Out-of-equilibrium CME from Lattice QCD

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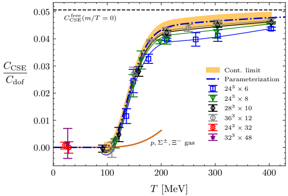
Anomalous transport

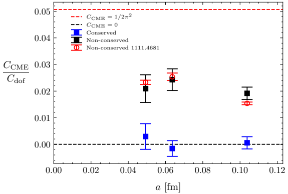
- Axial anomaly + EM fields / Vorticity → **Anomalous transport phenomena**
- Examples:
  - Chiral Magnetic Effect (CME) 🔗 Fukushima, Kharzeev, Warringa '08
  - Chiral Separation Effect (CSE)
  - Chiral Vortical Effect (CVE)
  - Chiral Magnetic Wave (CMW)
  - ...
- Probe of the topological nature of the QCD vacuum
- Experimental detection in heavy-ion collisions and condensed matter systems, review 🔗 Kharzeev, Liao,Tribedy '24
- Objective: use Lattice QCD to understand CME properties in QCD

Previous works

CSE conductivity in physical QCD

Brandt, Endrődi, EGV, Markó '23



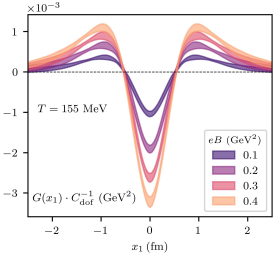


Absence of CME in equilibrium QCD

Brandt, Endrődi, EGV, Markó '24

Local CME with inhomogeneous B

Brandt, Endrődi, EGV, Markó, Valois '24

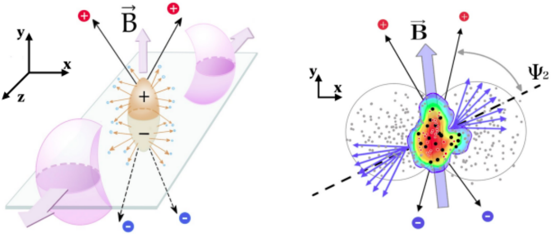


Out-of-equilibrium CME

- So far we concentrated on a *global thermal equilibrium* system.

- In experiments out-of-equilibrium effects can be very relevant

  Kharzeev, Liao,Tribedy '24



- What can we say from Euclidean lattice QCD about non-equilibrium effects?

