The axion-photon coupling

from lattice QCD

B. B. Brandt, G. Endrődi, J. J. H. Hernández*, G. Markó and L. Pannullo

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Outline

$$g_{A\gamma\gamma}^{\text{QCD}} = -1.72(7)\frac{\alpha}{2\pi f_A}$$

Preliminary

- Strong CP problem and axions
- The axion-photon coupling
- Lattice definitions and results
- Effect on current model bounds

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Why is $\bar{\theta}$ so small? $|\bar{\theta}| \lesssim 10^{-10}$

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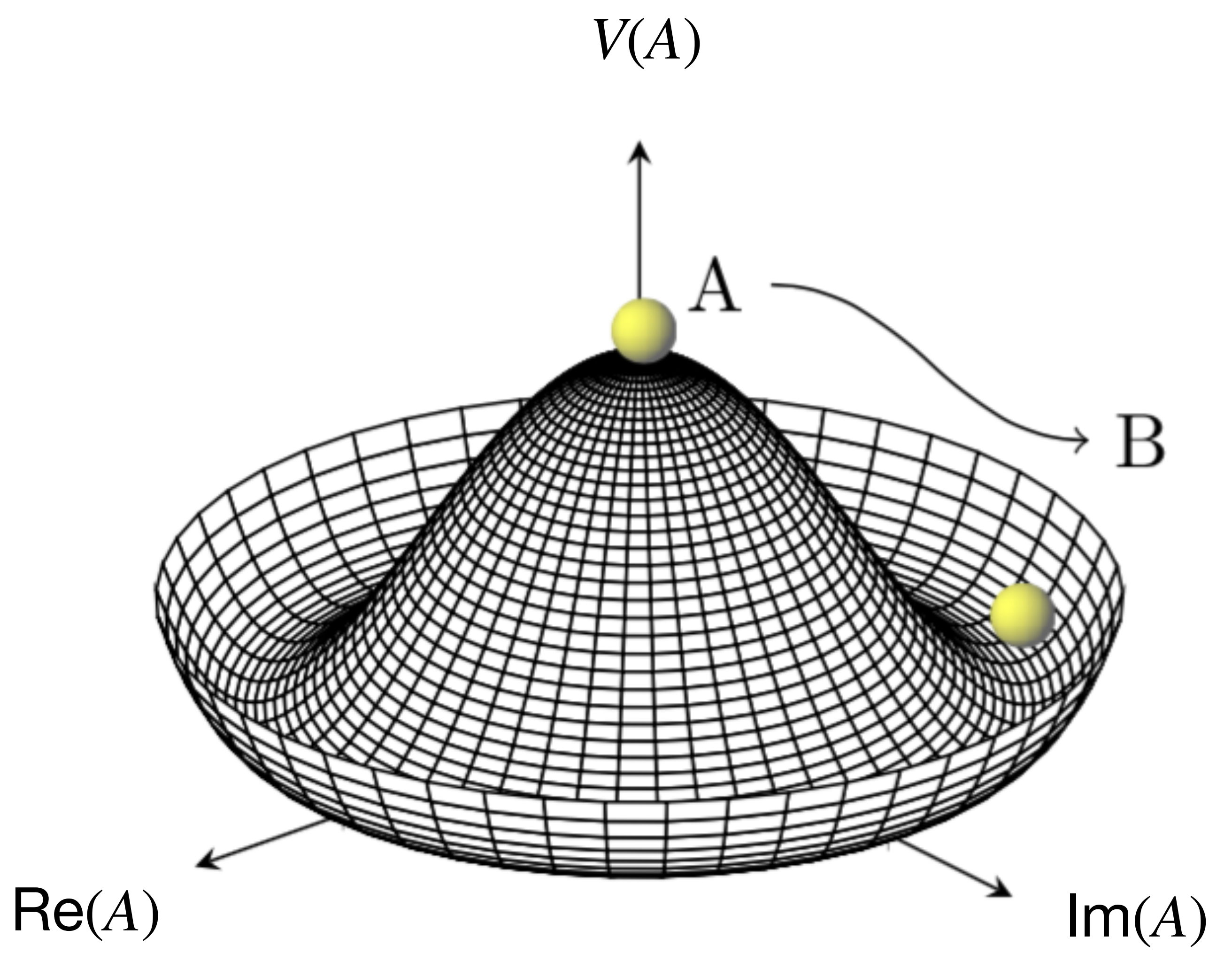

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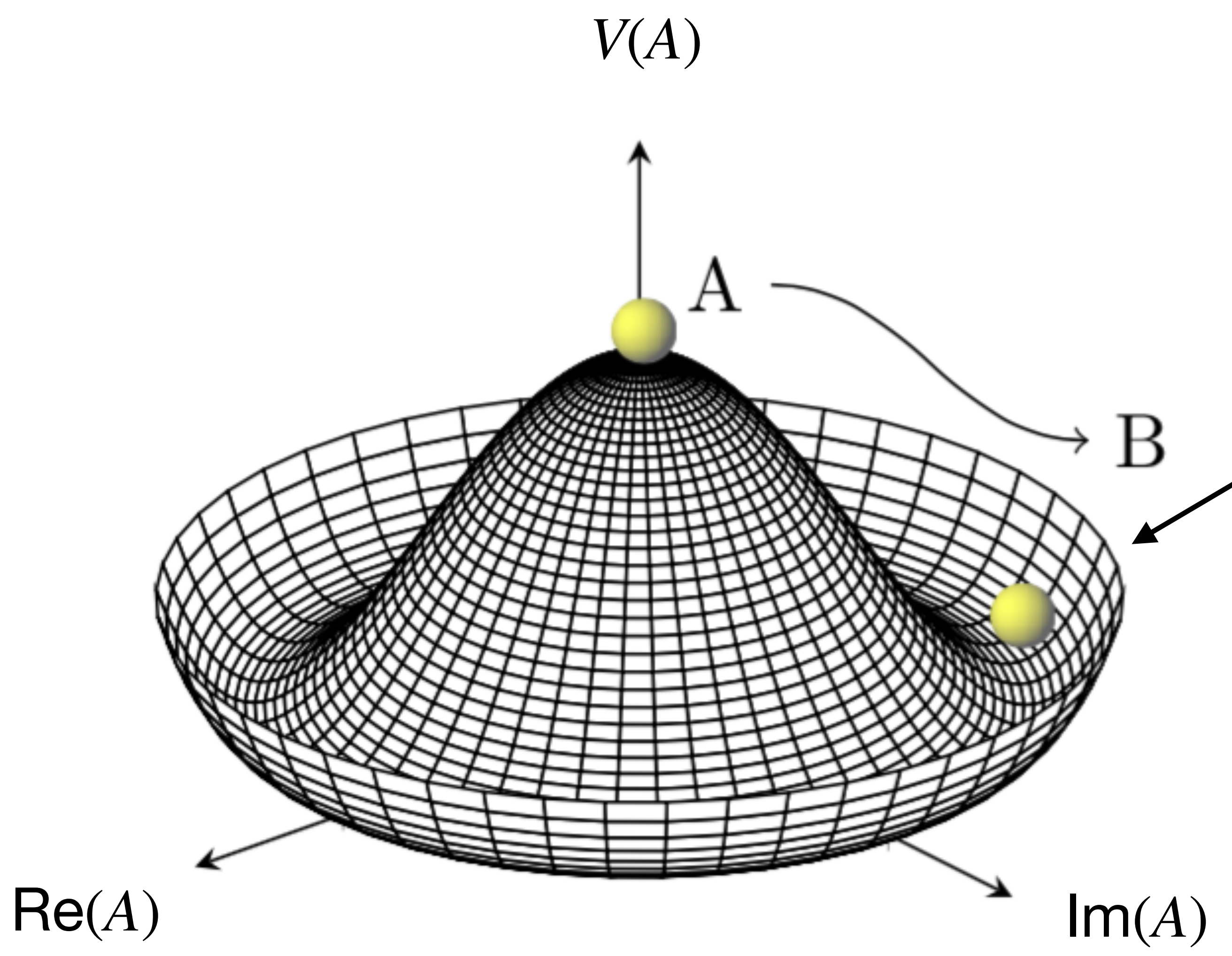

