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Electric Corrections to π-π Scattering lengths in the Linear Sigma Model and Something Else

M. Loewe

Universidad San Sebastián, Santiago, Chile

In collaboaration with R. Cádiz and R. Zamora

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This presentation is based on the article

Electric corrections to π-π scattering lengths in the linear sigma model.
R. Cádiz, M. Loewe and R. Zamora

Phys. Rev. D 109, 116004 (2024)

and

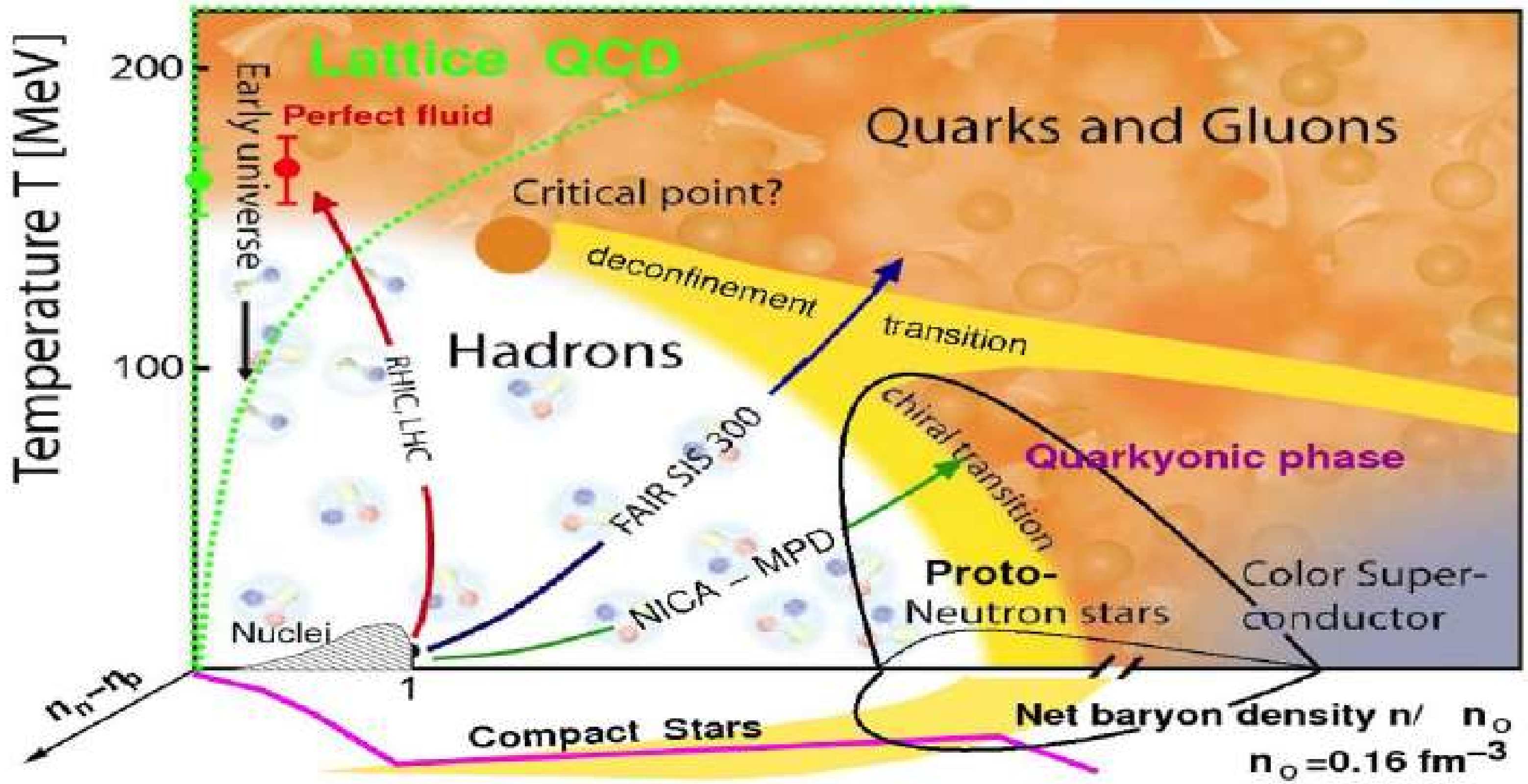
Thermal Regge Trayectories, arXiv: 2506.014979 [hep-ph] (submitted to PRD)

Relativistic heavy Ion collision experiments (RHIC, LHC, FAIR and NICA in the near future) open a window to explore initial stages of the universe.

A high temperature regime, huge magnetic fields, together with density effects, affect in a dramatic way the physics of strong interactions.

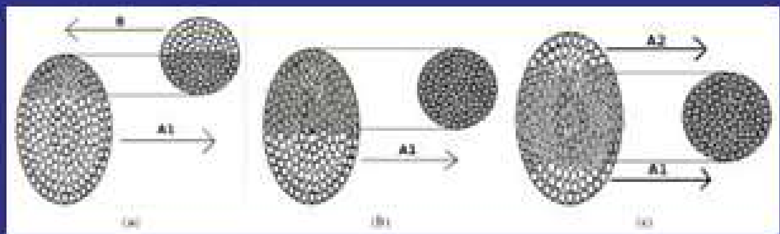
Several phase transitions occur: Deconfinement, Chiral Restoration, transition to a Quarkyonic phase

QCD phase diagram



Today I want to mention a different scenario where also huge electric fields appear: Asymmetric peripheral relativistic heavy ion collisions as, for example, Cu+Au, Cu+Pb collisions.

Due to the imbalance in the number of charges in each nuclei a strong electric field, $\perp$ to B, appears



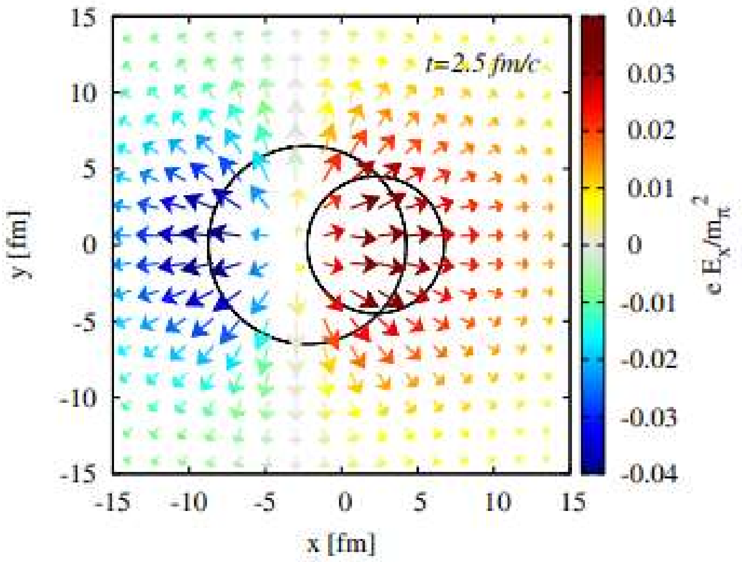


Fig. 1. Electric field generated in the transverse plane of Cu+Au collisions at $\sqrt{s_{NN}} = 9\,\text{GeV}$, $b = 4.5\,\text{fm}$ and $t = 2.5\,\text{fm}/c$. The direction of the arrows indicates the field direction projected onto the reaction plane and the length is proportional to the electromagnetic strength shown in color.

