Fermions in a fluctuating magnetic background: Possible implications in the HIC scenario

Phys. Rev. D 110, 056003 (2024); Phys. Rev. D 107, 096014 (2023);

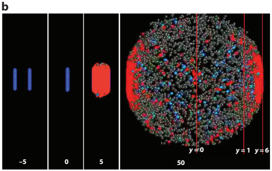
Phys. Rev. D 109, 056007 (2024); Phys. Rev. D 111, 076028 (2025)

Enrique Muñoz

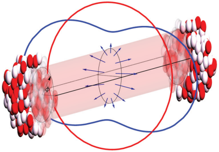
Pontifical Catholic University

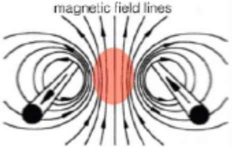
Collaboration: J. Castaño, M. Loewe, J. Rojas

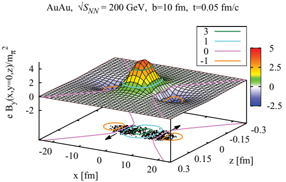
Heavy Ion Collisions

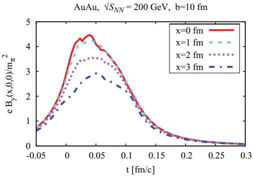


b) Snapshots of a central 2.76 TeV Pb+Pb collison. Blue and grey are hadrons, red is the quark-gluon plasma
http://web.mit.edu/mithig/movies/LHCanmation.mov









Voronyuk, Toneev, Cassing, Bratkovskaya, Konchakovski, Voloshin, Phys. Rev. C 83, 054911 (2011)

Very strong and inhomogeneous magnetic fields emerge at initial times

$B \sim 10^{18} - 10^{19}\, G$

Motivation

- A constant “classical” magnetic field background has been studied since the seminal work of Schwinger (Phys. Rev. 82, 664 (1951))
- In most theoretical studies, the **background magnetic field** is idealized as **static and uniform**
- In HIC scenarios, specially for non-central collisions, **strong magnetic fields** emerge in comparatively small regions of space, with **spatial anisotropies and fluctuations**
- We here propose a **statistical model** to study the physical effects of such fluctuations

The Schwinger propagator

J. Schwinger, Physical Review 82, 664 (1951)

- The propagator for the BG magnetic field $\mathbf{B} = \nabla \times \mathbf{A}_{\rm BG}$

$$S_F(x,x') = e^{i\Phi_{A_{BG}}(x,x')} \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-x')} S_F(k)$$

- The Schwinger phase $\Phi_{A_{BG}}(x,x') = \int_x^{x'} d\xi^{\mu} \left[A_{\mu} + \frac{1}{2}F_{\mu\nu}(\xi - x')^{\nu}\right]$

$$S_{\rm F}(k) = -\mathrm{i} \int_0^{\infty} \frac{d\tau}{\cos(eB\tau)} e^{\mathrm{i}\tau\left(k_{\parallel}^2 - \mathbf{k}_{\perp}^2 \frac{\tan(eB\tau)}{eB\tau} - m^2 + \mathrm{i}\epsilon\right)}$$
$$\times \left\{ \left[\cos(eB\tau) + \gamma^1\gamma^2 \sin(eB\tau)\right](m + \not{k}_{\parallel}) + \frac{\not{k}_{\perp}}{\cos(eB\tau)} \right\}$$

- The metric tensor splits into two subspaces $g^{\mu\nu} = g_{\parallel}^{\mu\nu} + g_{\perp}^{\mu\nu}$, such that $g_{\parallel}^{\mu\nu} = \mathrm{diag}\,(1,0,0,-1)$ and $g_{\perp}^{\mu\nu} = \mathrm{diag}(0,-1,-1,0)$

$$\begin{aligned} \not{k} &= \not{k}_{\perp} + \not{k}_{\parallel} \\ k^2 &= k_{\parallel}^2 - \mathbf{k}_{\perp}^2 \end{aligned}$$

An alternative representation Physical Review D 107, 096014 (2023)

- The propagator in terms of a single "master" integral and its derivatives

$$\boxed{S_{\mathrm{F}}(k) = -\mathrm{i}\left[\left(m + \not{k}_{\parallel}\right)\mathcal{A}_1 + \gamma^1\gamma^2\left(m + \not{k}_{\parallel}\right)\mathcal{A}_2 + \mathcal{A}_3\not{k}_{\perp}\right]}$$

$$\begin{aligned}
\mathcal{A}_1(k,B) &= \int_0^{\infty} d\tau\, e^{\mathrm{i}\tau\left(k_{\parallel}^2 - m^2 + \mathrm{i}\epsilon\right) - \mathrm{i}\frac{\mathbf{k}_{\perp}^2}{eB}\tan(eB\tau)} \\
\mathcal{A}_2(k,B) &= \mathrm{i}eB\frac{\partial\mathcal{A}_1}{\partial(\mathbf{k}_{\perp}^2)}, \quad \mathcal{A}_3(k,B) = \mathcal{A}_1 + (\mathrm{i}eB)^2\frac{\partial^2\mathcal{A}_1}{\partial(\mathbf{k}_{\perp}^2)^2}
\end{aligned}$$

- Landau levels ($x = \mathbf{k}_{\perp}^2/eB$)

$$\mathcal{A}_1(k,B) = \mathrm{i}e^{-x}\left[\frac{1}{k_{\parallel}^2 - m^2} + \sum_{n=1}^{\infty}\frac{(-1)^n\left[L_n^0(2x) - L_{n-1}^0(2x)\right]}{k_{\parallel}^2 - m^2 - 2n\,eB}\right]$$

- The inverse propagator

$$\boxed{\hat{S}_{\mathrm{F}}^{-1}(k) = \frac{\mathrm{i}}{\mathcal{D}(k)}\left[\left(m - \not{k}_{\parallel}\right)\mathcal{A}_1 - \gamma^1\gamma^2\left(m - \not{k}_{\parallel}\right)\mathcal{A}_2 - \mathcal{A}_3\not{k}_{\perp}\right]}$$

Inhomogeneus magnetic fields: Statistical model

J. Castaño, M. Loewe, E. Muñoz and J. Rojas, Phys. Rev. D 107, 096014 (2023)

Assumption: At initial stages prior to thermalization, classical ionic currents exhibit stochastic fluctuations $J^{\mu}_{cl}(x) + \delta J^{\mu}_{cl}(x)$, acting as sources for **classical background "BG"** gauge fields $A^{\mu}_{BG}(x)$ (in contrast with photons $A^{\mu}(x)$).

$$\boxed{\Box\left(A^{\mu}_{\rm BG} + \delta A^{\mu}_{\rm BG}\right) = J^{\mu}_{cl}(x) + \delta J^{\mu}_{cl}(x)}$$

- Statistical properties

$$\begin{aligned}
\overline{\delta A^{j}_{\rm BG}(\mathbf{x})\delta A^{k}_{\rm BG}(\mathbf{x}')} &= \Delta_B \delta_{j,k}\delta^{(3)}(\mathbf{x}-\mathbf{x}') \\
\overline{\delta A^{\mu}_{\rm BG}(\mathbf{x})} &= 0
\end{aligned}$$

$$\boxed{dP\left[\delta A^{\mu}_{\rm BG}\right] = \mathcal{N} e^{-\int d^3x \frac{\left[\delta A^{\mu}_{\rm BG}(\mathbf{x})\right]^2}{2\Delta_B}} \mathcal{D}\left[\delta A^{\mu}_{\rm BG}(\mathbf{x})\right]}$$

- The ensemble-average of over such fluctuations is defined by

$$\boxed{\overline{\mathcal{O}(x; A_{BG})} = \int dP[\delta A^{\mu}_{BG}]\mathcal{O}(x; A_{BG} + \delta A_{BG})}$$

The Lagrangian

Physical Review D 107, 096014 (2023)

