Universiteit Utrecht

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Spin-hydrodynamics and Hyperon polarisation in Trajectum

Raúl Wolters (Utrecht U.)

Umut Gürsoy †2025 (UU), Raimond Snellings (UU), Wilke van der Schee (CERN/UU),
Govert Nijs (CERN), Giuliano Giacalone (CERN), Enrico Speranza (CERN)

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The polarisation puzzle

- The polarisation of Λ hyperons provides a tantalising clue for the existence of spin-hydrodynamics
- We have many different interpretations and models for spin-hydrodynamics: from QFT in rotating systems [1] to gradient expansions in gravity with torsion [2]

[1]: Beccatini, Chandra, del Zanna & Gossi Ann. o. Phys. 338(2013) pp.32–49 | [2]: Gallegos, Gürsoy & Yarom, JHEP05(2023)139

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- Accompanying these models are some successful fits [3]

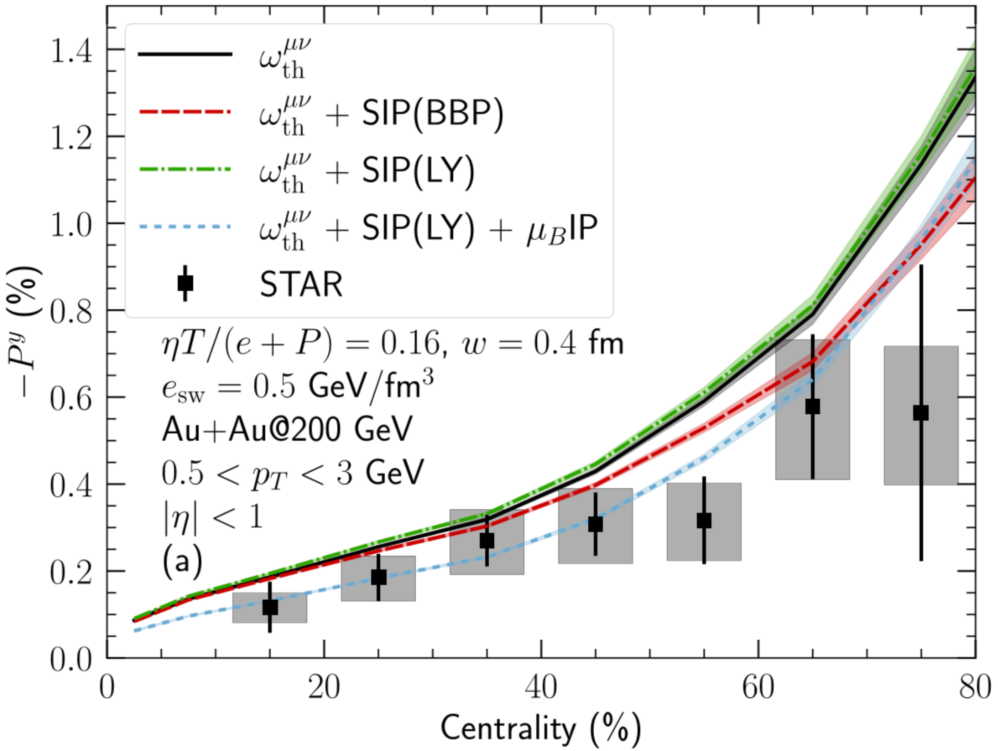

Figure 2: Global polarisation fit at RHIC from [3]

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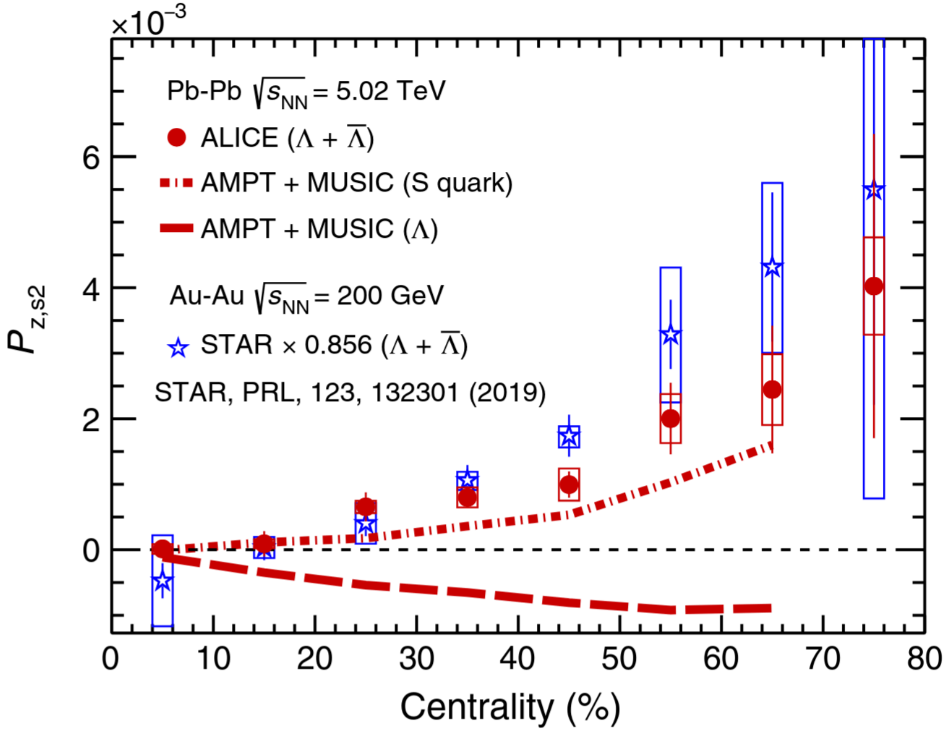

Figure 3: Local polarisation fit at LHC from [5] with MUSIC fit based on [4]

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- None of these estimates take systematic theory uncertainties into account
- A Bayesian fit (without polarisation) of 3+1D initial conditions to RHIC and LHC data can provide such a postdiction

This work (WIP): (update on) Bayesian 3+1D postdiction for hyperon polarisation with *Trajectum*

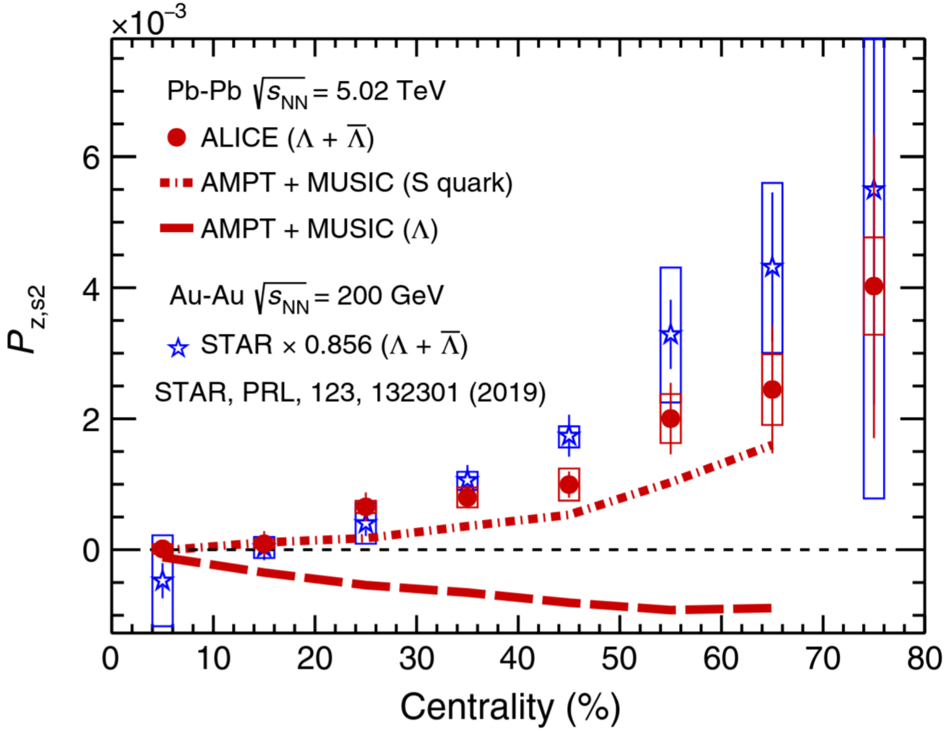
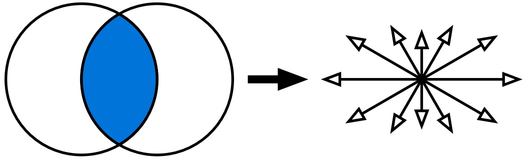


Figure 4: Local polarisation fit at LHC from [5] with MUSIC fit based on [4]

Hydrodynamics and Geometry

- Hydrodynamics translates initial-state geometry to momentum-space



- This works because energy-momentum is conserved and must therefore be **transported** (rather than dissipated)
- This is a two-way street:
  - well-known hydro $\Rightarrow$ constrains initial state geometry
  - well-known initial state $\Rightarrow$ constrains hydro parameters

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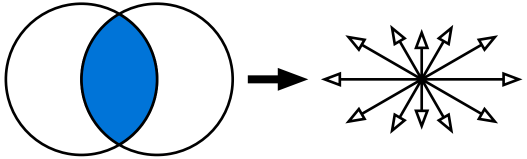

