Pion mass in a magnetized medium

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Overview

- Motivation.
- State of the art
- Linear sigma model with quarks (LSMq)
- Analysis of the neutral and charged pion pole masses
- Preliminary results
- Summary and perspectives

Why to study magnetic effects on the masses of pions?

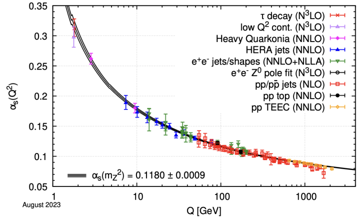
1) Electromagnetic fields provide a powerful probe to explore the properties of the QCD vacuum.

Why to study magnetic effects on the masses of pions?

2) Since the dynamics of chiral symmetry breaking is dominated by pions, the lightest of all quark-antiquark bound states, it then becomes important to explore how the pion properties like the pole mass and screening masses are affected by the presence of magnetic fields.

In this work we will study the effect of a constant magnetic field B in the pole masses of neutral and charged pions in the LSMq.

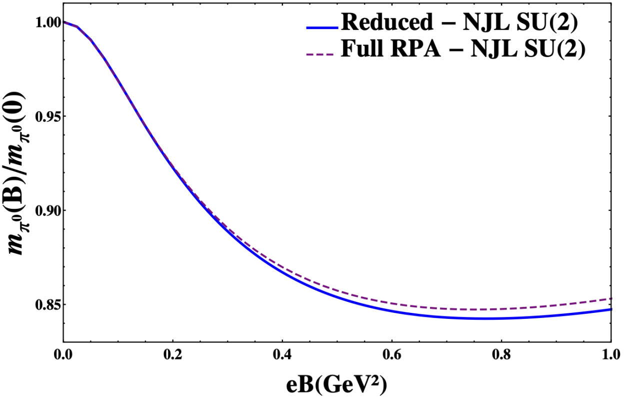
From non-perturbative QCD to effective models and LQCD



Since the determination of pole pion masses involves calculations in the non-perturbative regime of QCD, we need to follow other alternatives:

- Effective models: NJL, ChPT, LSMq
- LQCD

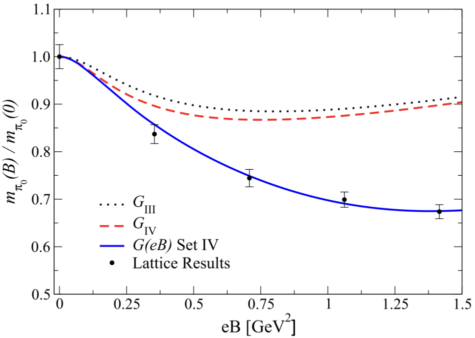
Previous results



[1]

[1] S. S. Avancini, W. R. Tavares and M. B. Pinto, Phys. Rev. D **93**, no.1, 014010 (2016).

Previous results



$$G(eB) = \alpha + \beta e^{-\gamma (eB)^2}$$

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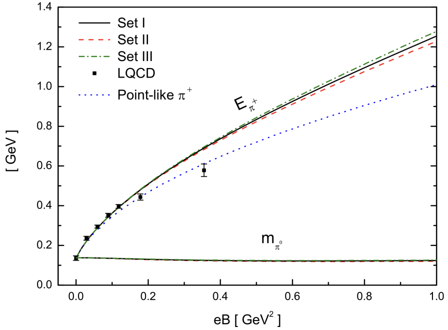
[2] S. S. Avancini, R. L. S. Farias, M. Benghi Pinto, W. R. Tavares and V. S. Timóteo, Phys. Lett. B **767**, 247-252 (2017).

[3] B. B. Brandt, G. Bali, G. Endrödi and B. Glässle, PoS **LATTICE2015**, 265 (2016).

Previous results



$$E_{\pi^+} = \sqrt{m_{\pi^+}^2 + eB}$$

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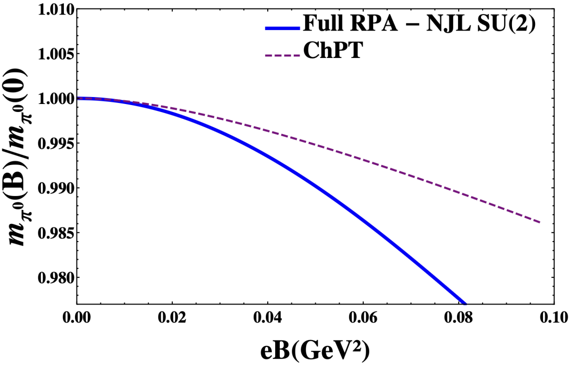
[4]M. Coppola, D. Gómez Dumm and N. N. Scoccola, Phys. Lett. B **782**, 155-161 (2018).

[5]G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz, S. Krieg, A. Schafer and K. K. Szabo, JHEP **02**, 044 (2012).