The Lagrangian for this model is a superposition of two terms

$$\boxed{\mathcal{L} = \bar{\psi}\left(\mathrm{i}\not{\partial} - e(\not{A}_{\mathrm{BG}} + \delta\not{A}_{\mathrm{BG}}) - e\not{A} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \equiv \mathcal{L}_{\mathrm{FBG}} + \mathcal{L}_{\mathrm{NBG}}}$$

Fermions immersed in the average BG

$$\boxed{\mathcal{L}_{\mathrm{FBG}} = \bar{\psi}\left(\mathrm{i}\not{\partial} - e\not{A}_{\mathrm{BG}} - e\not{A} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

Fermions interacting with the classical background noise (NBG), represented by the spatial fluctuations $\delta A^{\mu}_{BG}(x)$

$$\boxed{\mathcal{L}_{\mathrm{NBG}} = \bar{\psi}\left(-e\delta\not{A}_{\mathrm{BG}}\right)\psi}$$

- The ensemble-average over the magnetic background fluctuations with respect to its mean value $A_{BG}^{\mu} + \delta A_{BG}^{\mu}$

$$\boxed{\overline{G(x_1,\ldots,x_{2k};A_{BG})} = \left(-i\frac{\delta}{\delta\bar{J}(x_1)}\right)\ldots\left(i\frac{\delta}{\delta J(x_{2k})}\right)\overline{\ln Z[\bar{J},J;A_{BG}]}\Big|_{J=\bar{J}=0}}$$

- Clearly $\overline{\ln Z[\bar{J},J;A_{BG}]} \neq \ln\overline{Z[\bar{J},J;A_{BG}]}$
- The Replica Method [Mèzard and Parisi, (1991); Kardar, Parisi and Zhang, (1986)]

$$\overline{\ln Z[A_{BG}]} = \lim_{n\rightarrow 0}\frac{\overline{Z^n[A_{BG}]}-1}{n}$$

- In our scenario, n-replicas of the original fermion fields are defined

$$\psi(x)\rightarrow\psi^a(x)\qquad 1\leq a\leq n$$

The ensemble-averaged action

Physical Review D 107, 096014 (2023)

The statistical average

$$\overline{Z^n[A_{BG}]} = \int \prod_{a=1}^{n} \mathcal{D}[\bar{\psi}^a, \psi^a] e^{i\bar{S}[\bar{\psi}^a, \psi^a; A_{BG}]}$$

leads to an effective fermion-fermion interaction

$$\bar{S}\left[\bar{\psi}^a, \psi^a; A_{BG}\right] = \int d^4x \left(\sum_a \bar{\psi}^a \left(i\partial\!\!\!/ - eA\!\!\!/_{\mathrm{BG}} - eA\!\!\!/ - m \right) \psi^a - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right)$$

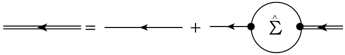
$$+ i\frac{e^2 \Delta_B}{2} \underbrace{\int d^4x \int d^4y \sum_{a,b} \sum_{j=1}^{3} \bar{\psi}^a(x)\gamma^j \psi^a(x) \bar{\psi}^b(y) \gamma_j \psi^b(y) \delta^3(\mathbf{x} - \mathbf{y})}_{Effective\ \ Fermion-Fermion\ \ interaction}$$

In what follows, we shall first focus on the fermions in the classical BG magnetic field $\mathbf{B} = \hat{e}^3 B$

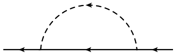
$$\boxed{A^{\mu}_{BG} = \frac{B}{2}(0, -x^2, x^1, 0)}$$

Perturbation Theory Physical Review D 107, 096014 (2023)

- Dyson equation for the dressed propagator $\boxed{\hat{S}_{\Delta}^{-1}(k) = \hat{S}_{\mathrm{F}}^{-1}(k) - \hat{\Sigma}_{\Delta}(k)}$



- The self-energy diagram at first-order in $\Delta = e^2\Delta_B$



$$\hat{\Sigma}_{\Delta}(q)=(\mathrm{i}\Delta)\int\frac{d^3p}{(2\pi)^3}\gamma^j\hat{S}_F(p+q;p_0=0)\gamma_j=\frac{\mathrm{i}(\mathrm{i}\Delta)}{(2\pi)^3}\left[3\left(\gamma^0q_0-m\right)\widetilde{\mathcal{A}}_1(q_0)-\gamma^1\gamma^2(\mathrm{i}\pi eB)\left(m-q_0\gamma^0\right)\widetilde{\mathcal{A}}_2(q_0)\right]$$

- Dressed propagator

$$\boxed{S_{\Delta}(q)=-\mathrm{i}z^{-1}\frac{\mathcal{D}(q)}{\tilde{\mathcal{D}}(q)}\left[\left(m+\tilde{\not{q}}_{\parallel}\right)\mathcal{A}_1(q)+z_3\gamma^1\gamma^2\left(m+\tilde{\not{q}}_{\parallel}\right)\mathcal{A}_2(q)+\mathcal{A}_3(q)\tilde{\not{q}}_{\perp}\right]}$$

- The momenta $\tilde{q}^{\mu} = (q^0, z^{-1}\mathbf{q})$, with an effective refractive index $v'/c = z^{-1}(q)$ due to magnetic fluctuations.

Renormalization factors

Physical Review D 107, 096014 (2023)

- Wavefunction renormalization factor and refractive index

$$z(q) = 1 + \frac{3\mathrm{i}\Delta}{(2\pi)^3}\frac{\widetilde{\mathcal{A}}_1(q_0)}{\mathcal{A}_1(q)}\mathcal{D}(q) \qquad \frac{v'}{c} = z^{-1}$$

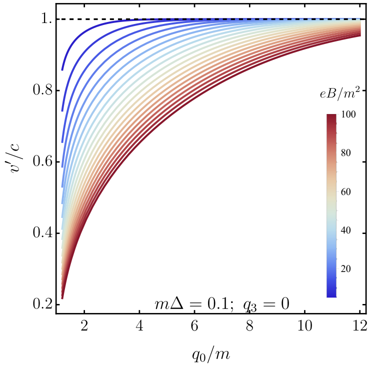
- Charge renormalization factor

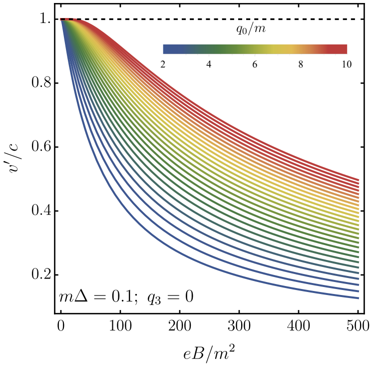
$$z_3(q) = \frac{1 - \frac{\mathrm{i}\pi(\mathrm{i}\Delta)(eB)}{(2\pi)^3}\frac{\widetilde{\mathcal{A}}_2(q_0)}{\mathcal{A}_2(q)}\mathcal{D}(q)}{1 + \frac{3\mathrm{i}\Delta}{(2\pi)^3}\frac{\widetilde{\mathcal{A}}_1(q_0)}{\mathcal{A}_1(q)}\mathcal{D}(q)}$$

- Dressed propagator

$$\boxed{S_\Delta(q) = -\mathrm{i}z^{-1}(q)\frac{\mathcal{D}(q)}{\tilde{\mathcal{D}}(q)}\left[\left(m+\tilde{\not{q}}_{\parallel}\right)\mathcal{A}_1(q) + z_3\gamma^1\gamma^2\left(m+\tilde{\not{q}}_{\parallel}\right)\mathcal{A}_2(q) + \mathcal{A}_3(q)\tilde{\not{q}}_{\perp}\right]}$$

Refractive Index $\frac{v'}{c} = z^{-1}$ Physical Review D 107, 096014 (2023)