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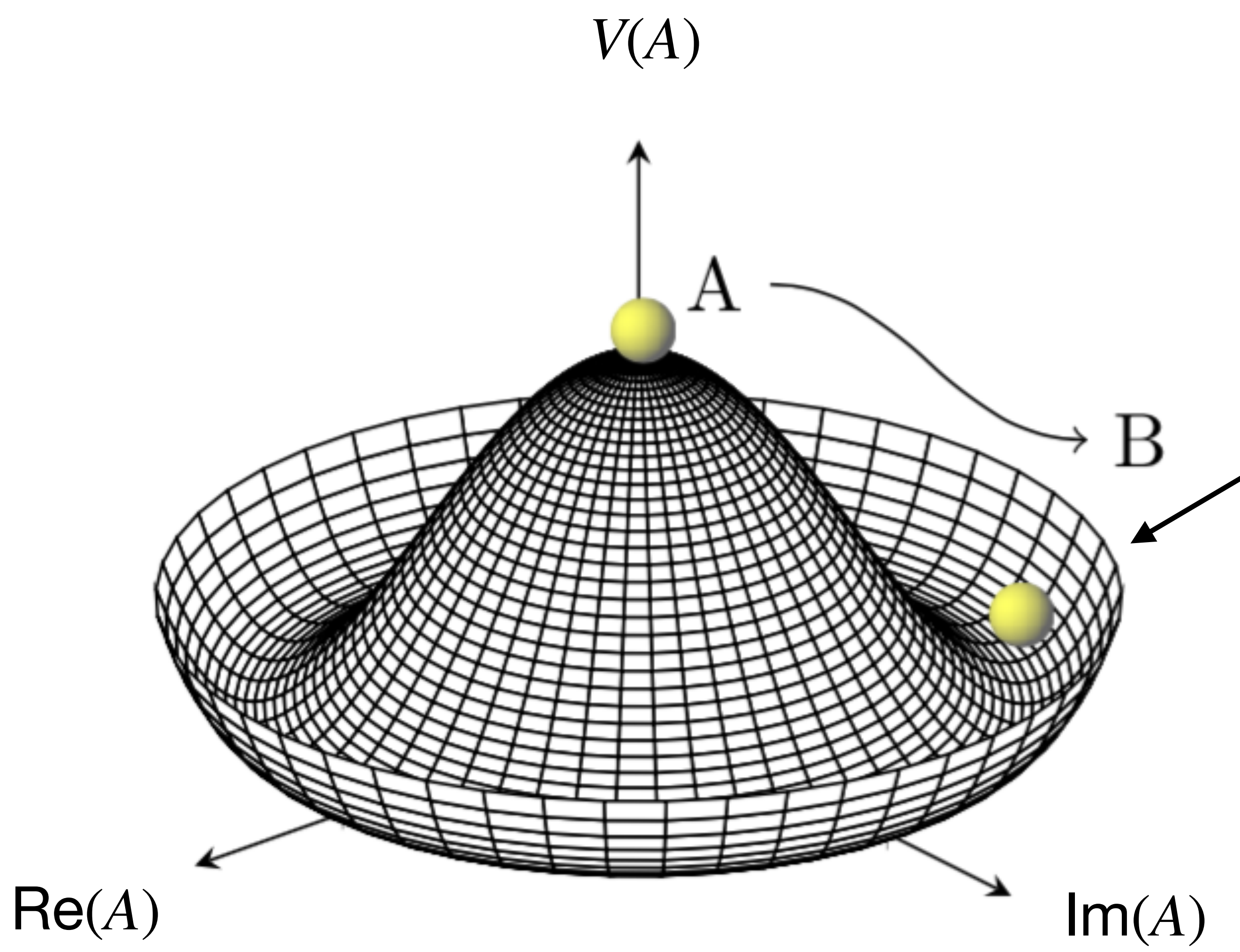

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We know its mass : $m_A = \frac{\sqrt{\chi_{\text{top}}}}{f_A} = 5.691(51)\left(\frac{10^9\,\text{GeV}}{f_A}\right)$ meV [8]

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8. Gorghetto, Marco, and Giovanni Villadoro. "Topological susceptibility and QCD axion mass: QED and NNLO corrections." *Journal of High Energy Physics* 2019.3 (2019).

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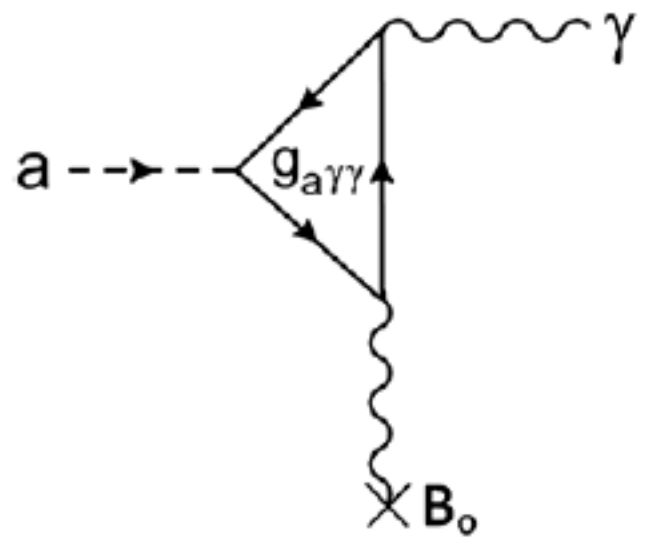

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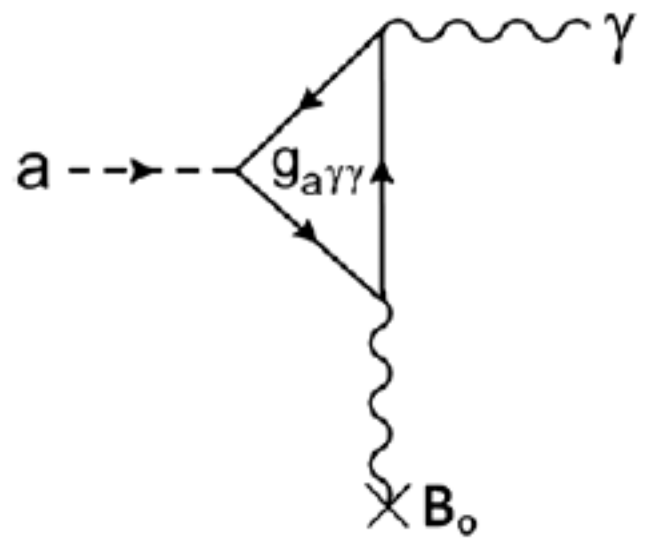

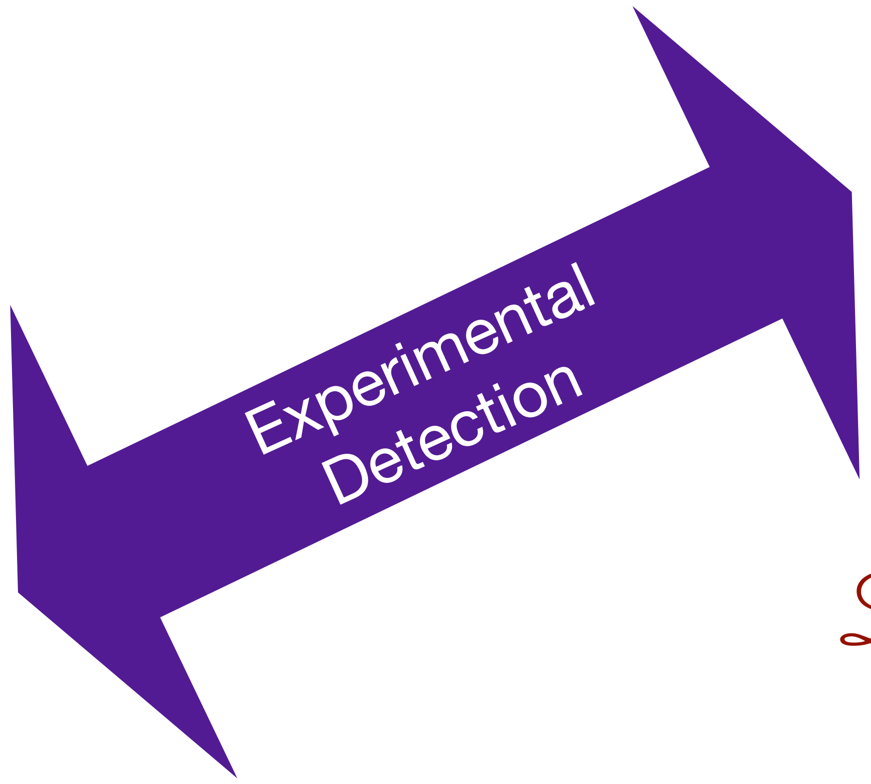

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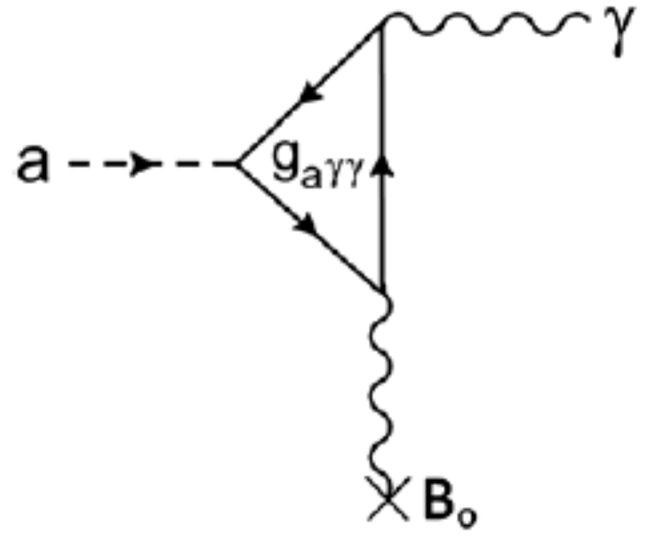

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$$g_{A\gamma\gamma} = g_{A\gamma\gamma}^{\text{model}} + g_{A\gamma\gamma}^{\text{QCD}} = \frac{\alpha}{2\pi f_A}\left(\frac{E}{N} - 1.92(4)\right)$$ [10]

Chiral Perturbation Theory

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10. Di Cortona, Giovanni Grilli, et al. "The QCD axion, precisely." *Journal of High Energy Physics* 2016.1 (2016): 1-37.