Previous results



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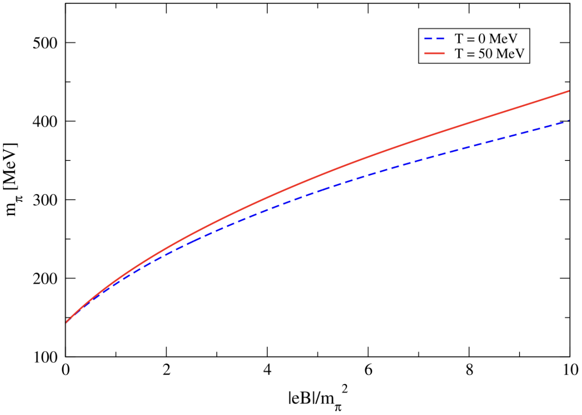
7

[6] S. S. Avancini, W. R. Tavares and M. B. Pinto, Phys. Rev. D **93**, no.1, 014010 (2016).

[7] J. O. Andersen, JHEP **10**, 005 (2012).

Previous results



$$E_{\pi^+} = \sqrt{m_{\pi^+}^2 + eB}$$

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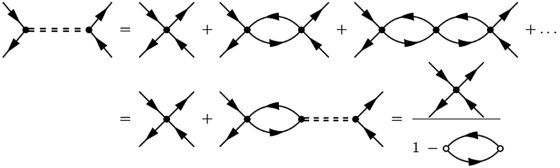
[8] G. Colucci, E. S. Fraga and A. Sedrakian, Phys. Lett. B **728**, 19-24 (2014).

NJL prescription



$$(ig_{\pi^0 qq})^2 iD_{\pi_0}(k^2) = \frac{2iG}{1 - 2G\Pi_0(k^2)}$$

$$-i\Pi_0(k^2) = -\sum_{q=u,d}\int \frac{d^4p}{(2\pi)^4} Tr[i\gamma^5 iS_q(p+k/2) i\gamma^5 iS_q(p-k/2)]$$

$$iS(p) = \int_0^\infty \frac{ds}{\cos(qBs)} \exp\left[is\left(p_\parallel^2 - p_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon\right)\right]$$
$$\times \left\{(m_f + \not{p}_\parallel)\Big(\cos(qBs) + \gamma^1\gamma^2 \sin(qBs)\Big) - \frac{\not{p}_\perp}{\cos(qBs)}\right\}$$

NJL prescription

The pole mass is obtained by solving

$$1 - 2G\Pi_0(m_{\pi^0}^2, 0) = 0$$

$$\Pi_0(k_\parallel^2, k_\perp^2) = \Pi_{0,B=0}^{reg}(k^2) + \Pi_0^{mag}(k_\parallel^2, k_\perp^2)\,, \quad (MFIR)$$

$$M = m - 2G < \bar{\psi}\psi >$$

NJL prescription

For the charged pion the same procedure follows, but we need to be careful with the Schwinger phase

$$iS(x, x') = e^{i\Phi_f(x,x)} \int \frac{d^4p}{(2\pi)^4} iS(p) e^{-ip\cdot(x-x)}$$

$$1 - 2G\Pi_+(0, m_{\pi^+}^2) = 0$$

$$\Pi_+(k, \Pi^2) = \Pi_{+,B=0}^{reg}(\Pi^2) + \Pi_+^{mag}(k, \Pi^2)\,, \quad (MFIR)$$

$$\Pi^2 = (2k+1)eB + q_\parallel^2$$

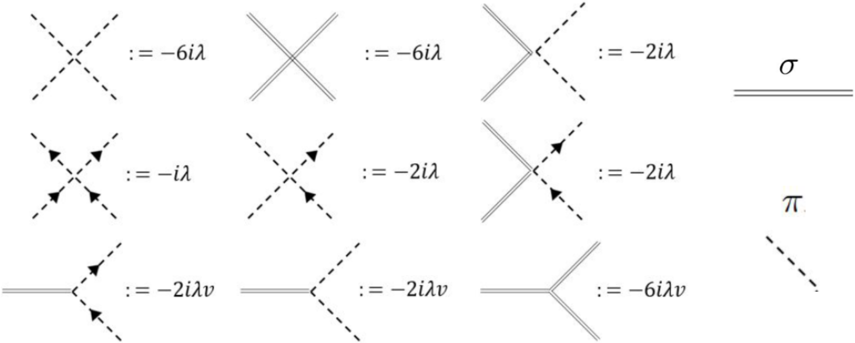
LSMq Lagrangian and features

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + i\bar{\psi}\gamma^\mu \partial_\mu \psi$$
$$- ig\bar{\psi}\gamma^5\bar{\psi}\vec{\tau} \cdot \vec{\pi}\psi - g\bar{\psi}\psi\sigma \, .$$

