Spin polarization in off-equilibrium medium at weak and strong coupling

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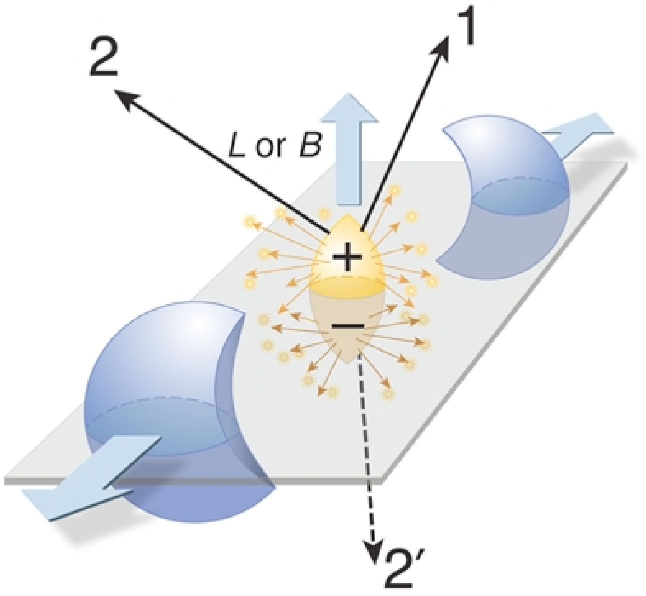
Li, SL, 2505.21883
SL, Tian, 2410.22935

Outline

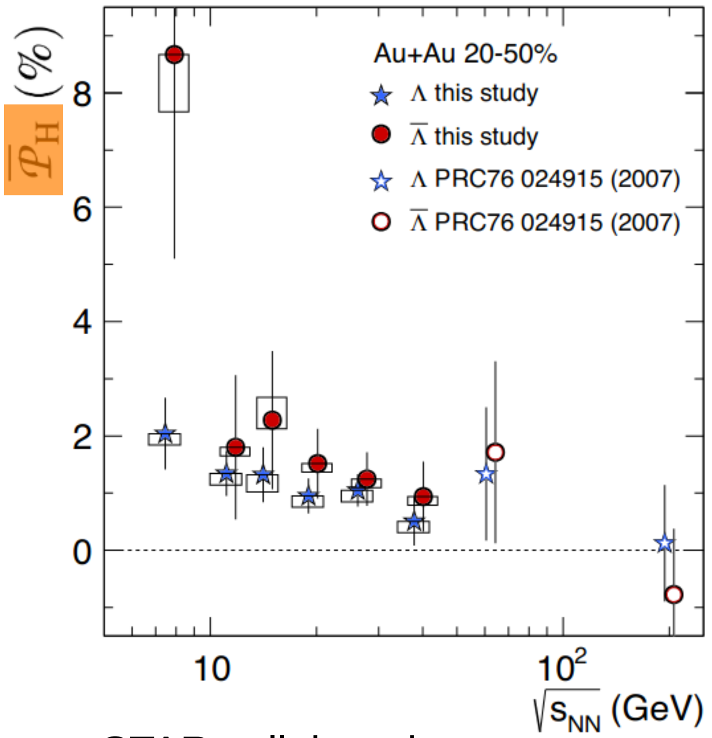
- Spin polarization in heavy ion collisions: sensitivity on structure/interaction of particles
- Spectral function of particle in off-equilbrium state
- Polarized spectral function at weak coupling
- Polarized spectral function at strong coupling
- Summary and outlook

