Signatures of Local Acceleration of Quark-Gluon Plasma in Dilepton Production

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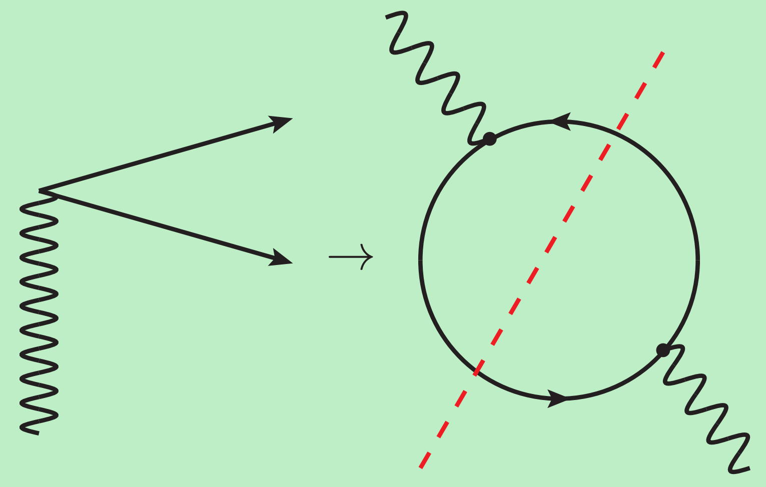


9th Conference on Chirality, Vorticity and Magnetic Fields in Quantum Matter

Signatures of Quark-gluon-plasma

Heavy Ion Collisions create an isolated Quantum system, which is :

- Initially far away from equilibrium
- Self - interacting
- Expanding against the vacuum

To know more about this system, we need to probe it.
These probes are also known as Signatures.

Electromagnetic probes as QGP signatures

Electromagnetic probes, i.e. Dileptons, Photons are uniquely built as candidates for QGP signatures

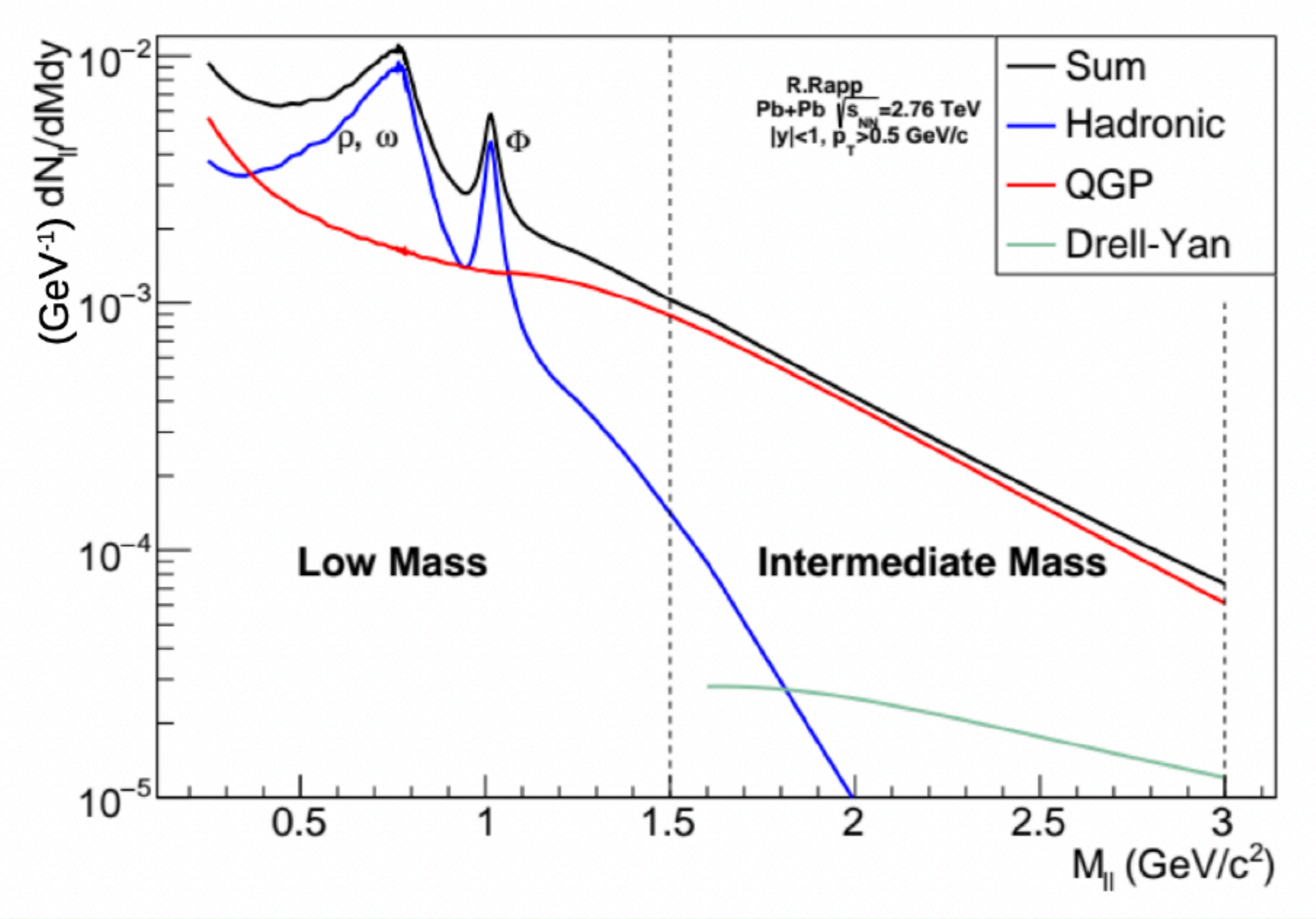
- Carries information from all the important stages of Heavy-Ion-Collisions.
- No strong interaction.
- Mean free path in medium > medium size.
- Escape the medium virtually unscathed.

More information !
Least contamination !!