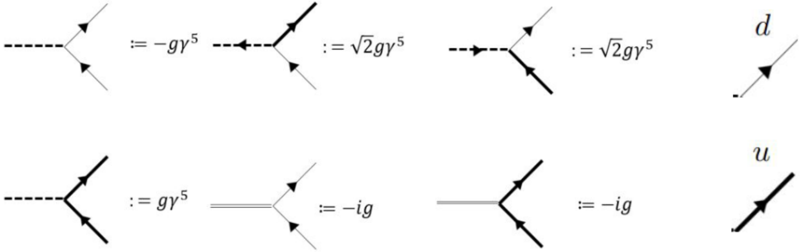
- it implements the SSB of: $SU(2)_L \times SU(2)_R \to SU(2)_V$.
- $m_\pi(v) = \sqrt{\lambda v^2 - a^2} = 0$ at VEV.
- $m_f(v) = gv$.
- $m_\sigma(v) = \sqrt{3\lambda v^2 - a^2}$

$\mathcal{L} \to \mathcal{L} + h(\sigma + v)$ in order to give the correct vacuum pion mass.

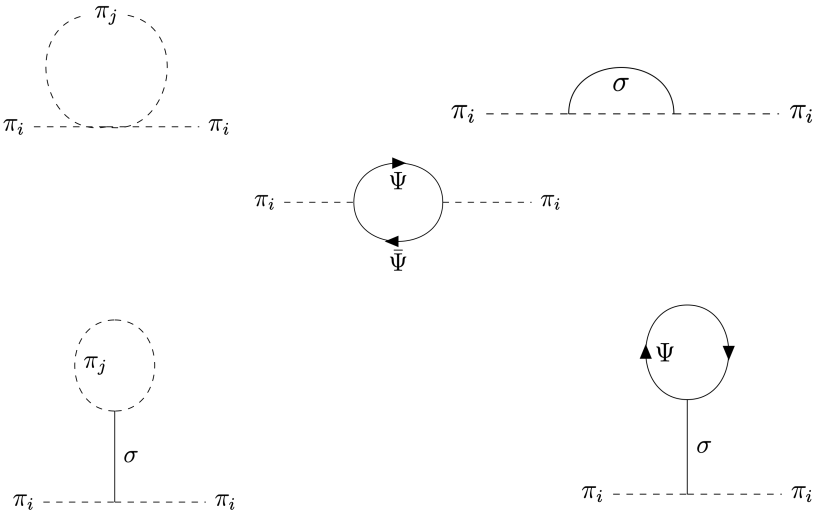
Feynman rules for boson-boson interactions



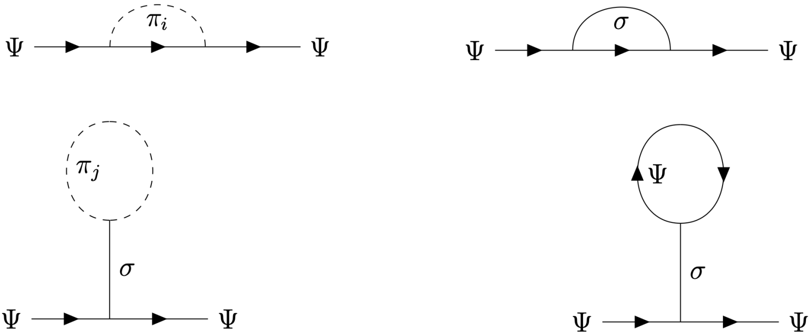
Feynman rules for boson-fermion interactions



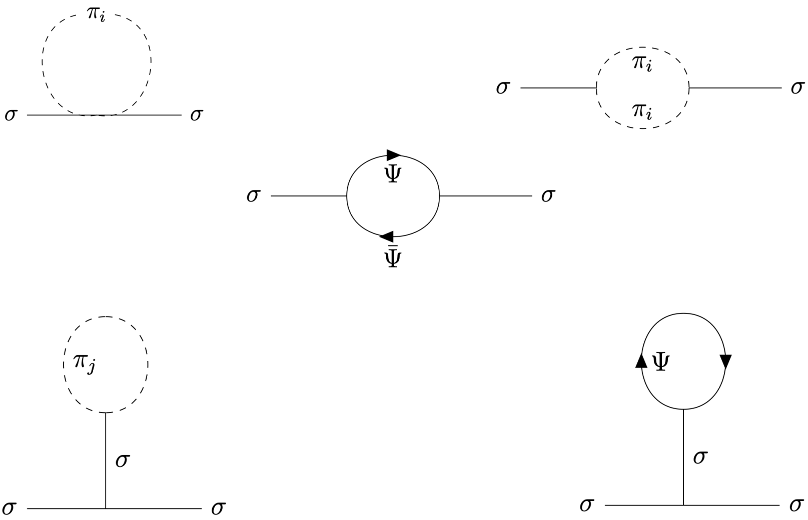
Relevant Feynman diagrams: pion case



Relevant Feynman diagrams: fermion case



Relevant Feynman diagrams: sigma case



Self-energies

$$M_\pi^2 = m_\pi^2 + \Pi_\pi(M_\sigma, M_\pi, M_f; v, B)$$
$$M_\sigma^2 = m_\sigma^2 + \Pi_\sigma(M_\sigma, M_\pi, M_f; v, B)$$
$$M_f = m_f + \Pi_f(M_\sigma, M_\pi, M_f; v, B)$$

$$M_\pi^2 = (\lambda v^2 - a^2) + \Pi_\pi(M_\sigma, M_\pi, M_f; v, B)$$
$$M_\sigma^2 = (3\lambda v^2 - a^2) + \Pi_\sigma(M_\sigma, M_\pi, M_f; v, B)$$
$$M_f = gv + \Pi_f(M_\sigma, M_\pi, M_f; v, B)$$