Linear response theory

Real-time correlator

$$G^R_{AB}(t) = i\theta(-t)\left\langle A(0), B(t)\right\rangle$$

$$\rho_{AB}(\omega) = 2\,\mathrm{Im}\,G^R_{AB}(\omega)$$

Constituive equation

Depends on
transport coefficient ξ

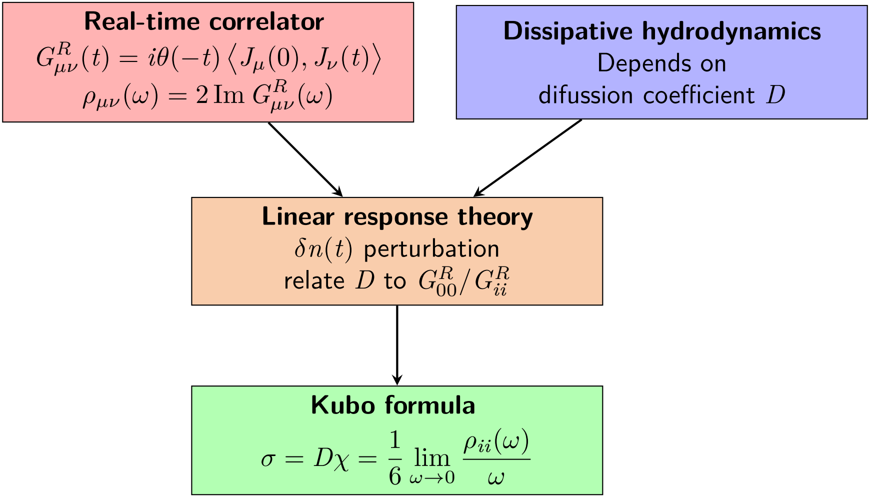
Linear response theory

$\delta B(t)$ perturbation
relate ξ to G^R_{AB}

Kubo formula

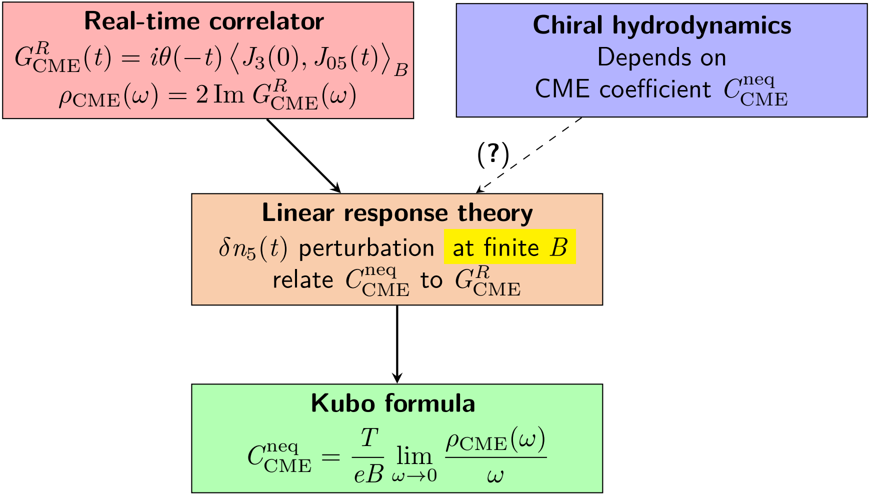
$$\xi = \lim_{\omega\to 0}\frac{\rho(\omega)}{\omega}$$

- Electric conductivity, bulk/shear viscosities,... in microscopic theory (QCD)
- Used in hydrodynamic simulations



- Well-known derivation
- Used to calculate σ in lattice QCD

CME



- Missing precise connection with chiral hydrodynamics!
- Correct normalization of Kubo formula?

1-loop spectral function

- Free fermions → 1-loop exact ρ

- **Electric conductivity**: e.g. Aarts, Nikolaev 21'

$$\rho_{ii}(\omega) = \chi\left(\frac{m}{T}\right)\omega\delta(\omega) + \Theta(\omega^2 - 4m^2)\frac{m^2}{4\pi^2}\sqrt{\frac{\omega^2 - 4m^2}{\omega^2}}(\omega^2 + 2m^2)\left[1 - 2n_F\left(\frac{\omega}{2}\right)\right]$$

- $\sigma \to \infty \Rightarrow$ resummation required Arnold, Moore, Yaffe '00

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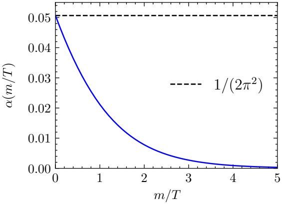
- **CME**: Brandt, Endrődi, EGV, Markó, Valois, Lattice '24

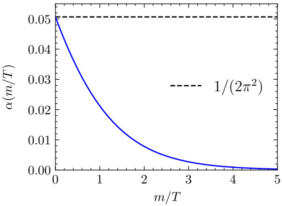
$$\frac{1}{eB}\rho_{\mathrm{CME}}(\omega) = \alpha\left(\frac{m}{T}\right)\omega\delta(\omega) + \Theta(\omega^2 - 4m^2)\frac{m^2}{\pi^2}\frac{\tanh[|\omega|/(4T)]}{\omega\sqrt{\omega^2 - 4m^2}}$$



- $C_{\mathrm{CME}}^{\mathrm{neq}} \to \infty \Rightarrow$ resummation required ↔ relation to the anomaly?

- **CME**:

$$\frac{1}{eB}\,\rho_{\mathrm{CME}}(\omega) = \alpha\left(\frac{m}{T}\right)\omega\delta(\omega) + \Theta(\omega^2 - 4m^2)\frac{m^2}{\pi^2}\,\frac{\tanh[|\omega|/(4T)]}{\omega\sqrt{\omega^2 - 4m^2}}$$



1-loop spectral function

- **CME**:

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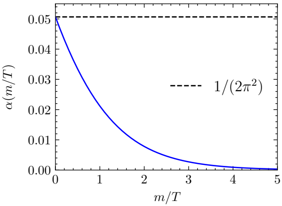


- **CSE**: Brandt, Endrődi, EGV, Markó, Valois, Lattice '24

$$\frac{1}{eB}\,\rho_{\text{CSE}}(\omega) = \alpha\left(\frac{m}{T}\right)\omega\delta(\omega)$$

- CSE correlator is τ-independent!