Global polarization of Λ



$$L_{ini} \sim 10^5 \hbar \rightarrow S_{final}$$

Liang, Wang, PRL 2005, PLB 2005



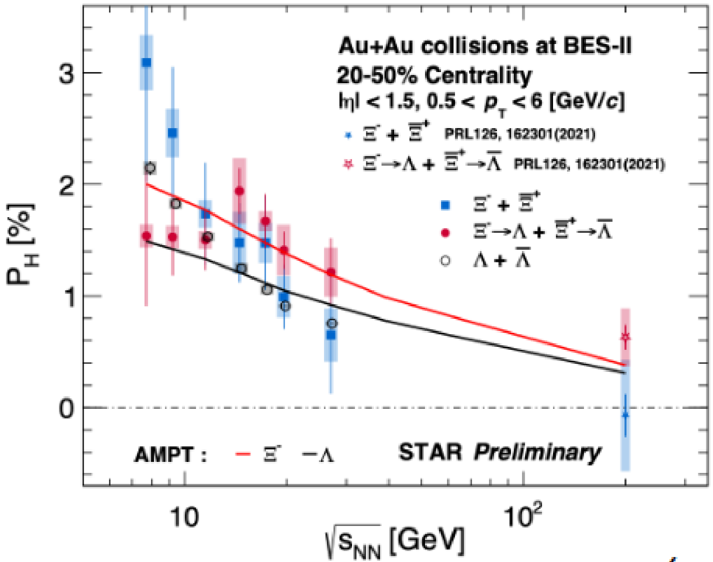
STAR collaboration, Nature 2017

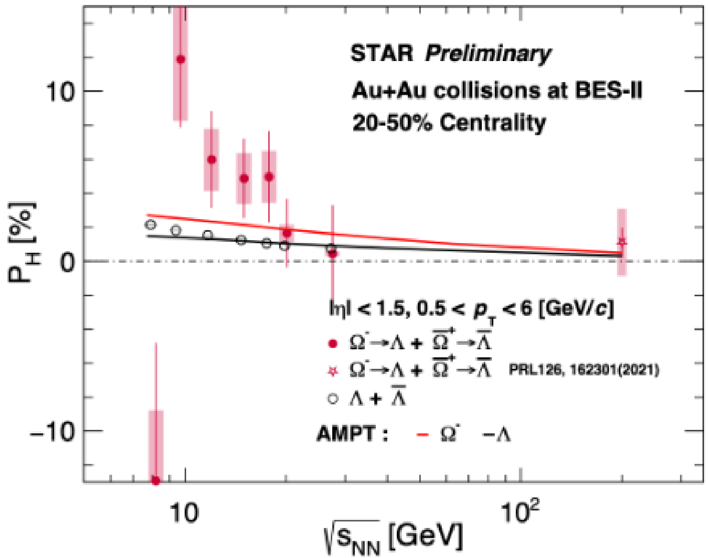
$$e^{-\beta(H_0 - \mathbf{S}\cdot\omega)}$$

Voloshin's talk

Becattini et al, PRC 2017

Global polarization of Ξ, Ω





point particle $$P_H \simeq \frac{(s+1)}{3}\frac{\omega}{T}$$ Becattini et al, PRC 2017

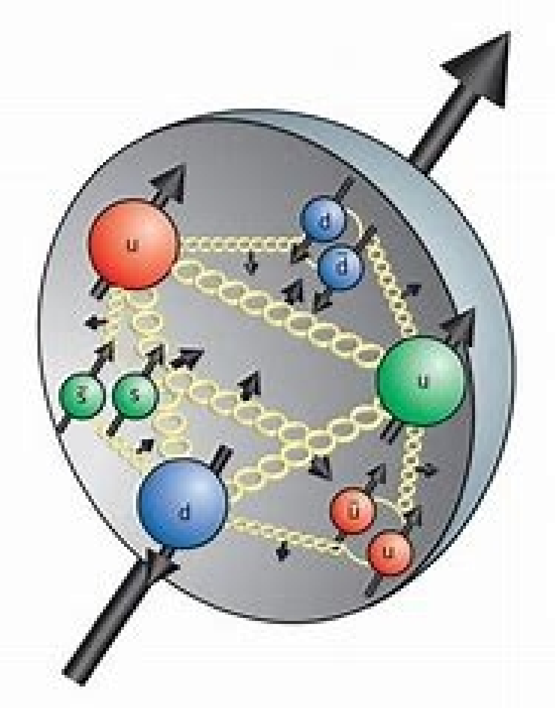
quark model $$P_\Lambda = P_s \quad P_\Xi \simeq \frac{4P_s - P_d}{3} \quad P_\Omega \simeq \frac{5P_s}{3}$$ Liang, Wang, PRL 2005

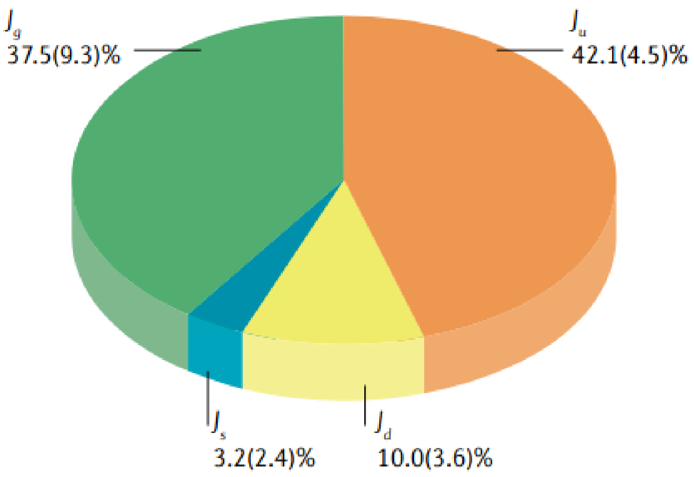
→ $$P_\Omega \simeq \frac{5P_\Lambda}{3} \quad P_\Xi \simeq P_\Lambda$$

same prediction from non-interacting nature of models

disfavored by data?