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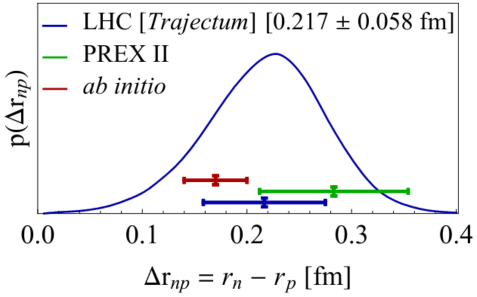
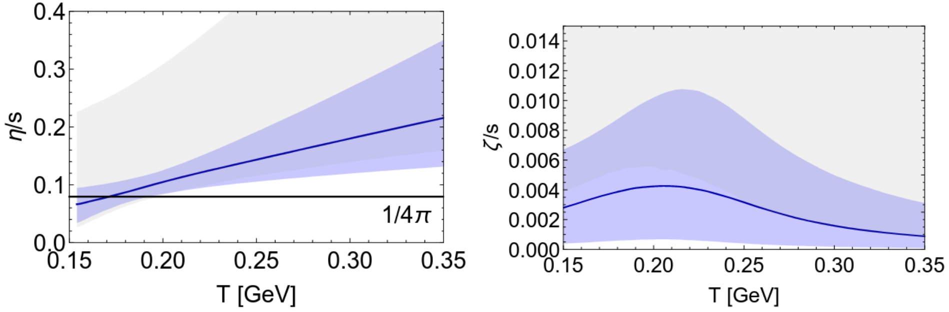
Examples using *Trajectum*:

1. Neutron skin of $^{208}\mathrm{Pb}$ from ALICE data [6]



2. Bulk and shear viscosities from global p_T-analysis [7]



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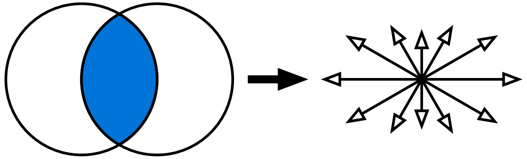

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- In principle, one could construct hydrodynamical theories of other conserved currents
- ... such as angular momentum:

$$J^{\rho,\mu\nu} = x^{\mu}T^{\nu\rho} - x^{\nu}T^{\mu\rho} + S^{\rho,\mu\nu}$$
$$\nabla_{\rho}J^{\rho,\mu\nu} = 0$$

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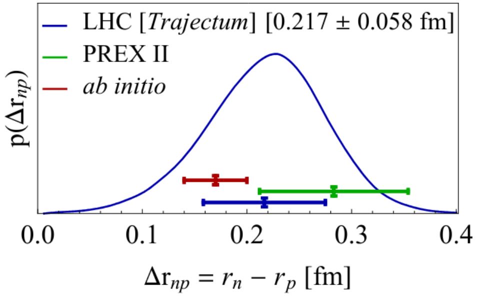

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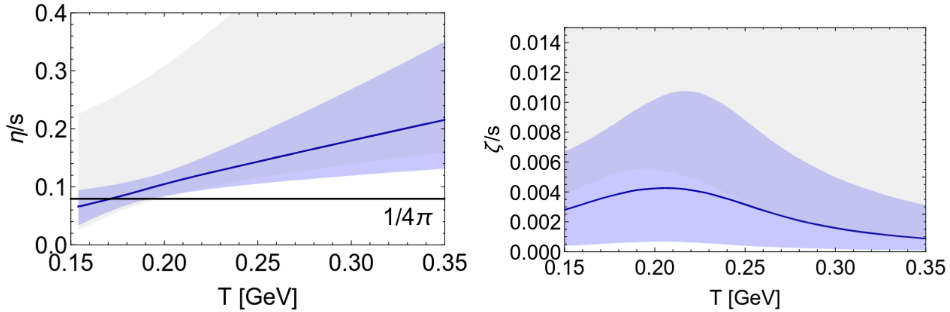

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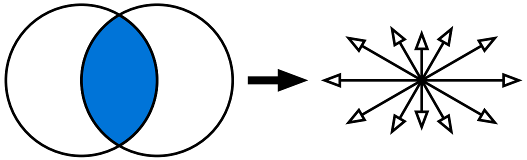

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Can we explain hyperon polarisation with geometric arguments?

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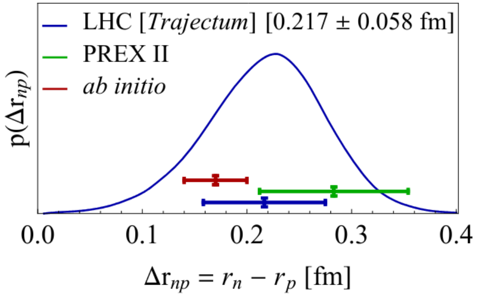

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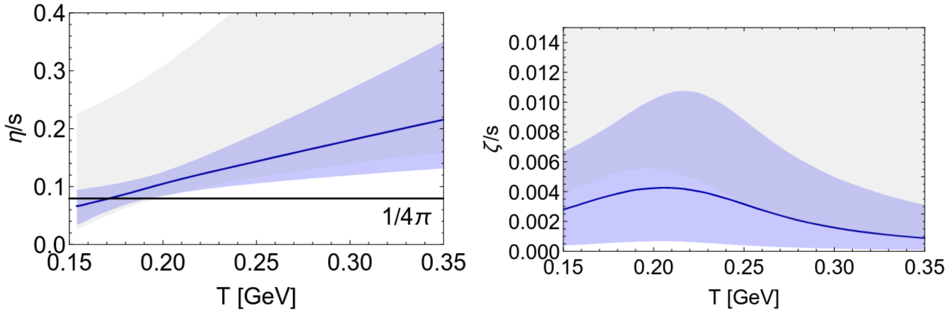

Outline

Observables

Λ–Hyperon polarisation

- Hyperon polarisation provides an experimental probe of angular momentum dynamics
- Spin in hyperon rest-frame accessible through weak decay: correlation between angle between decay products and the spin of the hyperon S^μ (or equivalently, $P^\mu = S^\mu/|S|$).

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- Spin in hyperon rest-frame accessible through weak decay: correlation between angle between decay products and the spin of the hyperon S^μ (or equivalently, $P^\mu = S^\mu/|S|$).
- Two geometric polarisation effects:

Global Polarisation

- Due to net angular momentum of the fluid
- Monopole
- Strong $\sqrt{s_{\mathrm{NN}}}$ dependence: very small at LHC energies, present at RHIC energies

Local Polarisation

- Due to local vorticity of the fluid
- Quadrupole
- Weak $\sqrt{s_{\mathrm{NN}}}$ dependence: present at both RHIC and LHC energies

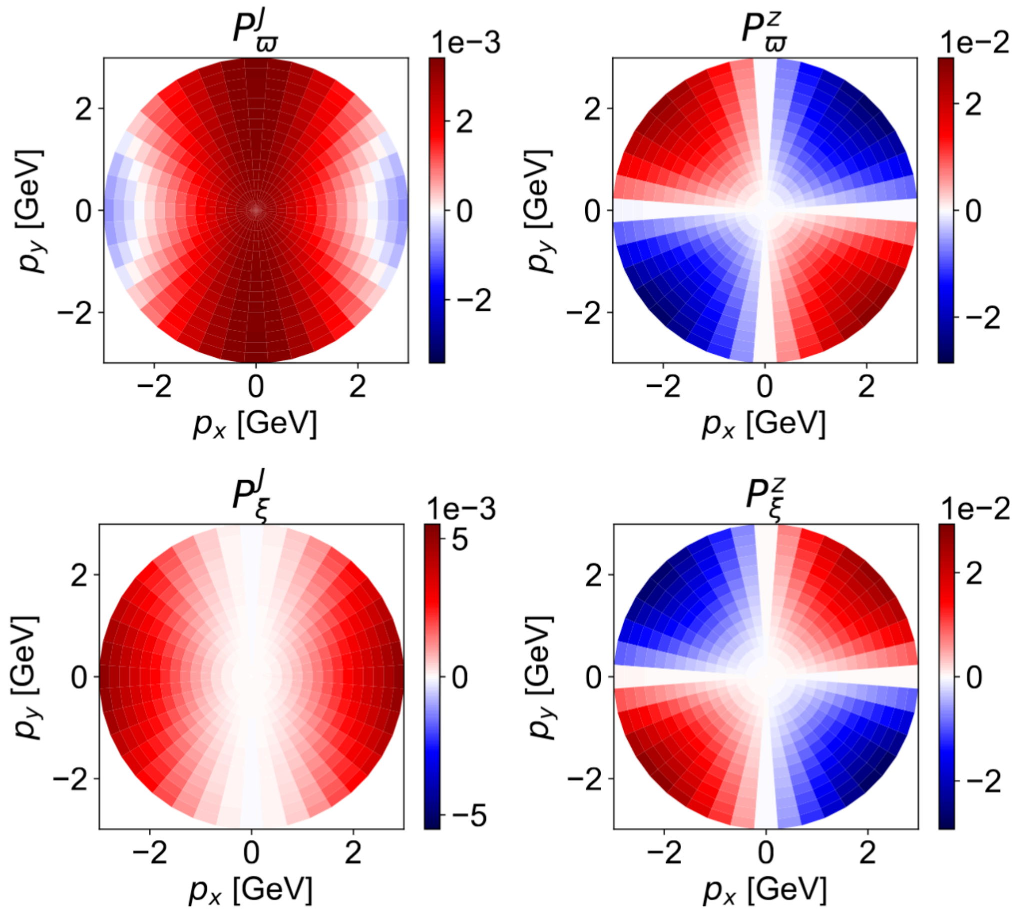

Figure 5: Global (P_J) and local (P_z) polarisation induced by thermal vorticity ϖ and thermal shear ξ. Taken from [8]