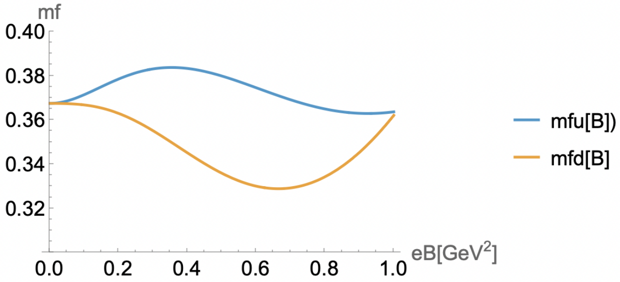
$$M_\pi^2 = \lambda'(v')^2 - (a')^2$$
$$M_\sigma^2 = 3\lambda'(v')^2 - (a')^2$$
$$M_f = g'v'$$

Parameters

Parameters at zero magnetic field

$$g = \frac{m_f}{v} \approx 4$$

Magnetic field dependence of masses



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[9] S. S. Avancini, M. Coppola, N. N. Scoccola and J. C. Sodré, Phys. Rev. D **104**, no.9, 094040 (2021) .

Feynman rules for the fermionic contribution to $\pi_{f\bar{f}}$ (vertices and propagator)

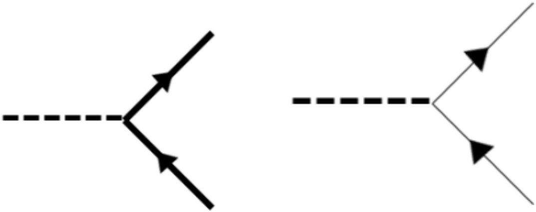


Figure: π_0-u quark vertex=gγ^5; π_0-d quark vertex=-gγ^5

$$iS(p) = \int_0^\infty \frac{ds}{\cos(qBs)} \exp\left[is\left(p_\parallel^2 - p_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon \right)\right]$$

$$\times \left\{ (m_f + \not{p}_\parallel)\Big(\cos(qBs) + \gamma^1\gamma^2 \sin(qBs) \Big) - \frac{\not{p}_\perp}{\cos(qBs)} \right\}$$

$$iS(p) \to i\frac{(m_f + \not{p})}{p^2 - m_f^2 + i\epsilon}$$

Neutral pion self-energy (fermion contribution)

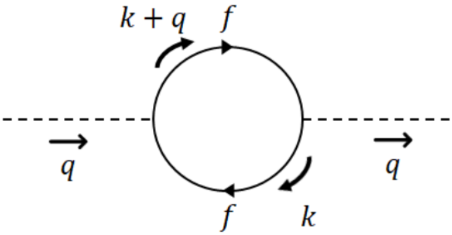


Figure: Neutral pion self-energy

$$-i\Pi_0 = -g^2 \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma^5 iS(k)\gamma^5 iS(k+q)\right] + c.c..$$

Charged pion self-energy (fermion contribution)

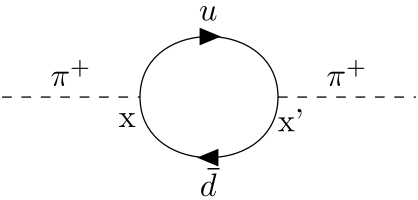


Figure: Charged pion self-energy

$$-i\Pi_+(x,x') = -\,g^2 e^{i\Phi_u(x,x')} e^{i\Phi_d(x',x)} e^{i(p-p')(x-x)} \times \int \frac{d^4p}{(2\pi)^4}\frac{d^4p'}{(2\pi)^4} Tr\left[\gamma^5 iS(p)\gamma^5 iS(p')\right] + c.c..$$

Analysis

$$f_0(p_{\|}, p_{\perp}, B) = \Pi_0 - \lim_{eB\to 0} \Pi_0$$

For obtaining the neutral pion pole mass we solve

$$p_0^2 - p_{\perp}^2 - p_3^2 - m_\pi^2 - \Re f_0(p_0^2, p_{\perp}^2, p_3^2, B)\Big|_{p_3^2=p_{\perp}^2=0} = 0$$

$$f_+(p_{\|}, k, B) = \Pi_+ - \lim_{eB\to 0} \Pi_+$$

For obtaining the charged pion pole mass we solve

$$p_0^2 - p_3^2 - (2k+1)eB - m_\pi^2 - \Re f_+(p_0^2, k, p_3^2, B)\Big|_{p_3^2=0,k=0} = 0$$