Interaction effect significant



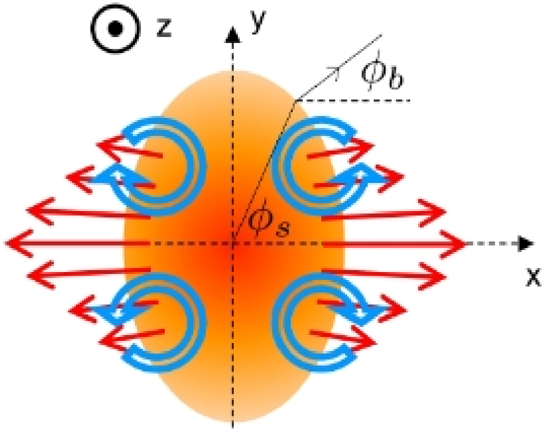


$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

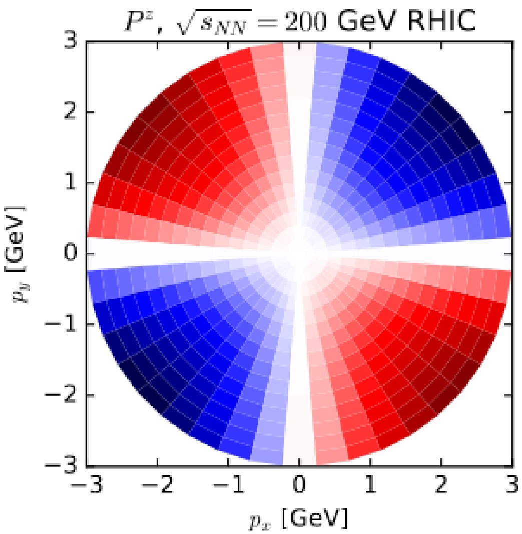
~40%

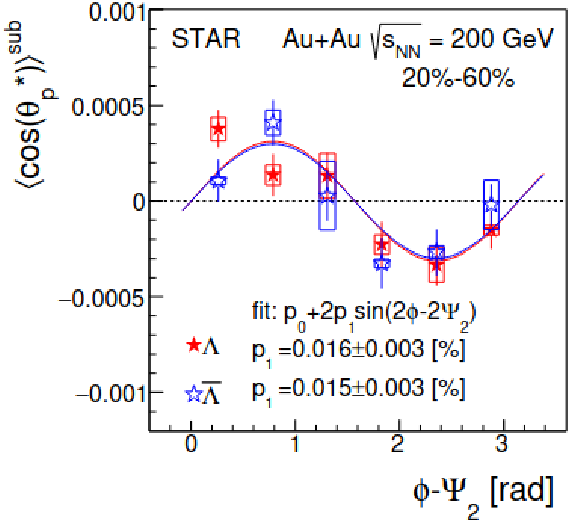
Alexandrou et al, PRD 2020

Local polarization of Λ: vorticity only



$$S^i \sim \omega^i$$



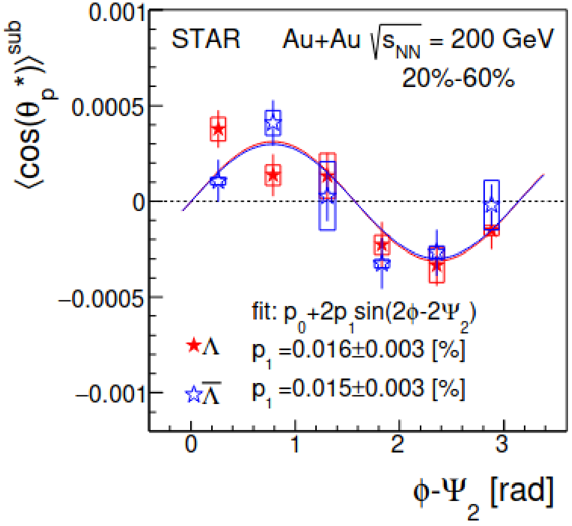


Becattini, Karpenko, PRL 2018
Wei, Deng, Huang, PRC 2019
Wu, Pang, Huang, Wang, PRR 2019
Fu, Xu, Huang, Song, PRC 2021

STAR collaboration, PRL 2019

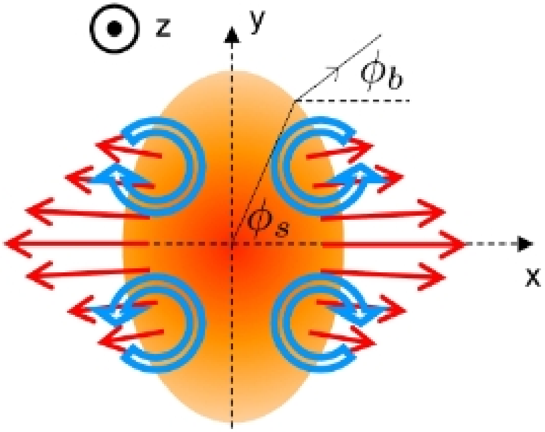
wrong sign

Local polarization of Λ: vorticity + shear



STAR collaboration, PRL 2019

vorticity



$$S^i \sim \omega^i$$

wrong sign

shear

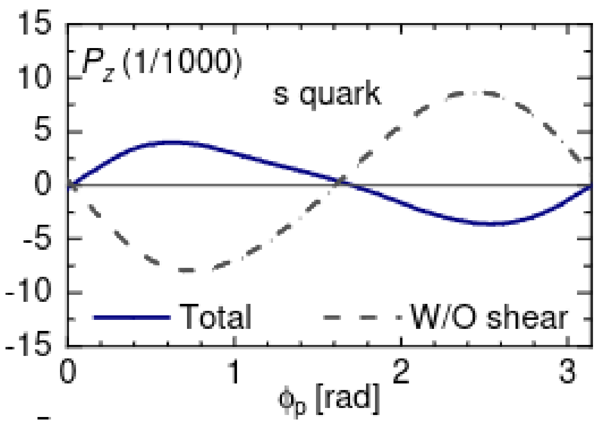
$$S^i \sim \epsilon^{ijk}\hat{p}_k\hat{p}_l\sigma_{jl}$$

