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Relaxation time for Λ hyperons coming from rotating Corona region in heavy ion collisions

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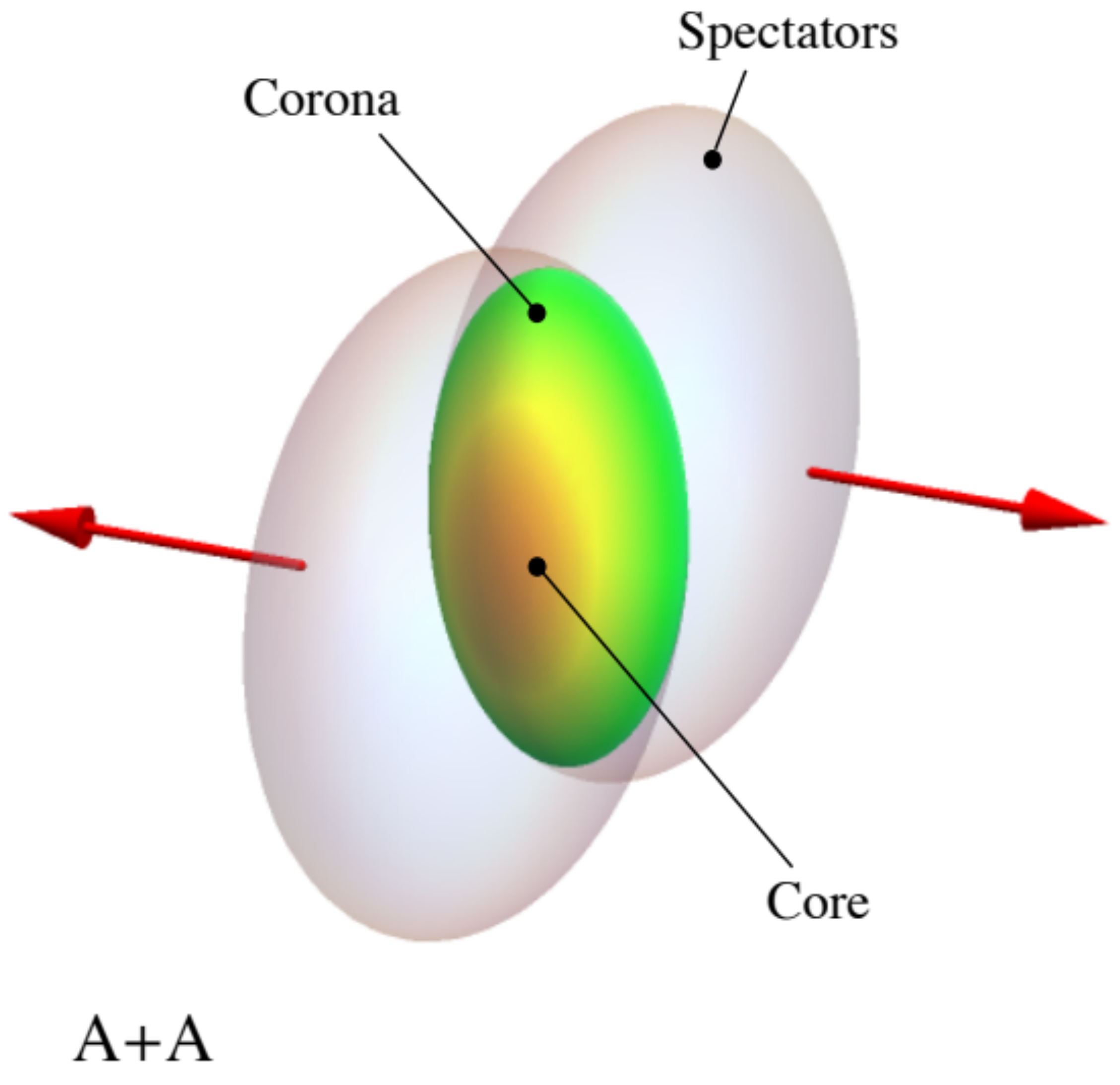
Core-Corona model for polarization



$$\mathcal{P}^{\Lambda} = \frac{(N^{\uparrow}_{\Lambda\ QGP} - N^{\downarrow}_{\Lambda\ QGP}) + (N^{\uparrow}_{\Lambda\ REC} - N^{\downarrow}_{\Lambda\ REC})}{(N^{\uparrow}_{\Lambda\ QGP} + N^{\uparrow}_{\Lambda\ REC}) + (N^{\downarrow}_{\Lambda\ QGP} + N^{\downarrow}_{\Lambda\ REC})}.$$

Intrinsic polarization for the core

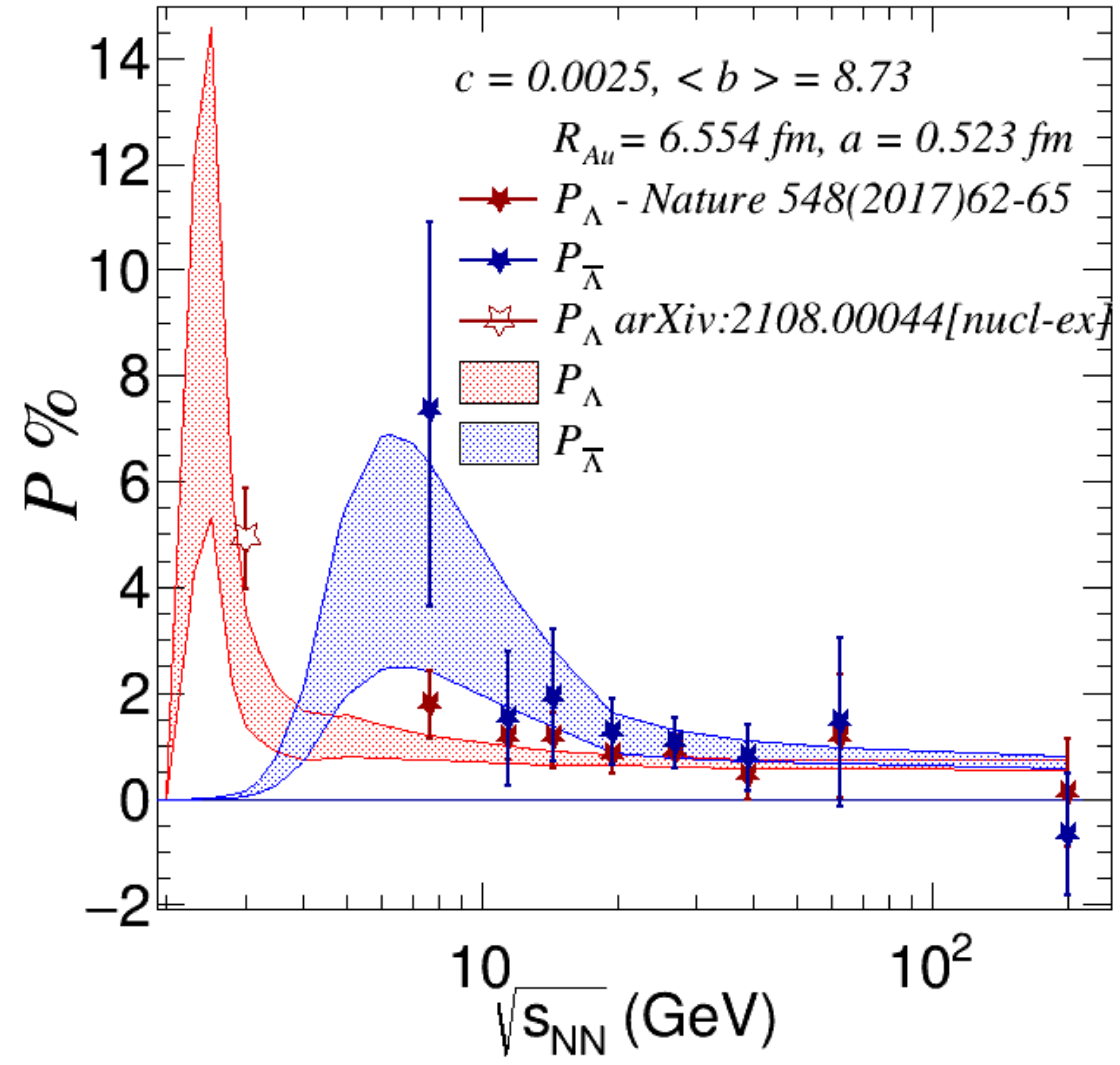
$$z_{QGP} = \frac{N^{\uparrow}_{\Lambda\ QGP} - N^{\downarrow}_{\Lambda\ QGP}}{N_{\Lambda\ QGP}}$$

Intrinsic polarization for the corona

$$z_{REC} = \frac{N^{\uparrow}_{\Lambda\ REC} - N^{\downarrow}_{\Lambda\ REC}}{N_{\Lambda\ REC}}$$

$$\mathcal{P}^{\Lambda} = \frac{\frac{N_{\Lambda\ QGP}}{N_{\Lambda\ REC}}}{1 + \frac{N_{\Lambda\ QGP}}{N_{\Lambda\ REC}}} z_{QGP} + \frac{1}{1 + \frac{N_{\Lambda\ QGP}}{N_{\Lambda\ REC}}} z_{REC}$$