Contributing processes

Electromagnetic probes in HIC

- Drell-Yan
- Pre-equilibrium
- Thermal / QGP
- Hadronic decays
- Others

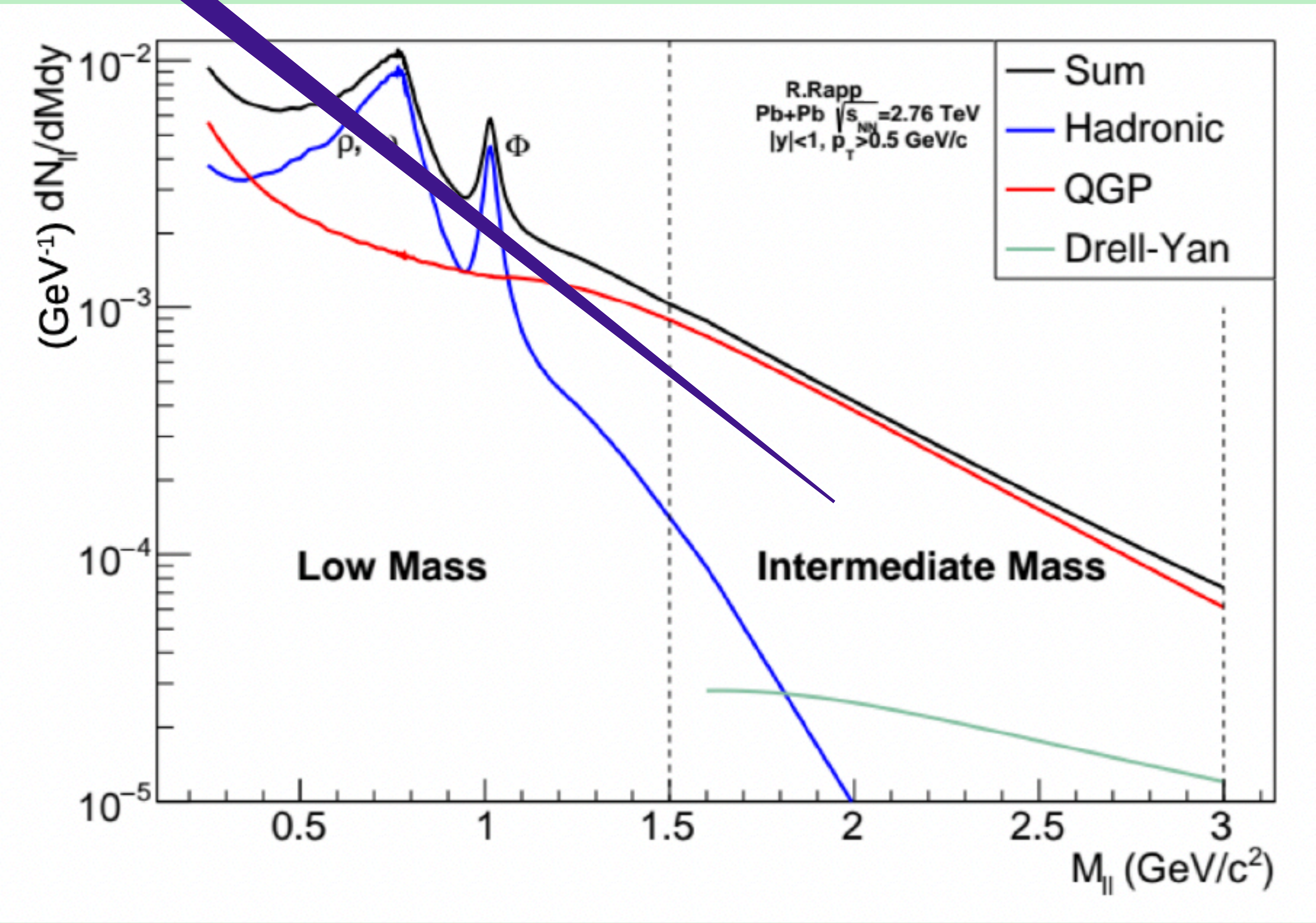


Contributing processes

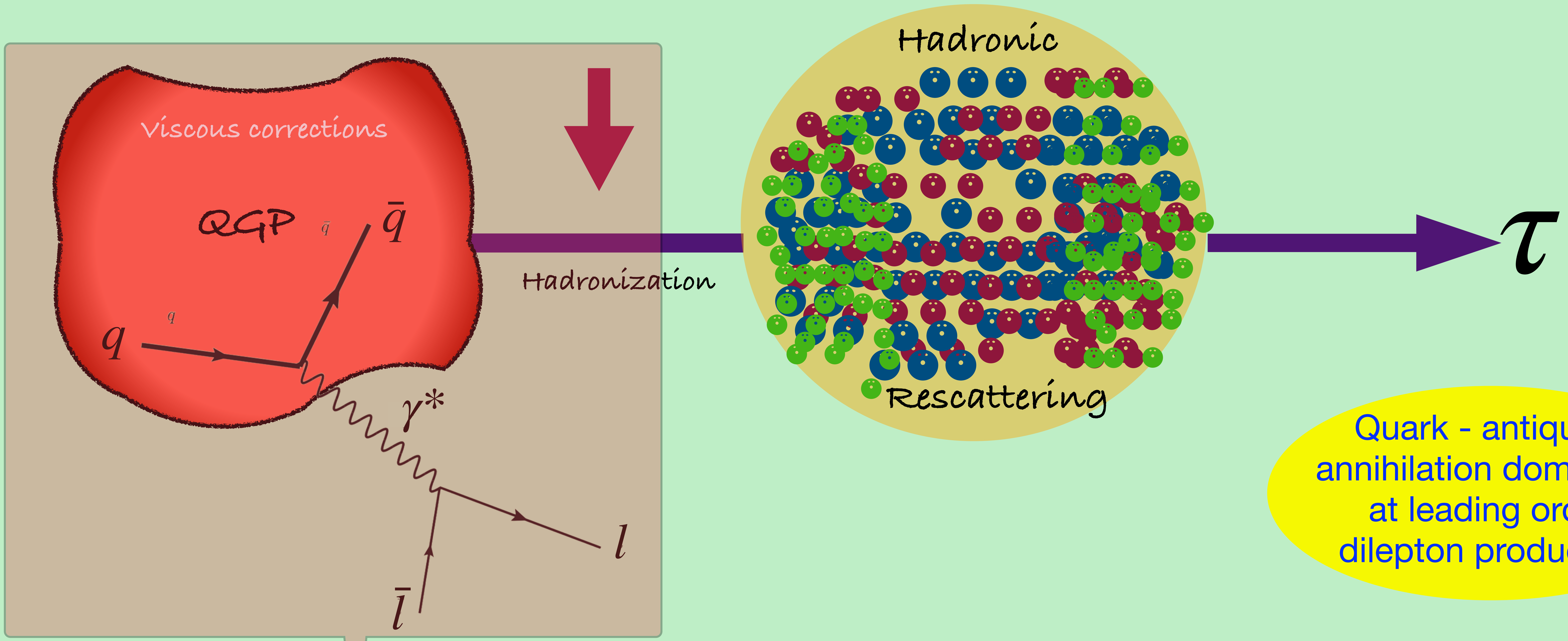
Electromagnetic probes in HIC

We are interested in

- Drell-Yan
- Pre-equilibrium
- Thermal / QGP
- Hadronic decays
- Others



Leading order Dilepton production



Quark - antiquark annihilation dominates at leading order dilepton production

$$\frac{dN}{d^4Q d^4X} \equiv$$ Dilepton Rate (Static/snapshot property)

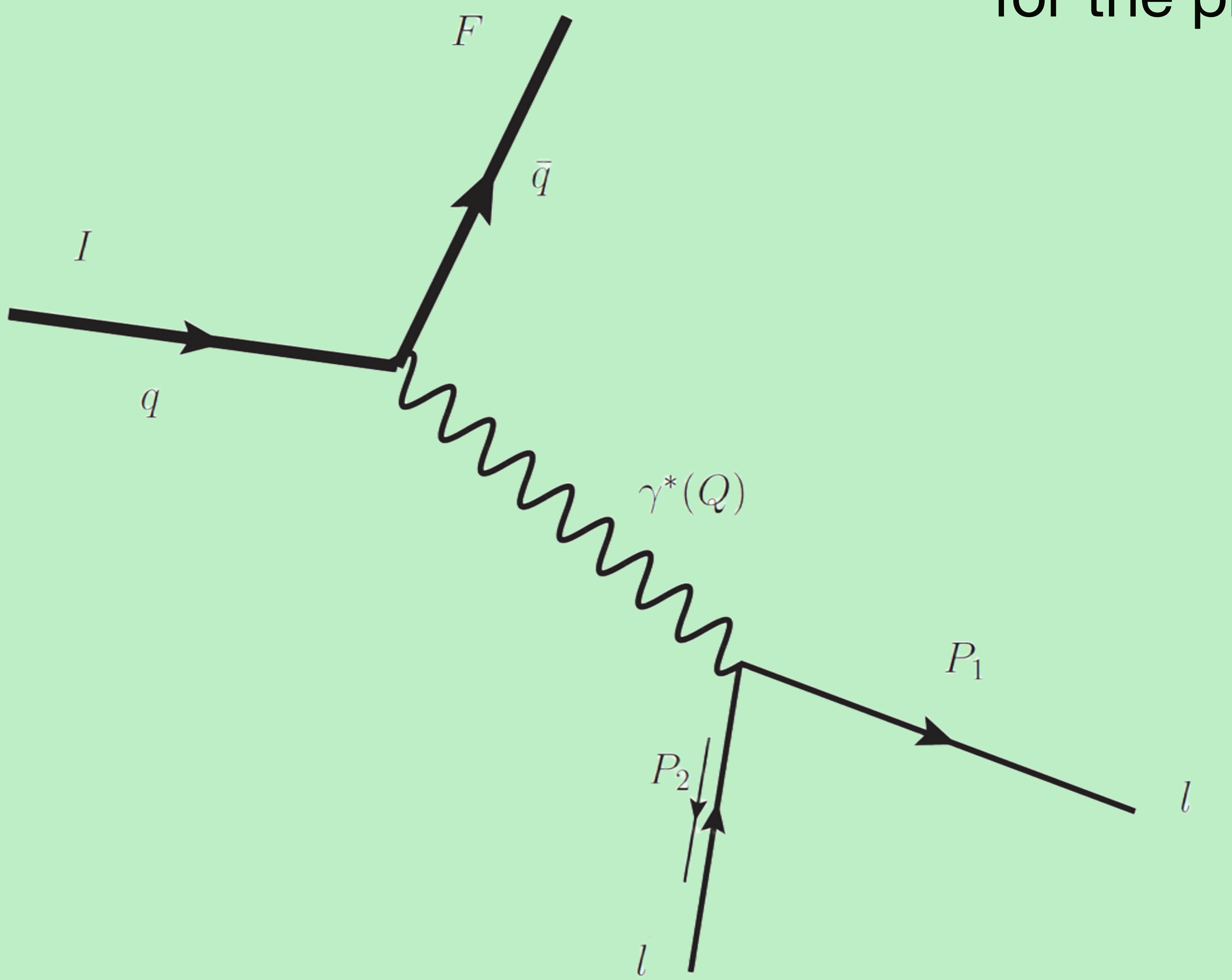
Spacetime evolution →

Dilepton Spectra (Dynamic property) $$\equiv \frac{dN}{d^4Q}$$

Dilepton Rate - Formalism

Weldon ; PRD 42 (1990)

The thermally averaged dilepton multiplicity in the local rest frame of the plasma for the process $I \to l(P_2)\bar{l}(P_1) + F$



