Relativistic Magnetohydrodynamics in Heavy-Ion Collisions

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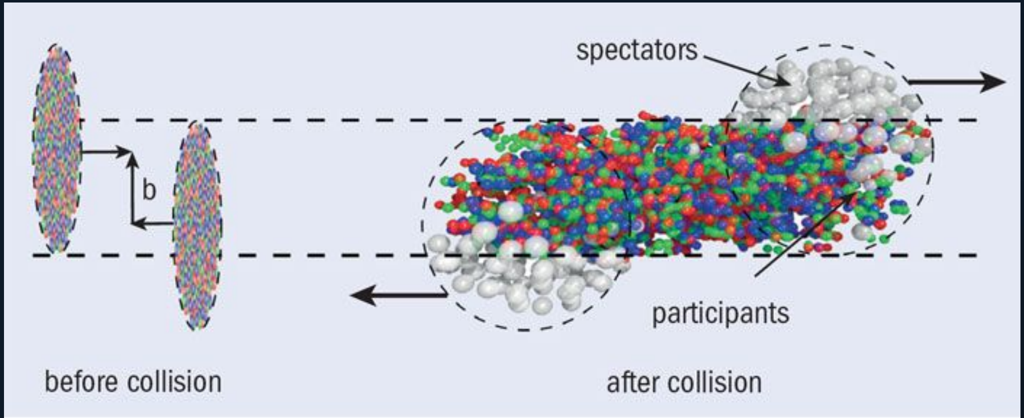
Outline

1. Motivation
2. Framework of Relativistic Magnetohydrodynamics
3. Equations of motion in Strong Magnetic Field
4. Equations of motion in Strong Electric Field
5. Conclusion

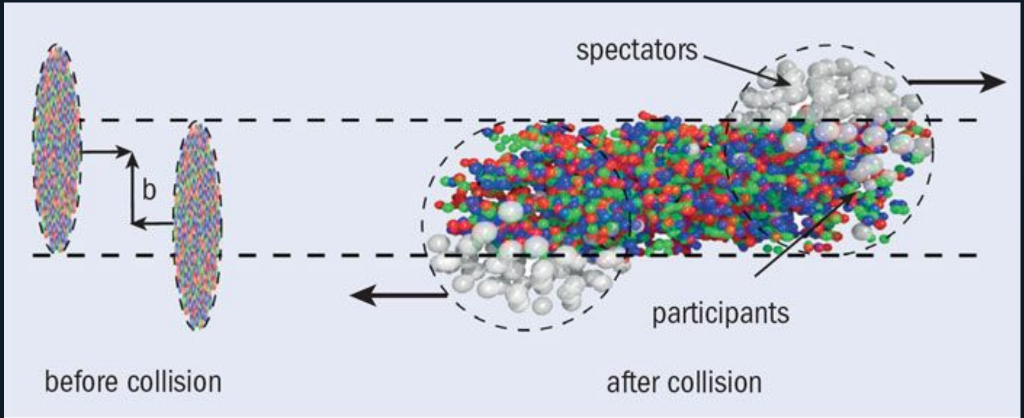
Why Relativistic Magnetohydrodynamics?



Heavy Ion Collisions ($\sim 10^{19}$ G)

Neutron stars ($\sim 10^{15}$ G)

Why Relativistic Magnetohydrodynamics?



Heavy Ion Collisions ($\sim 10^{19}$ G)

Neutron stars ($\sim 10^{15}$ G)

Maxwell's equations + Hydrodynamics

Relativistic Magnetohydrodynamics → Conducting fluids, e.g., Quark-Gluon Plasma (QGP)

Low freq., long wavelength approx.

Relativistic Magnetohydrodynamics framework

Locally neutral *non-resistive* fluid with no magnetization

Conservation Law

$$\partial_\mu T^{\mu\nu} = 0$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

$$T^{\mu\nu} = T^{\mu\nu}_{EM+Fluid} = \left(\varepsilon + \frac{B^2}{2}\right) u^\mu u^\nu - \Delta^{\mu\nu}\left(P + \frac{B^2}{2}\right) - B^\mu B^\nu + \pi^{\mu\nu}.$$

Maxwell's Equations

$$\partial_\mu \tilde{F}^{\mu\nu} = 0,$$

where, $\tilde{F}^{\mu\nu} = \mathcal{B}^\mu u^\nu - \mathcal{B}^\nu u^\mu.$

Relativistic Magnetohydrodynamics framework

Locally neutral *non-resistive* fluid with no magnetization

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Israel-Stewart Theory

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\langle\mu\nu\rangle}}{\tau_\pi} = \frac{2\eta\sigma^{\mu\nu}}{\tau_\pi} + \mathcal{O}(2),$$

Usual Hydro

What are the transient equations at finite B?