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Previous work showed that the other correlators are noisier [11,12]

11. Brandt, Bastian, et al. "QCD topology with electromagnetic fields and the axion-photon coupling." *arXiv preprint arXiv:2212.03385* (2022).
12. Brandt, Bastian B., et al. "Electromagnetic effects on topological observables in QCD." *arXiv preprint arXiv:2312.14660* (2023).

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By measuring this we can obtain the topological charges!

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$$\longrightarrow$$

$$\frac{N_c}{4\pi^2} q^2 \left(\frac{\langle \bar{\psi}\gamma_5\psi \rangle_{EB}}{\langle \bar{\psi}\gamma_5\psi \rangle_0} - 1 \right) = g_{A\gamma\gamma}^{\mathrm{QCD}} f_A + \dots$$

Yielding a purely fermionic expression for the coupling!

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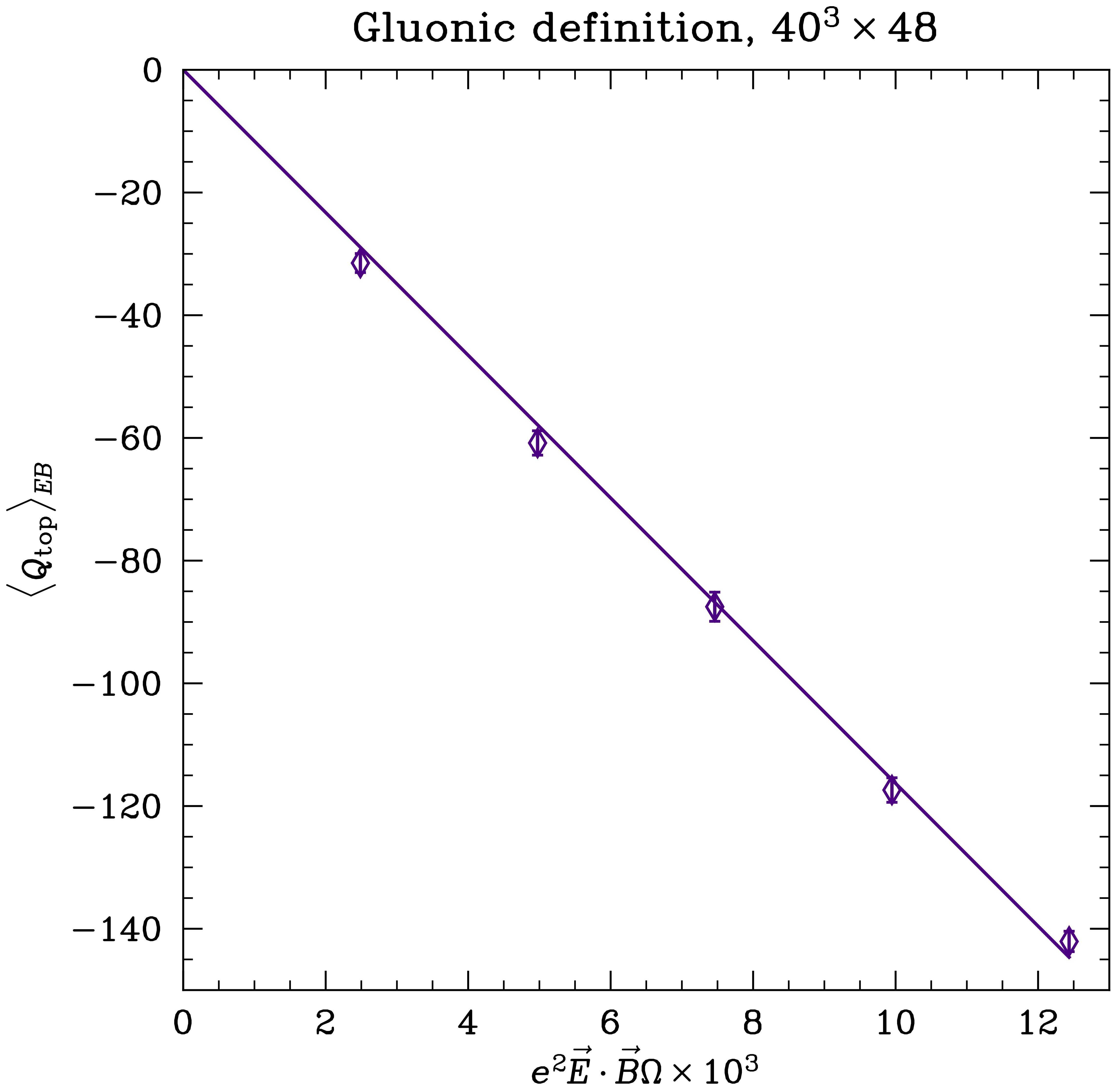
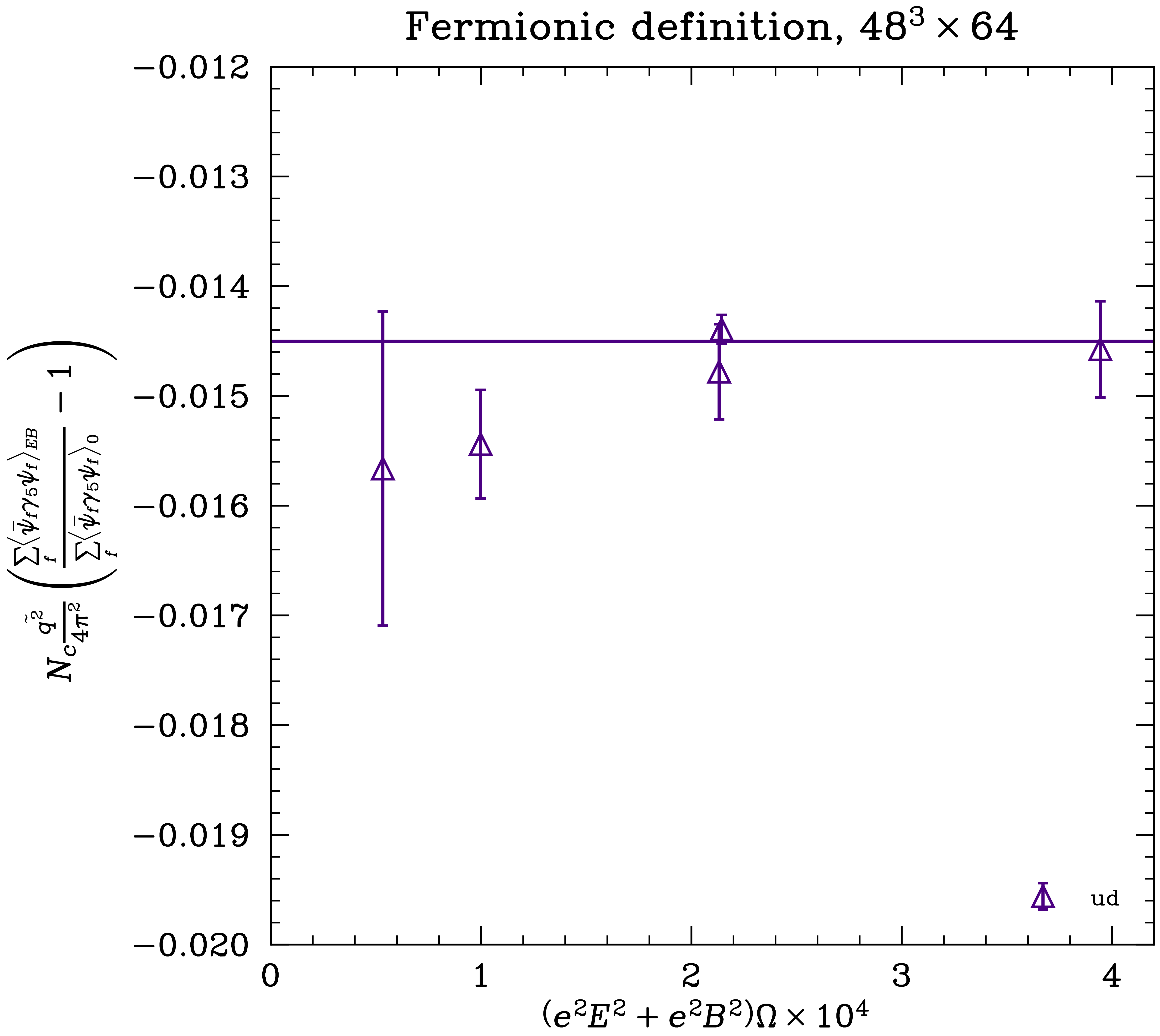
2 + 1 staggered quarks
at the physical point





