Neutrino emission from quark cores of magnetars

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July 10, 2025

[Ghosh, Shovkovy, JHEP **04** (2025) 110]
[Ghosh, Shovkovy, arXiv:2504.21083]

THE 9TH CONFERENCE ON CHIRALITY, VORTICITY AND MAGNETIC FIELDS IN QUANTUM MATTER
JULY 7 – 11, 2025, PRINCIPIA INSTITUTE, SÃO PAULO, BRAZIL

Neutrons stars: physics of extremes

- Natural laboratories of matter under extreme conditions
- Extreme density:

$$\rho \lesssim 10\rho_0$$

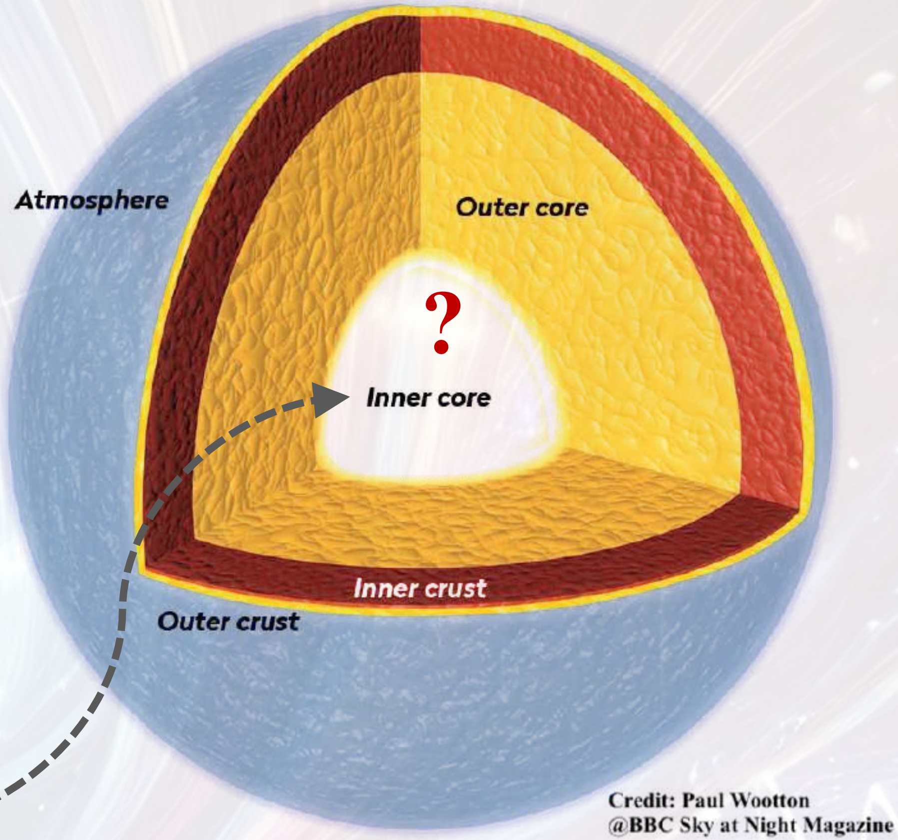
where $\rho_0 \approx 2.5\times10^{17}\text{kg/m}^3$

- Extreme magnetic fields:

$$B \lesssim 10^{14} - 10^{15}\ \text{G (surface)}$$

$$B \lesssim 10^{17} - 10^{18}\ \text{G (inside)}$$

- What is the inner core made of
  - meson condensates (?)
  - hyperons & other exotics (?)
  - quarks (?)



Quark matter cores?

EoS of most massive stars indicates presence of (≈conformal) quark matter

- Matter in conformal limit: $c_s^2 = \frac{1}{3}$ (sound speed) & γ=1 (polytropic index)

[Annala et al., Nat Commun 14, 8451 (2023)]

The "conformality" measure:

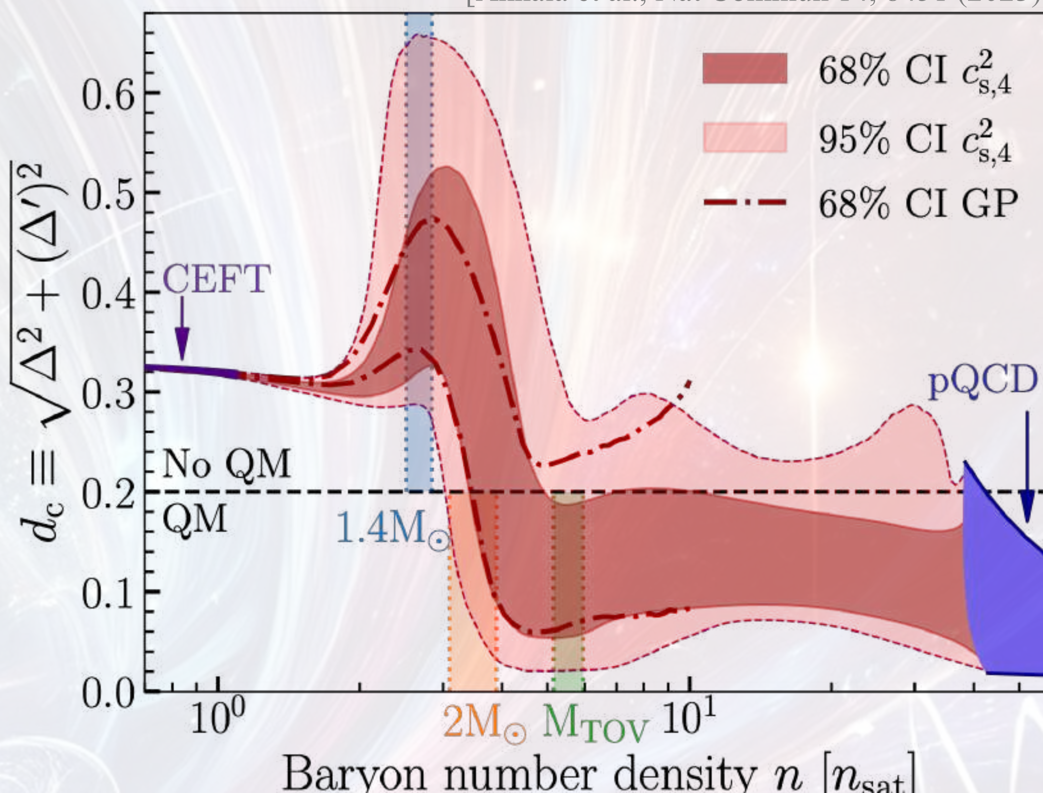
$$d_c \equiv \sqrt{\Delta^2 + (\Delta')^2}$$

where

$$\Delta = \frac{1}{3} - \frac{c_s^2}{\gamma}$$

and

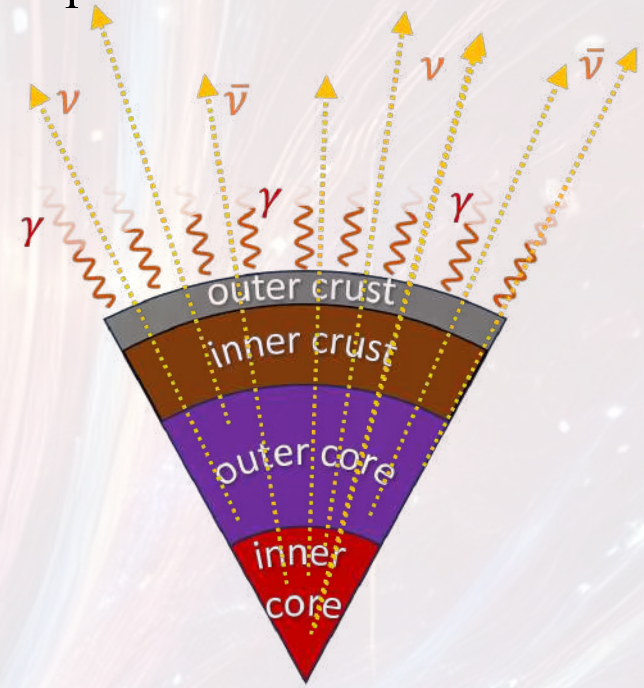
$$\Delta' = c_s^2 \left(\frac{1}{\gamma} - 1\right)$$



Cooling of compact stars

- The key factors affecting stellar cooling
  - The composition and heat capacity of matter in each phase
    - normal degenerate matter $C_V \sim \mu^2 T$
    - gapped phases $C_V \sim \mu^2 T e^{-\Delta/T}$
  - Available cooling mechanisms and their interplay
    - neutrino emission $\dot{E}_\nu \propto R^3 T^6$ (direct Urca)

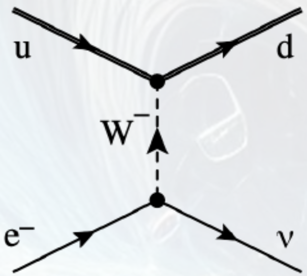
      or $\dot{E}_\nu \propto R^3 T^8$ (modified Urca)
    - surface photon emission $\dot{E}_\gamma \propto R^2_{\text{surface}} T^4_{\text{surface}}$
  - Thermal conductivity
  - Possible reheating mechanisms
- Strong **magnetic fields** may affect all of the above

