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Finite spin density effects on the chiral phase transition in the linear sigma model

Pracheta Singha

with Victor E. Ambrus, Sergiu Busuioc, Aritra Bandyopadhyay and Maxim N. Chernodub .

9th Conference on Chirality, Vorticity and Magnetic Fields in Quantum Matter .

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Motivation

- Quark-Gluon Plasma (QGP) is expected to form at extremely high temperatures and/or densities — conditions that existed shortly after the Big Bang or can be recreated in high-energy heavy-ion collisions.

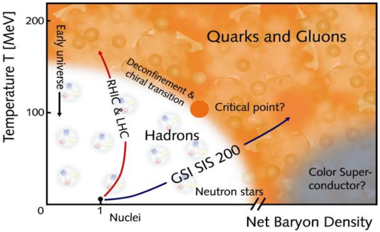

Figure: Source :arXiv:1412.0847

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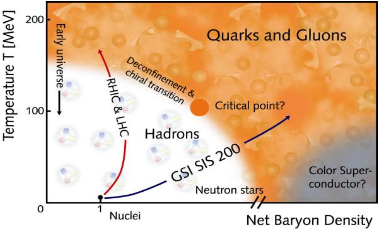
- This initial angular momentum results in a nonzero polarization of the produced particles.



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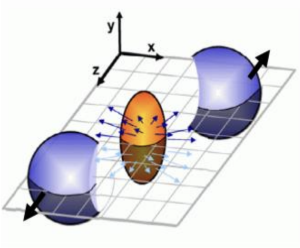

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- This initial angular momentum results in a nonzero polarization of the produced particles.
- The experimental results of Λ, $\bar{\Lambda}$ polarisation from STAR collaboration confirms QGP as the most vortical fluid.

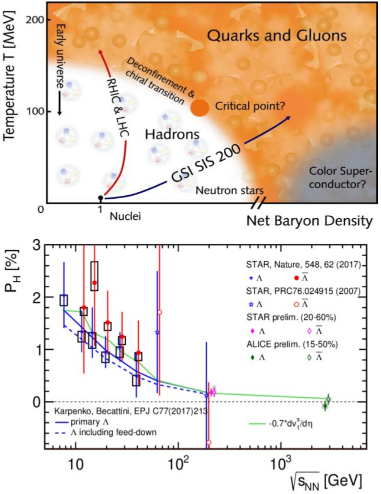
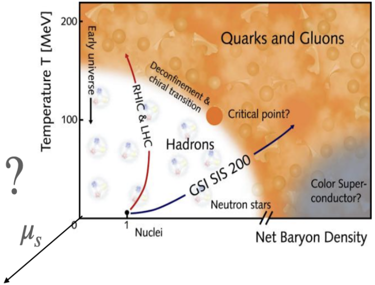


Figure: Source : arXiv:1412.0847 and 10.1103/PhysRevC.101.044611

Motivation

- Objective : Constructing an effective QCD model description with nonzero spin polarized quarks and antiquarks and study the phase structure.
- Methodology :
  - Quark spin density can be described by introducing a spin potential (μ_s) in the Lagrangian . Chernodub et al. Phys.Rev.D 111 (2025) 11, 114508



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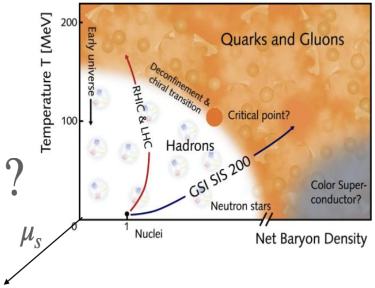

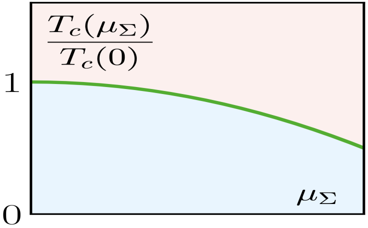

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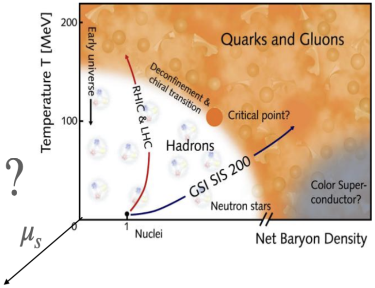
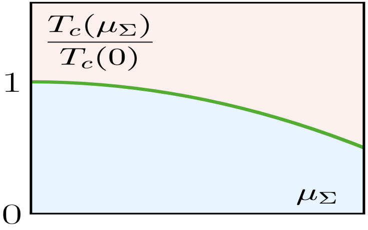
- Expectations:
  - Phase structure at arbitrary T and μ_s.
  - The agreement with the lattice data at small μ_s limit.