Emission amplitude · Thermal weight · Phase space

$$N = \sum_I \sum_F |\langle F, l(P_1), \bar{l}(P_2) | S | I \rangle|^2 \frac{e^{-\beta E_I}}{Z} \frac{V d^3 p_1}{(2\pi)^3} \frac{V d^3 p_2}{(2\pi)^3},$$

$$\langle F, l(P_1), \bar{l}(P_2) | S | I \rangle = \frac{e_0 \; \bar{u}(P_2)\gamma_\mu v(P_1)}{V\sqrt{4E_1E_2}} \int d^4x \; e^{iq\cdot x} \langle F | A^\mu(x) | I \rangle,$$

$$N = e_0^2 \; L_{\mu\nu} \; \mathscr{M}^{\mu\nu} \; \frac{d^3 p_1}{E_1(2\pi)^3} \frac{d^3 p_2}{E_2(2\pi)^3}$$

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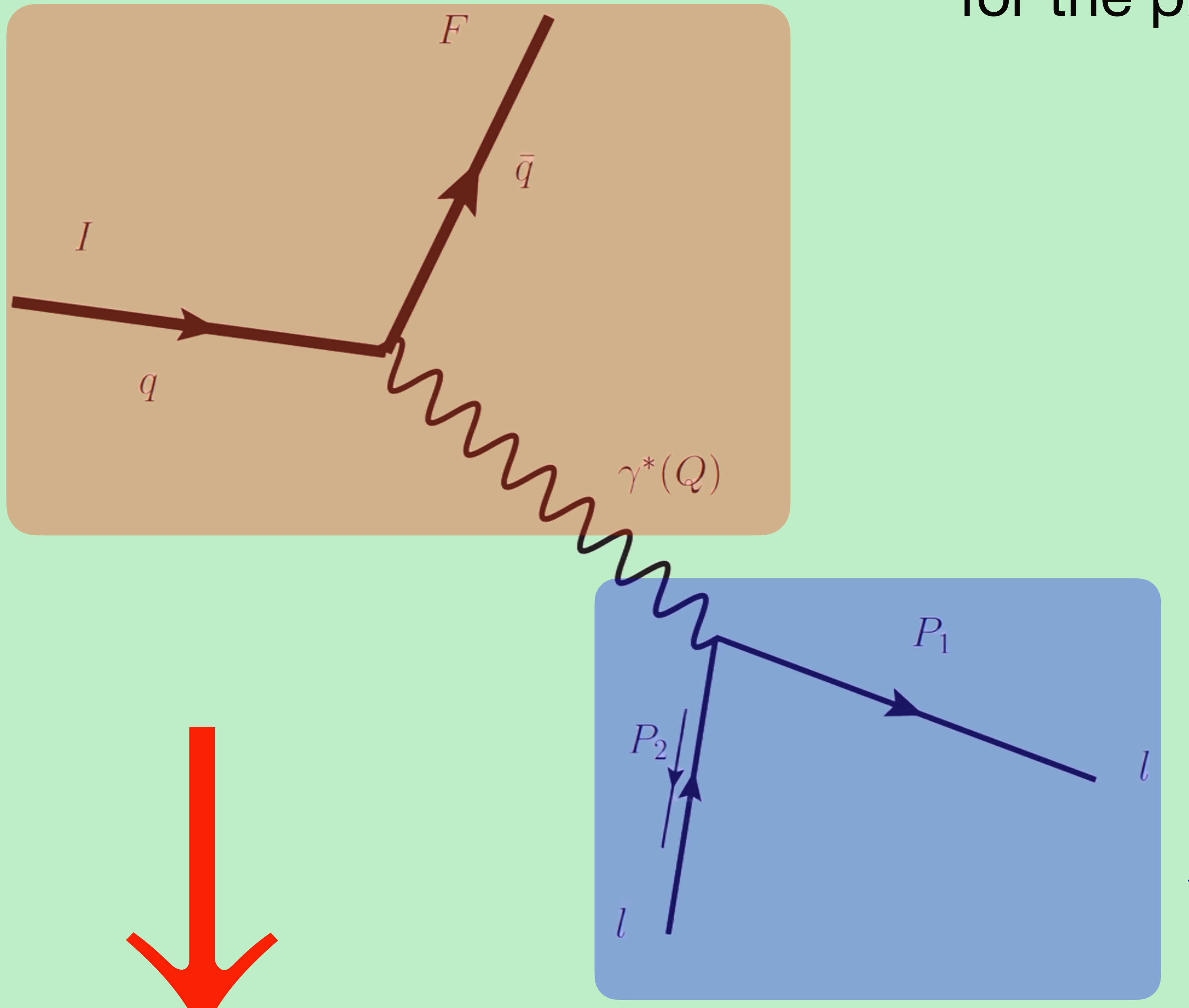

Emission amplitude | Thermal weight | Phase space

$$N = \sum_I \sum_F |\langle F, l(P_1), \bar{l}(P_2)|S|I\rangle|^2 \frac{e^{-\beta E_I}}{Z} \frac{Vd^3p_1}{(2\pi)^3}\frac{Vd^3p_2}{(2\pi)^3},$$

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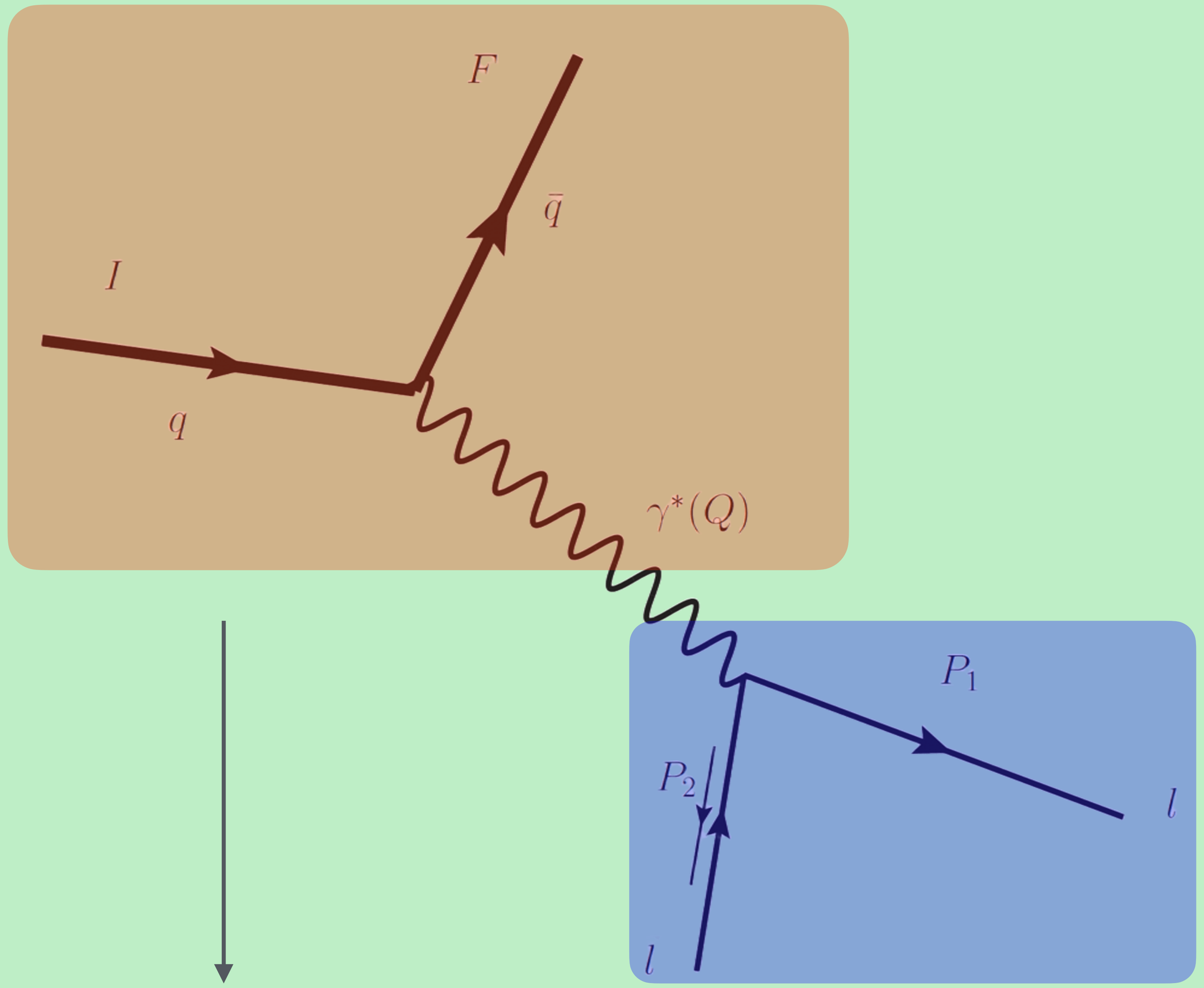
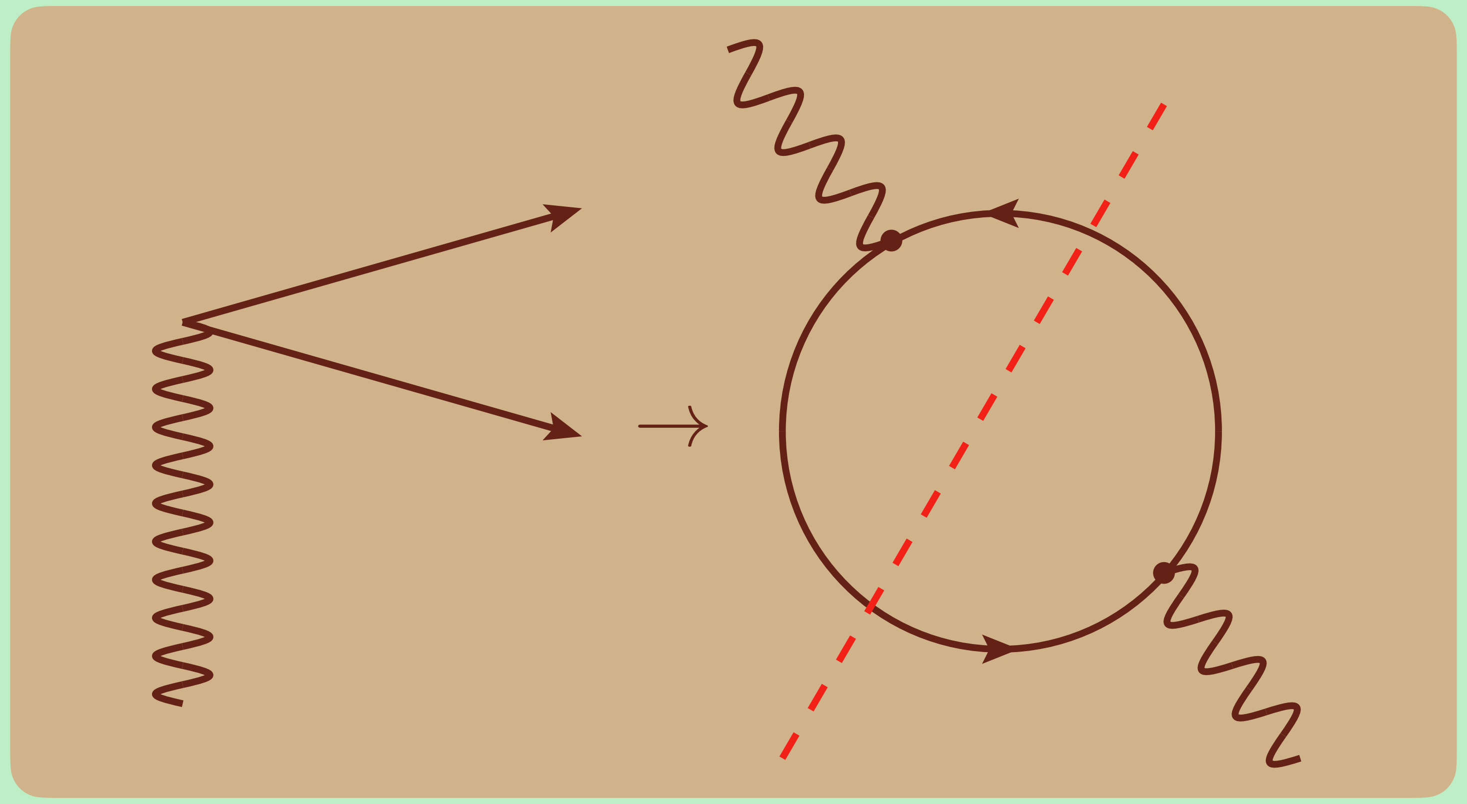
$$N = e_0^2\, L_{\mu\nu}\, \mathcal{M}^{\mu\nu} \frac{d^3p_1}{E_1(2\pi)^3}\frac{d^3p_2}{E_2(2\pi)^3}$$

