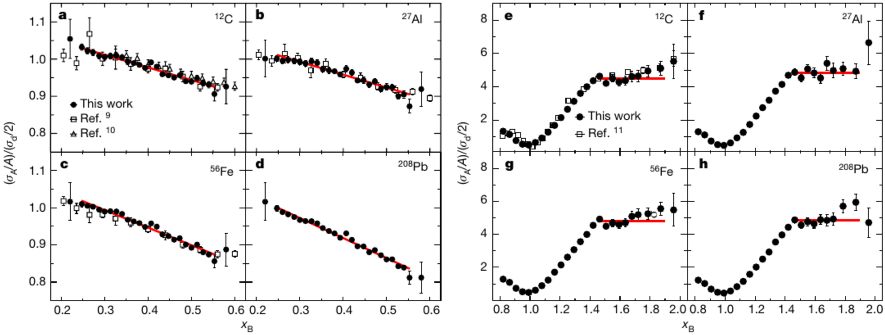
- Kinematical considerations:

  For $n - 1 < x_B \leq n$

  the **"active subsystem"** must include n-nucleons.

- The cross-section ratio $a_2 \equiv \sigma_A/\sigma_D$ is (almost) flat for $1.4 < x_B \leq 2$.

Electron scattering - scaling



B. Schmookler, M. Duer, et al. (CLAS collaboration), Nature 566, 354 (2019)

np dominance

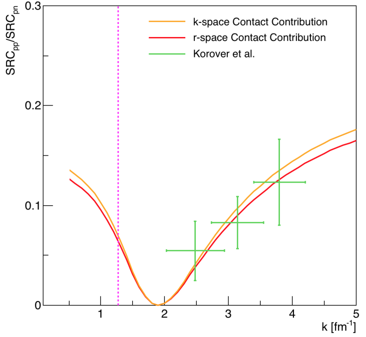
Electron scattering

The ratio of short range pp and np pairs is given by

$$\frac{SRC_{pp}(k)}{SRC_{pn}(k)} = \frac{\rho_{pp}(k)}{\rho_{pn}(k)} = \frac{C_{pp}^{s=0}|\tilde{\varphi}_{pp(nn)}^{s=0}(k)|^2}{C_{pn}^{s=0}|\tilde{\varphi}_{pn}^{s=0}(k)|^2 + C_{pn}^{s=1}|\tilde{\varphi}_{pn}^{s=1}(k)|^2}$$

np dominance

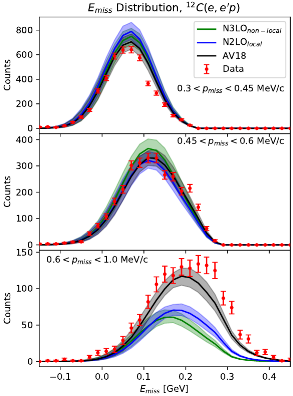
- np pairs much more common than pp or nn pairs
- The pp/np ratio is attributed to the node in the pp w.f.
- The tensor force fills this node in the np w.f.



Korover et al., PRL 113, 022501 (2014)

Electron Scattering $1.4 < x_B \leq 2$

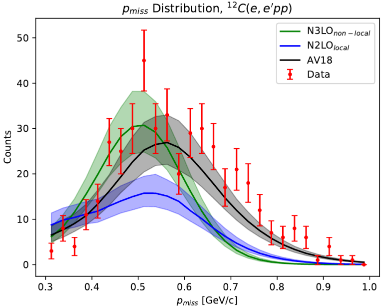
Generalized Contact Formalism



Contacts taken from **ab-initio** calculations

σ_{CM} taken from previous **experiments**.

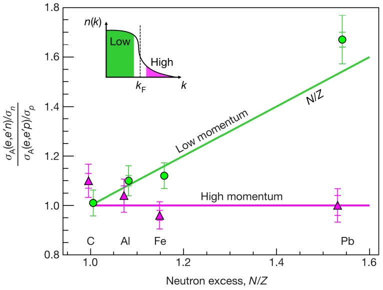
E^*_{A-2} is modified in the range $(0,30)$MeV.

