The effective QCD Phase diagram: Parameter fixing and the speed of sound.

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9th Conference on Chirality, Vorticity and Magnetic Fields in Quantum Matter

July 11th, 2025

Second Latin American Workshop on Electromagnetics Effects in QCD

24-28 November 2025
Pontificia Universidad Católica de Chile; Campus San Jaoquín
America/Mexico_City timezone

The scientific program includes contributed talks with a length of 25+5 minutes, review lectures, as well as poster sessions in the following topics:

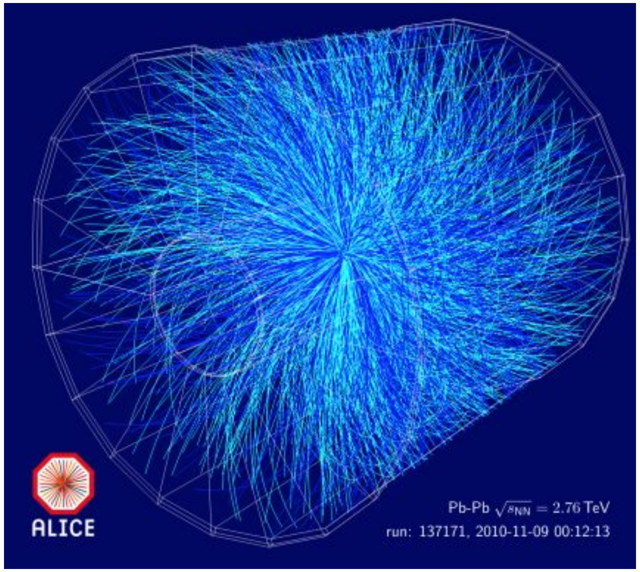
- Lattice QCD
- Non-perturbative QCD
- Holographic Ads/QCD models
- Dynamical Chiral Symmetry breaking
- Hadron Structure and Interactions
- Schwinger-Dyson equations
- Effective field Theory
- Heavy-Ion collisions
- Electromagnetic fields as probes of strongly interacting matter

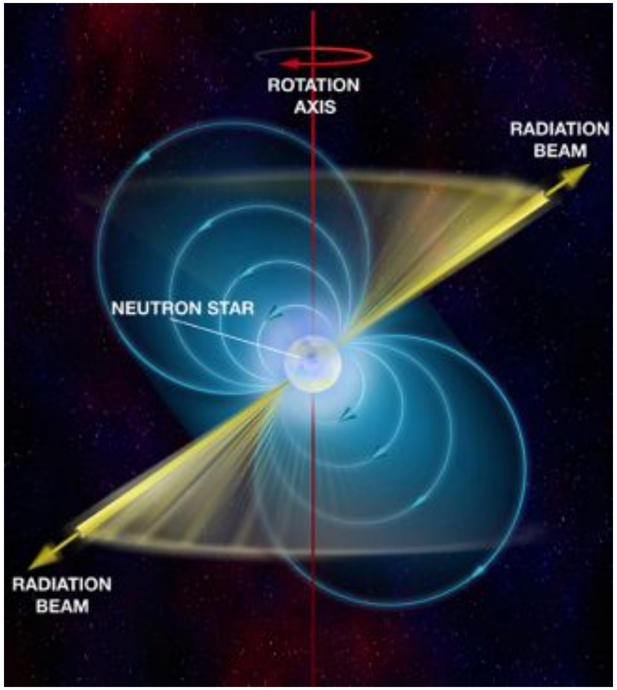
Outline

- Motivation
- The Linear Sigma model
- Results and comments so far...
- What's next???
- speed of sound
- final comments

