Causal and stable magnetohydrodynamics for an ultrarelativistic two-component gas

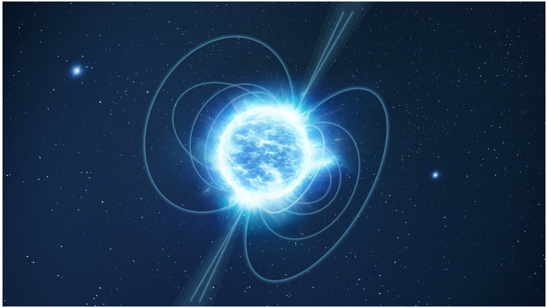
Caio V. P. de Brito *(with K. Kushwah and G. S. Denicol)*

Universidade Federal Fluminense, Brazil

9th Conference on Chirality, Vorticity and Magnetic Fields in Quantum Matter – São Paulo, Brazil

WHY RELATIVISTIC MAGNETOHYDRODYNAMICS?

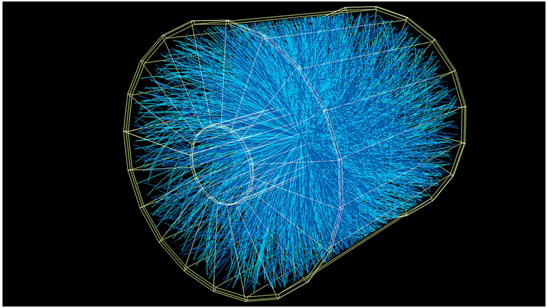
Magnetars



European Space Agency

$B \sim 10^{15}\,\mathrm{G}$

Heavy-ion collisions



ALICE Collaboration

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Locally neutral, non-resistive plasma

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- **Equations for the dissipative currents:**
  - Additional relations are required for closure.
  - Phenomenology, microscopic calculations, ...

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- Traditional Israel-Stewart theory with corrected energy-momentum tensor

$$\dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \cdots .$$

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G. S. Denicol *et al.*, Phys. Rev. D **98**, 076009 (2018).

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- Longitudinal approximation for the shear-stress tensor

$$\pi^{\mu\nu} \approx \pi_{bb}\left(b^\mu b^\nu + \frac{1}{2}\Xi^{\mu\nu}\right).$$

M. Chandra *et al.*, Astrophys. J. **810**, 162 (2015).

Bounds on nonlinear causality have been investigated.

I. Cordeiro *et al.*, Phys. Rev. Lett. **133**, 091401 (2024).

MICROSCOPIC DERIVATION OF FLUID DYNAMICS

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Assuming a two-component gas of classical particles described by a Boltzmann equation

$$k^\mu \partial_\mu f_{\mathbf{k}}^+ + q^+ k_\nu F^{\mu\nu} \frac{\partial}{\partial k^\mu} f_{\mathbf{k}}^+ = C[f_{\mathbf{k}}^+, f_{\mathbf{k}}^-],$$
$$k^\mu \partial_\mu f_{\mathbf{k}}^- + q^- k_\nu F^{\mu\nu} \frac{\partial}{\partial k^\mu} f_{\mathbf{k}}^- = C[f_{\mathbf{k}}^-, f_{\mathbf{k}}^+].$$

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Method of moments

Replace the equation for $f_{\mathbf{k}}$ by equations for its *irreducible* moments.

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14-moment approximation

Express the distribution function solely in terms of the usual hydrodynamic variables.

W. Israel and J. M. Stewart, Ann. Phys. (N.Y.) **118**, 341 (1979). G. S. Denicol *et al.*, Eur. Phys. J. A **48**, 170 (2012).

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Factorize the single-particle distribution as

$$f_{\mathbf{k}}^{\pm} = f_{0\mathbf{k}}^{\pm} + \delta f_{\mathbf{k}}^{\pm} \implies \delta f_{\mathbf{k}}^{\pm} = \frac{f_{0\mathbf{k}}\left(\pi_{\mu\nu}^{\pm}\, k^{\mu} k^{\nu}\right)}{2\left(\varepsilon^{\pm} + P^{\pm}\right) T^{2}}.$$

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Obtain coupled equations for the ***total*** and ***relative*** shear-stress tensor

$$\dot{\pi}^{\langle\mu\nu\rangle} + \Sigma \pi^{\mu\nu} + \frac{2|q|B}{5T} b^{\lambda\langle\mu} \delta\pi_{\lambda}^{\nu\rangle} = \frac{8}{15}\varepsilon\sigma^{\mu\nu} - \frac{4}{3}\pi^{\mu\nu}\theta - \frac{10}{7}\sigma^{\lambda\langle\mu}\pi_{\lambda}^{\nu\rangle} - 2\omega^{\lambda\langle\nu}\pi_{\lambda}^{\mu\rangle},$$

$$\delta\dot{\pi}^{\langle\mu\nu\rangle} + \Sigma' \delta\pi^{\mu\nu} + \frac{2|q|B}{5T} b^{\lambda\langle\mu} \pi_{\lambda}^{\nu\rangle} = -\frac{4}{3}\delta\pi^{\mu\nu}\theta - \frac{10}{7}\sigma^{\lambda\langle\mu}\delta\pi_{\lambda}^{\nu\rangle} - 2\omega^{\lambda\langle\nu}\delta\pi_{\lambda}^{\mu\rangle}.$$

where $\pi^{\mu\nu} = \pi_{+}^{\mu\nu} + \pi_{-}^{\mu\nu}$ and $\delta\pi^{\mu\nu} = \delta\pi_{+}^{\mu\nu} + \delta\pi_{-}^{\mu\nu}$. K. Kushwah and G. S. Denicol, Phys. Rev. D **109**, 096021 (2024).