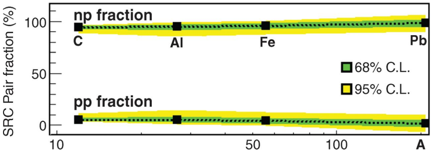


A. Schmidt *et al.* (CLAS Collaboration), Nature 578 (2020) 7796, 540-544

SRCs and Combinatorics

- The number of SRC pairs scales differently from the number of pairs
- np pairs with $s = 1$ are the **majority** of SRCs pairs.
- **Conclusion:** the number of high momentum protons is the same as neutrons
- This conclusion also holds when $N \neq Z$.





CLAS Collaboration, Nature **560**, 617 (2018) O. Hen, *et al.*, Science **346**, 614 (2014)

Mean-field theory - Shell Model

Contacts - Scale and Scheme dependence

Model (in)dependence of contact ratios

$$\frac{C^{\alpha}_{NN}(A)}{C^{\alpha}_{NN}(B)} = \frac{\langle \Psi_A | \hat{O}^{\alpha} | \Psi_A \rangle}{\langle \Psi_B | \hat{O}^{\alpha} | \Psi_B \rangle}$$

ratios weakly depend on interaction

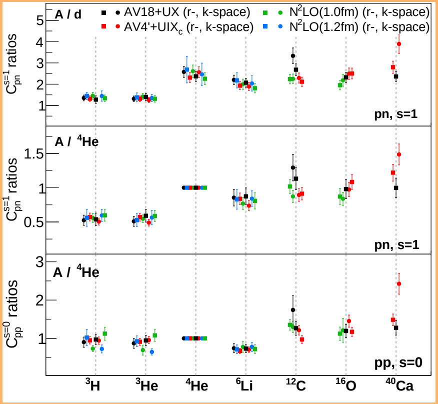
Conjecture (I. Talmi):

$$\frac{C^{\alpha}_{NN}(A)}{C^{\alpha}_{NN}(B)} \cong \frac{\langle \Phi_A | \hat{O}^{\alpha} | \Phi_A \rangle}{\langle \Phi_B | \hat{O}^{\alpha} | \Phi_B \rangle} .$$

with $|\Phi_A\rangle, |\Phi_B\rangle$ single Slater determinant **shell-model** w.f.

Cruz-Torres, Lonardoni, et al.

Nature Phys. 17, 306 (2021)



Contacts - Shell Model (LS)

The single particle states

$$\Phi_{nlm}(\boldsymbol{x}) = R_{nl}(x)Y_{lm}(\theta,\phi)$$

The matrix-element we need are:

$$\langle n_1,l_1,m_1,n_2,l_2,m_2|\delta(\boldsymbol{x}_1-\boldsymbol{x}_2)|n_1',l_1',m_1',n_2',l_2',m_2'\rangle =$$
$$= \int d\boldsymbol{x}_1 d\boldsymbol{x}_2\ \Phi^*_{n_1l_1m_1}(\boldsymbol{x}_1)\Phi^*_{n_2l_2m_2}(\boldsymbol{x}_2)$$
$$\times\ \delta(\boldsymbol{x}_1-\boldsymbol{x}_2)\,\Phi_{n_1'l_1'm_1'}(\boldsymbol{x}_1)\Phi_{n_2'l_2'm_2'}(\boldsymbol{x}_2)$$

CoM and relative coordinates,

$$\boldsymbol{x}_1 = \boldsymbol{X} + \frac{\boldsymbol{x}}{2} \quad ; \quad \boldsymbol{x}_2 = \boldsymbol{X} - \frac{\boldsymbol{x}}{2}\ ,$$

$$\langle n_1,l_1,m_1,n_2,l_2,m_2|\delta(\boldsymbol{x}_1-\boldsymbol{x}_2)|n_1',l_1',m_1',n_2',l_2',m_2'\rangle =$$
$$= \int d\boldsymbol{X}\ \Phi^*_{n_1l_1m_1}(\boldsymbol{X})\Phi^*_{n_2l_2m_2}(\boldsymbol{X})\Phi_{n_1'l_1'm_1'}(\boldsymbol{X})\Phi_{n_2'l_2'm_2'}(\boldsymbol{X})$$