$$L_{\mu\nu} = \frac{1}{4}\mathrm{Tr}\left[\bar{u}(P_2)\gamma_\mu v(P_1)\bar{v}(P_1)\gamma_\nu u(P_2)\right] = P_{1\mu}P_{2\nu} + P_{1\nu}P_{2\mu} - (P_1\cdot P_2 + m_l^2)g_{\mu\nu}$$

$$\mathcal{M}^{\mu\nu} = \sum_F\sum_I \int d^4x d^4y\, e^{iq\cdot(x-y)}\langle F|A^\mu(x)|I\rangle \times \frac{1}{Z}\langle I|\hat{\rho}\, A^\nu(y)|F\rangle$$

Dilepton Rate - Formalism





$$\mathcal{M}^{\mu\nu} = \sum_F \sum_I \int d^4x\, d^4y\, e^{iq\cdot(x-y)} \langle F | A^\mu(x) | I \rangle \times \frac{1}{Z} \langle I | \hat{\rho}\, A^\nu(y)\, \hat{\rho}^{-1} \hat{\rho} | F \rangle$$

$$= \int d^4x\, d^4y\, e^{iq\cdot(x-y)} \sum_F \langle F | A^\mu(x)\, A^\nu(y_0 + i\beta, \mathbf{y})\, \hat{\rho} | F \rangle \frac{1}{Z}$$

$$= e^{-\beta q_0} \int d^4x\, d^4y\, e^{iq\cdot(x-y)} \sum_F \langle F | A^\mu(x) A^\nu(y) | F \rangle \frac{e^{-\beta E_F}}{Z}$$

$$= 2\pi\, e^{-\beta q_0}\, \Omega \int \frac{d^4x}{2\pi}\, e^{iq\cdot x} \sum_F \langle F | A^\mu(x) A^\nu(0) | F \rangle \frac{e^{-\beta E_F}}{Z}$$

$$\rho^{\mu\nu}(q) = -\frac{1}{\pi} \frac{e^{\beta q_0}}{e^{\beta q_0} - 1} \frac{e_f^2}{q^4} \,\mathrm{Im}\left[C^{\mu\nu}(q)\right]$$

$$\frac{dN}{d^4x d^4q} = \frac{\alpha_{em}}{12\pi^3} \frac{e_f^2}{q^2} \frac{1}{e^{\beta q_0} - 1} \sum_{f=u,d} \frac{1}{\pi}\, \mathrm{Im}\, C^\mu_{\mu,f}(q)$$

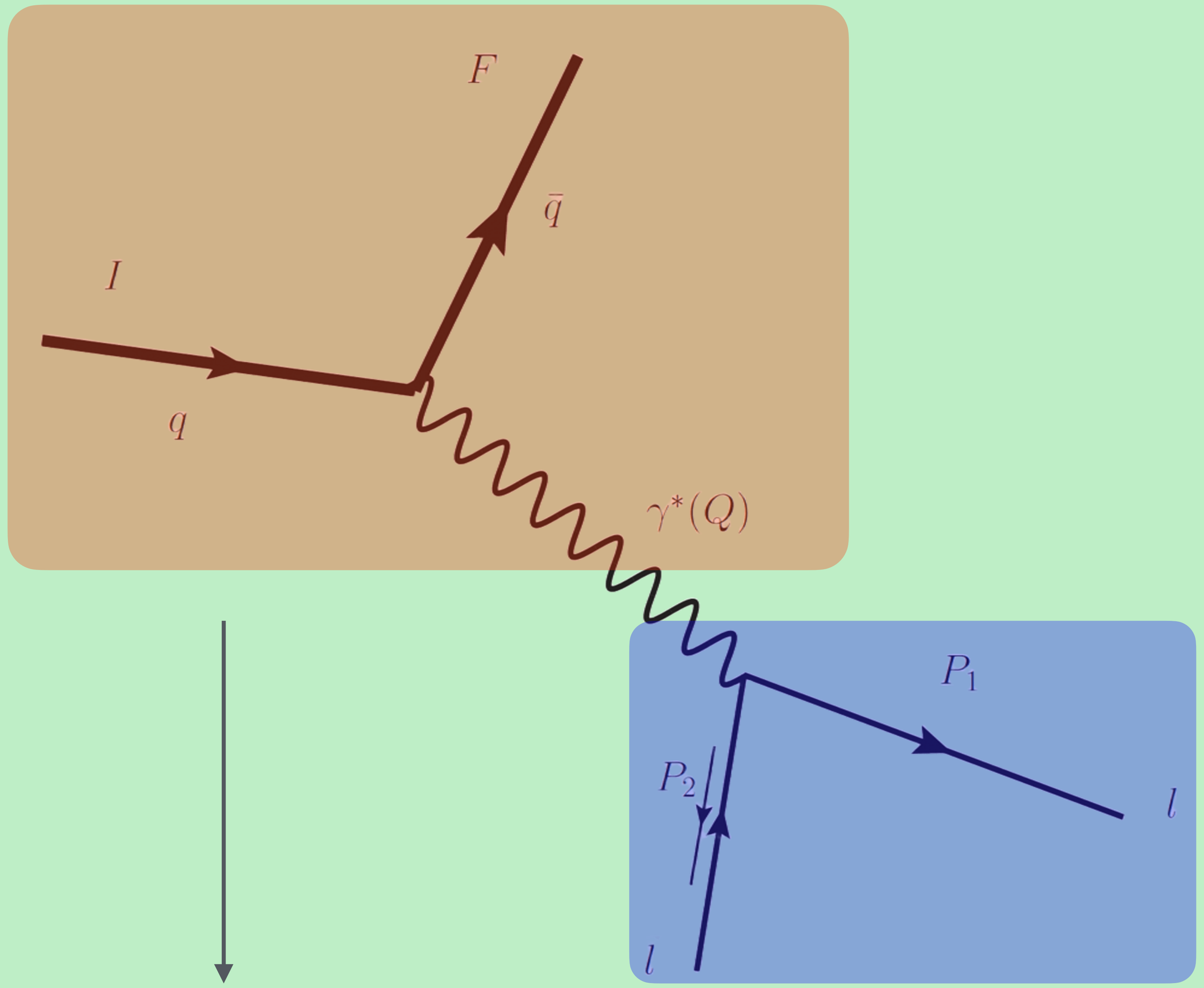
*Disclaimer - Formalism derived for **static medium**

