UNIVERSIDAD AUTÓNOMA METROPOLITANA
Casa abierta al tiempo
Unidad Iztapalapa

Departamento de Física
División de Ciencias Básicas e Ingeniería

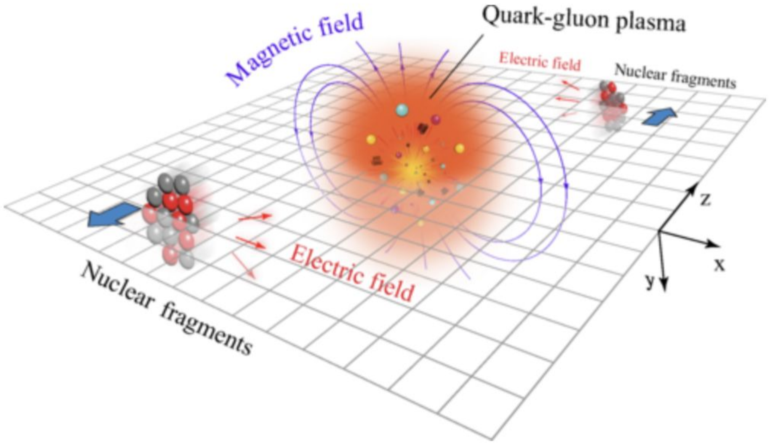
A Journey Beyond Mean Field: Toward Inverse Magnetic Catalysis and the Critical Point in the LSMq

Luis Alberto Hernández Rosas

9th Conference on Chirality, Vorticity and Magnetic Fields in Quantum Matter

Outline

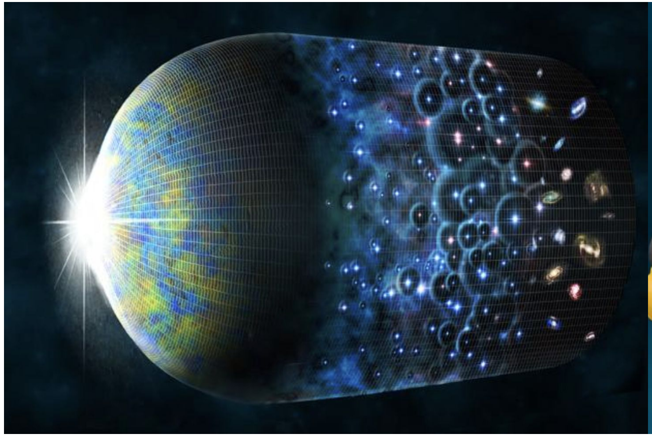
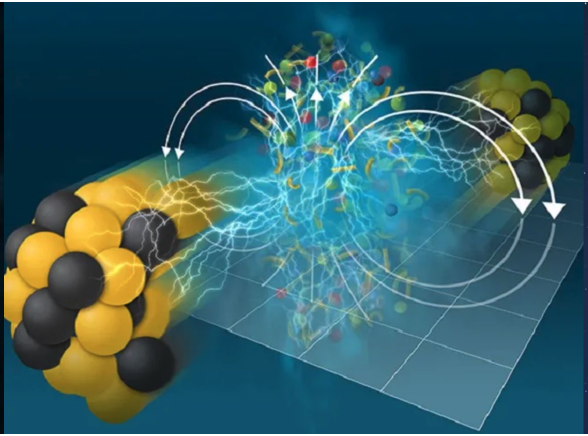
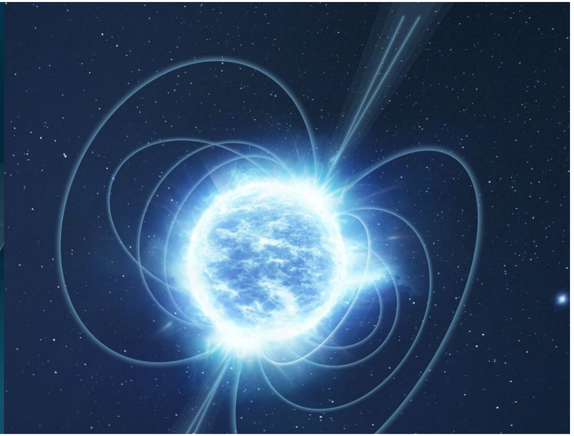
- Motivation.
- Brief recap.
- One more attempt.
- First test.
- Analysis and results.
- Future work.



Phys. Rev. X ***14***, 011028

Why study magnetic effects in nuclear matter?

Physics systems



Linear Sigma Model with quarks

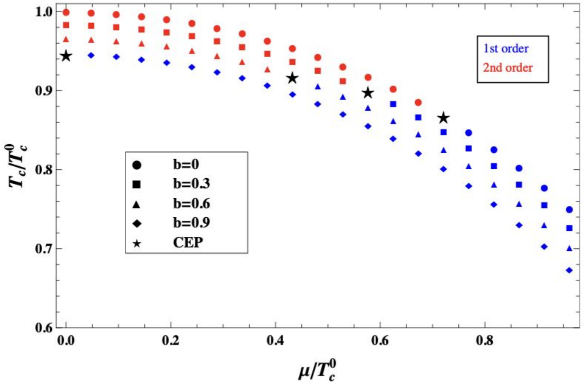
- Effective model for low-energy QCD.
- Renormalizable.
- Symmetry spontaneously broken.
- Quarks and mesons involved in the chiral phase transition

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(D_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + i\bar{\psi}\gamma^\mu D_\mu \psi - g\bar{\psi}(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi})\psi,$$

Strongly interacting matter in extreme conditions explored with the LSMq

✓ Inverse Magnetic Catalysis.

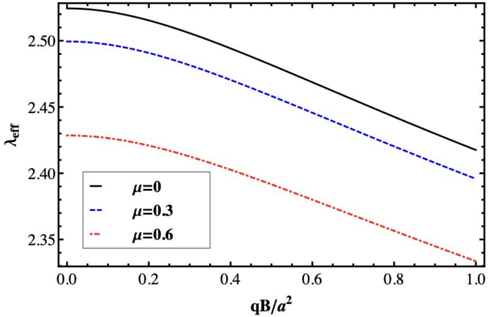
✓ Critical End Point.

