Chiral and deconfinement thermal transitions at finite quark spin polarization in lattice QCD simulations

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General theoretical aim:
to understand the effect of finite spin polarization on QCD thermodynamics

A more focused question:
$\ell : L = \text{(de)confinement}, \psi = \text{chiral}$

how do the QCD crossover temperatures T_c^{ℓ} depend on the <u>quark</u> spin potential μ_Σ ?

- polarizes spins of quarks along certain axis
- introdices no baryon/electric/axial charge

Quantities we calculate from first principles:
the curvatures of the crossover temperatures

$$\frac{T_c^{\ell}(\mu_\Sigma)}{T_c(0)} = 1 - \boxed{\kappa_\Sigma^{\ell}} \left(\frac{\mu_\Sigma}{T_c(0)}\right)^2 + \dots\,, \qquad [\ell = L, \psi]$$

based on

V. V. Braguta, A. A. Roenko, M. Ch., Phys.Rev.D 111 (2025) 11, 114508 • e-Print: 2503.18636

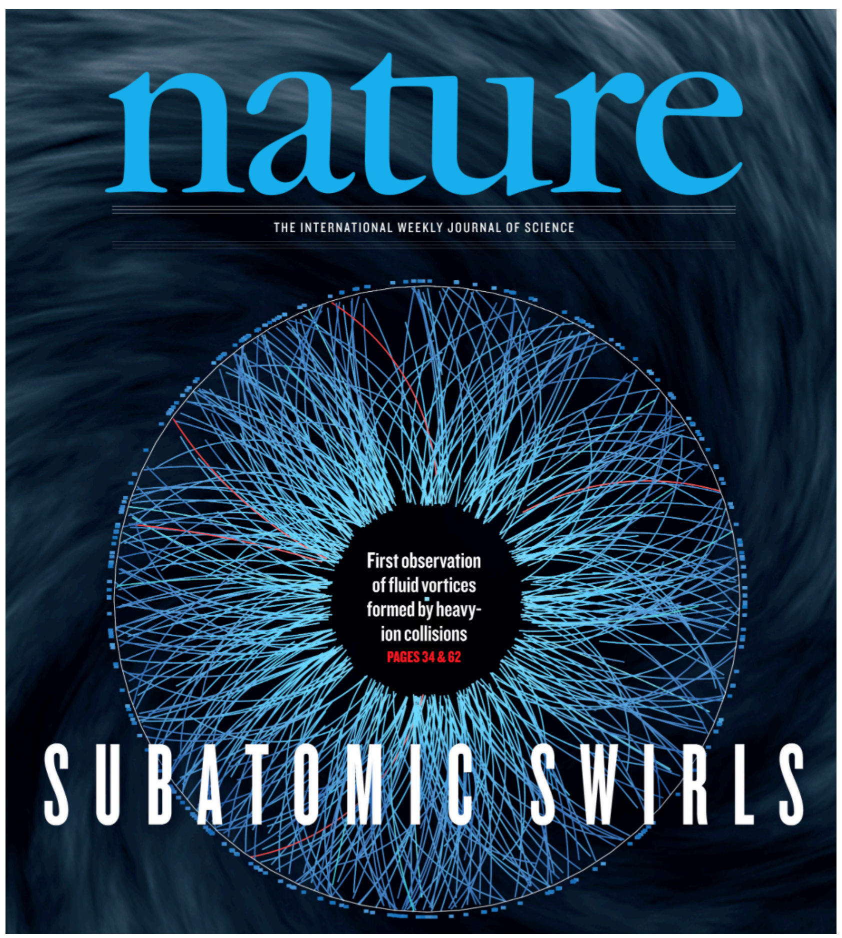
Motivation by rotation

Experiment

The experimental result for the global vorticity:

$$\omega \approx (9 \pm 1) \times 10^{21}\,\mathrm{s}^{-1}$$

The most vortical fluid ever observed



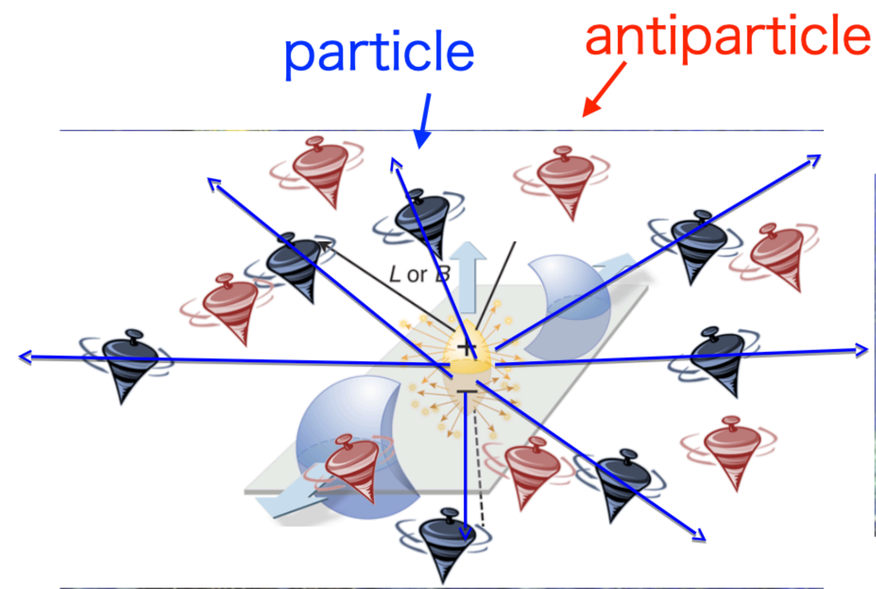
The STAR Collaboration, Nature 62, 548 (2017)