Model Formalism

- Model Lagrangian:

$$\mathcal{L}(\sigma, \vec{\pi}, \psi) = \mathcal{L}_{\mathcal{M}}(\sigma, \vec{\pi}) + \mathcal{L}_q(\sigma, \vec{\pi}, \psi) \, .$$

- Quark contribution:

$$\mathcal{L}_q = \bar{\psi} \left[i \partial\!\!\!/ - g(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) \right] \psi + \bar{\psi} \mu_{\alpha\beta} \mathcal{S}^{0,\alpha\beta} \psi$$

Where we introduce the coupling to the spin charge in the quark part of the model Lagrangian, with the relativistic spin density matrix,

$$\mathcal{S}^{\lambda,\alpha\beta} = \frac{1}{2} \{ \gamma^\lambda, \Sigma^{\alpha\beta} \} \, , \qquad \Sigma^{\alpha\beta} = \frac{i}{4} [\gamma^\alpha, \gamma^\beta] \, ,$$

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- When only the rotation part of the spin potential is non-vanishing and the spins of quarks polarized along the z axis, quark contribution:

$$\mathcal{L}_q = \bar{\psi}(i\partial\!\!\!/)\psi - g\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi + 2\mu_s\psi^\dagger\Sigma^{12}\psi$$

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- Mesonic contribution:

$$\mathcal{L}_\mathcal{M}(\phi) = \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - V_\mathcal{M}(\sigma,\vec{\pi}),$$
$$V_\mathcal{M}(\sigma,\vec{\pi}) = \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2 - h\sigma\,.$$

  - $h\sigma \equiv$ explicit chiral symmetry breaking $\rightarrow m_\pi \neq 0$.
  - Model parameters: $\lambda,\ v,\ g,\ h$ can be fitted to reproduce vacuum physics.

Formalism

- Under the path integral formalism, we have $\mathcal{Z} = \mathcal{Z}_\sigma \mathcal{Z}_q$, with

$$\mathcal{Z}_q = \int [id\psi^\dagger][d\psi]\, e^{S_E}, \quad S_E = \int_0^\beta d\tau \int d^3x\, \mathcal{L}_q.$$

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- The free energy $\mathcal{F}_q = -T \ln \mathcal{Z}_q = V(F_q^{\mathrm{ZP}} + F_q)$ can be decomposed as:

$$F_q^{\mathrm{ZP}} = -N_c N_f \sum_s \int \frac{d^3p}{(2\pi)^3} E_{\mathbf{p}}^{(s)}, \quad F_q = -2N_c N_f T \sum_s \int \frac{d^3p}{(2\pi)^3} \ln(1 + e^{-\beta E_{\mathbf{p}}^{(s)}}).$$

- While the mesonic contribution to the free energy under the mean field approximation , ($\sigma = \sigma_{\mathrm{mf}}, \vec{\pi} = \vec{\pi}_{\mathrm{mf}} = 0$), reads,

$$F_\sigma = V_\sigma(\sigma, 0) = \frac{\lambda}{4}\left(\sigma^2 - v^2\right)^2 - h\sigma$$

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- Chiral order parameter $\sigma_{\mathrm{mf}}(T, \mu_s)$ can be obtained as $\frac{\partial(F_q^{\mathrm{ZP}} + F_q + F_\sigma)}{\partial\sigma}\Big|_{\sigma=\sigma_{\mathrm{mf}}} = 0$.

Vacuum renormalisation at $\mu_s = 0$

- The zero point energy for $\mu_s \to 0$ has a diverging contribution.

$$F_q^{\mathrm{ZP}} = -2N_c N_f \int \frac{d^3p}{(2\pi)^3} E_{\mathbf{p}}, \quad \text{with } E_p = \sqrt{\mathbf{p}^2 + g^2\sigma^2}$$

$$= \frac{N_c N_f g^4 \sigma^4}{16\pi^2}\left[\frac{1}{\epsilon} + \frac{3}{2} - \gamma_E + \ln\left(\frac{4\pi\mu^2}{g^2\sigma^2}\right)\right].$$