Relativistic Magnetohydrodynamics Framework

Non-resistive, finite Magnetic field

2 Species-Particle fluid

The system in consideration

- 2 particle species fluid of massless point like particles
- Locally neutral system, Vanishing dipole moment
- No spin: classical particles
- Only Shear Stress Tensor
- Using Method of Moments to solve the equation of motion.

Gabriel S. Denicol, Dirk H. Rischke (2021)
Microscopic foundations of relativistic fluid dynamics

Israel, Stewart, Annals Phys. 118 (1979) 341-372

Equations of motion for the Shear Stress Tensor

Boltzmann Equations

$$k^\mu \partial_\mu f_k^+ + q^+ k_\nu \mathcal{B}^{\mu\nu} \frac{\partial}{\partial k^\mu} f_k^+ = C[f_k^+, f_k^-]$$

$$k^\mu \partial_\mu f_k^- + q^- k_\nu \mathcal{B}^{\mu\nu} \frac{\partial}{\partial k^\mu} f_k^- = C[f_k^-, f_k^+]$$

$$\mathcal{B}^{\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta} u_\alpha B_\beta .$$

Positive species $\longrightarrow \pi_+^{\mu\nu}$

Negative species $\longrightarrow \pi_-^{\mu\nu}$

$+$: $\pi^{\mu\nu} = \pi_+^{\mu\nu} + \pi_-^{\mu\nu}$

$-$: $\delta\pi^{\mu\nu} = \pi_+^{\mu\nu} - \pi_-^{\mu\nu}$

Binary elastic collisions:
interaction between same species +particles and also interspecies particles.

Equations of motion for the Shear Stress Tensor

Boltzmann Equations

$$k^\mu \partial_\mu f_k^+ + q^+ k_\nu \mathcal{B}^{\mu\nu} \frac{\partial}{\partial k^\mu} f_k^+ = C[f_k^+, f_k^-]$$

$$k^\mu \partial_\mu f_k^- + q^- k_\nu \mathcal{B}^{\mu\nu} \frac{\partial}{\partial k^\mu} f_k^- = C[f_k^-, f_k^+]$$

$$\mathcal{B}^{\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta} u_\alpha B_\beta .$$

Positive species $\dashrightarrow \pi_+^{\mu\nu}$

Negative species $\dashrightarrow \pi_-^{\mu\nu}$

$$\begin{cases} \pi^{\mu\nu} = \pi_+^{\mu\nu} + \pi_-^{\mu\nu} & (+) \\ \delta\pi^{\mu\nu} = \pi_+^{\mu\nu} - \pi_-^{\mu\nu} & (-) \end{cases}$$

- 14-moment approximation *Israel, Stewart, Annals Phys.* 118 (1979) 341-372

$$f_k^\pm = f_{0k}^\pm \left(1 + \varphi_k^\pm\right)$$

$$\varphi_k^\pm \approx \frac{\pi_{\mu\nu}^\pm k^\mu k^\nu}{2(\epsilon_0^\pm + P_0^\pm)T^2}$$

➢ Exact equations for shear stress tensor

$$\Delta^{\mu\nu}_{\alpha\beta}\dot{\pi}^{\alpha\beta} + \Sigma\pi^{\mu\nu} + \boxed{\frac{2qB}{5T} q b^{\lambda\langle\mu}\delta\pi^{\nu\rangle}_{\lambda}} = \frac{8}{15}\varepsilon\sigma^{\mu\nu} - \frac{4}{3}\pi^{\mu\nu}\theta - \frac{10}{7}\sigma^{\lambda\langle\mu}\pi^{\nu\rangle}_{\lambda} - 2\omega^{\lambda\langle\nu}\pi^{\mu\rangle}_{\lambda},$$

$$\Delta^{\mu\nu}_{\alpha\beta}\delta\dot{\pi}^{\alpha\beta} + \Sigma'\delta\pi^{\mu\nu} + \boxed{\frac{2qB}{5T} q b^{\lambda\langle\mu}\pi^{\nu\rangle}_{\lambda}} = -\frac{4}{3}\delta\pi^{\mu\nu}\theta - \frac{10}{7}\sigma^{\lambda\langle\mu}\delta\pi^{\nu\rangle}_{\lambda} - 2\omega^{\lambda\langle\nu}\delta\pi^{\mu\rangle}_{\lambda}.$$

$$\Sigma = \frac{3P}{5T}\left(\sigma_T^{+-} + \sigma_T\right)$$

$$\Sigma' = \frac{P}{5T}\left(5\sigma_T^{+-} + 3\sigma_T\right)$$

$$\sigma_T^{++} = \sigma_T^{--} = \sigma_T$$

➢ Projected equation for shear stress tensor

$$\pi^{\mu\nu} = \left(b^\mu b^\nu + \frac{1}{2}\Xi^{\mu\nu}\right)\pi_{||} - b^\mu\pi^\nu_\perp - b^\nu\pi^\mu_\perp + \pi^{\mu\nu}_\perp$$

$$b^\mu \equiv \frac{B^\mu}{B}$$

- → $\pi_{||}$ (longitudinal),
- → $\pi_\perp^{\mu}$ (semi - transverse),
- → $\pi_\perp^{\mu\nu}$ (transverse).