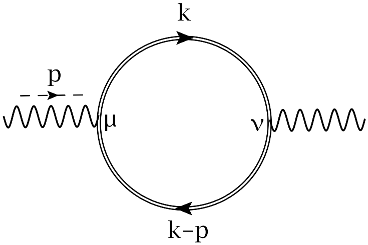




- In the very weak field limit $eB/m^2 \ll 1$, $z, z_3 \to 1$, and hence $v'/c \to 1$
- Ultra-intense field $eB/m^2 \gg 1$ (LLL)

$$z = 1 + \frac{3}{4}\frac{\Delta(eB)e^{-\mathbf{q}_\perp^2/eB}}{\pi\sqrt{q_0^2 - m^2}} \qquad z_3 = \frac{1 + \frac{\Delta(eB)e^{-\mathbf{q}_\perp^2/(eB)}}{4\pi\sqrt{q_0^2 - m^2}}}{1 + \frac{3}{4}\frac{\Delta(eB)e^{-\mathbf{q}_\perp^2/(eB)}}{\pi\sqrt{q_0^2 - m^2}}}$$

$$\lim_{eB/m^2 \to \infty} z_3 = 1/3$$



$$\Pi^{\mu\nu}(p) = -\frac{(\mathrm{i}q_f)^2}{2}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left\{\gamma^{\nu}\mathrm{i}S_{\Delta}\left(k\right)\gamma^{\mu}\mathrm{i}S_{\Delta}(k-p)\right\} + \text{c.c.}$$

Fermion propagator (first-order in noise)

$$\mathrm{i}S(p) = \mathrm{i}S_0(p) + \Delta\cdot\mathrm{i}S_1(p) + O(\Delta^2)$$

Fermion propagator

J. Castaño and E. Muñoz Physical Review D 110, 056003 (2024)

- Noiseless propagator in LLL ($|q_f B|/m^2 \gg 1$)

$$\mathrm{i}S_0(p) = 2\mathrm{i}\frac{e^{-p_\perp^2/|q_f B|}}{p_\parallel^2 - m^2 + i\epsilon}(\not{p}_\parallel + m)\mathcal{O}^{(\uparrow)}$$

- Noise contribution

$$\mathrm{i}S_1(p) = \mathrm{i}\left(\frac{|q_f B|}{2\pi}\right)\left[\Theta_1(p)(\not{p}_\parallel + m)\mathcal{O}^{(\uparrow)} - \Theta_2(p)\gamma^3\mathcal{O}^{(\uparrow)} + \Theta_3(p)\mathrm{i}\gamma^1\gamma^2(\not{p}_\parallel + m)\right]$$

- Spin projectors $\mathcal{O}^{(\uparrow,\downarrow)} = \frac{1}{2}\left[1 \mp \mathrm{sign}(q_f B)\mathrm{i}\gamma^1\gamma^2\right]$

$$\Theta_1(p) = \frac{3(p_\parallel^2 + m^2)e^{-2p_\perp^2/|q_f B|}}{(p_\parallel^2 - m^2)^2\sqrt{p_0^2 - m^2}}, \quad \Theta_2(p) = \frac{3p_3 e^{-2p_\perp^2/|q_f B|}}{(p_\parallel^2 - m^2)\sqrt{p_0^2 - m^2}}$$

$$\Theta_3(p) = \frac{e^{-2p_\perp^2/|q_f B|}}{(p_\parallel^2 - m^2)\sqrt{p_0^2 - m^2}}.$$

The photon polarization tensor for $\Delta > 0$

J. Castaño and E. Muñoz Phys. Rev. D 109, 056007 (2024)

- At first order in noise Δ

$$\mathrm{i}\Pi_{\Delta}^{\mu\nu} = \mathrm{i}\Pi_{0}^{\mu\nu} + \mathrm{i}\frac{q_f^2|q_f B|\Delta}{4\pi}\sum_{i=1}^{3} T_i^{\mu\nu}$$

$$\mathrm{i}\Pi_0^{\mu\nu} = \frac{\mathrm{i}q_f^2|q_f B|}{4\pi^2} e^{-p_\perp^2/2|q_f B|} p_\parallel^2 \mathcal{I}_0(p_\parallel^2)\left(g_\parallel^{\mu\nu} - \frac{p_\parallel^\mu p_\parallel^\nu}{p_\parallel^2}\right)$$

$$\begin{aligned}
\mathcal{I}_0(p_\parallel^2) &= \int_0^1 dx \frac{x(1-x)}{x(1-x)p_\parallel^2 - m^2} \\
&= \frac{1}{p_\parallel^2}\left[1 + \frac{2m^2/p_\parallel^2}{\sqrt{1-\frac{4m^2}{p_\parallel^2}}}\log\left[\frac{1+\sqrt{1-\frac{4m^2}{p_\parallel^2}}}{1-\sqrt{1-\frac{4m^2}{p_\parallel^2}}}\right]\right]
\end{aligned}$$

The photon polarization tensor for $\Delta > 0$

J. Castaño and E. Muñoz Phys. Rev. D 109, 056007 (2024)

$$
\begin{aligned}
T_1^{\mu\nu} &= 16\mathrm{i}\int \frac{d^4k}{(2\pi)^4}\frac{e^{-k_\perp^2/eB}}{k_\parallel^2 - m^2}\Theta_1(k-p)\Big[(m^2 + k_\parallel\cdot(p_\parallel - k_\parallel))(g_\parallel^{\mu\nu} - g_\perp^{\mu\nu}) \\
&\quad + (k_\parallel^\mu - p_\parallel^\mu)k_\parallel^\nu + k_\parallel^\mu(k_\parallel^\nu - p_\parallel^\nu)\Big]
\end{aligned}
$$

$$
T_2^{\mu\nu} = 16\mathrm{i}\int \frac{d^4k}{(2\pi)^4}\frac{e^{-k_\perp^2/eB}}{k_\parallel^2 - m^2}\Theta_2(k-p)\left(k^3 g_\parallel^{\mu\nu} + k_\parallel^\mu\delta_3^\nu + k_\parallel^\nu\delta_3^\mu\right)
$$

$$
\begin{aligned}
T_3^{\mu\nu} &= 16\mathrm{i}\int \frac{d^4k}{(2\pi)^4}\frac{e^{-k_\perp^2/eB}}{k_\parallel^2 - m^2}\Theta_3(k-p)\Big[(m^2 + k_\parallel\cdot(p_\parallel - k_\parallel))(g_\parallel^{\mu\nu} - g_\perp^{\mu\nu}) \\
&\quad + (k_\parallel^\mu - p_\parallel^\mu)k_\parallel^\nu + k_\parallel^\mu(k_\parallel^\nu - p_\parallel^\nu)\Big].
\end{aligned}
$$

Noise-induced photon mass for $\Delta > 0$?

J. Castaño and E. Muñoz Phys. Rev. D 109, 056007 (2024)

- Dyson equation for the noise-dressed photon propagator

$$\left[D_{\Delta}^{\mu\nu}(p)\right]^{-1} = \left[D_{0}^{\mu\nu}(p)\right]^{-1} - \mathrm{i}\Pi_{\Delta}^{\mu\nu}(p)$$

$$D_0^{\mu\nu}(p) = \frac{-\mathrm{i}\, g^{\mu\nu}}{(p^2 + \mathrm{i}\epsilon)}$$

$$\lim_{\substack{p_0\to 0\\ \mathbf{p}\to 0}} \mathrm{i}\Pi_0^{\mu\nu} = 0$$

- The noise contribution

$$\lim_{\substack{p_0\to 0\\ \mathbf{p}\to 0}} \mathrm{i}\Pi_{\Delta}^{\mu\nu} = \mathrm{i}\frac{q_f^2|q_f B|\Delta}{4\pi}\sum_{i=1}^{3}\lim_{\substack{p_0\to 0\\ \mathbf{p}\to 0}} T_i^{\mu\nu}$$

$$
\begin{aligned}
\lim_{\substack{p_0\to 0\\ \mathbf{p}\to 0}} T_1^{\mu\nu} &= \frac{4\mathrm{i}|q_f B|}{\pi m}\int_{\mathbb{R}^2}\frac{dydx}{(2\pi)^2}\frac{(x^2+y^2-1)g_{\parallel}^{\mu\nu}}{(x^2+y^2+1)^3\sqrt{y^2+1}}\\
&= -\frac{\mathrm{i}|q_f B|}{32\pi m}g_{\parallel}^{\mu\nu}
\end{aligned}
$$