Spectral reconstruction

- G_R, ρ not directly accessible from lattice QCD
- Spectral representation of Euclidean correlators

$$G_E(\tau) = \int \mathrm{d}\omega \, \frac{\rho(\omega)}{\omega} \, K(\tau, \omega)$$

Lattice Want Known

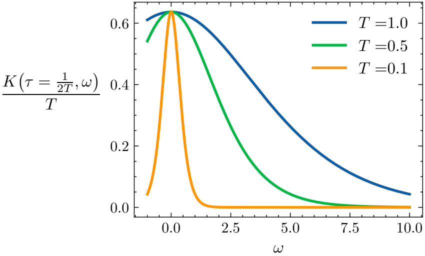
- On the lattice: $N_t \sim \mathcal{O}(10)$ ill-posed inverse problem
- Many methods on the market $\rightarrow$ applied to get other transport coefficients
- Examples: Maximum entropy method, Backus-Gilbert, machine learning, ...

Midpoint method

- Kernel evaluated at the midpoint

$$K(\tau,\omega) = \frac{\omega}{\pi}\frac{\cosh[\omega(\tau - 1/(2T))]}{\sinh[\omega/(2T]} \to K(\tau = 1/(2T),\omega) = \frac{\omega}{\pi\sinh[\omega/(2T)]}$$

- Behaves as a smeared $\delta(\omega)$ when $T \to 0$

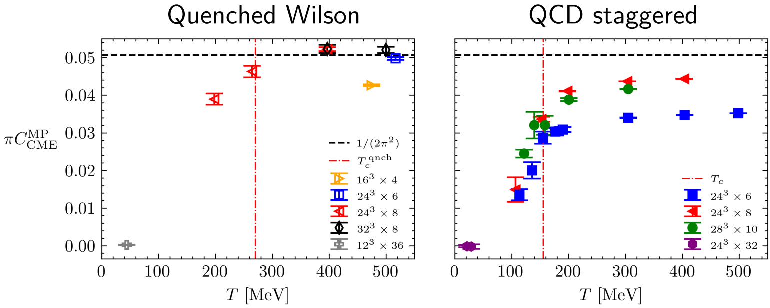


- $G_E(a\tau = N_t/2)$ carries first estimate! Buividovich '24

$$C_{\mathrm{CME}}^{\mathrm{MP}} = \frac{G_E(N_t/2)}{eB\;T\,\pi}$$

Midpoint in QCD

- Estimation of $C_{\mathrm{CME}}^{\mathrm{neq}}$ in QCD: Brandt, Endrődi, EGV, Markó, Valois, Lattice '24



- Estimation reliable when $T \to 0$
- Suppression at low temperatures
- Intuition at high $T \to$ at 1-loop:

$$\frac{G_E(N_t/2)}{eB\,T} = \pi C_{\mathrm{CME}}^{\mathrm{MP}}(m/T) = C_{\mathrm{CSE}}^{\mathrm{free}}(m/T) \xrightarrow{m/T \to 0} \frac{1}{2\pi^2}$$

Multipoint estimator

- Improve midpoint estimator by using several points

- Expanding $K(\tau,\omega)$ and $\rho(\omega) = \sum_{n=1}^{\infty} \frac{c_n}{n!}\omega^n$ to obtain

$$G(N_t/2) = \pi T^2 c_1 + \frac{14}{\pi} T^3 \zeta(3) c_2 + \frac{30}{\pi} T^4 \zeta(4) c_3 + \ldots$$

Multipoint estimator

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- Expanding $K(\tau,\omega)$ and $\rho(\omega) = \sum_{n=1}^{\infty} \frac{c_n}{n!}\omega^n$ to obtain

$$G(N_t/2) = \underbrace{\pi T^2}_{A_{11}} c_1 + \underbrace{\frac{14}{\pi} T^3 \zeta(3)}_{A_{12}} c_2 + \underbrace{\frac{30}{\pi} T^4 \zeta(4)}_{A_{13}} c_3 + \ldots$$

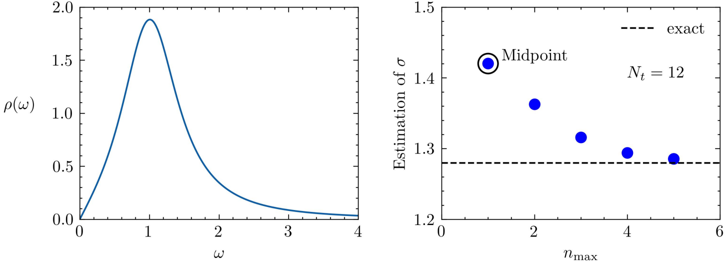
- Similar expression for other τ values $\rightarrow$ system of equations

$$\begin{pmatrix} A_{11} & A_{12} & \ldots & A_{1n_{\max}} \\ A_{21} & A_{22} & \ldots & A_{2n_{\max}} \\ \vdots & \ddots & \ddots & \vdots \\ A_{n_{\max}1} & A_{n_{\max}2} & \ldots & A_{n_{\max}n_{\max}} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_{n_{\max}} \end{pmatrix} = \begin{pmatrix} G(N_t/2) \\ G(N_t/2-1) \\ \vdots \\ G(N_t/2-(n_{\max}-1)) \end{pmatrix}$$

- c_1 ($\sim$ conductivity) improved estimation!