Global Λ–Hyperon polarisation

- Off-central collisions have non-zero angular momentum:

$$\vec{J} \propto \vec{b} \times \boldsymbol{e}_3$$

- This geometric angular momentum should be transferred to the hyperons upon freezeout via spin-orbit coupling
- Thus, the observable of interest is the average polarisation along the axis of net global angular momentum:

$$P_H := \frac{\left\langle \left(\vec{b}_{\psi_1} \times \boldsymbol{e}_3 \right) \cdot \vec{P} \right\rangle}{\mathrm{Res}(\psi_1)} = \frac{\langle \sin\psi_1 P^x - \cos\psi_1 P^y \rangle}{\mathrm{Res}(\psi_1)}$$

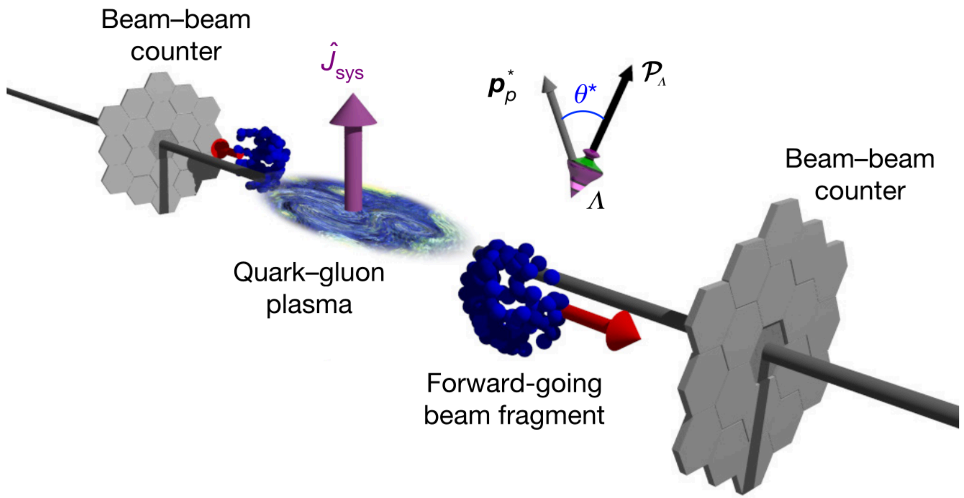

Figure 6: Sketch of $\mathrm{Au} + \mathrm{Au}$ collisions at RHIC. Taken from [9]

Global polarisation – Experiments

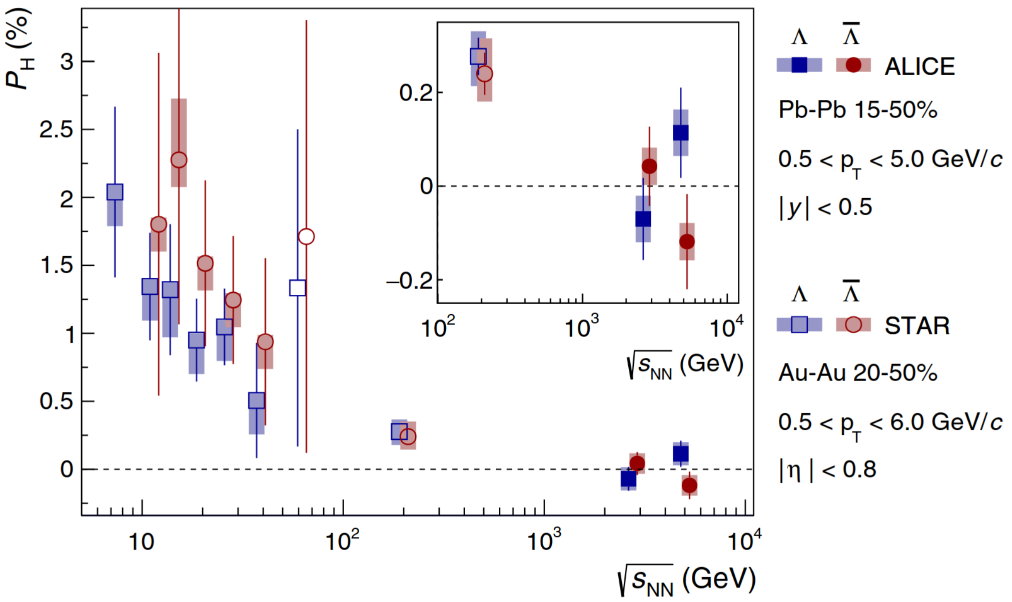

Figure 7: $\sqrt{s_{\mathrm{NN}}}$ dependence of global hyperon polarisation as measured by ALICE and STAR. From: [10]

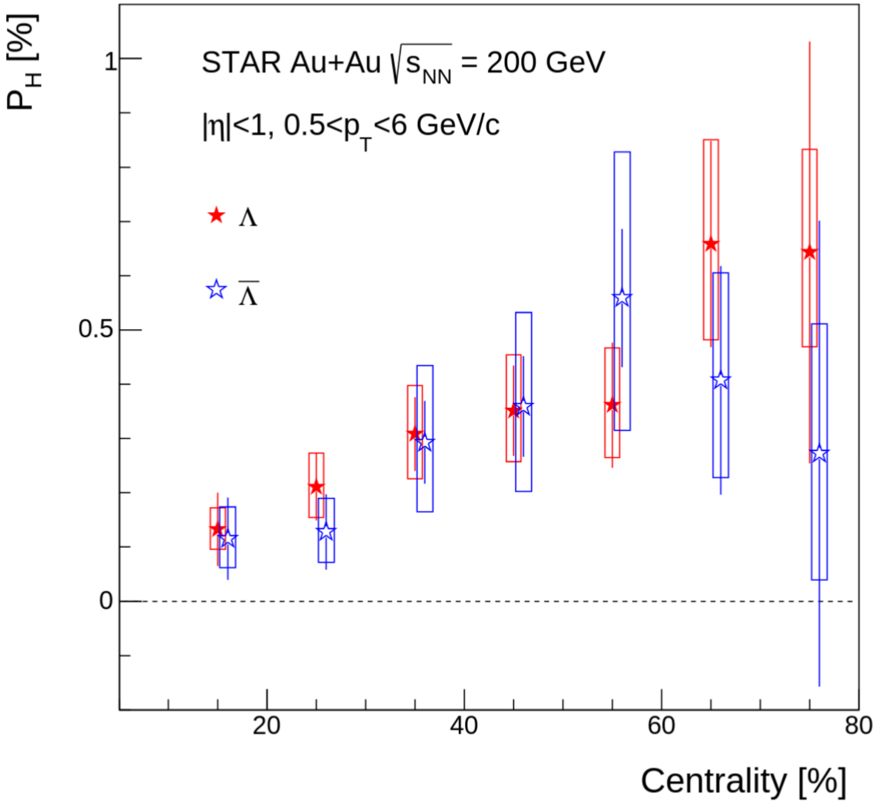

Figure 8: Centrality dependence of global hyperon polarisation at as measured by STAR. From: [11]

Local Λ–Hyperon polarisation

- Anisotropic expansion of the QGP in an off-central collision leads to quadrupole vorticity pattern in the plane transverse the beam direction
- Again, by spin-orbit coupling, this should be imprinted in the average longitudinal polarisation (perpendicular to the transverse plane)
- Relevant observable is thus the quadrupole longitudinal polarisation $P_z^{(2)}$:

$$P_z^{(2)} := \frac{\langle P^z \sin(2(\varphi - \psi_2)) \rangle}{\mathrm{Res}(\psi_2)}$$

- Furthermore, we expect a correlation between $P_z^{(2)}$ and v_2:

$$\rho\left(v_2, P_z^{(2)}\right) = \frac{\mathrm{cov}\left(v_2, P_z^{(2)}\right)}{\sqrt{\mathrm{var}(v_2)\mathrm{var}\left(P_z^{(2)}\right)}}$$

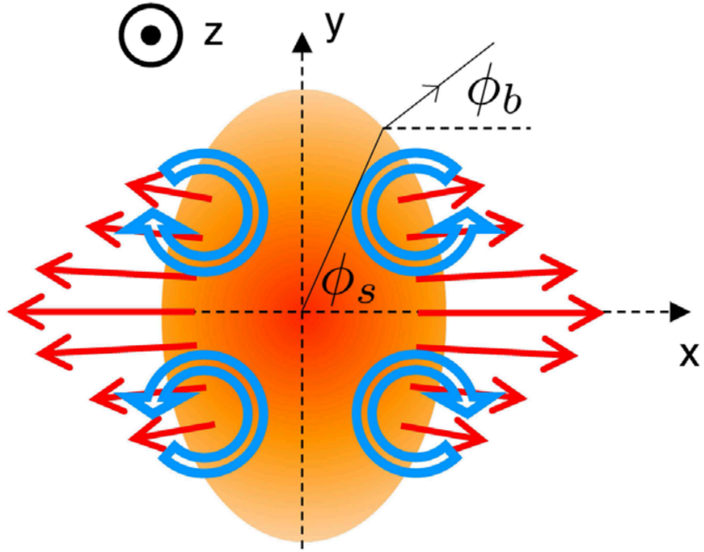

Figure 9: Sketch of vorticity due to anisotropic expansion of the fluid in an off-central collision. Taken from [12]