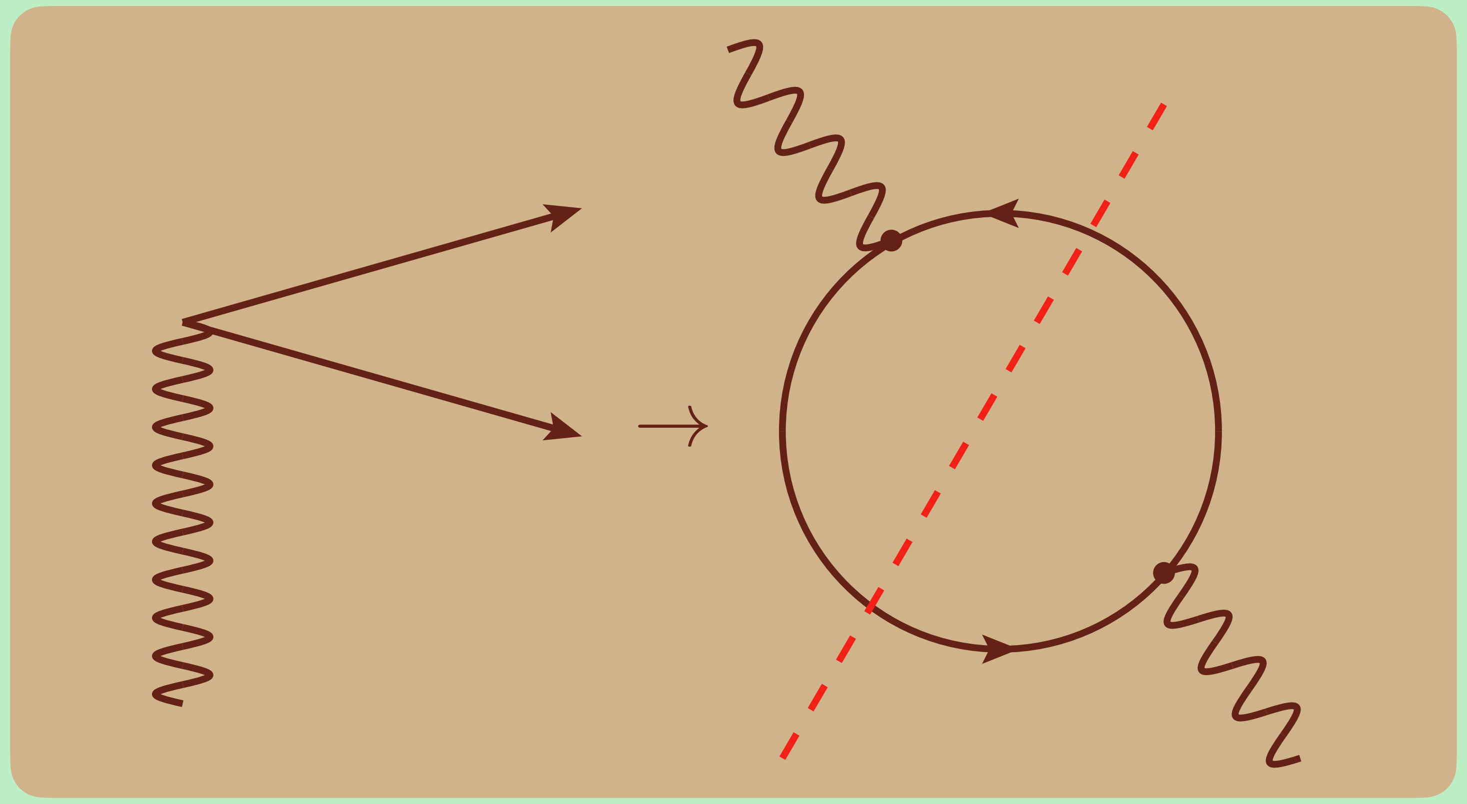
For the present work → Considered modifications only in the **correlation function** C^μ_μ

$$C^\mu_\mu(q) = \int d^4x\, e^{iq\cdot x}\, \mathrm{Tr}_{Dfc}\left[\gamma^\mu\, S(x,0)\, \gamma_\mu\, S(0,x)\right]$$

Weldon ; PRD 42 (1990)

Dilepton Rate - Formalism





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$$\left[C^\mu_\mu\right]^\alpha(q) = \int d^4x\, e^{iq\cdot x}\, \mathrm{Tr}_{Dfc}\left[\gamma^\mu\, S^\alpha(x,0)\, \gamma_\mu\, S^\alpha(0,x)\right]$$

Weldon ; PRD 42 (1990)

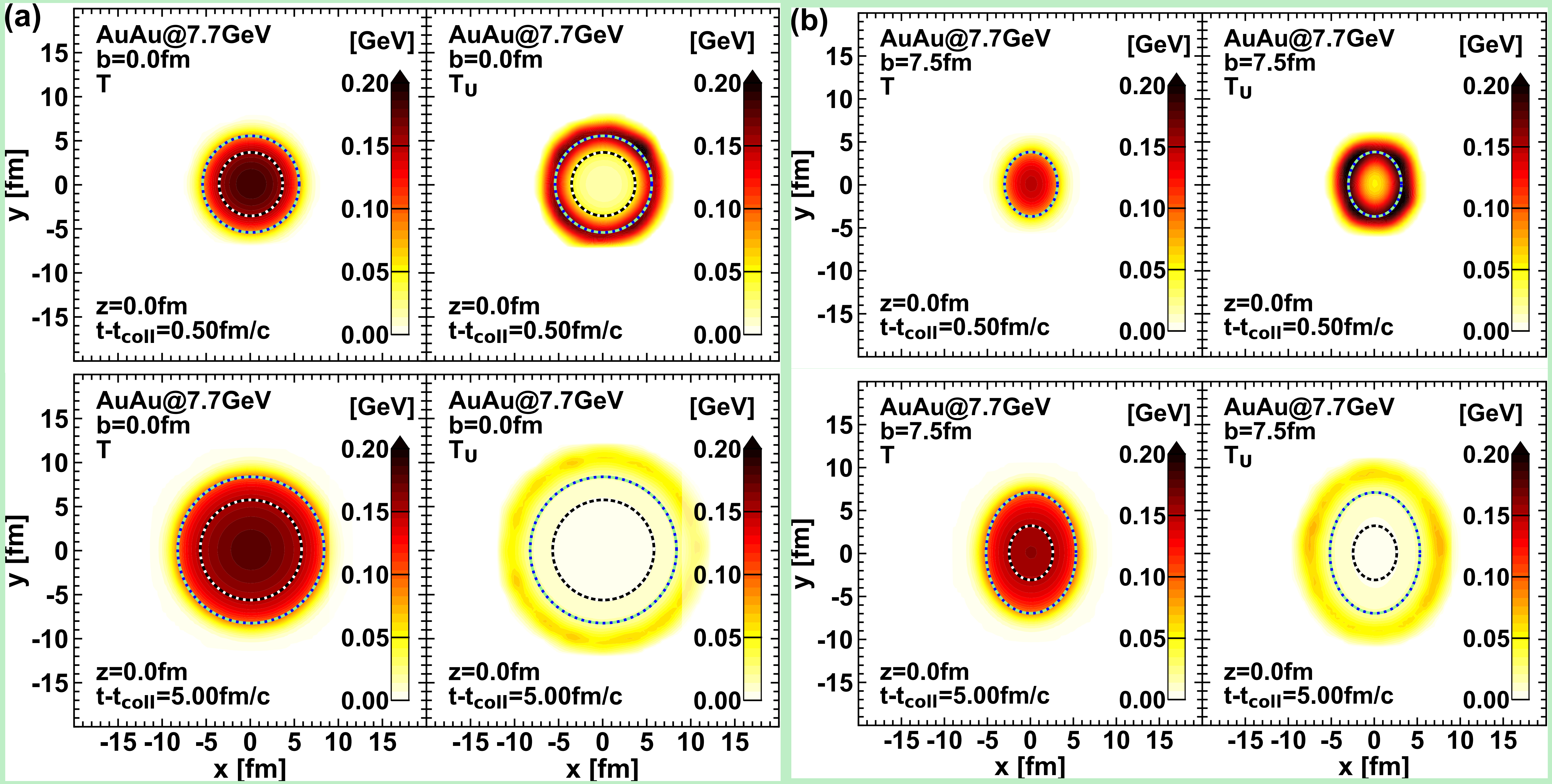
Local acceleration in HIC

Unruh ; PRD 14 (1976)

Unruh effect : an observer detects an apparent thermal radiation in an uniformly accelerated frame -

$$T_U = \frac{a}{2\pi}$$

Ultra-relativistic heavy Ion collisions → Large acceleration immediately after the collisions → Perfect place to test the effect due to acceleration



Prokhorov et. al. ; 2502.10146

A natural question : How acceleration affects the observables / signatures from different stages of HIC ?