Lattice results II: continuum limit

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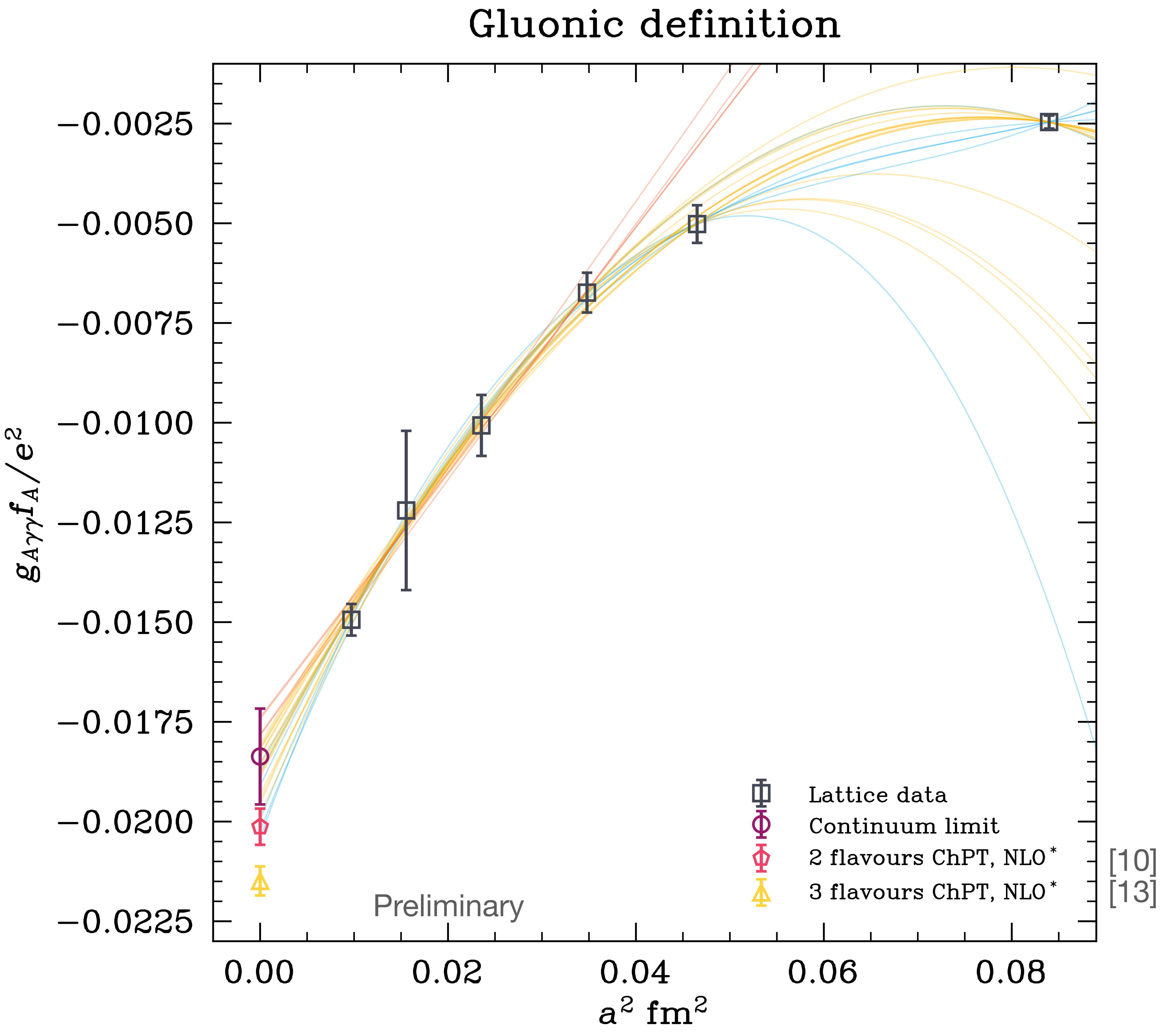

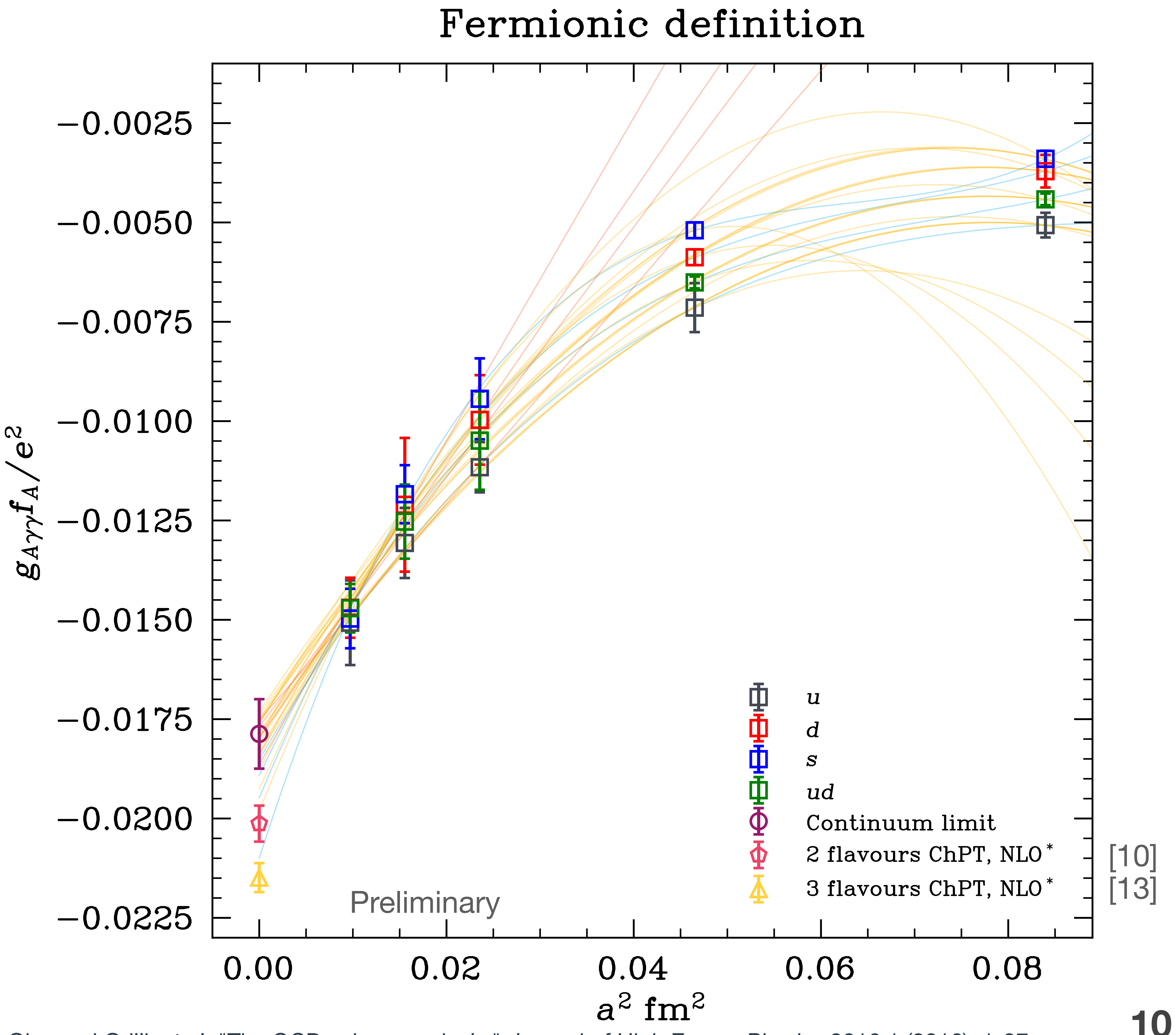

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10. Di Cortona, Giovanni Grilli, et al. "The QCD axion, precisely." *Journal of High Energy Physics* 2016.1 (2016): 1-37.
13. Lu, Zhen-Yan, et al. "QCD θ-vacuum energy and axion properties." *Journal of high energy physics* 2020.5 (2020): 1-23.
14. Borsanyi, Sz, et al. "Lattice QCD for cosmology." *arXiv preprint arXiv:1606.07494* (2016).
15. Gao, Rui, et al. "Axion-meson mixing in light of recent lattice η–η′ simulations and their two-photon couplings within U (3) chiral theory." *Journal of High Energy Physics* 2023.4 (2023): 1-25.