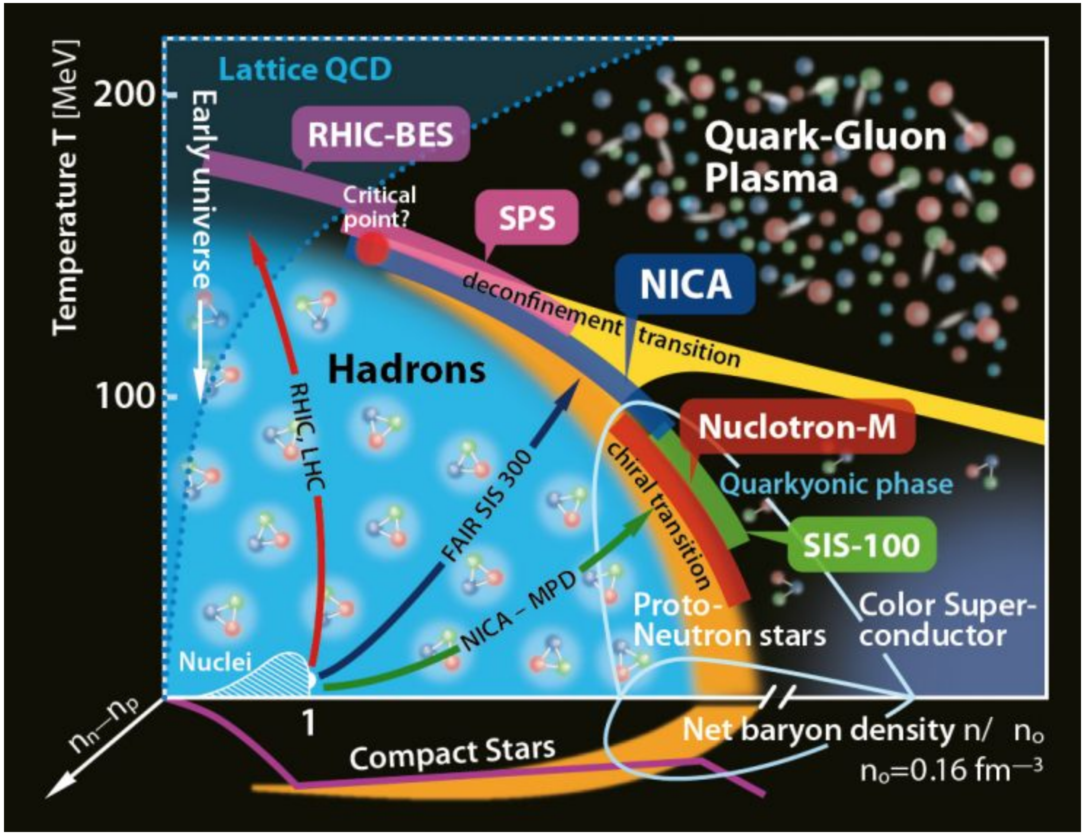
Motivation

- QCD under extreme conditions (temperature and finite quark density) play an important role in understanding the transitions that took place in the early universe.