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$$V_\sigma^{\mathrm{bare}} + F_q^{\mathrm{ZP}} = V_\sigma(\sigma, 0) \implies \quad V_\sigma^{\mathrm{bare}} = \frac{\lambda_{\mathrm{b}}}{4}(\sigma^2 - v_{\mathrm{b}}^2)^2 - h_{\mathrm{b}}\sigma + L_{\mathrm{b}}\sigma^4 \ln\frac{f_\pi^2}{\sigma^2} + V_0^{\mathrm{b}}.$$

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  - The free energy $\left(F_q + V_\sigma(\sigma, 0)\right)$ remains finite and no additional term appears.
  - The bare parameters $(\lambda_{\mathrm{b}},\ v_{\mathrm{b}}^2,\ h_{\mathrm{b}},\ L_{\mathrm{b}} \cdots)$ contain the counter terms to cancel the divergent contribution from F_q^{ZP}.
  - The bare parameters are not medium dependent.

Let's turn on μ_s!

- The zero point energy has a diverging contribution.

$$F_q^{\mathrm{ZP}} = -2N_cN_f \int \frac{d^3p}{(2\pi)^3} E_{\mathbf{p}}^{(s)}, \quad \text{with } E_p^s = \sqrt{\mathbf{p}^2 + g^2\sigma^2 - 2s\mu_s\sqrt{p_z^2 + g^2\sigma^2} + \mu_s^2}$$
$$= -2N_cN_f \left[I_1 + \mu_s^2 I_2 + \mu_s^4 I_3\right]$$

After dimensional regularization, these integrals reduce to

$$I_2 = -\frac{g^2\sigma^2}{8\pi^2}\left[\frac{1}{\epsilon} - \gamma_E + \ln\frac{4\pi\mu^2}{g^2\sigma^2}\right]$$
$$I_3 = \frac{1}{24\pi^2}.$$

Let's turn on μ_s!

- The zero point energy has a diverging contribution.

$$F_q^{\mathrm{ZP}} = -2N_c N_f \int \frac{d^3p}{(2\pi)^3} E_{\mathbf{p}}^{(s)}, \quad \text{with,} \;\; E_p^{(s)} = \sqrt{\mathbf{p}^2 + g^2\sigma^2 - 2s\mu_s\sqrt{p_z^2 + g^2\sigma^2} + \mu_s^2}$$

- Scheme I : If we follow the exact same method as before, we get,

$$V_\sigma^{\mathrm{bare}}(\mu_s) + F_q^{\mathrm{ZP}}(\mu_s) = V_\sigma(\sigma, 0) \;,$$

$$\text{where,} \quad V_\sigma^{\mathrm{bare}}(\mu_s) = \frac{\lambda_{\mathrm{b}}}{4}(\sigma^2 - v_{\mathrm{b}}^2)^2 - h_{\mathrm{b}}\sigma + L_{\mathrm{b}}^{(4)}\sigma^4 \ln\frac{f_\pi^2}{\sigma^2} + L_{\mathrm{b}}^{(2)}\sigma^2 \ln\frac{f_\pi^2}{\sigma^2} + V_0^{\mathrm{b}}.$$

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  - The free energy $\left(F_q + V_\sigma(\sigma, 0)\right)$ remains finite and no additional medium dependent term appears, although $F_q^{\rm ZP}(\mu_s)$ is μ_s dependent.
  - The bare parameters $(\lambda_{\rm b},\, v_{\rm b}^2,\, h_{\rm b},\, L_{\rm b}^{(2)},\, L_b^{(4)} \cdots)$ are now medium dependent.
  - Can we have an alternative scheme?

Let's turn on μ_s!

- Scheme II : We impose

$$V_\sigma^{\text{bare}}\Big|_{\mu_s=0} + F_q^{\text{ZP}}\Big|_{\mu_s=0} = V_\sigma, \qquad \frac{\partial F}{\partial \sigma}\Big|_{\substack{T=0\\ \sigma=f_\pi}} = 0, \qquad \frac{\partial^2 F}{\partial \sigma^2}\Big|_{\substack{T=0\\ \sigma=f_\pi}} = M_\sigma^2,$$

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- These constraints give us,

$$V_\sigma^{\text{bare}}(\mu_s) + F_q^{\text{ZP}}(\mu_s) = \frac{\lambda}{4}(\sigma^2 - v^2)^2 - h\sigma + \frac{N_c N_f g^2 \mu_s^2}{8\pi^2 f_\pi^2}\left(\sigma^4 + 2f_\pi^2 \sigma^2 \ln\frac{f_\pi^2}{\sigma^2}\right)(\ell_{\text{b}}^{(2)} + 1)$$