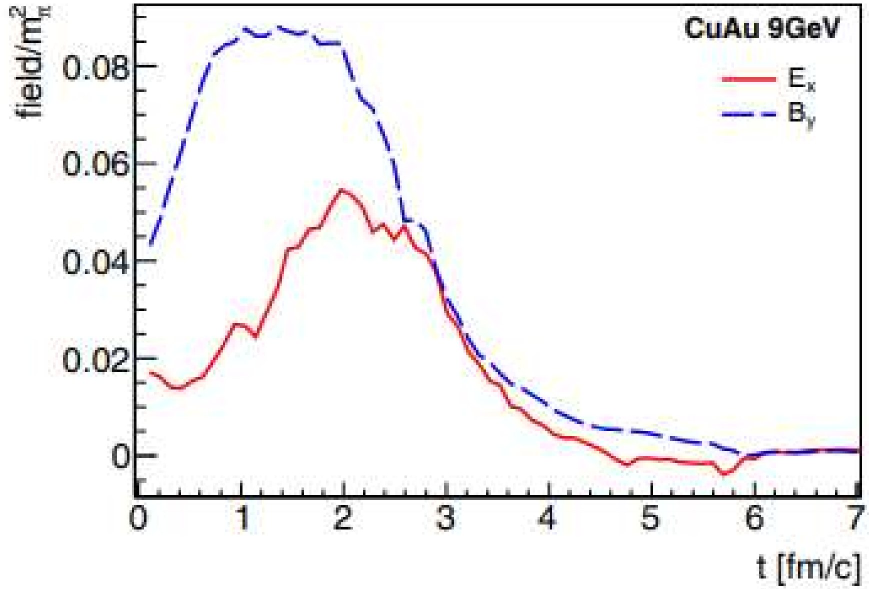


Fig. 2. The time dependent components of the average electric E_x and magnetic B_y field generated in the center of Cu+Au collision at $b = 4.5\,\text{fm}$ and $\sqrt{s_{NN}} = 9\,\text{GeV}$.

Extracted from V. Toneev, O. Rogachevsky and V. Voronyuk: Eur. Phys. J. A. (2016) 53: 264

In general, a π-π scattering amplitude has the form (greek indeces refer to Isospin indeces)

$$T_{\alpha\beta\,\gamma\delta} = A(s,t,u)\delta_{\alpha\beta}\delta_{\delta\gamma} + A(t,s,u)\delta_{\alpha\gamma}\delta_{\beta\gamma} + A(u,t,s)\delta_{\alpha\delta}\delta_{\beta\gamma}$$

Using the following projectors

$$P_0 = \frac{1}{3}\delta_{\alpha\beta}\delta_{\gamma\delta},$$

$$P_1 = \frac{1}{2}\left(\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}\right),$$

$$P_2 = \frac{1}{2}\left(\delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma} - \frac{2}{3}\delta_{\alpha\beta}\delta_{\gamma\delta}\right)$$

In this way we find the following Isospin dependent amplitudes

$$T^0 = 3A(s,t,u) + A(t,s,u) + A(u,t,s),$$
$$T^1 = A(t,s,u) - A(u,t,s),$$
$$T^2 = A(t,s,u) + A(u,t,s).$$

The elastic regime refers to the region $4M_\pi^2 < s < 16M_\pi^2$

See J. Gasser and H. Leutwyler: Ann. Phys. 158, 142 (1984); J. Donoghue, E. Golowich, and B. Holstein: The Dynamics of the Standard Model, Cambridge 1996

Scattering lengths were introduced as appropriate paramaters for the description of nucleon-nucleon and pion nucleon scattering in the good old times.
Consider a partial wave expansion, projected into some isospin channel I.

$$T^I(s,t,u) = 32\pi \sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\cos\theta) T_\ell^I(s)$$

Coming close to the threshold

$$T_\ell^I = \left(\frac{s}{s-4m_\pi^2}\right)^{1/2} \frac{1}{2i} \left(e^{2i\delta_\ell^I(s)} - 1\right)$$

In fact, the previous expression can be expanded as

$$\Re\left(T_\ell^I\right) = \left(\frac{p^2}{m_\pi^2}\right)^\ell \left(a_\ell^I + \frac{p^2}{m_\pi^2} b_\ell^I + \cdots\right)$$

$$p^2 \equiv (s - 4m_\pi^2)/4$$

Where a_l^I are the so called scattering lengths. The next coefficients, b_l^I are the scattering slopes. It can be shown that

$$a = \lim_{p\to 0} \frac{\tan\delta_0}{p} = -\lim_{p\to 0} \frac{\delta_0}{p}$$

This definition can be extended to the other angular momentum channels

$$a_\ell = \lim_{p\to 0} \frac{\tan\delta_\ell}{p} = -\lim_{p\to 0} \frac{\delta_\ell}{p}$$

In general we have the following order

$$|a_0| > |a_1| > |a_2| > \cdots$$

At very low energies, scattering is dominated by the s-channel amplitude (isotropic scattering) and the total cross section is given by $\sigma = 4\pi a_0^2$. Notice that the Martin inequalities are satisfied

A. Martin, Nuovo Cimento A47 (1967) 265

$$a^I_{l+2} \leqslant a^I_l \frac{(l+1)(l+2)}{4(2l+3)(2l+5)}$$

π-π scattering lengths were measured first by L. Rossellet et al, PRD 15, 574 (1977) using pions coming from the decay

$$K^+ \to \pi^+ \pi^- e^+ \nu$$

The interaction occurs between two real pions, the only hadrons in the final state. The quantum states of the di-pion in the decay are L = 0, I = 0 and L =1, I = 1. The invariant mass distribution of the di-pions has a maximum relatively close to the π-π threshold.

Later: B. Adeva et al, Phys. Lett. B 704 (2011) 24 (Dirac experiment)

The DIRAC experiment at CERN has achieved a sizeable production of $\pi^+\pi^-$ atoms and has significantly improved the precision on its lifetime determination. From a sample of 21 227 atomic pairs, a 4% measurement of the S-wave $\pi\pi$ scattering length difference $|a_0 - a_2| = (0.2533^{+0.0080}_{-0.0078}|_{\text{stat}}{}^{+0.0078}_{-0.0073}|_{\text{syst}})M^{-1}_{\pi^+}$ has been attained, providing an important test of Chiral Perturbation Theory.