Motivation

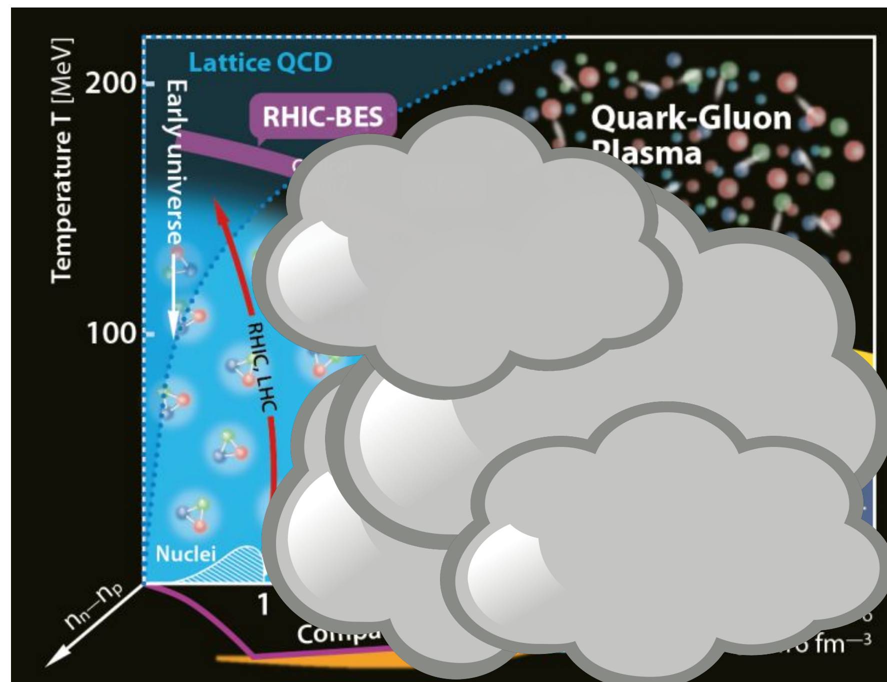


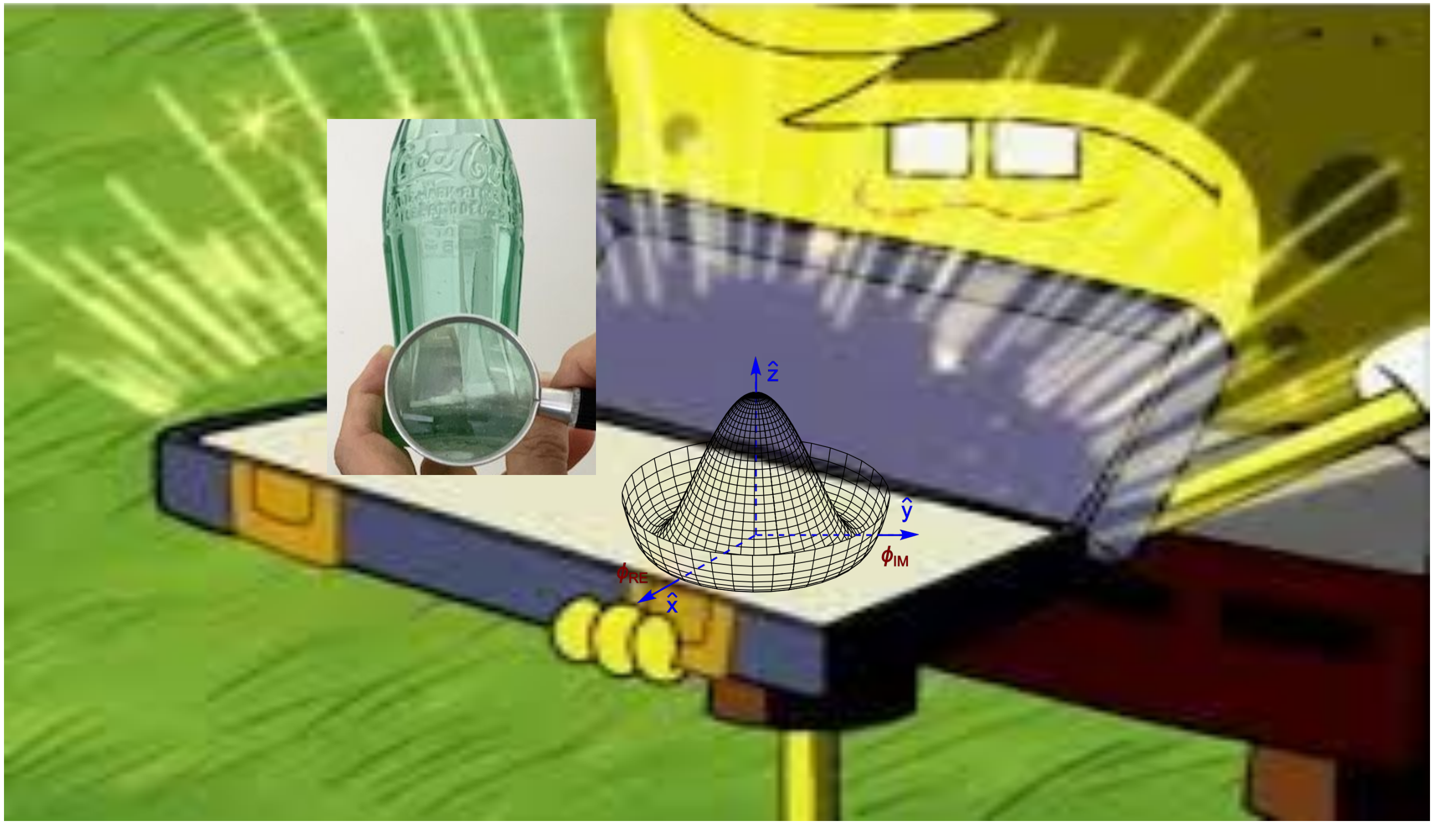
Phase Transition ⇔ Restore/Broke Symmetry

Confinement y/o Chiral Symmetry restauration

Motivation

- There is only reliable information at low densities.
- There are experimental efforts to dissipate doubts at higher densities.
  - NICA
  - RHIC(BESII)
  - JPARC
  - HADES





Old Reliable!!!

Linear Sigma Model

- Effective model for low-energy QCD.

- Effects of quarks and mesons on the chiral phase transition.