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  - The free energy remains finite but now we have medium (μ_s) dependent terms as one expects since $F_q^{\rm ZP}(\mu_s)$ is μ_s dependent.
  - The bare parameters $(\lambda_{\rm b},\ v_{\rm b}^2,\ h_{\rm b},\ L_{\rm b}^{(2)},\ L_b^{(4)}\cdots)$ are now medium dependent.
  - $\ell_{\rm b}^{(2)}(\geq -1)$ is a free parameter introduced to match with lattice result. $\ell_{\rm b}^{(2)} = -1$ gives back scheme I.

Let's turn on μ_s!

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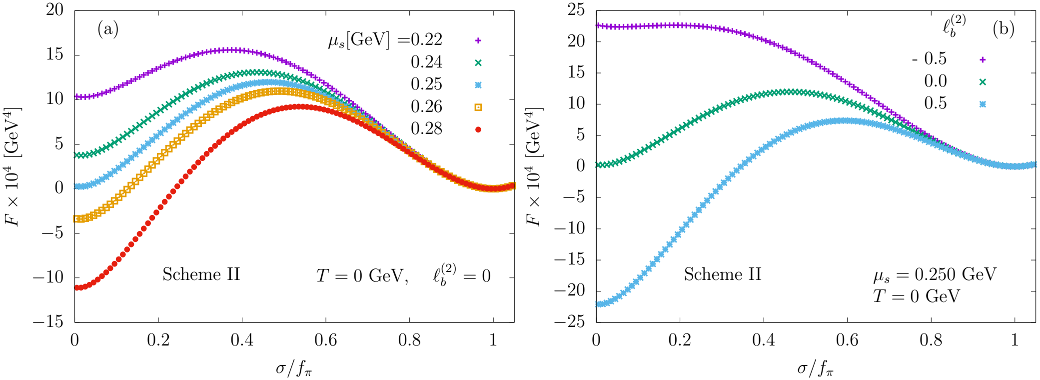
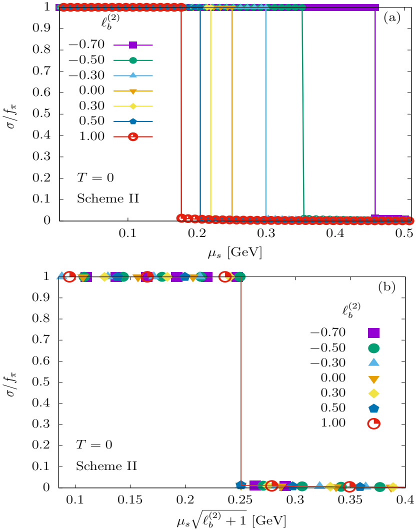
  - $\ell_{\rm b}^{(2)}(\geq -1)$ is a free parameter introduced to match with lattice result. $\ell_{\rm b}^{(2)} = -1$ gives back scheme I.
- How to choose the appropriate $\ell_{\rm b}^{(2)}$?

Results at $T = 0$



- In scheme I the vacuum potential is μ_s independent.
- For scheme II global minima moves towards $\sigma/f_\pi \to 0$ with increasing $\ell_b^{(2)}$ or μ_s .

