Dissipative corrections to the spin polarization vector

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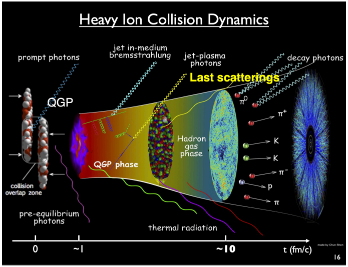
Heavy-ion collision and the quark gluon plasma

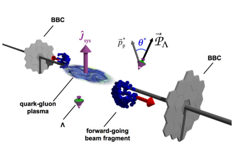
Spin polarization in QGP arises from hydrodynamic gradients in non-central heavy-ion collisions.

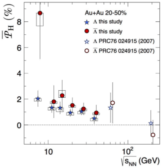
QGP is a system dominated by QCD in non-perturbative regime with very high temperature and vorticity.

Local and global polarization of fermions explained by:

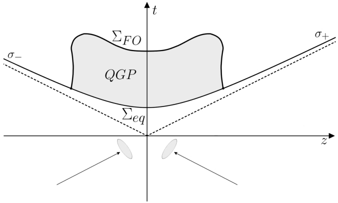
- Local thermodynamic equilibrium.
- Hydrodynamic evolution.
- Isothermal decoupling at high energies.







- QGP assumed in local thermodynamic equilibrium (LTE) at Σ_{EQ}.
- Freeze-out: transition from strongly interacting QCD to quasi-free hadrons.



Initial LTE state:

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma_{EQ}} \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu} \beta_\nu \right],$$

$$\beta_\mu = \frac{u_\mu}{T}, \quad \text{Four-temperature}$$

Using Gauss theorem between Equilibrium (Σ_{EQ}) and the Freeze-out (Σ_{FO}):

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu \widehat{T}^{\mu\nu} \beta_\nu + \int_\Omega \mathrm{d}\Omega \widehat{T}^{\mu\nu} \nabla_\mu \beta_\nu \right]$$

Local Equilibrium

Dissipation

Dissipative correction for observable $\widehat{O}(x)$ in linear response theory:

$$\Delta O(x) \equiv \int_0^1 \mathrm{d}z \int \mathrm{d}\Omega(y)\nabla_\mu\beta_\nu(y)\mathrm{Tr}\left(\widehat{O}(x)e^{-z\beta(x)\widehat{P}}\widehat{T}^{\mu\nu}(y)e^{z\beta(x)\widehat{P}}(1/Z)e^{-\beta(x)\widehat{P}}\right),$$

Example: $\widehat{O}(x)=\widehat{T}^{\mu\nu}(x)$:

$$\Delta T^{\sigma\lambda}(x) \sim \int \mathrm{d}^4y\, \partial_\mu\beta_\nu(y)\langle\widehat{T}^{\sigma\lambda}(x)\widehat{T}^{\mu\nu}(y)\rangle_{\beta(x),C}, \quad \langle\bullet\rangle_{\beta(x),C} \equiv \mathrm{Tr}\left[\frac{e^{-\beta(x)\cdot\widehat{P}}}{Z}\bullet\right]_{Connected}$$

Separation of scales between gradients and correlators:

- $\partial_\mu\beta_\nu(y)$varies on macroscopic lengths.
- $\langle\widehat{T}(x)\widehat{T}(y)\rangle$ different from zero only if $y-x$ is a microscopic length.

We take $\partial_\mu\beta_\nu(y)$ out of the integral in y and ontain the Kubo formulae:

$$\Delta T^{\sigma\lambda}(x) \sim \partial_\mu\beta_\nu(x)\left(\widehat{T}^{\sigma\lambda},\widehat{T}^{\mu\nu}\right)$$

1. Viscous coefficient, shear viscosity: $\eta \sim \left(\widehat{T}^{ij},\widehat{T}^{ij}\right)$.
2. Proportional to the gradient in the same space-time point x.

Our goal: calculate dissipative correction to the spin polarization vector S^μ.

S^μ can be express using the free-Wigner function W, for spin 1/2:

$$S^\sigma(k) = \frac{1}{2}\frac{\int_{\Sigma_{FO}} \mathrm{d}\Sigma \cdot k \,\mathrm{tr}\left[\gamma^\sigma\gamma^5 W(x,k)\right]}{\int_{\Sigma_{FO}} \mathrm{d}\Sigma \cdot k \,\mathrm{tr}\left[W(x,k)\right]} \equiv \frac{1}{2}\frac{\int_{\Sigma_{FO}} \mathrm{d}\Sigma \cdot k \,\mathcal{A}^\sigma(x,k)}{\int_{\Sigma_{FO}} \mathrm{d}\Sigma \cdot k \,\mathcal{S}(x,k)}$$