Multipoint Breit-Wigner

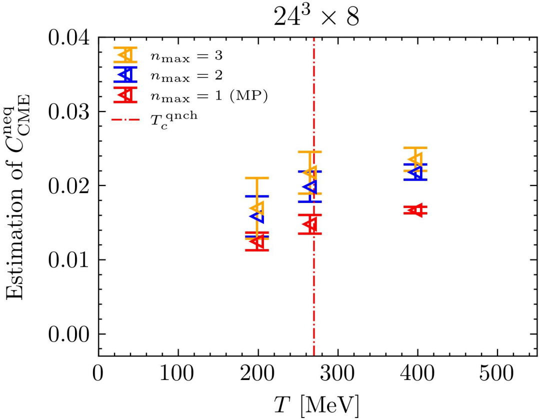
- Toy model: Breit-Wigner spectral function
- Conductivity $\sigma \equiv \lim_{\omega \to 0} \rho(\omega)/\omega$



- Multipoint improves the conductivity estimation

Multipoint quenched QCD wilson

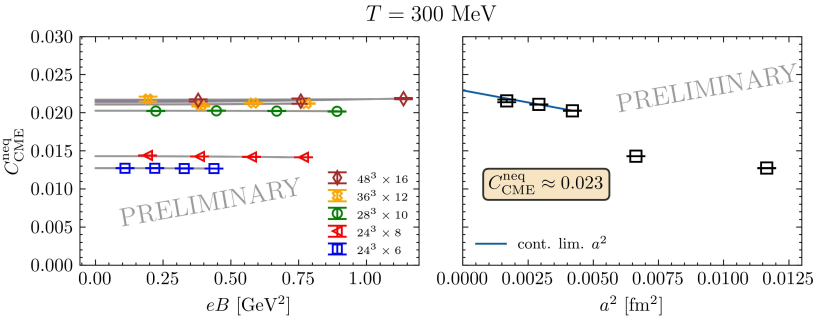
- Use multipoint estimation with quenched QCD Wilson data



- Estimation of $C_{\mathrm{CME}}^{\mathrm{neq}} \approx 0.022$ - 0.025 at high T

Conductivity in QCD

- Reconstructing staggered QCD data using Gaussian proccesses Horak et al. '21



- Precise normalization of Kubo formula missing!
- Detailed systematic error analysis required
- Consistent with multipoint quenched Wilson estimation

Summary

Summary

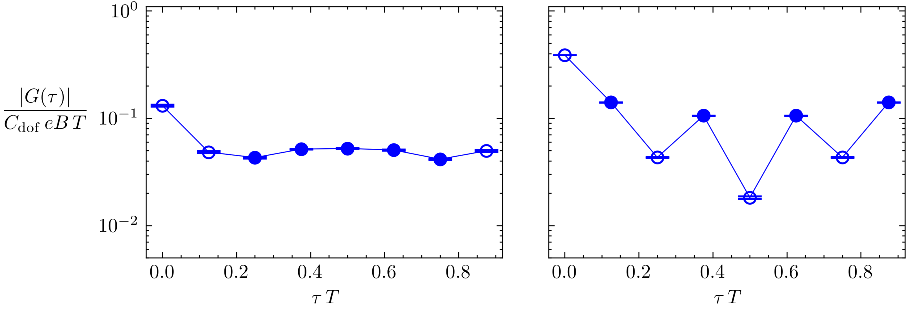
- Study of out-of-equilibrium CME in QCD using lattice simulations
- Spectral function calculated at 1-loop in perturbation theory
- First estimation yields $C_{\mathrm{CME}}^{\mathrm{neq}}$ suppressed at low T
- Preliminary results towards continuum limit at $T = 300$ MeV

Outlook

- How to obtain Kubo formula from chiral hydrodynamics?
- Relation between required resummation and axial anomaly?
- Apply several different reconstruction methods
- Systematic scan in T

Backup slides

Correlators



Wilson Staggered