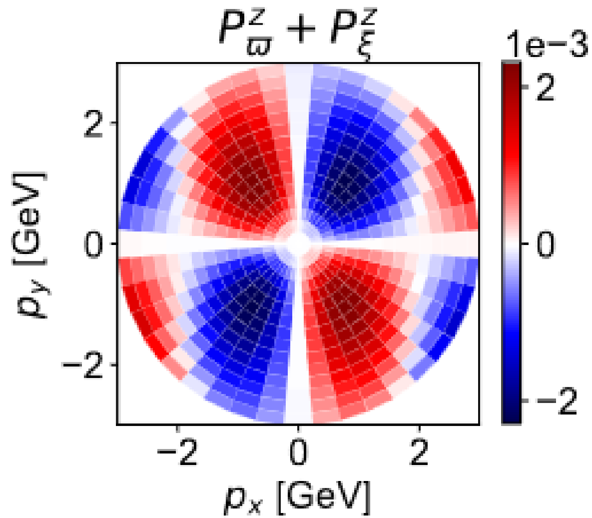
$$S^z \sim \left(\langle p_y^2\rangle - \langle p_x^2\rangle\right)\partial_y u_x$$

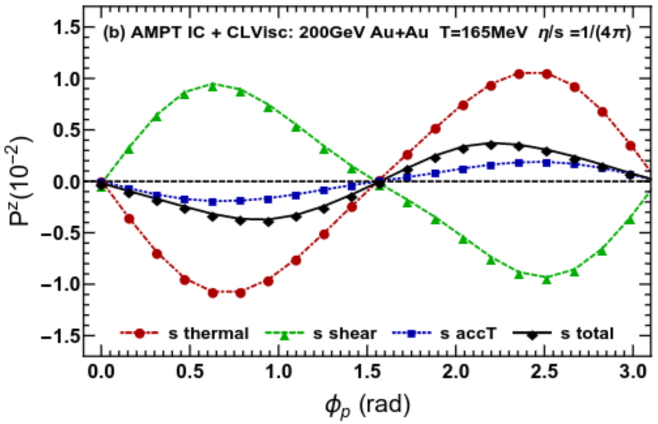
right sign

Hidaka, Pu, Yang, PRD 2018
Liu, Yin, JHEP 2021
Becattini, et al, PLB 2021

Sensitivity on interaction: shear







two scenarios

Λ: point particle

Λ: quark model

$$S^\mu(p^\alpha) = \frac{1}{4m}\frac{\int d\Sigma \cdot p\, n_0(1-n_0)\mathcal{A}^\mu}{\int d\Sigma \cdot p\, n_0}$$

Interaction effect?

Fu, Liu, Pang, Song, Yin, PRL 2021

Becattini, et al, PRL 2021

Yi, Pu, Yang, PRC 2021

C. Yi's talk

Interaction effect in medium: weak coupling picture

free theory $$S^i \sim \left(1\,\omega^i + 1\,\epsilon^{ijk}\,\hat{p}_j\,\hat{p}_l\,\sigma_{kl} + 1\,\epsilon^{ijk}\,\hat{p}_j\,\frac{\partial_k\beta}{\beta}\right)\delta(p^2 - m^2)$$

- distribution function correction

$$(\partial_t + \hat{p}\cdot\nabla_x) f(t, x, p) = -C[f] \qquad \delta f \propto \frac{\nabla_x f}{g^4}$$

deviation of steady state from local equilibrium

$$S^i \sim \left((1 + O(1))\epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{kl}\right)\delta(P^2 - m^2)$$

SL, Wang, JHEP 2022, PRD 2025
Fang, Pu, Yang, PRD 2024
Fang, Pu, PRD 2025

- spectral function correction in off-equilibrium state

focus of this talk

SL, Tian, 2410.22935
Fang, Pu, Yang, 2503.13320

Spectral function of probe "particle" in medium

$$\rho_{\alpha\beta}(\omega,\vec{p}) = \int d^4x e^{ip\cdot x} \left\langle \Psi_\alpha(x)\Psi^\dagger_\beta(0) + \Psi^\dagger_\beta(0)\Psi_\alpha(x) \right\rangle$$

$\omega, p \gg \partial_X$ particle localized in fluid element

$\langle \cdots \rangle = \mathrm{Tr}[D \cdots]$ D: density matrix

probe particle in off-equilibrium fluid $D = D_{\text{probe}} \otimes D_{\text{fluid}}.$

$D^{(0)}{}_{\text{probe}} \otimes D^{(1)}{}_{\text{fluid}}$ $D^{(1)}{}_{\text{probe}} \otimes D^{(0)}{}_{\text{fluid}}$

$\rho = 2\,\mathrm{Im}\,G^R \longrightarrow S^i \sim \mathrm{tr}[\rho\,\sigma^i]$

Off-equilibrium spectral function at weak coupling

- Probe quark

Spectral function from self-energy

$$\frac{i}{2}\partial\!\!\!/ S_R(X,P) + P\!\!\!/ S_R(X,P) - \left(\Sigma_R(X,P)S_R(X,P) + \frac{i}{2}\{\Sigma_R(X,P), S_R(X,P)\}_{\rm PB}\right) = -1$$

$$S_R = S_R^{(0)} + S_R^{(1)} + \cdots\,,$$

$$\{A,B\}_{\rm PB} = \partial_P A\cdot\partial_X B - \partial_X A\cdot\partial_P B$$

$$S_R^{(0)} = -\frac{1}{P\!\!\!/} - \frac{1}{P\!\!\!/}\Sigma_R\frac{1}{P\!\!\!/}$$

$$S_R^{(1)} = -\frac{1}{P\!\!\!/}\boxed{\delta\Sigma_R}\frac{1}{P\!\!\!/} + \gamma^5\gamma^\beta P^\nu T^{\mu\lambda}\epsilon_{\beta\lambda\mu\nu}\frac{-1}{(P^2)^2}$$