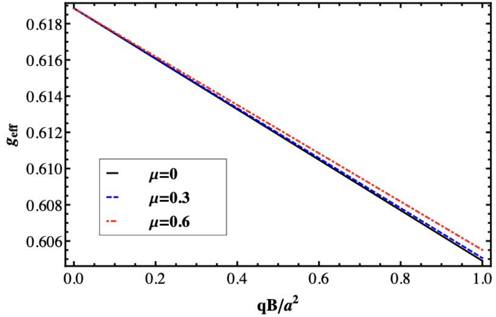


- λ=0.4, g=0.63, a=154 MeV
- λ=0.36, g=0.51, a=132 MeV

Effective Couplings

Phys.Rev.D **92** (2015) 9, 096011



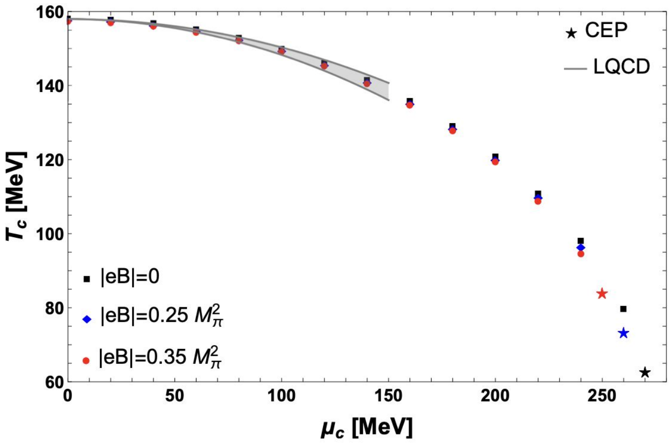


Strongly interacting matter in extreme conditions explored with the LSMq

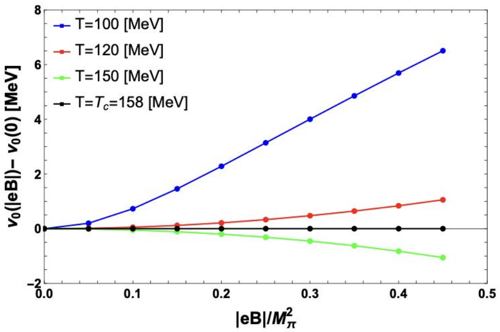
✓ Inverse Magnetic Catalysis.

✓ Critical End Point.

✓ LQC [1]



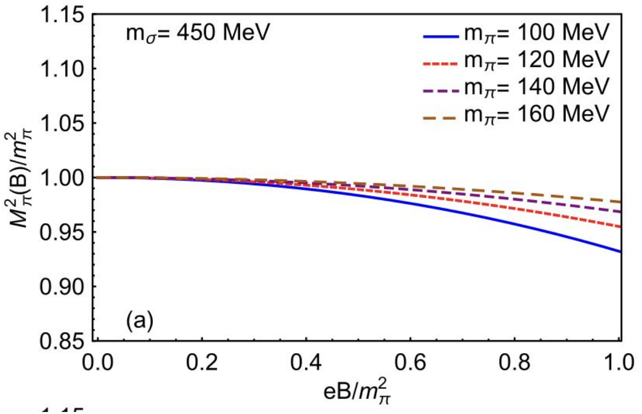
Eur.Phys.J.A 57 (2021) 7, 234

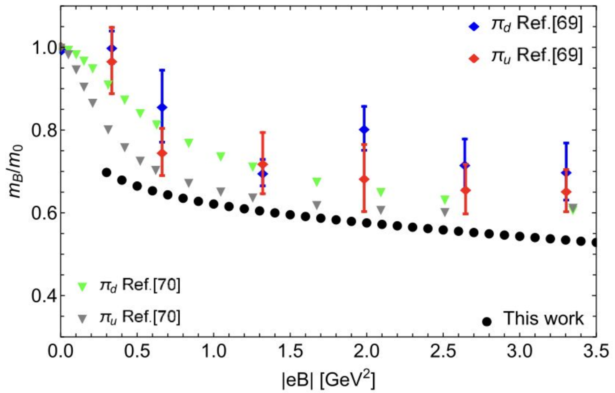


λ=1.6, g=0.794, a=133.5 MeV
λ=2, g=0.484, a=80.5 MeV

Improved Ring Diagrams contribution

Strongly interacting matter in extreme conditions explored with the LSMq





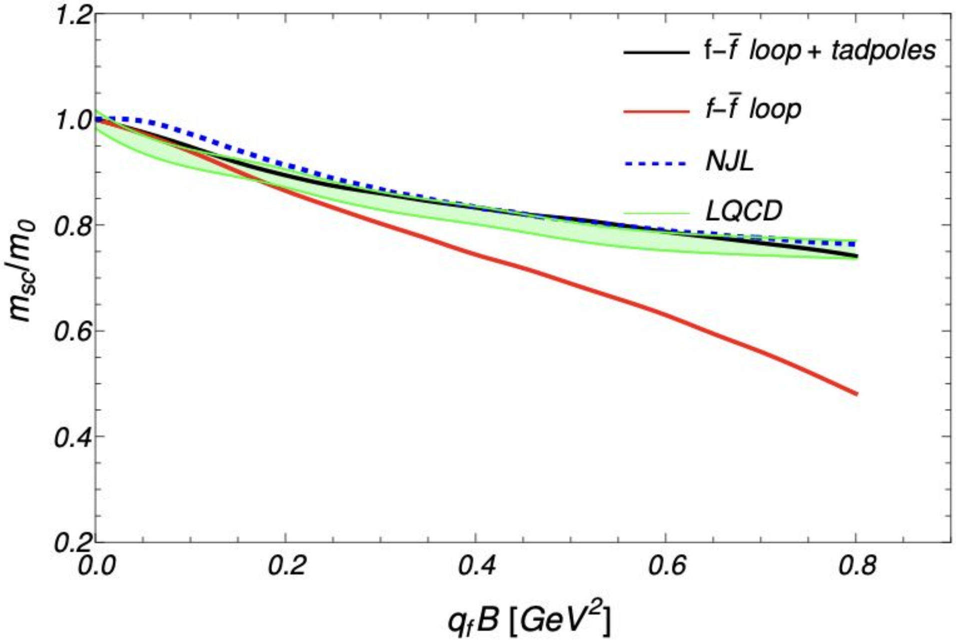
- Weak field approximation
- Effective couplings
- Free parameters fixed in vacuum

