# Spin alignment of the $\phi$ meson in the hadronic phase of the Quark Gluon Plasma

#### Based on v.C Phys.Re111 (2025) 1, 014914 and 25xx.xxxx In collaboration with E. Grossi and I. Zahed



Center for Frontiers in Nuclear Science



## **Motivation**

Spin vector polarization



STAR Phys.Rev.Lett. 126 (2021) 16, 162301

Alignment = spin tensor polarization



STAR, Nature 614 (2023) 7947

Leading theoretical model: strong force field fluctuations fitted from alignment data (first principle calculations are ongoing, see L. Oliva, QM2025). Sheng, Oliva, Liang, Wang, Wang Phys.Rev.Lett. 131 (2023) 4, 042304; Phys.Rev.D 109 (2024) 3, 036004, Sheng, Pu, Wang Phys.Rev.C 108 (2023) 5, 054902



Hydrodynamic evolution of the medium is not taken into account. Independent probes would be desirable.

#### **Results**

We compute the polarized  $\phi$  production rate in the hadronic phase, due to interactions with Kaons and dissipation.

We test the model in a Bjorken flow.

Then we take into account scatterings with baryons and the hydrodynamic evolution of the medium.



Grossi, AP, Zahed Phys.Rev.C 111 (2025) 1, 014914

#### $\phi$ meson production rate

The number of  $\phi$  mesons produced in the hadronic phase is computed similarly to an electromagnetic rate

$$N_{\phi} = \sum_{I,F} |\mathcal{M}_{I,F}|^2 \times \frac{e^{-\beta E_I}}{Z} \times \frac{V d^3 q}{(2\pi)^3}$$
$$|\mathcal{M}_{I,F}|^2 = |\langle F\phi|\mathcal{S}|I\rangle|^2$$

With a vector meson-current interaction with strange quarks

$$\mathcal{S} = \int \mathrm{d}^4 x \; G_V \phi_\mu \bar{\psi}_s \gamma^\mu \psi_s = \int \mathrm{d}^4 x \; G_V \phi_\mu J_s^\mu$$

The meson state can be defined as

$$\langle \phi(q,\sigma) | J_s^{\mu} | 0 \rangle = \frac{f_{\phi} m_{\phi} \epsilon_{\sigma}^{\mu}(q)}{\sqrt{2\varepsilon_q V}}$$

And eventually the production rate reads

$$E_q \frac{\mathrm{d}R_{\sigma\sigma'}}{\mathrm{d}^3 q} = \frac{1}{e^{\beta \cdot q} + 1} \frac{G_V^2 f_\phi^2 m_\phi^2}{(2\pi)^3} \epsilon_\sigma^\mu(q) \epsilon^{*\nu}{}_{\sigma'}(q) \mathrm{Im}\left(iW_\phi^F{}_{\mu\nu}(q)\right)$$

With the Feynman propagator

$$W_F(k) = \int \mathrm{d}^4 x \, e^{ik \cdot x} \langle T\phi(x)\phi(0) \rangle$$

## To compute the thermal propagator, one can organize an expansion in the most abundant stable states.

Steele, Yamagishi, Zahed Phys.Lett.B 384 (1996) 255-262

Dealing with the  $\phi$  meson, the most relevant one are Kaon states, and we consider baryon resonances that may become relevant at larger densities

$$\begin{split} W^{F}_{\mu\nu}(q) &= \int \mathrm{d}^{4}x e^{iq \cdot x} \langle 0|T(\phi_{\mu}(x)\phi_{\nu}(0))|0\rangle \\ &+ \sum_{a} \int \frac{\mathrm{d}^{3}k}{(2\pi)^{3}} \frac{n_{K}(k)}{2k^{0}} \int \mathrm{d}^{4}x e^{iqx} \langle K^{a}(k)|T(\phi_{\mu}(x)\phi_{\nu}(0))|K^{a}(k)\rangle_{\mathrm{conn.}} \\ &+ \int \frac{\mathrm{d}^{3}p}{(2\pi)^{3}} \frac{n_{N}(p)}{2p^{0}} \int \mathrm{d}^{4}x e^{iqx} \sum_{spin} \langle N(p,s)|T(\phi_{\mu}(x)\phi_{\nu}(0))|N(p,s)\rangle_{\mathrm{conn.}} + \dots \end{split}$$

#### **Leading contribution**

$$iW_F^{\mu\nu}(q)_0 = i\langle 0|\phi^{\mu}(q)\phi^{\nu}(0)|0\rangle = \left(-q^2g^{\mu\nu} + q^{\mu}q^{\nu}\right)\Pi_V^{\phi}(q)$$

$$E_{q}\frac{dR_{\sigma,\sigma'}^{0}}{d^{3}q} = \delta_{\sigma,\sigma'} \left[\frac{1}{e^{\beta E_{q}^{\phi}} + 1}G_{V}^{2}f_{\phi}^{2}m_{\phi}^{2}\frac{q^{2}}{(2\pi)^{3}}\operatorname{Im}\,\Pi_{V}^{\phi}(q)\right]_{q^{2}=m_{\phi}^{2}}$$