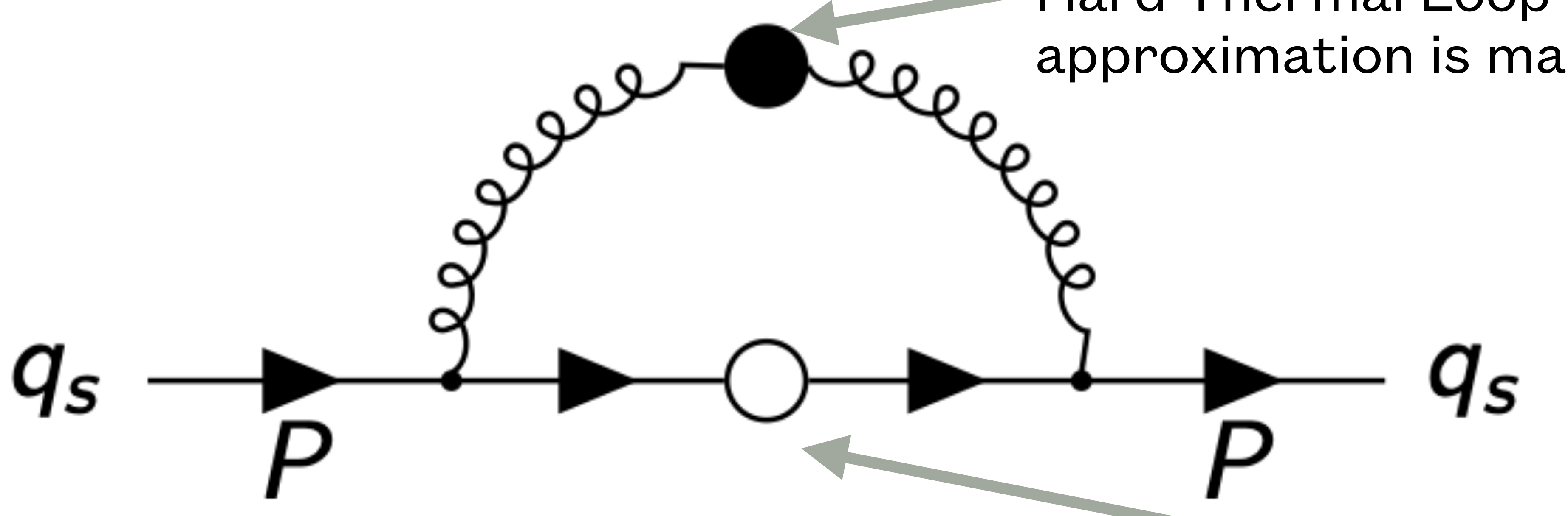
$$= \mathcal{P}_{QGP} + \mathcal{P}_{REC}$$



Ayala et al, 10.1103/PhysRevC.105.034907

Self-energy

Self energy of a quark s in the core



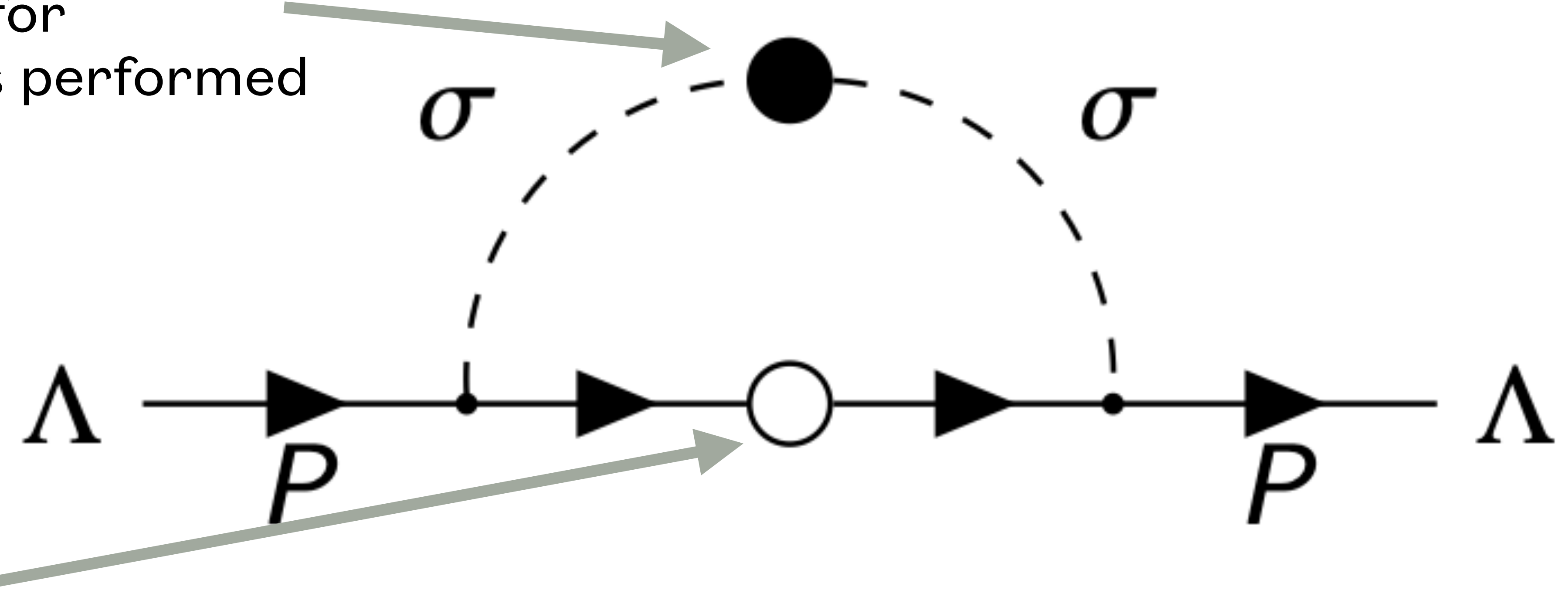
Hard Thermal Loop approximation is made

- In the corona, the QDC reactions domain.
- The quark s carries the spin information.
- After hadronization, the quark polarization is transferred to the Λ

Fermion propagator under a rigid cylinder is used

Self energy of a Λ in the corona

An approximation for low temperature is performed



- The core behaves as an hadronic gas, then effective models contribute.
- The Λ hyperon carries the spin.
- Since the corona is formed by hadrons, a low temperature approach is necessary.

Intrinsic polarizations

$$z = 1 - e^{-\frac{t}{\tau}}$$

Relaxation time

If the lifetime of the system is lower than the relaxation time, the system shows small or zero polarization

Relaxation time is closely related to the interaction rate of a particle to align its its spin along the angular velocity

$$\Gamma^{\pm}(p_0) = f_F(p_0)\ \mathrm{Tr}\left[\mathrm{Im}\Sigma^{\pm}\right]$$

Self-energy of
a rotating fermion

$$S^{\pm}(p) = \frac{\gamma^0(p_0 \pm \Omega/2) - \vec{\gamma}\cdot\vec{p} + m}{(p_0 \pm \Omega/2)^2 - p^2 - m^2 + i\epsilon}\mathcal{O}^{\pm}$$

The propagator for a fermion with its spin aligned/anti-aligned along the angular velocity under a rigid cylinder approximations

Ayala et al, 10.1103/PhysRevC.105.034907

Polarization rate

$$\Gamma = V \int \frac{d^3p}{(2\pi)^3} \left(\Gamma^+(p_0) - \Gamma^-(p_0)\right)$$

Volume of the system

Finally, the polarization rate for core/corona can be obtained as de difference of the alignment rate and the anti alignment rate integrated over all the phase space.

$$\frac{1}{\tau} = \Gamma$$

The relation between the relaxation time and the polarization rate is inverse

Modeling the corona region

Relativistic Mean Field Model

$$\mathscr{L} = \bar{\psi}\left[i\gamma^{\mu}\partial_{\mu} - M_N - g_{\sigma}\sigma\right]\psi$$