- Implement ideas of chiral symmetry and spontaneous symmetry breaking

Linear Sigma Model

- After the shift

$$\mathcal{L} = -\frac{1}{2}[\sigma(\partial_\mu + iqA_\mu)^2\sigma] \underline{- \frac{1}{2}\left(3\lambda v^2 - a^2\right)\sigma^2}$$
$$- \frac{1}{2}[\vec{\pi}(\partial_\mu + iqA_\mu)^2\vec{\pi}] \underline{- \frac{1}{2}\left(\lambda v^2 - a^2\right)\vec{\pi}^2}$$
$$+ i\bar{\psi}\gamma^\mu D_\mu \psi \underline{- gv\bar{\psi}\psi} + \frac{a^2}{2}v^2 - \frac{\lambda}{4}v^4$$
$$-\frac{\lambda}{4}[(\sigma^2 + \pi_0^2)^2 + 4\pi^+\pi^-(\sigma^2 + \pi_0^2 + \pi^+\pi^-)]$$
$$-g\hat{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi$$

with masses

$$m_\sigma^2 = 3\lambda v^2 - a^2$$
$$m_\pi^2 = \lambda v^2 - a^2$$
$$m_f = gv$$

which increases with the order parameter

Linear Sigma Model

- We calculate the effective potential for fermions and bosons at temperature and finite chemical potential

$$V_b = s_b T \sum_n \int \frac{d^3k}{(2\pi)^3} \ln\left(D^{-1}\right)^{1/2}, \qquad V_f = s_f T \sum_n \int \frac{d^3k}{(2\pi)^3} \ln\left(S^{-1}\right)$$

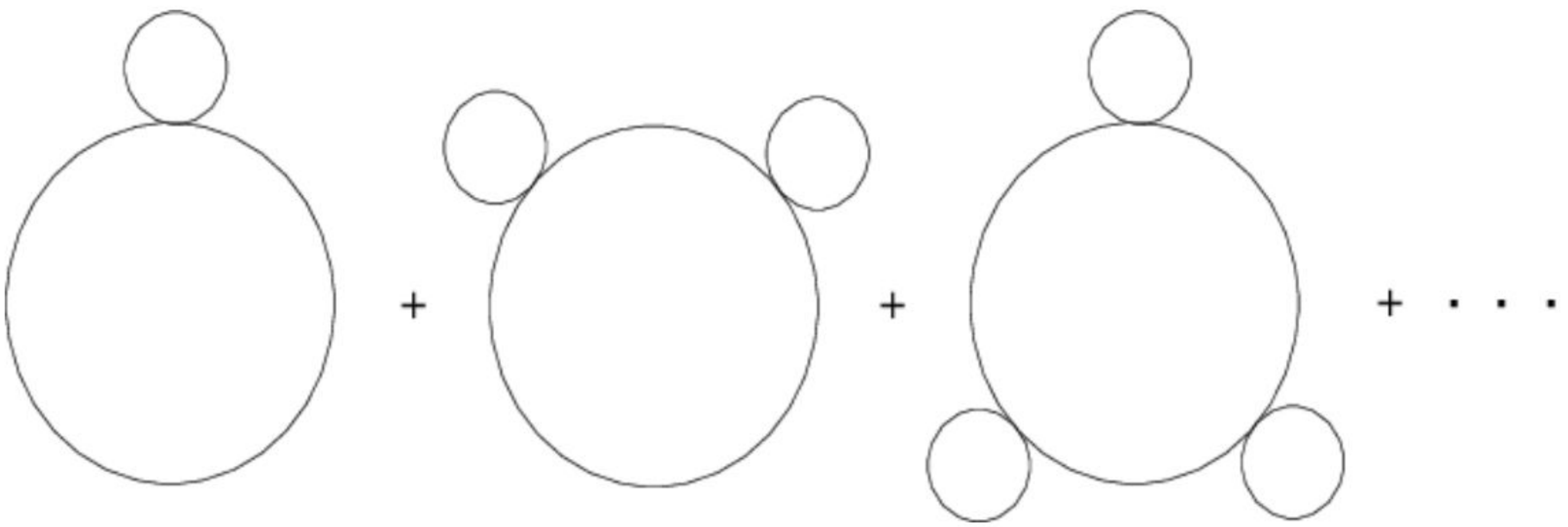
.

where the thermal boson and fermion propagators are given by

$$D = \frac{1}{k^2 + m_b^2 + \omega_n^2}, \qquad S = \frac{\not{k} + m_f}{k^2 + m_f^2 + (\omega_n - i\mu)^2}.$$