__3 components__ of shear viscosity.

Each component relaxes differently.

➢ New basis

Individual components in new basis for simplicity. Taking projection with respect to $\ell^{\mu}_{\pm}$.

$$\pi_{\perp}^{\mu\nu} = \pi_{\perp}^{++}\ell_{+}^{\mu}\ell_{+}^{\nu} + \pi_{\perp}^{--}\ell_{-}^{\mu}\ell_{-}^{\nu}$$

where

$$\ell_{\pm}^{\mu} = \frac{1}{\sqrt{2}}(x^{\mu} \pm iy^{\mu})$$

➢ *Transverse component*

$$\dot{\pi}_{\perp}^{\pm\pm} + \frac{1}{\tau_{\pi}}\pi_{\perp}^{\pm\pm} \mp \boxed{i\frac{2|q|B}{5T}\delta\pi_{\perp}^{\pm\pm}} = \frac{8}{15}\epsilon\sigma_{\perp}^{\pm\pm} - \frac{4}{3}\theta\pi_{\perp}^{\pm\pm} + \cdots$$

$$\delta\dot{\pi}_{\perp}^{\pm\pm} + \frac{1}{\tau_{\delta\pi}}\delta\pi_{\perp}^{\pm\pm} \mp \boxed{i\frac{2|q|B}{5T}\pi_{\perp}^{\mp\mp}} = -\frac{4}{3}\theta\delta\pi_{\perp}^{\pm\pm} + \cdots$$

Shear Stress Tensor in Linear Regime: Oscillatory dynamics ?

➔ *Linearising equations around equilibrium and in constant magnetic field* (Homogeneous case)

$$\ddot{\pi}_{\perp}^{\mp\mp} + \left(\Sigma + \Sigma'\right)\dot{\pi}_{\perp}^{\mp\mp} + \left(\Sigma\Sigma' + 4\Omega^2\right)\pi_{\perp}^{\mp\mp} = \frac{8}{15}\varepsilon\Sigma'\sigma_{\perp}^{\mp\mp} + \frac{8}{15}\varepsilon\dot{\sigma}_{\perp}^{\mp\mp}.$$

$$\Sigma = \frac{3P}{5T}\left(\sigma_T^{+-} + \sigma_T\right)$$

$$\Sigma' = \frac{P}{5T}\left(5\sigma_T^{+-} + 3\sigma_T\right)$$

Harmonic Oscillator type equation!

$$\Omega = \frac{qB}{5T}$$

➔ *Mode for the transverse component*

$$\omega = \frac{i}{2}\left[\Sigma + \Sigma' \pm \sqrt{\left(\Sigma - \Sigma'\right)^2 - 4\Omega^2}\right]$$

$$\Omega < \frac{\Sigma' - \Sigma}{4} \Longrightarrow \frac{2qB}{T} < n_0\sigma_T^{+-}$$

Overdamped

Relaxing to equilibrium

B = 0

Critically damped

$\omega = i\Sigma$ ***and*** $\omega = i\Sigma'$

Shear Stress Tensor in Linear Regime: Oscillatory dynamics ?

➔ *Linearising equations around equilibrium and in constant magnetic field* (Homogeneous case)

$$\ddot{\pi}_{\perp}^{\mp\mp} + \left(\Sigma + \Sigma'\right)\dot{\pi}_{\perp}^{\mp\mp} + \left(\Sigma\Sigma' + 4\Omega^2\right)\pi_{\perp}^{\mp\mp} = \frac{8}{15}\varepsilon\Sigma'\sigma_{\perp}^{\mp\mp} + \frac{8}{15}\varepsilon\dot{\sigma}_{\perp}^{\mp\mp}.$$

$$\Sigma = \frac{3P}{5T}\left(\sigma_T^{+-} + \sigma_T\right)$$

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$$\Omega > \frac{\Sigma' - \Sigma}{4} \Longrightarrow \frac{2qB}{T} > n_0\sigma_T^{+-}$$

Oscillatory dynamics

Can't be described by IS theory!