A pionium atom, with a radius of 378 fm, decays into a pair of neutron pions.

$$\Gamma_{2\pi^0} = \frac{2}{9}\alpha^3 p^\star (a_0 - a_2)^2 (1+\delta) M_{\pi^+}^2$$

$$p^\star = \sqrt{M_{\pi^+}^2 - M_{\pi^0}^2 - (1/4)\alpha^2 M_{\pi^+}^2}$$

More recentely: X.-H Liu, F.-K. Guo and E. Epelbaum, Eur. Phys. J. C73, 2284, (2013) were able to extract the scattering lengths from heavy qurakonium decays

Charge-exchange rescattering $\pi^+\pi^- \to \pi^0\pi^0$ leads to a cusp effect in the $\pi^0\pi^0$ invariant mass spectrum of processes with $\pi^0\pi^0$ in the final state which can be used to measure $\pi\pi$ S-wave scattering lengths. Employing a non-relativistic effective field theory, we discuss the possibility of extracting the scattering lengths in heavy quarkonium $\pi^0\pi^0$ transitions. The transition $\Upsilon(3S) \to \Upsilon(2S)\pi^0\pi^0$ is studied in details. We discuss the precision that can be reached in such an extraction for a certain number of events.

$$\psi'(P_{\psi'}) \to \pi^0(p_1)\pi^0(p_2)J/\psi(p_3),$$

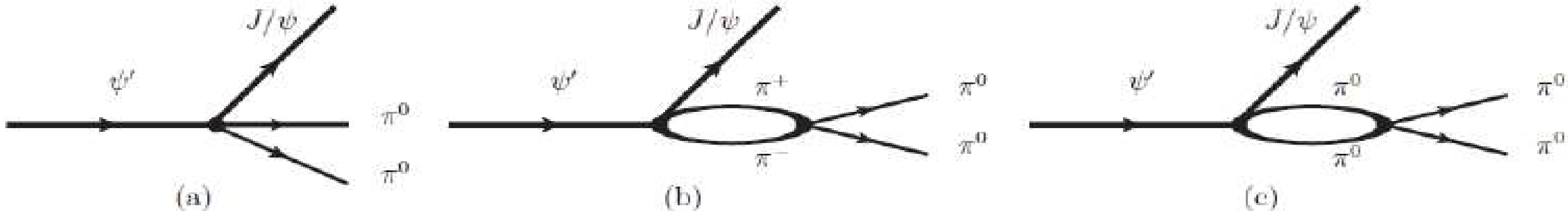


Figure 1: $\psi' \to J/\psi\pi^0\pi^0$ via tree diagram and $\pi\pi$ rescattering diagrams.

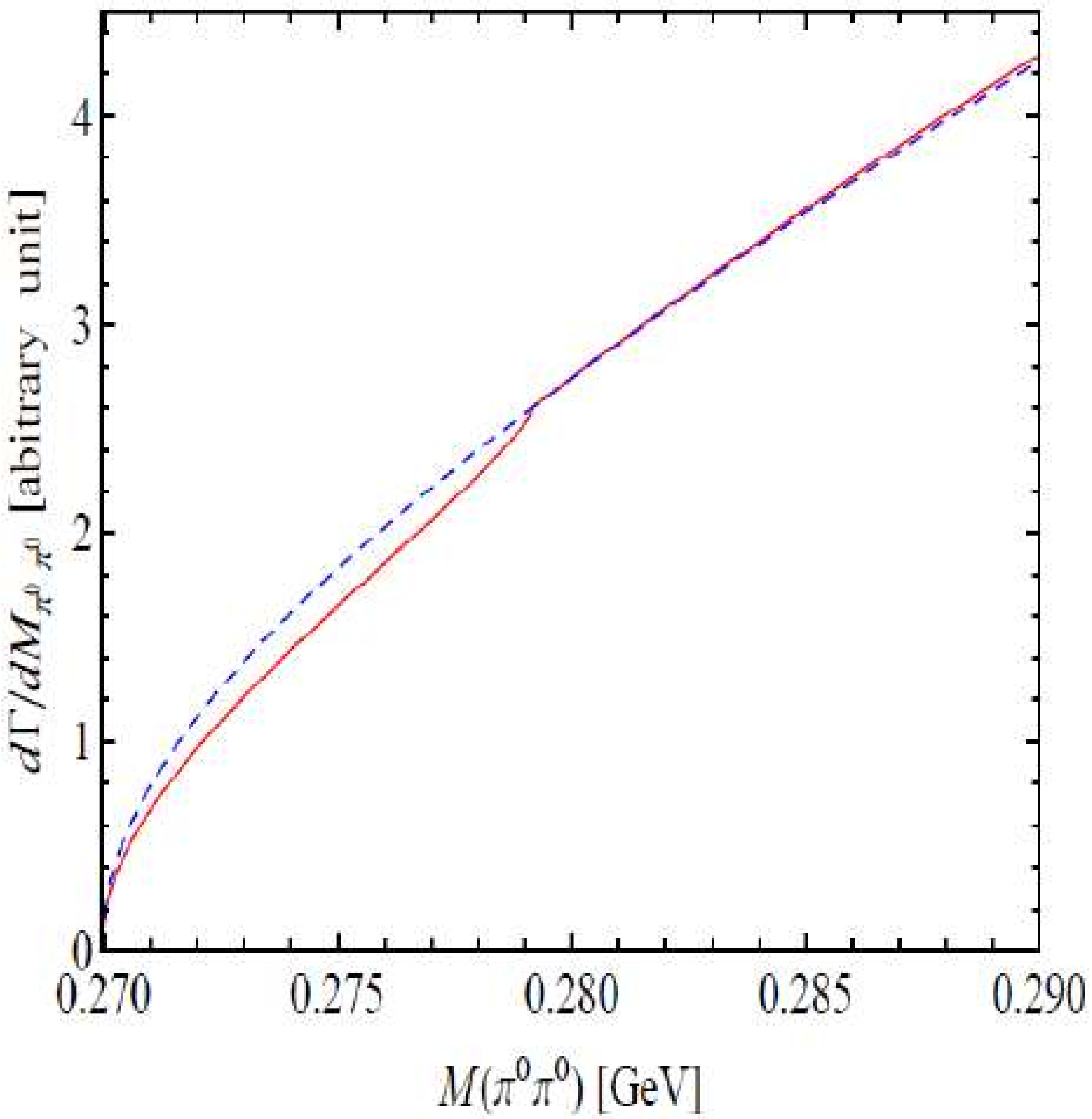


Figure 4: The cusp effect at the $\pi^+\pi^-$ threshold in the reaction $\Upsilon(3S) \to \Upsilon(2S)\pi^0\pi^0$ calculated in the NREFT framework (solid line). The dashed line shows the result without charge-exchange rescattering.