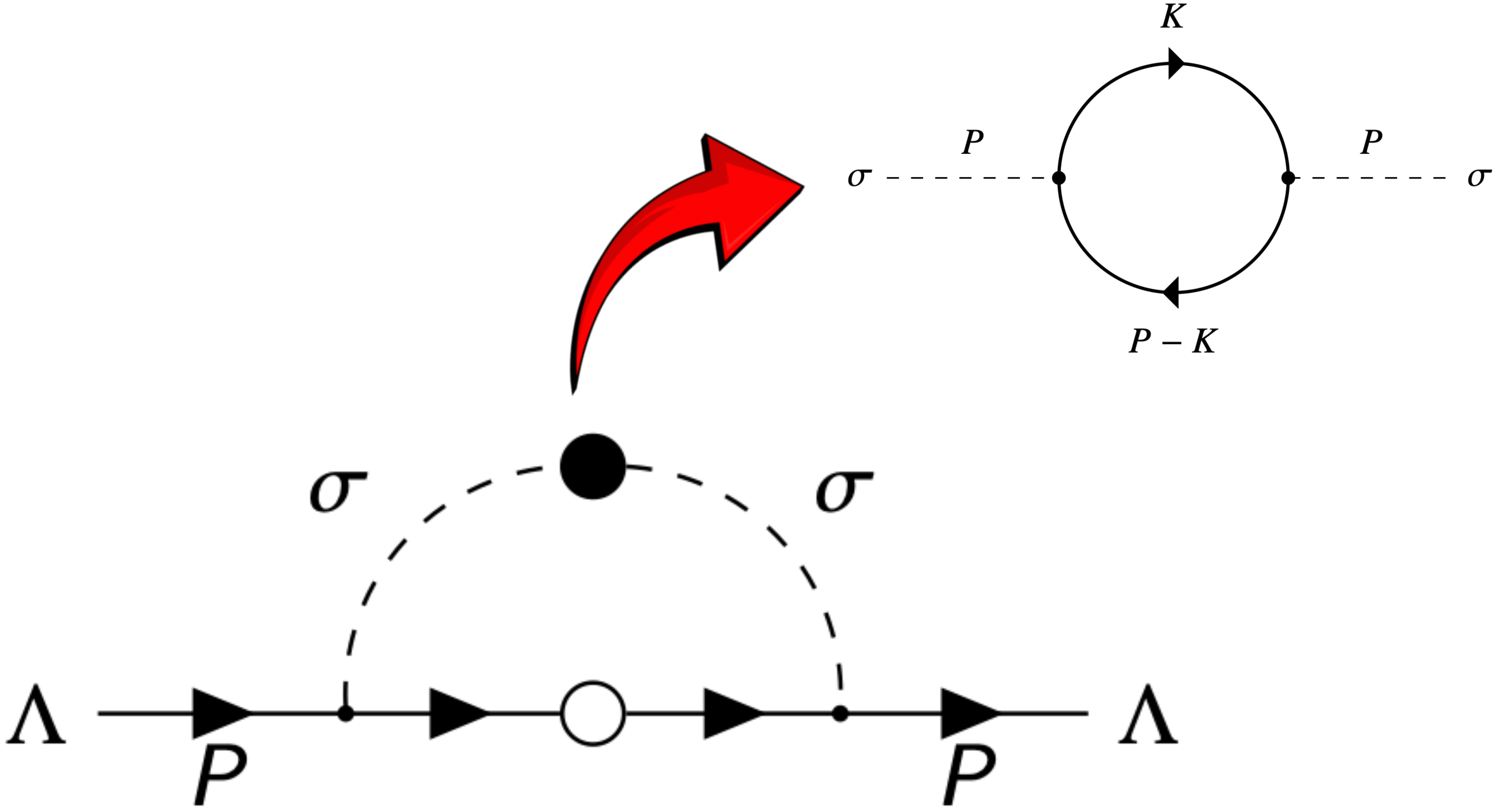
Nucleon part

$$+\bar{\psi}_{\Lambda}\left[i\gamma^{\mu}\partial_{\mu} - M_{\Lambda} - g_{\sigma\Lambda}\sigma\right]\psi_{\Lambda}$$

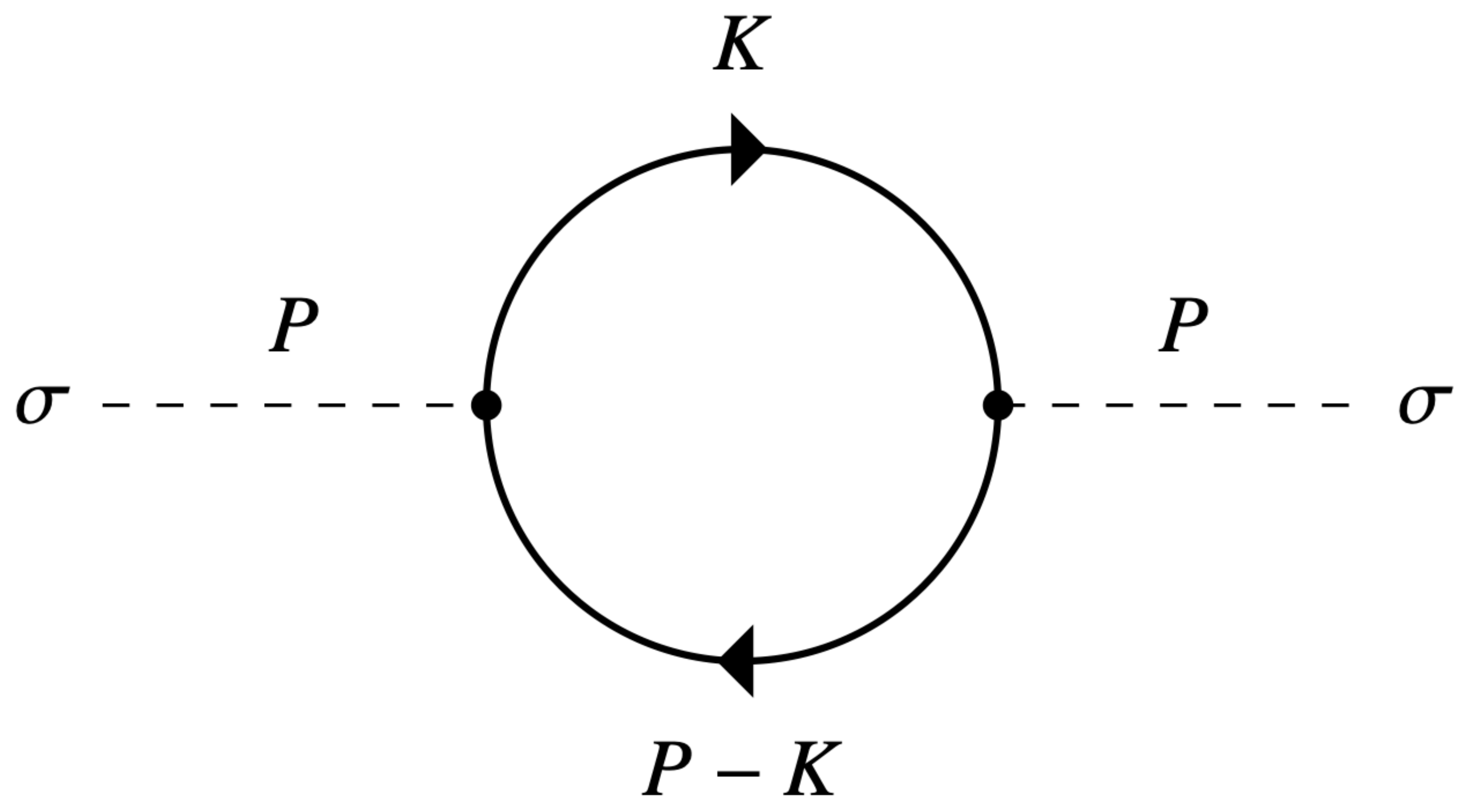
Λ part

$$+\frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma - \frac{1}{2}m_{\sigma}^{2}\sigma^{2}$$

σ part



Thermal modification to σ propagator



$$\Pi = -\,g_\sigma^2 \sum_n \int \frac{d^3k}{(2\pi)^3} \mathrm{Tr}\left[(M_N - \not{k})(M_N - (\not{k} - \not{p}))\right] \tilde{\Delta}(K)\tilde{\Delta}(K-P)$$

$$\Pi = M_T^2 + \gamma_T(i\omega)\left\{ 10x^2 - \frac{4}{3} + \left(5x^2+1\right)\left[x \log\left(\frac{x+1}{x-1}\right) - \frac{2x^2}{\sqrt{3x^2+1}} \log\left(\frac{x^2+\sqrt{3x^2+1}+1}{x^2-\sqrt{3x^2+1}+1}\right)\right]\right\}$$

$$\Delta^*(i\omega, p) = \frac{1}{\omega^2 + p^2 + m_\sigma^2 + \Pi}$$

$$M_T^2 \equiv \frac{2g_\sigma^2 M_N T}{\pi^2} K_1\left(\frac{M_N}{T}\right)\cosh\left(\frac{\mu}{T}\right) \qquad \gamma_T = \frac{M_T^2 \tanh\left(\frac{\mu}{T}\right)}{T} \qquad x \equiv \frac{p_0}{p}$$

JJ Medina et al, arXiv:2502.14248

Spectral density

The propagator can be separated in a real part and an explicit imaginary part

$$A(x) = \left(5x^2+1\right)\left(x - \frac{2x^2}{\sqrt{3x^2+1}}\right)$$

$$\Delta^*(p_0, p) = \frac{-1}{P^2 - M_\sigma^2 - \gamma_T p_0 F(x) - i\pi A(x)\theta(p^2 - p_0^2)}$$

$$F(x) = 10x^2 - \frac{4}{3} + \left(5x^2+1\right)\left[x \log\left|\frac{x+1}{x-1}\right| - \frac{2x^2}{\sqrt{3x^2+1}} \log\left|\frac{x^2+\sqrt{3x^2+1}+1}{x^2-\sqrt{3x^2+1}+1}\right|\right]$$

$$\rho_\sigma(p_0, p) = 2\mathrm{Im}\Delta^*(q_0 + i\eta, q) \longrightarrow$$

Quasi-particle term

$$\rho_\sigma(p_0, p) = 2\pi\, Z(\omega(p))\left[\delta(p_0 - \omega(p)) - \delta(p_0 + \omega(p))\right]$$
$$+\beta(p_0, p)$$

Landau damping term

$$\beta(p_0, p) = 2\pi\gamma_T p_0\, A(x)\theta\left(1 - x^2\right)\left[\left(P^2 - M_\sigma^2 - \gamma_T p_0 F\left(x\right)\right)^2 - \left(\gamma_T p_0 A\left(x\right)\pi\right)^2\right]^{-1}$$