Theory

Rotation (vorticity) polarizes the spins of quarks

→ gives an imprint in the spins of produced hadrons

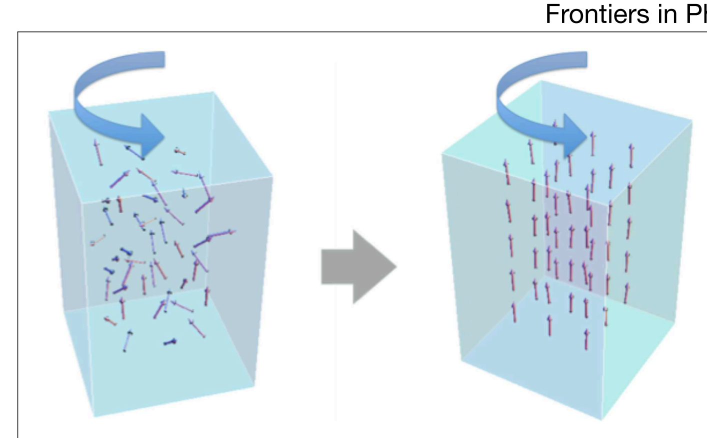
→ can be detected experimentally



[Illustration: T.Niida, 2019]

The Barnett effect:

[Illustration: Matsuo, Ieda, Maekawa, Frontiers in Physics 3, 54 (2015)]



Barnett effect. Magnetization is induced by applying mechanical rotation since an effective magnetic field, emerges in a rotating body.

S. J. Barnett, “Magnetization by rotation,” Phys. Rev. 6, 239 (1915).

A special kind of (quark) matter:

1. Consider QGP with a non-zero spin density of quarks

   (generated, for example, by rotation/vorticity or other effects)

2. Take a clean example: a uniform, time-independent spin density

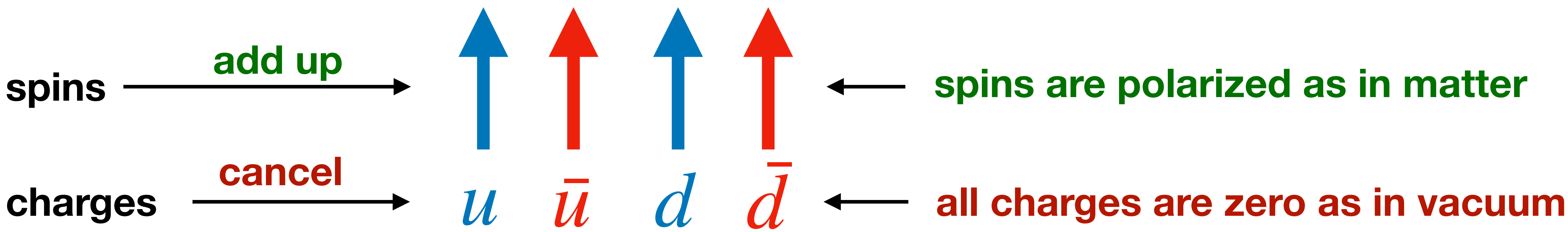
   (in a spirit of uniform steady rotation, uniform magnetic field, etc)

3. Assume that all charge densities are vanishing.

   (so that baryon, isospin, electric, axial, etc chemical potentials are zero)

4. Can we create such a configuration? Yes, we can!

A simplified example:



Each quark appears always in a company together with a corresponding anti-quark to make all global charges vanishing again, as in vacuum.

Disclaimer: valid in quark-gluon plasma phase (neutral, but not really a vacuum)

Introducing spins of quarks

The quark spin density term in the Dirac Lagrangian

$$\delta_\Sigma \mathcal{L}_q = \mu_{\alpha,\mu\nu}\, \overline{\psi} \mathcal{S}^{\alpha,\mu\nu} \psi$$

Spin potential

introduces a finite spin density and
a finite spin current in the system

the relativistic spin
density matrix

$$\mathcal{S}^{\alpha,\mu\nu} = \frac{1}{2}\left\{\gamma^\alpha, \Sigma^{\mu\nu}\right\}$$

$$\Sigma^{\mu\nu} = \frac{i}{4}\left[\gamma^\mu, \gamma^\nu\right]$$

the canonical relativistic spin current
operator for spin-1/2 Dirac fermions

A particular choice

(a component of the)
spin potential

$$\mu_{\alpha,\mu\nu} = \frac{\mu_\Sigma}{2}\delta_{\alpha 0}\left(\delta_{\mu 1}\delta_{\nu 2} - \delta_{\nu 1}\delta_{\mu 2}\right)$$

makes the spins of quarks polarized along
the z-axis in certain reference frame

There is well-discussed ambiguity
(a pseudo-gauge symmetry) in the
definition of the local spin operator.