Linear sigma model. Gell-Mann-Levy 1960

$$\mathcal{L} = \bar{\psi}[i\gamma^\mu \partial_\mu - m_\psi - g(s + i\vec{\pi}\cdot\vec{\tau}\gamma_5)]\psi + \frac{1}{2}[(\partial\vec{\pi})^2 + m_\pi^2 \vec{\pi}^2] + \frac{1}{2}[(\partial\sigma)^2 + m_\sigma^2 s^2] - \lambda^2 v s(s^2 + \vec{\pi}^2) - \frac{\lambda^2}{4}(s^2 + \vec{\pi}^2)^2 + (\varepsilon c - v m_\pi^2)s.$$

The σ field has been shifted $\sigma = s + v$

$\langle s \rangle = 0$

$v = \langle \sigma \rangle$

cσ is the term that breaks the SU(2)xSU(2) symmetry explicitly

All masses are determined by v, the expectation value of the σ field

$m_\psi = gv$ $m_\pi^2 = \mu^2 + \lambda^2 v^2$ $m_\sigma^2 = \mu^2 + 3\lambda^2 v^2$

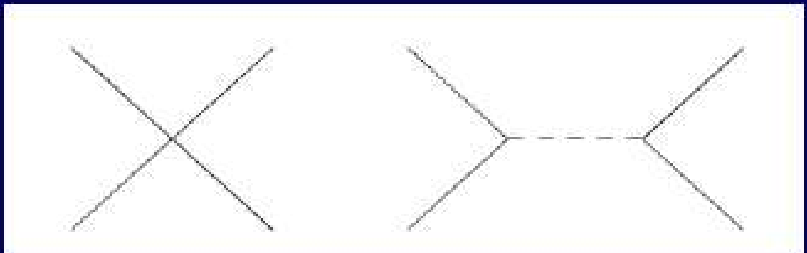
Perturbation theory at the tree level allows to identify $f_\pi = v$

$$T^0(s,t,u) = -10\lambda^2 - \frac{12\lambda^4 v^2}{s - m_\sigma^2} - \frac{4\lambda^4 v^2}{t - m_\sigma^2} - \frac{4\lambda^4 v^2}{u - m_\sigma^2}$$

$$T^1(s,t,u) = \frac{4\lambda^4 v^2}{u - m_\sigma^2} - \frac{4\lambda^4 v^2}{t - m_\sigma^2},$$

$$T^2(s,t,u) = -4\lambda^2 - \frac{4\lambda^4 v^2}{t - m_\sigma^2} - \frac{4\lambda^4 v^2}{u - m_\sigma^2}.$$

Tree level diagrams for the π-π scattering lengths



	Experimental results	Chiral perturbation theory	Linear sigma model
a_0^0	0.218 ± 0.02	$\frac{7m_\pi^2}{32\pi f_\pi^2} = 0.16$	$\frac{10m_\pi^2}{32\pi f_\pi^2} = 0.22$
b_0^0	0.25 ± 0.03	$\frac{m_\pi^2}{4\pi f_\pi^2} = 0.18$	$\frac{49m_\pi^2}{128\pi f_\pi^2} = 0.27$
a_0^2	-0.0457 ± 0.0125	$\frac{-m_\pi^2}{16\pi f_\pi^2} = -0.044$	$\frac{-m_\pi^2}{16\pi f_\pi^2} = -0.044$
b_0^2	-0.082 ± 0.008	$\frac{-m_\pi^2}{8\pi f_\pi^2} = -0.089$	$\frac{-m_\pi^2}{8\pi f_\pi^2} = -0.089$
a_1^1	0.038 ± 0.002	$\frac{m_\pi^2}{24\pi f_\pi^2} = 0.030$	$\frac{m_\pi^2}{24\pi f_\pi^2} = 0.030$
b_1^1	...	0	$\frac{m_\pi^2}{48\pi f_\pi^2} = 0.015$

The (charged) bosonic propagator in the present of a constant electric field

$$D(p) = \int_0^\infty ds \frac{e^{-s(\frac{\tanh(qiEs)}{qiEs}p_\parallel^2 + p_\perp^2 + m^2)}}{\cosh(qiEs)}$$

For small values of the electric field the propagator goes into

$$D(p) \approx \frac{1}{p^2 + m^2} - (qE)^2 \left(-\frac{1}{(p^2 + m^2)^3} + \frac{2p_{\parallel}^2}{(p^2 + m^2)^4} \right).$$

We will use this propagator for the charged pions in our calculation

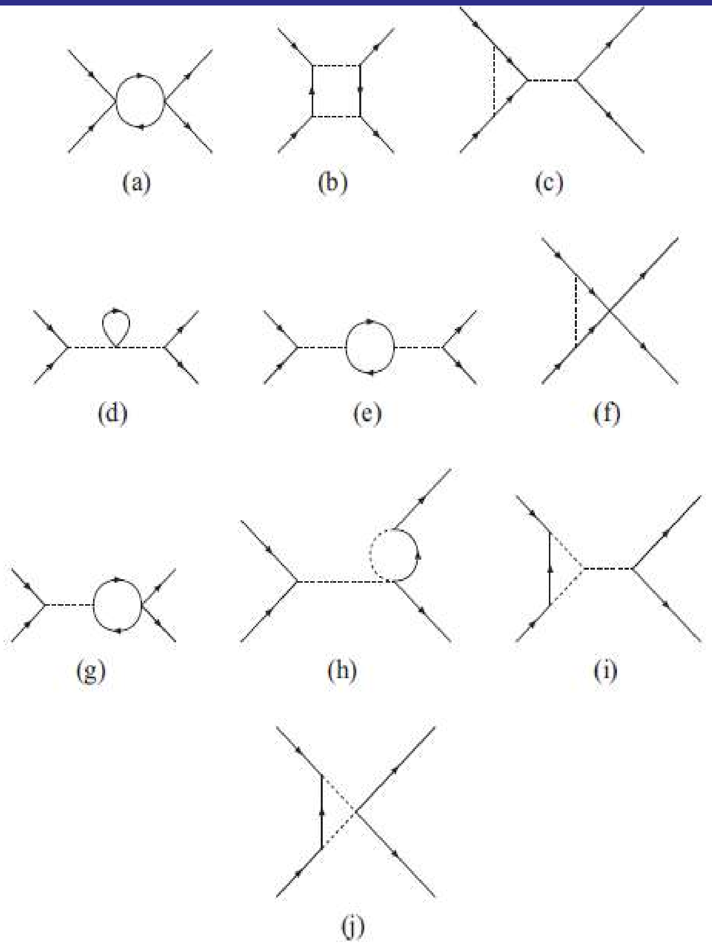


FIG. 1. (a)–(j) s channel diagrams.

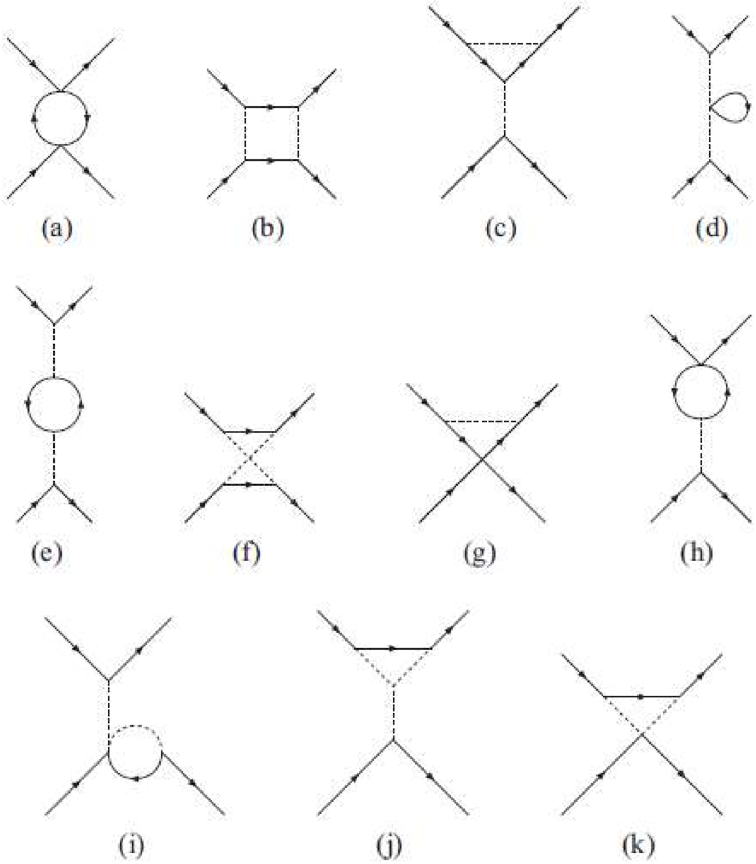


FIG. 2. (a)–(k) t channel diagrams.