Phys.Rev. D **98** (2018) 11, 114008

- Strong field approximation
- Effective couplings
- Free parameters fixed in vacuum

Phys.Rev. D **103** (2021) 5, 054038

Strongly interacting matter in extreme conditions explored with the LSMq



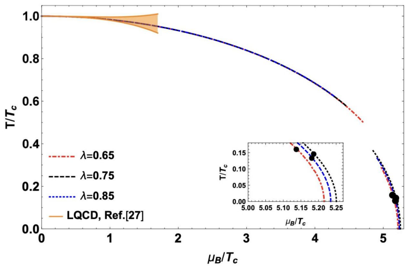
Longitudinal screening mass for the neutral pion

- λ=2.5, g=0.33, a=40 MeV
- No effective couplings
- Free parameters fixed with finite |eB|.

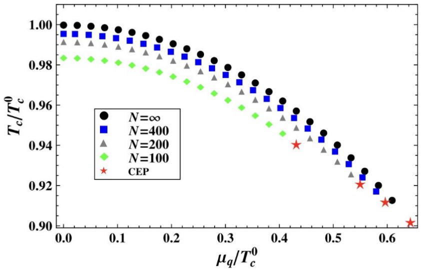
Phys.Rev. D **109** (2024) 7, 074019

Free parameters of the theory and their determination

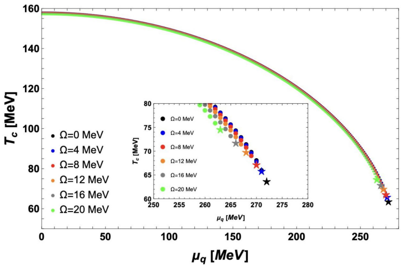
- These parameters are fixed in vacuum, to take well-determined values.
    - But we are not studying the vacuum. We are in a situation far from that state.
- We determine the values of these parameters under the studied conditions.
    - But we do not have enough physical information to determine them.
- The region under study is broad, so the parameters should not be constant.



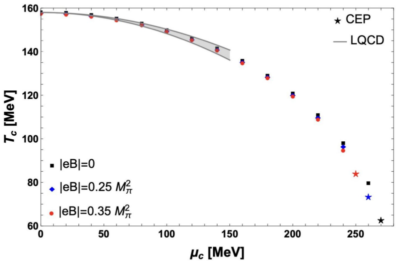
Eur. Phys. J. A **56** (2020) 2, 71



Phys. Rev. D **98** (2018) 11, 114002



Phys. Rev. D **111** (2025) 3, 036003



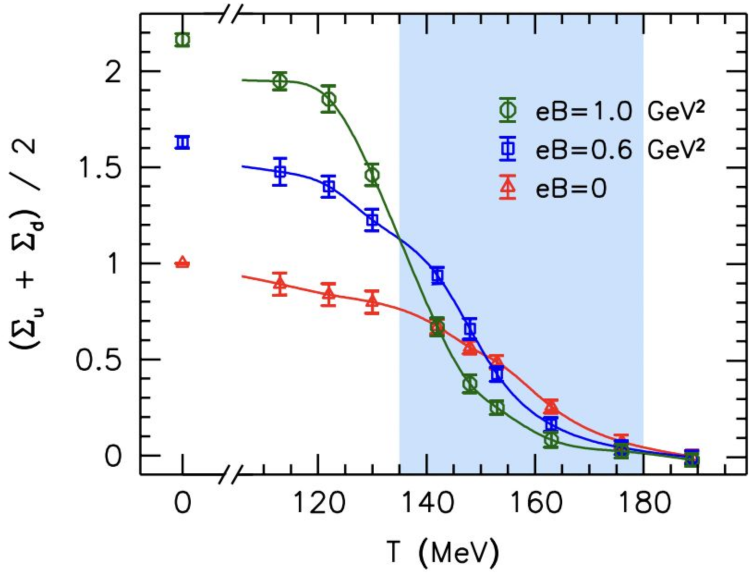
Eur. Phys. J. A 57 (2021) 7, 234

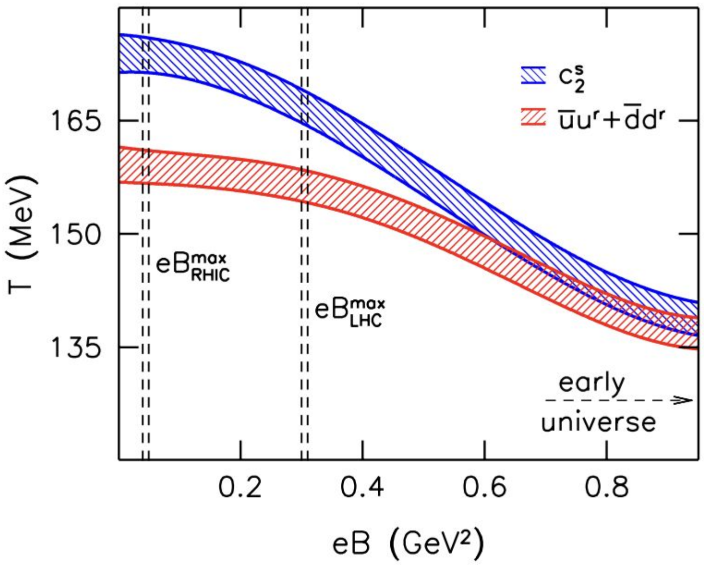
Effective Magnetic QCD phase diagram

Work in collaboration with A. J. Mizher and G. Fernández

(in preparation)