Local polarisation – Experiments

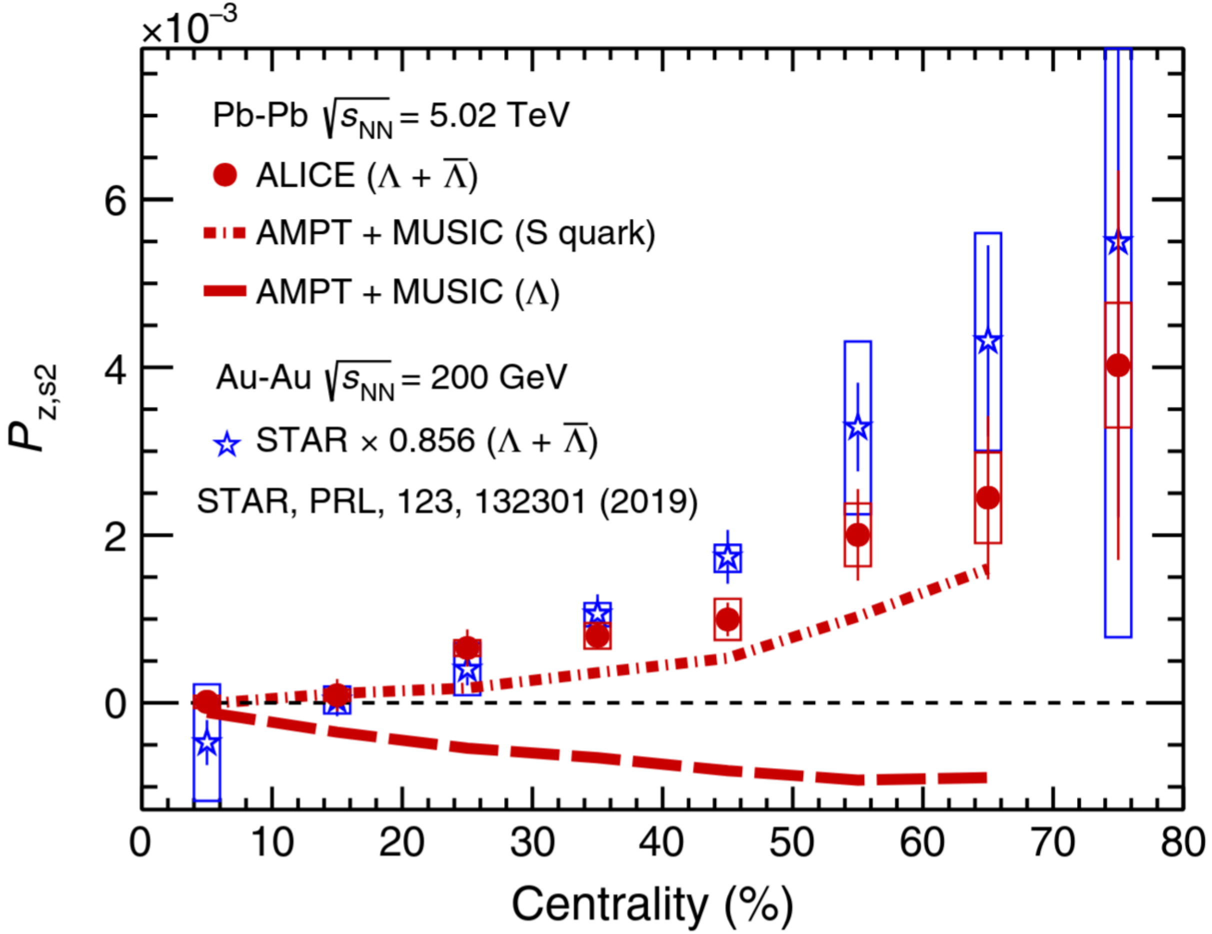

Figure 10: Centrality dependence of global hyperon polarisation as measured by ALICE [5] and STAR [12]. From: [5]

Theory

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Polarisation from spin-orbit coupling

Key observable: the *Pauli-Lubański pseudo-vector*

$$W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} \underbrace{p^\nu}_{\int d\Sigma_\lambda T^{\nu\lambda}} \underbrace{J^{\rho\sigma}}_{\int d\Sigma_\lambda J^{\lambda\rho\sigma}}$$

Well-defined single-particle observable:

- Spin pseudo-gauge independent
- Generator of the orthochronous Lorentz group
- Casimir of Poincaré group, $W^2 = m^2 J(J+1)$

  $\Rightarrow$ we'll use the polarisation $P^\mu = W^\mu/|W|$

How to compute $\langle P^\mu \rangle$?

- We use result of QFT computation by Beccatini [1], [13]
- Compue trace in LTE $\hat{\rho}_{\mathrm{LE}}$ state in Belinfante pseudogauge

Key assumptions:

1. Spin degrees of freedom are non-dynamical
2. There is no dissipation of spin DOFs ($\Omega = \varpi$)

Not covered:

- Non-trivial spin in the initial state
- Spin hydrodynamics

Result:

$$P^\mu(k) = \frac{1}{8m}\epsilon^{\mu\alpha\beta\gamma}(n_F(k) - 1)\left[2\frac{k_\beta \Sigma_\gamma}{\Sigma \cdot k}\xi_{\alpha\delta}k^\delta + \varpi_{\alpha\beta}k_\gamma\right]$$

Two terms:

1. Thermal vorticity $\varpi_{\mu\nu} = -\partial_{[\mu}\beta_{\nu]}$
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Essentially only considering polarisation due to spin-orbit coupling

3+1D Initial Conditions & Bayesian Analysis

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Overview of Trajectum

- Hydrodynamical nuclear collision framework [14]
- Two primary components:
  1. Hydrodynamical simulation of heavy-ion collisions
  2. Data analysis framework
- Holistic approach allows us to make it **fast**
  - Example: we can hugely oversample unlikely events to improve difficult statistics and then correct for the introduced bias in the analysis framework
- Fast enough to do **Bayesian analyses** with it at a reasonable cost
- Quickly increasing feature set (WIP):
  - **Spin freeze-out (this work)**
  - EM fields
  - Charged hydro: baryon transport

DEVELOPED ON Codeberg

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Trajectum Event Generator

Initial Condition $\tau = 0$

- Specify projectile and collision geometry
- Set-up initial state variables (entropy density)

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Trajectum Event Generator

Initial Condition $\tau = 0$

Prehydrodynamic stage $\tau = \tau_{\text{hydro}}$

- Compute initial hydrodynamical variables ε, u^{μ}, Π, $\pi^{\mu\nu}$
- Current implementation: linear interpolation of free-streaming $T^{\mu\nu}$ and AdS/CFT inspired $T^{\mu\nu}$

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Prehydrodynamic stage $\tau = \tau_{\text{hydro}}$

↓

Hydro evolution $\tau_{\text{hydro}} < \tau < \tau_{\text{fs}}(x)$

- Evolve hydrodynamical variables ε, u^μ, Π, $\pi^{\mu\nu}$
  - Viscous **DNMR** hydro implemented
  - Temperature-dependent transport coefficients (e.g. EoS from Lattice QCD) available
- Finite-volume flux-limiter PDE solver: **MUSCL** scheme + midpoint for time evolution

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Trajectum Event Generator

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Hydro evolution $\tau_{\text{hydro}} < \tau < \tau_{\text{fs}}(x)$

Hadronisation $\tau = \tau_{\text{fs}}(x)$

- Hadronisation via **Cooper-Frye** freezeout with viscous corrections

$$\underbrace{p^0 \frac{d^3N}{dp^3}}_{\text{particles}} = \underbrace{\frac{g_\star}{(2\pi)^3} \int \frac{d\Sigma_\mu p^\mu}{e^{u_\nu p^\nu / k_B T_{\text{fs}}} \pm 1}}_{\text{fluid}}$$

Download from https://sites.google.com/view/govertnijs/trajectum | [14]: Nijs, van der Schee, Gürsoy & Snellings, PRC 103(2021) 054909

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Hadronisation $\tau = \tau_{\text{fs}}(x)$

Afterburner (*external*) $\tau > \tau_{\text{fs}}(x)$

- Decay and scatter produced hadrons
- **SMASH** and **UrQMD** formats supported

Bayesian Analysis

- Bayes' theorem:

$$\mathcal{P}(\theta|x) = \frac{\mathcal{L}(x|\theta)\pi(\theta)}{\mathcal{E}(x)} \approx \mathcal{L}(x|\theta)\pi(\theta)$$

- $\mathcal{P}(\theta \mid x)$: **posterior** distribution over parameters θ

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- We sample the **likelihood** $\mathcal{L}(x|\theta)$ for different values of $\vec{\theta}$
- To do this, we have to assume the distribution of $\vec{x}$.
  Standard ansatz is a Gaußian:

$$\mathcal{L}\left(\vec{x} \,\middle|\, \vec{\theta}\right) = \frac{1}{\sqrt{(2\pi)^D \ \det\Sigma}} \exp\left\{-\frac{1}{2}\left[\overrightarrow{\mathcal{M}}\left(\vec{\theta}\right) - \vec{x}\right]^{\mathsf{T}} \Sigma^{-1} \left[\overrightarrow{\mathcal{M}}\left(\vec{\theta}\right) - \vec{x}\right]\right\}$$

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- Trajectum plays the role of the **model** $\vec{\mathcal{M}}\left(\vec{\theta}\right)$, which produces a prediction for each parameter set.
- In practice, this is much too slow: need to compute $\vec{\mathcal{M}}\left(\vec{\theta}\right)$ millions of times
  $\Rightarrow$ use a **surrogate model** instead. We use a Gaußian Process Emulator (GPE).