Present work → We are interested in QGP phase → The acceleration becomes reasonably weak!

Dirac propagator with weak acceleration

Ambruș & Chernodub ; PLB 855 (2024)

Density matrix in an accelerated medium : $\hat{\rho} = \exp\left[-\beta(\widehat{H} - a\widehat{K}^{z})\right]$

$$\hat{\rho}\hat{\phi}(x_\perp; t, z)\hat{\rho}^{-1} = e^{-\alpha\mathcal{S}^{0z}}\hat{\phi}(x_\perp; \tilde{t}, \tilde{z})$$

$$\alpha = \frac{a}{T}$$

$$T(x) = \frac{1}{\beta\sqrt{(1+az)^2 - (at)^2}},$$

$$u^\mu\partial_\mu = \frac{(1+az)\partial_t + (at)\partial_z}{\sqrt{(1+az)^2 - (at)^2}},$$

$$a^\mu\partial_\mu = a \times \frac{(at)\partial_t + (1+az)\partial_z}{(1+az)^2 - (at)^2}.$$

KMS relation : $S_E(x_\perp, \tilde{\tau}, \tilde{z}; x') = -\, e^{\alpha\mathcal{S}^{0z}} S_E(x_\perp, \tau, z; x')$

$$S_E^{(\alpha)}(x_\perp, \tau, z; x') = \sum_{j=-\infty}^{\infty} (-1)^j\, e^{-j\alpha S^{0z}}\, S_E^{\text{vac}}(x_\perp, \tau_{(j)}, z_{(j)}; x')$$

$$e^{-j\alpha S^{0z}} = \cos\frac{j\alpha}{2} - i\gamma^0\gamma^3 \sin\frac{j\alpha}{2}$$

$$S_E^{\text{vac}}(x, x') = \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot(x-x')} \frac{m - \not{p}}{p^2 + m^2}$$

$$\tau_{(j)} = \tau\cos(j\alpha) - \frac{1}{a}(1+az)\sin(j\alpha)\,,$$

$$z_{(j)} = \tau\sin(j\alpha) + \frac{1}{a}(1+az)\cos(j\alpha) - \frac{1}{a}\,,$$

Dirac propagator with weak acceleration

Ambruș & Chernodub ; PLB 855 (2024)

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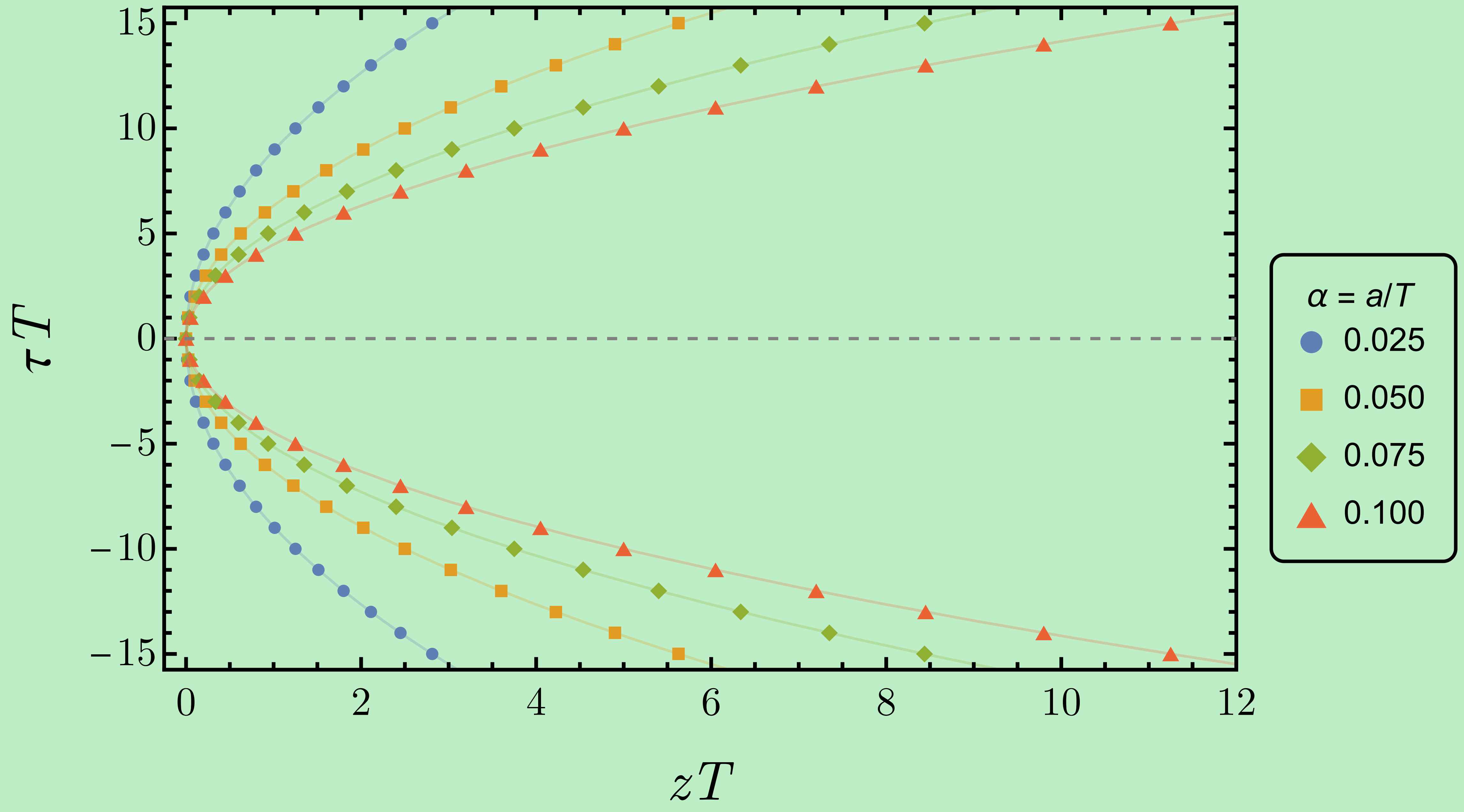
$$\tau_{(j)} = \tau - j\beta - zj\alpha + O(\alpha^2)$$
$$z_{(j)} = z + \tau j\alpha - \frac{\alpha\beta j^2}{2} + O(\alpha^2)$$



Dirac propagator with weak acceleration

$$S_E^{(\alpha)}(x_\perp, \tau, z; x') = \sum_{j=-\infty}^{\infty} (-1)^j \, e^{-j\alpha S^{0z}} \, S_E^{\rm vac}(x_\perp, \tau_{(j)}, z_{(j)}; x')$$

Ambruș & Chernodub ; PLB 855 (2024)

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$$\tau_{(j)} = \tau - j\beta - zj\alpha + O(\alpha^2)$$

$$z_{(j)} = z + \tau j\alpha - \frac{\alpha\beta j^2}{2} + O(\alpha^2)$$

↓

Translationally non-invariant contribution

↓

Need for coordinate transformation

$$\{x, x'\} \to \{X, \Delta x\} \quad X = \frac{x + x'}{2}; \quad \Delta x = x - x'$$

←

$$S_E^{(\alpha)}(X, \Delta x) = S_E^0(\Delta x) + \alpha \, S_E^1(X, \Delta x) + \mathcal{O}[\alpha^2]$$

$$S_E^0(\Delta x) = \frac{1}{\beta_T}\sum_n \int \frac{d^3p}{(2\pi)^3} e^{ip\cdot\Delta x} \frac{m - \not{p}}{p^2 + m^2}\bigg|_{p_\tau = \omega_n}$$

$$S_E^1(X, \Delta x) = \frac{1}{\beta_T^2}\sum_n \int \frac{d^3p}{(2\pi)^3} \frac{\partial}{\partial p_\tau}\left[e^{ip\cdot\Delta x}\left\{p_\tau\left(Z + \frac{\Delta z}{2}\right) - p_z\left(T + \frac{\Delta\tau}{2}\right) + \frac{\gamma^0\gamma^3}{2}\right\}\frac{m - \not{p}}{p^2 + m^2}\right]_{p_\tau = \omega_n} + \frac{i}{2\beta_T^2}\sum_n \int \frac{d^3p}{(2\pi)^3} \, p_z \frac{\partial^2}{\partial p_\tau^2}\left[e^{ip\cdot\Delta x}\frac{m - \not{p}}{p^2 + m^2}\right]_{p_\tau = \omega_n}$$