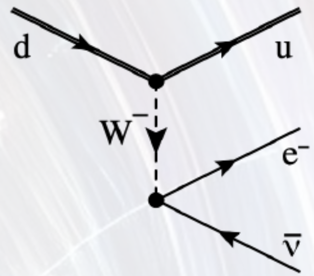


Quark matter: ν ($\bar{\nu}$) emission

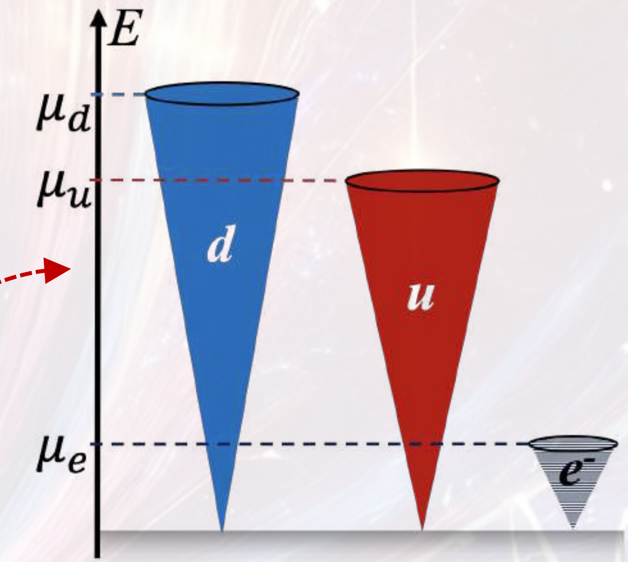
- Neutrino emission comes from direct Urca (weak) processes:

$$u + e^- \to d + \nu_e \quad \& \quad d \to u + e^- + \bar{\nu}_e$$





$n_u \approx 0.5 n_d,\ n_e \lesssim 0.003 n_d$



- Constraints in quark matter:
  - electrically neutrality: $\frac{2}{3} n_u - \frac{1}{3} n_d - n_e = 0$
  - β-equilibrium: $\mu_d = \mu_u + \mu_e$

Iwamoto's rate at $B = 0$

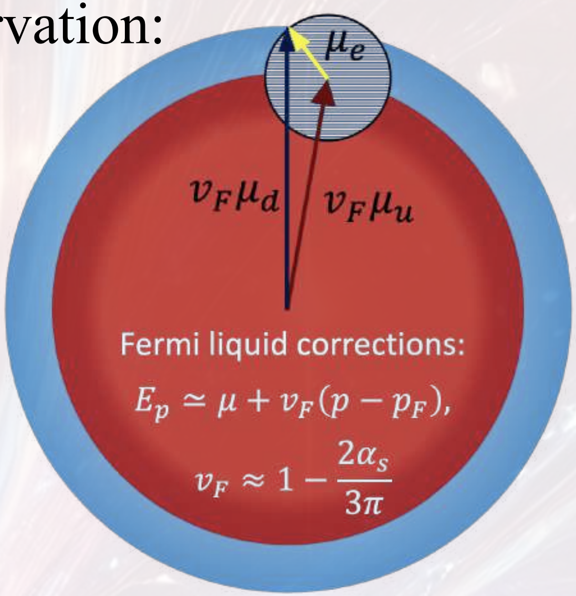
At $B = 0$, the rate was calculated by Iwamoto in the 1980s

$$\dot{\mathcal{E}}_{\nu}^{(\mathrm{Iwamoto})} \simeq \frac{457}{630} \alpha_s G_F^2 \cos^2\theta_C \mu_u \mu_d \mu_e T^6 + O\left(\alpha_s^2, \frac{\mu_e}{\mu_u}\right)$$

Kinematics is restricted by energy-momentum conservation:

$$\vec{k}_d = \vec{k}_u + \vec{k}_e + \vec{k}_{\bar{\nu}}$$

where $k_d \approx v_F \mu_d$, $k_u \approx v_F \mu_u$, $k_e \approx \mu_e$, $k_{\bar{\nu}} \approx T$



Scales: $T \lesssim 1$ MeV, $\mu_e \sim 50$ MeV, $\mu_u \lesssim \mu_d \sim 300$ MeV

i.e., $T \ll \mu_e$ and $\mu_e \ll \mu_u \lesssim \mu_d$

Note: $\vec{k}_d$, $\vec{k}_u$, $\vec{k}_e$ would be ($\approx$) collinear without Fermi liquid corrections

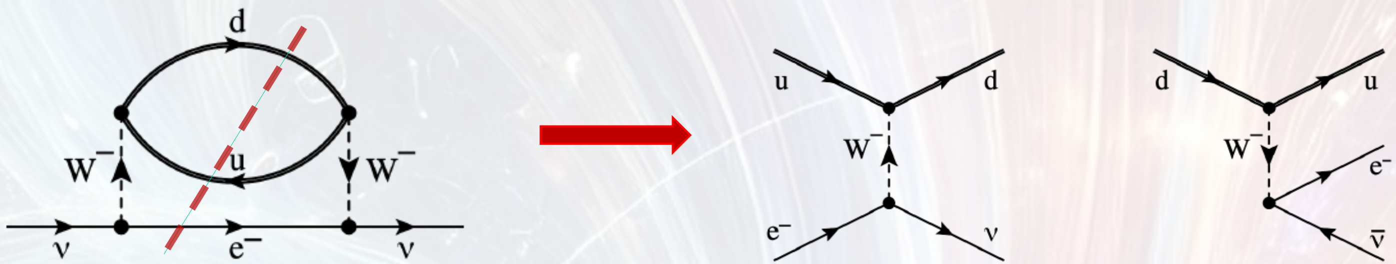
[Iwamoto, Phys. Rev. Lett. **44** (1980) 1637], [Iwamoto, Annals Phys. **141** (1982) 1]

Kadanoff-Baym formalism ($B \neq 0$)

- Use Kadanoff-Baym transport equation for neutrinos (~kinetic equation)

$$i\partial_t \mathrm{Tr}[\gamma^0 G_\nu^<(t,P)] = -\mathrm{Tr}[G_\nu^>(t,P)\Sigma_\nu^<(t,P) - \Sigma_\nu^>(t,P)G_\nu^<(t,P)]$$

- The main information comes from the neutrino self-energy:



- This leads to an explicit expression for neutrino-number production rate

$$\frac{\partial f_\nu(t,\boldsymbol{p}_\nu)}{\partial t} = -\frac{G_F^2\cos^2\theta_C}{2}\sum_{\lambda=\pm}\sum_{n=0}^{\infty}(-1)^n\int\frac{d^3\boldsymbol{p}_e e^{-p_{e,\perp}^2\ell^2}}{(2\pi)^3 p_\nu E_{e,n}} n_F(E_{e,n}-\mu_e) n_B(p_\nu+\mu_e-E_{e,n}) L_{n,\lambda}^{\delta\sigma}(\boldsymbol{p}_e,\boldsymbol{p}_\nu)\mathrm{Im}\left[\Pi_{\delta\sigma}^R(Q)\right]$$

where $\mathrm{Im}\left[\Pi_{\delta\sigma}^R(Q)\right]$ is the absorptive part of the W^- self-energy

[Ghosh, Shovkovy, JHEP **04** (2025) 110; arXiv:2501.03318]

Energy & momentum rates

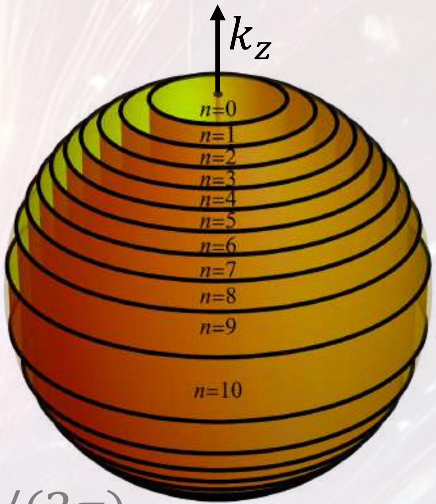
- Then, the energy and net longitudinal momentum emission rates are

$$\dot{\mathcal{E}}_{\nu} = 2\int \frac{d^3\vec{p}}{(2\pi)^3} p_0 \frac{\partial f_{\nu}(t,\vec{p})}{\partial t}$$

$$\dot{\mathcal{P}}_{\nu,z} = 2\int \frac{d^3\vec{p}}{(2\pi)^3} p_z \frac{\partial f_{\nu}(t,\vec{p})}{\partial t}$$

Note: $|eB| \simeq \frac{B}{1.69\times10^{14}\ \mathrm{G}} \mathrm{MeV}^2$

i.e., $|eB| \lesssim 1000\ \mathrm{MeV}^2$



- Approximations made in calculations:
  - Neglect the LL quantization of quarks ($\delta\epsilon_B \simeq |e_f B|/\mu_q$)
  - Account for the exact LL quantization of electrons
  - Include Fermi liquid corrections for quarks, $v_F \approx 1 - 2\alpha_s/(3\pi)$
  - Include exactly nonzero temperature effects

Formal results for $\dot{\mathcal{E}}_\nu$

- General expression ($\ell \equiv 1/\sqrt{|eB|}$)

$$\dot{\mathcal{E}}_\nu = \frac{12 N_c G_F^2 \cos^2\theta_C T^5}{\pi^5} v_F \mu_u \mu_d \sum_{n=0}^{\infty} \frac{(-1)^n}{\ell^2} \int_0^\infty \int_0^\infty \frac{\Theta(u,v)\, du\, dv}{\sqrt{u}\sqrt{u+v}} \frac{e^{-v}}{e^{\epsilon_n^u}+1} \left(\mathrm{Li}_5\left(e^{\epsilon_n^u}\right) - \frac{\epsilon_n^u}{4}\mathrm{Li}_4\left(e^{\epsilon_n^u}\right)\right)$$
$$\times \left[L_n(2v) - L_{n-1}(2v)\right]\left[1 + \frac{\sqrt{2n+u}}{2\ell\mu_u}\left(1 - \frac{v_F^2 \mu_e (\mu_d+\mu_u)\ell^2}{u+v}\right)\right]$$

where $\epsilon_n^u = \dfrac{\sqrt{2n+u} - \mu_e \ell}{T\ell}$, $\Theta(u,v) \equiv \theta\left(u + v - v_F^2 \mu_e^2 \ell^2\right) \theta\left[v_F^2 (\mu_d + \mu_u)^2 \ell^2 - u - v\right]$