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- Sample $\mathcal{P}$ with **Markov Chain Monte Carlo** methods: MCMC yields ensamble $\left\{\vec{\theta}_i \sim \mathcal{P}\left(\vec{\theta}\right)\right\}$ that follows the posterior

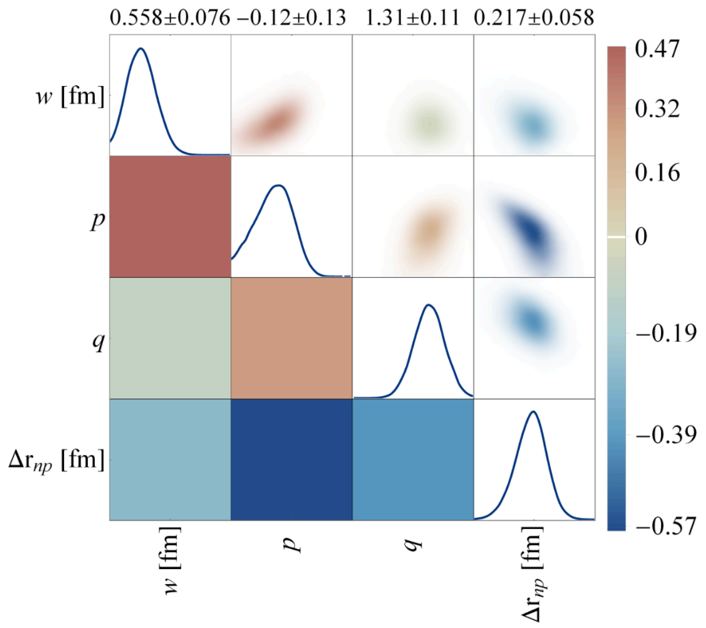

Figure 11: Posterior on nuclear structure parameters of $^{208}\mathrm{Pb}$, from: [6]

Bayesian Prediction

- Bayes' theorem:

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- We sample the **likelihood** $\mathcal{L}(x|\theta)$ for different values of $\vec{\theta}$
- To do this, we have to assume the distribution of $\vec{x}$. Standard ansatz is a Gaußian:

$$\mathcal{L}\left(\vec{x}\,\middle|\,\vec{\theta}\right) = \frac{1}{\sqrt{(2\pi)^D\ \det\Sigma}} \exp\left\{-\frac{1}{2}\left[\vec{\mathcal{M}}\left(\vec{\theta}\right) - \vec{x}\right]^{\mathsf{T}} \Sigma^{-1} \left[\vec{\mathcal{M}}\left(\vec{\theta}\right) - \vec{x}\right]\right\}$$

  where $\Sigma = \mathrm{cov}(\vec{x}, \vec{x}) + \mathrm{cov}\left(\vec{\mathcal{M}}, \vec{\mathcal{M}}\right)$
- Trajectum plays the role of the **model** $\vec{\mathcal{M}}\left(\vec{\theta}\right)$, which produces a prediction for each parameter set.
- In practice, this is much too slow: need to compute $\vec{\mathcal{M}}\left(\vec{\theta}\right)$ millions of times
  $\Rightarrow$ use a **surrogate model** instead. We use a Gaußian Process Emulator (GPE).

- Sample $\mathcal{P}$ with **Markov Chain Monte Carlo** methods: MCMC yields ensamble $\left\{\vec{\theta}_i \sim \mathcal{P}\left(\vec{\theta}\right)\right\}$ that follows the posterior
- The global maximum (also: **Maximum à Posteriori**, MAP) of $\mathcal{P}$ gives the best fit value of a parameter
- The spread around the MAP yields confidence contours

Predictions are easy once you have the MCMC chain:
- Randomly select samples from posterior $\vec{\theta}_i \sim \mathcal{P}\left(\vec{\theta}\right)$
- Distribution of $\vec{\mathcal{M}}\left(\vec{\theta}_i\right)$ is your prediction
  $\Rightarrow$ with systematic (measurement & theory) errors!

3+1D initial conditions (WIP)

- We use a 3D extension of the TRENTO model [15], which differs slightly from the extension [16] proposed by the original authors
- Initial entropy density written as product of longitudinal and transverse distributions:

$$s(x,\eta) = f(x,\eta) g(x,\eta)$$

- $f(x,\eta)$ is the familiar TRENTO ansatz:

$$f(x,\eta) \propto \left(\frac{T_A^p + T_B^p}{2} \right)^{\frac{q}{p}}$$

- Thickness functions deposit entropy where nucleons overlap:

$$T_A(x,\eta) = \sum_{\substack{\text{wounded} \\ \text{nucleons}}} [\gamma \sim \Gamma(\eta)] \exp\left\{ -\frac{(\vec{x}-\vec{x}_i)^2}{2w^2} \right\}$$

- Fluctuations are modelled with γ, drawn from a gamma distribution $\Gamma[\mu = 1, \sigma_{\text{fluct}}]$
- Fluctuations are chosen such that their correlator is η-dependent:

$$\rho(\gamma(\eta_1), \gamma(\eta_2)) = e^{-|\eta_1 - \eta_2|/\eta_{\text{corr}}}$$

- For $g(x,\eta)$ we choose a log-normal distribution with moments dependent on the thickness function:

$$\mu = \mu_{\eta,0} \ln \frac{\tilde{T}_A(x)}{\tilde{T}_B(x)}$$

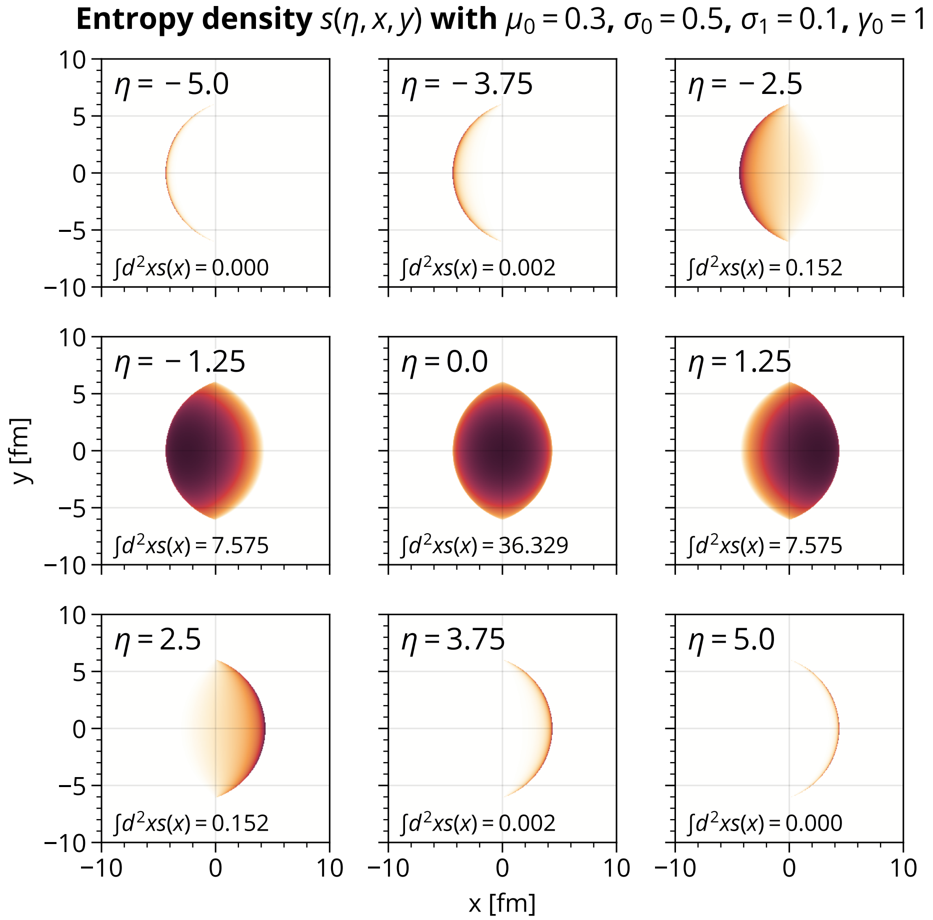
$$\sigma = \sigma_{\eta,0} + \sigma_{\eta,1} \sqrt{\tilde{T}_A(x) \tilde{T}_B(x)}$$