$$T_{\mu\lambda} = \boxed{\partial_{[\mu}\Sigma^R_{\lambda]}}$$

off-equilibrium self-energy

gradient of equilibrium self-energy

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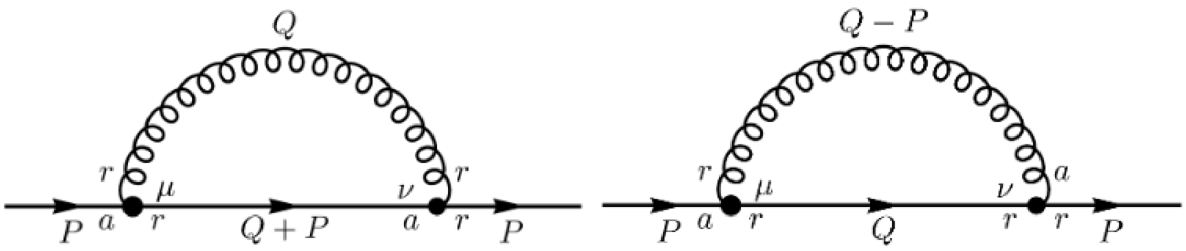
Equilibrium self-energy



$P \gg T$, $P^2 \ll p/\beta$,

energetic quark
nearly on-shell

$$\frac{\Sigma_{ar}}{g^2 C_F} = 2i\not{P}(A+B) + 4ip_0\gamma^0 A.$$

$$p_0 = p(1 + 8A)$$

$$A = \frac{1}{2(2\pi)^2}\frac{-i\pi}{2p\beta}.$$

modified dispersion:

finite damping

$$S_R^{(1)} = \gamma^5\gamma_\beta P_\nu \partial_\mu \Sigma_\lambda^R \frac{-1}{(P^2)^2}\epsilon^{\beta\lambda\mu\nu} \sim O(T\partial)$$

$D^{(0)}_{\text{probe}} \otimes D^{(1)}_{\text{fluid}}$

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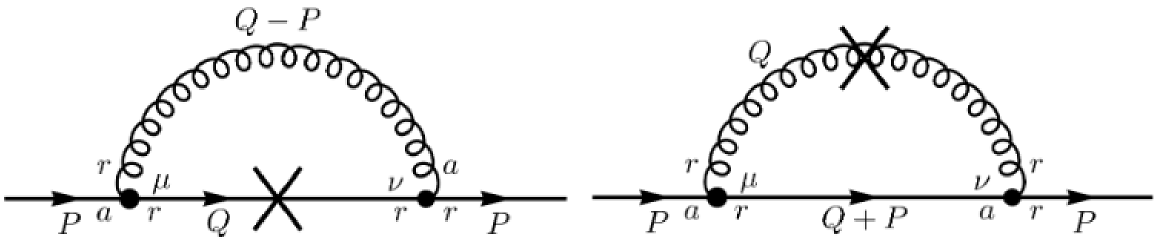
Off-equilibrium self-energy



off-equilibrium propagators

Hidaka, Pu, Yang 2017

Huang et al 2020

Hattori et al 2020

$$\frac{\delta\Sigma_{ar}}{g^2 C_F} = \gamma^5\gamma^\mu \mathcal{A}_\mu,$$

$$\mathcal{A}^0 = i\omega^i p_i \beta(-4\delta A - 2\delta B),$$

$$\mathcal{A}^k = i\omega_{\parallel}^k \beta p(-4\delta A - 2\delta B) + \omega_\perp^k \beta p(-4\delta A - \delta B) + \epsilon^{ijk}\hat{p}_i\hat{p}_l\sigma_{jl}\beta p(-\delta B) + \epsilon^{ijk} p_i \partial_j \beta(-\delta C)$$

$$\delta S_R^{(0)} = -\frac{1}{\not{p}}\delta\Sigma_R \frac{1}{\not{p}} \sim O(p\partial)$$

dominant contribution $\quad P \gg T$

$D^{(1)}{}_{\text{probe}} \otimes D^{(0)}{}_{\text{fluid}}$

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Weak coupling summary

free $$\gamma^5\gamma_i\frac{2\pi}{2}\left(\omega^i+\epsilon^{ijk}\hat{p}_k\frac{\partial_j\beta}{\beta}+\epsilon^{ijk}\hat{p}_k\hat{p}_l\sigma_{lj}\right)\beta\tilde{f}(p).$$

interaction $$\int dp_0\,\delta\rho(P)\,\tilde{f}(p_0)$$

$$O(\lambda)\quad=\frac{g^2C_F}{2(2\pi)^2}\frac{\pi}{2p}\gamma^5\gamma_i\left[0.95\omega^i_{\parallel}+1.48\omega^i_{\perp}-0.52\epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{kl}-0.02\epsilon^{ijk}\hat{p}_j\frac{\partial_k\beta}{\beta}\right]\tilde{f}(p)$$

Polarized spectral function

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Off-equilibrium spectral function at strong coupling

- Probe baryon

Holographic model for baryon

$$S = i \int d^{D+1}x \sqrt{-g}\,\bar{\psi}\left(\Gamma^M \nabla_M - m\right)\psi,$$

Iqbal, Liu 2009

5D Dirac fermion ⟷ 4D Weyl fermion

$$\psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$$

“lump of quark/gluon”