- Low-T approximation, $T \ll |eB|/\mu_e$ ($u_n \equiv \mu_e^2 \ell^2 - 2n$)

$$\dot{\mathcal{E}}_\nu^{(0)} = \frac{457\pi N_c G_F^2 \cos^2\theta_C}{5040} v_F \mu_u \mu_d \mu_e T^6 \left(1 + \frac{\mu_e}{2\mu_u}\right) \sum_{n=0}^{n_{\max}} \frac{(-1)^n}{\sqrt{u_n}} \int \frac{\Theta(u_n, v) e^{-v} dv}{\sqrt{u_n + v}} \left(1 - \frac{v_F^2 \mu_e^2 \ell^2}{u_n + v}\right) \left[L_n(2v) - L_{n-1}(2v)\right]$$

- LLL approximation ($|eB| \gtrsim \mu_e^2$)

$$\dot{\mathcal{E}}_\nu^{(\mathrm{LLL})} \simeq \frac{457\pi^{3/2} N_c G_F^2 \cos^2\theta_C}{5040\ell} v_F \mu_u \mu_d T^6 \left(1 + \frac{\mu_e}{2\mu_u}\right) \left(\left(1 + 2 v_F^2 \mu_e^2 \ell^2\right) e^{\mu_e^2 \ell^2} \mathrm{erfc}(\mu_e \ell) - \frac{2 v_F^2 \mu_e \ell}{\sqrt{\pi}}\right)$$

Energy emission: $T \ll |eB|/\mu_e$

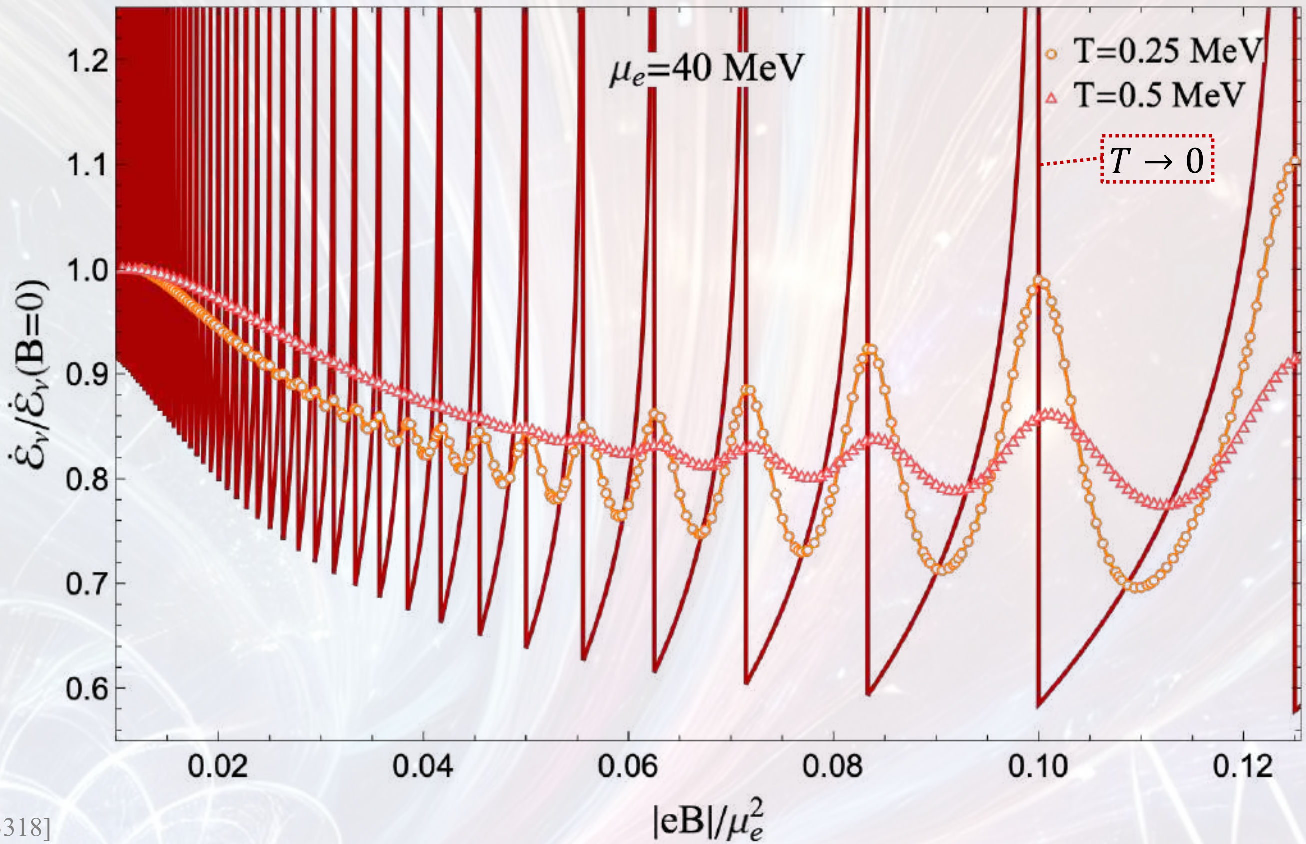
- Energy emission rate exhibits a sawtooth dependence on $|eB|/\mu_e^2$ when $T \to 0$, with divergences at LL thresholds

Divergences as $T \to 0$:

$$\frac{|eB|}{\mu_e^2} = \frac{1}{2n}$$

Oscillations of $\dot{\mathcal{E}}_\nu$ are similar to de Haas–van Alphen (or Shubnikov–de Haas) effect

Increasing T attenuates the oscillations

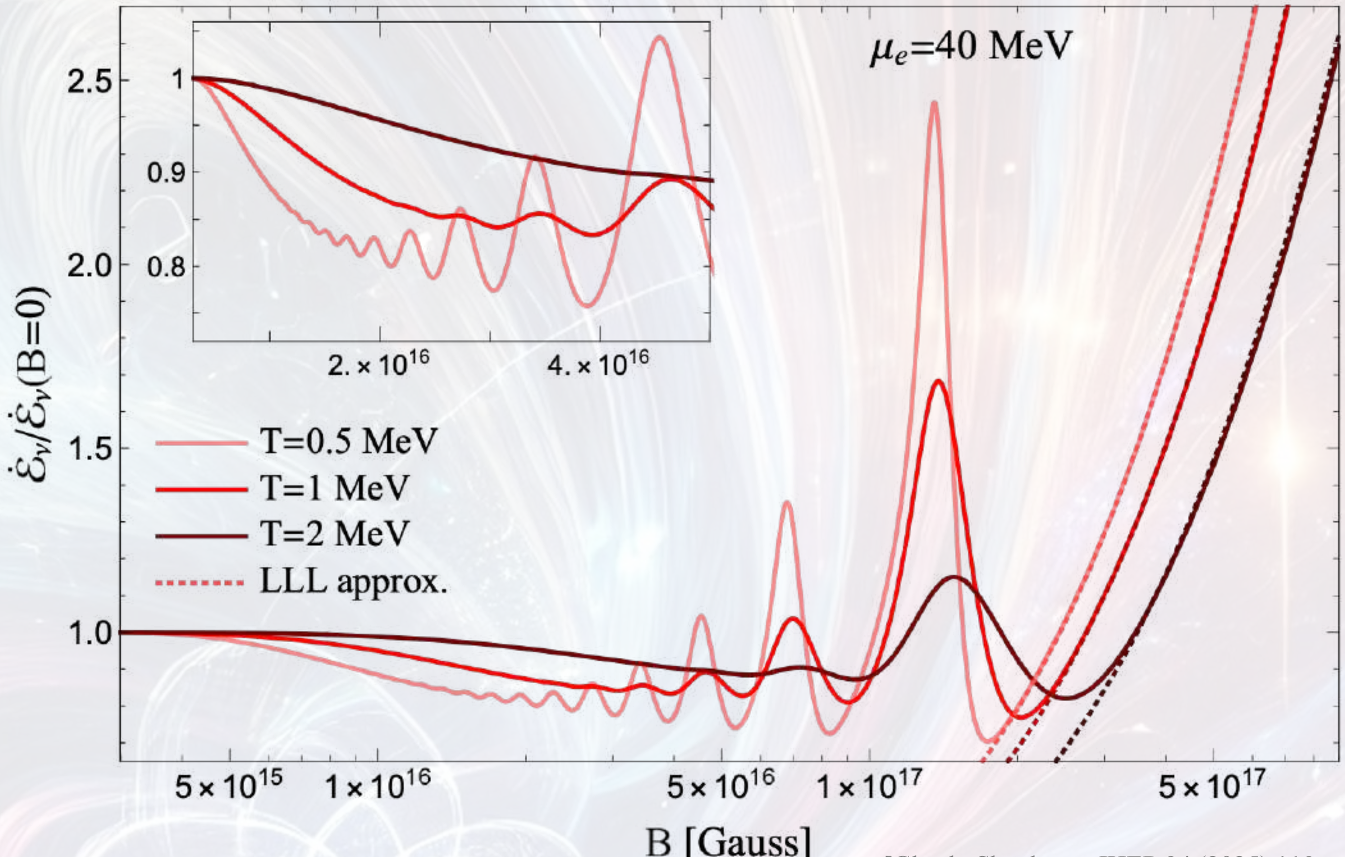


[Ghosh, Shovkovy, JHEP **04** (2025) 110; arXiv:2501.03318]

B-dependence of $\dot{\mathcal{E}}_\nu$

- Energy emission rate:

$$|eB| \simeq \frac{B}{1.69\times10^{14}\ \mathrm{G}}\ \mathrm{MeV}^2 \lesssim 1000\ \mathrm{MeV}^2$$



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Momentum emission: $T \ll |eB|/\mu_e$