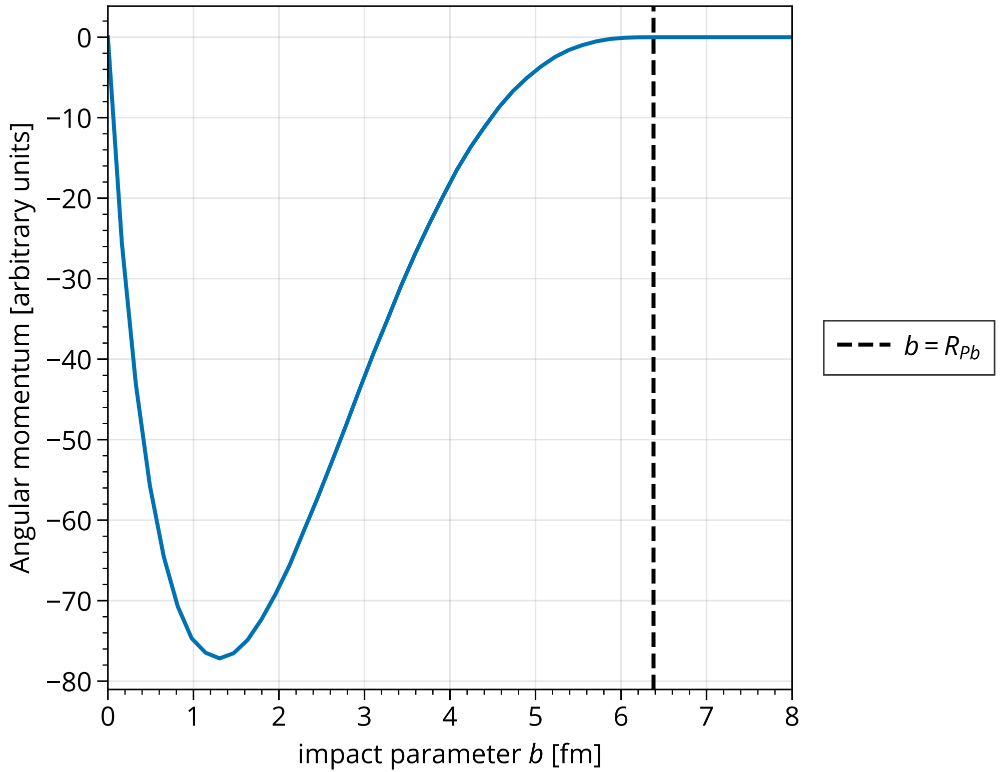
$$\gamma = \gamma_{\eta,0} \frac{\tilde{T}_A(x) - \tilde{T}_B(x)}{\tilde{T}_A(x) + \tilde{T}_B(x)}$$

- $\tilde{T}$ are the thickness functions marginalised over η:

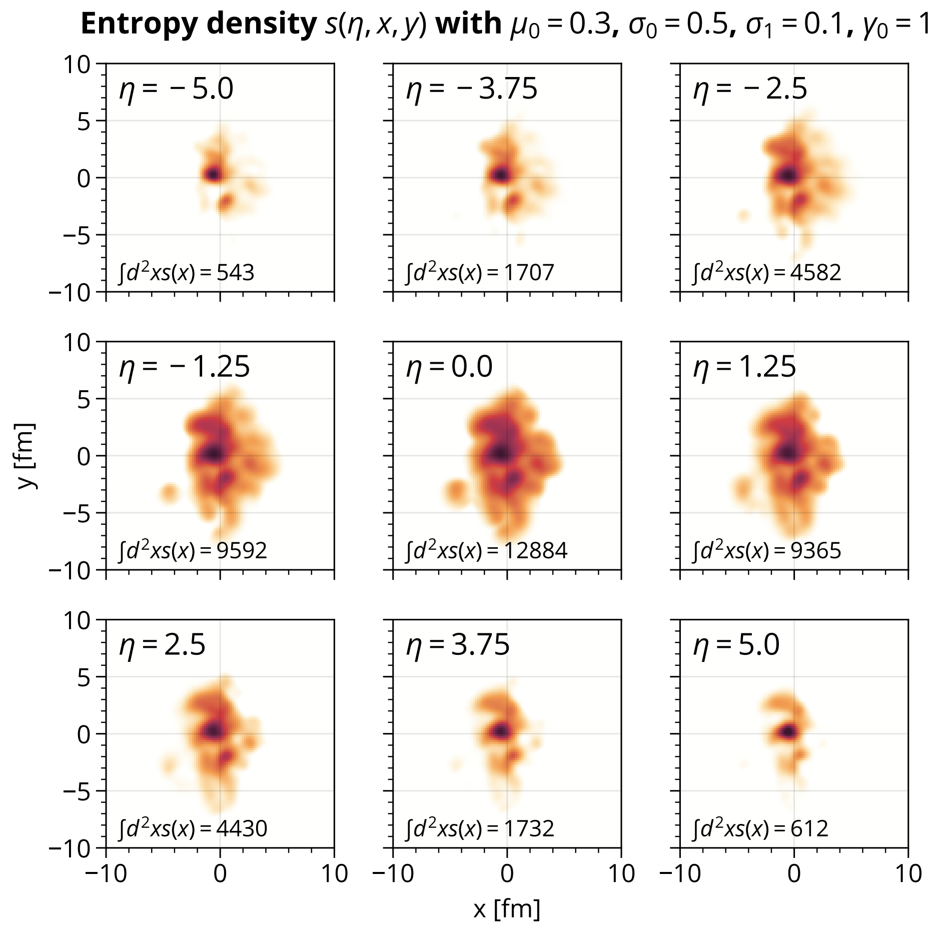
$$\tilde{T}_A(x) \propto \int d\eta T_A(x,\eta) e^{-\eta^2/2\sigma_s}$$