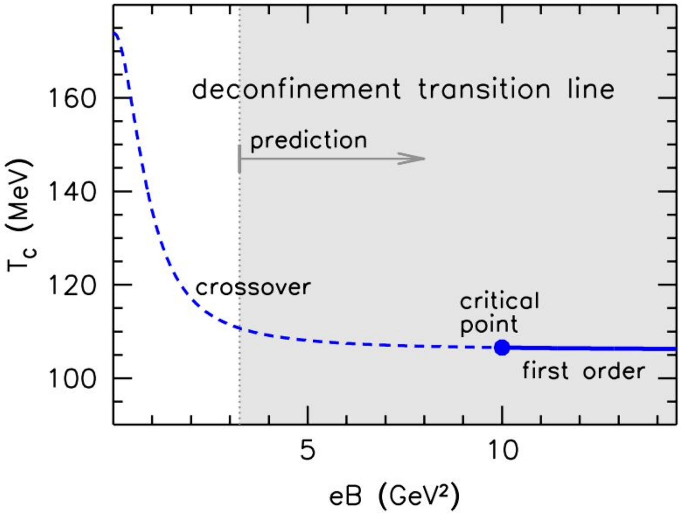
Inverse magnetic catalysis



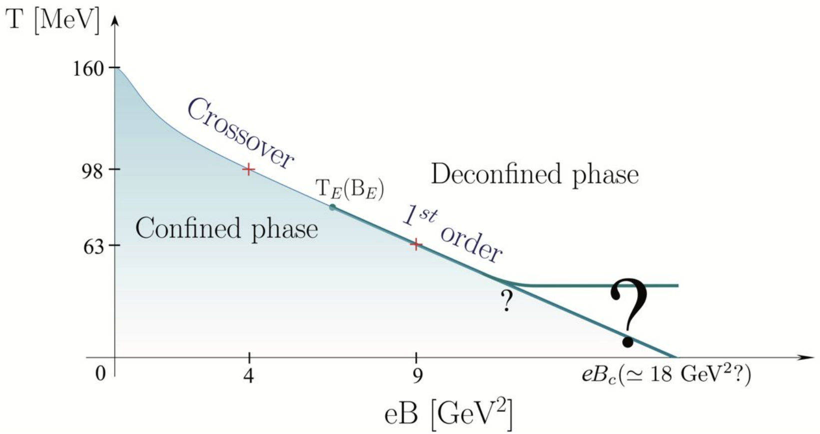


JHEP **02** (2012) 044

Change in the phase transition



JHEP **07** (2015) 173



Phys. Rev. D **105** (2022) 3, 034511

However, ...

Phys.Lett.B **731** (2014) 154-158

ABSTRACT

We explore the parameter space of the two-flavor thermal quark–meson model and its Polyakov loop-extended version under the influence of a constant external magnetic field B. We investigate the behavior of the pseudo critical temperature for chiral symmetry breaking taking into account the likely dependence of two parameters on the magnetic field: the Yukawa quark–meson coupling and the parameter T_0 of the Polyakov loop potential. Under the constraints that magnetic catalysis is realized at zero temperature and the chiral transition at $B = 0$ is a crossover, we find that the quark–meson model leads to thermal magnetic catalysis for the whole allowed parameter space, in contrast to the present picture stemming from lattice QCD.

Linear Sigma Model with quarks

- Effective model for low-energy QCD.
- Renormalizable.
- Symmetry spontaneously broken.
- Quarks and mesons involved in the chiral phase transition

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(D_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + i\bar{\psi}\gamma^\mu D_\mu \psi - g\bar{\psi}(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi})\psi,$$

Free parameters

$$\lambda \qquad g \qquad a^2 > 0$$

Covariant derivative

$$D_\mu = \partial_\mu + iq_{f,b} A_\mu$$

$$A^\mu = \frac{B}{2}(0, -y, x, 0).$$

Spontaneous symmetry breaking

$$\sigma \to \sigma + v.$$

Linear Sigma Model with quarks

- As a result of the *shift*

$$\mathcal{L} = -\frac{1}{2}[\sigma(\partial_\mu + iqA_\mu)^2\sigma] - \frac{1}{2}\left(3\lambda v^2 - a^2\right)\sigma^2$$
$$-\frac{1}{2}[\vec{\pi}(\partial_\mu + iq_bA_\mu)^2\vec{\pi}] - \frac{1}{2}\left(\lambda v^2 - a^2\right)\vec{\pi}^2$$
$$+\frac{a^2}{2}v^2 - \frac{\lambda}{4}v^4 + i\bar{\psi}\gamma^\mu D_\mu\psi - gv\bar{\psi}\psi + \mathcal{L}_I^b + \mathcal{L}_I^f,$$

$$\mathcal{L}_I^b = -\frac{\lambda}{4}\left[(\sigma^2 + (\pi^0)^2)^2 + 4\pi^+\pi^-(\sigma^2 + (\pi^0)^2 + \pi^+\pi^-)\right],$$
$$\mathcal{L}_I^f = -g\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi.$$

- Dynamical masses

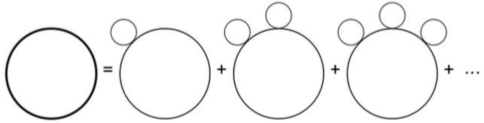
$$m_\sigma^2 = 3\lambda v^2 - a^2,$$
$$m_\pi^2 = \lambda v^2 - a^2,$$
$$m_f = gv.$$

- Classical potential

$$V^{\rm tree}(v) = -\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4 - hv,$$

Free energy beyond mean field approximation

Lowest Landau Level



$V^{\text{eff}} = \text{classic} + \text{1-loop} + \text{Ring diagrams}$

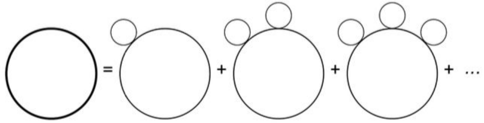
$$V_b^{1,0} = \frac{T}{2}\sum_n \int \frac{d^3k}{(2\pi)^3} \ln\left[G(\omega_n, \vec{k})^{-1}\right], \quad V_b^{1,B} = \frac{T}{2}\sum_n \int \frac{d^3k}{(2\pi^3)} \ln\left[G^{\text{LLL}}(\omega_n, \vec{k}, |eB|)^{-1}\right], \quad V_f^1 = -N_c T \sum_n \int \frac{d^3k}{(2\pi)^3} \text{Tr}[S^{LLL}(\tilde{\omega}_n, \vec{k})^{-1}],$$

$$iG(\omega_n, \vec{k}) = -\frac{i}{\omega_n^2 + \vec{k}^2 + m_b^2}, \quad iG^{LLL}(k) = 2i\frac{e^{-\frac{k_\perp^2}{|eB|}}}{\omega_n^2 + k_3^2 + m_b^2 + |eB|}. \quad iS_f^{LLL}(\tilde{\omega}_n, \vec{k}) = -2ie^{-\frac{k_\perp^2}{2|qB|}} \frac{i\gamma_0\tilde{\omega}_n - \gamma_3 k^3 + m_f}{\tilde{\omega}_n^2 + k_3^2 + m_f^2} \mathcal{O}^{\pm}$$

$$\boxed{V^{ring} = \frac{T}{2}\sum_n \int \frac{d^3k}{(2\pi)^3} \ln\left(1 + \Pi\, G\right),}$$