Neutral pion pole masses for different g values, preliminary results

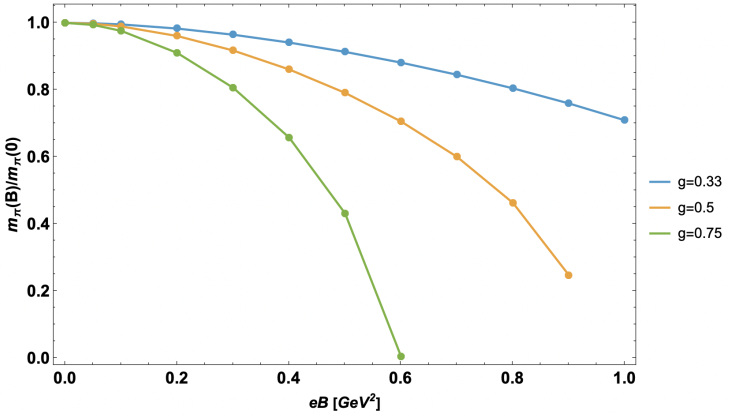


Figure: Neutral pion pole masses as a function of B for different values of g.

Neutral pion pole mass

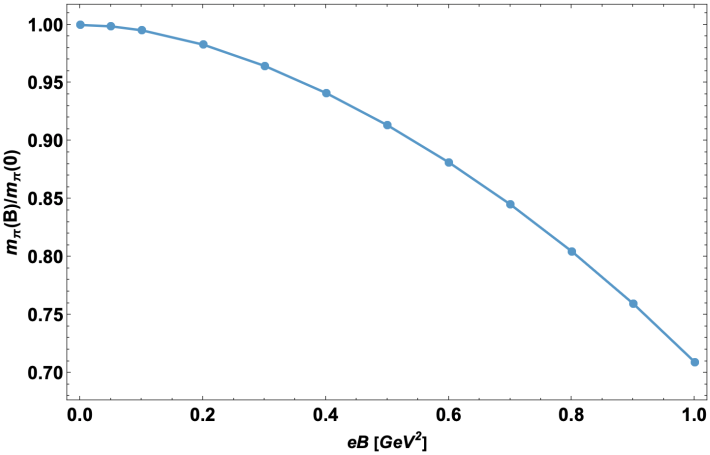


Figure: Neutral pion pole mass as a function of B for $g = 0.33$.

Charged pion pole mass

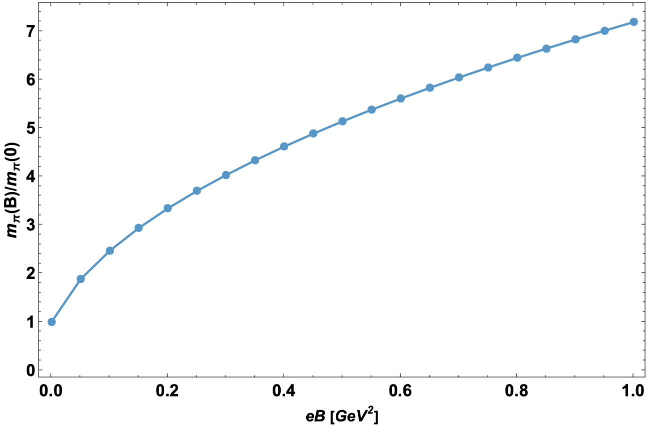


Figure: Charged pion effective pole mass as a function of B for $g = 0.33$.

$$E_{\pi^+} = \sqrt{m_{\pi^+}^2 + eB}$$

Summary and perspectives

Summary:

- We have calculated the neutral and charged pion self-energies in the LSMq.
- We have obtained preliminary results for the pole masses for neutral and charged pions as a function of B
- We have compared our results with LQCD and NJL, and we have found a qualitative agreement only when there is a magnetic field dependence on the coupling g.

Perspectives:

- We need to complete the calculation in a full consistent way with the LSMq only.

$$M_\pi^2 = \lambda'(v')^2 - (a')^2$$

$$M_\sigma^2 = 3\lambda'(v')^2 - (a')^2$$

$$M_f = g'v'$$

- We will study the case where $T \neq 0$.

Thank You

Interplay between strong magnetic fields and QCD

- Magnetic catalysis at zero temperature. enhancement of the condensate $<\bar{q}q>$ with B
- Inverse magnetic catalysis around T_C. decrease of the condensate $<\bar{q}q>$ as a function of B near T_C
- Chiral magnetic effect. Creation of a current J which is collinear to B as a consequence of the axial anomaly
- Electromagnetic fields provide a powerful probe to explore the properties of the QCD vacuum.