Wigner function W: expectation value of the Wigner operator:

$$\widehat{W}^A_{B+}(x,k) = \frac{\theta(k^2)\theta(k^0)}{(2\pi)^3}\int \frac{\mathrm{d}^3 p}{2E_p}\int \frac{\mathrm{d}^3 p'}{2E_{p'}} e^{\mathrm{i}x\cdot(p-p')}\delta^4\left(k-\frac{p+p'}{2}\right)$$
$$\times \sum_{r,r'=\pm} u^A_{r'}(p')\bar{u}_{B\,r}(p)\widehat{a}^\dagger_r(p)\widehat{a}_{r'}(p'). \quad \{\text{Free Dirac fields}\}$$

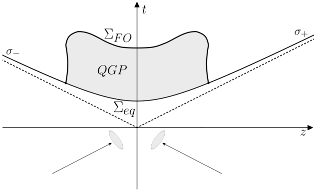
In linear response: dissipative corrections to S from dissipative corrections to W

$$\Delta S^\sigma(k) = \frac{1}{2}\frac{\int_{\Sigma_{FO}} \mathrm{d}\Sigma \cdot k \,\Delta\mathcal{A}^\sigma(x,k)}{\int_{\Sigma_{FO}} \mathrm{d}\Sigma \cdot k \,\mathcal{S}_{LE}(x,k)}$$

Using linear response theory for dissipative correction to the Wigner function:

$$\Delta W^A_B(k,x) = \int_0^1 \mathrm{d}z \int_\Omega \mathrm{d}\Omega(y) \nabla_\mu \beta_\nu(y) \langle \widehat{W}^A_B(x,k)\, e^{-z\beta(x)\widehat{P}} \widehat{T}^{\mu\nu}(y) e^{z\beta(x)\widehat{P}} \rangle_{\beta(x),C}$$

- At FO Σ_{FO} in x we have quasi-free hadrons.
- In the volume Ω system is strongly coupled.
- W is Wigner function of free theory
- $\widehat{T}^{\mu\nu}(y)$ is the SEMT of the interacting plasma



$W(x,k)$ is a function of coordinate x and momentum k.

We found: correlator $\langle \widehat{W}(x,k)\, \widehat{T}^{\mu\nu}(y)\rangle$ is different from 0 on a $k-$dependent world line over which the gradient $\partial_\mu \beta_\nu(y)$ may vary significantly.
Even in hydrodynamic limit:

$$\boxed{\Delta W(x,k) \neq \partial_\mu \beta_\nu(x) \left(\widehat{W}, \widehat{T}^{\mu\nu}\right)}$$

Using free field expansion for $\widehat{W}_B^A$ and the homogeneous equilibrium:

$$\Delta W_B^A(x,k) = \frac{\theta(k^0)\theta(k^2)}{(2\pi)^6}\int \mathrm{d}^4 Q\,\delta\left(k\cdot Q\right)\int \mathrm{d}\Omega(y) e^{-\mathrm{i}Q(y-x)}\nabla_\mu\beta_\nu(y)$$
$$\times\,\delta\left(k^2-m^2+\frac{Q^2}{4}\right)\left[F^{\mu\nu}(k,Q,\beta)\right]_B^A$$

where

$$Q = p - p', \quad k = \frac{p+p'}{2}, \quad k\cdot p = 0.$$

All information about interaction in:

$$\left[F^{\mu\nu}(k,Q,\beta)\right]_B^A \equiv \frac{e^{\beta(x)\cdot Q}-1}{\beta(x)\cdot Q}\sum_{r,r'=\pm} u_{r'}^A(p')\overline{u}_{B\,r}(p)\langle \widehat{a}_r^\dagger(p)\widehat{a}_{r'}(p')\widehat{T}^{\mu\nu}(0)\rangle_{\beta(x),C}$$

Dependence on y: Fourier transform of the gradient of the TD field .
Q^0 is constrained: $Q^0 = \mathbf{Q}\cdot\mathbf{k}/k^0$.

Expand interaction term on a base of the Clifford algebra:

$$\langle \widehat{a}_r^\dagger(p)\widehat{a}_{r'}(p')\widehat{T}^{\mu\nu}(0)\rangle_{\beta(x),C} = \sum_{C,D=1}^{4} u_r^C(p)\,[\Gamma^{\mu\nu}(k,Q,\beta)]_C^D\ \overline{u}_{D\,r'}(p'),$$

where:

$$[\Gamma^{\mu\nu}]_C^D = \Gamma_1^{\mu\nu}\mathbb{I}_C^D + \Gamma^{\mu\nu}_{2\ \lambda}[\gamma^\lambda]_C^D + \Gamma^{\mu\nu}_{3\ \lambda\rho}[\Sigma^{\lambda\rho}]_C^D + \Gamma_4^{\mu\nu}[\mathrm{i}\gamma^5]_C^D + \Gamma^{\mu\nu}_{5\ \lambda}[\gamma^\lambda\gamma^5]_C^D$$

Each $\Gamma_i^{\mu\nu}$:

- $\Gamma_i^{\mu\nu} = \Gamma_i^{\nu\mu}$ (we assume Belinfante p.g),
- Depends on vectors k^μ, Q^μ, β^μ, pseudo-vector $a^\mu \equiv \varepsilon^{\mu\rho\sigma\gamma}k_\rho Q_\sigma \beta_\gamma$,
- Depends on all the scalars $S = k^2, Q^2, \beta^2, Q\cdot\beta, k\cdot\beta$,
- Does not depend on any pseudo-scalars.
- Is an analytic function of its variables.