Results at $T = 0$

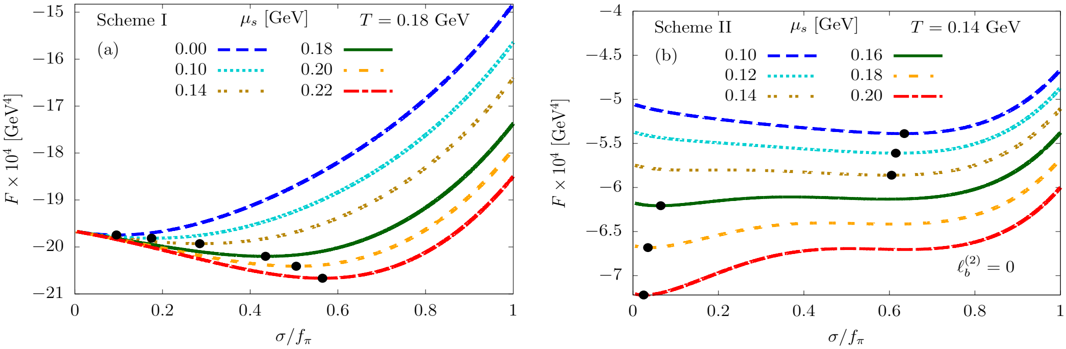


- At zero temperature the system goes through a first order phase transition as we increase μ_s.
- As $\ell_b^{(2)}$ increases the critical μ_s^c corresponding to the first order phase transition decreases.
- The vacuum potential reads, $\frac{N_c N_f g^2 \mu_s^2}{8\pi^2 f_\pi^2}\left(\sigma^4 + 2f_\pi^2\sigma^2 \ln\frac{f_\pi^2}{\sigma^2}\right)(\ell_\mathrm{b}^{(2)} + 1) +$ μ_s independent terms.
- There exist a scaling in terms $\ell_b^{(2)}$ as,

$$\mu_s^c(\ell_b^{(2)}) = \mu_s^c(0)/\sqrt{\ell_b^{(2)} + 1}\,.$$

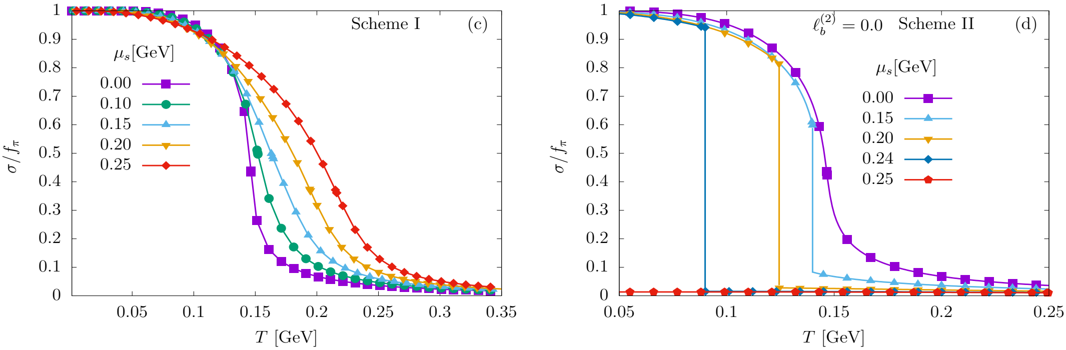
.

Comparing 2 schemes at $T \neq 0$



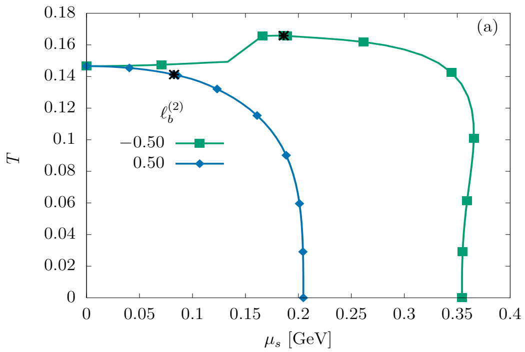
- For scheme I, with a fixed temperature, the minima of the free energy moves towards the higher value of σ with increasing μ_s.
- For scheme II, with a fixed temperature, the minima of the free energy moves towards the lower value of σ with increasing μ_s. $\rightarrow$ In agreement with lattice.

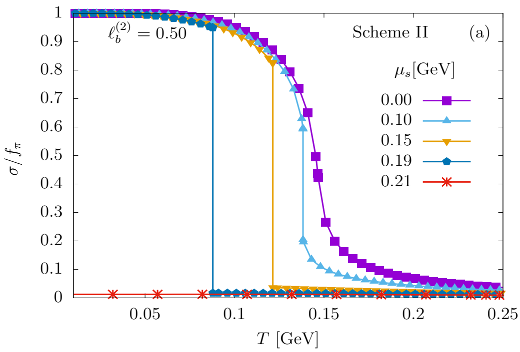
Comparing 2 schemes at $T \neq 0$



- For scheme I, as μ_s increases the transition temperature (T_c) increases $\rightarrow$ μ_s inhibits the phase transition.
- For scheme II, as μ_s increases the transition temperature (T_c) decreases $\rightarrow$ μ_s drives the system towards the phase transition.

Phase transition properties with different $\ell_b^{(2)}$.