JJ Medina et al, arXiv:2502.14248

Polarization rate

The final result for the alignment rate is

$$\Gamma^{\pm}_{REC}(p_0) = \frac{g_\Lambda^2 M_\Lambda \pi}{2}\int_0^\infty \frac{dk\,k^2}{(2\pi)^3}\int_{\mathscr{R}^\pm} dk_0 \frac{f(k_0)}{pk} \times \tilde{f}\left(p_0 - k_0 - \mu_B \mp \Omega/2\right)\rho_\sigma(k_0)$$

For corona

The region $\mathscr{R}^\pm$ is defined by

$$k_0 \ \geq \ p_0 \pm \Omega/2 - \sqrt{(p+k)^2 + M_\Lambda^2}$$

$$k_0 \ \leq \ p_0 \pm \Omega/2 - \sqrt{(p-k)^2 + M_\Lambda^2}$$

$$\Gamma^{\pm}_{QGP}(p_0) = \frac{g^2 m_q C_F \pi}{2}\int_0^\infty \frac{dk\,k^2}{(2\pi)^3}\int_{\mathscr{R}^\pm} dk_0 \frac{f(k_0)}{2pk} \times \left(4\rho_L(k_0) + 8\rho_T(k_0)\right)\tilde{f}\left(p_0 - k_0 - \mu_q \mp \Omega/2\right)$$

For core

JJ Medina et al, Phys.Rev.D 109 (2024) 7, 074018

Preliminar numerical results

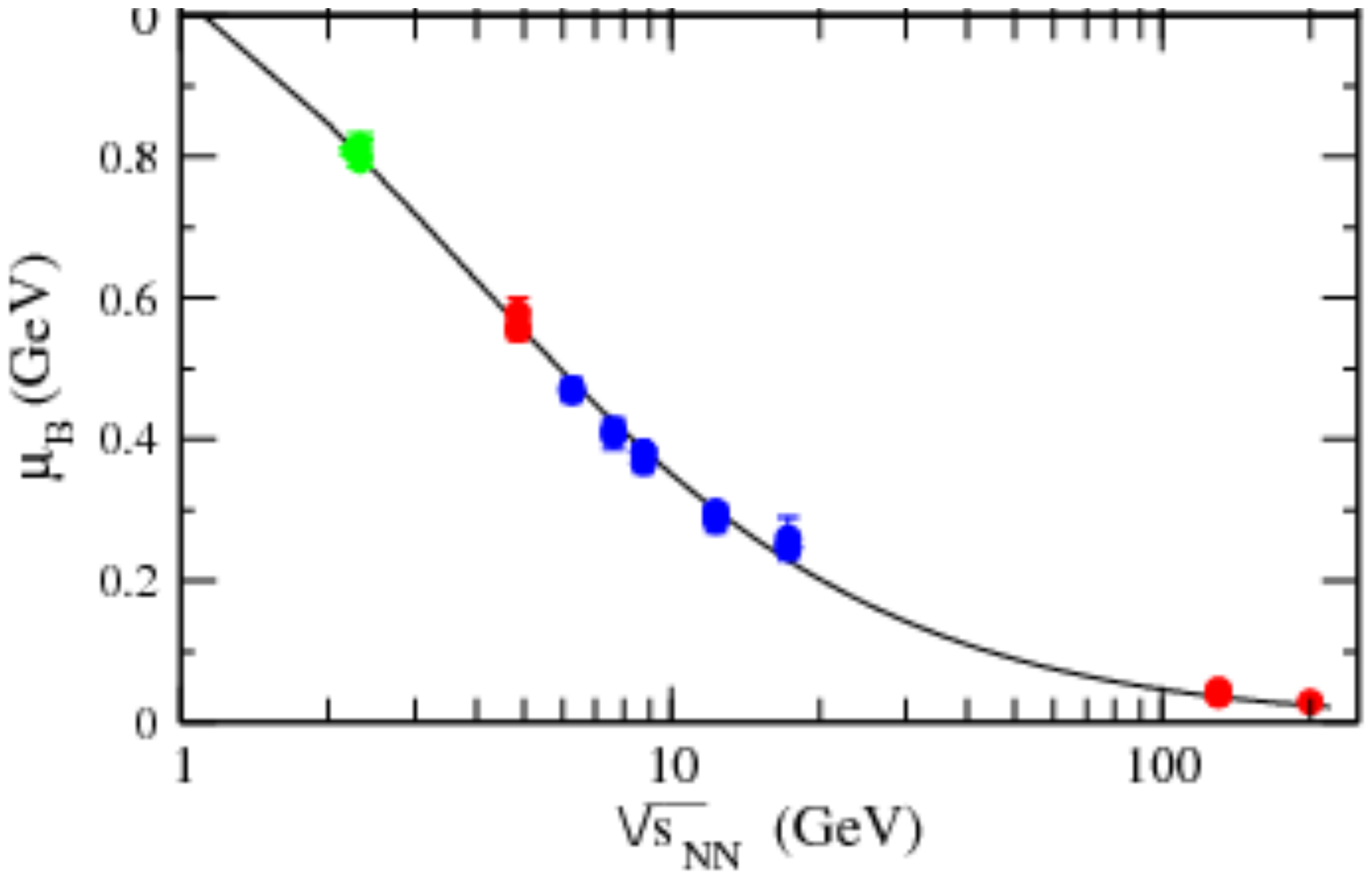
To obtain some numerical results with our model, we use the following values of volume and expansion velocity

	Volume (fm^3)	**Expansion velocity** β
0 - 10 %	2667	0.246
10 - 20 %	2550	0.240
20 - 40 %	2470	0.233
40 - 80 %	2409	0.229

- These values are obtained for the whole system, core and corona.
- For an estimation of the core region, we asume a 1% of the system volume for the volume.
- We estimate the life time of the system as $\tau = (\frac{3V}{4\pi})^{1/3}\frac{1}{\beta}$

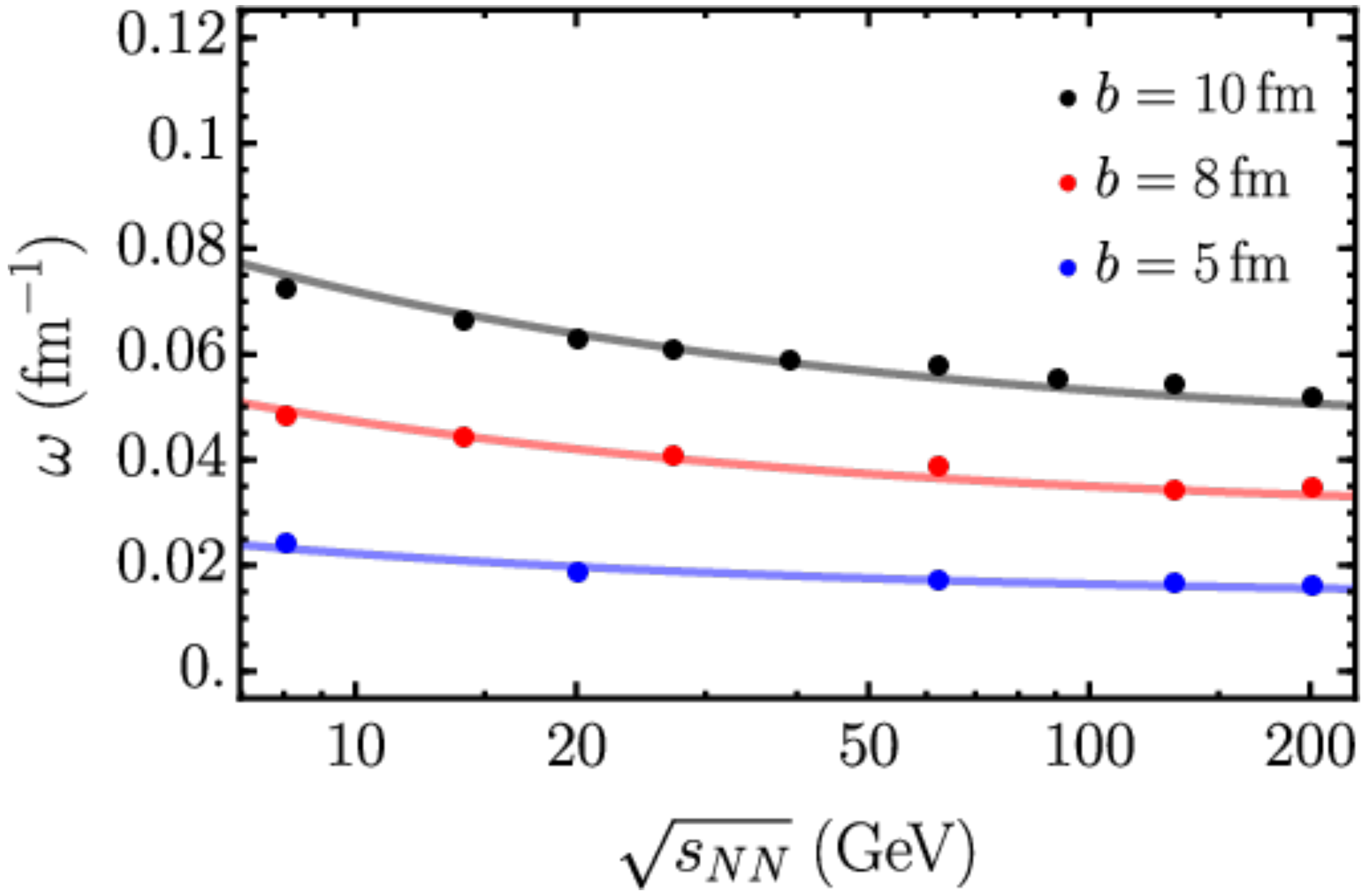
M. Waqas et al, *Results Phys.* 64 (2024) 107894