B = 0

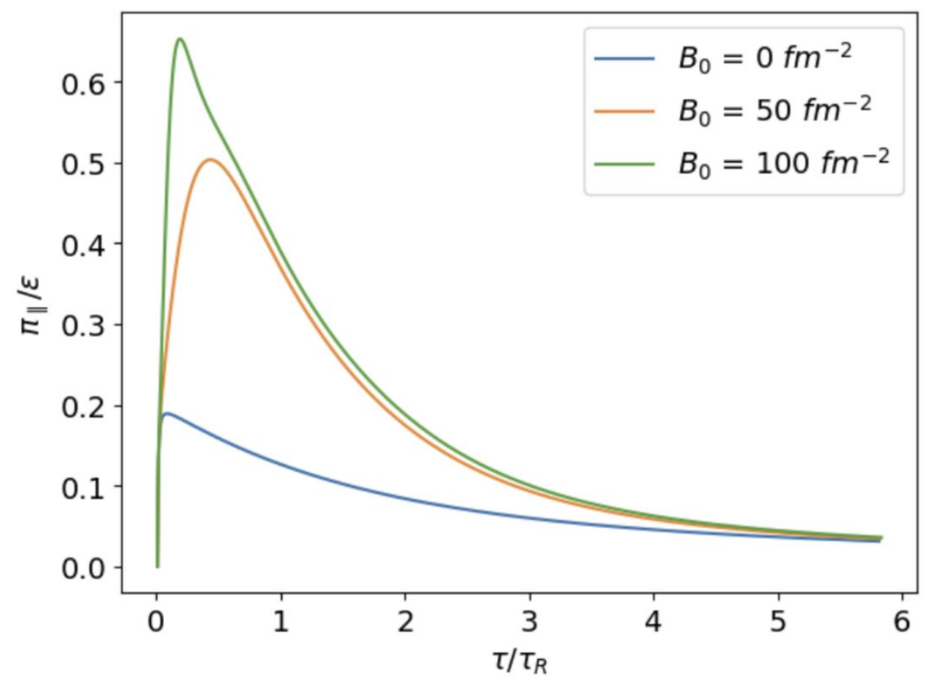
Critically damped

$\omega = i\,\Sigma$ ***and*** $\omega = i\,\Sigma'$

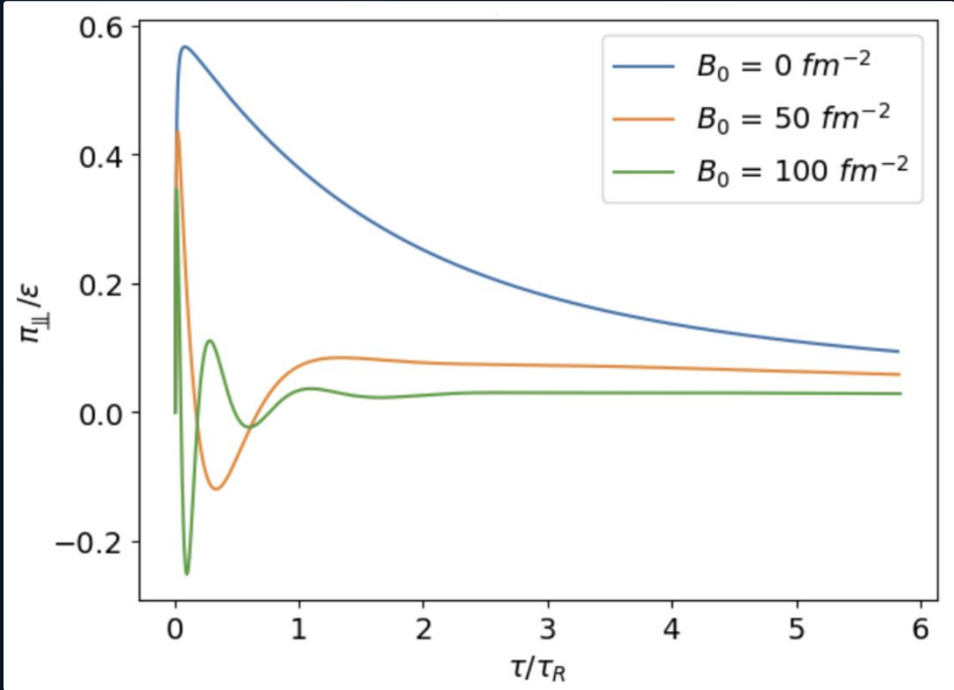
Our System in Bjorken flow

$$B \sim \left(\frac{\tau_0}{\tau}\right)$$

Longitudinal component



Transverse component



Relativistic Magnetohydrodynamics Framework

Resistive case, No Magnetic field,

<u>Homogeneous system</u>

Relativistic Magnetohydrodynamics framework

Locally neutral *resistive* fluid with *no magnetic field*

$$T^{\mu\nu} = T^{\mu\nu}_{EM+Fluid} = \left(\varepsilon + \frac{E^2}{2}\right) u^\mu u^\nu - \Delta^{\mu\nu}\left(P + \frac{E^2}{2}\right) - E^\mu E^\nu + \pi^{\mu\nu}.$$

$$N^\mu = n u^\mu + J^{\langle\mu\rangle}$$

Conservation Law

$$\partial_\mu T^{\mu\nu} = 0,$$
$$\partial_\mu N^\mu = 0.$$

Maxwell's Equations

$$\partial_\mu F^{\mu\nu} = 0,$$
where, $F^{\mu\nu} = E^\mu u^\nu - E^\nu u^\mu.$