- For $\ell_b^{(2)} = 0.5$ the phase diagram in $T - \mu_s$ plane is qualitatively similar to the phase diagram in $T - \mu_B$ plane.
- starting from a small T and μ_s system moves towards the chiral symmetry restored phase with increasing T and/or μ_s .

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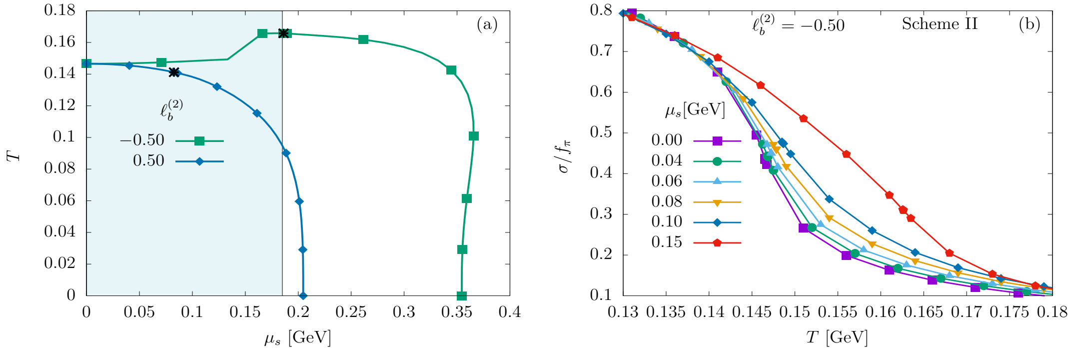

- for $\ell_b^{(2)} = -0.5$ for small $\mu_s (< 0.186 \text{ GeV})$, transition temperature increases with increasing μ_s, which contradicts the lattice results.

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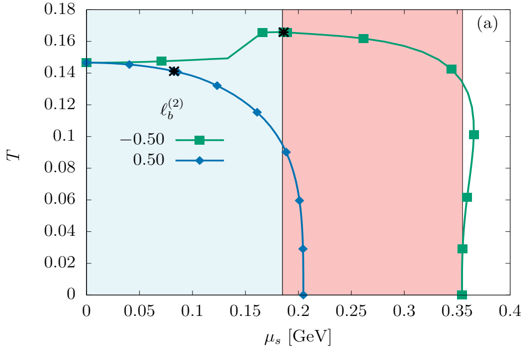

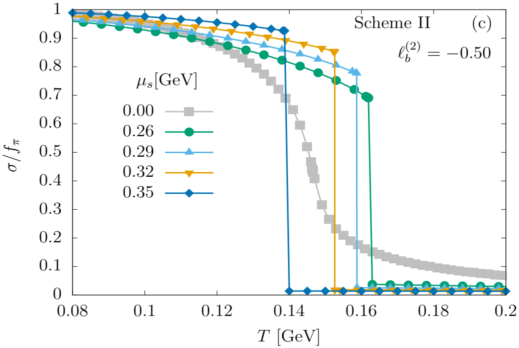

- for $\ell_b^{(2)} = -0.5$ for higher values of $\mu_s(> 0.186\,\text{GeV})$, transition temperature starts to decrease with increasing μ_s, and only for large $\mu_s(> 0.350\,\text{GeV})$ the transition temperature becomes smaller than $T_c(\mu_s = 0)$.