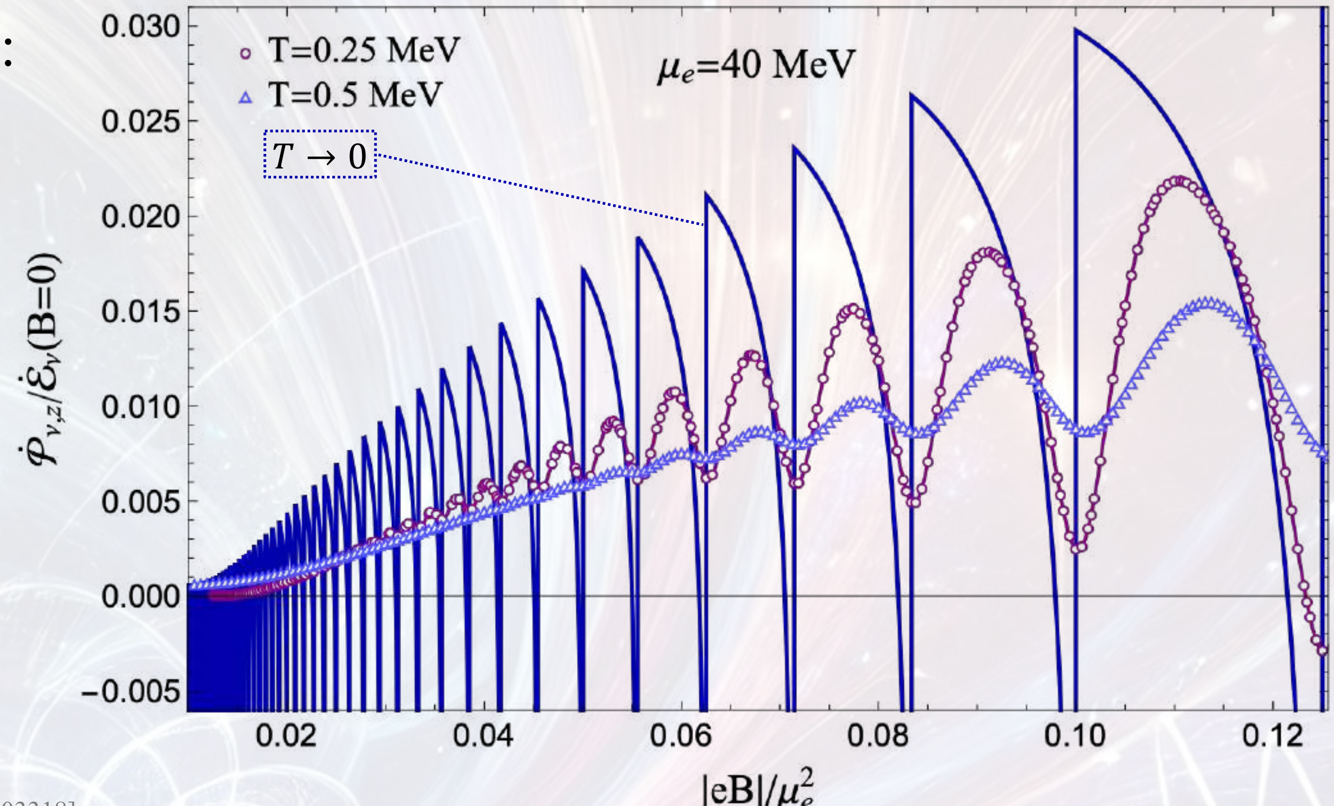
- Momentum emission rate: $\dot{\mathcal{P}}_{\nu,z} = 2\int \frac{d^3\vec{p}}{(2\pi)^3} p_z \frac{\partial f_\nu(t,\vec{p})}{\partial t}$

Divergences as $T = 0$:

$$\frac{|eB|}{\mu_e^2} = \frac{1}{2n}$$

Sign of $\dot{\mathcal{P}}_{\nu,z}$ is not fixed

Increasing T attenuates the oscillations



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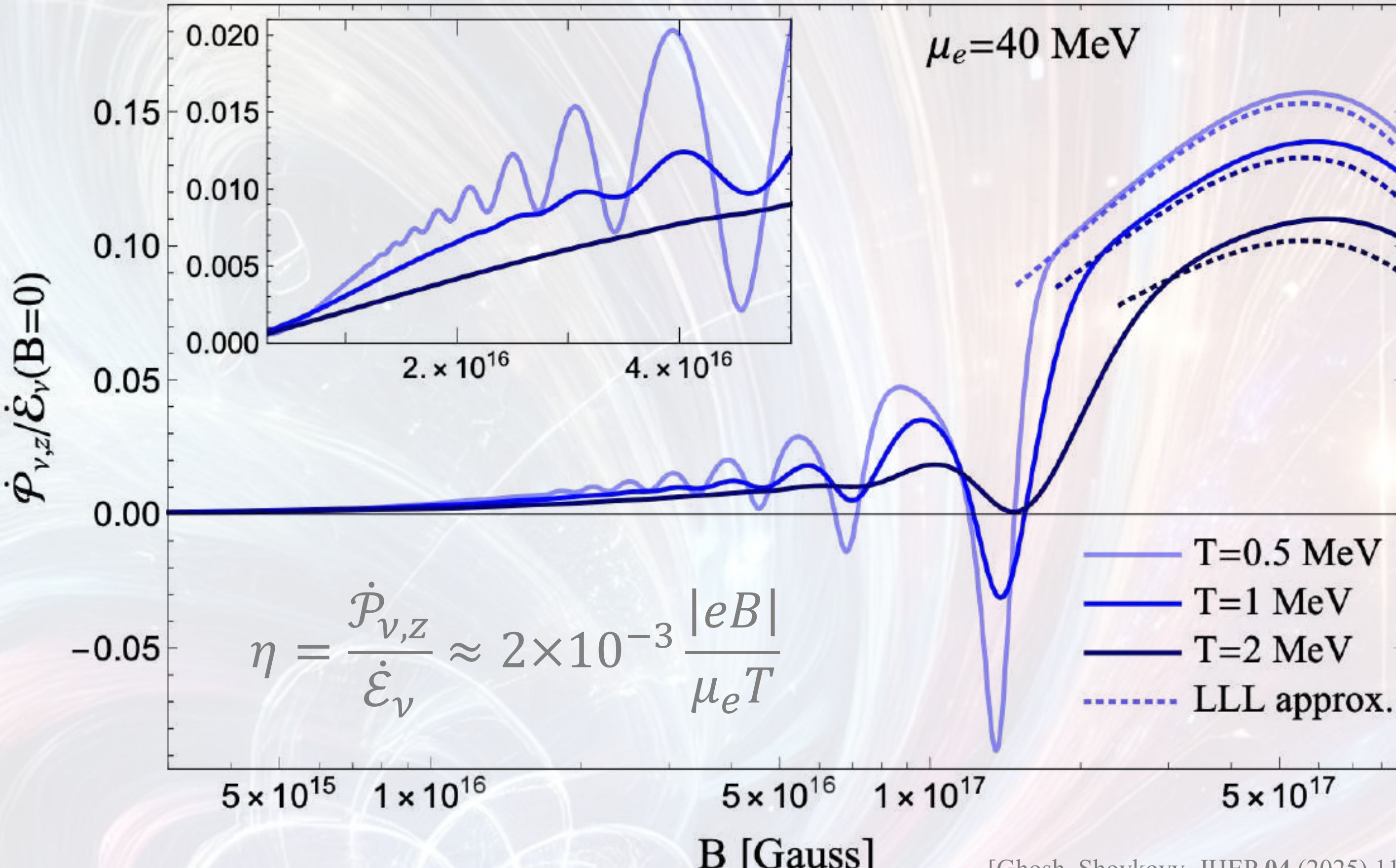
B-dependence of $\dot{\mathcal{P}}_{\nu,z}$

- Momentum emission rate:

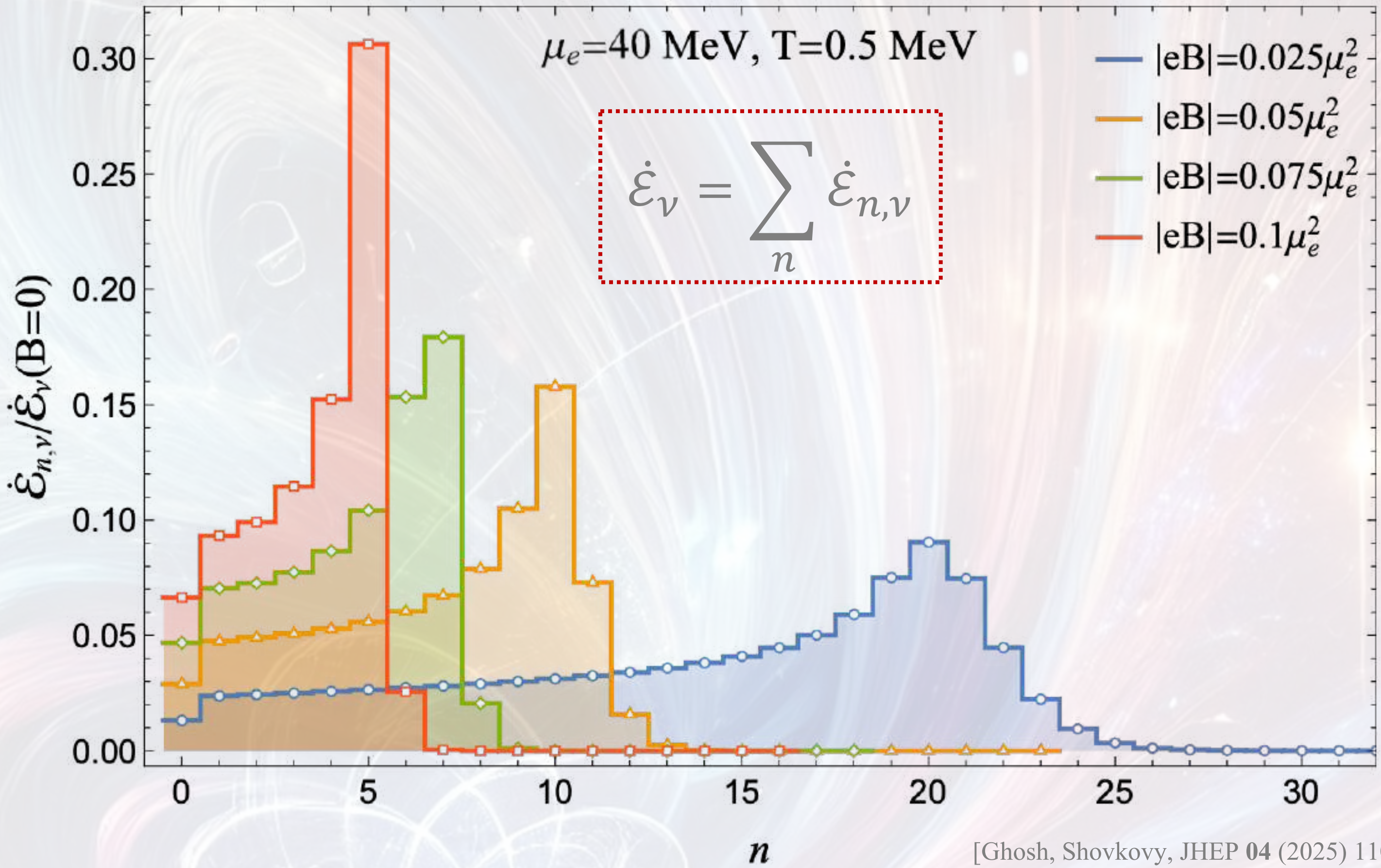


$$\eta = \frac{\dot{\mathcal{P}}_{\nu,z}}{\dot{\mathcal{E}}_\nu} \approx 2\times 10^{-3}\frac{|eB|}{\mu_e T}$$

[Ghosh, Shovkovy, JHEP **04** (2025) 110; arXiv:2501.03318]

LL contributions to $\dot{\mathcal{E}}_\nu$

- Partial contributions of the *n*th Landau-level electron states



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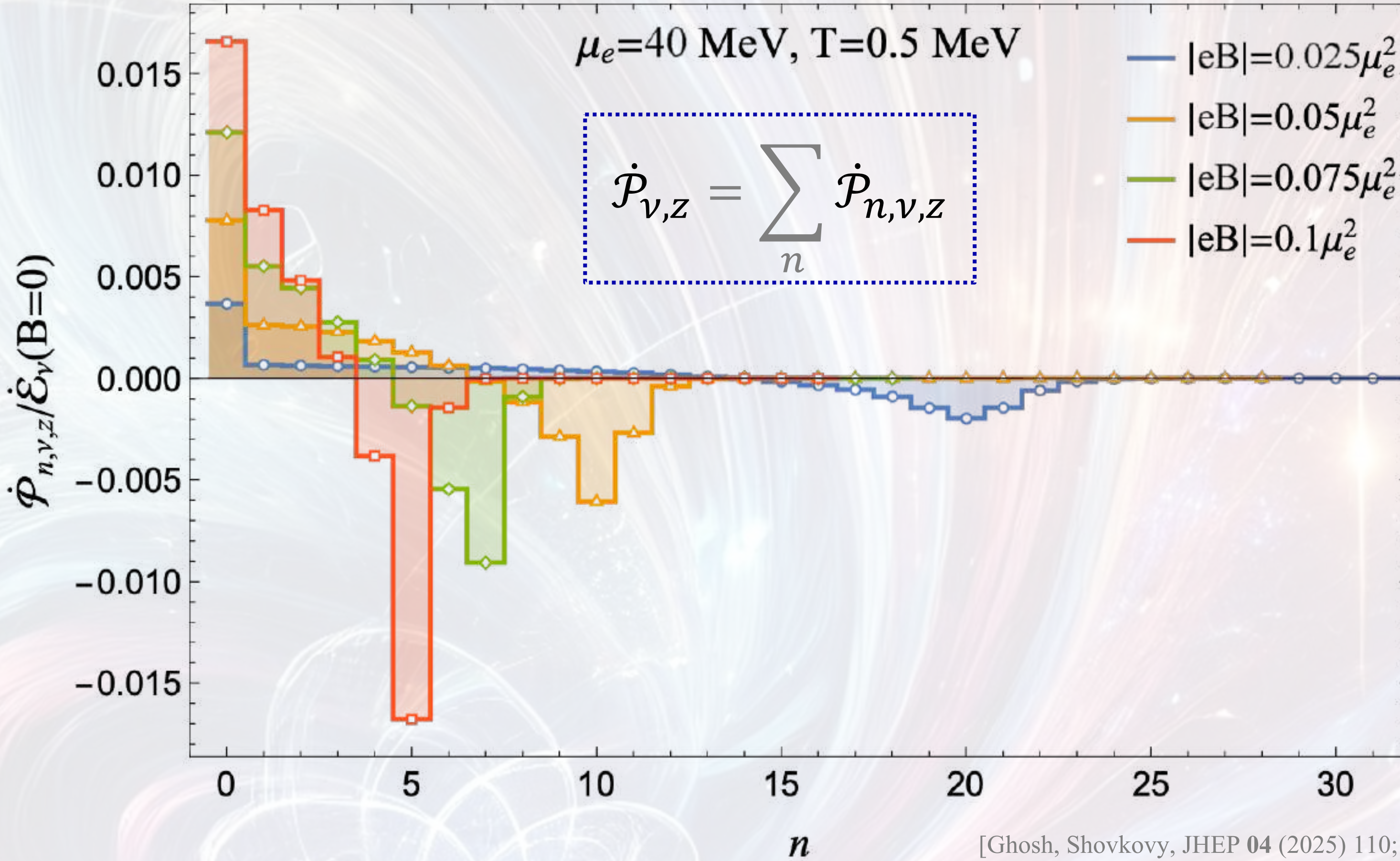

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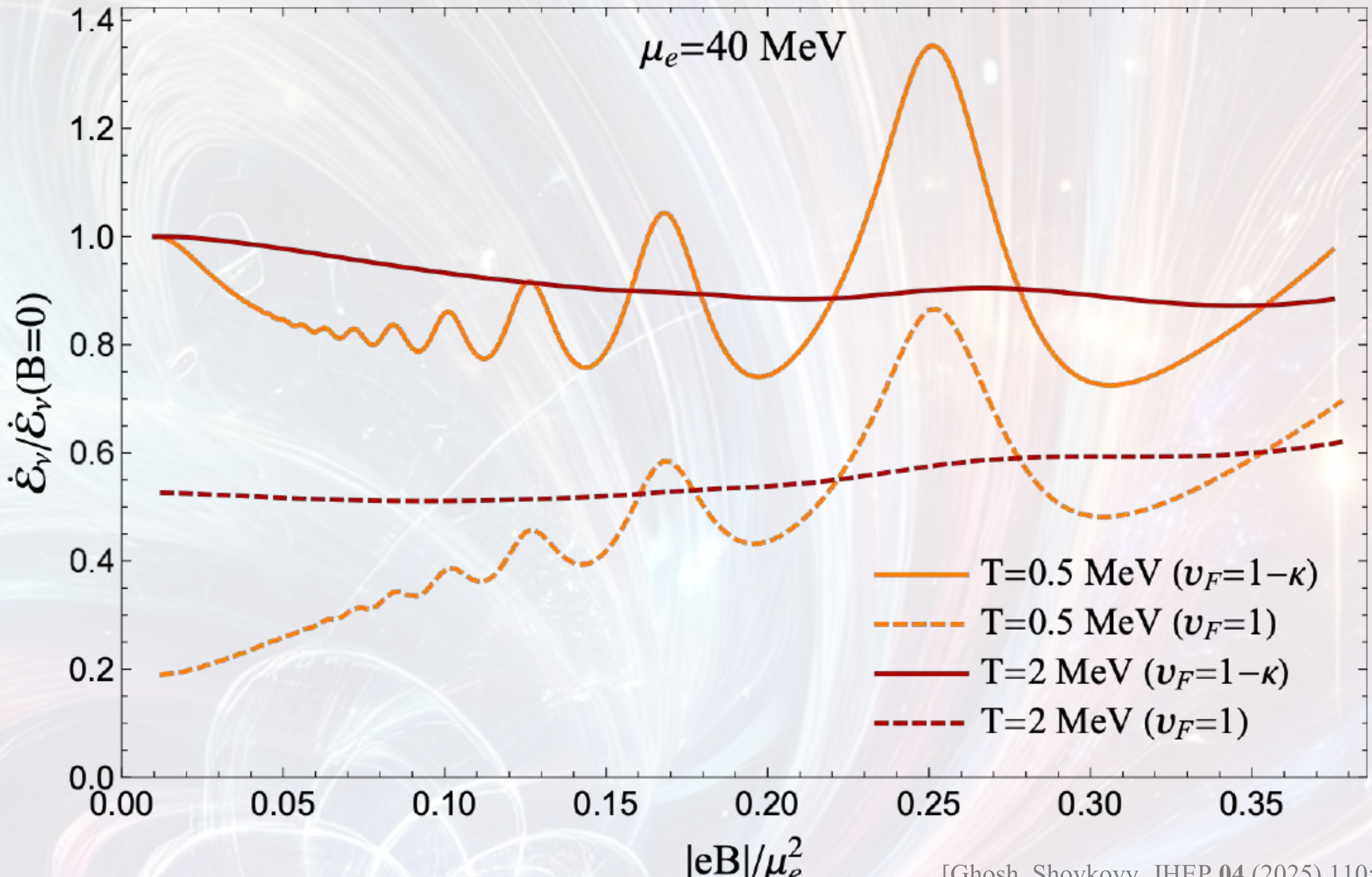
B vs. Fermi liquid corrections