Important in spin (hydro)dynamics:

[F. Becattini, Rept. Prog. Phys. 85, 122301 (2022); W. Florkowski,
K. Fukushima, S. Pu, Phys.Lett.B 817 (2021) 136346; J.-H. Gao,
G.-L. Ma, S. Pu, Q. Wang, *Nucl.Sci.Tech.* 31 (2020) 9, 90;
B. Friman, A. Jaiswal, E. Speranza, Phys. Rev. C 97, 041901 (2018);
D. Montenegro, L. Tinti, G. Torrieri, Phys. Rev. D 96, 056012 (2017), …]

Can be fixed in interacting theories; example:
[M. Buzzegoli, A. Palermo, *Phys.Rev.Lett.* 133 (2024) 26, 262301]

Polarization and rotation of fermions

("spin polarization = rotation - orbital motion")

The quark spin density term:

$$\delta_\Sigma \mathcal{L}_q = \mu_\Sigma \underbrace{\overline{\psi}\gamma^0\Sigma^{12}\psi}_{\text{spin polarization}}$$

$$\delta_\Sigma \mathcal{L}_q = \mu_{\alpha,\mu\nu}\, \overline{\psi}\mathcal{S}^{\alpha,\mu\nu}\psi$$

for the specific choice of the spin potential:

$$\mu_{\alpha,\mu\nu} = \frac{\mu_\Sigma}{2}\delta_{\alpha 0}\left(\delta_{\mu 1}\delta_{\nu 2} - \delta_{\nu 1}\delta_{\mu 2}\right)$$

Rotation (about z-axis with the angular frequency Ω)

$$\delta_\Omega \mathcal{L}_q = \Omega\overline{\psi}\Big[\underbrace{i(-x\partial_y + y\partial_x)}_{\text{orbital rotation}} + \underbrace{\gamma^0\Sigma^{12}}_{\text{spin polarization}}\Big]\psi$$

"$\Omega = \mu_\Sigma$"

No orbital rotation - no infrared (causality) problem!

Behaves well in the infrared! (good thermodynamic limit)

The spin potential μ_Σ is a thermodynamically conjugated quantity to the fermionic spin polarization density $S_z = \bar{\psi}\gamma^0\Sigma^{12}\psi$

"The spin potential μ_Σ plays a role for the spin polarization S_z similar to the role of the baryon chemical potential μ_B for the baryon density ρ_B"

N.B. - The spin potential μ_Σ is a not a "chemical" potential as spin is not conserved.

Setting up the lattice simulations

1. Add the quark spin density term $\delta_\Sigma \mathcal{L}_q = \mu_\Sigma \overline{\psi}\gamma^0 \Sigma^{12}\psi$

   to the fermionic lattice action of the $N_f = 2$ QCD

2. Make a Wick rotation to the Euclidean spacetime.

3. The sign-problem: make the spin potential imaginary, $\mu_\Sigma = i\mu_\Sigma^{\mathrm{I}}$

4. The Euclidean fermionic action becomes real-valued:

$$S_F = \int d^4x\ \bar{\psi}\Big[\gamma^x D_x + \gamma^y D_y + \gamma^z D_z + \gamma^\tau \left(D_\tau + i\mu_\Sigma^{\mathrm{I}}\Sigma^{12}\right) + m\Big]\psi$$

→ we need an analytical continuation back to the real world

5. The lattice QCD action:

   Gluons: the renormalization-group improved (Iwasaki) lattice action

$$S_G = \beta \sum_x \Big(c_0 \sum_{\mu<\nu} W_{\mu\nu}^{1\times1} + c_1 \sum_{\mu\neq\nu} W_{\mu\nu}^{1\times2}\Big)$$

   Quarks: clover-improved action for Wilson fermions

$$S_F = \sum_{f=u,d} \sum_{x_1,x_2} \bar{\psi}^f(x_1) M_{x_1,x_2} \psi^f(x_2)$$

$$\begin{aligned} M_{x_1,x_2} = \delta_{x_1,x_2} - & \\ & - \kappa\Big[\sum_{\mu=x,y,z}\Big((1-\gamma^\mu)T_{\mu+} + (1+\gamma^\mu)T_{\mu-}\Big) + \\ & + (1-\gamma^\tau)\ \exp\Big(ia\mu_\Sigma^{\mathrm{I}}\Sigma^{12}\Big)T_{\tau+} + \\ & + (1+\gamma^\tau)\ \exp\Big(-ia\mu_\Sigma^{\mathrm{I}}\Sigma^{12}\Big)T_{\tau-}\Big] - \\ & - \delta_{x_1,x_2} c_{SW}\kappa \sum_{\mu<\nu}\sigma_{\mu\nu}F_{\mu\nu}\,, \end{aligned}$$

Lattice observables

Confinement

Polyakov loop

$$L(\boldsymbol{r}) = \left\langle \frac{1}{3} \operatorname{Tr} \left[\prod_{\tau=0}^{N_t-1} U_4(\tau, \boldsymbol{r}) \right] \right\rangle$$

and its susceptibility

$$\chi_L = N_s^3 \left(\langle |L_{\text{bulk}}|^2 \rangle - \langle |L_{\text{bulk}}| \rangle^2 \right)$$

Chiral symmetry breaking

Susceptibility of the chiral condensate

$$\chi_{\bar\psi\psi}^{\text{disc}} = \frac{N_f T}{V} \left[\left\langle \operatorname{Tr}(M^{-1})^2 \right\rangle - \left\langle \operatorname{Tr}(M^{-1}) \right\rangle^2 \right]$$

(a disconnected part of)

Effect of the finite spin density on crossover temperatures

$$\frac{T_c^\ell(\mu_\Sigma^{\mathrm{I}})}{T_c^\ell(0)} = 1 + \kappa_\Sigma^\ell \left(\frac{\mu_\Sigma^{\mathrm{I}}}{T_c(0)} \right)^2 + \dots, \qquad [\ell = L, \psi]$$

Imaginary spin potential

the curvatures of the confinement ($\ell = L$) and chiral ($\ell = \psi$) crossover transitions

$$\frac{T_c^\ell(\mu_\Sigma)}{T_c(0)} = 1 - \kappa_\Sigma^\ell \left(\frac{\mu_\Sigma}{T_c(0)} \right)^2 + \dots, \qquad [\ell = L, \psi]$$