Relativistic Magnetohydrodynamics framework

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$$N^\mu = n u^\mu + J^{\langle\mu\rangle}$$

Conservation Law

$$\partial_\mu T^{\mu\nu} = 0,$$
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Israel-Stewart Theory

$$\tau_J \dot{J}^{\langle\mu\rangle} + J^{\langle\mu\rangle} = \sigma_{eff} E^\mu$$
$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\langle\mu\nu\rangle}}{\tau_\pi} = \frac{2\eta\sigma^{\mu\nu}}{\tau_\pi} + \mathcal{O}(2),$$

What are the transient equations of motion at finite E and no B?

Relativistic Magnetohydrodynamics framework

Locally neutral *resistive* fluid with *no magnetic field*

Boltzmann Equations

$$k^\mu \partial_\mu f_k^+ + q^+ k_\nu \mathcal{E}^{\mu\nu} \frac{\partial}{\partial k^\mu} f_k^+ = C[f_k^+, f_k^-]$$

$$k^\mu \partial_\mu f_k^- + q^- k_\nu \mathcal{E}^{\mu\nu} \frac{\partial}{\partial k^\mu} f_k^- = C[f_k^-, f_k^+]$$

$$\mathcal{E}^{\mu\nu} = E^\mu u^\nu - E^\nu u^\mu$$

$$\sigma_T^{++} = \sigma_T^{--} = \sigma_T$$

$$\frac{\sigma_T^{+-}}{\sigma_T} >> 1$$

$$J^\mu = J_+^\mu + J_-^\mu$$

$$\tau_J \dot{j}^{\langle\mu\rangle} + J^\mu = \sigma_{eff} E^\mu + \boxed{\lambda \pi^{\mu\nu} E_\nu}$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{1}{\tau_\pi} \pi^{\mu\nu} = \boxed{\frac{8}{5} E^{\langle\mu} J^{\nu\rangle}}$$

➔ *Electric field triggers shear stress tensor → contributes to charge four-current.*

Relativistic Magnetohydrodynamics framework

Locally neutral *resistive* fluid with *no magnetic field*

➔ *The modes of the equations of motion*

$$\omega_{\pm} = \frac{1}{2}\left(\frac{1}{\tau_J} + \frac{1}{\tau_\pi} \pm \sqrt{-\frac{64\lambda E^2}{15} + \frac{1}{\tau_J^2} + \frac{1}{\tau_\pi^2} - \frac{2}{\tau_J \tau_\pi}}\right)$$

$$\frac{\sigma_T^{+-}}{\sigma_T} >> 1$$

➔ *Onset for oscillatory dynamics*

$$E^2 > \frac{15}{64\lambda}\left(\frac{1}{\tau_J^2} + \frac{1}{\tau_\pi^2} - \frac{2}{\tau_J \tau_\pi}\right)$$

For large enough E, electric charge four-current can not be approximated as a simple Ohm's law given by Israel-Stewart-like theories.

Conclusion

- We derived MHD equations for a binary gas of classical massless particles.
- Equations of motion change drastically for this system.
- Shear stress tensor splits into three non degenerate components. Each component relaxes differently, with their own dynamical equation.
- For sufficiently large values of B, shear stress tensor will show oscillatory dynamics.
- For purely resistive case, in absence of B, electric charge four current exhibits similar oscillatory dynamics for sufficiently large values of E.
- For large E, simple ohm's law, $J^{\mu} = \sigma_{eff} E^{\mu}$, will not be able to capture accurate dynamics.
- Outlook: Astrophysical systems like accretion disks.