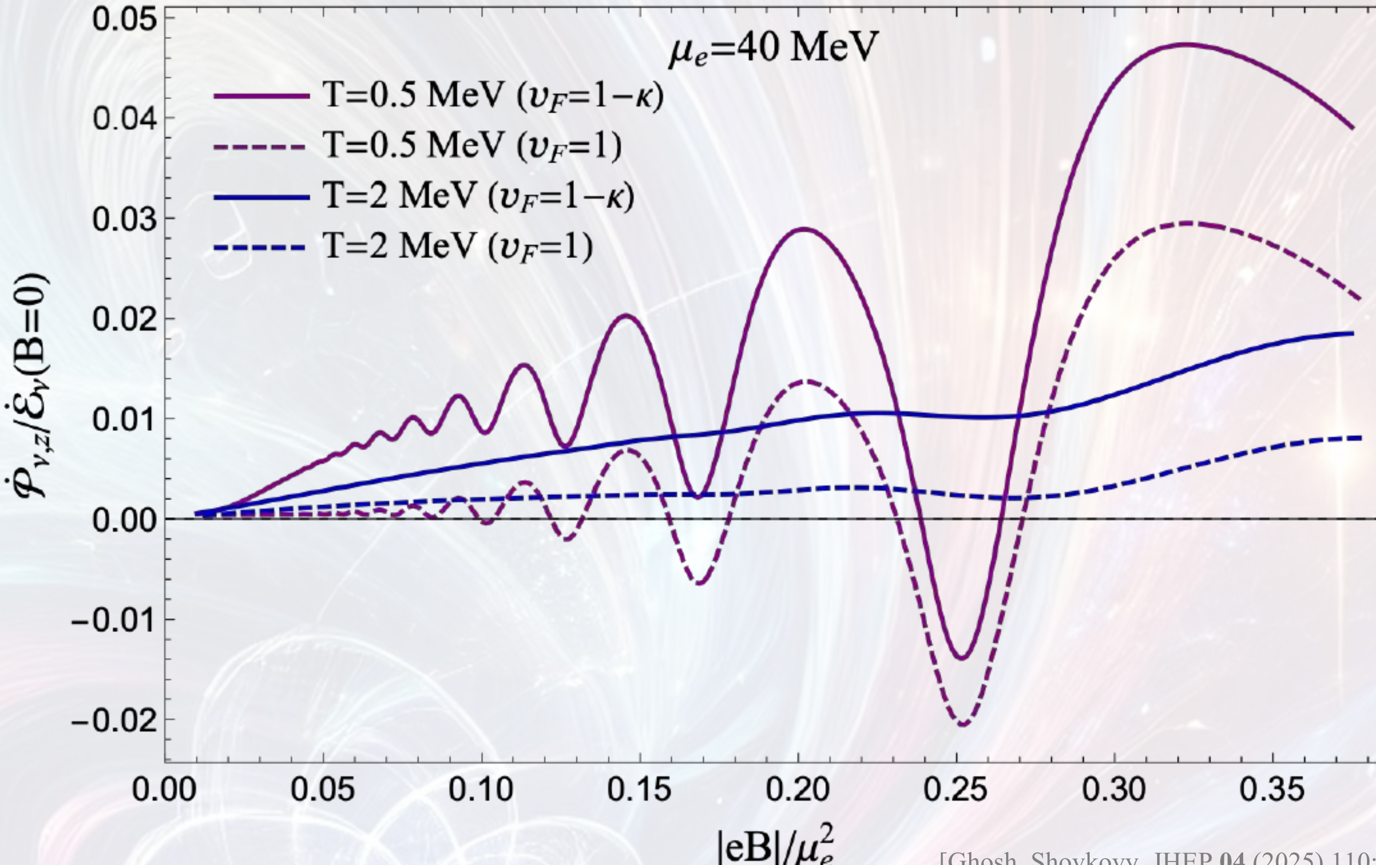
- Strong magnetic field resembles some effects of Fermi liquid corrections



[Ghosh, Shovkovy, JHEP **04** (2025) 110; arXiv:2501.03318]

B vs. Fermi liquid corrections

- Without Fermi liquid corrections, the average $\dot{\mathcal{P}}_{\nu,z}$ is ≈ 0



[Ghosh, Shovkovy, JHEP **04** (2025) 110; arXiv:2501.03318]

Pulsar kicks?

- Net nonzero longitudinal momentum emission leads to a stellar recoil or a pulsar kick
- The relative kick strength is determined by

$$\eta = \frac{\dot{\mathcal{P}}_{\nu,z}}{\dot{\mathcal{E}}_{\nu}} \approx 2\times10^{-3}\frac{|eB|}{\mu_e T}$$

c.f., early naïve studies: $\eta \approx (n_- - n_+)/(n_- + n_+) \sim 0.1$ to 1

[Sagert, Schaffner-Bielich, Astron. Astrophys. 489 (2008) 281], [Ayala et al., Phys. Rev. D 97 (2018) 103008]

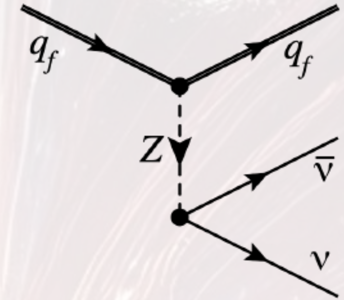
- Estimated pulsar kick velocity:

$$v_k \simeq \frac{4\pi R_c^3}{3M}\int \dot{\mathcal{P}}_{\nu,z}\,dt \approx \frac{4\pi R_c^3}{3M}\int \eta\,\dot{\mathcal{E}}_{\nu}\,dt \approx -\frac{4\pi R_c^3}{3M}\int_{T_i}^{T_f} \eta\; C_V\,dT$$

which leads to

$$v_k \simeq 1.9\ \mathrm{km/s}\left(\frac{B}{10^{16}\ \mathrm{G}}\right)\left(\frac{M_\odot}{M}\right)\left(\frac{R_c}{10\ \mathrm{km}}\right)^3\left(\frac{40\ \mathrm{MeV}}{\mu_e}\right)\left(\frac{\mu_f}{300\ \mathrm{MeV}}\right)^2\left(\frac{\Delta T}{10\ \mathrm{MeV}}\right)$$

$\nu_i\bar{\nu}_i$ synchrotron emission

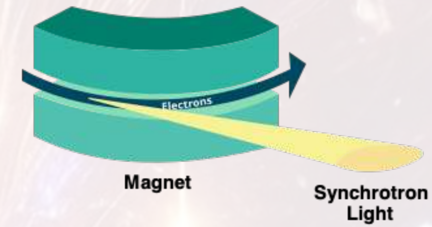


- Other types of emission operating **only** when $B \neq 0$?
- Synchrotron-like emission:

$$q_f \to q_f + \nu_i + \bar{\nu}_i$$

c.f., the usual synchrotron emission: $e^- \to e^- + \gamma$

$q_f \to q_f + \gamma$ in heavy-ion physics: [Wang, Shovkovy, Phys. Rev. D **104** (2021) 056017; arXiv:2103.01967]
[Wang, Shovkovy, Huang, Yu, Phys. Rev. D **102** (2020) 076010; arXiv:2006.16254]



- Previously, $e^- \to e^- + \nu_i + \bar{\nu}_i$ was studied in other phases of matter in compact stars

[Yakovlev, Tschaepe, Astronomische Nachrichten 302 (1981) 167]
[Kaminker et al., Phys. Rev. D 46 (1992) 3256]
[Kaminker, Yakovlev, J. Exp. Theor. Phys. 76 (1993) 229]
[Vidaurre et al., Astrophys. J. 448 (1995) 264]
[Bezchastnov et al., Astron. Astrophys. 328 (1997) 409]

- In some temperature/density regimes of core/crust of compact stars, synchrotron emission may dominate the total rate

Formalism

- Using the Kadanoff-Baym formalism, we obtain

$$\dot{\mathcal{E}}_\nu = \frac{2N_c G_F^2 |e_f B|^2}{3(2\pi)^5}\left((c_V^f)^2 + (c_A^f)^2\right)\sum_{s=1}^{\infty}\int_0^{\pi}\sin\theta d\theta \int_{-s\delta\epsilon_B/(1+\cos\theta)}^{s\delta\epsilon_B/(1-\cos\theta)} dq_z\, q_0^2\, n_B(q_0)(q_0^2 - q_z^2) J_{s+1}(w)\, J_{s-1}(w)$$

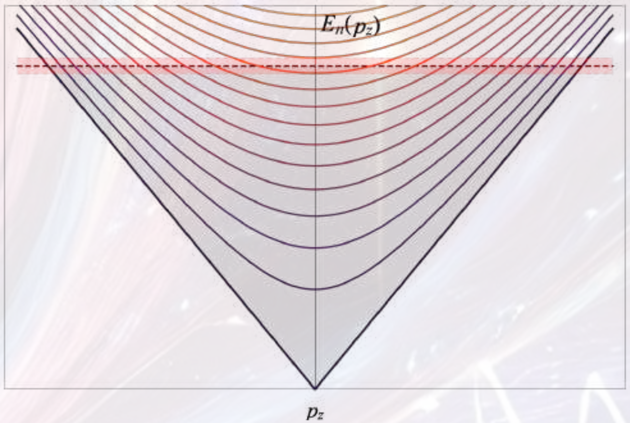
where q is the $\nu_i\bar{\nu}_i$ momentum, $s = n' - n$, and $w = \frac{\mu_q}{|eB|}\sqrt{q_0^2 - q_z^2}\sin\theta$

- Since $\mu_q \gg T$, the only relevant low-energy parameters are

$$T \quad \& \quad \delta\epsilon_B \simeq |eB|/\mu_q$$

- The emission rate is determined by their ratio:

$$b = \frac{|eB|}{\mu_q T}$$



[Ghosh, Shovkovy, arXiv:2504.21083]