Real spin potential

Lattice strategy

1. **Many temperature points in the crossover region $T \sim T_c$**

   → check the effect of the spin density on T_c

2. **Various meson mass ratios $m_{\rm PS}/m_{\rm V} = 0.60, \ldots, 0.85$**

   → extrapolate to the physical value $m_\pi/m_\rho \simeq 0.175$

3. **Several lattice extensions in the imaginary time, $N_\tau = 4, 5, 6$**

   → check relevance of UV lattice cutoff

4. **Several spatial extensions, $N_s = 16, 20, 24$**

   → check the thermodynamic (IR) limit

5. **We do not renormalize susceptibilities:**

   a renormalization is important for actual values condensates, currents, densities in the presence of the axial chemical potential

   [V. V. Braguta, V. A. Goy, E. M. Ilgenfritz, A. Yu. Kotov, A. V. Molochkov, M. Muller-Preussker, and B. Petersson, JHEP 06, 094 (2015);
   V. V. Braguta, E. M. Ilgenfritz, A. Yu. Kotov, B. Petersson, and S. A. Skinderev, Phys. Rev. D 93, 034509 (2016);
   B. B. Brandt, G. Endrődi, E Garnacho-Velasco, G. Markó, JHEP 09 (2024) 092; G. Endrődi, Prog.Part.Nucl.Phys. 141 (2025) 104153.]

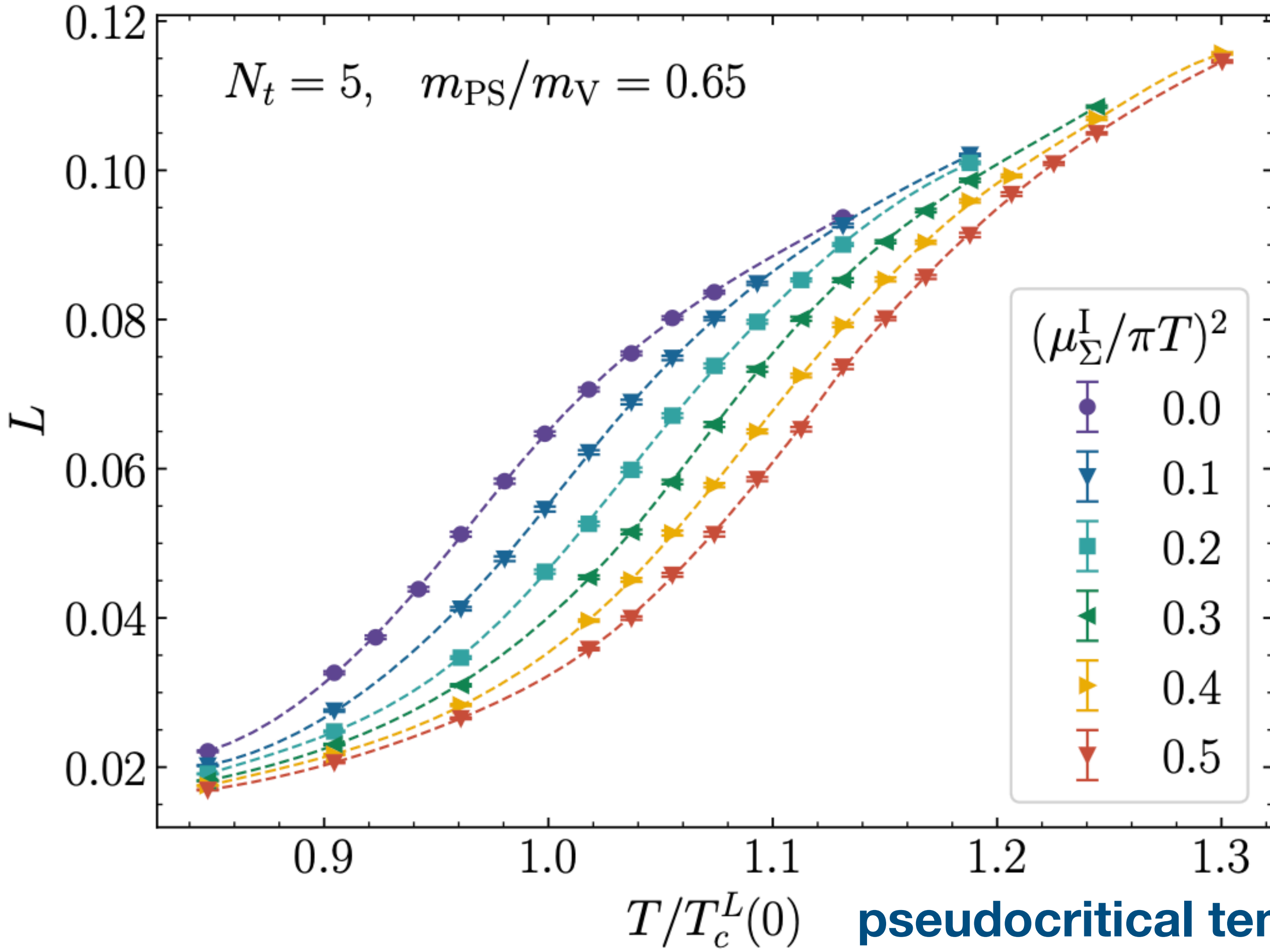
   Talks by Gergely Endrodi and Eduardo Garnacho on Tuesday

   not essential effect in temperature ratios $T_c(\mu_\Sigma)/T_c(0)$

Qualitative effect of polarization of quarks on gluons!