$$
\begin{aligned}
\lim_{\substack{p_0\to 0\\ \mathbf{p}\to 0}} T_2^{\mu\nu} &= -\frac{4\mathrm{i}|q_f B|}{\pi m}\int_{\mathbb{R}^2}\frac{dydx}{(2\pi)^2}\frac{x^2\left(g_{\parallel}^{\mu\nu}+2\delta_3^{\mu}\delta_3^{\nu}\right)}{(x^2+y^2+1)^2\sqrt{y^2+1}}\\
&= -\frac{\mathrm{i}|q_f B|}{2\pi m}\left(g_{\parallel}^{\mu\nu}+2\delta_3^{\mu}\delta_3^{\nu}\right)
\end{aligned}
$$

$$
\begin{aligned}
\lim_{\substack{p_0\to 0\\ \mathbf{p}\to 0}} T_3^{\mu\nu} &= \frac{4\mathrm{i}|q_f B|}{3\pi m}\int_{\mathbb{R}^2}\frac{dydx}{(2\pi)^2}\frac{1}{(x^2+y^2+1)\sqrt{y^2+1}}\left[\frac{1}{(x^2+y^2+1)^2}g_{\parallel}^{\mu\nu}-g_{\perp}^{\mu\nu}\right]\\
&= \frac{\mathrm{i}|q_f B|}{3\pi m}\left[\frac{1}{4}g_{\parallel}^{\mu\nu}-g_{\perp}^{\mu\nu}\right]
\end{aligned}
$$

Photon propagator with $\Delta > 0$

J. Castaño and E. Muñoz Physical Review D 109, 056007 (2024)

- Thus we obtain

$$\lim_{\substack{p_0 \to 0 \\ \mathbf{p} \to 0}} \mathrm{i}\Pi^{\mu\nu}_{\Delta} = \frac{\alpha_{\mathrm{em}}(q_f B)^2}{m\pi}\Delta\left(\frac{59}{96} g_{\parallel}^{\mu\nu} + \frac{1}{3} g_{\perp}^{\mu\nu} + \delta_3^{\mu}\delta_3^{\nu}\right),$$

- Photon propagator with noise

$$D^{\mu\nu}(p) = \frac{-\mathrm{i} g_{\parallel}^{\mu\nu}}{p^2 + M_{\parallel}^2 + \mathrm{i}\epsilon} + \frac{-\mathrm{i} g_{\perp}^{\mu\nu}}{p^2 + M_{\perp}^2 + \mathrm{i}\epsilon} + \frac{3\mathrm{i} M_{\perp}^2 \delta_3^{\mu}\delta_3^{\nu}}{\left(p^2 + M_{\perp}^2 + \mathrm{i}\epsilon\right)\left(p^2 + \left(M_{\parallel}^2 - M_{\perp}^2\right) + \mathrm{i}\epsilon\right)}$$

- Noise-induced masses (complex)

$$M_{\parallel}^2 = \mathrm{i}\frac{59\alpha_{em}}{96\pi}\frac{(q_f B)^2\Delta}{m} \quad M_{\perp}^2 = \mathrm{i}\frac{\alpha_{em}}{3\pi}\frac{(q_f B)^2\Delta}{m}$$

Conclusions

- We studied the effects of white noise spatial fluctuations in an otherwise uniform background magnetic field, over the QED fermion propagator
- At first order in Δ, the propagator retains its free form, thus representing renormalized quasi-particles with the same mass $m' = m$, but propagating in a "dispersive medium" with an index of refraction $v'/c = z^{-1}$, and effective charge $e' = z_3 e$, where z and z_3 depend on the average field and its noise
- Low energy components in the propagator (long-wavelength) are more sensitive to the spatial distribution of the magnetic fluctuations, and hence experience a higher degree of decoherence, thus reducing $v'/c = z^{-1}$. In contrast, the high-energy Fourier modes are less sensitive to magnetic fluctuations.
- The photon propagator develops a complex mass that leads to damping in the dispersion relation across the effective magnetized medium

Anisotropic photon yield under magnetic fluctuations

J. Castaño and E. Muñoz Phys. Rev. D 111, 076028 (2025)

- Direct photons are commonly estimated as the sum of prompt photons produced in the hard process at the collision instant, plus thermal photons emitted from the soft, thermalized portion of the medium.
- Nuclear matter undergoes several stages after the collision, from initial color glass condensate to glasma, then a hydrodynamic QCD regime, and finally hadronic gas.
- The common picture is that of QGP as a nearly perfect hydrodynamic fluid, but **asymmetries** in photon production rates have been experimentally detected by the PHENIX Collaboration at the RHIC (Phys. Rev. Lett. 109, 122302 (2012), Phys. Rev. C 94, 064901 (2016)) and by the ALICE Collaboration at the LHC (Phys. Lett. B 789, 308 (2019)).

Anisotropic flow coefficients

X. Wang, I. Shovkovy, L. Yu, and M. Huang, Phys. Rev. D 102, 076010 (2020)

X. Wang and I. Shovkovy, Phys. Rev. D 109, 056008 (2024).

- Angular distribution for the emission rate

$$p^0 \frac{d^3R}{dp_x dp_y dp_z} = \frac{d^3R}{p_T dp_T d\phi dy} = \frac{1}{2\pi}\mathcal{R}_0 \left[1 + 2\sum_{n=1}^{\infty} v_n \cos(n\phi)\right].$$

- Anisotropic flow coefficients

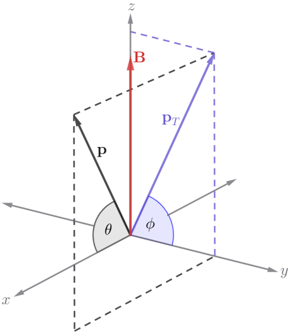
$$\begin{aligned} v_n &= \frac{1}{\mathcal{R}_0}\int_0^{2\pi} d\phi \cos(n\phi) \frac{d^3R}{p_T dp_T d\phi dy}, \\ \mathcal{R}_0 &= \frac{1}{2\pi}\int_0^{2\pi} d\phi \frac{d^3R}{p_T dp_T d\phi dy}. \end{aligned}$$

Anisotropic photon yield

X. Wang and I. Shovkovy, Phys. Rev. D 109, 056008 (2024)

J. Castaño and E. Muñoz Phys. Rev. D 111, 076028 (2025)

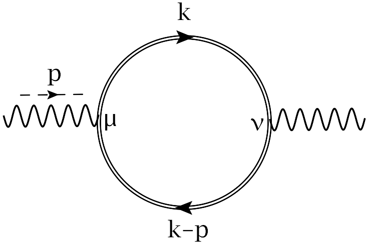
- Coordinates system $p_y = p_T \cos\phi, \quad p_z = p_T \sin\phi, \quad y = \frac{1}{2}\ln\frac{p_0+p_x}{p_0-p_x}$



- Photon emission rate $\frac{d^3R}{p_T dp_T d\phi dy} = -\frac{n_B(\omega)}{(2\pi)^3}\mathrm{Im}\left\{g_{\mu\nu}\Pi_R^{\mu\nu}(p)\right\}$

The photon polarization tensor at $T > 0$ and $\Delta > 0$

J. Castaño and E. Muñoz Phys. Rev. D 111, 076028 (2025)



$$\Pi^{\mu\nu}(p) = -\frac{(\mathrm{i}q_f)^2}{2\beta}\sum_{\omega_n}\int\frac{d^3k}{(2\pi)^3}\mathrm{Tr}\left\{\gamma^{\nu}\mathrm{i}S_{\Delta}(k)\,\gamma^{\mu}\mathrm{i}S_{\Delta}(k-p)\right\} + \text{c.c.}$$