Contacts - Shell Model (continue)

A product of an **angular** and **radial** integrals, i.e.

$$\langle n_1, l_1, m_1, n_2, l_2, m_2 | \delta(\boldsymbol{x}_1 - \boldsymbol{x}_2) | n'_1, l'_1, m'_1, n'_2, l'_2, m'_2 \rangle = I_{n_1 l_1 n_2 l_2 n'_1 l'_1 n'_2 l'_2} \, A^{l_1 m_1 l_2 m_2}_{l'_1 m'_1 l'_2 m'_2}$$

The Radial Integral

$$I_{n_1 l_1 n_2 l_2 n'_1 l'_1 n'_2 l'_2} = \int_0^\infty dX \; X^2 R_{n'_1 l'_1}(X) R_{n'_2 l'_2}(X) R_{n_1 l_1}(X) R_{n_2 l_2}(X)$$

The Angular Integral

$$A^{l_1 m_1 l_2 m_2}_{l'_1 m'_1 l'_2 m'_2} = \int d\hat{\boldsymbol{X}} \; Y^*_{l_1 m_1}(\hat{\boldsymbol{X}}) Y^*_{l_2 m_2}(\hat{\boldsymbol{X}}) Y_{l'_1 m'_1}(\hat{\boldsymbol{X}}) Y_{l'_2 m'_2}(\hat{\boldsymbol{X}}).$$

2-body operators - single Slater determinant

$$\langle O\rangle = \frac{1}{2}\sum_{a,b}^{F}\left[\langle a_1 a_2|O_{12}|a_1 a_2\rangle - \langle a_1 a_2|O_{12}|a_2 a_1\rangle\right]$$

a_1, a_2 - single particle states, F - the Fermi level.

The nuclear basis states are

$$|a\rangle = |nlm\nu\tau\rangle,$$

ν - spin projection, τ - isospin

$$C = \frac{1}{2}\sum_{a_1,a_2}^{F}\left[\langle a_1 a_2|\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)|a_1 a_2\rangle - \langle a_1 a_2|\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)|a_2 a_1\rangle\right]$$
$$= \frac{1}{2}\sum^{F} I_{n_1 l_1 n_2 l_2 n_1 l_1 n_2 l_2}\; A^{l_1 m_1 l_2 m_2}_{l_1 m_1 l_2 m_2}\left[1 - \langle \nu_1\tau_1\nu_2\tau_2|\nu_2\tau_2\nu_1\tau_1\rangle\right].$$

Turns out (R. Yankovich) that $A^{l_1 m_1 l_2 m_2}_{l_1 m_1 l_2 m_2} \approx \frac{1}{4\pi}$

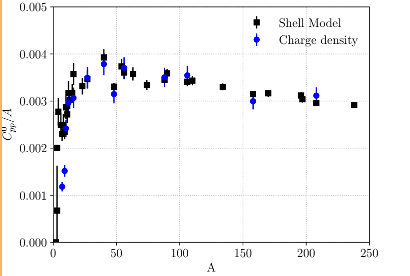
Verification

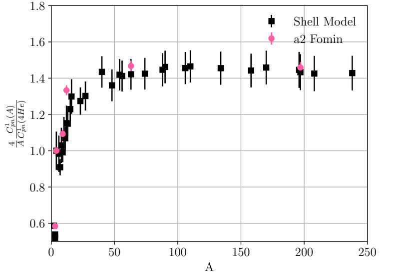
Conjecture:

$$\frac{C^{\alpha}_{NN}(A)}{C^{\alpha}_{NN}(B)} \cong \frac{\langle \Phi_A | \hat{O}^{\alpha} | \Phi_A \rangle}{\langle \Phi_B | \hat{O}^{\alpha} | \Phi_B \rangle} \, .$$

$|\Phi_A\rangle, |\Phi_B\rangle$ single Slater determinant
shell-model w.f.