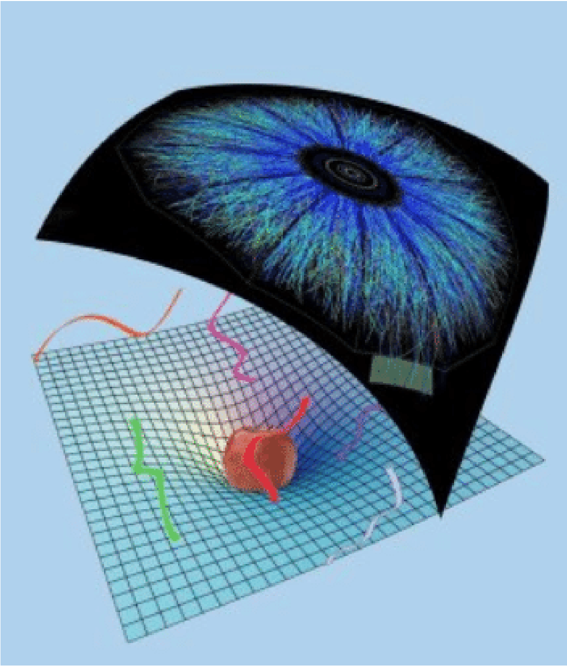
One baryon = R + L Weyl fermions

focus on spectral function in off-equilibrium QGP

Holographic model for QGP

$$ds^2 = -2u_\mu(x)dx^\mu dr - r^2 f(b(x)r)u_\mu u_\nu dx^\mu dx^\nu + r^2 P_{\mu\nu}dx^\mu dx^\nu$$ local equilibrium

$$+\, 2r^2 bF(br)\sigma_{\mu\nu}dx^\mu dx^\nu + \frac{2}{3} r u_\mu u_\nu \partial_\lambda u^\lambda dx^\mu dx^\nu - r u^\lambda \partial_\lambda (u_\mu u_\nu) dx^\mu dx^\nu$$ steady state



Bhattacharyya et al, JHEP 2008

$$\begin{array}{cc} & \quad\quad O(\partial^0) \quad\quad\quad\quad\quad\quad O(\partial) \\ T^{\mu\nu} = & (\pi T)^4 (\eta^{\mu\nu} + 4u^\mu u^\nu) - 2\boxed{(\pi T)^3}\sigma^{\mu\nu} \end{array}$$

local equilibrium

η

steady state

$$\sigma_{ij} \quad \cancel{\partial_0 b = \frac{1}{3}\partial_i u_i} \quad \partial_i b = \partial_0 u_i$$

T-grad = acceleration

Gradient corrections

$$\left(\Gamma^M \nabla_M - m\right) \psi = 0$$

gradient correction to Dirac field → gradient correction to baryon

$$D^{(0)}_{\text{fluid}} \otimes D^{(1)}_{\text{baryon}}$$

$$\left(\Gamma^M \nabla_M - m\right) \psi = 0$$

gradient correction to Dirac operator → gradient correction to medium

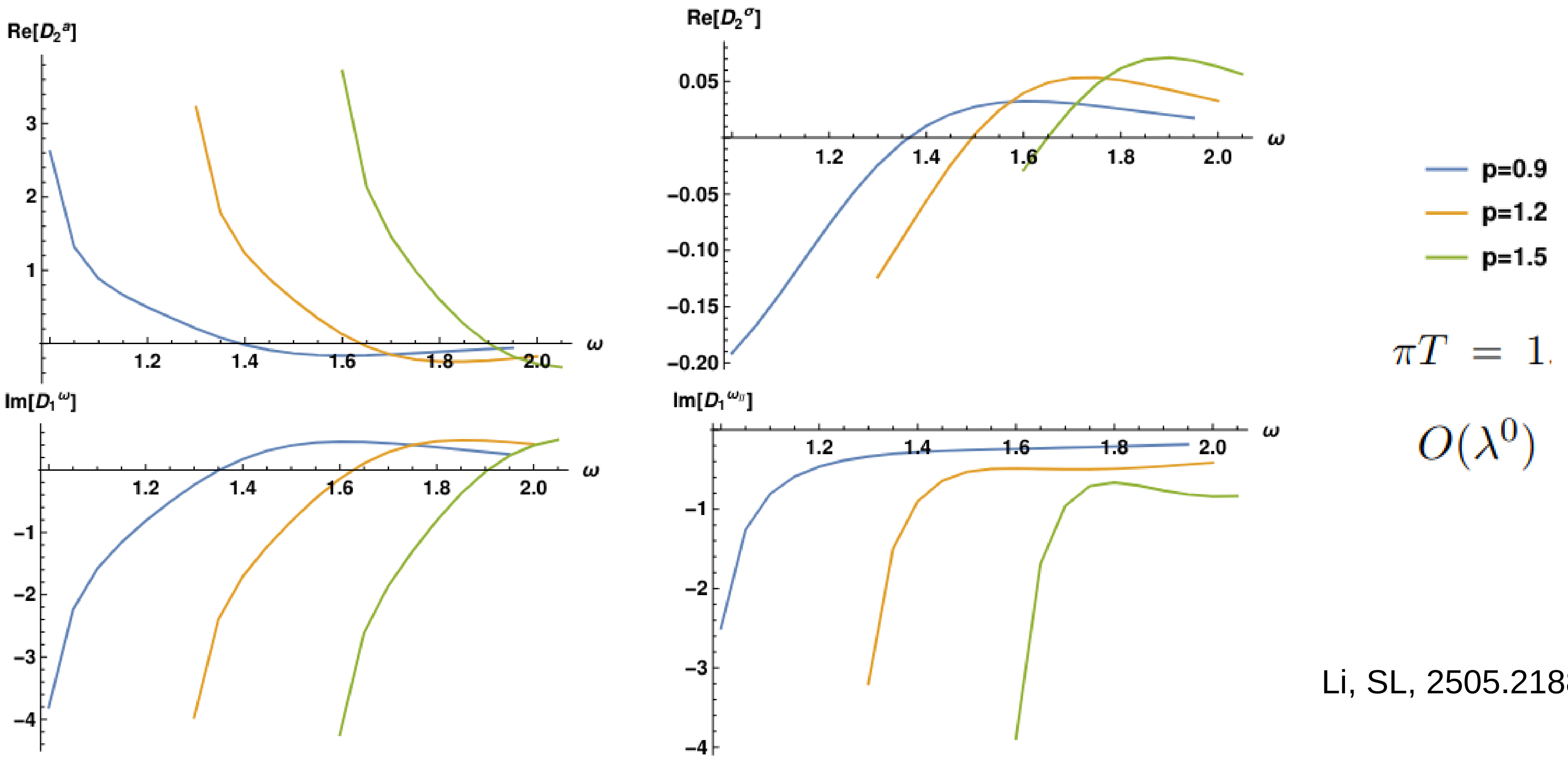
$$D^{(1)}_{\text{fluid}} \otimes D^{(0)}_{\text{baryon}}$$