The retarded polarization tensor is obtained by analytic continuation

$$\Pi_R^{\mu\nu}(p^0 = \omega, \mathbf{p}) = \Pi^{\mu\nu}(\mathrm{i}\nu_n \to \omega + \mathrm{i}\epsilon, \mathbf{p}).$$

$$\frac{d^3 R_\gamma^{(\text{noise})}}{p_T dp_T d\phi dy} = \frac{d^3 R_\gamma^0}{p_T dp_T d\phi dy} + \frac{d^3 R_\gamma^\Delta}{p_T dp_T d\phi dy}.$$

The noiseless contribution

$$\frac{d^3 R_\gamma^0}{p_T dp_T d\phi dy} = \sum_{f=u,d} \frac{q_f^2 m_f^2 |q_f B| N_c n_B(\omega)}{32\pi^4} e^{-\frac{p_\perp^2}{2|q_f B|}} \theta\left(\omega - \sqrt{p_z^2 + 4m_f^2}\right) \mathcal{I}_0$$

$$\mathcal{I}_0 = -\sum_{s_1=\pm} \sum_{s_2=\pm} \sum_{s=\pm} \theta\left(s_1 E_+^{(s)}\right) \theta\left(s_2 E_-^{(s)}\right) \frac{\left[n_F(E_-^{(s)}) - n_F(E_+^{(s)})\right]}{E_+^{(s)} E_-^{(s)} \left| \frac{k_s}{E_+^{(s)}} - \frac{(k_s - p)}{E_-^{(s)}} \right|}.$$

Here we defined (for $s = \pm$)

$$E_\pm^{(s)} = \pm\frac{\omega}{2} + s\frac{p_z}{2}\sqrt{1 - \frac{4m_f^2}{\omega^2 - p_z^2}}, \quad k_s = \frac{p_z}{2} + s\frac{|\omega|}{2}\sqrt{1 - \frac{4m_f^2}{\omega^2 - p_z^2}}.$$

$$\frac{d^3R_\gamma^\Delta}{p_T dp_T d\phi dy} = \frac{n_B(\omega)\Delta N_c}{32\pi^5}\sum_{f=u,d} q_f^2 eB^2 m_f^2 e^{-\frac{2\mathbf{p}_\perp^2}{3eB}}\theta\left(\omega - \sqrt{p_z^2 + 4m_f^2}\right)\left\{\mathcal{I} + \frac{3}{m_f^2}\mathcal{J}\right\}.$$

Here, we define

$$\mathcal{I} \equiv -\sum_{s_1, s_2, s=\pm} \frac{\theta\left(s_1 E_+^{(s)}\right)\theta\left(s_2 E_-^{(s)}\right)\left(\left[E_-^{(s)}\right]^4 - 3m_f^2\left[E_-^{(s)}\right]^2 + m_f^4\right)\left\{n_F\left(E_-^{(s)}\right) - n_F\left(E_+^{(s)}\right)\right\}}{E_+^{(s)}\left[E_-^{(s)}\right]^3\left(\left[E_-^{(s)}\right]^2 - m_f^2\right)^{3/2}\left|\frac{k_s}{E_+^{(s)}} - \frac{(k_s - p_z)}{E_-^{(s)}}\right|}$$

$$\mathcal{J} \equiv \sum_{s_1, s_2, s=\pm} \frac{\theta\left(s_1 E_+^{(s)}\right)\theta\left(s_2 E_-^{(s)}\right)\left[E_+^{(s)}E_-^{(s)} - k_s(k_s - p_z)\right]\left\{n_F\left(E_-^{(s)}\right) - n_F\left(E_+^{(s)}\right)\right\}}{E_+^{(s)}E_-^{(s)}\sqrt{\left[E_-^{(s)}\right]^2 - m_f^2}\left|\frac{k_s}{E_+^{(s)}} - \frac{(k_s - p_z)}{E_-^{(s)}}\right|}$$

Photon Emission Rate with magnetic noise

J. Castaño s and E. Muñoz Phys. Rev. D 111, 076028 (2025)

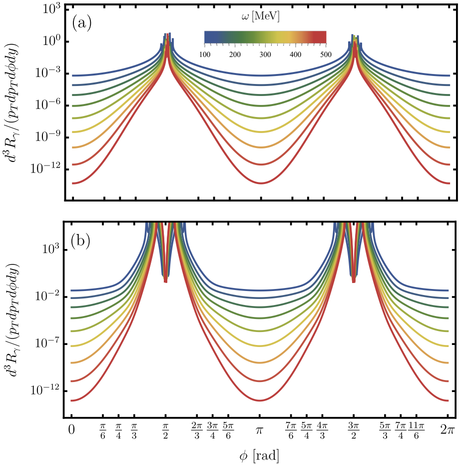
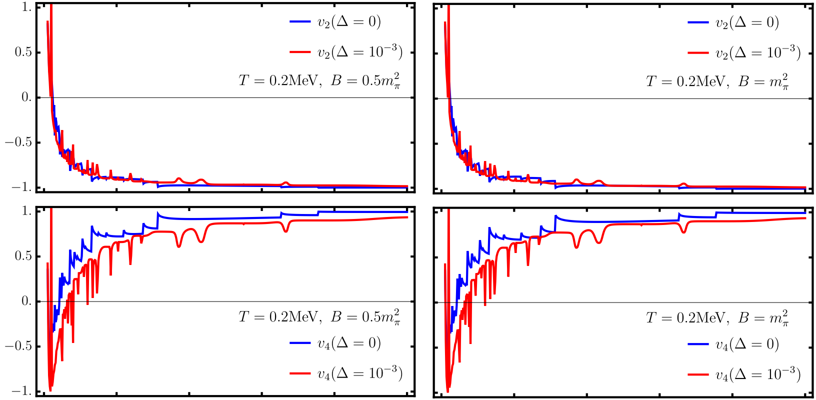


Figure: $B = 0.5 m_\pi^2$. (a) $\Delta = 0$, (b) $\Delta = 10^{-3}$ MeV^{-1}.

Anisotropic flow coefficients with magnetic noise

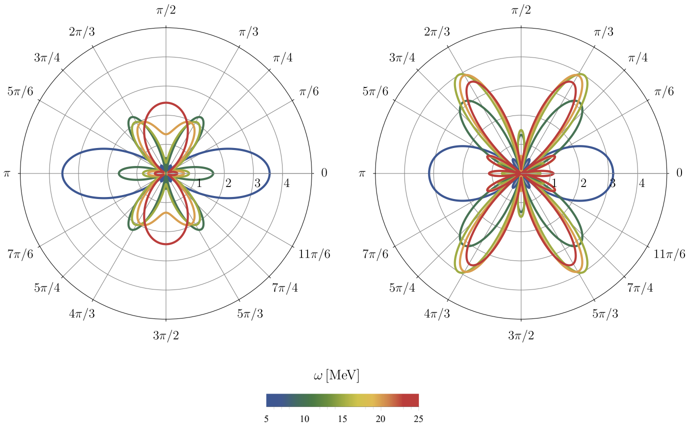
J. Castaño and E. Muñoz Phys. Rev. D 111, 076028 (2025)



$T = 0.2$ MeV, Blue: $\Delta = 0$, Red: $\Delta = 10^{-3}\,\text{MeV}^{-1}$.

Total angular distribution for emitted photons in polar coordinates

J. Castaño and E. Muñoz Phys. Rev. D 111, 076028 (2025)



Normalized by $\mathcal{R}_0/2\pi$, with v_n (up to $n = 6$), $T = 0.2$ MeV, $B = 0.5m_\pi^2$. Left: $\Delta = 0$. Right: $\Delta = 10^{-3}\text{MeV}^{-1}$.

Conclusions

- Our results complement previous studies, based on the assumption of a constant magnetic field background (Phys. Rev. D 102, 076010 (2020); Phys. Rev. D. 109, 056008 (2024)), by incorporating the effects of stochastic fluctuations.
- The anisotropic flow coefficients are affected by the presence of stochastic noise. Secondary lobules emerge in addition to the main directions $\phi = \pi/2$ and $\phi = 3\pi/2$.
- The elliptic flow coefficient v_2 is weakly affected, whereas the higher-order components $v_4 \cos(4\phi)$ and $v_6 \cos(6\phi)$ are significantly modified by stochastic noise.
- Low-frequency photons are more affected by the magnetic noise effects.
- Our results suggest that magnetic noise effects should be taken into account in the theoretical analysis of experimental signals.