(an example for the deconfinement transition)

the value of the expectation value of the Polyakov loop



imaginary spin potential

pseudocritical temperature

An increasing imaginary spin potential:

→ leads to a decrease in the value of the Polyakov loop for all studied temperatures

→ finite *imaginary* spin density favors confinement and increases the crossover temperature

→ finite *real-valued* (= real) spin density favors deconfinement and lowers the crossover temperature

Let's make it quantitative

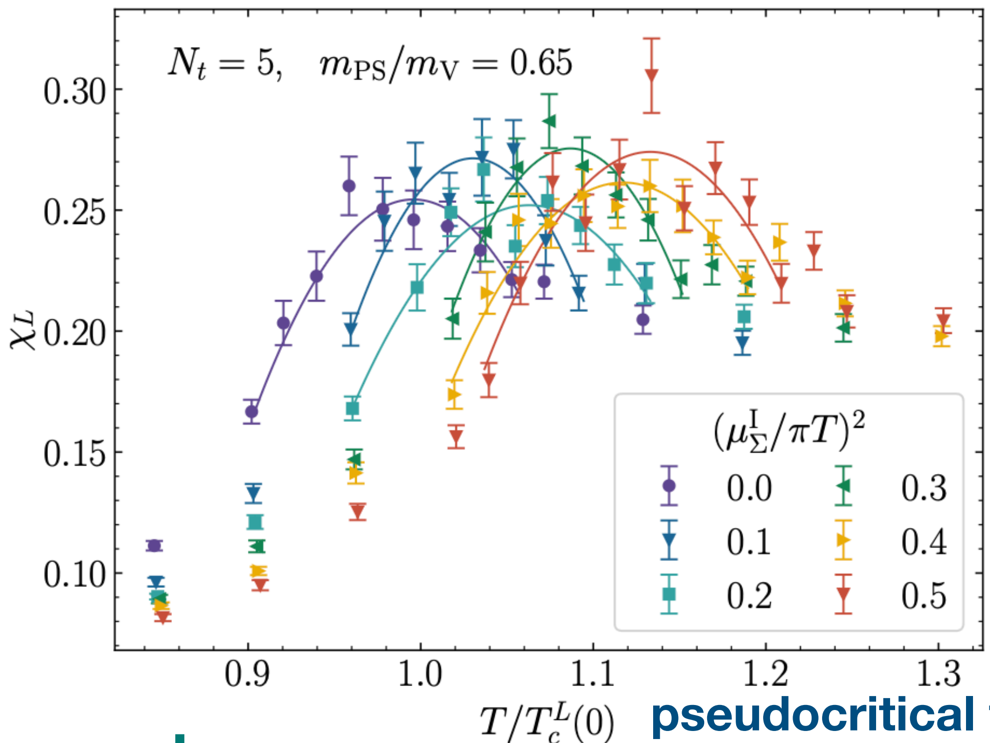
Quantitative effect

a parabolic dependence of crossover temperatures on the (*imaginary*) spin potential