Correlation function with weak acceleration

$$C^{\mu\nu}_{(\alpha)}(q) = \frac{1}{\Omega}\int d^4X \int d^4\Delta x \; e^{-iq\cdot\Delta x} \; \mathrm{Tr}_{Dfc}\left[\gamma^\mu S_E^{(\alpha)}(X,\Delta x)\gamma^\nu S_E^{(\alpha)}(X,-\Delta x)\right],$$

$$= \frac{1}{\Omega}\int d^4X \int d^4\Delta x \; e^{-iq\cdot\Delta x} \; \mathrm{Tr}_{Dfc}\left[\gamma^\mu \left(S_E^0(\Delta x)+\alpha \; S_E^1(X,\Delta x)\right)\gamma^\nu \left(S_E^0(-\Delta x)+\alpha \; S_E^1(X,-\Delta x)\right)\right],$$

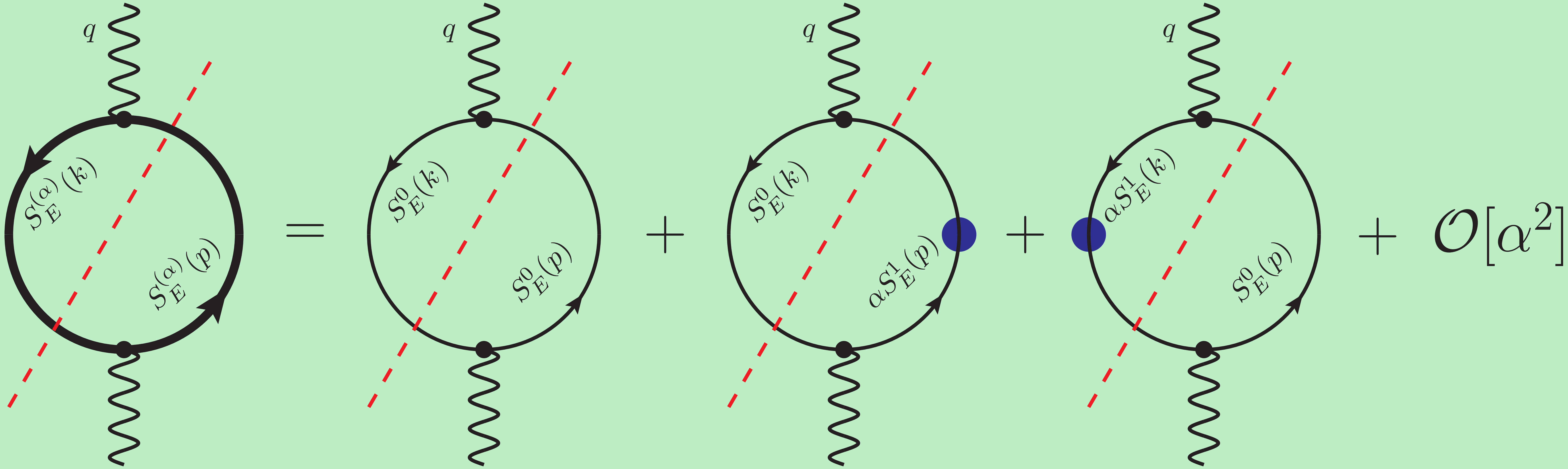
$$= \qquad C_0^{\mu\nu}(q) \qquad + \qquad \alpha \; C_1^{\mu\nu}(q) \qquad + \qquad \mathcal{O}[\alpha^2] \, .$$

Correlation function with weak acceleration

$$\text{Im }C^{\mu\nu}_{(\alpha)}(q) = \text{Im}\left(\frac{1}{\Omega}\int d^4X\int d^4\Delta x\ e^{-iq\cdot\Delta x}\ \mathrm{Tr}_{Dfc}\left[\gamma^\mu S^{(\alpha)}_E(X,\Delta x)\gamma^\nu S^{(\alpha)}_E(X,-\Delta x)\right]\right),$$

$$= \text{Im}\left(\frac{1}{\Omega}\int d^4X\int d^4\Delta x\ e^{-iq\cdot\Delta x}\ \mathrm{Tr}_{Dfc}\left[\gamma^\mu\left(S^0_E(\Delta x)+\alpha\ S^1_E(X,\Delta x)\right)\gamma^\nu\left(S^0_E(-\Delta x)+\alpha\ S^1_E(X,-\Delta x)\right)\right]\right)$$

$$= \text{Im }C^{\mu\nu}_0(q) + \alpha\ \text{Im }C^{\mu\nu}_1(q) + \mathcal{O}[\alpha^2].$$