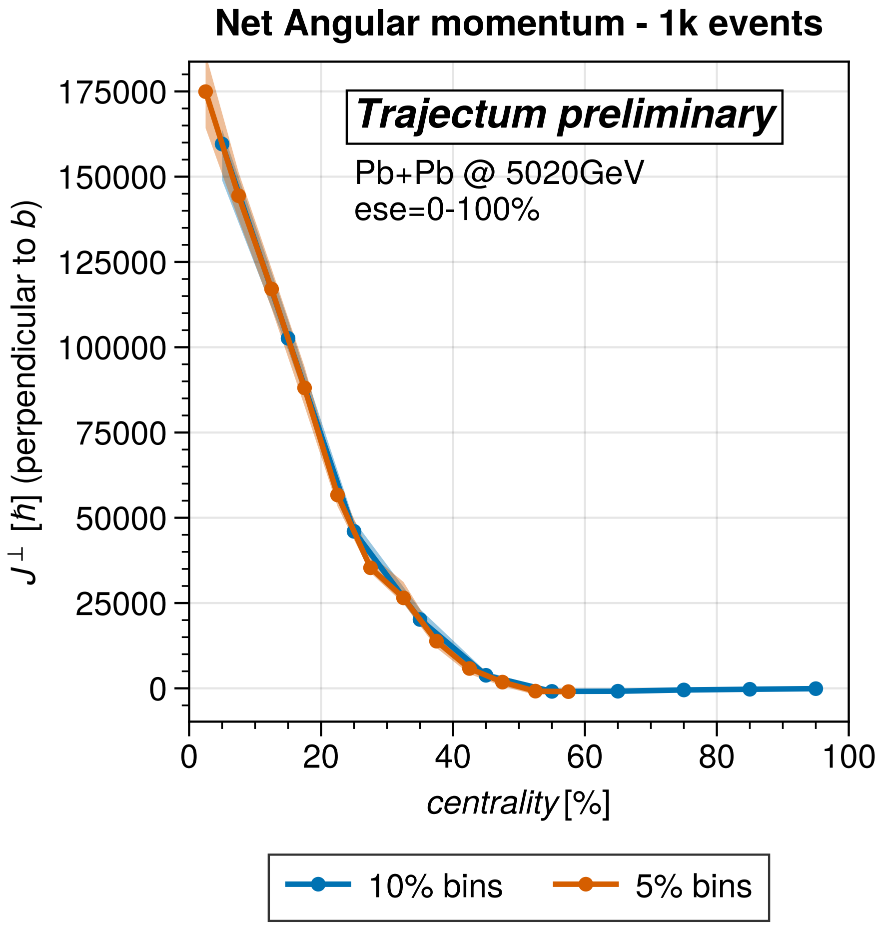
Hard-shell thickness functions



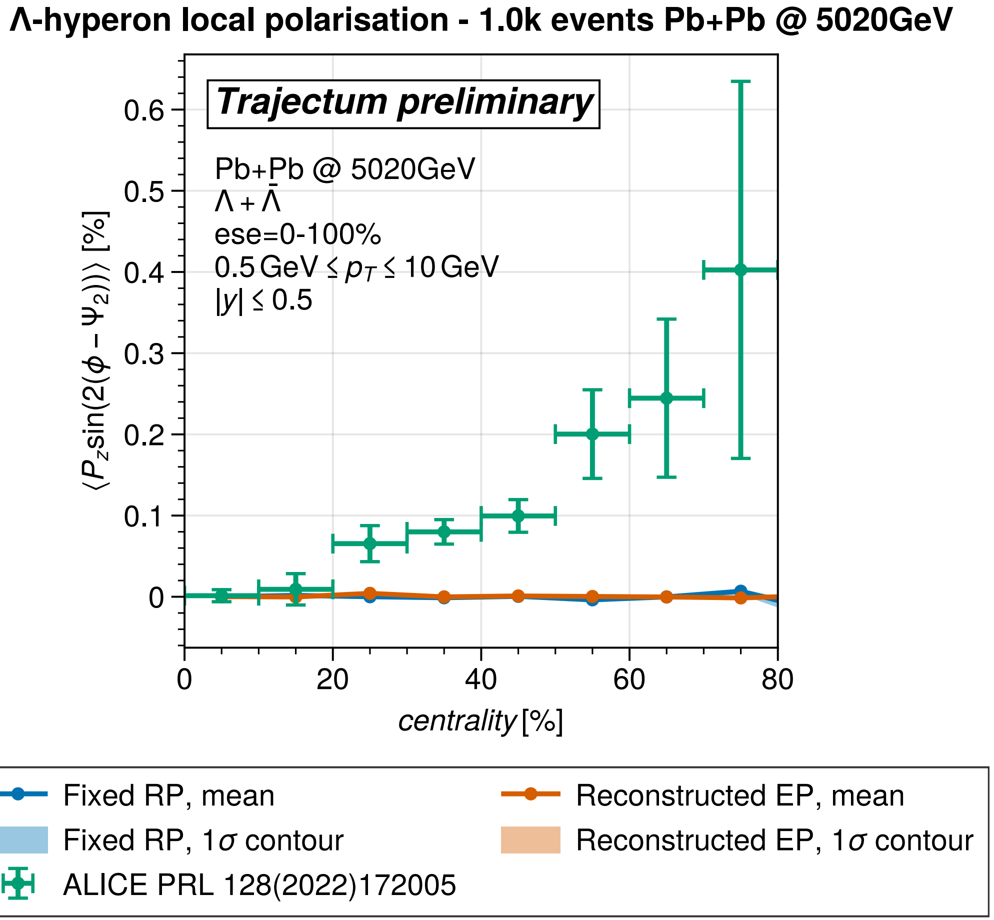


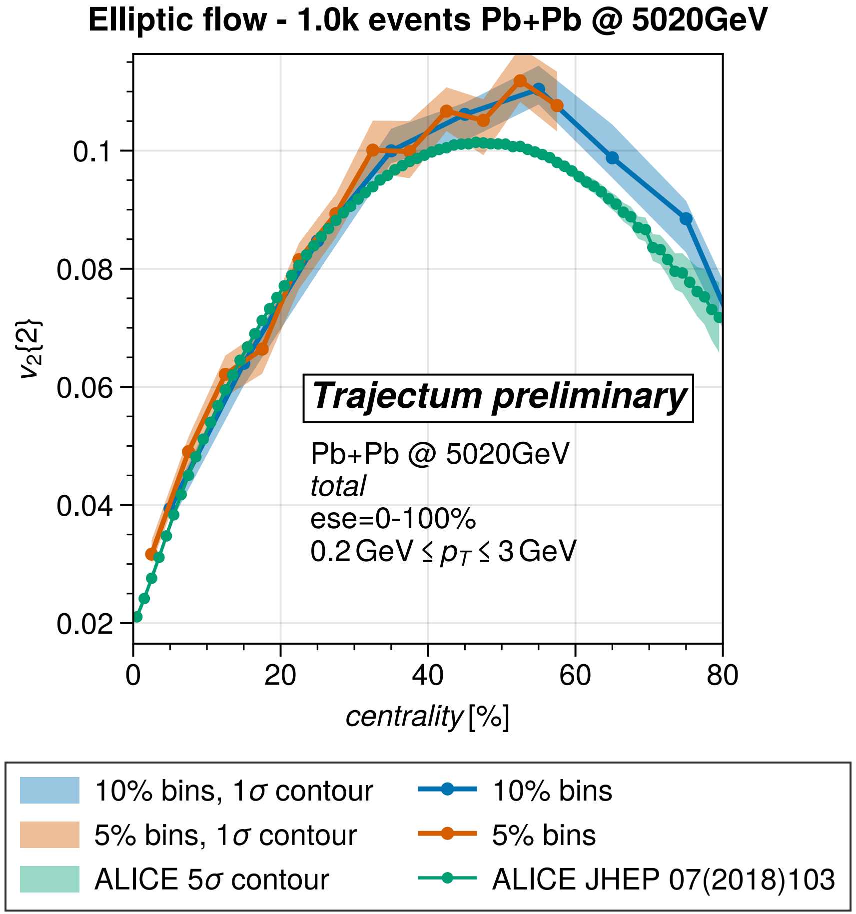
Realistic thickness functions



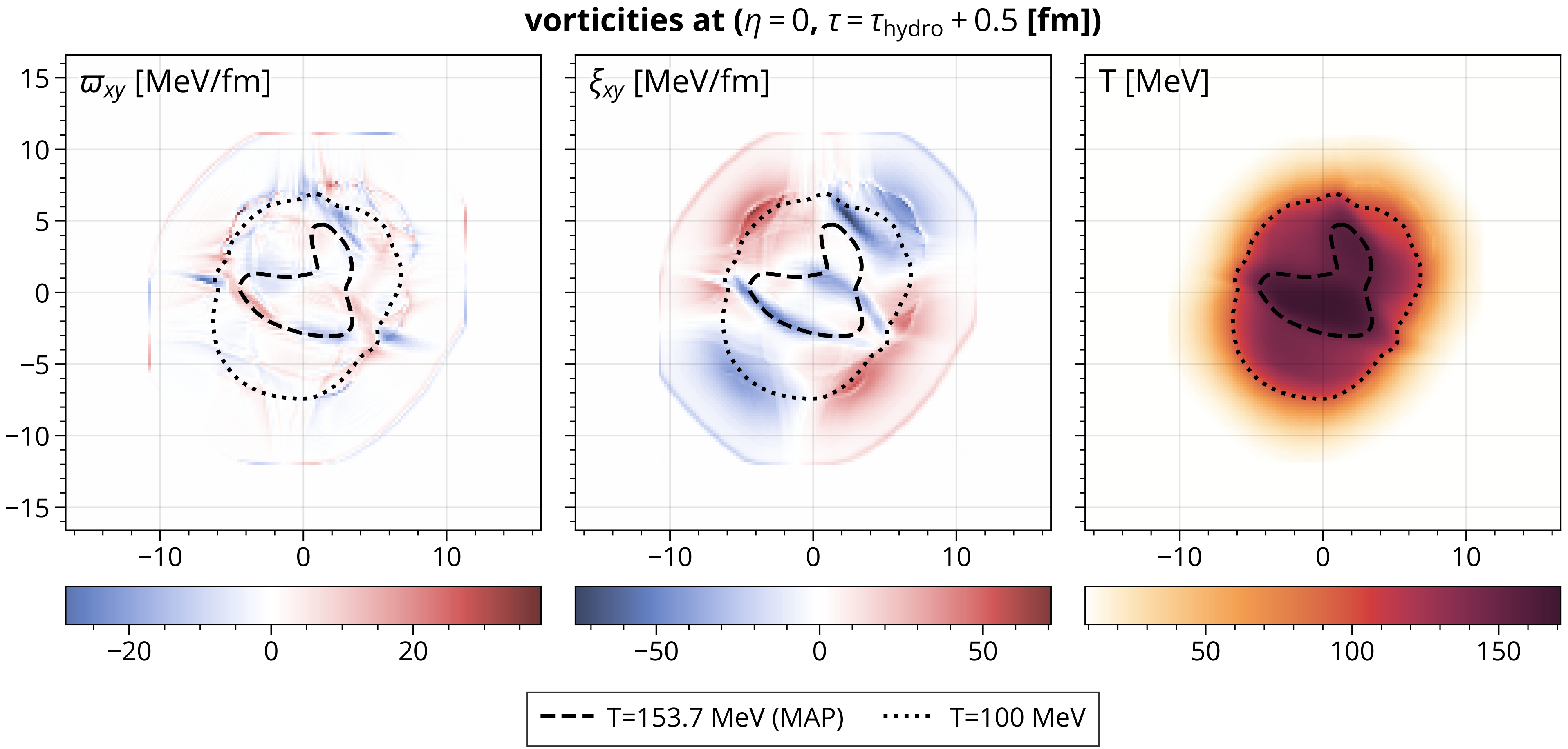


Status quo (WIP) – Local Polarisation @5.02 TeV





Vorticity



vorticities at ($\eta = 0$, $\tau = \tau_{\mathrm{hydro}} + 0.5$ [fm])
ϖ_{xy} [MeV/fm]
ξ_{xy} [MeV/fm]
T [MeV]
15
10
5
0
−5
−10
−15
−10
0
10
−20
0
20
−50
0
50
50
100
150
T=153.7 MeV (MAP)
T=100 MeV

Outlook

Outlook – Work in Progress

Initial Conditions:

- Get effect of fluctuations under control
- Another round of bughunting (vorticity generation, observables etc)

Bayesian fits:

Collaboration with GSI heavy-ion group: Andrea Dubla, Ilya Selyuzhenkov, Alp Tibet Sayıklı, Marwah Zaki

- Do 3+1D Bayesian fit at LHC energies ($\sqrt{s_{\mathrm{NN}}} = 5.02\ \mathrm{TeV}$ and $2.67\ \mathrm{TeV}$)
- Do 3+1D Bayesian fit for STAR $\mathrm{Au} + \mathrm{Au}$ at $\sqrt{s_{\mathrm{NN}}} = 200\ \mathrm{GeV}$

$\Rightarrow$ Maximum A-Posteriori (MAP) postdictions for polarisations

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$\Rightarrow$ Maximum A-Posteriori (MAP) postdictions for polarisations

Outlook:

- Looking forward to the results of the postdiciton:
  - A postdiction that fits the data well indicates that spin-orbit coupling effects (no dynamical spin DOFs) is the leading contribution for the hyperon polarisation
  - ... a poor fit on would *hint* at the necessity of something beyond Becattini's formula
- Future directions for trajectum:
  - vector-meson alignment
  - dynamical spin-hydro
  - spin in the initial conditions, eg.: [17]

[17]: Giacalone, Speranza (*preprint*), `arXiv:2502.13102`

In Memoriam

Universiteit Utrecht

prof. dr. Umut Gürsoy

September 20th 1975 – April 24th 2025

Umut has been my PhD supervisor during the last two years, contributing to this project (amongst other projects).

He tragically and unexpectedly passed away earlier this year at the age of 49 while visiting IAS in Princeton.