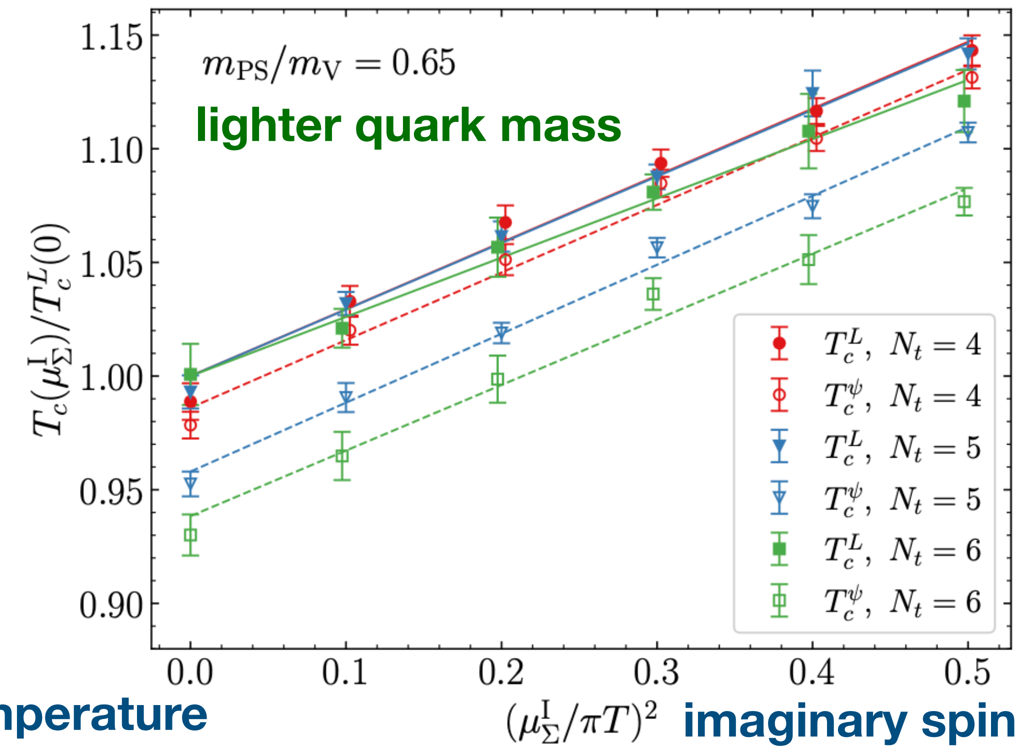
Confinement

Suseptibilities

Crossover temperatures



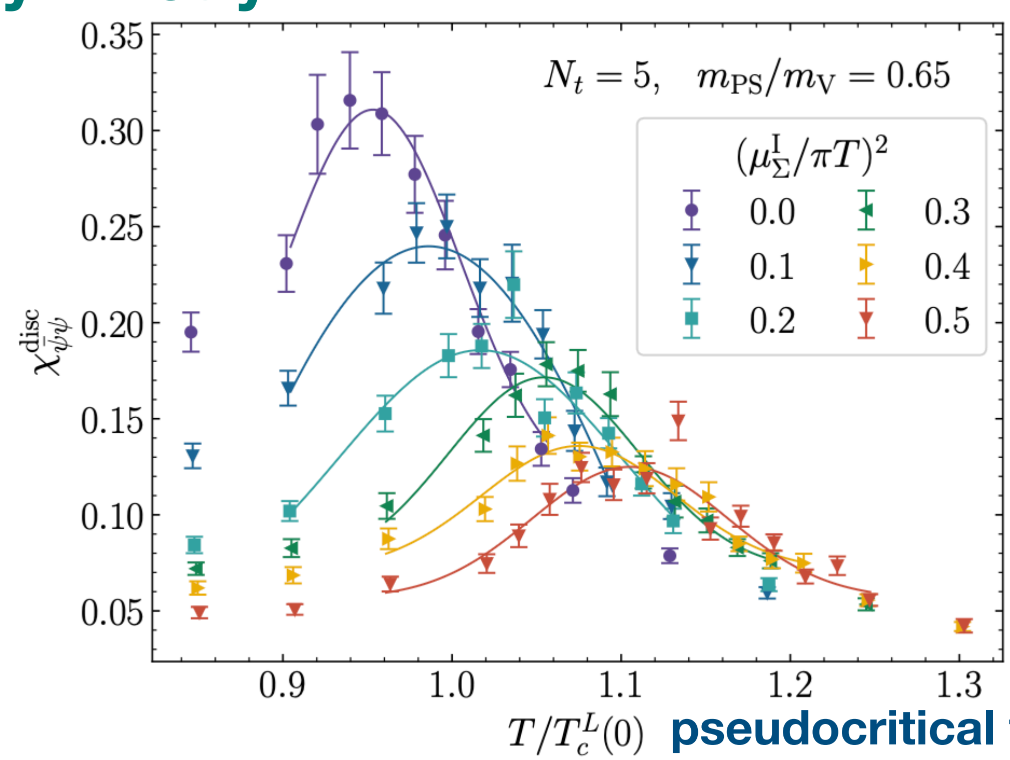
pseudocritical temperature



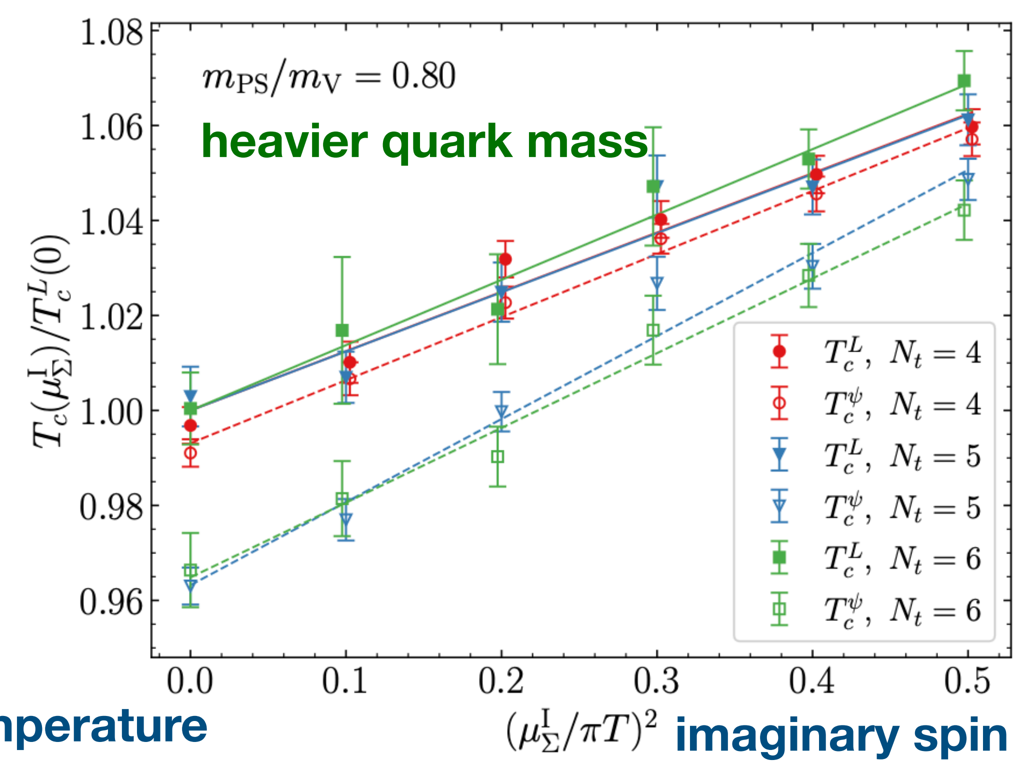
lighter quark mass

imaginary spin potential squared

Chiral symmetry



pseudocritical temperature

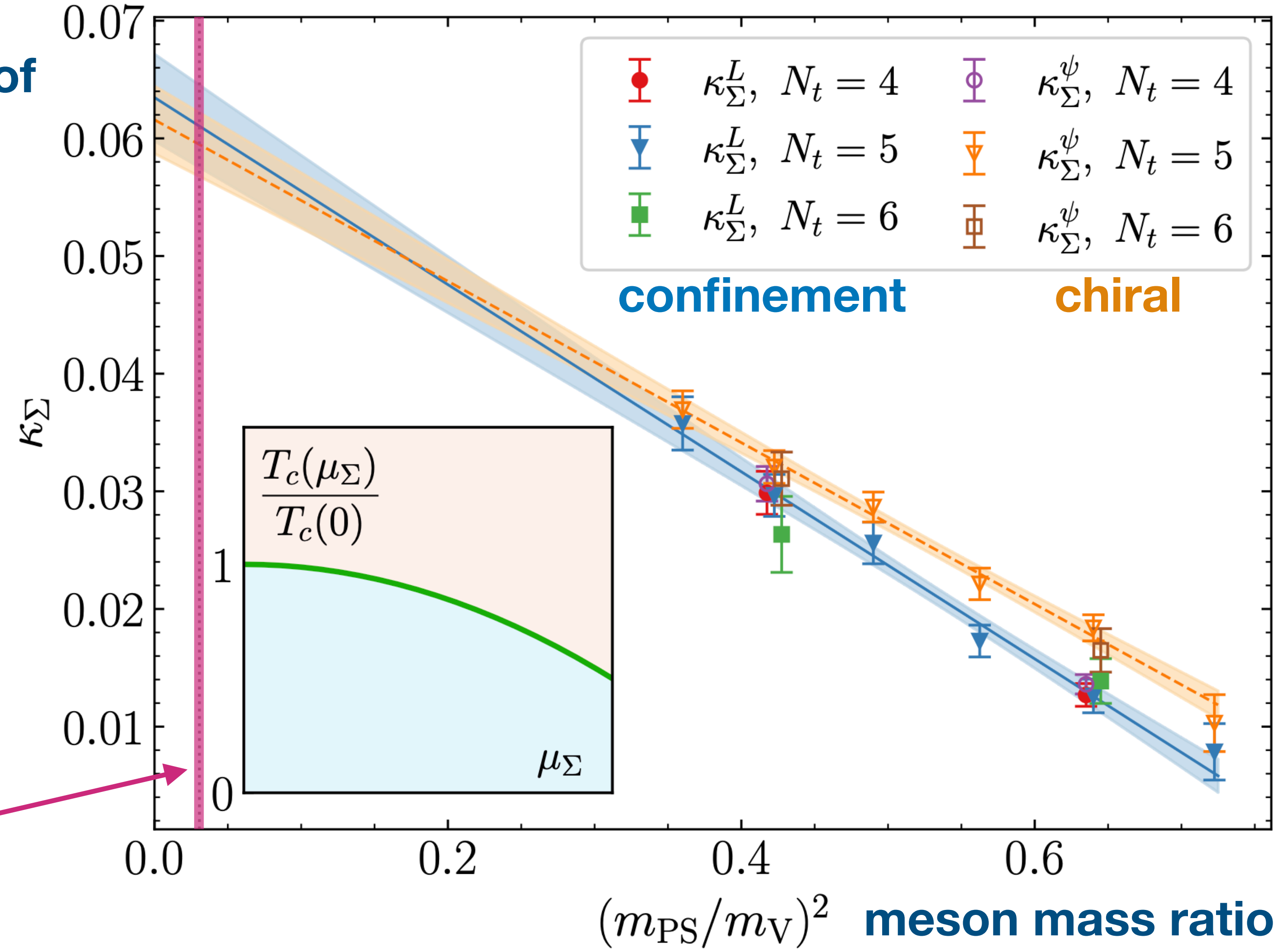


heavier quark mass

imaginary spin potential squared

Extrapolation of the curvatures

the curvatures of the transitions



physical point

meson mass ratio squared

$$\kappa_\Sigma^\ell(\xi) = \varkappa_\Sigma^\ell + \gamma_\Sigma^\ell\,\xi^2$$

$$\xi = \frac{m_{\mathrm{PS}}}{m_{\mathrm{V}}}$$