More parameterizations



Parametrization for the chemical potencial[1]

$$\mu_B(\sqrt{s_{NN}}) = \frac{1.308}{1 + 0.273\sqrt{s_{NN}}}$$



Parametrization for the angular velocity[2]

$$\Omega = \frac{b^2}{2V_N}\left[1 + 2\left(\frac{m_N}{\sqrt{s_{NN}}}\right)^{1/2}\right]$$

Parameters of the RMF model[3]

M_σ [MeV]	508.194
g_σ	10.217
$g_{\sigma\Lambda}/g_\sigma$	0.6188

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[1] J. Claymans et al, *Phys.Rev.C* 73 (2006) 034905

[2] Ayala et al, *Phys.Rev.D* 102 (2020) 5, 056019

[3] Ting-Ting Sun et al, *Chin.Phys.C* 42 (2018) 2, 025101

Number of Λ

Number of Λ from the core

$$N_{p\ QGP} = \int ds^2\ n_p(\vec{s}, \vec{b})\ \theta[n_p(\vec{s}, \vec{b}) - c_n]$$

$$n_p(\vec{s}, \vec{b}) = T_A(\vec{s})\left[1 - e^{-\sigma_{NN}(\sqrt{s_{NN}})T_B\vec{s}-\vec{b}}\right]$$

$$+T_B(\vec{s} - \vec{b})\left[1 - e^{-\sigma_{NN}(\sqrt{s_{NN}})T_A(\vec{s})}\right]$$

$$T_A(\vec{s}) = \int_{\infty}^{\infty} \rho_A(z, \vec{s})\ dz$$

$$N_{\Lambda\ QGP} = cN^2_{p\ QGP}$$

Number of Λ from the corona

$$N_{N\ QGP} = \int ds^2\ T_A(\vec{s})T_B(\vec{b} - \vec{s})\ \theta[c_n - n_p(\vec{s}, \vec{b})]$$

The key ingredient is the critical density. Densities above c_n allows the production of QGP, while densities below c_n do not.

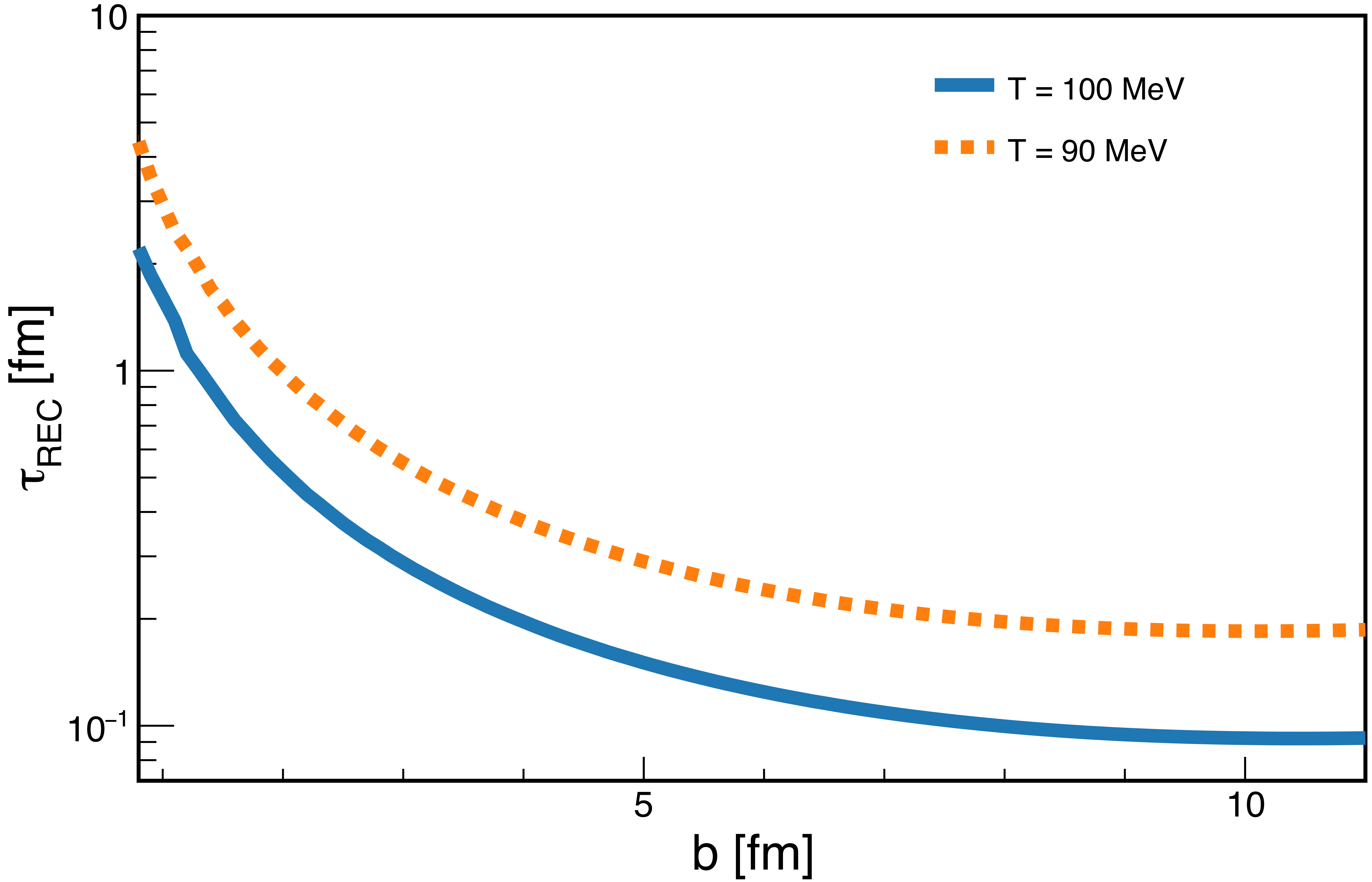
$c_n = 3.8$ fm^{-2}

$c = 0.0026$

Ayala et al, 10.1103/PhysRevC.105.034907

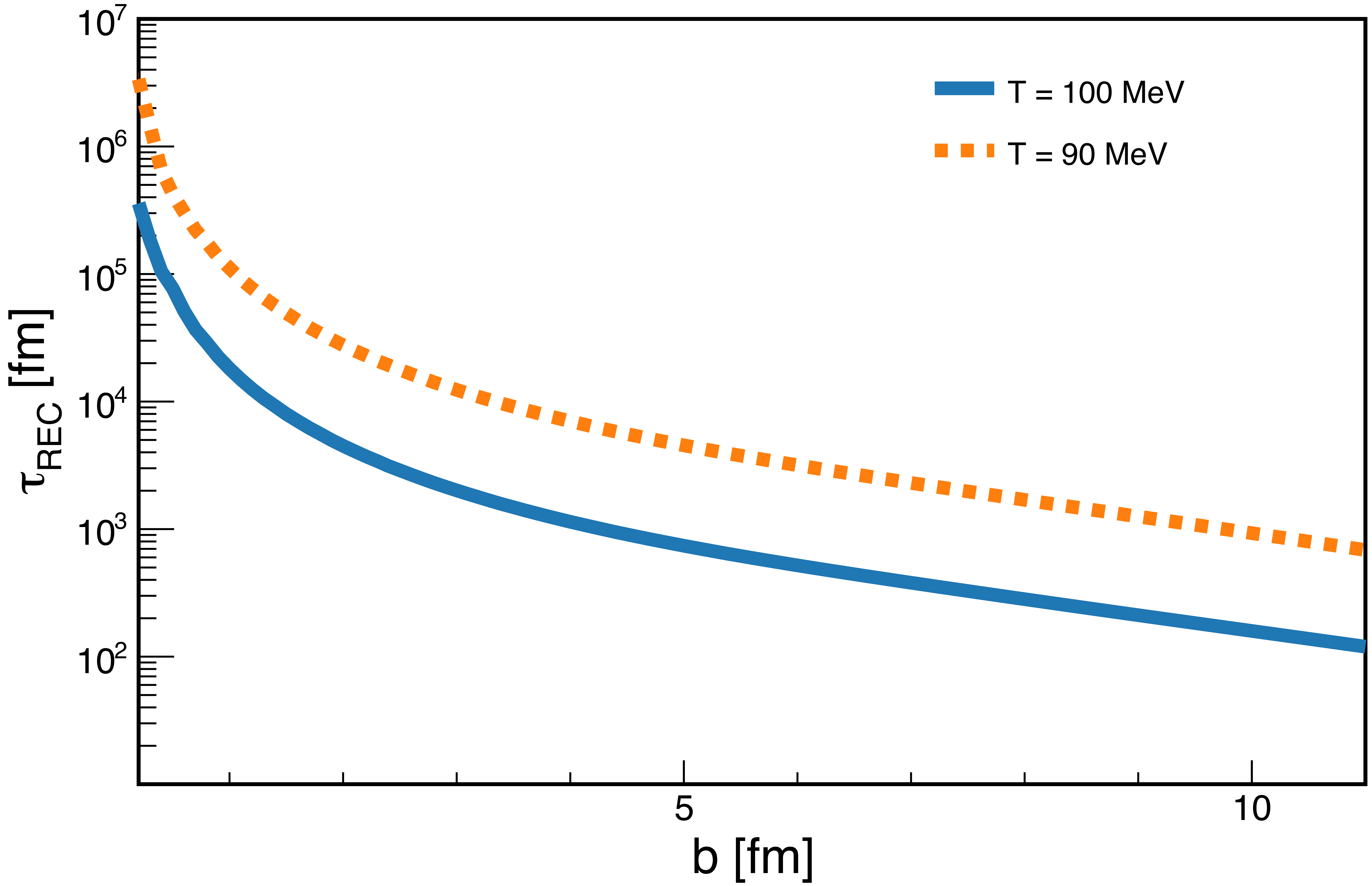
Blaizot J. et al, 10.1103/PhysRevLett.77.1703