After obtaining his undergrad in Turkey, Umut started his career with a PhD at MIT under supervision of Daniel Friedman. Afterwards, he held postdoctoral positions in Paris, Utrecht and at CERN before finally settling on a permanent position in Utrecht. He was promoted to full professor (Hoogleraar) in Utrecht just last March.

His most well known contributions were in holographic QCD, his computation of Δv_1 from electromagnetic fields and more recent efforts in deriving the gauge-string duality from microscopic considerations.

Universiteit Utrecht

Sharing science,
shaping tomorrow

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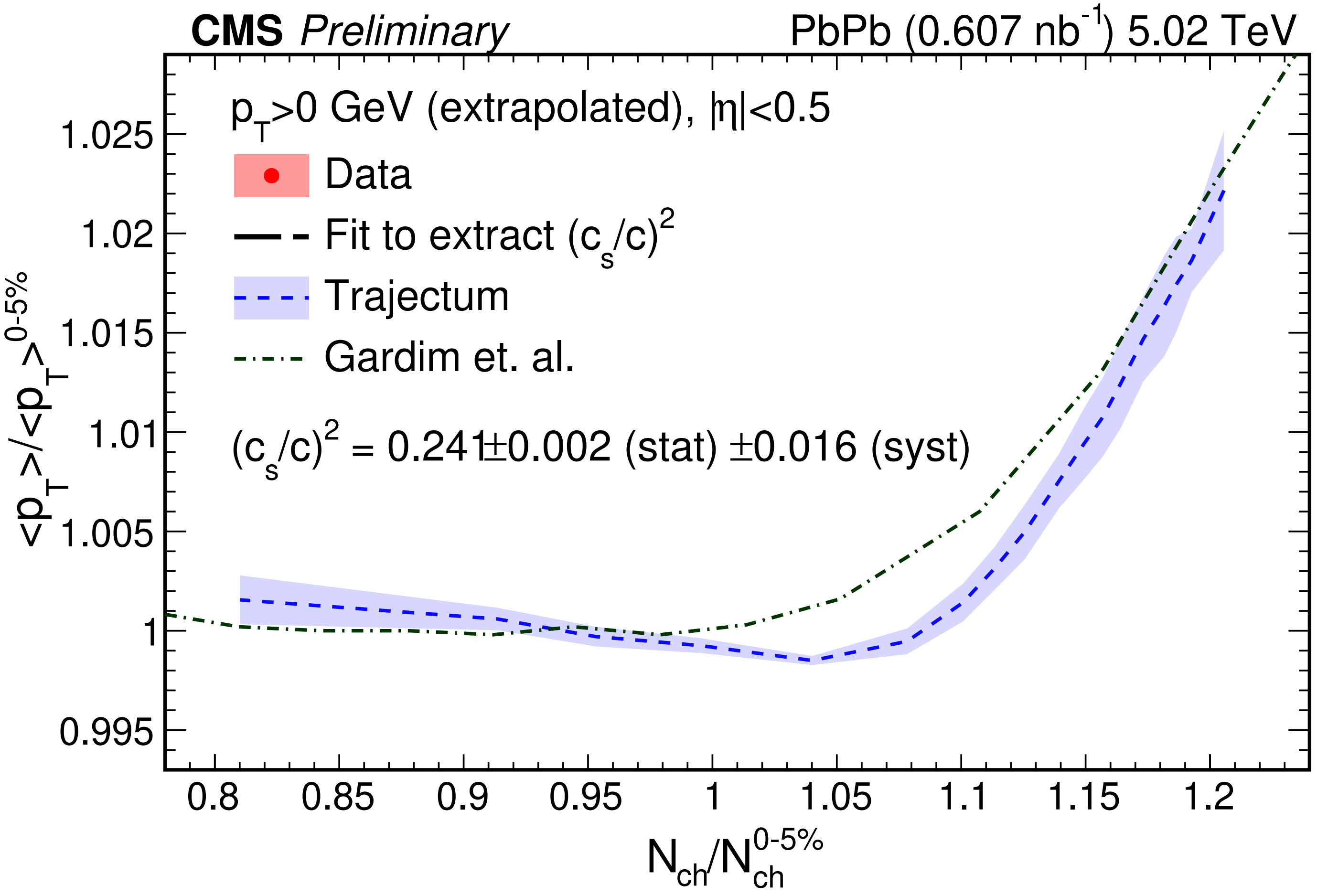
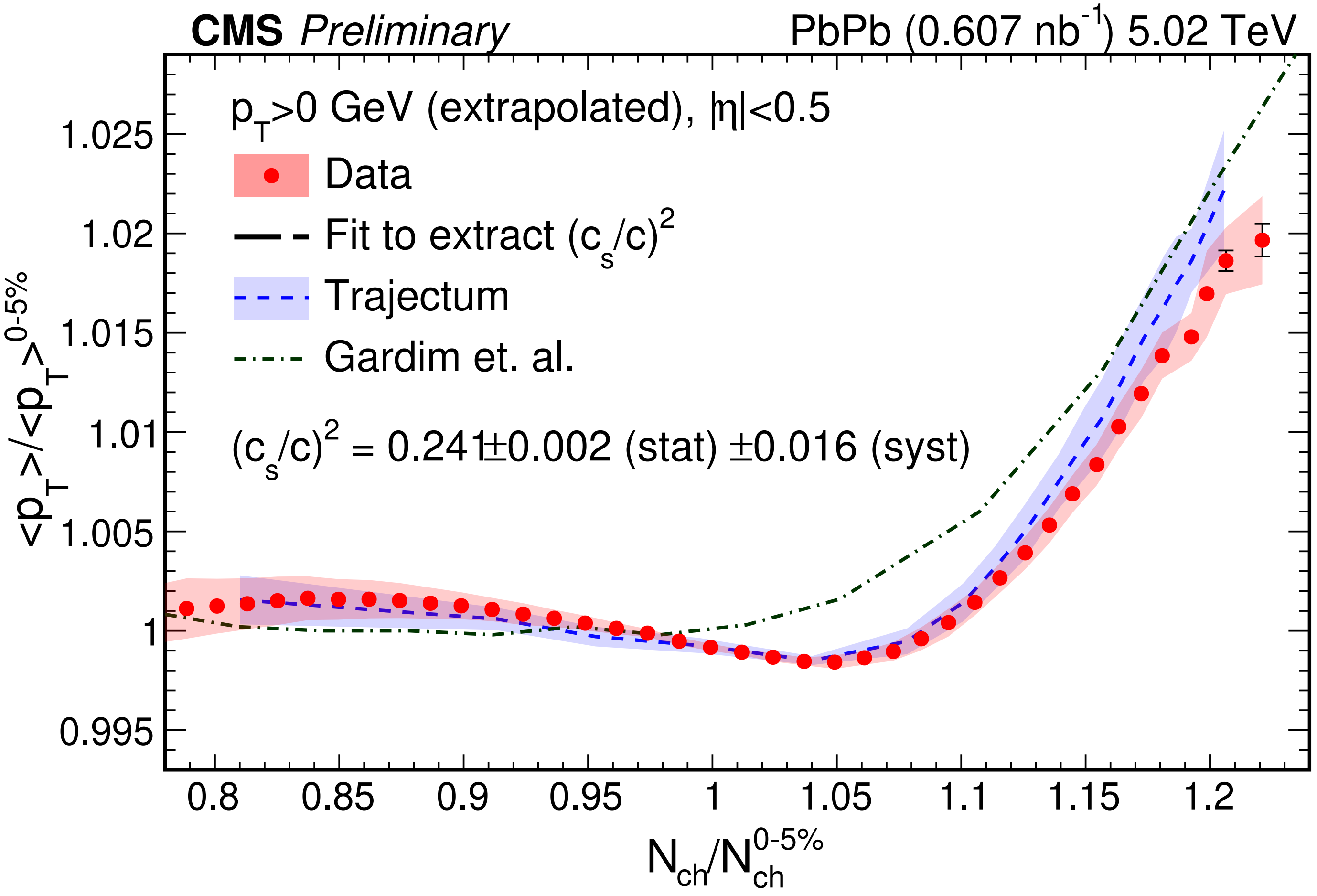
Bayesian prediction: c_s



Bayesian prediction: c_s



Hydrodynamics

Ideal Hydrodynamics

1. EoM is conservation law:

$$\partial_\mu T^{\mu\nu} = 0$$

2. 4 equations for 4 hydro variables and EoS $P(\varepsilon)$:

$$T^{\mu\nu}_{\text{ideal}} = \varepsilon u^\mu u^\nu - \Delta^{\mu\nu} P$$

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Viscous Hydrodynamics (Müller-Israel-Stewart theory)

1. Add viscous terms (expansion in derivatives of DOFs)

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + T^{\mu\nu}_{\text{visc}} = T^{\mu\nu}_{\text{ideal}} - \Pi\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$\Pi = -\zeta\nabla_\alpha u^\alpha + \dots$$

$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle} + \dots \quad \langle\dots\rangle := \text{TT projection}$$

2. Promote viscosities Π and $\pi^{\mu\nu}$ to DOFs that relax to their value from the gradient expansion ($\mathsf{D} := u^\mu\partial_\mu$):

$$\tau_\Pi \mathsf{D}\Pi = -\left[\Pi + \Delta_{\mu\nu}T^{\mu\nu}_{\text{visc}}\right]$$

$$\tau_\pi \Delta^\mu{}_\alpha \Delta^\nu{}_\beta \mathsf{D}\pi^{\alpha\beta} = -\left[\pi^{\mu\nu} - T^{\langle\mu\nu\rangle}_{\text{visc}}\right]$$