Free energy beyond mean field approximation

~~Lowest Landau Level~~



$V^{\mathrm{eff}} = \text{classic} + \text{1-loop} + \text{Ring diagrams}$

Latin Landau Level

$$V_b^{1,0} = \frac{T}{2}\sum_n \int \frac{d^3k}{(2\pi)^3} \ln\left[G(\omega_n, \vec{k})^{-1}\right], \quad V_b^{1,B} = \frac{T}{2}\sum_n \int \frac{d^3k}{(2\pi^3)} \ln\left[G^{\mathrm{LLL}}(\omega_n, \vec{k}, |eB|)^{-1}\right], \quad V_f^1 = -N_c T \sum_n \int \frac{d^3k}{(2\pi)^3} \mathrm{Tr}[S^{LLL}(\tilde{\omega}_n, \vec{k})^{-1}],$$

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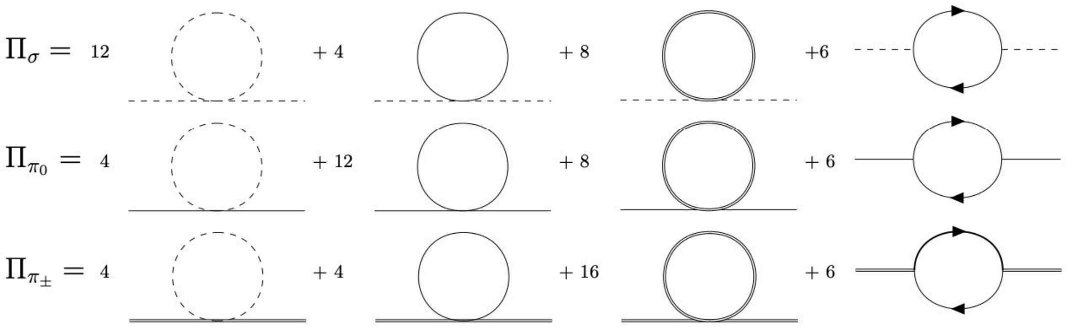
Self-energy of the bosonic fields



$$\Pi_\sigma = \frac{3\lambda}{2}\left(\frac{T^2}{6} - \frac{T\sqrt{m_\sigma^2}}{2\pi} - \frac{m_\sigma^2}{8\pi^2}\left(1 - 2\gamma_E - \ln\left(\frac{m_\sigma^2}{(4\pi T)^2}\right)\right)\right) + \lambda\left(\frac{T^2}{6} - \frac{T\sqrt{m_\pi^2}}{2\pi} - \frac{m_\pi^2}{8\pi^2}\left(1 - 2\gamma_E - \ln\left(\frac{m_\pi^2}{(4\pi T)^2}\right)\right)\right)$$
$$+ \frac{\lambda|eB|}{2\pi^2}\ln\left(\frac{\mu^2}{m_\pi^2 + |eB|}\right) + \frac{\lambda|eB|}{2\pi^2}\sum_{n=1}^{\infty} K_0\left(\frac{n\sqrt{m_\pi^2 + |eB|}}{T}\right) - 6\left(\frac{4g^2|qB|}{\pi^2}\sum_{n=1}^{\infty}(-1)^n K_0\left(\frac{ngf_\pi}{T}\right)\right), \qquad (39)$$

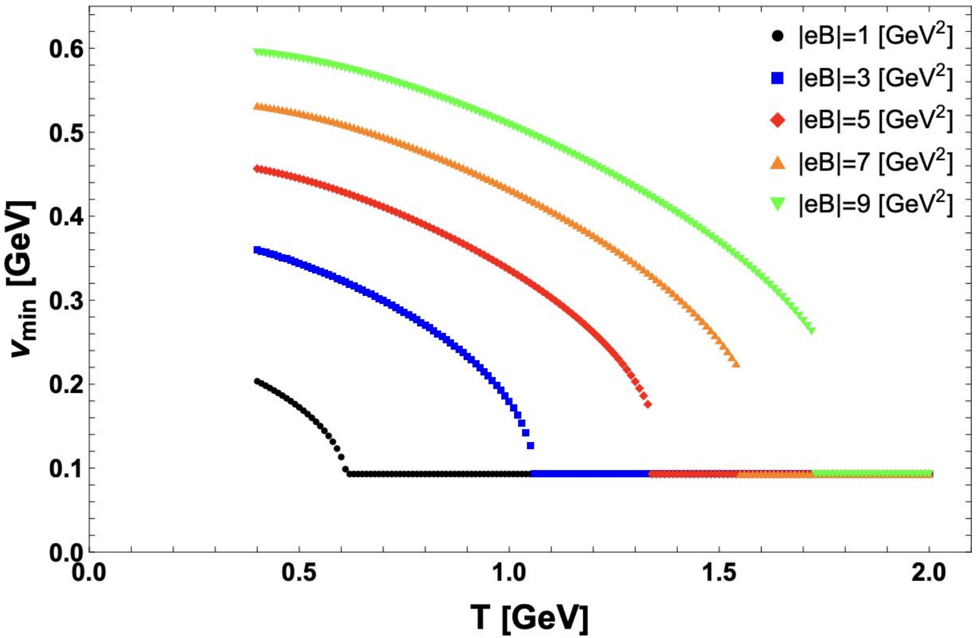
$$\Pi_{\pi_0} = \lambda\left(\frac{T^2}{6} - \frac{T\sqrt{m_\sigma^2}}{2\pi} - \frac{m_\sigma^2}{8\pi^2}\left(1 - 2\gamma_E - \ln\left(\frac{m_\sigma^2}{(4\pi T)^2}\right)\right)\right) + \frac{3\lambda}{2}\left(\frac{T^2}{6} - \frac{T\sqrt{m_\pi^2}}{2\pi} - \frac{m_\pi^2}{8\pi^2}\left(1 - 2\gamma_E - \ln\left(\frac{m_\pi^2}{(4\pi T)^2}\right)\right)\right)$$
$$+ \frac{\lambda|eB|}{2\pi^2}\ln\left(\frac{\mu^2}{m_\pi^2 + |eB|}\right) + \frac{\lambda|eB|}{2\pi^2}\sum_{n=1}^{\infty} K_0\left(\frac{n\sqrt{m_\pi^2 + |eB|}}{T}\right) - 6\left(\frac{4g^2|qB|}{\pi^2}\sum_{n=1}^{\infty}(-1)^n K_0\left(\frac{ngf_\pi}{T}\right)\right), \qquad (40)$$