Phenomenological scenario for the magnetic fluctuations

- Very strong magnetic fields $\mathbf{B} = \nabla \times \mathbf{A}_{BG}$ are generated locally within a small spatial region $L \sim \sqrt{\sigma}$
- On average $\langle \mathbf{B} \rangle = \hat{e}_3 B$, but smaller transverse components δB_x and δB_y exist such that field fluctuations are estimated on the order of $(\delta B)^2 \sim (\delta B_x)^2 + (\delta B_y)^2$.
- Therefore, by dimensional analysis

$$\boxed{\Delta_B \sim (\delta B)^2 \; L^5 \sim (\delta B)^2 \; \sigma^{5/2}}$$

Further estimations

- The fraction f of the geometrical cross-section σ_{geom}, defined by a circle with a radius of $r_1 + r_2 = 2R$ in a maximum peripheral collision, and the cross-section σ_b for a peripheral collision with impact parameter b

$$f = \frac{\sigma_b}{\sigma_{\text{geom}}} = \left(\frac{N_{\text{part}}}{2N}\right)^{2/3}$$

- The nuclear radius $r_A = r_0 N^{1/3}$, where N is the number of nucleons per ion and $r_0 \sim 1.25$ fm.

- From the previous expressions

$$\Delta_B \sim \pi^{5/2}\,(\delta B)^2\, r_0^5 N^{5/3}\left(\frac{N_{\text{part}}}{2N}\right)^{5/3}$$

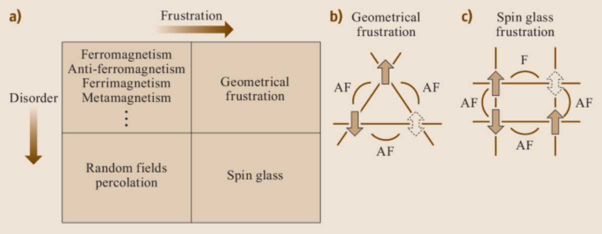
- In peripheral heavy-ion collisions, the magnetic fluctuations in the transverse plane $|e\,\delta B| \sim m_\pi^2/4$
- For an Au+Au collision with $N = 197$, and if $N_{\text{part}}/N = 1/2$,

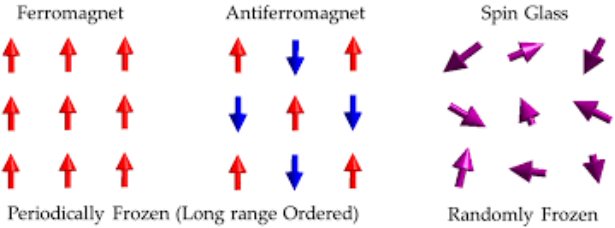
$$\Delta \equiv e^2\Delta_B \sim 2.6\text{MeV}^{-1}$$

- For less central collisions with $N_{\text{part}}/N = 1/8$

$$\Delta \sim 0.26\ \text{MeV}^{-1}$$

Statistical mechanics of disordered magnets: Spin glasses





The Sherrington-Kirkpatrick model: Thermodynamic properties Phys. Rev. Lett. 35 (26), 1792 (1975)

- For a **given realization** of the set of values of the couplings, the **Free Energy** is

$$
\begin{aligned}
F[T,N,B;\{J_{ij}\}] &= -T\ln\left[\mathrm{Tr}\, e^{-H[\{\sigma_i\},\{J_{ij}\}]/T}\right] \\
&= -T\ln\left[\sum_{\{\sigma_i=\pm\}} e^{\sum_{<i,j>}\frac{J_{ij}}{T}\sigma_i\sigma_j+\frac{B}{T}\sum_{i=1}^{N}\sigma_i}\right] \\
&\equiv -T\ln\mathcal{Z}[T,N,B;\{J_{ij}\}]
\end{aligned}
$$

- The above is a random variable (via the $\{J_{ij}\}$). The **physical** Free Energy is the statistical average over the distribution of couplings

$$
F[T,N,B]\equiv\overline{F[T,N,B;\{J_{ij}\}]}=-T\int dP[J_{ij}]\ln\mathcal{Z}[T,N,B;\{J_{ij}\}]
$$

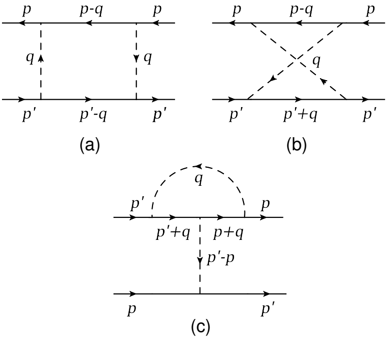
- Apply the basic identity

$$\ln \mathcal{Z}[T,B;\{J_{ij}\}] = \lim_{n\to 0}\frac{\mathcal{Z}^n[T,B;\{J_{ij}\}]-1}{n}$$

- The degrees of freedom $\sigma_i \to \sigma_i^a,\ 1 \leq a \leq n$

$$\begin{aligned}\overline{\mathcal{Z}^n[T,B;\{J_{ij}\}]} &= \sum_{\{\sigma_i^a=\pm\}}\left[\int \prod_{<i,j>}\frac{dJ_{ij}}{\sqrt{\pi\Delta}}e^{-\frac{J_{ij}^2}{\Delta}}\right]e^{\sum_{a,<i,j>}\frac{J_{ij}}{T}\sigma_i^a\sigma_j^a - \frac{B}{T}\sum_{a,i}\sigma_i^a}\\ &= \sum_{\{\sigma_i^a=\pm\}} e^{-\frac{\Delta^2}{4T^2}\sum_{a,b}\sum_{<i,j>,<k,l>}\sigma_i^a\sigma_j^a\sigma_k^b\sigma_l^b - \frac{B}{T}\sum_{a=1}^{n}\sum_{i=1}^{N}\sigma_i^a}\end{aligned}$$

- Diagrams contributing to the 4-point vertex



$$
\begin{aligned}
\hat{\Gamma}_{(a)} &= \int \frac{d^3q}{(2\pi)^3}\, \gamma^i S_F(p-q)\gamma^j \otimes \gamma_i S_F(p'-q)\gamma_j \\
\hat{\Gamma}_{(b)} &= \int \frac{d^3q}{(2\pi)^3}\, \gamma^i S_F(p-q)\gamma^j \otimes \gamma_i S_F(p'+q)\gamma_j \\
\hat{\Gamma}_{(c)} &= \int \frac{d^3q}{(2\pi)^3}\, \gamma^i S_F(p+q)\gamma^j \otimes \gamma_i S_F(p'-q)\gamma_j
\end{aligned}
$$

Vertex corrections

Physical Review D 107, 096014 (2023)

$$\hat{\Gamma} = 2\hat{\Gamma}_{(a)} + 2\hat{\Gamma}_{(b)} + 4\hat{\Gamma}_{(c)} = \tilde{\Delta}(\bar{\psi}\gamma^i\psi)(\bar{\psi}\gamma^i\psi)$$

- Renormalized $\tilde{\Delta}$

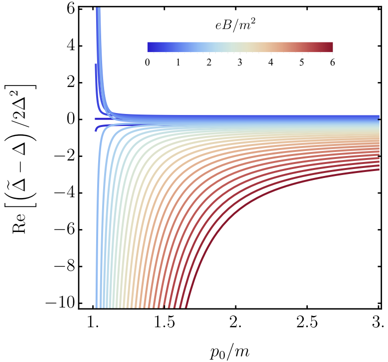
$$\tilde{\Delta}=\Delta+2\Delta^2\Big(\mathcal{J}_2^{(-,-)}+\mathcal{J}_2^{(-,+)}+2\mathcal{J}_2^{(+,-)}+\big(1-\partial_x^2\big)(1-\partial_y^2)\mathcal{J}_3^{(-,-)}$$
$$+\big(1-\partial_x^2\big)(1-\partial_y^2)\mathcal{J}_3^{(-,+)}+2\big(1-\partial_x^2\big)(1-\partial_y^2)\mathcal{J}_3^{(+,-)}\Big)$$