Pole and screening masses ($B \neq 0$, $T = 0$)

- Pole mass

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_3^2 = p_\perp^2 = 0} = 0$$

- Screening mass B breaks Lorentz invariance and defines $\parallel$ and $\perp$
  - Longitudinal

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2 = p_\perp^2 = 0} = 0$$

  - Transverse

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_0^2 = p_3^2 = 0} = 0$$

Pole and screening masses ($B \neq 0$, $T = 0$)

- Pole mass of the neutral pion

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_3^2=p_\perp^2=0} = 0$$

- Pole mass of the charged pion

$$p_0^2 - p_\perp^2 - p_3^2 - m_\pi^2 - \Re f(p_0^2, p_\perp^2, p_3^2, B)\Big|_{p_3^2=p_\perp^2=0} = 0$$

Pole Mass

$$C(t) = \int d^3x < \mathcal{O}(\mathbf{x}, t)\mathcal{O}(\mathbf{0}, 0) >$$

$$C(t) = \sum_n | < 0|\mathcal{O}|n > |^2 e^{-E_n t}$$

$$C(t) \approx Z_0 e^{-m_{pole} t}$$

Screening Mass

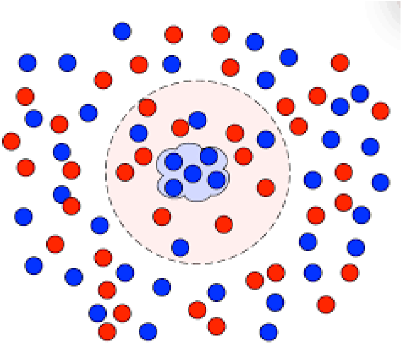


Figure: Debye mass (inverse of Debye length)

$$V \propto \frac{1}{r} \to \frac{1}{r} e^{-r/\lambda}$$

Screening Mass

In QFT we don't see the screening from the potential like in the previous example from classical electrodynamics, instead we pay attention to correlation functions:

$$G(x) \propto e^{-m_{scr}x}$$

Pole and screening masses (definitions)

$$E^2 = u_\perp^2 \mathbf{q}_\perp^2 + u_\parallel^2 q_3^2 + m_{\pi^0,pole}^2$$

$$m_{\pi^0,scr.\perp} = \frac{m_{\pi^0,pole}}{u_\perp}$$

$$m_{\pi^0,scr.\parallel} = \frac{m_{\pi^0,pole}}{u_\parallel}$$

$$u_\perp \equiv u_\perp(B,T) \ , \ \ u_\parallel \equiv u_\parallel(T)$$

Pole and screening masses (special cases)

- **(i)T=0, B=0**

$$u_\perp = u_\parallel = 1$$

$$m_{\pi^0,pole} = m_{\pi^0,scr,\parallel} = m_{\pi^0,scr,\perp}$$

- **(ii)T≠0, B=0**

$$u_\perp = u_\parallel = u \neq 1$$

$$m_{\pi^0,pole} \neq m_{\pi^0,scr,\perp} = m_{\pi^0,scr,\parallel}$$

$u < 1$ in order to satisfy causality

Pole and screening masses (special cases)

- **(iii)B≠0, T=0**

$$u_\perp \neq u_\parallel \ \ but \ \ u_\parallel = 1$$

$$m_{\pi^0,pole} = m_{\pi^0,scr,\parallel} < m_{\pi^0,scr,\perp}$$

- **(iv)B≠0, T≠0**

$$u_\perp < u_\parallel < 1$$

$$m_{\pi^0,pole} < m_{\pi^0,scr,\parallel} < m_{\pi^0,scr,\perp}$$

Comparison with LQCD and the NJL model

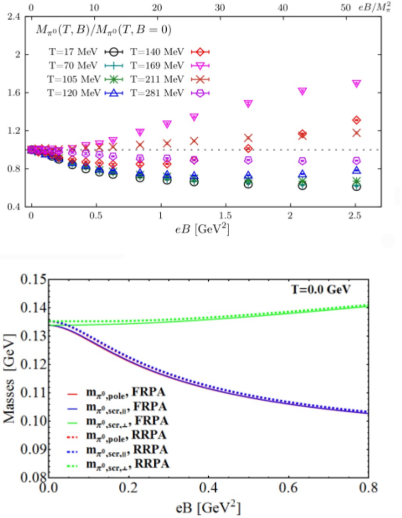
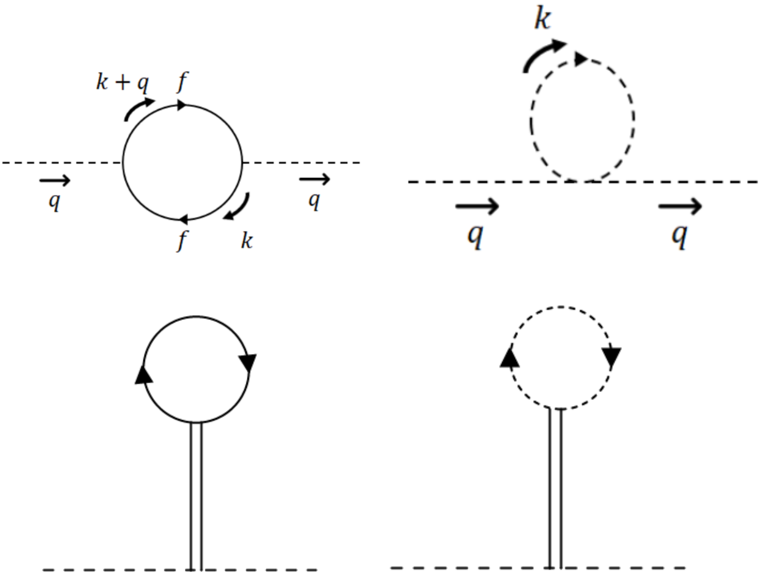