$D^{(1)\,\text{le}}{}_{\text{fluid}}$ $D^{(1)\,\text{ss}}{}_{\text{fluid}}$

Li, SL, 2505.21883

$$\frac{1}{2}\mathrm{tr}\left[(\delta\rho^L+\delta\rho^R)\sigma_k\right]=4\left(-\mathrm{Re}[D_2^a]\epsilon^{ijk}\hat{p}_j\partial_0 u_i-\mathrm{Re}[D_2^\sigma]\epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{il}+\mathrm{Im}[D_1^\omega]\omega_k+\mathrm{Im}[D_1^{\omega^\parallel}]\omega_k^\parallel\right)$$

$$D^{(0)}_{\rm fluid}\otimes D^{(1)}_{\rm baryon}$$



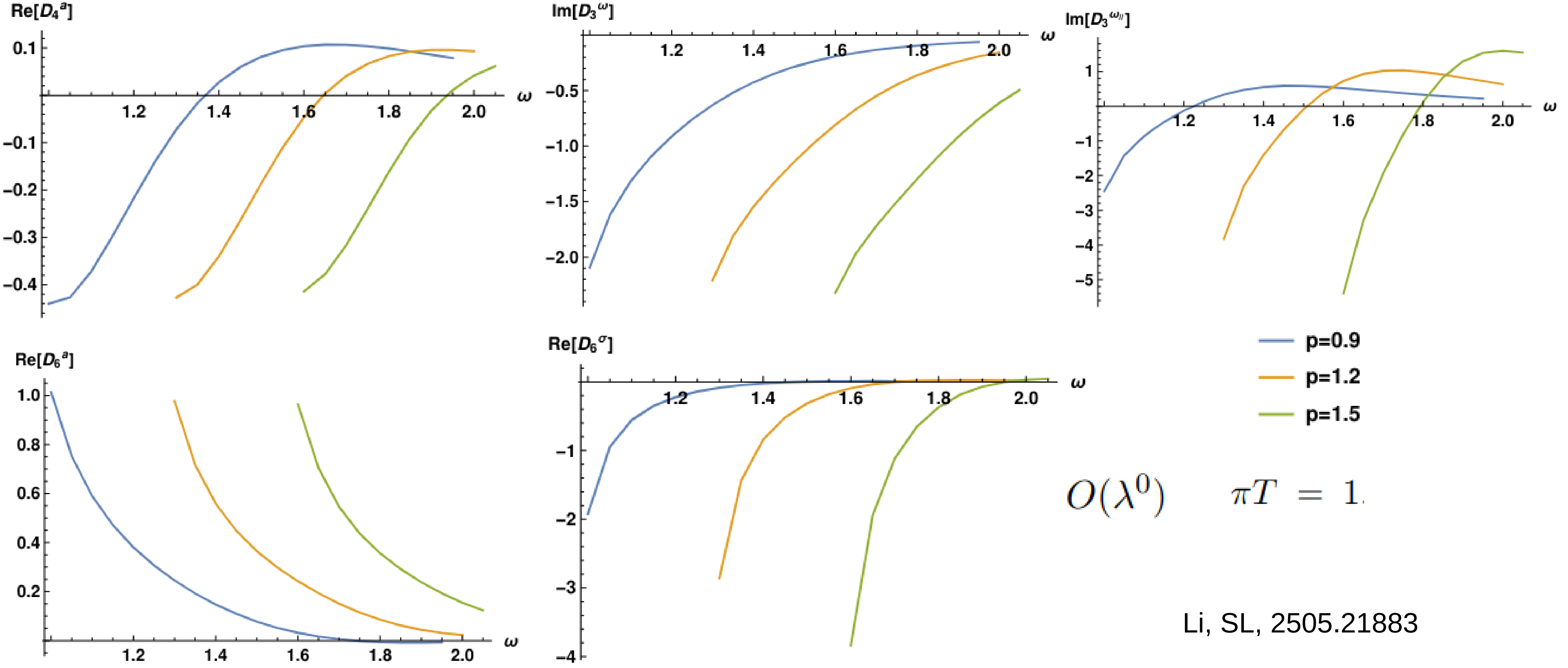
$\pi T = 1.$

$O(\lambda^0)$

Li, SL, 2505.21883

$$\frac{1}{2}\mathrm{tr}[(\delta\rho^L+\delta\rho^R)\sigma_k] = 4\Big[-\mathrm{Re}[D_4^a + D_6^a]\epsilon^{ijk}\hat{p}_j\partial_0 u_i - \mathrm{Re}[D_6^\sigma]\epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{il} + \mathrm{Im}[D_3^\omega]\omega_k + \mathrm{Im}[D_3^{\omega\parallel}]\omega_k^\parallel\Big]$$

$D^{(1)\,le}_{fluid}$ $D^{(1)\,ss}_{fluid}$



$O(\lambda^0)$ $\pi T = 1$.

Li, SL, 2505.21883

Strong coupling summary

$\mathrm{tr}[(\delta\rho^L+\delta\rho^R)\sigma_k]$	$\epsilon^{ijk}\hat{p}_j\partial_0 u_i$	$\epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{il}$	$\omega_k^\perp$	$\omega_k^\parallel$
$D^{(1)}_{\mathrm{probe}}$	$-4\mathrm{Re}[D_2^a]$	$-4\mathrm{Re}[D_2^\sigma]$	$4\mathrm{Im}[D_1^\omega]$	$4\mathrm{Im}[D_1^\omega+D_1^{\omega^\parallel}]$