$$\Pi_{\pi_\pm} = \lambda\left(\frac{T^2}{6} - \frac{T\sqrt{m_\sigma^2}}{2\pi} - \frac{m_\sigma^2}{8\pi^2}\left(1 - 2\gamma_E - \ln\left(\frac{m_\sigma^2}{(4\pi T)^2}\right)\right)\right) + \lambda\left(\frac{T^2}{6} - \frac{T\sqrt{m_\pi^2}}{2\pi} - \frac{m_\pi^2}{8\pi^2}\left(1 - 2\gamma_E - \ln\left(\frac{m_\pi^2}{(4\pi T)^2}\right)\right)\right)$$
$$+ \frac{\lambda|eB|}{\pi^2}\ln\left(\frac{\mu^2}{m_\pi^2 + |eB|}\right) + \frac{\lambda|eB|}{\pi^2}\sum_{n=1}^{\infty} K_0\left(\frac{n\sqrt{m_\pi^2 + |eB|}}{T}\right). \qquad (41)$$

Effective potential

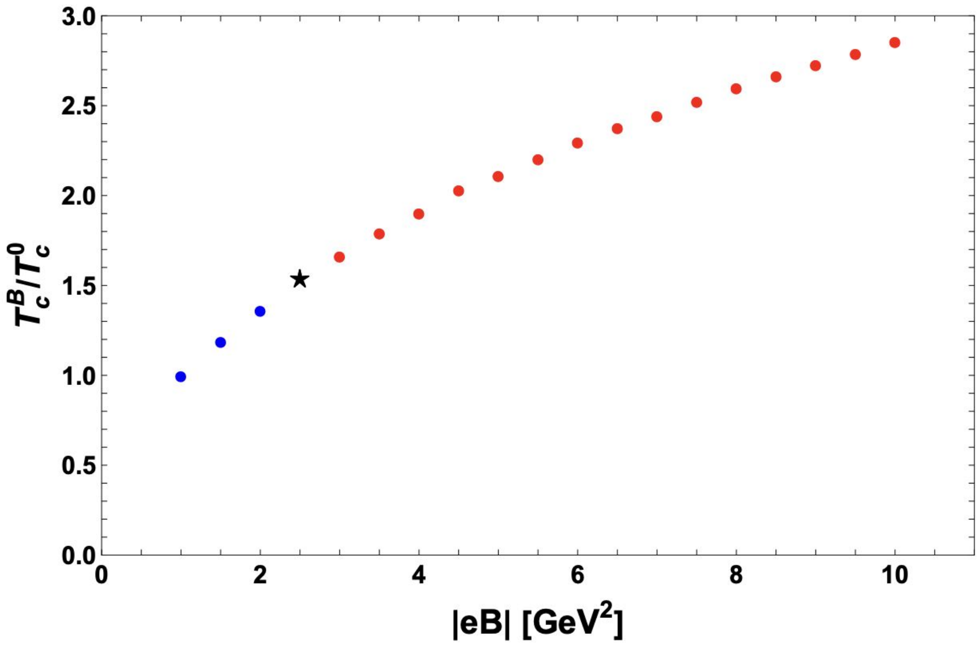
$$
\begin{aligned}
V_N^{\text{eff}} =& -\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4 - hv - \frac{m_\sigma^4}{64\pi^2}\left(\frac{3}{2} + \ln\left(\frac{\mu^2}{m_\sigma^2}\right)\right) - \frac{m_\pi^4}{64\pi^2}\left(\frac{3}{2} + \ln\left(\frac{\mu^2}{m_\pi^2}\right)\right) \\
&+ \frac{T}{2\pi^2}\int_0^\infty dk\, k^2 \ln\left(1 - e^{-\sqrt{k^2+m_\sigma^2+\Pi_\sigma}/T}\right) + \frac{T}{2\pi^2}\int_0^\infty dk\, k^2 \ln\left(1 - e^{-\sqrt{k^2+m_\pi^2+\Pi_{\pi_0}}/T}\right) \\
&+ \frac{|eB|}{2\pi^2}(m_\pi^2 + |eB|)\left(1 + \ln\left(\frac{\mu^2}{m_\pi^2 + |eB|}\right)\right) - \frac{T|eB|}{\pi^2}\sqrt{m_\pi^2 + |eB|}\sum_{n=1}^\infty \frac{1}{n}K_1\left(\frac{n\sqrt{m_\pi^2 + |eB|}}{T}\right) \\
&- \frac{3|qB|}{4\pi^2}m_f^2\left(1 + \ln\left(\frac{\mu^2}{m_f^2}\right)\right) - \frac{6|qB|T}{\pi^2}m_f\sum_{n=1}^\infty \frac{(-1)^{n-1}}{n}K_1\left(\frac{nm_f}{T}\right),
\end{aligned}
$$

vev as an order parameter



λ=13.32, g=2.58 y a=0.309 GeV (vacuum parameters)