Bounds on the model dependent part I [16]

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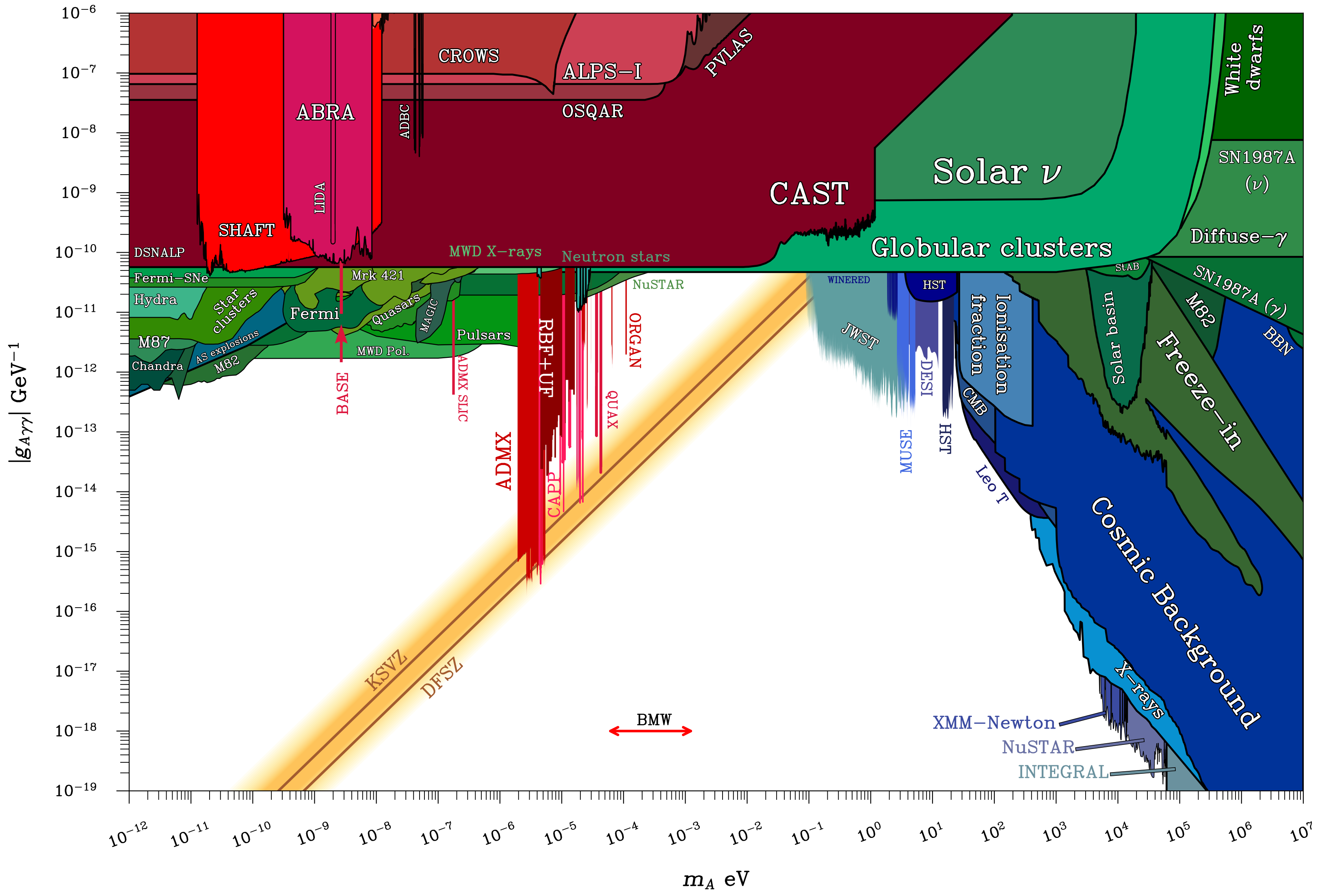

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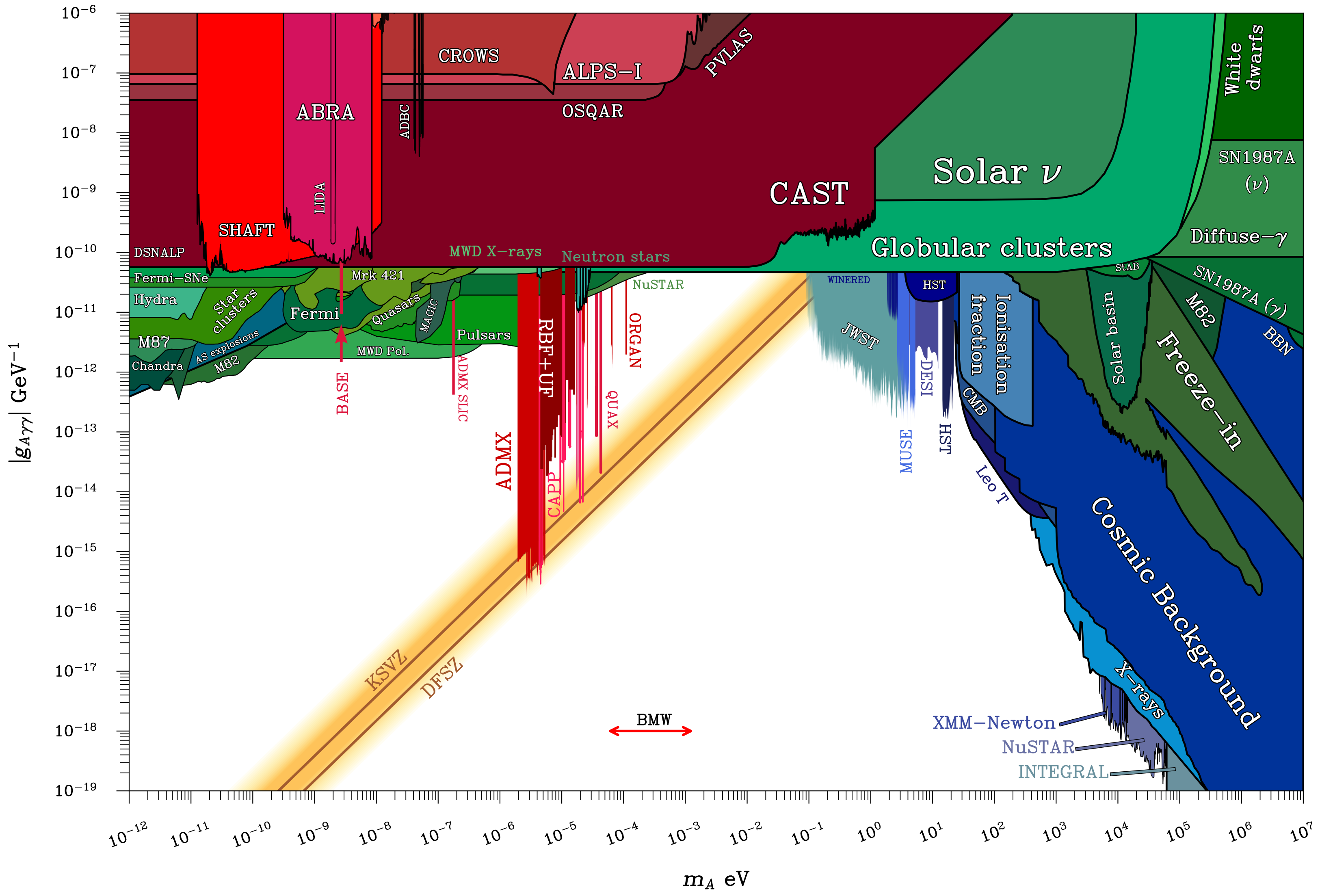