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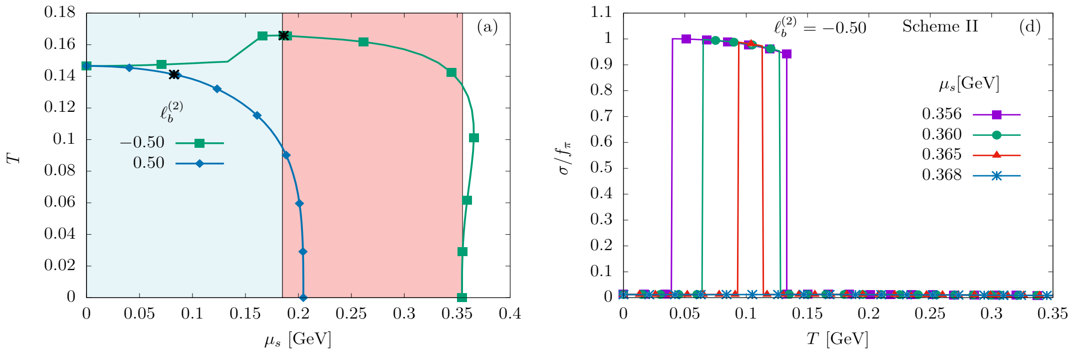
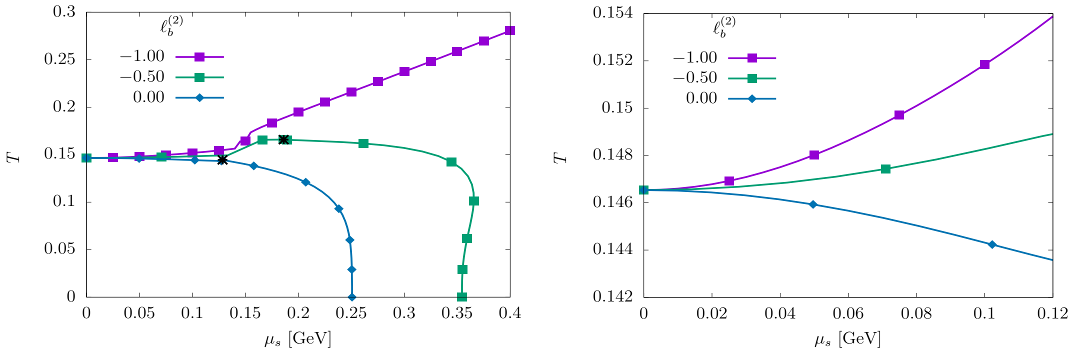


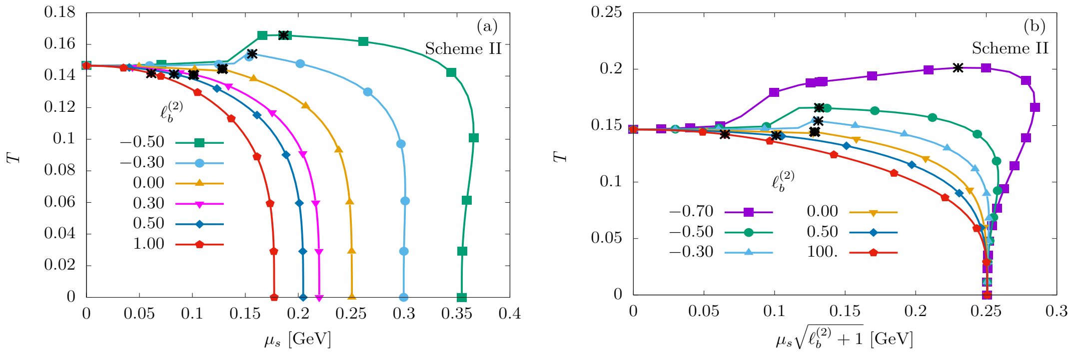
- For even higher $\mu_s(> \mu_s^c(T=0))$, system can exhibit double transitions, as it starts from the chiral symmetry restored phase , goes to chiral symmetry broken phase and the symmetry is restored again, as we move along the T axis.

Phase transition properties with different $\ell_b^{(2)}$.



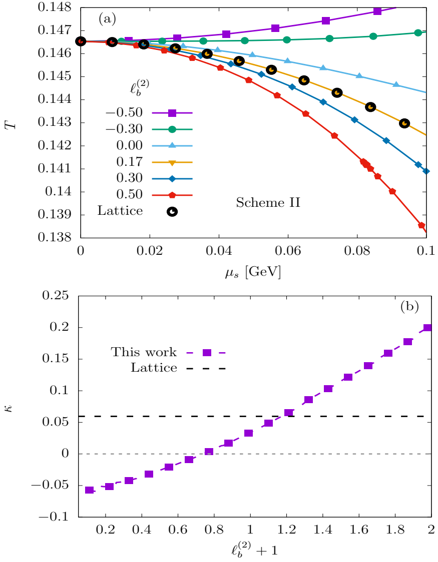
- $F_{T=0} = \frac{N_c N_f g^2 \mu_s^2}{8\pi^2 f_\pi^2}\left(\sigma^4 + 2f_\pi^2\sigma^2 \ln\frac{f_\pi^2}{\sigma^2}\right)(\ell_{\mathrm{b}}^{(2)} + 1) + \mu_s$ independent terms.
- $\ell_b^{(2)}$ controls the strength of the μ_s dependent vacuum potential term, and $-1 < \ell_b^{(2)} < 0$ gradually moves the phase diagram from positive curvature to negative curvature.