Relaxation time for the core region



- We assume a collision energy of Au+Au, $\sqrt{s_{NN}}$ = 3 GeV
- We take a value of chemical potential μ_B = 700 MeV.
- We take the angular velocity as function of collision energy and impact parameter b.

Relaxation time for the corona region



- We assume a collision energy of Au+Au, $\sqrt{s_{NN}}$ = 3 GeV
- We take a value of chemical potential μ_B = 700 MeV.
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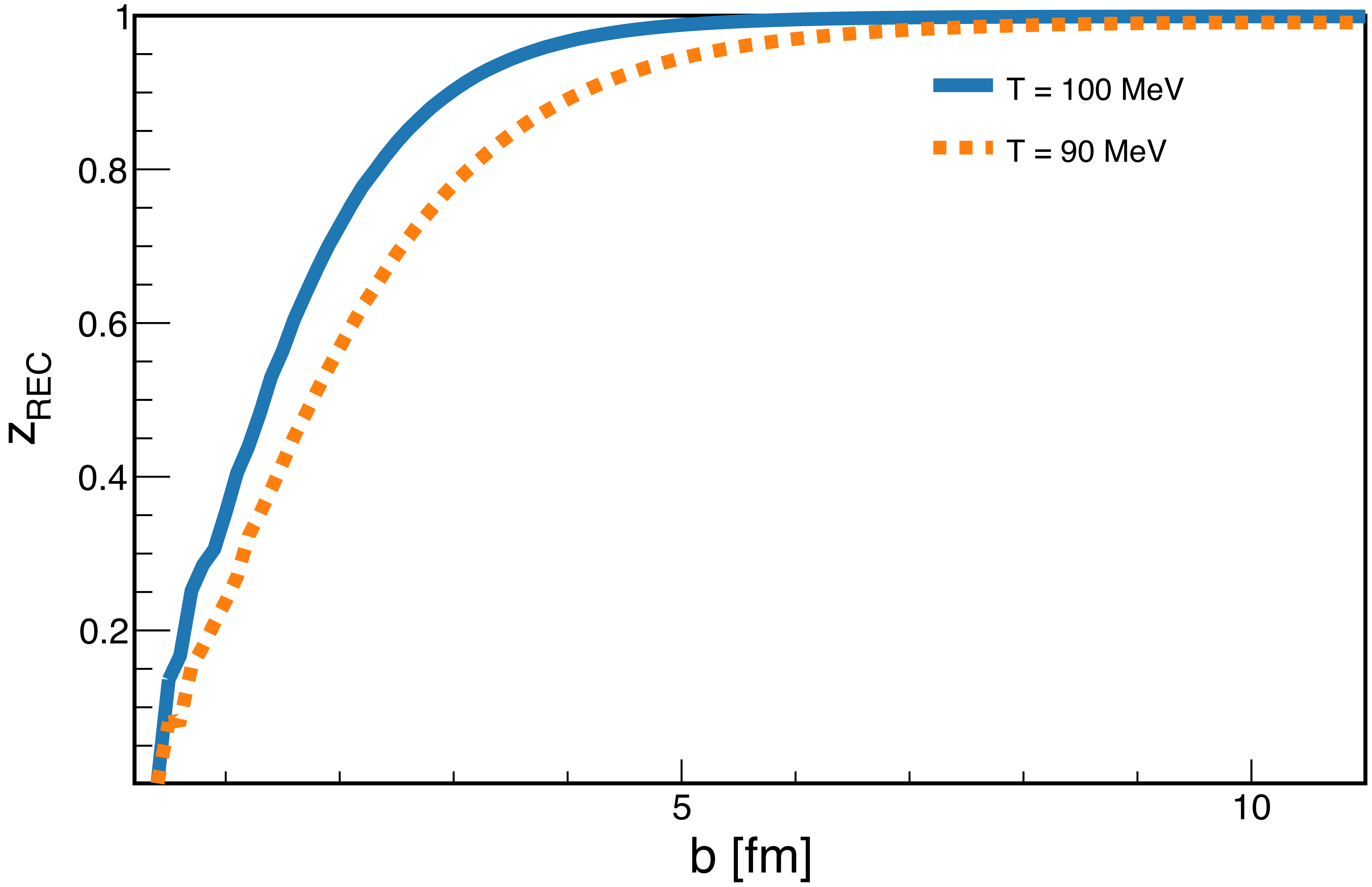
Intrinsic polarization for the core region