Figure: H. T. Ding, S. T. Li, J. H. Liu, and X. D. Wang, Phys. Rev D105, 034514 (2022), and B. Sheng, Y. Wang, X. Wang, and L. Yu, Phys. Rev. D103 (2021) 9, 094001.

Relevant Feynman diagrams



Important integrals and convenient change of variables

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$$

$$u = s + s'$$

$$s = u(1 - v)$$

$$s' = uv$$

$$\frac{\partial(s, s')}{\partial(u, v)} = u$$

Small magnetic field approximation

$$iS(p) = \int_0^\infty \frac{ds}{\cos(qBs)} \exp\left[is\left(p_\parallel^2 - p_\perp^2 \frac{\tan(qBs)}{qBs} - m_f^2 + i\epsilon\right)\right]$$

$$\times \left\{(m_f + \not{p}_\parallel)\Big(\cos(qBs) - \gamma^1\gamma^2 \sin(qBs)\Big) + \frac{\not{p}_\perp}{\cos(qBs)}\right\}$$

$$iS(p) \to iS^0(p) + iS^1(p) + iS^2(p)$$

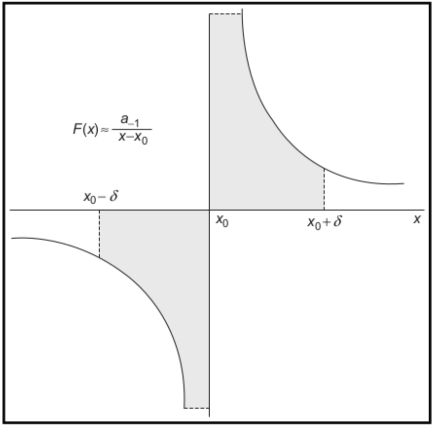
,

$$iS^0(p) = i\frac{m_f + \not{p}}{p^2 - m^2}$$

$$iS^1(p) = |q_f B|\gamma^1\gamma^2 \frac{m_f + \not{p}_\parallel}{(p^2 - m^2)^2} sign(q_f B)$$

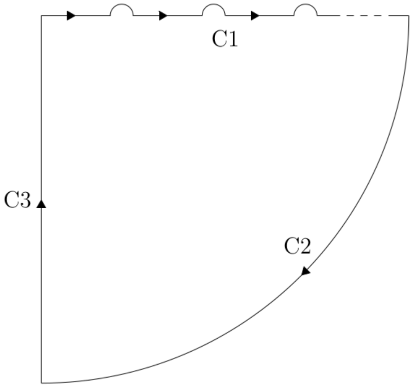
$$iS^2(p) = -2i|q_f B|^2 \frac{p_\perp^2(\not{p}_\parallel + m_f) - (m_f^2 - p_\parallel^2)\not{p}_\perp}{(p^2 - m_f^2)^4}$$

Principal value prescription for evaluating contours 1



$$f(z) = \frac{a_1}{(z-z_0)} + a_0 + \ldots$$

Typical contour of integration



$$I \equiv \int_0^\infty du e^{-u\epsilon} e^{-iau} \left(\cot(|q_f B|u) - \frac{1}{|q_f B|u} \right) .$$

$$I_C = \oint_C du e^{-u\epsilon} e^{-iau} \left(\cot(|q_f B|u) - \frac{1}{|q_f B|u} \right) ,$$

Neutral pion self-energy (fermion contribution)

$$\pi_{\bar{f}f} = \frac{-4g^2 qB}{(4\pi)^2} \int_0^1 dv \int_0^\infty du \exp\left(-ix\right) \exp\left(-u\epsilon\right) \Bigg[\frac{m_f^2}{\tan\left(qBu\right)} - \frac{qB}{\sin^2\left(qBu\right)} \left(\frac{p_\perp^2}{|qB|} \frac{\sin\left(qBu(1-v)\right) \sin\left(qBuv\right)}{\sin\left(qBu\right)} \right) - \frac{v(1-v)(p_3^2 - p_0^2)}{\tan(qBu)} - \frac{iqB}{\sin\left(qBu\right)} - \frac{i}{u \tan\left(qBu\right)} \Bigg]$$

where x is given by:

$$x = \frac{p_\perp^2}{qB} \frac{\sin(qBu(1-v)) \sin(qBuv)}{\sin(qBu)} + p_3^2 uv(1-v) - p_0^2 uv(1-v) + m_f^2 u$$