Yankovich, Pazy, Barnea, PRC (2025)



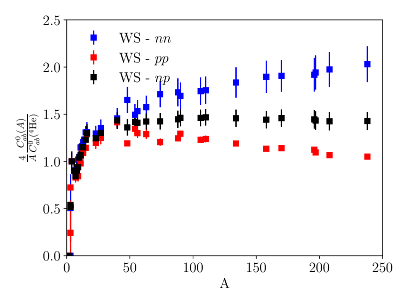


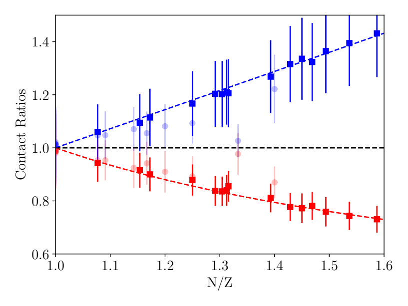
Spin zero contacts

For nuclei with $A > 50$ the contact ratio $C^0_{nn}(A)/C^0_{np}(A)$ presents a rather linear dependence on N/Z, Same for $C^0_{pp}(A)/C^0_{np}(A)$ with Z/N.

$$\frac{C^0_{nn}}{C^0_{np}} \approx \left[1 + \beta\left(\frac{N}{Z} - 1\right)\right]$$
$$\frac{C^0_{pp}}{C^0_{np}} \approx \left[1 + \beta\left(\frac{Z}{N} - 1\right)\right] ,$$

with $\beta \approx 0.72$.





Conjecture

- Assume N_a, N_b particles of species a, b in volume $\Delta\Omega$
- Further assume that the **correlation volume** is v_{ab}

Then averege number of correlated pairs is

$$\Delta C_{ab} = N_a N_b \frac{v_{ab}}{\Delta\Omega} = \rho_a \rho_b v_{ab} \Delta\Omega$$

ρ_a, ρ_b - densities of types a, b

Replacing v_{ab} by L^s_{ab} (honoring Levinger)

$$\boxed{C^S_{ab}(A) = L^S_{ab} \int d\Omega \; \rho_a \rho_b \approx L^S_{ab} \rho_a \rho_b \Omega_{ab}}$$

Ω_{ab} - the relevant nuclear volume

Contacts - A simple model

Yankovich, et al (2025)

$$C^s_{ab}(A) = L^s_{ab}\ \rho_a \rho_b \Omega_{ab}$$

ρ_a, ρ_b - densities of types a, b

Ω_{ab} - the relevant nuclear volume

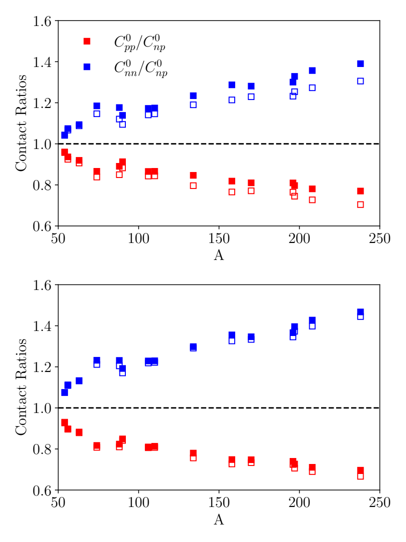
$$C^0_{nn}(A) = L^0_{nn} \frac{N^2}{R_N^3}$$

$$C^0_{np}(A) = L^0_{np} \frac{NZ}{R_N^3 R_P^3} R^3$$

$$C^0_{pp}(A) = L^0_{pp} \frac{Z^2}{R_P^3}$$

top - Universal WS parameters

bottom - SWV parameters.