Outlook– Astrophysical systems ~ Accretion Disk

Massive case: Electrons and Ions

Causal Magnetohydrodynamics Theory

~ Strong Magnetic field, Large mean free path

$$T^{\mu\nu} = T^{\mu\nu}_{EM+Fluid} = \left(\varepsilon + \frac{B^2}{2}\right) u^\mu u^\nu - \Delta^{\mu\nu}\left(P + \frac{B^2}{2}\right) - B^\mu B^\nu + \boxed{\pi^{\mu\nu}}.$$

Chandra, Gammie, Foucart, Quataert, Astrophys.J. 810 (2015) 2, 162

$$\pi^{\mu\nu} \approx -\boxed{\Delta P}\left(\hat{b}^\mu \hat{b}^\nu + \frac{1}{3}\Delta^{\mu\nu}\right)$$

Relaxation type (Israel-Stewart-like) equation for ΔP

Causal Magnetohydrodynamics Theory

~ Strong Magnetic field, Large mean free path

$$T^{\mu\nu} = T^{\mu\nu}_{EM+Fluid} = \left(\varepsilon + \frac{B^2}{2}\right) u^\mu u^\nu - \Delta^{\mu\nu}\left(P + \frac{B^2}{2}\right) - B^\mu B^\nu + \boxed{\pi^{\mu\nu}}$$

Chandra, Gammie, Foucart, Quataert, Astrophys.J. 810 (2015) 2, 162

$$\pi^{\mu\nu} \approx -\Delta P \left(\hat{b}^\mu \hat{b}^\nu + \frac{1}{3}\Delta^{\mu\nu}\right)$$

Good approximation for very large B and small T (~ 20 MeV).

In heavy-ion collisions: large B (~10^{19} G) and large T (~ 150 MeV) – Approximation will not work.

Linear stability analysis

Very large Magnetic field

$$\varepsilon = \varepsilon_0 + \Delta\varepsilon, \;\; n = \Delta n, \;\; u^\mu = u_0^\mu + \Delta u^\mu, \;\; \boxed{\pi^{\mu\nu} = \pi_0^{\mu\nu} + \Delta\pi^{\mu\nu},} \;\; B^\mu = B_0 b_0^\mu + b_0^\mu \Delta B + B_0 \Delta b^\mu,$$

$$\pi_0^{\mu\nu} = \boxed{\pi_\parallel(\tau)} \left(b_0^\mu b_0^\nu + \frac{\Xi^{\mu\nu}}{2} \right)$$

- Coupled equations of motion for π_{bk}, $\delta\pi_{bk}$, π_{bq} and $\delta\pi_{bq}$

Navier-Stokes Limit:

b → Along magnetic field
q → Transverse to b and k

$$\Delta\tilde{\pi}_{bq} \approx \frac{3}{2}\pi_\parallel \Delta\tilde{b}_q$$

$$\Delta\tilde{\pi}_{bk} \approx \frac{3}{2}\pi_\parallel \Delta\tilde{b}_k$$

If parallel component grows large enough, other components CAN NOT be ignored.

Conclusion

- We derived MHD equations for a binary gas of classical massless particles.
- Equations of motion change drastically for this system.
- Shear stress tensor splits into three non degenerate components. Each component relaxes differently, with their own dynamical equation.
- For sufficiently large values of B, shear stress tensor will show oscillatory dynamics.
- For purely resistive case, in absence of B, electric charge four current exhibits similar oscillatory dynamics for sufficiently large values of E.
- For large E, simple ohm's law, $J^\mu = \sigma_{eff} E^\mu$, will not be able to capture accurate dynamics.
- Outlook: Astrophysical systems like accretion disks.

Backup

14-moment approximation

Israel, Stewart, Annals Phys. 118 (1979) 341-372

$$f_k^{\pm} = f_{0k}^{\pm}\left(1 + \varphi_k^{\pm}\right) \Longrightarrow \varphi_k^{\pm} \approx \epsilon^{\pm} + \epsilon_{\mu}^{\pm} k^{\mu} + \epsilon_{\mu\nu}^{\pm} k^{\mu} k^{\nu}$$

Truncated at second order in momentum

We consider only the Shear Stress Tensor $\Longrightarrow$ $\epsilon^{\pm} \to 0, \quad \epsilon_{\mu}^{\pm} \to 0$

$$\epsilon_{\mu\nu}^{\pm} \sim \frac{\pi_{\mu\nu}^{\pm}}{(\epsilon_0^{\pm} + P_0^{\pm})T^2} \Longrightarrow \varphi_k^{\pm} \approx \frac{\pi_{\mu\nu}^{\pm}\, k^{\mu} k^{\nu}}{2(\epsilon_0^{\pm} + P_0^{\pm})T^2}$$

14 moments and 14 coefficients in distribution function to be identified.