$B \to 0$ limit of $\pi_{\bar{f}f}$

$$\lim_{B\to 0} \pi_{\bar{f}f} = -\frac{4g^2}{(4\pi)^2} \int_0^1 dv \int_0^\infty \frac{du}{u} e^{-ix_0} e^{-u\epsilon} \times \left\{ m_f^2 - v(1-v)(p_\perp^2 - p_3^2) + v(1-v)p_0^2 - \frac{2i}{u} \right\}$$

where

$$x_0 = uv(1-v)(p_\perp^2 + p_3^2) - p_0^2 uv(1-v) + m_f^2$$

Fermionic contribution for different g values

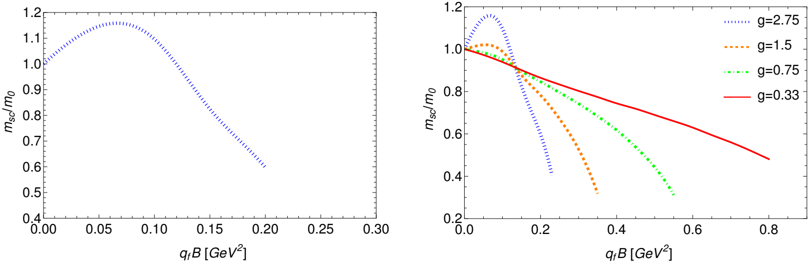


Figure: 'Longitudinal' SC mass as function of B for $g = 2.75$ (left). 'Longitudinal' SC mass as function of B for different values of g.

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[10] A. Ayala, R. L. S. Farias, L. A. Hernández, A. J. Mizher, J. Rendón, C. Villavicencio and R. Zamora, Phys. Rev. D **109**, no.7, 074019 (2024).

Fermionic contribution + Tadpoles

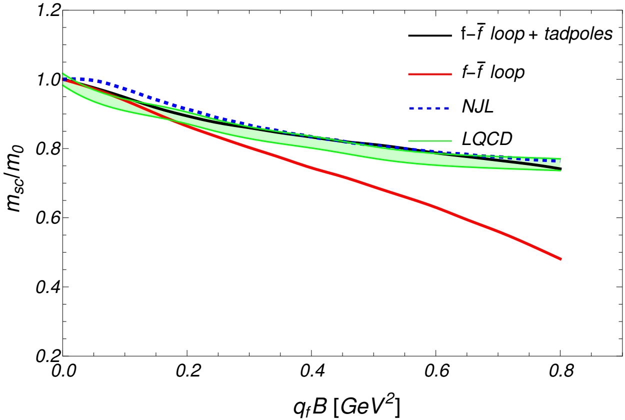

Figure: 'Longitudinal' Screening mass as a function of B, $g = 0.33, \lambda = 2.5$.

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[11] A. Ayala, R. L. S. Farias, L. A. Hernández, A. J. Mizher, J. Rendón, C. Villavicencio and R. Zamora, Phys. Rev. D **109**, no.7, 074019 (2024).

Fermionic contribution with g_{eff} as a function of B

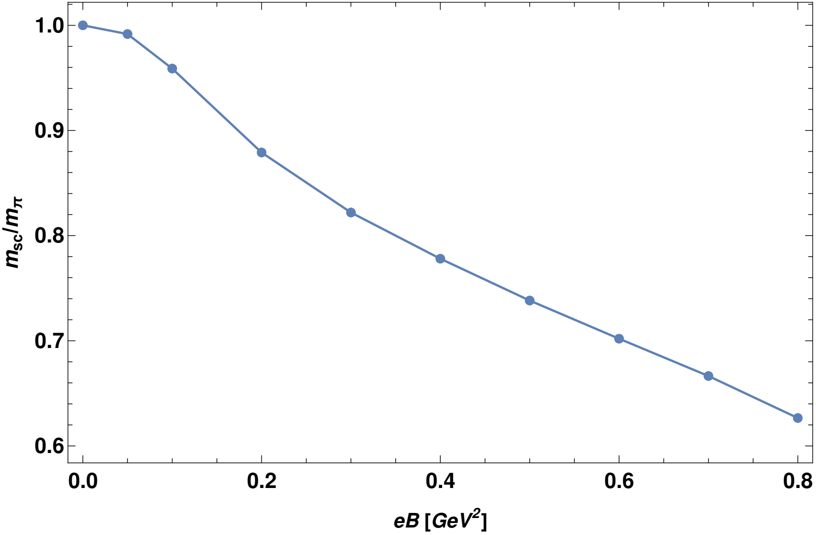

Figure: 'Longitudinal' Screening mass as a function of B, $g_{eff}(B) = 0.3 + 1.2\exp[-(7B)^2]$.