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Is this theory suitable to describe physical systems?

CAUSALITY AND STABILITY ANALYSIS

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What are the minimal requirements a theory must satisfy?

- **Causality:** perturbations travel with subluminal speed.
- **Stability:** perturbations decay exponentially with time.

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L. Gavassino, Phys. Rev. X **12**, 041001 (2022).

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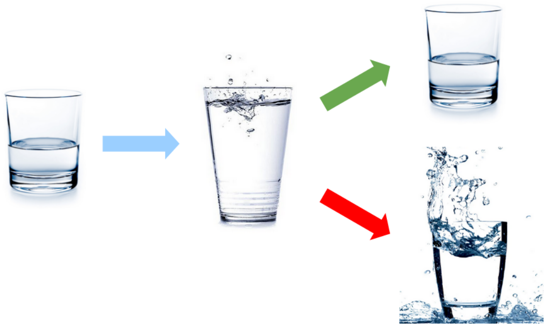

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- Constrain the transport coefficients such that the following conditions are satisfied:

$$\mathrm{Im}\,(\omega) > 0, \qquad \lim_{k\to\infty} \left| \frac{\partial \mathrm{Re}\,(\omega)}{\partial k} \right| \le 1.$$

Stability — Causality

TRADITIONAL ISRAEL-STEWART THEORY

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For small k,

$$\hat{\omega}_{\rm nh}(\hat{k}) = \frac{i}{\hat{\tau}_\pi} - i\hat{k}^2 + \mathcal{O}(\hat{k}^4),$$

$$\hat{\omega}_{\rm nh}(\hat{k}) = \frac{i}{\hat{\tau}_\pi} - \frac{4}{3}i\hat{k}^2 + \mathcal{O}(\hat{k}^4),$$

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Hydrodynamic and nonhydrodynamic modes

- **Hydrodynamic modes:** long-lived, associated with conserved quantities.
- **Nonhydrodynamic modes:** short-lived, necessary for linear causality and stability.

TWO-COMPONENT MAGNETOHYDRODYNAMICS

Longitudinal perturbations

In the small wave number limit, the modes are

$$\omega_{\mathrm{nh}}(k) = i\Sigma - \frac{4i}{15\Sigma}k^2 + \mathcal{O}(k^3),$$

$$\omega_{\mathrm{nh}}(k) = \frac{i}{2}\left[\Sigma + \Sigma' \pm \sqrt{(\Sigma - \Sigma')^2 - \frac{4q^2B_0^2}{25T^2}}\right] + \mathcal{O}(k^2),$$

$$\omega_{\mathrm{h}}(k) = \pm\frac{1}{\sqrt{3}}k + \frac{2i}{15\Sigma}k^2 + \mathcal{O}(k^3),$$

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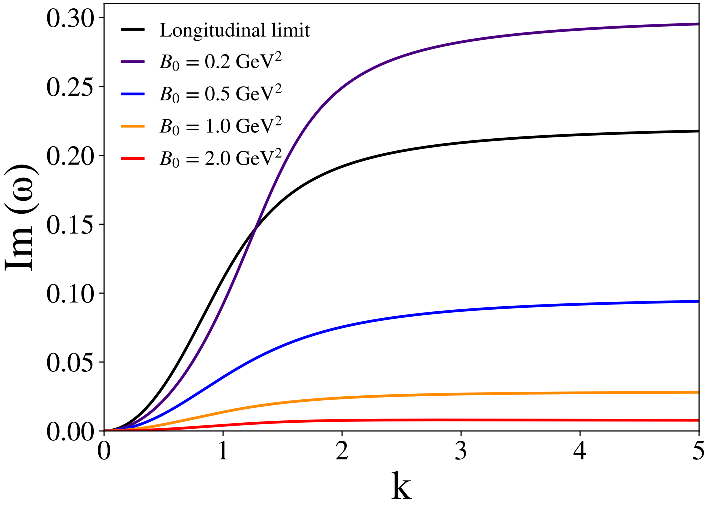
- Terms that are absent in the longitudinal approximation.

LONGITUDINAL PERTURBATIONS

Comparison of the hydrodynamic modes



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Longitudinal approximation

$$\omega = i\Sigma,$$

$$\omega = \pm\sqrt{\frac{1 + 3\mathcal{B}}{3(1 + \mathcal{B})}}k.$$

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- Modes are significantly distinct from Israel-Stewart-like theories.
- Causality and stability remain being fulfilled in the longitudinal approximation.
- Longitudinal approximation is justified in the limit of large B.
- Longitudinal approximation fails to capture the dynamics when B is not sufficiently large.