$$\varkappa_\Sigma^L = 0.0635(37)$$
$$\varkappa_\Sigma^\psi = 0.0616(29)$$

$$\gamma_\Sigma^L = -0.0795(67)$$
$$\gamma_\Sigma^\psi = -0.0686(53)$$

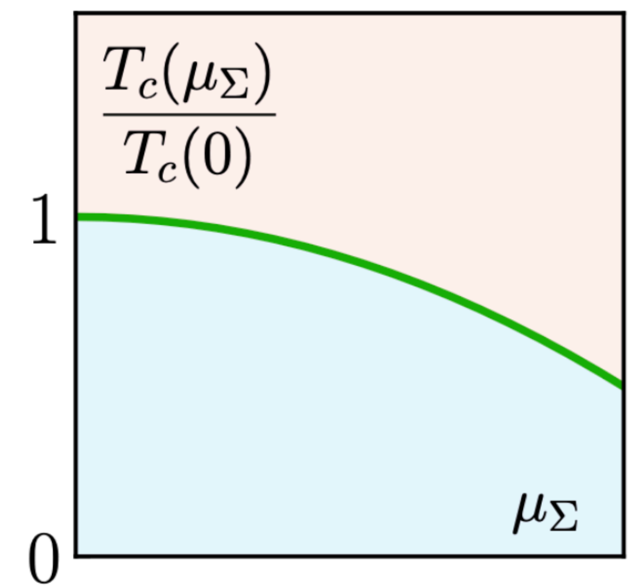
$$\frac{T_c^\ell(\mu_\Sigma^{\mathrm{I}})}{T_c^\ell(0)} = 1 + \kappa_\Sigma^\ell\left(\frac{\mu_\Sigma^{\mathrm{I}}}{T_c(0)}\right)^2 + \ldots\,, \qquad [\ell = L, \psi]$$

the curvatures of the confinement ($\ell = L$) and chiral ($\ell = \psi$) crossover transitions

$$\frac{T_c^\ell(\mu_\Sigma)}{T_c(0)} = 1 - \kappa_\Sigma^\ell\left(\frac{\mu_\Sigma}{T_c(0)}\right)^2 + \ldots\,, \qquad [\ell = L, \psi]$$