The relevant diagrams

$$T^0_{1L} = 3A(s,t,u) + 2A(t,s,u),$$

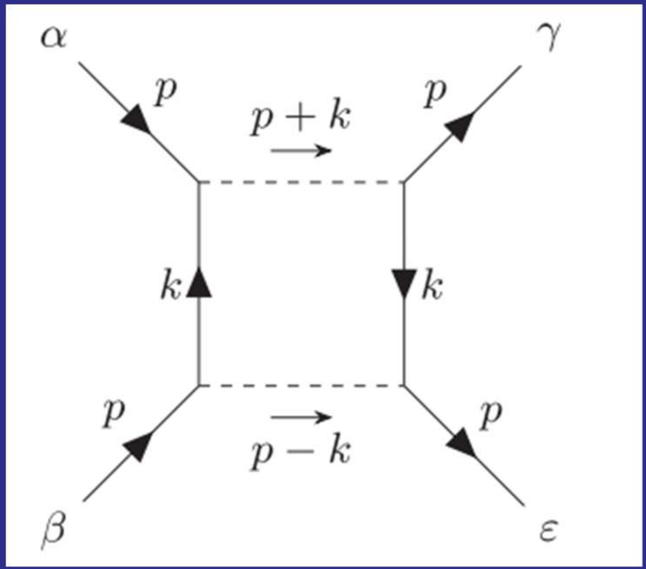
$$T^1_{1L} = 0,$$

$$T^2_{1L} = 2A(t,s,u).$$

$$T^0_{1L} = 32\lambda^8 v^4\left(\frac{9}{2}I_{B1} + I_{B2} + I_9\right) + I_3\left(\frac{144\lambda^8 v^4}{4m_\pi^2 + m_\sigma^2} + 120\lambda^6 v^2\right) + I_4\left(\frac{32\lambda^8 v^4}{m_\sigma^2} + 80\lambda^6 v^2\right) + I_6\left(\frac{432\lambda^8 v^4}{4m_\pi^2 + m_\sigma^2} + 72\lambda^6 v^2\right)$$
$$+ I_5\left(\frac{96\lambda^8 v^4}{m_\sigma^2} + 16\lambda^6 v^2\right) + I_7\left(\frac{216\lambda^6 v^3}{(4m_\pi^2 + m_\sigma^2)^2} + \frac{48\lambda^6 v^3}{m_\sigma^4}\right) + I_8\left(\frac{72\lambda^6 v^2}{4m_\pi^2 + m_\sigma^2} + \frac{16\lambda^6 v^2}{m_\sigma^2}\right)$$
$$+ I_2\left(300\lambda^4 + \frac{432\lambda^8 v^4}{(4m_\pi^2 + m_\sigma^2)^2} + \frac{360\lambda^6 v^3}{4m_\pi^2 + m_\sigma^2}\right) + I_1\left(120\lambda^4 + \frac{96\lambda^8 v^4}{m_\sigma^4} + \frac{80\lambda^6 v^3}{m_\sigma^2}\right), \quad (2$$

$$T^2_{1L} = 160\lambda^8 v^4(I_{B2} + I_9) + I_4\left(\frac{160\lambda^8 v^4}{m_\sigma^2} + 160\lambda^6 v^2\right) + I_5\left(\frac{480\lambda^8 v^4}{m_\sigma^2} + 80\lambda^6 v^2\right) + \frac{240\lambda^6 v^3}{m_\sigma^4} I_7 + \frac{80\lambda^6 v^2}{m_\sigma^2} I_8$$
$$+ I_1\left(360\lambda^4 + \frac{480\lambda^8 v^4}{m_\sigma^4} + \frac{400\lambda^6 v^3}{m_\sigma^2}\right). \quad (2$$

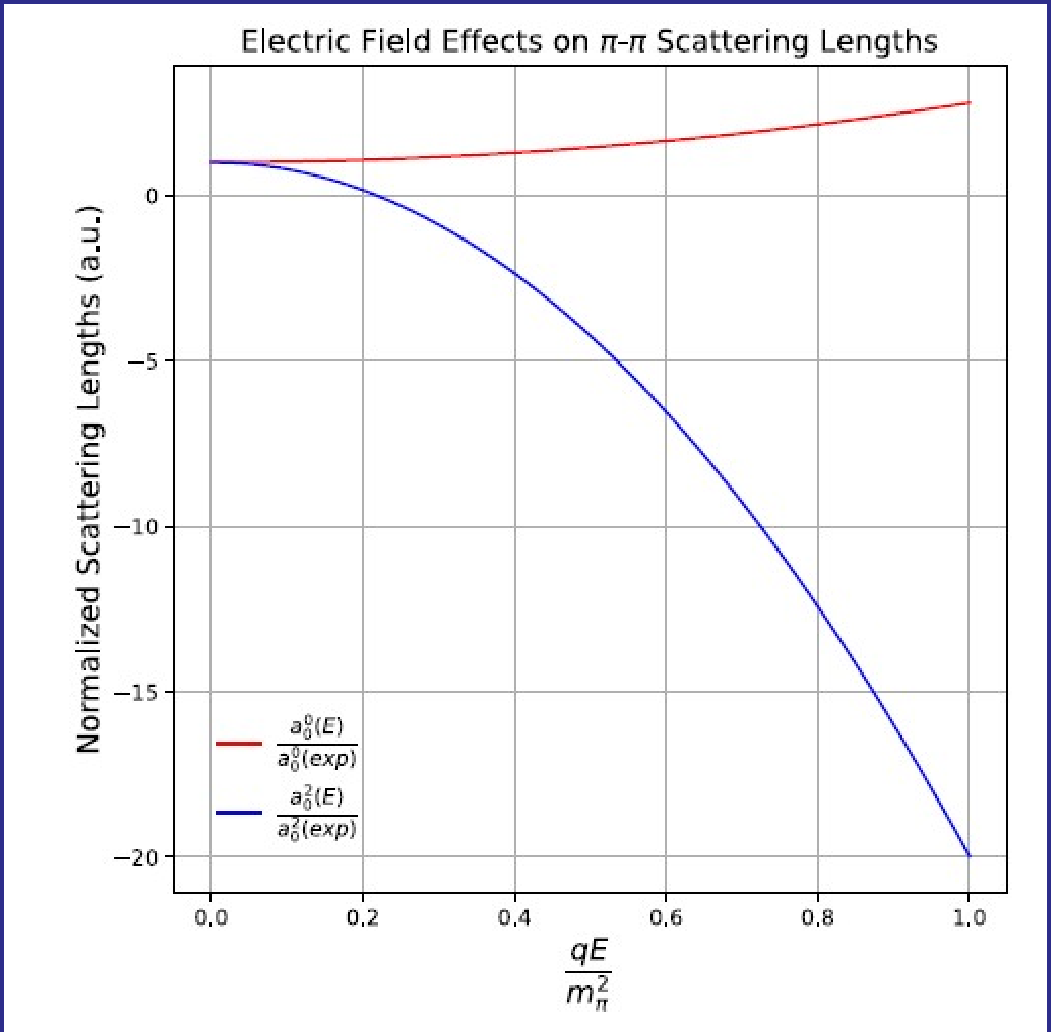
A novelty of our article was the calculation of the box diagram