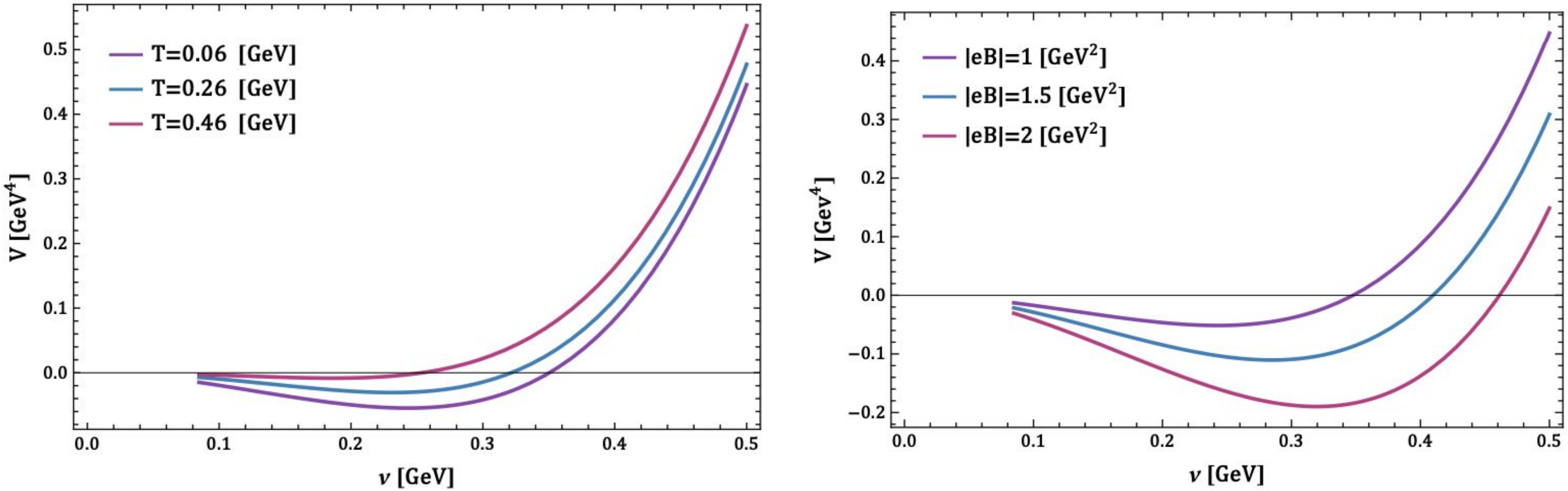
Effective phase diagram



✓ Critical End Point.

✗ Inverse Magnetic Catalysis.

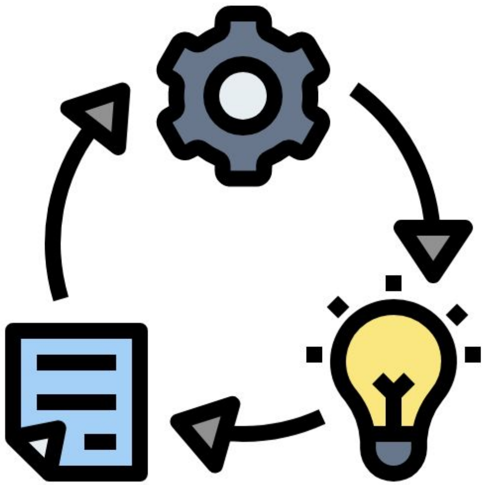
Without approximations, but something is not right



$$m_\sigma^2 = 3\lambda v^2 - a^2,$$
$$m_\pi^2 = \lambda v^2 - a^2, \quad \longleftrightarrow \quad -\frac{m_\sigma^4}{64\pi^2}\left(\frac{3}{2}+\ln\left(\frac{\mu^2}{m_\sigma^2}\right)\right)-\frac{m_\pi^4}{64\pi^2}\left(\frac{3}{2}+\ln\left(\frac{\mu^2}{m_\pi^2}\right)\right)$$
$$m_f = gv.$$

A new alternative

- We will allow all the free parameters of the theory to become variable, without a functional form for each one of them.
- We will look for a new functional form of the masses.



Self-consistent masses

System of coupled equations

$$M_\sigma^2(T,B) = 3\lambda v^2 - a^2 + \frac{3}{2}\Pi_b^0\left(M_\sigma^2, T\right) + \Pi_b^0\left(M_{\pi_0}^2, T\right) + 2\Pi_b^B\left(M_{\pi_\pm}^2, T, B\right) + 6\Pi_f(M_f, T, B),$$

$$M_{\pi_0}^2(T,B) = \lambda v^2 - a^2 + \Pi_b^0\left(M_\sigma^2, T\right) + \frac{3}{2}\Pi_b^0\left(M_{\pi_0}^2, T\right) + 2\Pi_b^B\left(M_{\pi_\pm}^2, T, B\right) + 6\Pi_f(M_f, T, B),$$

$$M_{\pi_\pm}^2(T,B) = \lambda v^2 - a^2 + \Pi_b^0\left(M_\sigma^2, T\right) + \Pi_b^0\left(M_{\pi_0}^2, T\right) + 6\Pi_b^B\left(M_{\pi_\pm}^2, T, B\right). \quad (4$$

Self-energies

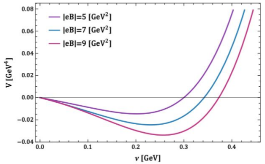
$$\Pi_b^0 = \frac{\lambda}{\pi^2}\int_{-\infty}^{\infty} dk \, \frac{k^2}{\sqrt{k^2 + M_b^2}} \frac{1}{e^{\sqrt{k^2+M_b^2}/T} - 1}$$

$$\Pi_b^B = \frac{\lambda|eB|}{4\pi^2}\ln\left(\frac{\mu^2}{M_b^2 + |eB|}\right) + \frac{\lambda|eB|}{2\pi^2}\sum_{n=1}^{\infty} K_0\left(\frac{n\sqrt{M_b^2 + |eB|}}{T}\right)$$

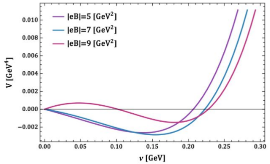
$$\Pi_f = -\frac{4g^2|qB|}{\pi^2}\sum_{n=1}^{\infty}(-1)^n K_0\left(\frac{nM_f}{T}\right)$$