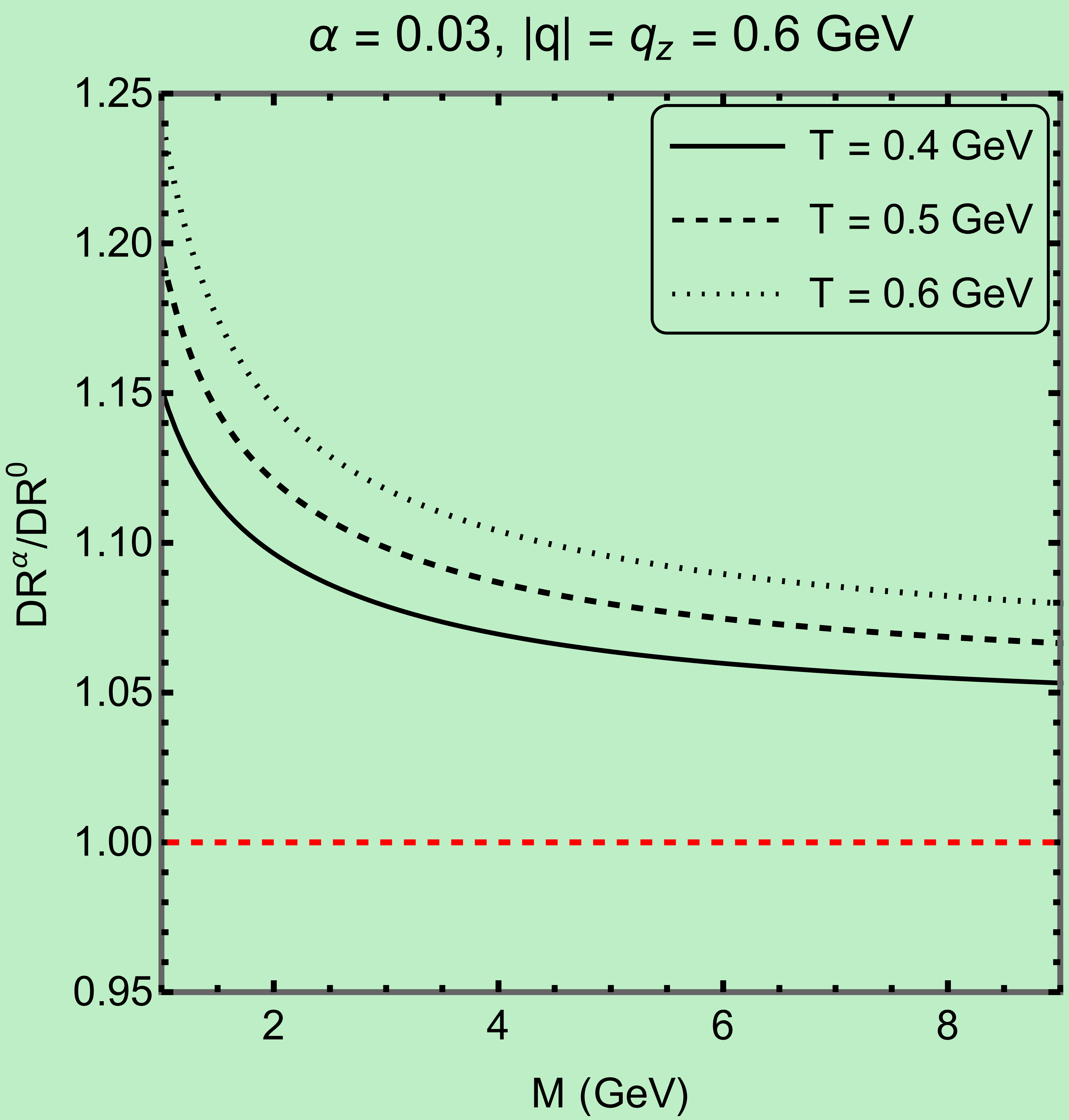
Results

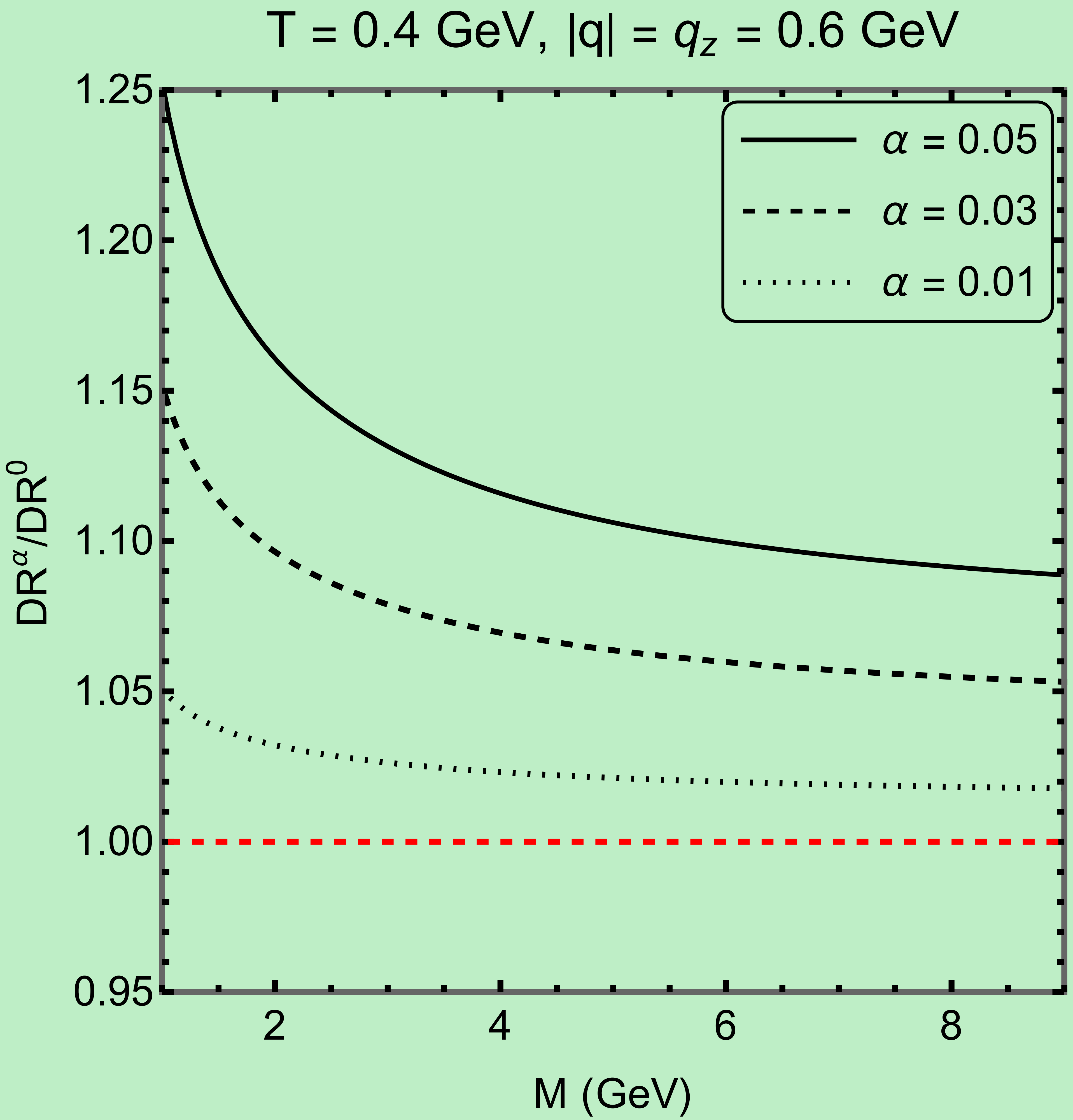
$$\text{Im } C^{\mu}_{0\mu}(\omega, \mathbf{q}) = \frac{1}{2\pi|\mathbf{q}|\beta} N_c N_f (Q^2 + 2m^2) \log \left[\frac{(e^{-\beta\omega} + e^{-\beta\omega_-})(1 + e^{-\beta\omega_+})}{(e^{-\beta\omega} + e^{-\beta\omega_+})(1 + e^{-\beta\omega_-})} \right].$$

$$\text{Im} C^{\mu}_{1\mu}(\omega, \mathbf{q}) = \frac{N_c N_f}{2\pi\beta} \Bigg[\frac{2Q^2}{\beta|\mathbf{q}|^2} \log \left[\frac{(e^{-\beta\omega} + e^{-\beta\omega_-})(1 + e^{-\beta\omega_+})}{(e^{-\beta\omega} + e^{-\beta\omega_+})(1 + e^{-\beta\omega_-})} \right] - \frac{\omega}{|\mathbf{q}|} \mathscr{M}(\omega, \omega_+) \left[\sqrt{\omega_+^2 - m^2} + \sqrt{\omega_-^2 - m^2} \right]$$
$$+ \mathscr{M}(\omega, \omega_+)\big(\omega_+ - \omega_-\big) - 2(Q^2 + 2m^2) \frac{\omega}{|\mathbf{q}|} \frac{\partial}{\partial m^2} \Big(\mathscr{M}(\omega, \omega_+)\big(\omega_+ - \omega_-\big) \Big)$$
$$+ 2\mathscr{M}(\omega, \omega_+) \left[\omega_+ \left(\omega_+ - \frac{\omega}{|\mathbf{q}|} \sqrt{\omega_+^2 - m^2} \right) + \omega_- \left(\omega_- - \frac{\omega}{|\mathbf{q}|} \sqrt{\omega_-^2 - m^2} \right) \right] \frac{\partial \omega_+}{\partial m^2} \Bigg].$$

$$\omega_{\pm} = \frac{1}{2} \left(\omega \pm |\mathbf{q}| \sqrt{1 - \frac{4m^2}{Q^2}} \right), \quad \mathscr{M}(\omega, \omega_+) = 1 - n_F(\omega_+) - n_F(\omega - \omega_+)$$

Results





DR as a function of invariant mass for a weakly accelerating medium (DR^α) shown in comparison with the Born dilepton rate (DR^0), i.e. DR with vanishing acceleration. With varying T (left panel) and α (right panel).

Summary

In this talk we have

- Discussed about the signatures of different stages of Heavy - Ion collisions.
- Emphasised about the importance of Dileptons as a QGP signature.
- Formulated the basic expression for the Dilepton rate.
- Discussed the necessity of incorporating local acceleration in the system.
- Evaluated the Dilepton rate in a weakly accelerated medium.

We found encouraging enhancement of the Dilepton rate in the intermediate invariant mass (M) range in comparison with the Born rate.

Outlook :

- Changes in the formalism of DPR due to local acceleration → incorporate the modified density matrix.
- Extending to arbitrary values of acceleration.
- Incorporating arbitrary polarisation for the outgoing lepton pairs etc.

Thank you for your kind attention

Summary

In this talk we have

- Discuss
- Emphas
- Formula
- Discuss
- Evaluate

$$\hat\rho\,\widehat{A}^{\mu}(t,\mathbf{x}_\perp,z)\hat\rho^{-1} = [e^{-\alpha\mathcal{J}^{0z}}]^{\mu}{}_{\nu}\,\widehat{A}^{\nu}(\tilde t,\mathbf{x}_\perp,\tilde z),$$

$$\tilde x^{\mu} = \mathcal{L}^{\mu}{}_{\nu}x^{\nu} + A^{\mu},$$

$$\mathcal{L}^{\mu}{}_{\nu} = \begin{pmatrix} \cos\alpha & 0 & 0 & i\sin\alpha \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ i\sin\alpha & 0 & 0 & \cos\alpha \end{pmatrix}, \qquad [e^{-\alpha\mathcal{J}^{0z}}]^{\mu}{}_{\nu} = (\mathcal{L}^{-1})^{\mu}{}_{\nu} = \begin{pmatrix} \cos\alpha & 0 & 0 & -i\sin\alpha \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\sin\alpha & 0 & 0 & \cos\alpha \end{pmatrix}.$$

$$A^{\mu}\partial_{\mu} = \frac{i}{a}\sin\alpha\partial_t - \frac{1}{a}(1-\cos\alpha)\partial_z\,.$$

Outlook :

- Changes
- Extendir
- Incorpor

$$\mathcal{M}^{\mu\nu}(Q) = (\mathcal{L}^{-1})^{\nu}{}_{\lambda}\int d^4y\, d^4y'\, e^{iQ\cdot(y-y')}\left\langle A^{\mu}(y)A^{\lambda}(\mathcal{L}y'+A)\right\rangle_{\beta,\alpha} \qquad \langle\,\widehat{A}\,\rangle_{\beta,\alpha} = \sum_F \langle F|\,\widehat{A}\,\hat\rho\,|F\rangle$$

Thank you for your kind attention

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