Causal Magnetohydrodynamics Theory

Current approach ~ Strong Magnetic field

$$T^{\mu\nu} = T^{\mu\nu}_{EM+Fluid} = \left(\varepsilon + \frac{B^2}{2}\right) u^\mu u^\nu - \Delta^{\mu\nu}\left(P + \frac{B^2}{2}\right) - B^\mu B^\nu + \pi^{\mu\nu}$$

Chandra, Gammie, Foucart, Quataert, Astrophys.J. 810 (2015) 2, 162

$$\pi^{\mu\nu} \approx -\Delta P\left(\hat{b}^\mu \hat{b}^\nu + \frac{1}{3}\Delta^{\mu\nu}\right)$$

Relaxation type equation for ΔP

$$\frac{d\Delta P}{d\tau} = -\frac{\Delta P - \Delta P_0}{\tau_R} - \frac{\Delta P}{2}\frac{d}{d\tau}\log\left(\frac{\tau_R}{\rho\nu P}\right)$$

$$\Delta P_0 \equiv 3\rho\nu(\hat{b}^\mu \hat{b}^\nu \nabla_\mu u_\mu - \frac{1}{3}\nabla_\mu u^\mu)$$

Navier-Stokes Value

ϱ = Rest mass density; ν = Kinematic viscosity

Final equations for shear stress tensor

Iterating order by order in gradient & truncating upto 2nd order in gradient.

Longitudinal component

$$\dot{\pi}_{\parallel} + \pi_{\perp}^{\mu}\dot{b}_{\mu} + \Sigma\pi_{\parallel} = \frac{8}{15}\varepsilon\sigma_{\parallel} - \frac{4}{3}\pi_{\parallel}\theta - \frac{10}{7}\left(-\frac{1}{2}\pi_{\parallel}\sigma_{\parallel} + \frac{1}{3}\sigma_{\perp}^{\mu}\pi_{\perp\mu} + \frac{1}{3}\pi_{\perp\alpha\beta}\sigma_{\perp}^{\alpha\beta}\right) - \frac{2}{3}\left(\omega_{\perp}^{\mu}\pi_{\perp\mu} + \omega_{\perp}^{\alpha\beta}\pi_{\perp\alpha\beta}\right)$$

Transverse component

$$\begin{aligned}(1-4\varphi^2)\,\dot{\pi}_{\perp}^{\mp\mp} + \left(\Sigma + 4\Sigma'\varphi^2\right)\pi_{\perp}^{\mp\mp} \\ = \frac{8}{15}\varepsilon\sigma_{\perp}^{\mp\mp} - \left[(1-4\varphi^2)\left(2\ell_{\mp}^{\beta}\dot{\ell}_{\beta}^{\pm} + \frac{4}{3}\theta + \frac{5}{7}\sigma_{\parallel}\right) - 4\varphi\dot{\varphi}\right]\pi_{\perp}^{\mp\mp} \\ + (1+2\varphi^2)\left(2\ell_{\beta}^{\pm}\dot{b}^{\beta} + \frac{10}{7}\sigma_{\perp}^{\mp} + 2\omega_{\perp}^{\mp}\right)\pi_{\perp}^{\mp} - \left(\frac{5}{7}\sigma_{\perp}^{\mp\mp} + \omega_{\perp}^{\mp\mp}\right)\pi_{\parallel}\end{aligned}$$

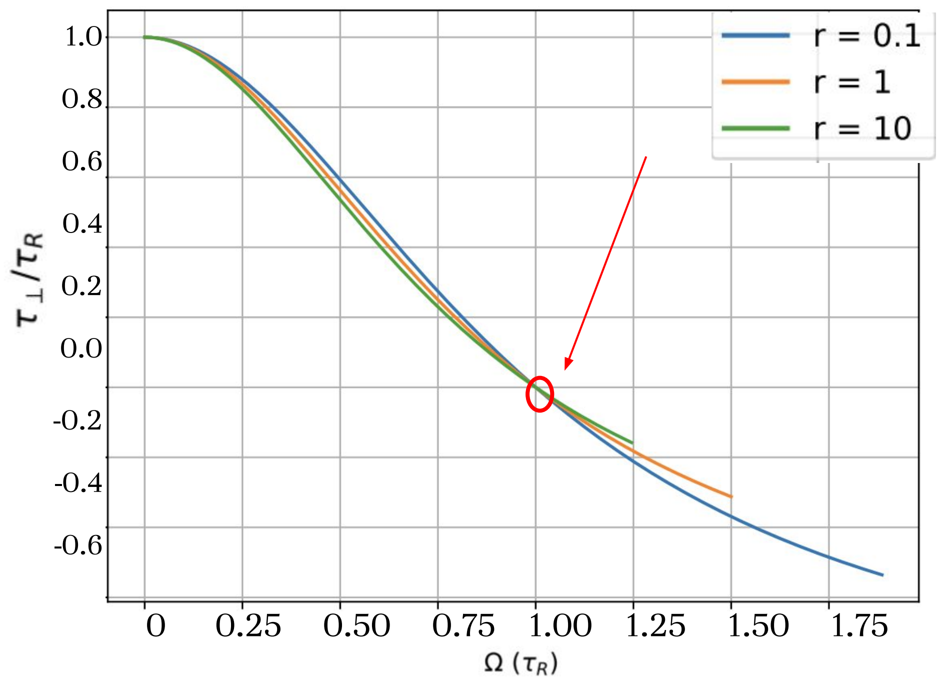
Relaxation time as function of magnetic field