$D^{(1)\mathrm{le}}_{\mathrm{fluid}}$	$-4\mathrm{Re}[D_4^a]$		$4\mathrm{Im}[D_3^\omega]$	$4\mathrm{Im}[D_3^\omega+D_3^{\omega^\parallel}]$
$D^{(1)\mathrm{ss}}_{\mathrm{fluid}}$	$-4\mathrm{Re}[D_6^a]$	$-4\mathrm{Re}[D_6^\sigma]$		

Polarized spectral function

Li, SL, 2505.21883

Weak (quark) vs Strong (baryon)

Common

- Polarized spectral function in off-equilibrium state

  $D^{(1)}_{probe}$ $D^{(1)\,le}_{fluid}$ $D^{(1)\,ss}_{fluid}$

- Correction suppressed for energetic particles

Difference

- Weak $O(\lambda)$ vs strong $O(\lambda^0)$

Summary

- Sensitivity of polarization to baryon structure
- Spectral function in off-equilibrium state
- vorticity, shear, T-grad generically lead to (interaction dependent) polarized spectral function

Outlook

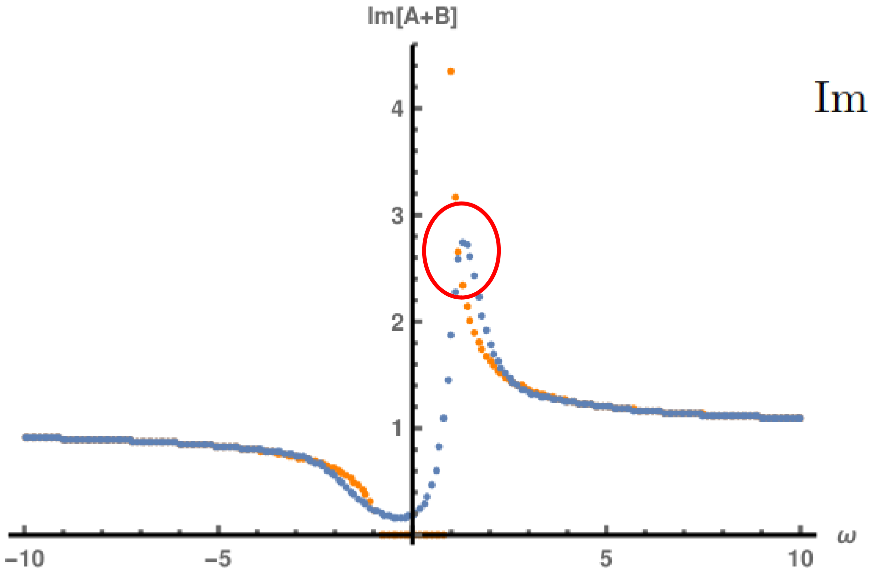
- Mass dependence of polarized spectral function

The 10th International Conference on Chirality, Vorticity and Magnetic Field in Quantum Matter, Zhuhai, Nov 16-20, 2026

Holography: equilibrium spectral function

$$G_R = A + B\hat{p}\cdot\vec{\sigma}$$

$$\rho = 2\mathrm{Im}G_R = \mathrm{Im}[A+B]\,(1-\hat{p}\cdot\vec{\sigma}) + \mathrm{Im}[A-B]\,(1+\hat{p}\cdot\vec{\sigma})$$



orange: vacuum

Iqbal, Liu 2009

$$\mathrm{Im}[A+B] = \mathrm{sgn}(\omega)\theta(\omega^2-p^2)\left(\frac{\omega+p}{\omega-p}\right)^{1/2}$$

no spacelike spectral

blue: equilibrium QGP

soften the singularity

develop spacelike spectral

focus on timelike spectral for baryon