For example:

$$\Gamma_1^{\mu\nu} = G_1(S)k^\mu k^\nu + G_2(S)Q^\mu Q^\nu + m^2 G_3(S)\beta^\mu\beta^\nu + \dots$$

All information about interaction in the form factors $G_i(S)$.

Constraining the interaction term

Dynamical constraints:

- Dirac equation: $\overline{u}(p')\left(\not{p}'-m\right)=0$ and $\left(\not{p}-m\right)u(p)=0$,
- Transverse condition: $Q_\mu \overline{u}(p)\Gamma^{\mu\nu}u(p)=0$.

Only the vector and pseudo-vector contributions are independent:

$$\overline{u}(p')\Gamma^{\mu\nu}u(p)=\Gamma^{\mu\nu}_{2\ \lambda}(Q)\overline{u}(p')\gamma^\lambda u(p)+\tilde{\Gamma}^{\mu\nu}_{5\ \lambda}\overline{u}(p')\gamma^\lambda\gamma^5 u(p)$$

The expectation value is calculated in the homogeneous equilibrium.

Thermostatic constraints:

- KMS condition in momentum space: $[\Gamma^{\mu\nu}(Q)]^\dagger=\gamma^0\Gamma^{\mu\nu}(-Q)\gamma^0 e^{-\beta(x)\cdot Q}$,
- Accordance with parity and time-reversal transformation.

Preliminary result: only contribution from $\Gamma^{\mu\nu}_{2\ \lambda}$,

$$\Gamma^{\mu\nu}_{2\ \lambda}=X_1(S)k^\mu k^\nu k_\lambda+X_2(S)\left(Q^\mu Q^\nu-Q^2g^{\mu\nu}\right)k_\lambda+\dots$$

In total 14 possible independent scalar form factors: $X_1,\dots,X_{14}$.

$$\Delta W^A_B(x,k) \propto \int_\Omega \mathrm{d}^4 y\, \partial_\mu \beta_\nu(y) \int \mathrm{d}^3\mathbf{Q} e^{-\mathrm{i}Q\cdot(y-x)} \left[F^{\mu\nu}(Q)\right]^A_B \delta\left(k^2 - m^2 + \frac{Q^2}{4}\right).$$

If the exponential in Q is peaked in $y-x$, we can extract the gradient and calculate it in $y=x$:

$$\Delta W^A_B(x,k) \propto \partial_\mu \beta_\nu(x) \underbrace{\int_\Omega \mathrm{d}^4 y \int \mathrm{d}^3\mathbf{Q} e^{-\mathrm{i}Q\cdot(y-x)} \left[F^{\mu\nu}(Q)\right]^A_B \delta\left(k^2 - m^2 + \frac{Q^2}{4}\right)}_{\text{Viscous coefficient}}$$

Dissipative correction proportional to gradient calculated in the same point x.

However we have:

$$Q^0 = \frac{\mathbf{k}\cdot\mathbf{Q}}{Q^0} \implies -\mathrm{i}Q\cdot(y-x) = \mathrm{i}\mathbf{Q}\cdot\left[\mathbf{y} - \mathbf{x} - \frac{\mathbf{k}}{k^0}\left(y^0 - x^0\right)\right].$$

We should compute the gradient on the $k-$dependent world line and not only on x:

$$\boxed{\Delta W(x,k) \neq \partial_\mu \beta_\nu(x)\left(\widehat{W}, \widehat{T}^{\mu\nu}\right)}$$

Dissipative correction for W: can't be separated in gradient times viscous coefficient. Gradient must be integrated together with the expectation value:

$$[F^{\mu\nu}(Q)]^A_B = \left\{ \left(\not{k} + \frac{\not{Q}}{2} + m \right) [X_1(S) k^\mu k^\nu k_\lambda + \ldots] \gamma^\lambda \left(\not{k} - \frac{\not{Q}}{2} + m \right) \right\}^A_B \frac{e^{\beta(x)\cdot Q} - 1}{\beta(x) \cdot Q}.$$

The function F contains all information about interactions. Dissipative corrections:

- Do not depend on a single viscous coefficient,
- Depend on a set of unknown form factors $X_i(S)$.

For spin 1/2 field: 14 independent scalar form factors.

Their functional dependence on S is fixed buy the underlying microscopic theory.

We have studied the dissipative correction of the Wigner function of a free Dirac field:

1. Depend, in principle, on 14 unknown scalar functions X_i determined by the underlying microscopic interaction.
2. Are not proportional to the gradients of the thermodynamic field calculated in the same space-time point.

Future work:

1. Estimate the correction for the spin polarization vector.
2. Test different interaction models to constrain the X_i.

Thank you!