HFB Calculations

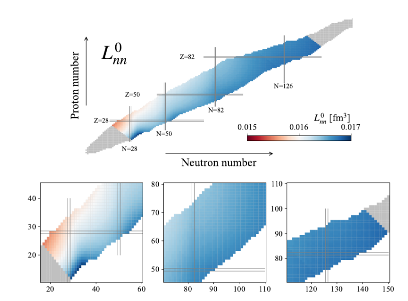
T. Liang et al., PLB 863 139351 (2025)

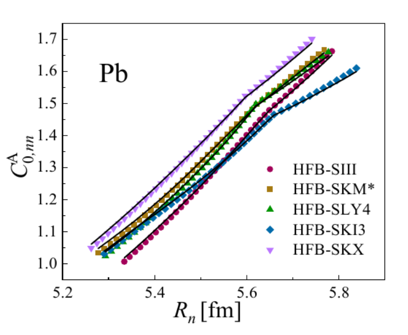
HFB many-body approch to calculate the nuclear w.f.
Used the matching method to calculate the contacts.

$$C^0_{nn}(A) = L^0_{nn}\frac{N^2}{R_N^3}$$

$$C^0_{np}(A) = L^0_{np}\frac{NZ}{R_N^3 R_P^3}R^3$$

$$C^0_{pp}(A) = L^0_{pp}\frac{Z^2}{R_P^3}$$





The YPB laws

Phys. Lett. B 863 (2025) 139351

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Letter

Universal laws for nuclear contacts

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ABSTRACT

The nuclear contact characterizes the nucleon-nucleon pairs in close proximity and serves as an important tool for studying the short-range correlations (SRCs) within atomic nuclei. While they have been extracted for selected nuclei, the investigation of their behavior across the nuclear chart remains limited. Very recently, Yankovich, Pazy, and Barnea have proposed a set of universal laws (YPB laws) to describe the correlation between nuclear contacts and nuclear radii and tested their laws for a small number of nuclei by using the Woods-Saxon mean-field model Yankovich et al. (2021) [32]. In this Letter, we extend their study to a majority part of the nuclear chart within the framework of the Skyrme Hartree-Fock-Bogolyubov model, which incorporates several essential beyond-mean-field features and offers a more accurate description of the bulk properties of atomic nuclei. Our results suggest that the YPB laws hold as a good approximation for different nuclear mass regions, with minor deviations attributed to, e.g., isospin-breaking effects. Our work lays a firm foundation for future applications of the YPB laws in finite nuclei and provides new evidence for the long-range nature of the relative abundance of short-range pairs.

The YPB laws

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Physics Letters B

Abstract

The nuclear contact characterizes the nucleon-nucleon pairs in close proximity and serves as an important tool for studying the short-range correlations (SRCs) within atomic nuclei. While they have been extracted for selected nuclei, the investigation of their behavior across the nuclear chart remains limited. Very recently, Yankovich, Pazy, and Barnea have proposed a set of universal laws (YPB laws) to describe the correlation between nuclear contacts and nuclear radii and tested their laws for a small number of nuclei by using the Woods-Saxon mean-field model [R. Yankovich, E. Pazy, and N. Barnea, arXiv:2407.15068 (2021)]. In this Letter, we extend their study to a majority part of the nuclear chart within the framework of the Skyrme Hartree-Fock-Bogolyubov model, which incorporates several essential beyond-mean-field features and offers a more accurate description of the bulk properties of atomic nuclei. Our results suggest that the YPB laws hold as a good approximation for different nuclear mass regions, with minor deviations attributed to, e.g., isospin-breaking effects. Our work lays a firm foundation for future applications of the YPB laws in finite nuclei and provides new evidence for the long-range nature of the relative abundance of short-range pairs.

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Conclusions

Summary and Conclusions

Factorization and universality in nuclear physics

- The contact formalism was generalized and applied to NP.
- **Surprisingly, it seems to be working...**
- Many relations were derived. Many more still await.
- Coupled Cluster theory provides the **"missing link"** between the contact formalism and the underlying many-body physics.
- 3-body SRCs
- Relativistic effects not completely under control
- Different observables

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