- In terms of the integrals

$$\begin{aligned}
\mathcal{J}_1^{(\lambda,\sigma)}(p,p') &= \int \frac{d^3q}{(2\pi)^3}\mathcal{A}_1(p+\lambda q)\mathcal{A}_1(p'+\sigma q) \\
\mathcal{J}_2^{(\lambda,\sigma)}(p,p') &= \int \frac{d^3q}{(2\pi)^3}q_{\parallel}^2\mathcal{A}_1(p+\lambda q)\mathcal{A}_1(p'+\sigma q) \\
\mathcal{J}_3^{(\lambda,\sigma)}(p,p') &= \int \frac{d^3q}{(2\pi)^3}\mathbf{q}_{\perp}^2\mathcal{A}_1(p+\lambda q)\mathcal{A}_1(p'+\sigma q)
\end{aligned}$$

Vertex renormalization

Physical Review D 107, 096014 (2023)



Fermion self-energy and effective mass in a noisy magnetic background

J. Castaño and E. Muñoz Phys. Rev. D 110, 056003 (2024)

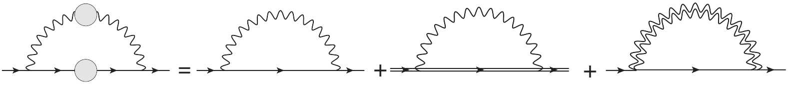
- The **fermion magnetic mass** exhibits a leading **double logarithm** (Tsai, PRD 10, 1342 (1974);Jancovici, PRD 187, 2275 (1969); Machet, Int. J. Mod. Phys. 31, 1650071 (2016); Ayala, Castaño, Muñoz, Loewe, PRD 104, 016006 (2021))

$$m_B - m \sim \alpha_{em} m \left[\ln\left(eB/m^2\right)\right]^2$$

- Is this behaviour modified by stochastic noise when radiative corrections are included?

Fermion self-energy and effective mass in a noisy magnetic background

J. Castaño and E. Muñoz Phys. Rev. D 110, 056003 (2024)



$$-i\Sigma(x,x') = (-ie)^2\gamma^\mu iS(x,x')\gamma^\mu D_{\mu\nu}(x-x') = \Phi(x,x')\int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x-x')}\left[-i\Sigma(p)\right]$$

Schwinger phase $\Phi(x,x') = e^{ie\int_x^{x'}d\xi^\mu A_\mu} = e^{\frac{ieB}{2}\epsilon_{ij}x_i x_j'}$

Translational-invariant self-energy $$\boxed{-i\Sigma(p) \equiv (-ie)^2\int\frac{d^4k}{(2\pi)^4}\gamma^\mu iS(k)\gamma^\nu D_{\mu\nu}(p-k)}$$

Noise-dressed Fermion propagator $iS(p) = iS_0(p) + \Delta\cdot iS_1(p) + O(\Delta^2)$

Noise-dressed photon propagator $D^{\mu\nu}(q) = D_0^{\mu\nu}(q) + \Delta\cdot D_1^{\mu\nu}(q) + O(\Delta^2)$

Fermion propagator

J. Castaño and E. Muñoz Physical Review D 110, 056003 (2024)

- Fermion propagator (first-order in noise)

$$\mathrm{i}S(p) = \mathrm{i}S_0(p) + \Delta \cdot \mathrm{i}S_1(p) + O(\Delta^2)$$

- Noiseless propagator in LLL ($|eB|/m^2 \gg 1$)

$$\mathrm{i}S_0(p) = 2\mathrm{i}\frac{e^{-p_\perp^2/eB}}{p_\parallel^2 - m^2 + i\epsilon}(\not{p}_\parallel + m)\mathcal{O}^{(\uparrow)}$$

- Noise contribution

$$\mathrm{i}S_1(p) = \mathrm{i}\left(\frac{eB}{2\pi}\right)\left[\Theta_1(p)(\not{p}_\parallel + m)\mathcal{O}^{(\uparrow)} - \Theta_2(p)\gamma^3\mathcal{O}^{(\uparrow)}\right.$$
$$\left. + \Theta_3(p)\mathrm{i}\gamma^1\gamma^2(\not{p}_\parallel + m)\right]$$

- Spin projectors

$$\mathcal{O}^{(\uparrow,\downarrow)} = \frac{1}{2}\left[1 \mp \mathrm{sign}(eB)\mathrm{i}\gamma^1\gamma^2\right]$$

Photon propagator

J. Castaño and E. Muñoz Physical Review D 109, 056007 (2024)

- Photon propagator with noise

$$D^{\mu\nu}(q) = \frac{-\mathrm{i}g_{\parallel}^{\mu\nu}}{q^2 + \mathrm{i}M_{\parallel}^2 + \mathrm{i}\epsilon} + \frac{-\mathrm{i}g_{\perp}^{\mu\nu}}{q^2 + \mathrm{i}M_{\perp}^2 + \mathrm{i}\epsilon} - \frac{3M_{\perp}^2\,\delta_3^{\mu}\delta_3^{\nu}}{(q^2 + \mathrm{i}M_{\perp}^2 + \mathrm{i}\epsilon)\left(q^2 + \mathrm{i}\left(M_{\parallel}^2 - M_{\perp}^2\right) + \mathrm{i}\epsilon\right)}$$

- Noise-induced masses

$$\mathrm{i}M_{\parallel}^2 = \frac{59\alpha_{em}}{96\pi}\frac{(eB)^2\Delta}{m} \quad \mathrm{i}M_{\perp}^2 = \frac{\alpha_{em}}{3\pi}\frac{(eB)^2\Delta}{m}$$

- At first-order in noise

$$D^{\mu\nu}(q) = D_0^{\mu\nu}(q) + \Delta \cdot D_1^{\mu\nu}(q) + O(\Delta^2)$$

- Noise contribution

$$D_1^{\mu\nu}(q) = -\frac{\alpha_{\mathrm{em}}(eB)^2}{96\pi m(q^2 + \mathrm{i}\epsilon)^2}\left(59g_{\parallel}^{\mu\nu} - 32g_{\perp}^{\mu\nu} + 64\delta_3^{\mu}\delta_3^{\nu}\right)$$

Fermion self-energy and magnetic mass

J. Castaño and E. Muñoz Phys. Rev.D 110, 056003 (2024)

- The fermion self-energy with magnetic noise $\Delta > 0$

$$\Sigma(p,B,\Delta) = \Sigma_0^r(p,B) + \Sigma_\Delta^r(p,B) + O(\Delta^2)$$

- The magnetic mass operator in the $\Delta \rightarrow 0$ limit (Phys. Rev. D 104, 016006 (2021)), $\mathcal{B} = |eB|/m^2$

$$\hat{M}_B(\Delta = 0) = m + \Sigma_0(p,B)|_{\not{p}_\parallel = m} = \mathcal{O}^\uparrow M_{B0}^{(\uparrow)} + \mathcal{O}^\downarrow M_{B0}^{(\downarrow)}$$

$$\begin{aligned} M_{B0}^{(\uparrow)} &= \frac{\alpha_{\rm em} m}{\pi}\left[\ln^2\mathcal{B} - \left(\gamma_e + \mathrm{i}\frac{\pi}{2}\right)\ln\mathcal{B} + \frac{\pi^2}{3}\right] + O(\mathcal{B}^{-1}) \\ M_{B0}^{(\downarrow)} &= \frac{\alpha_{\rm em} m}{\pi}\left[\ln^2\mathcal{B} - \left(1 + \gamma_e + \mathrm{i}\frac{\pi}{2}\right)\ln\mathcal{B}\right. \\ &\quad \left. - \left(2 - \gamma_e - \frac{\pi^2}{3} - \mathrm{i}\frac{\pi}{2}\right)\right] + O(\mathcal{B}^{-1}) \end{aligned}$$