Which was previously neglected (normally the σ-propagator is pinched due to the high mass)

In the previous expresión all I'$_i$ denote certain integrals you might find in the original article

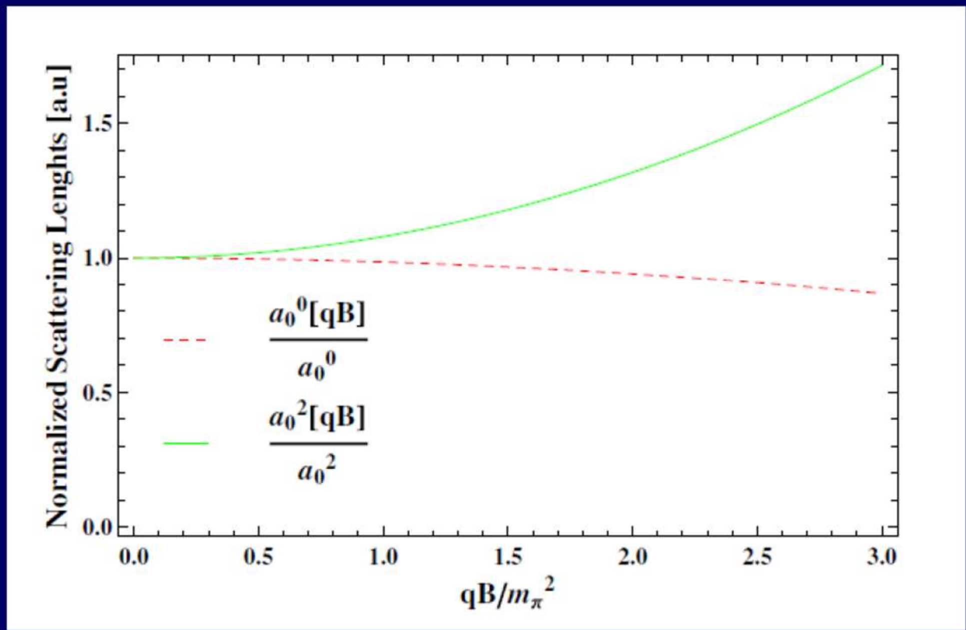
Behavior of the scattering lengths: we have used $m_\pi = 140$ MeV, $m_\sigma = 550\ MeV$ V = 89 MeV, λ



It is interesting to see that this result is opposite respect to the same calculation in the presence of a constant magnetic field. Also, the effects of the electric field are much bigger than the magnetic effects.

Temperature and electric field effects go in opposite direction respect to magnetic effects. Thermo- magnetic effects on Pi-pi scattering lengths were calculated in M. Loewe, E. Muñoz, and R. Zamora, Phys. Rev. D 100 (2019) 116006

Using m_σ= 550 MeV and m_π = 140 MeV



In general there is an agreement in the community that it will be extremely difficult to disantangle magnetic from termal effects.

Therefore, propositions like the one presentes by A. Ayala and Collaborators on the Z decay into muons are very attractive. (keeping in mind the cloud of the LLL approximation)

Collisions between asymmetric nuclei, due to the important role of the electric field, might be an interesting window for exploring this kind of physics.

Let us go now into “Something else”.
Here I want to mention some recent results for thermal effects in the Regge behavior of scattering amplitudes

In general, we refer to an amplitude as regeized when it becomes analytical in its dependence on J, in the high energy limit,

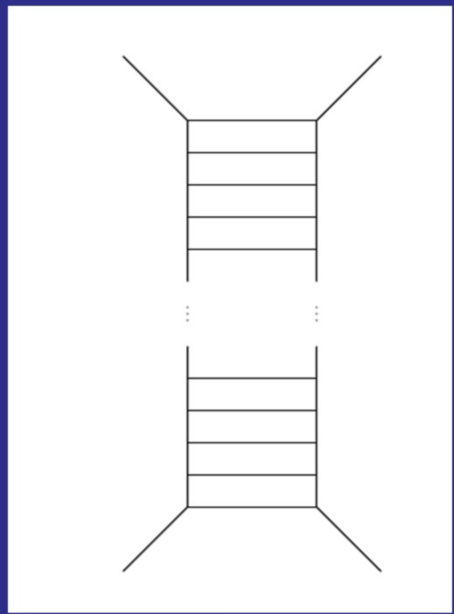
$$s \to \infty, t\ finite$$

becoming
$A(s, t) \sim s^{\alpha(t)}$

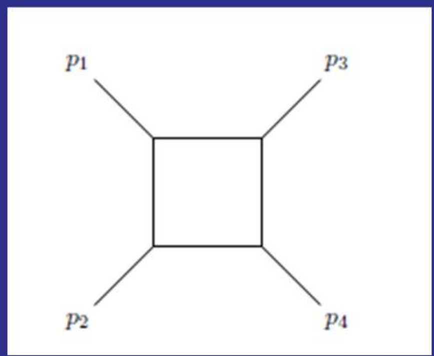
This happens, normally, when resumation of diagrams are considered.