$$\Omega = \frac{qB}{5T}$$

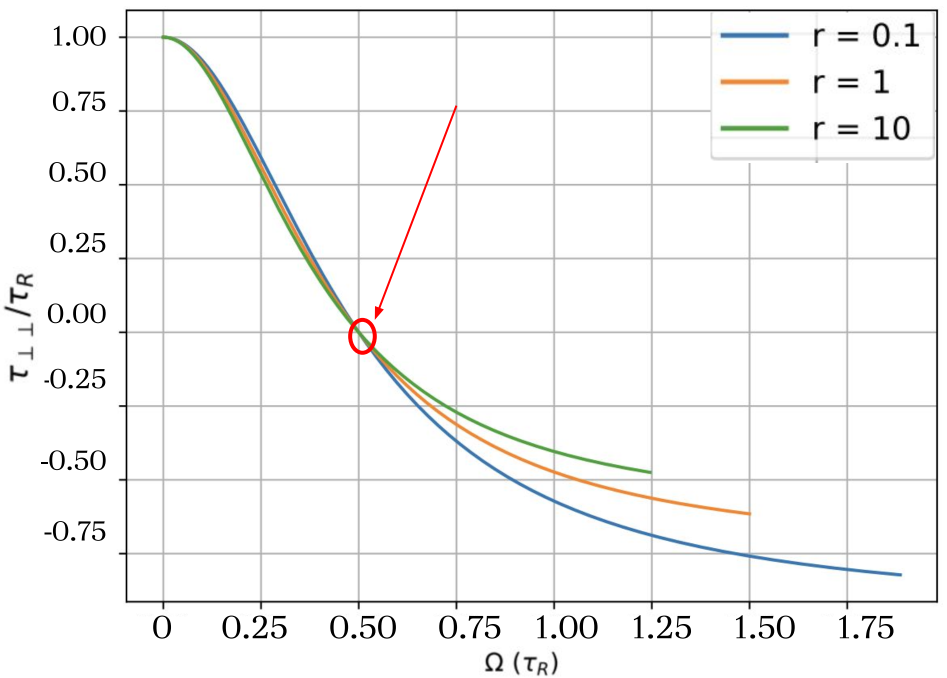
$$r = \frac{\sigma_T^{+-}}{\sigma_T^{++}}$$

- ➔ *Different component of shear relaxes differently.*
- ➔ *Theory fails at large B: Negative relaxation times.*

Semi transverse component



Fully transverse component



Relativistic Magnetohydrodynamics framework

$$\frac{\sigma_T^{+-}}{\sigma_T} >> 1$$

Locally neutral *resistive* fluid with *no magnetic field*

➔ *Electric field triggers shear stress tensor*
→ contributes to charge four-current.

$$J^\mu = J^\mu_+ + J^\mu_-$$

$$\delta J^\mu = J^\mu_+ - J^\mu_-$$

$$\tau_J \dot{j}^{\langle\mu\rangle} + J^\mu = \sigma_{eff} E^\mu + \boxed{\lambda \pi^{\mu\nu} E_\nu}$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{1}{\tau_\pi}\pi^{\mu\nu} = \boxed{\frac{8}{5} E^{\langle\mu} J^{\nu\rangle}}$$

$$\tau_{\delta J} \delta \dot{j}^{\langle\mu\rangle} + \delta J^\mu = \lambda \delta \pi^{\mu\nu} E_\nu$$

$$\delta \dot{\pi}^{\langle\mu\nu\rangle} + \frac{1}{\tau_{\delta\pi}} \delta\pi^{\mu\nu} = \frac{8}{5} E^{\langle\mu} \delta J^{\nu\rangle}$$

No coupling arises due to electric field

Relativistic Magnetohydrodynamics framework

$$\frac{\sigma_T^{+-}}{\sigma_T} >> 1$$

Locally neutral *resistive* fluid with *no magnetic field*

➔ *Electric field triggers shear stress tensor → contributes to charge four-current.*

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No coupling arises due to electric field

Relativistic Magnetohydrodynamics framework

Locally neutral *resistive* fluid with *no magnetic field*

$$\sigma_T^{++} = \sigma_T^{--} = \sigma_T$$
$$\sigma_T^{+-} = 0$$

Oscillatory dynamics for charge four current –

$$E > \frac{13\,\hat{n}_0 \sigma_T T}{12\sqrt{3}\,|q|}$$