Then the spin density matrix is computed

$$\rho_{00}(q_T) = \frac{\int \mathrm{d}\phi \mathrm{d}\eta_q \mathrm{d}^4 V \ E_q \frac{\mathrm{d}R_{00}}{\mathrm{d}^3 q}}{\sum_{\sigma=-1}^1 \int \mathrm{d}\phi \mathrm{d}\eta_q \mathrm{d}^4 V \ E_q \frac{\mathrm{d}R_{\sigma\sigma}}{\mathrm{d}^3 q}}$$

The vacuum spectral function yields an isotropic emission

$$\rho_{00} = 1/3$$

#### **Kaon contribution**

We use the reduction scheme developed in Yamagishi and Zahed, Annals Phys. 247, 292, Kamano Phys.Rev.D 81 (2010) 076004

It's based on algebraic properties of broken SU(3) flavor symmetry in QCD

It allows writing  $\phi$  spectral functions on a Kaon state in terms of vacuum spectral functions.

$$\begin{aligned} \langle K(k) | \phi_{\mu}(q) \phi_{\nu}(0) | K(k) \rangle &= \frac{G_{V}^{2}}{m_{\phi}^{4}} \langle K(k) | J_{\mu}^{s}(q) J_{\nu}^{s}(0) | K(k) \rangle = \\ &= (g_{\mu\nu}q^{2} - q_{\mu}q_{\nu}) \frac{4}{f_{K}^{2}} \operatorname{Im}\Pi_{V}^{88}(q) - (g_{\mu\nu}(k \pm q)^{2} - (k \pm q)_{\mu}(k \pm q)_{\nu}) \frac{2}{f_{K}^{2}} \operatorname{Im}\Pi_{A}^{U}(k \pm q) \\ &+ (2k_{\mu} + q_{\mu})(-k_{\nu}q^{2} + k \cdot qq_{\nu}) \frac{4}{f_{K}^{2}} \operatorname{Re}\Delta_{R}(k \pm q) \operatorname{Im}\Pi_{V}^{88}(q) \end{aligned}$$

The Kaon contribution to the production rate reads

$$\frac{dR_{L,\perp}^{1}}{d^{3}q} = \frac{1}{e^{\beta E_{q}^{\phi}} + 1} \frac{G_{V}^{4} f_{\phi}^{2}}{m_{\phi}^{2}} \frac{1}{(2\pi)^{3} E_{q}^{\phi}} \int \frac{d^{3}p}{(2\pi)^{3}} \frac{1}{2E_{p}^{K}} \frac{1}{e^{\beta E_{p}^{K}} - 1} \\
\times \left[ \frac{4q^{2}}{f_{K}^{2}} \operatorname{Im}\Pi_{V}^{88}(q) - ((p \pm q)^{2} - |\epsilon_{L,\perp} \cdot (p \pm q)|^{2}) \frac{2}{f_{K}^{2}} \operatorname{Im}\Pi_{A}^{U}((p \pm q)) \\
- q^{2} |\epsilon_{L,\perp} \cdot p|^{2} \frac{8}{f_{K}^{2}} \operatorname{Re} \Delta_{R}(p \pm q) \operatorname{Im}\Pi_{V}^{88}(q) \right]_{q^{2} = m_{\phi}^{2}}$$

It is suppressed by a "diluteness" factor

$$\kappa = \frac{1}{f_K^2} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{1}{e^{\beta \cdot k} - 1}$$



#### **Dissipative corrections**

Dissipative corrections can be computed in a Zubarev-like fashion We follow Yiu, Zahed Phys.Rev.D 96 (2017) 11, 116021

$$\rho(t_0) \simeq \left(1 - \partial_\mu \beta_\nu \int_{t_0}^t \mathrm{d}t' \int_0^1 \mathrm{d}z \mathrm{d}^3 x T^{\mu\nu}(x, t' + iz\beta)\right) \rho(t)$$
$$\langle \widehat{J}^s_\mu \widehat{J}^s_\nu \rangle \simeq \langle \widehat{J}^s_\mu \widehat{J}^s_\nu \rangle_0 - \int_{t_0}^t \mathrm{d}t' \partial_\rho \beta_\sigma \langle \widehat{T}^{\rho\sigma} \widehat{J}^s_\mu \widehat{J}^s_\nu \rangle$$