See, for example: P. D. Collins: “An Introduction to Regge Theory and High Energy Physics”, Cambridge University Press and references therein. , For this purpose we use the $\lambda\varphi^3$ theory

At T = 0, let us considered resummation of a Ladder diagram



The building element is the box diagram



It can be shown, see technical details in the article, that in the high energy limit, the box diagram (M) behaves like

$$\mathcal{M} = \lambda^2 K(t) \left(\frac{\ln s}{s} \right)$$

where

$$K(t) = \frac{\lambda^2}{24\pi^2 \sqrt{t(4m^2 - t)}} \arctan\left(\sqrt{\frac{t}{4m^2 - t}} \right).$$

For the ladder diagram with n rungs, we have

$$\mathcal{M}_n = \frac{\lambda^2}{s} \frac{[K(t) \ln s]^{n-1}}{(n-1)!}$$

Thus, summing an infinite series of Ladder diagrams yields the asymptotic behavior of the amplitude as

$$\begin{aligned}
A(s,t) &= \sum_{n=1}^{\infty} \mathcal{M}_n \\
&= \frac{\lambda^2}{s} \sum_{n=1}^{\infty} \frac{[K(t)\ln s]^{n-1}}{(n-1)!} \\
&= \frac{\lambda^2}{s} \cdot e^{K(t)\ln s} \\
&= \lambda^2 s^{\alpha(t)},
\end{aligned}$$

Identifying, then, the Regge trayectory as

$$\alpha(t) = K(t) - 1.$$

When temperature appears, we have to use the well known prescription

$$\int \frac{d^4q}{(2\pi)^4} f(q) \to \frac{i}{\beta} \sum_n \int \frac{d^3q}{(2\pi)^3} f(\omega_n, \mathbf{q}),$$

for calculating the box diagram. The result can be decomposed as $\mathcal{M}_0 + \mathcal{M}_T$

where

$$\mathcal{M}_0 = -\lambda^4 \int_0^1 dx\, dy\, dz\, dw\, \delta(x+y+z+w-1) \times \left(\frac{\partial}{\partial \Delta}\right)^3 \int \frac{d^3q}{(2\pi)^3} \frac{1}{2(\mathbf{q}^2+\Delta)^{1/2}}, \quad (19)$$

and

$$\mathcal{M}_T = -\lambda^4 \int_0^1 dx\, dy\, dz\, dw\, \delta(x+y+z+w-1) \times \left(\frac{\partial}{\partial \Delta}\right)^3 \int \frac{d^3q}{(2\pi)^3} \frac{1}{(\mathbf{q}^2+\Delta)^{1/2}} e^{-\beta\sqrt{\mathbf{q}^2+\Delta}}. \quad (20)$$

It is posible to show that

$$\mathcal{M}_T = \lambda^2 B(t,\beta)\left(\frac{\ln s}{s}\right)$$

$$B(t,\beta) = -\lambda^2 \int_0^1 dz\, \frac{\beta^2 e^{-\beta\sqrt{tz(1-z)+m}}}{32\pi}$$

Inserting the temperature independent part and summing over an infinite number of rungs we get

$$\begin{aligned} A(s,t,\beta) &= \sum_{n=1}^{\infty} \mathcal{M}_{0,T,n} \\ &= \frac{\lambda^2}{s}\sum_{n=1}^{\infty} \frac{[(K(t)+B(t,\beta))\ln s]^{n-1}}{(n-1)!} \\ &= \frac{\lambda^2}{s}\cdot e^{(K(t)+B(t,\beta))\ln s} \\ &= \lambda^2 s^{\alpha(t,\beta)}, \end{aligned}$$

$$\alpha(t,\beta) = K(t) - 1 + B(t,\beta).$$

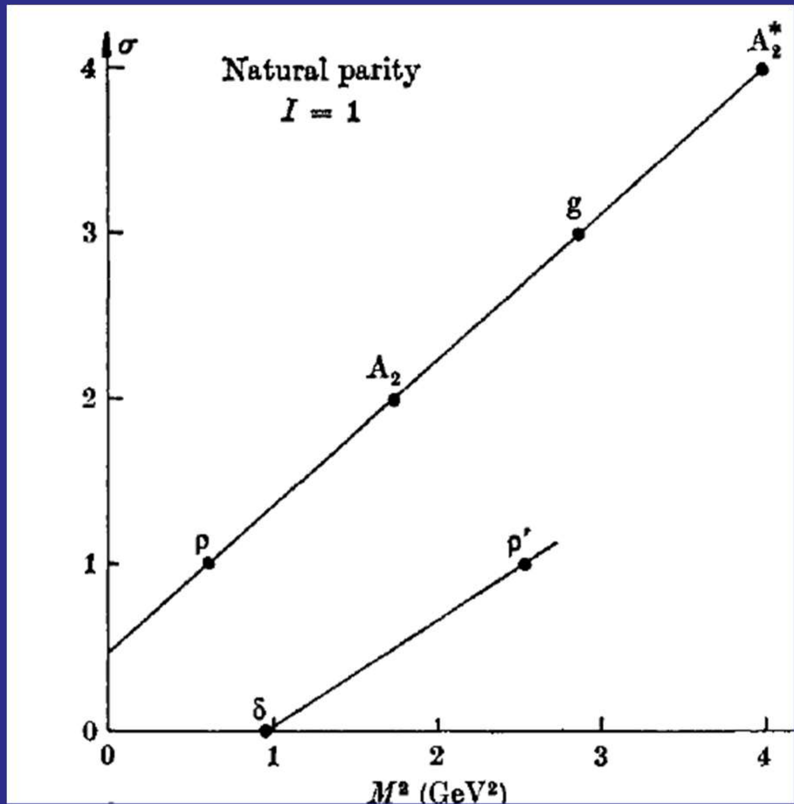
Expanding for small values of t

$$\alpha(t) = \alpha_0 + \alpha' t,$$

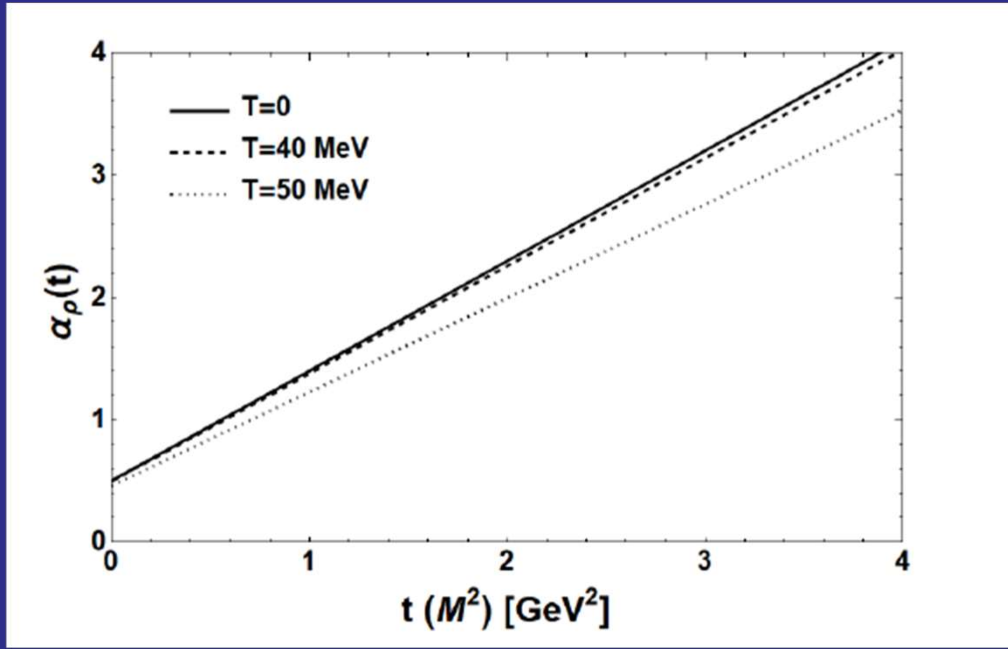
$$\alpha(t,\beta) = \frac{\lambda^2}{96\pi^2 m^2} - \lambda^2\beta^2\frac{e^{-m\beta}}{32\pi} + \lambda^2 t\left(\frac{1}{576\pi^2 m^4} - \beta^3\frac{e^{-m\beta}}{384\pi m}\right).$$