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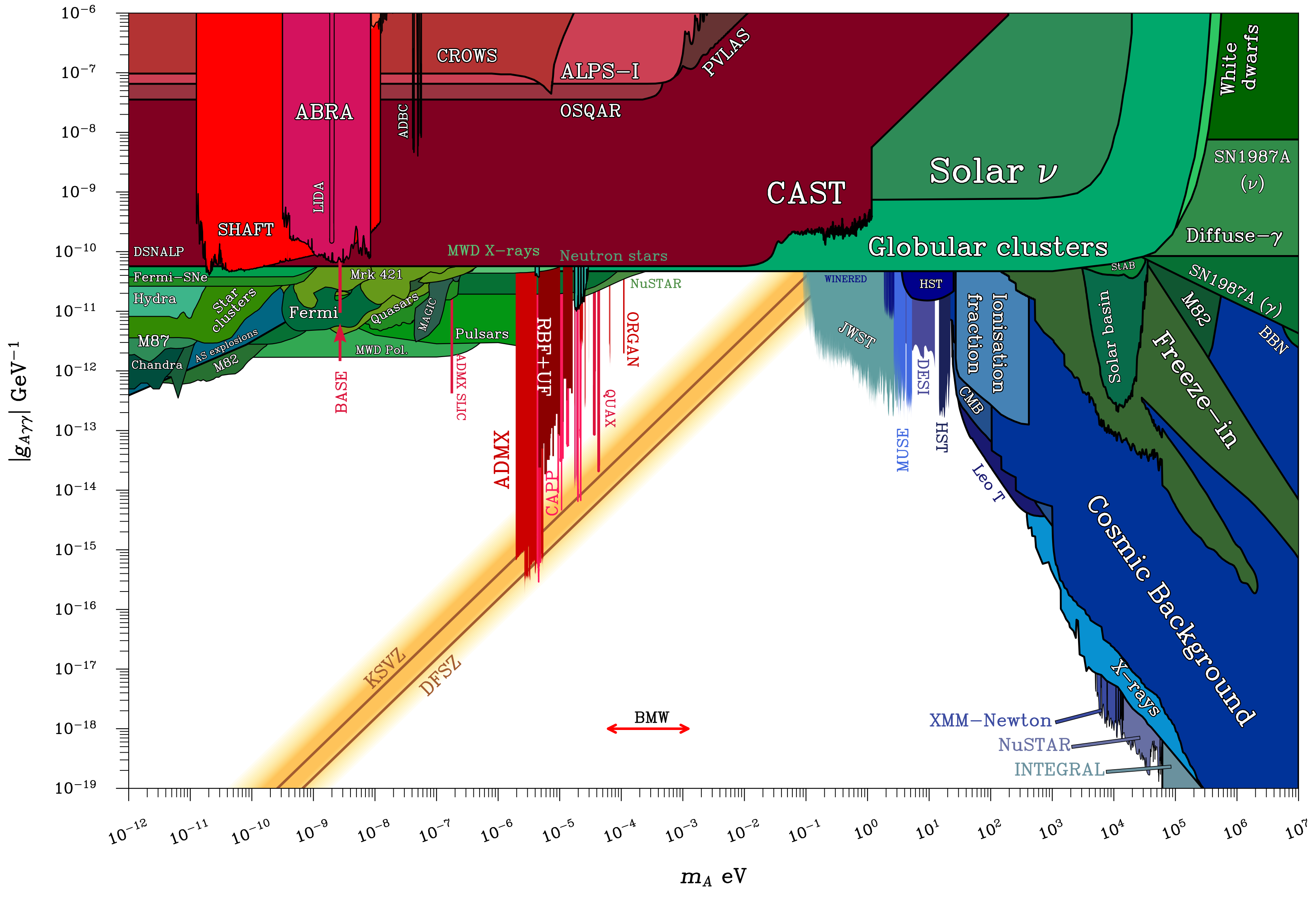

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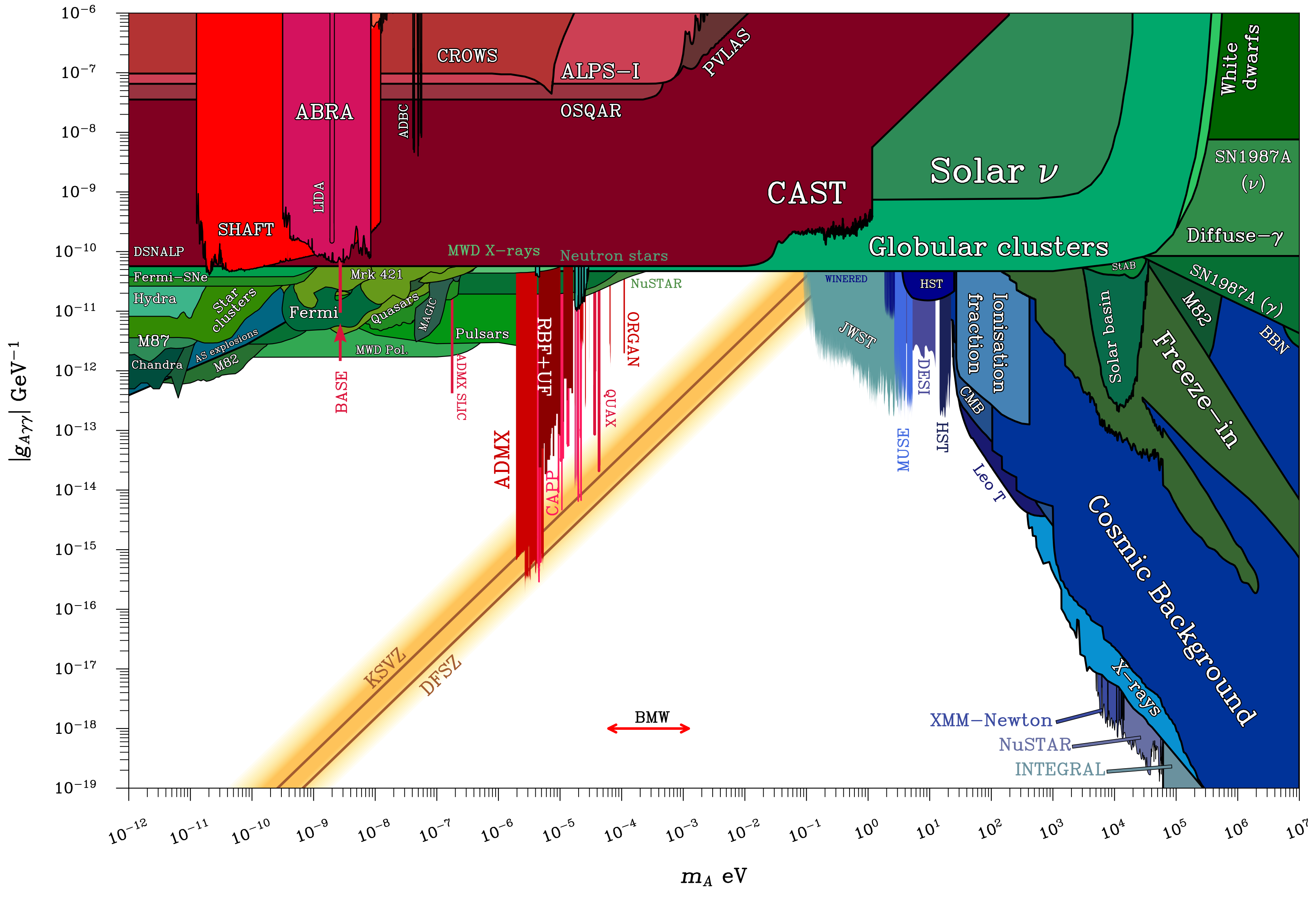