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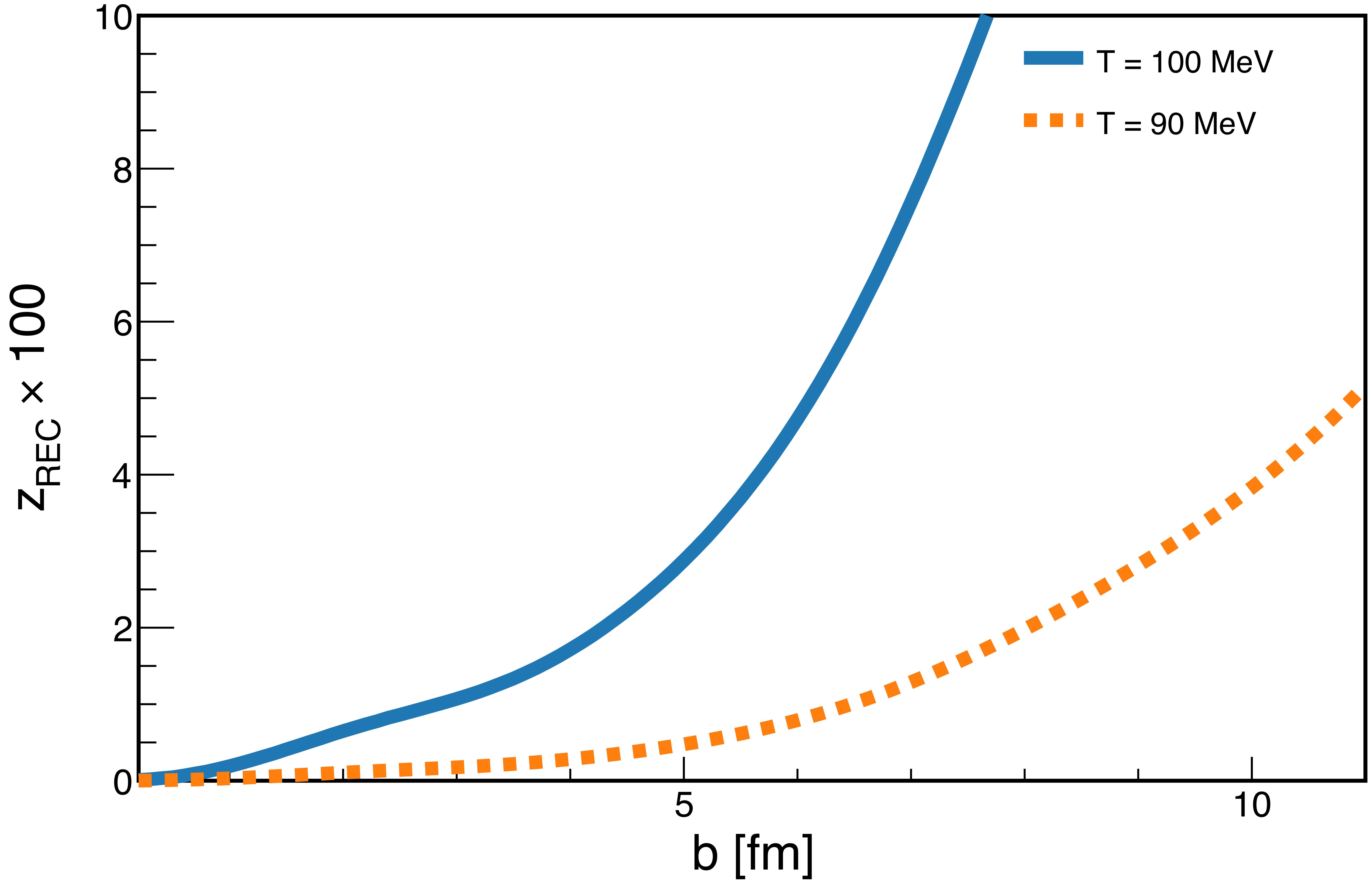
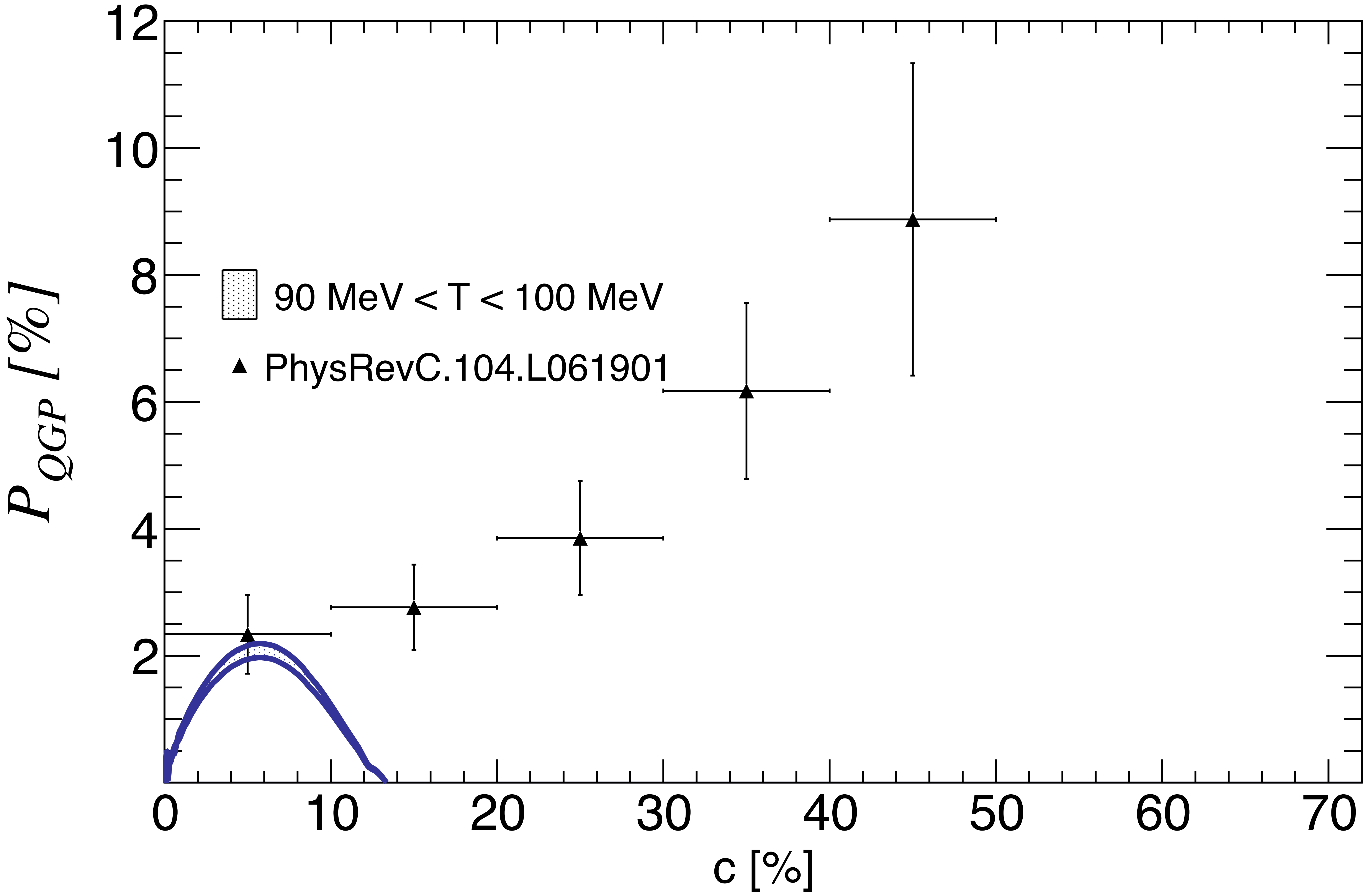


- We assume a collision energy of Au+Au, $\sqrt{s_{NN}}$ = 3 GeV
- We take a value of chemical potential μ_B = 700 MeV.
- We take the angular velocity as function of collision energy and impact parameter b.

Polarization contribution for the core region



- We assume a collision energy of Au+Au, $\sqrt{s_{NN}}$ = 3 GeV.
- The experimental data correspond to Au+Au, $\sqrt{s_{NN}}$ = 3 GeV.
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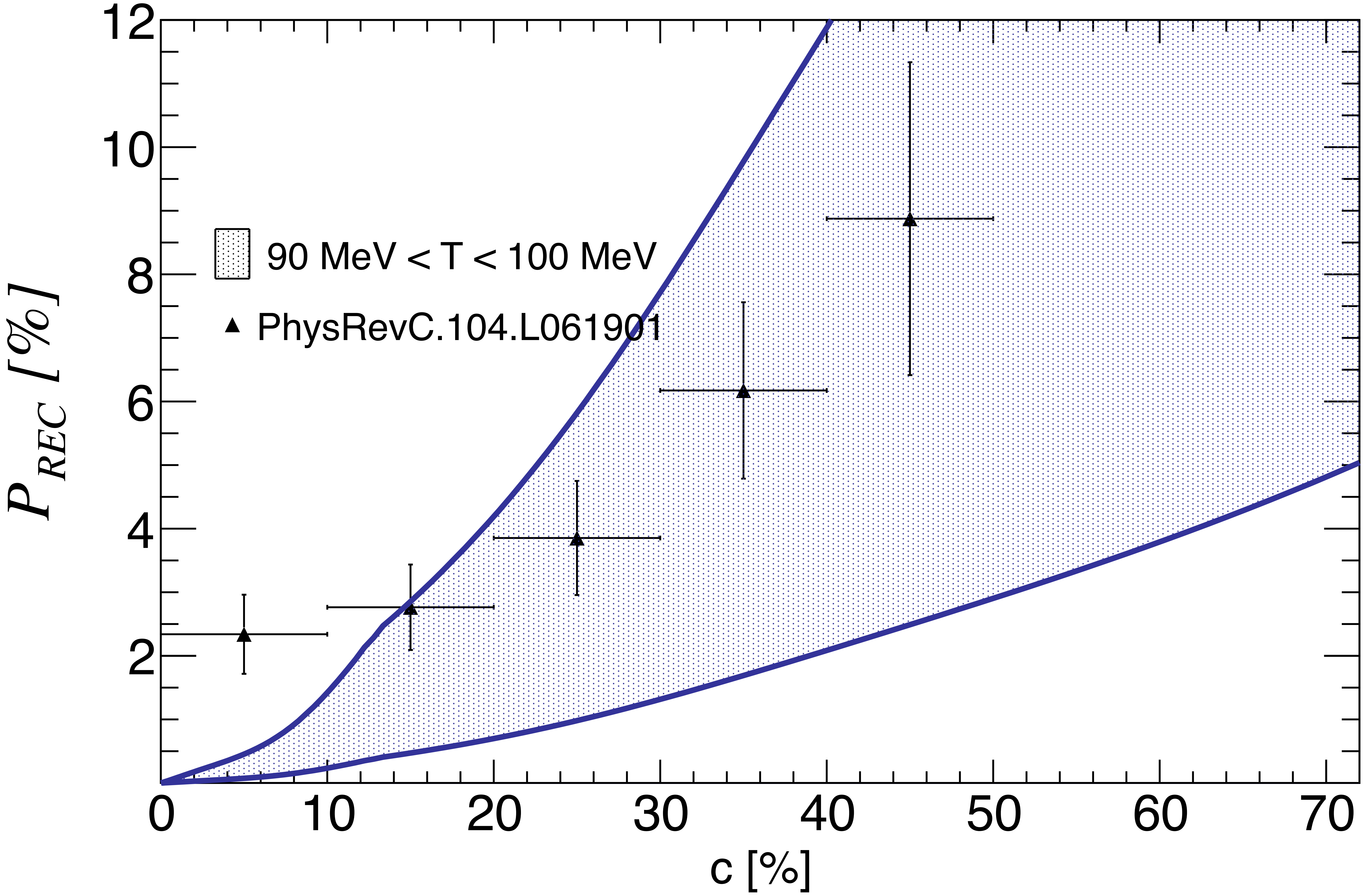
Polarization contribution for the corona region