The Regge trayectories for the rho meson

$$\alpha_\rho(t) \approx 0.5 + 0.9t$$

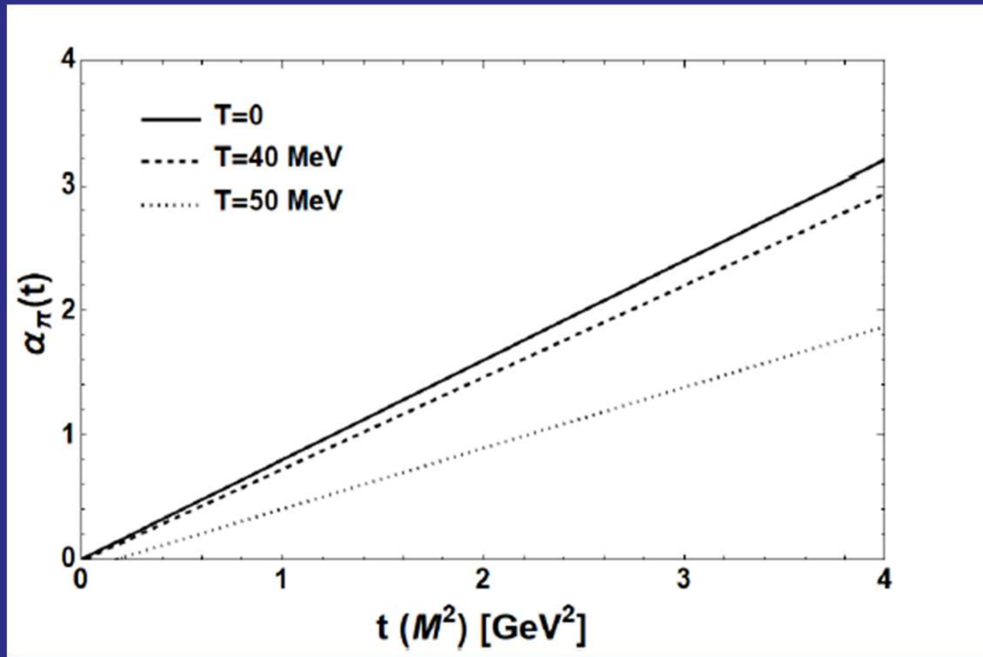


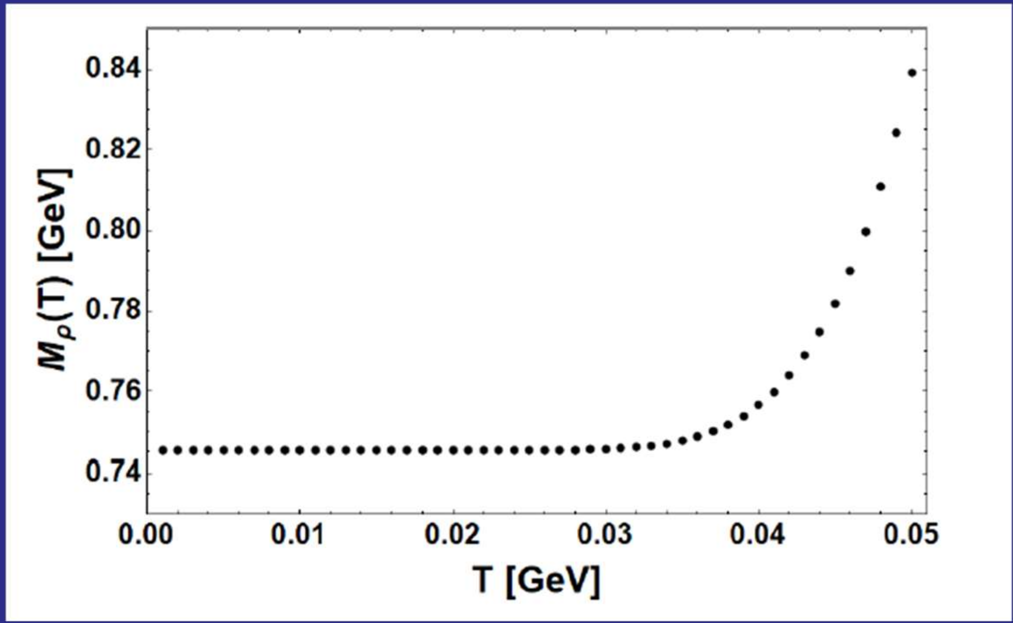
The slope diminishes with T



The same happens with other trayectories

This means that the mass increases with T





This behavior is surprisingly similar to the rho-mass evolution obtained from
Totally different approaches

Temperature dependence of the rho-meson mass and width

C.A. Dominguez[1], M. Loewe[2], J.C. Rojas[2]

[1] Institute of Theoretical Physics and Astrophysics, University of Cape Town, Rondebosch 7700, Cape, RSA
[2] Pontificia Universidad Católica de Chile, Santiago, Chile

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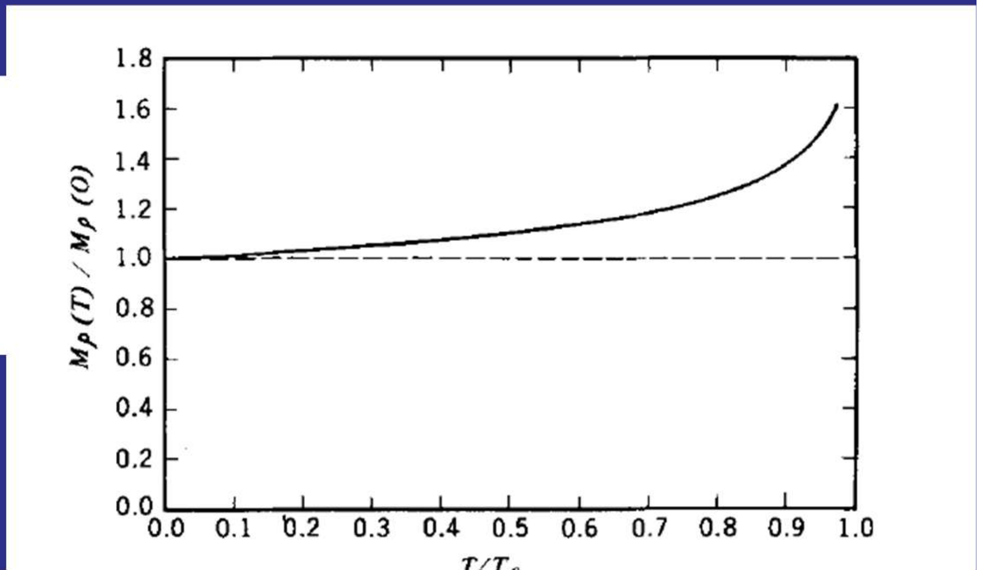

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As a kind of conclusión: New approaches or perspectives for physical problems are always welcome!!