Effective potential with self-consistent masses

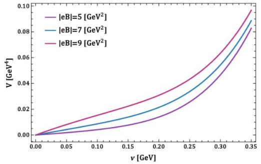
$$
\begin{aligned}
V_{sc} =& V_{sc}^{vac} + V_{0,T}(M_\sigma^2(T,B),T) + V_{0,T}(M_{\pi_0}^2(T,B),T) \\
&+ V_{b,vac}(M_{\pi_\pm}^2(0,B),B) + V_{b,T}(M_{\pi_\pm}^2(T,B),T,B) \\
&+ V_{f,vac}(gv+\eta_0 B,B) + V_{f,T}(gv+\eta B,T,B),
\end{aligned}
$$



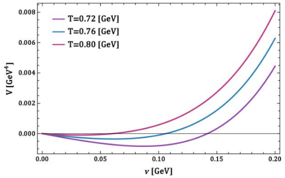
(a) $T = 0.48\,\mathrm{GeV}$



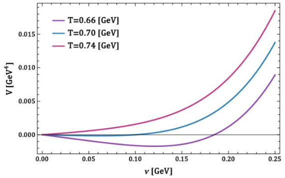
(b) $T = 0.62\,\mathrm{GeV}$



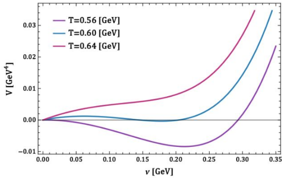
(c) $T = 0.78\,\mathrm{GeV}$



(a) $|eB| = 3\,\mathrm{GeV}^2$

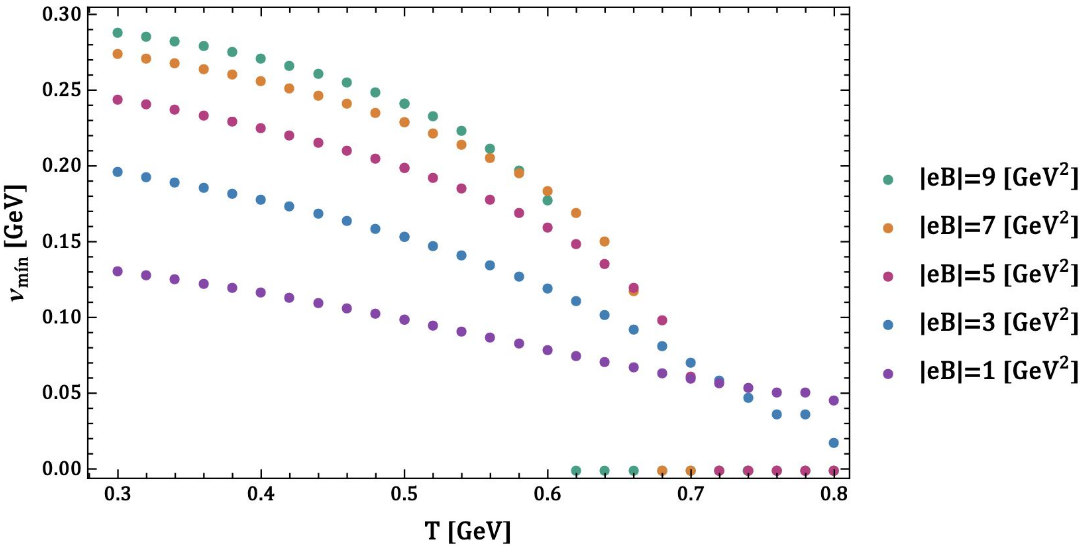


(b) $|eB| = 5\,\mathrm{GeV}^2$



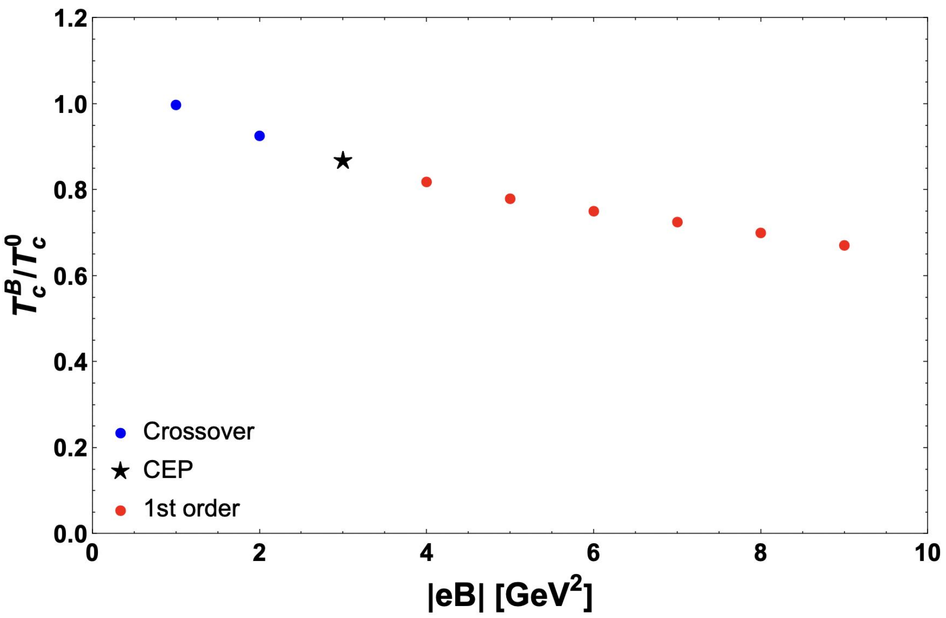
(c) $|eB| = 9\,\mathrm{GeV}^2$

Phase transition analysis



λ=13.32, g=0.36 and a=0.309 GeV

Effective phase diagram 2.0



✓ Critical End Point.

✓ Inverse Magnetic Catalysis.

A unique result due to its nature.

Future work

- Compute all contributions to all self-energies, especially fermionic ones.
- Use AI to find the general analytic expression for the masses.
- Valid calculation for any magnetic field strength.

Conclusions

- In the LSMq, the masses should capture collective effects of the medium without any restriction, and it allows going beyond mean field robustly, generating consistent results.
- The Linear Sigma Model with quarks is successful in exploring different phase diagrams of the strong interaction.
- A systematic study of the strong phase transition has been developed under different conditions, gradually approaching the physical conditions of the systems of interest.

Thank you for your attention!

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