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- The experimental data correspond to Au+Au, $\sqrt{s_{NN}}$ = 3 GeV.
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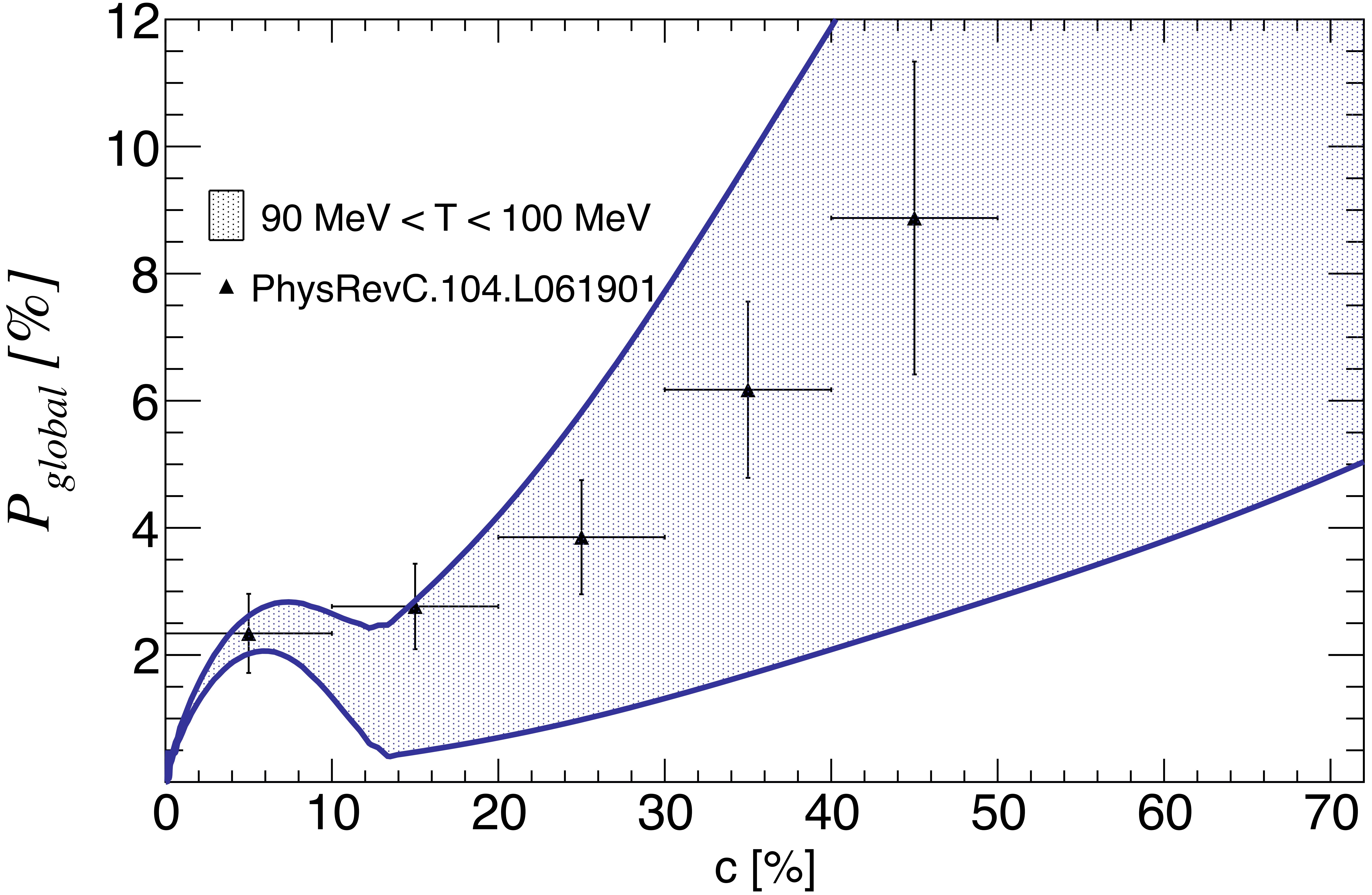
- We take the angular velocity as function of collision energy and impact parameter b.

Global polarization



- We assume a collision energy of Au+Au, $\sqrt{s_{NN}}$ = 3 GeV.
- The experimental data correspond to Au+Au, $\sqrt{s_{NN}}$ = 3 GeV.
- We take a value of chemical potential μ_B = 700 MeV.
- We take the angular velocity as function of collision energy and impact parameter b.

Summary

- The core-corona model allow us to identify two different contributions to global polarization.
- Intrinsic polarization is a key ingredient to produce polarization within our model.
- We obtained an estimation of the relaxation time for q_s from the core and Λ from the corona.
- τ_{QGP} is larger than τ_{REC}, therefore z_{QGP} is larger than z_{REC}.
- $\mathscr{P}_{QGP}$ vanishes for centralities larger than 11% because of the number of participant is smaller than the critical number of participants for QGP formation.
- In the range of 90 and 100 MeV of temperature, our model bounds upper and lower the experimental results for global polarization.

Thank you!!

Backup

Fermion propagator under rotation

Metric tensor under rigid rotations

$$g_{\mu\nu} = \begin{pmatrix} 1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\longrightarrow$$

Dirac equation under rigid rotations

$$\left[i\gamma^0\left(\partial_t + \Omega\hat{J}_z\right) + i\vec{\gamma}\cdot\vec{\nabla} - m\right]\Psi = 0$$

We can find solutions to Dirac equation in terms of a new function $\phi(x)$

$$\Psi(x) = \left[i\gamma^0\left(\partial_t + \Omega\hat{J}_z\right) + i\vec{\gamma}\cdot\vec{\nabla} + m\right]\phi(x)$$

And $\phi(x)$ satisfies a Klein-Gordon equation under rotation

$$\left[\left(i\partial_t + \Omega\hat{J}_z\right)^2 + \partial_x^2 + \partial_y^2 + \partial_z^2 - m^2\right]\phi(x) = 0.$$

Solutions

The solutions to Klein-Gordon equations are in the form

$$\phi(x) = \begin{pmatrix} J_\ell(k_\perp \rho) \\ J_{\ell+1}(k_\perp \rho) e^{i\varphi} \\ J_\ell(k_\perp \rho) \\ J_{\ell+1}(k_\perp \rho) e^{i\varphi} \end{pmatrix} e^{-Et+ik_z z+i\ell\varphi}$$

Therefore, the solutions to Dirac equation are in the form

$$\Psi(x) = \begin{pmatrix} \left[E + j\Omega + m - k_z + ik_\perp\right] J_\ell(k_\perp \rho) \\ \left[E + j\Omega + m + k_z - ik_\perp\right] J_{\ell+1}(k_\perp \rho) e^{i\varphi} \\ \left[-E + j\Omega + m - k_z + ik_\perp\right] J_\ell(k_\perp \rho) \\ \left[-E + j\Omega + m + k_z - ik_\perp\right] J_{\ell+1}(k_\perp \rho) e^{i\varphi} \end{pmatrix} e^{-(E+j\Omega)t+ik_z z+i\ell\varphi}$$

Closure relation for the solutions

Representation of the Klein-Gordon propagator →

$$G(x,x') = (-i)\int_{-\infty}^{0} d\tau\ \text{Exp}\left[-i\tau\mathscr{H}\right]\delta^4(x-x')$$

Fermion propagator from Klein-Gordon →

$$S(x,x') = \left[i\gamma^0\left(\partial_t + \Omega\hat{J}_z\right) + i\vec{\gamma}\cdot\vec{\nabla} + m\right]G(x,x')$$

Closure relation →

$$\sum_{l=-\infty}^{\infty}\int\frac{dE\,dk_z\,dk_\perp k_\perp}{(2\pi)^3}\phi_i(x)\phi_i^\dagger(x') = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\delta^4(x-x')$$

Propagator

$$S(p) = \frac{\gamma^0(p_0 + \Omega/2) - \vec{\gamma}\cdot\vec{p} + m}{(p_0 + \Omega/2)^2 - p^2 - m^2 + i\epsilon}\mathcal{O}^+ + \frac{\gamma^0(p_0 - \Omega/2) - \vec{\gamma}\cdot\vec{p} + m}{(p_0 - \Omega/2)^2 - p^2 - m^2 + i\epsilon}\mathcal{O}^-$$

Aligned term

$$\mathcal{O}^+ = \frac{1}{2}(1 + i\gamma^1\gamma^2)$$

Anti aligned term

$$\mathcal{O}^- = \frac{1}{2}(1 - i\gamma^1\gamma^2)$$

Real and imaginary part

Notice that for $-1 < x < 1$

$$\frac{x+1}{x-1} < 0 \quad \text{and} \quad \frac{x^2+\sqrt{3x^2+1}+1}{x^2-\sqrt{3x^2+1}+1} < 0$$

Then,

$$\log\left(\frac{x+1}{x-1}\right) = \log\left|\frac{x+1}{x-1}\right| + i\pi\theta\left(1-x^2\right),$$

$$\log\left(\frac{x^2+\sqrt{3x^2+1}+1}{x^2-\sqrt{3x^2+1}+1}\right) = \log\left|\frac{x^2+\sqrt{3x^2+1}+1}{x^2-\sqrt{3x^2+1}+1}\right| + i\pi\theta\left(1-x^2\right)$$

Therefore, the propagator with thermal modifications ca be written as

$$\Delta^*(p_0,p) = \frac{-1}{P^2 - M_\sigma^2 - \gamma_T p_0 F(x) - i\pi A(x)\theta(p^2-p_0^2)}$$

$$A(x) = \left(5x^2+1\right)\left(x - \frac{2x^2}{\sqrt{3x^2+1}}\right)$$

$$F(x) = 10x^2 - \frac{4}{3} + \left(5x^2+1\right)\left[x\log\left|\frac{x+1}{x-1}\right| - \frac{2x^2}{\sqrt{3x^2+1}}\log\left|\frac{x^2+\sqrt{3x^2+1}+1}{x^2-\sqrt{3x^2+1}+1}\right|\right]$$

JJ Medina et al, arXiv:2502.14248