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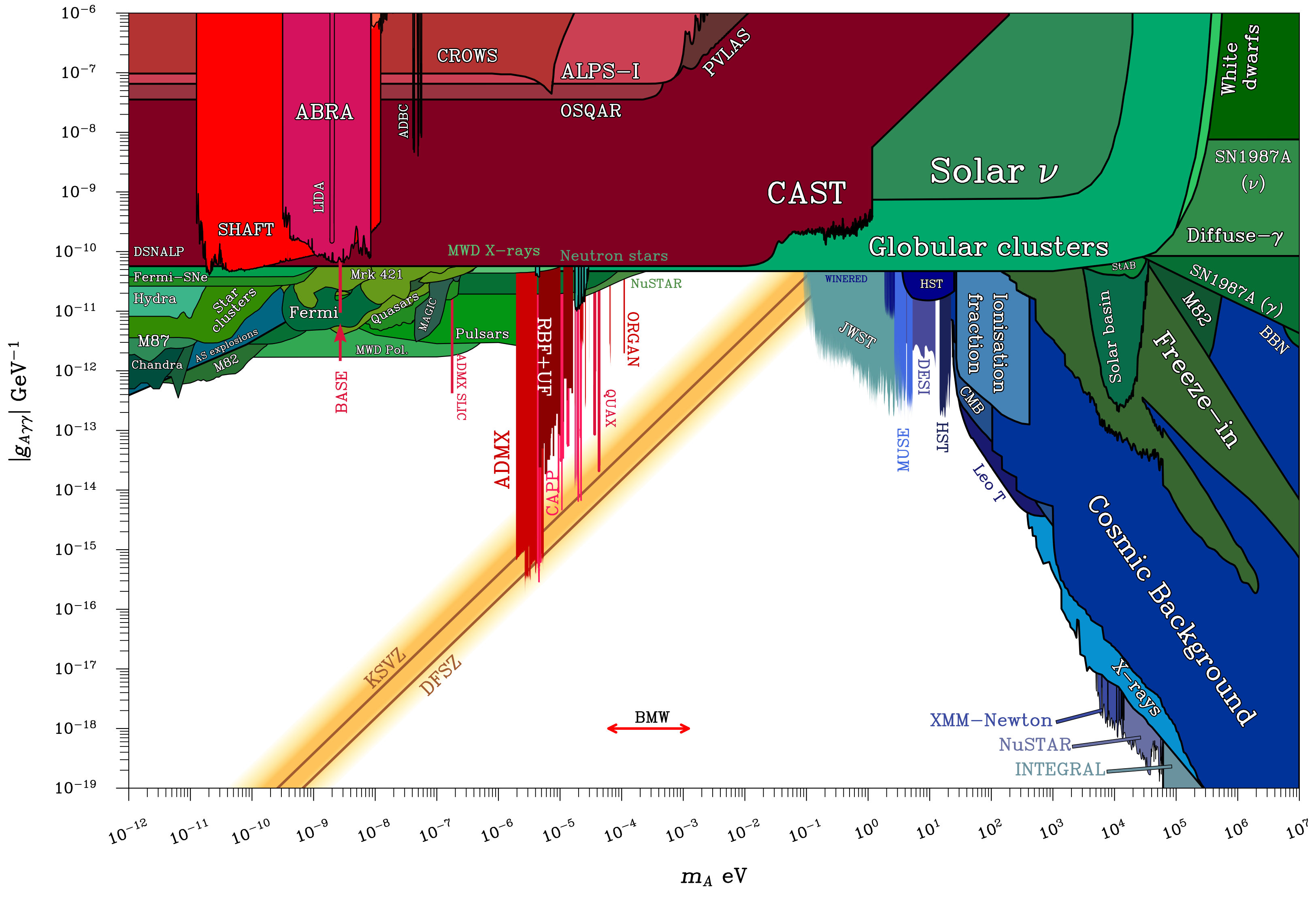

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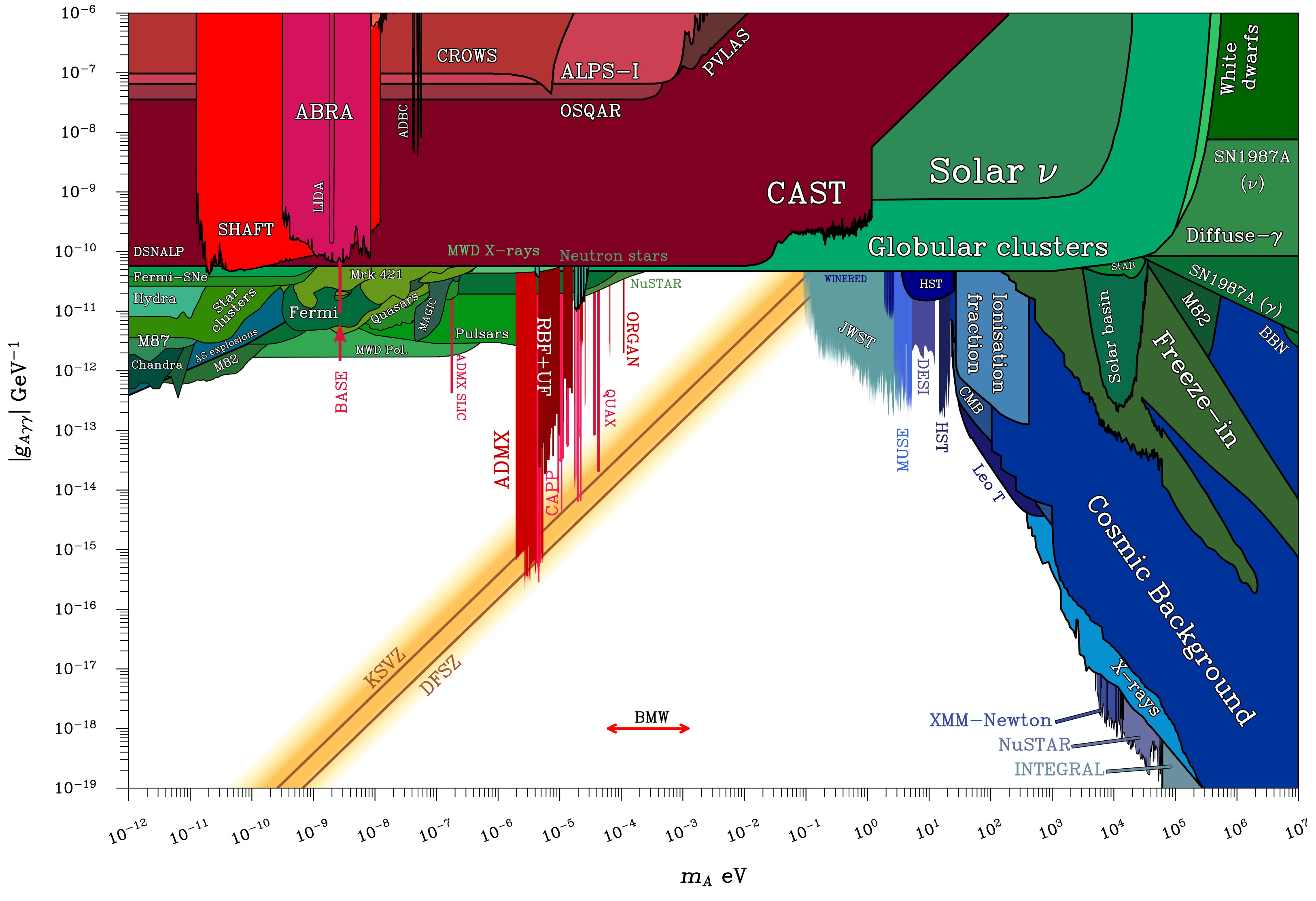

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Bounds on the model dependent part II

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14. Borsanyi, Sz, et al. "Lattice QCD for cosmology." *arXiv preprint arXiv:1606.07494* (2016).
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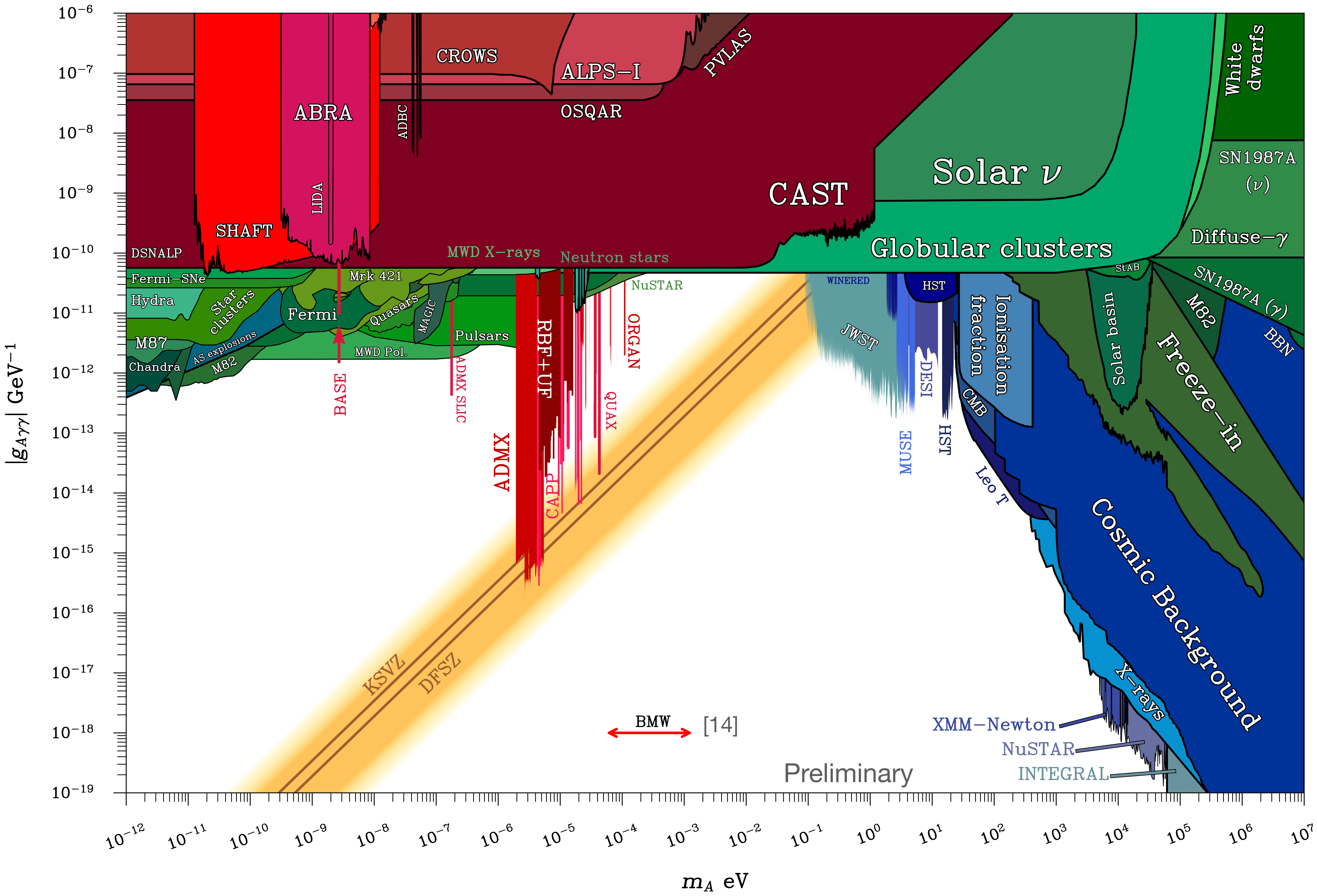