Curvatures of the confinement and chiral transitions

$$\frac{T_c^\ell(\mu_\Sigma)}{T_c(0)} = 1 - \kappa_\Sigma^\ell \left(\frac{\mu_\Sigma}{T_c(0)}\right)^2 + \dots, \qquad [\ell = L, \psi]$$



Spin-density curvature of the transitions:

$$\kappa_\Sigma^{L\ (\mathrm{phys})} = 0.0610(35)$$
$$\kappa_\Sigma^{\psi\ (\mathrm{phys})} = 0.0595(27)$$

at $\left(\frac{m_{\mathrm{PS}}}{m_{\mathrm{V}}}\right)_{\mathrm{phys}} = 0.175$

Compare with the effect of a finite baryonic density in $N_f = 2+1$ QCD:

baryon chemical potential

$$T_c(\mu_B)/T_c(0) = 1 - \kappa_B(\mu_B/T_c)^2$$
$$\kappa_B = 0.0135(20)$$

→ quark chemical potential

$$T_c(\mu_B)/T_c(0) = 1 - \kappa_q(\mu_q/T_c)^2$$
$$\mu_B = 3\mu_q$$

[C. Bonati, M. D'Elia, M. Mariti, M. Mesiti, F. Negro, Phys.Rev.D 92 (2015) 5, 054503;
C Bonati, M. D'Elia, F. Negro, F. Sanfilippo, K. Zambello, Phys.Rev.D 98 (2018) 5, 054510;
S. Borsanyi, Z. Fodor, J. N. Guenther, et al. Phys.Rev.Lett. 125 (2020) 5, 052001;
H.-T. Ding, O. Kaczmarek, P. Petreczky, M. Sarkar et al., Phys.Rev.D 109 (2024) 11, 114516]

→ get a comparable curvature to the usual quark chemical potential $\kappa_q = 0.12(2)$

→ Can be captured in effective infrared models

Talk by Pracheta Singha on Linear Sigma model coupled to quarks (on Monday)

[Pacheta Singha, Victor E. Ambrus, Sergiu Busuioc, Aritra Bandyopadhyay and M.Ch., in preparation]

Conclusions

1. Out first principle lattice simulations in lattice QCD show that the confinement and chiral crossover temperatures drop with the increase in the (canonical) quark spin density

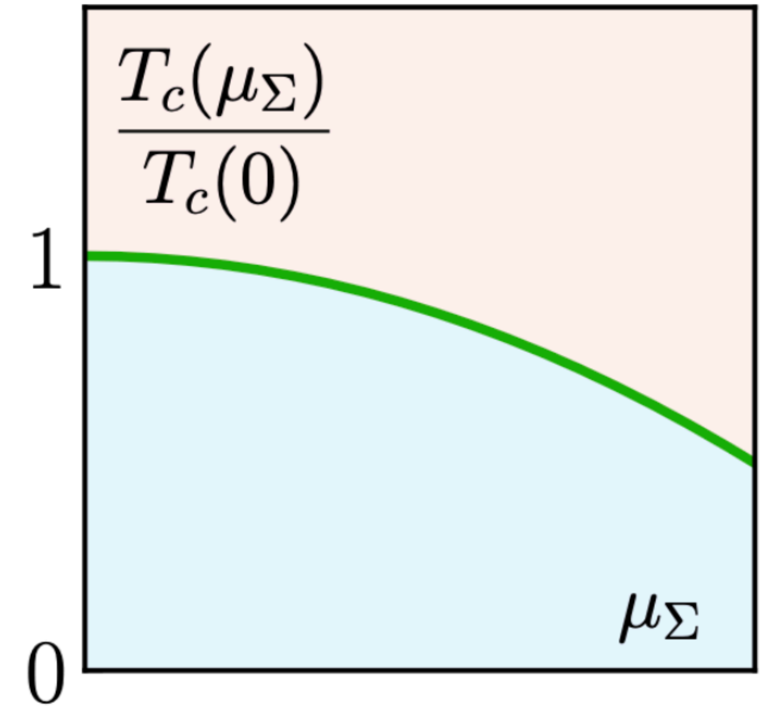
2. Spin polarization of quarks affects gluons

3. No problem with pseudogauge symmetry

4. Quantitatively,

$$\kappa_\Sigma^{L\ \mathrm{(phys)}} = 0.0610(35)$$
$$\kappa_\Sigma^{\psi\ \mathrm{(phys)}} = 0.0595(27)$$

$$\frac{T_c^\ell(\mu_\Sigma)}{T_c(0)} = 1 - \kappa_\Sigma^\ell \left(\frac{\mu_\Sigma}{T_c(0)}\right)^2 + \dots, \qquad [\ell = L, \psi]$$



4. The magnitude of the finite-spin curvature is comparable to (a factor of 2 smaller than) the curvature at a finite baryon density

5. No split in the deconfining and chiral transitions is found.

6. Despite the spin density of quarks is not a conserved quantity, the QCD thermodynamics is sensitive the quark spin polarization.