Self-energy and renormalization conditions

J. Castaño and E. Muñoz Phys. Rev. D 110, 056003 (2024)

- We define two counterterms

$$\Sigma_0^r(p,B) = \Sigma_0(p,B) + \delta_Z(\not{p}_{\parallel} - m\gamma^0) - \delta_m,$$

- Those are fixed by the renormalization conditions

$$\begin{aligned}
\lim_{\Delta\to 0^+} \frac{\partial}{\partial \not{p}_{\parallel}} \Sigma(p,B,\Delta)\Big|_{\substack{\not{p}_{\parallel}=m\gamma^0,\\ \mathbf{p}=0}} &= \frac{\partial}{\partial \not{p}_{\parallel}} \Sigma_0^r(p,B)\Big|_{\substack{\not{p}_{\parallel}=m\gamma^0,\\ \mathbf{p}=0}} \\
&= \frac{\partial}{\partial \not{p}_{\parallel}} \Sigma_0(p,B)\Big|_{\substack{\not{p}_{\parallel}=m,\\ \mathbf{p}_{\perp}=0}}
\end{aligned}$$

$$\begin{aligned}
\lim_{\Delta\to 0^+} \Sigma(p,B,\Delta)\big|_{\not{p}_{\parallel}=m\gamma^0,\mathbf{p}=0} &= \Sigma_0^r(p,B)\big|_{\not{p}_{\parallel}=m\gamma^0,\mathbf{p}=0} \\
&= \hat{M}_B(\Delta=0) - m \\
&= \Sigma_0(p,B)\big|_{\not{p}_{\parallel}=m,\not{p}_{\perp}=0}
\end{aligned}$$

Fermion self-energy and magnetic mass with noise

J. Castaño and E. Muñoz Phys. Rev. D 110, 056003 (2024)

- The self energy is projected onto spin $\mathcal{O}^{\uparrow,\downarrow}$ and $\mathcal{P}^{\pm} = \frac{1}{2}\left(\mathbb{1} \pm \gamma^0\right)$ particle/antiparticle subspaces

$$\lim_{\substack{p_0\to m\\ \mathbf{p}\to 0}} \left[-\mathrm{i}\Sigma_\Delta(p)\right]_r = \sum_{\sigma=\uparrow,\downarrow}\sum_{\lambda=\pm 1}\left[-\mathrm{i}\widetilde{\Sigma}_\Delta^{(\sigma,\lambda)}\mathcal{O}^\sigma\mathcal{P}^{(\lambda)}\right]$$

- The fermion magnetic mass operator, for $\Delta > 0$, possesses four different eigenvalues depending on the spin $\sigma = \uparrow,\downarrow$ and $\lambda = \pm$ projections

$$M_B^{(\sigma,\lambda)}(\Delta) = m + M_{B0}^{(\sigma)} + \widetilde{\Sigma}_\Delta^{(\sigma,\lambda)}$$

- The eigenvalues are complex, and hence we have

$$\begin{aligned} m_B^\sigma &= \mathrm{Re}\, M_B^{(\sigma,\lambda)}(\Delta) = m + \mathrm{Re}\, M_{B0}^{(\sigma)} \\ \Gamma^{(\sigma,\lambda)}(\Delta) &= -2\mathrm{Im}\, M_B^{(\sigma,\lambda)}(\Delta) \end{aligned}$$

Fermion self-energy and magnetic mass with noise

J. Castaño and E. Muñoz Phys. Rev. D 110, 056003 (2024)

- The real parts correspond to the fermion magnetic mass, which turns out to be noise-independent (in agreement with Phys. Rev. D 104, 016006 (2021)), for $\mathcal{B} = |eB|/m^2$

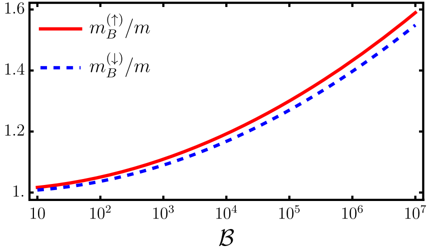
$$
\begin{aligned}
m_B^{(\uparrow)} &= m + \frac{\alpha_{\text{em}} m}{\pi}\left[\ln^2\mathcal{B} - \gamma_e \ln\mathcal{B} + \frac{\pi^2}{3}\right] + O(\mathcal{B}^{-1}) \\
m_B^{(\downarrow)} &= m + \frac{\alpha_{\text{em}} m}{\pi}\left[\ln^2\mathcal{B} - (1+\gamma_e)\ln\mathcal{B} + \frac{\pi^2}{3} + \gamma_e - 2\right] + O(\mathcal{B}^{-1})
\end{aligned}
$$

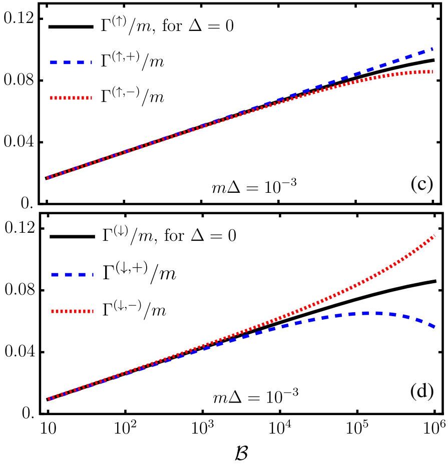
- The imaginary parts represent a Breit-Wigner resonance due to the combination of the field and the magnetic noise

$$
\begin{aligned}
\Gamma^{(\uparrow,\pm)}(\Delta) &= \alpha_{\text{em}} m\left(\ln\mathcal{B} - \frac{2\sqrt{2\mathcal{B}}m\Delta}{\pi^{3/2}}\left(3\ln(2) \mp 2\right)\right) \\
\Gamma^{(\downarrow,\pm)}(\Delta) &= \alpha_{\text{em}} m\left(\ln\mathcal{B} - 1 - \frac{2\sqrt{2\mathcal{B}}m\Delta}{\pi^{3/2}}\left(3\ln(2) \pm 8\right)\right)
\end{aligned}
$$

Fermion magnetic mass and spectral width

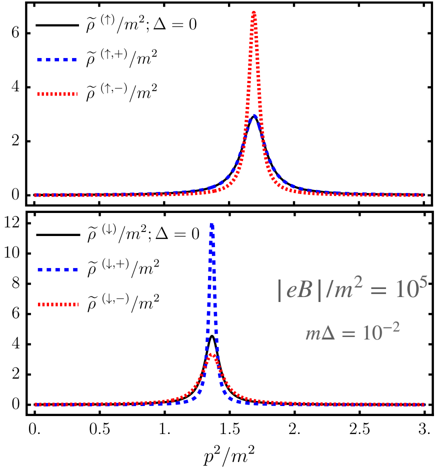
J. Castaño and E. Muñoz Phys. Rev. D 110, 056003 (2024)





Fermion spectral density and spectral width

J. Castaño and E. Muñoz Phys. Rev. D 110, 056003 (2024)



The imaginary part determines a spectral width in the spectral density

$$\boxed{\tilde{\rho}^{(\sigma,\lambda)}(p^2) = \frac{m_B^{(\sigma)}\Gamma^{(\sigma,\lambda)}/\pi}{\left[p^2 - (m_B^{(\sigma)})^2\right]^2 + \left[m_B^{\sigma}\Gamma^{(\sigma,\lambda)}\right]^2}}$$

In the limit of very strong magnetic fields

$$\frac{\Gamma^{(\sigma,\lambda)}}{m_B^{\sigma}} \sim [\ln \mathcal{B}]^{-1}$$

$$\tilde{\rho}^{(\sigma,\lambda)}(p^2) \rightarrow \delta(p^2 - m_B^{\sigma})$$