Synchrotron emission rate

- The final expression for the rate

$$\dot{\mathcal{E}}_\nu = \frac{2N_c G_F^2 |e_f B|^2 T^5}{3(2\pi)^5}\left((c_V^f)^2 + (c_A^f)^2\right) F(b) \qquad b = \frac{|eB|}{\mu_q T}$$

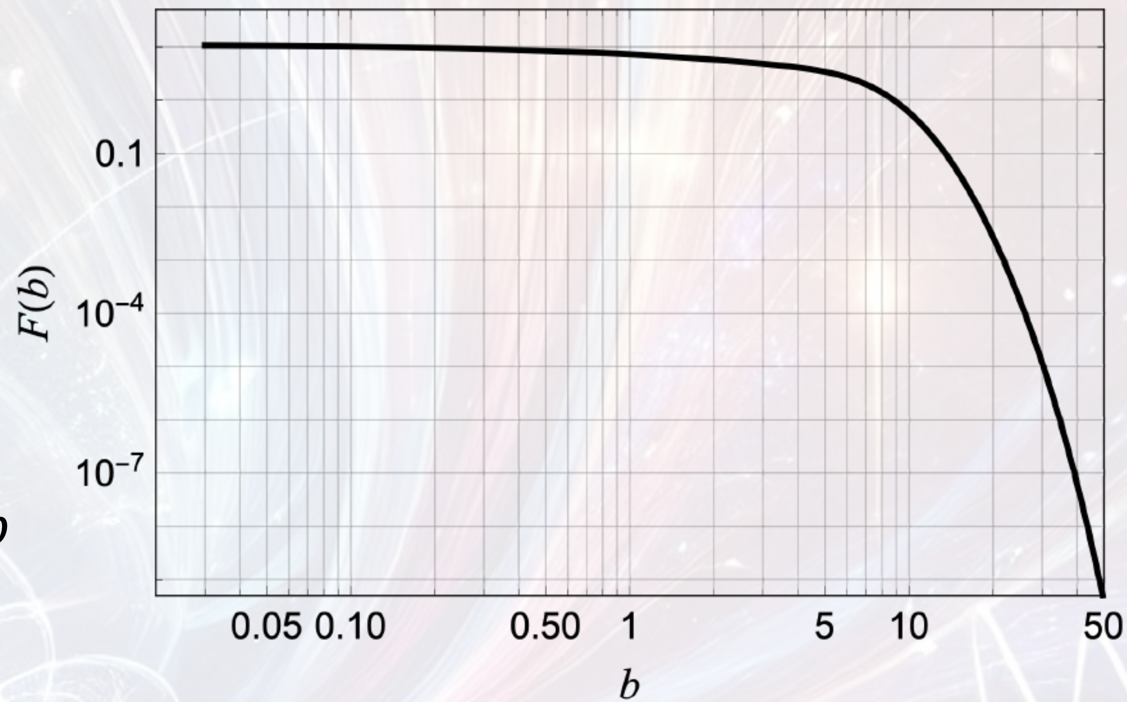
- Limit $b \to 0$:

$$F(0) = 11.06$$

- Limit $b \gg 1$:

$$F(b) \propto e^{-b/2}$$

- $\dot{\mathcal{E}}_\nu$ depends on μ_q only via b



[Ghosh, Shovkovy, arXiv:2504.21083]

Transitions with fixed $s = n' - n$

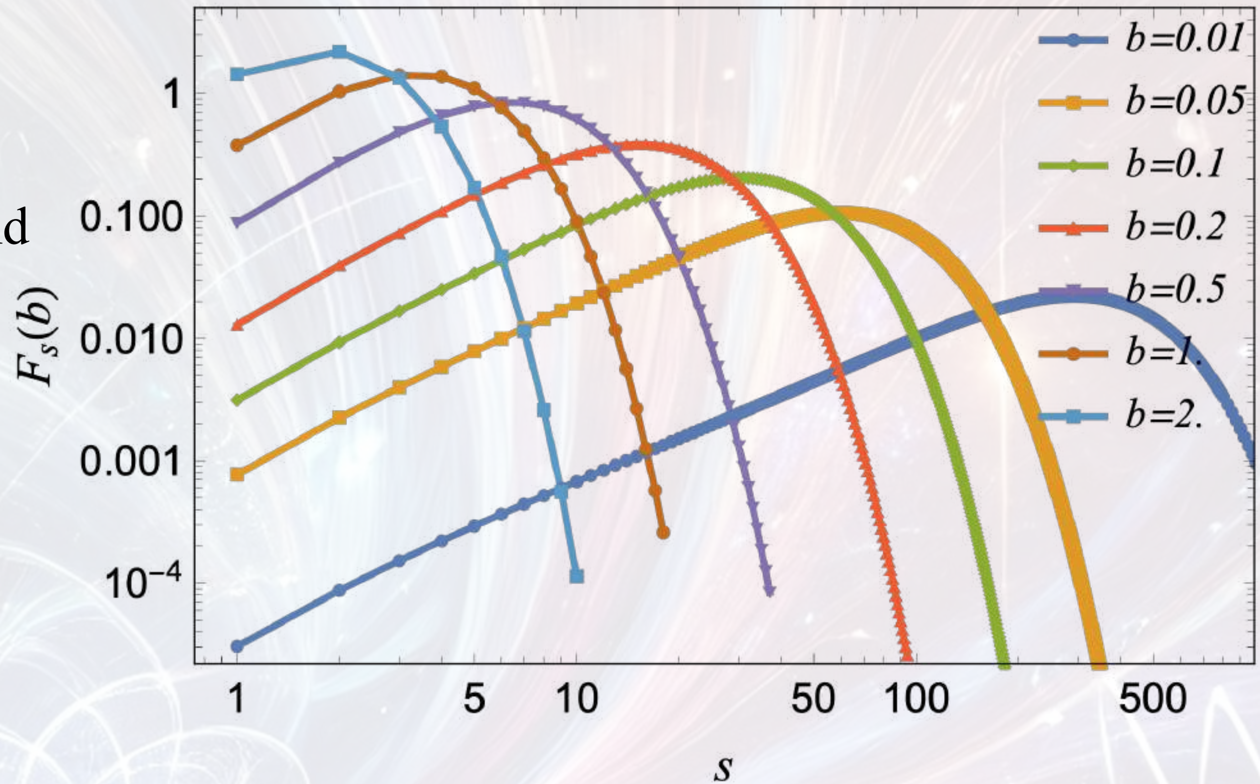
- Partial contributions of different series of quantum transitions with fixed values of $s = n' - n$:

$$F(b) = \sum_{s=1}^{\infty} F_s(b)$$

$F_s(b)$ has maximum around

$$s_{\max} \approx \frac{3}{b}$$

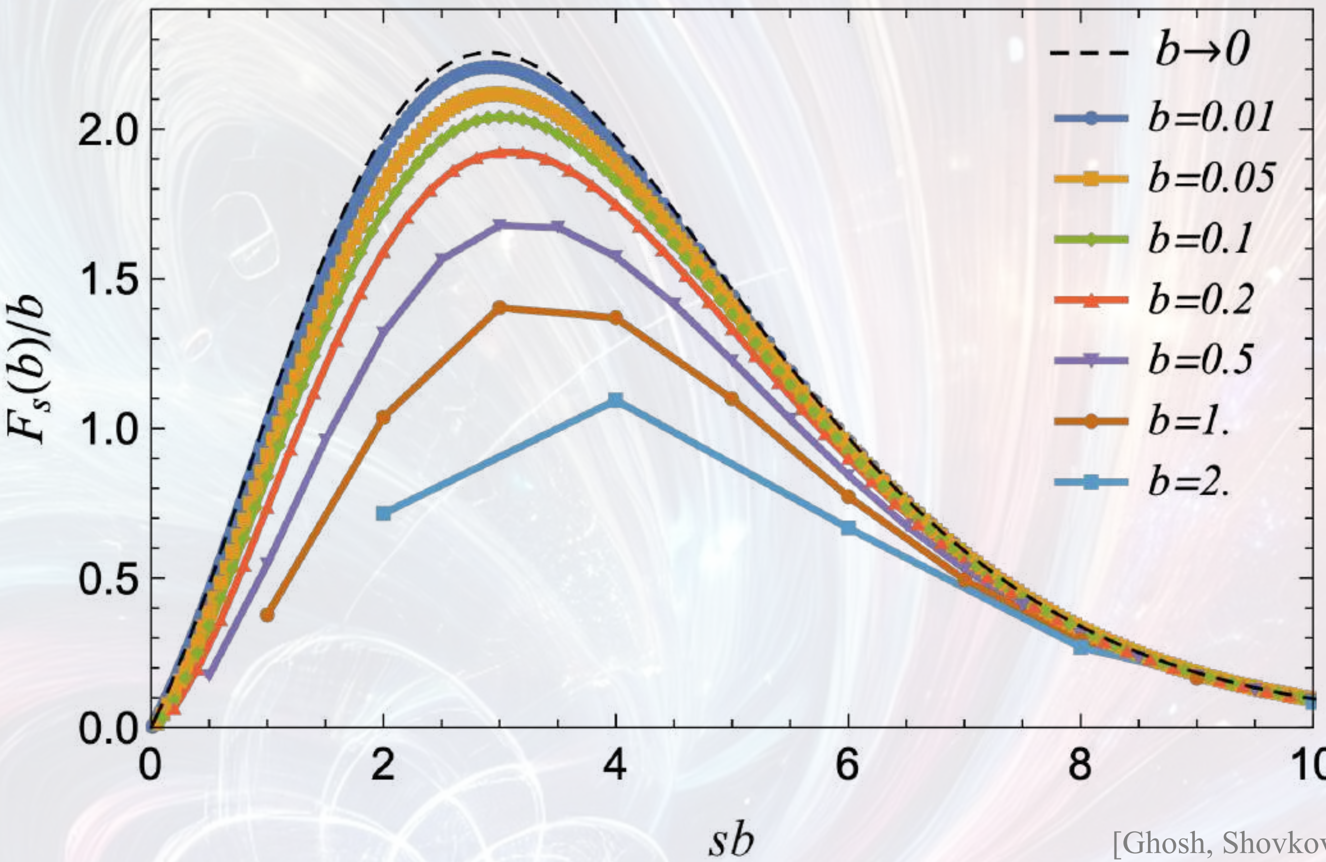
$$F_{s_{\max}}(b) \propto b$$



[Ghosh, Shovkovy, arXiv:2504.21083]