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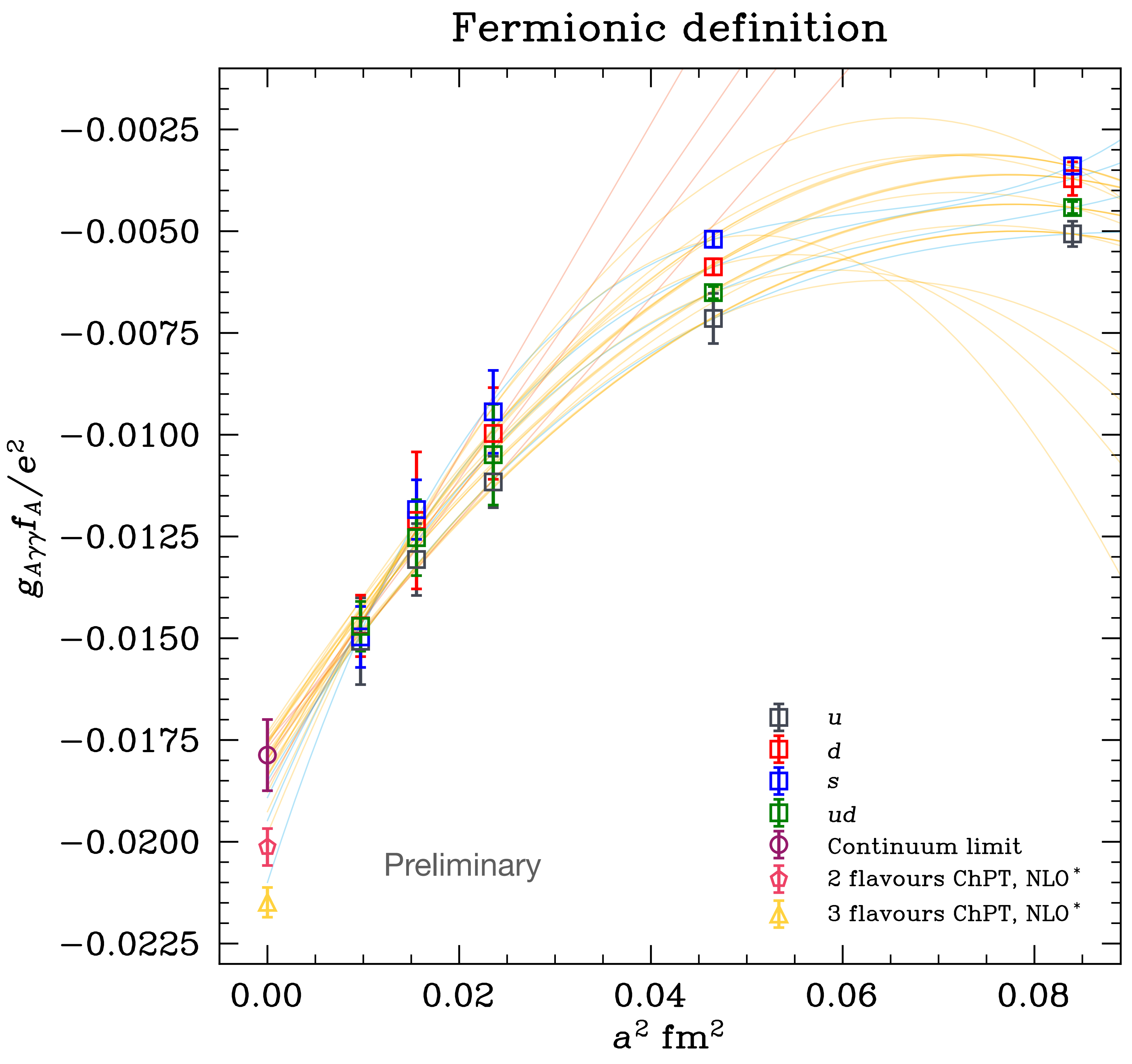

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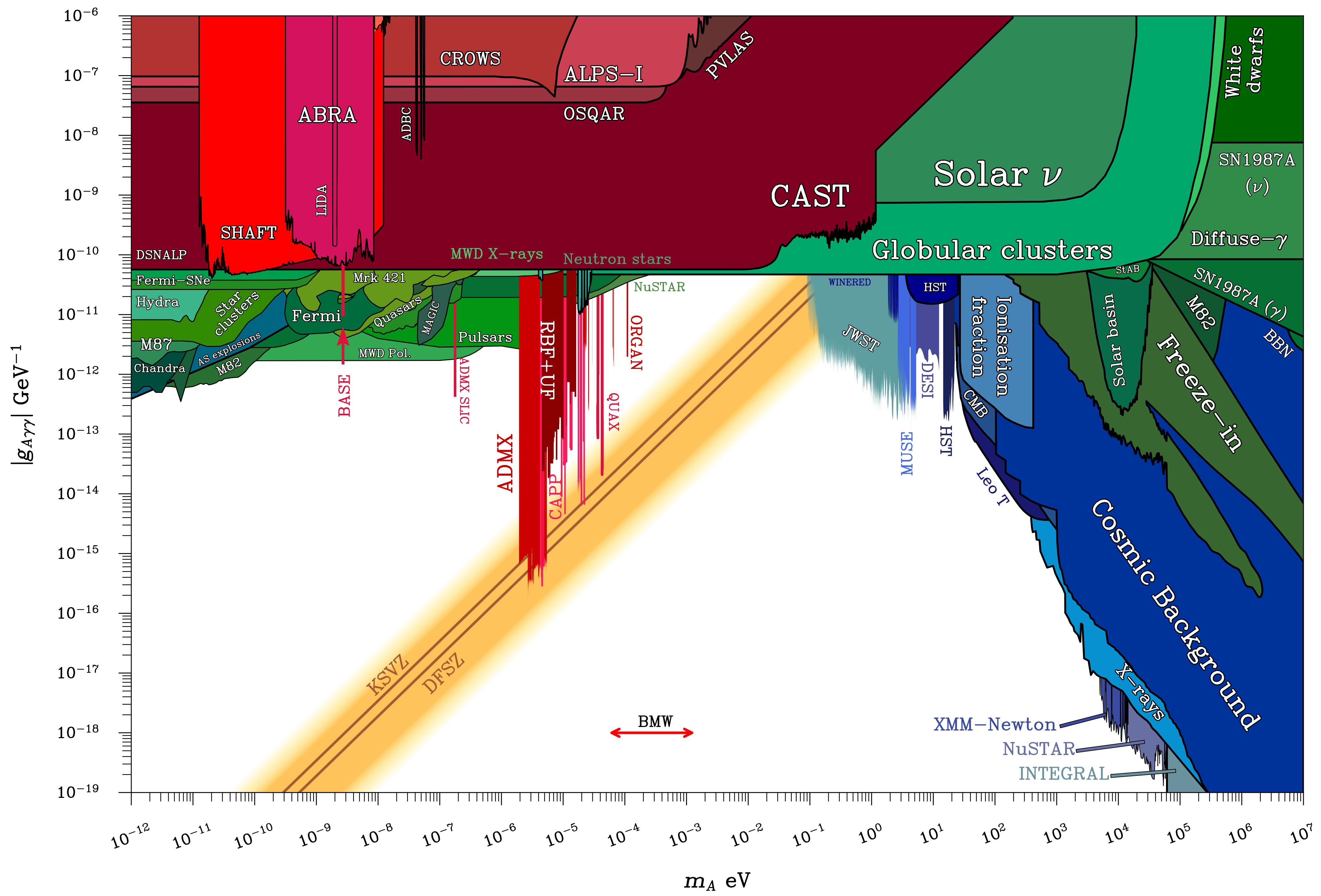

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