$$\frac{dR_{L,\perp}^{1}}{d^{3}q} = \frac{1}{e^{\beta E_{q}^{\phi}} + 1} \frac{G_{V}^{4} f_{\phi}^{2}}{m_{\phi}^{2}} \frac{1}{(2\pi)^{3} E_{q}^{\phi} T} \int \frac{d^{3}p}{(2\pi)^{3}} \frac{1}{2E_{p}^{K}} \frac{e^{\beta p \cdot u}}{(e^{\beta p \cdot u} - 1)^{2}} \frac{p_{\mu} p_{\nu} (\frac{\eta}{s} \sigma^{\mu\nu} + \frac{1}{3} \frac{\zeta}{s} \theta \Delta^{\mu\nu})}{p \cdot u} \\
\times \left[ \frac{4q^{2}}{f_{K}^{2}} \operatorname{Im} \Pi_{V}^{88}(q) - ((p \pm q)^{2} - |\epsilon_{L,\perp} \cdot (p \pm q)|^{2}) \frac{2}{f_{K}^{2}} \operatorname{Im} \Pi_{A}^{U}((p \pm q)) \\
- q^{2} |\epsilon_{L,\perp} \cdot p|^{2} \frac{8}{f_{K}^{2}} \operatorname{Re} \Delta_{R}(p \pm q) \operatorname{Im} \Pi_{V}^{88}(q) \right]_{q^{2} = m_{\phi}^{2}}$$

#### **Results in a Bjorken flow**



The effect is rather small due to the small kaon diluteness.

If Kaons are out-ofequilibrium and there is a Kaon inbalance the effect is enhanced

Grossi, AP, Zahed Phys.Rev.C 111 (2025) 1, 014914, data from STAR, Nature 614 (2023) 7947

#### **Nucleons and resonances**

We account for effective interactions with spin ½ baryons

$$\mathcal{L} = -g\bar{\psi}\left(\gamma^{\mu}\phi_{\mu} - \frac{i\kappa\sigma^{\mu\nu}}{2m}\partial_{\mu}\phi_{\nu}\right)\psi$$

The rate is modified as:

$$E_{q}\frac{\mathrm{d}R_{\sigma\sigma'}^{N}}{d^{3}q} = -\frac{1}{e^{\beta E_{q}} + 1}\frac{G_{V}^{2}m_{\phi}^{2}f_{\phi}^{2}}{(2\pi)^{3}}g^{2}(f(q)\delta_{\sigma\sigma'} + g_{\sigma\sigma'}(q))$$

In the s-channel g vanishes for stable particles, not in the u channel though. We consider protons, neutrons and N(1880) resonances.

## **Hydrodynamic Simulation**

#### **Preliminary results!**

We use the code Fluidum, with Trento IC, in 2+1D.

Hadronic phase is assumed to be between T=110 and T=170 MeV.

Integrals are computed over the entire fluid history via Monte Carlo methods.

Spectra are used to fix the coupling constant and the normalization of the initial state

AuAu  $\sqrt{s_{NN}} = 200 \,\, {
m GeV}$ 



0-10×10<sup>7</sup> 10-20×10<sup>6</sup>

20-30×10<sup>5</sup>

30-40×10

40-50×10<sup>3</sup> 50-60×10<sup>2</sup>

60-70×10 70-80×10

#### **Global alignment**



In-Plane alignment qualitatively in agreement.

The model doesn't capture the Out-of-Plane alignment: 3D hydrodynamic needed?





Scattering don't seem to play a significant role



#### **Conclusions and outlook**

Emission rate in the hadronic phase can be computed via chiral master formulae for SU(3) flavor symmetry breaking.

Rate are computed on top of 2+1D hydro evolution.

Scattering with nucleons are accounted for but their effect is negligible.

For the  $\rho$  meson the approximate flavor symmetry is more accurate. Can one describe its alignment with this method?

The results do not describe out of plane experimental data. Need for 3+1D hydro?

#### **Thanks for the attention!**

### Backup

### **Spectral functions**

$$\Pi_V^I(q^2) = \frac{f_V^2}{q^2} (\mathbf{F}_V(q^2) - 1) \qquad \Pi_A^I(q^2) = \frac{f_V^2}{m_V^2 - q^2 - im_V \Gamma(q^2)}$$

$$\mathbf{F}_{V}(q^{2}) = \frac{m_{V}^{2}}{m_{V}^{2} - q^{2} - im_{V}\Gamma(q^{2})} \qquad \Gamma(q^{2}) = \theta(q^{2} - M_{BR}^{2})\Gamma_{0}\frac{m_{V}}{\sqrt{q^{2}}} \left(\frac{q^{2} - M_{BR}^{2}}{m_{V}^{2} - M_{BR}^{2}}\right)^{\frac{3}{2}}$$

$$\Pi_V^{88} = \frac{1}{3} \Pi_V^{\omega} + \frac{2}{3} \Pi_V^{\phi} \qquad \Pi_A^U = \Pi_A^{K_1(1270)} + \Pi_A^{K_1(1400)}$$

$$\operatorname{Re}\Delta_{R}(p \pm q) = \frac{\operatorname{PP}}{(p+q)^{2} - m_{K}^{2}} + \frac{\operatorname{PP}}{(p-q)^{2} - m_{K}^{2}}$$