Phase diagram



- The critical point moves towards the smaller μ_s as $\ell_b^{(2)}$ increases.
- As $\ell_b^{(2)}$ increases the dependency of $T_c^{\rm crit}$ on $\ell_b^{(2)}$ decreases.
- For different values of $\ell_b^{(2)}$ the phase diagrams can be scaled with $\sqrt{\ell_b^{(2)}+1}$ at small T limit and small μ_s limit.

Comparison with Lattice results



- Lattice data in the plot is given as,

Chernodub et al. Phys.Rev.D 111 (2025) 11, 114508 .

$$\frac{T_c(\mu_s)}{T_c(0)} = 1 - \kappa_{\text{Lat}}\left(\frac{\mu_s}{T_c(0)}\right)^2 + \cdots$$

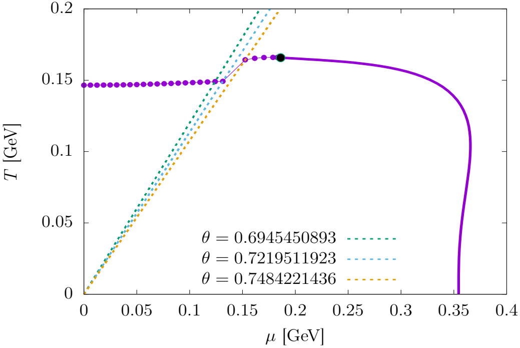
with $T_c(0) = 0.14653125$ and $\kappa_{\text{lat}} = 0.0595$ .Chernodub et al. Phys.Rev.D 111 (2025) 11, 114508 .

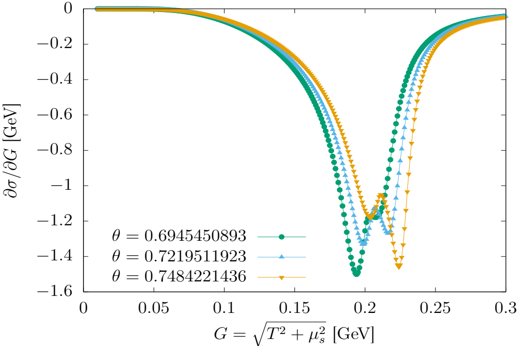
- Scheme II, $\ell_{\text{b}}^{(2)} = 0.17$ agrees the best with lattice.
- Scheme II covers a large range of curvature.

Conclusion

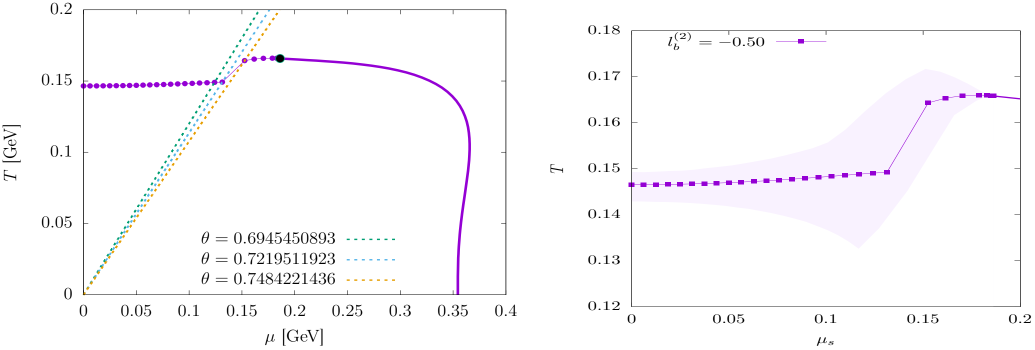
- We consider the effect of finite spin density on the QCD chiral phase transition using the Linear Sigma model coupled to quarks (LSM_q) .
- We introduce the finite spin density via a quark spin potential (μ_s) in the canonical formulation of the spin operator, leading to a nonlinear modification of the energy dispersion relation .
- The spin potential enters non-trivially in the zero-point, temperature independent part of the fermion loop. Employing renormalization techniques, we observe a substantial influence of this term on the phase structure of the system.
- We have discussed a proper vacuum renormalization scheme that qualitatively agrees with the first principle calculations.
- One can extend this study including the Polyakov potential, to study the deconfinement phase transition as well.

Thank You For Your Attention!!





- The slope for variation of σ with respect to the parameters shows a double peak structure.
- This jump can be attributed to the uncertainty related to defining a crossover transition temperature and spin potential.



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