Scaling of transitions with fixed "s"

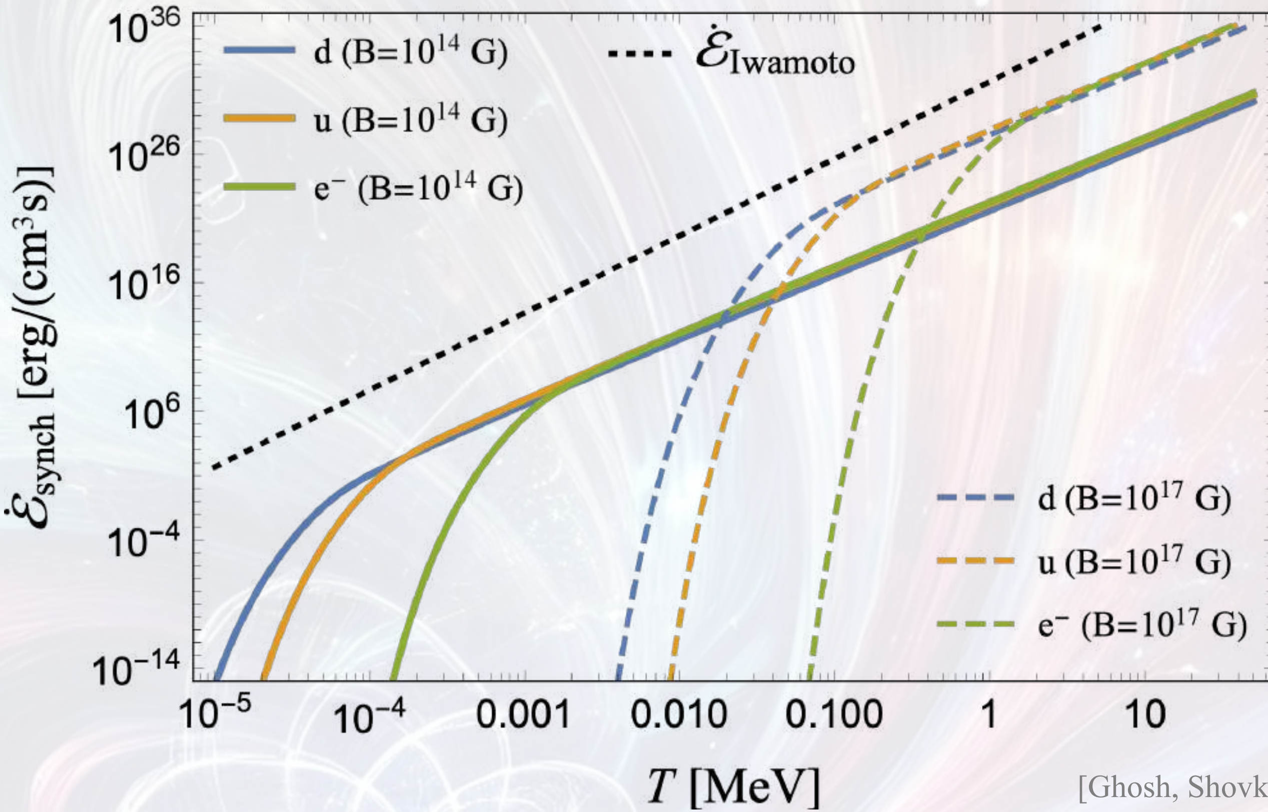
- $F_s(b)/b$ vs. sb approaches a fixed curve as $b \to 0$:



[Ghosh, Shovkovy, arXiv:2504.21083]

Quark & electron contributions

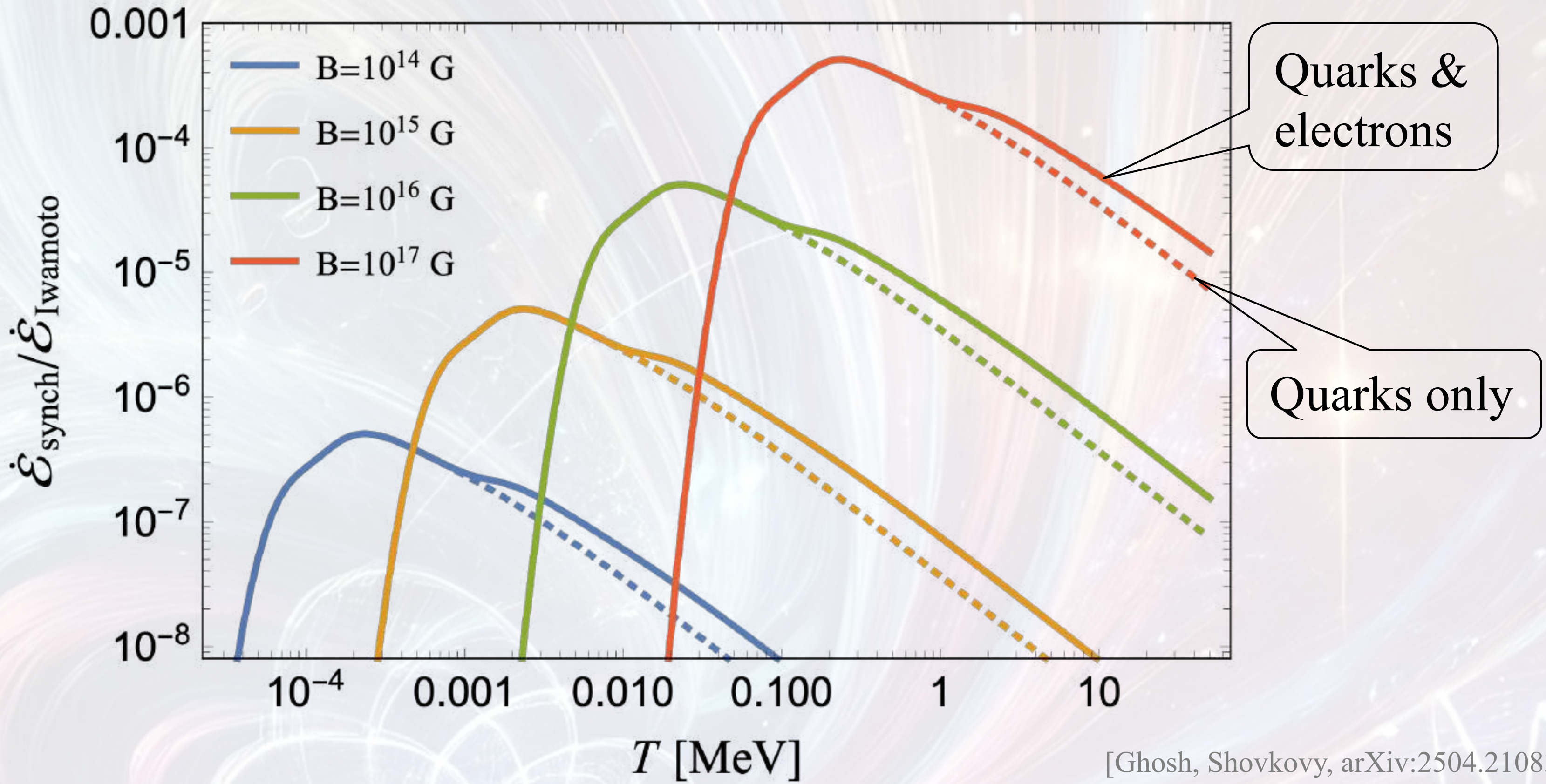
- In electrically neutral, β-equilibrated quark matter, small density of electrons must be present $(n_u \approx 0.5 n_d, n_e \lesssim 0.003 n_d)$



[Ghosh, Shovkovy, arXiv:2504.21083]

Rate vs. temperature

- Synchrotron rate $\dot{\mathcal{E}}_\nu$ is more than 1000 times smaller than $\dot{\mathcal{E}}_{\mathrm{Iwamoto}}$



[Ghosh, Shovkovy, arXiv:2504.21083]

Other phases of quark matter

- Dense quark matter is likely to be color superconducting
  - Gapped states contribute little to specific heat and neutrino emission:

$$C_V \propto e^{-\Delta/T}$$
$$\dot{\mathcal{E}}_\nu \propto e^{-\Delta/T}$$

  where $\Delta \sim O(10\text{ MeV})$ is the color superconducting gap

- 2-flavor color superconductor (2SC) has unpaired quark states
  - It is likely to cool similarly to the normal phase of quark matter
- 3-flavor superconductor (CFL) has all quark states gapped
  - It is likely to play little role in stellar cooling (no heat, no emission)
- 1-flavor (spin-one) color superconducting phases are nontrivial
  - Some but not all states at the Fermi surface are ungapped (gapless nodes & lines)

Summary and Outlook

- Detailed analysis of direct Urca & synchrotron neutrino emission from quark matter is performed
- Direct Urca emission from quark matter is affected by the magnetic field, but not very dramatically (~20%)
- Direct Urca emission is asymmetric, but $\dot{\mathcal{P}}_{\nu,z}$ is not large enough to contribute to pulsar kicks
- Compared to direct Urca, synchrotron emission is significantly weaker
- 2SC & CFL color superconducting phases of quark matter are unlikely to affect cooling (albeit for different reasons)
- Additional studies of spin-1 color superconducting phases are needed