Linear Sigma Model

- In order to include media effects on the mesons we need to go beyond mean field and include the Ring diagrams to the boson contribution

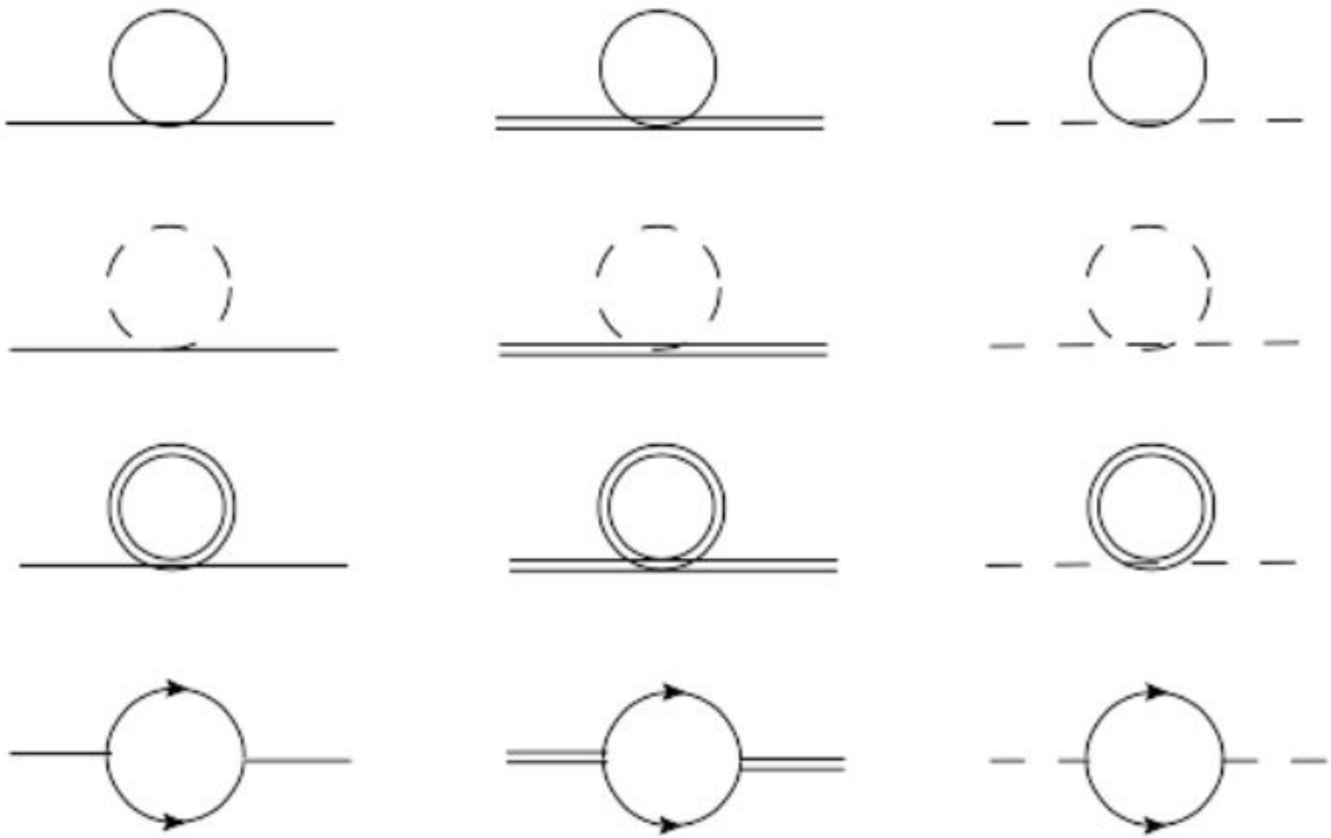


Then the full effective potential is:

$$
\begin{aligned}
V^{\mathrm{eff}} &= V^{\mathrm{tree}} + V^{\mathrm{b}} + V^{\mathrm{f}} + V^{\mathrm{Ring}} \\
V^{\mathrm{tree}}(v) &= -\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4 \\
V^{\mathrm{b}}(v, T) &= T\sum_n \int \frac{d^3k}{(2\pi)^3} \ln D_{\mathrm{b}}(\omega_n, \vec{k})^{1/2} \\
V^{\mathrm{f}}(v, T, \mu) &= -T\sum_n \int \frac{d^3k}{(2\pi)^3} \mathrm{Tr}[\ln S_{\mathrm{f}}(\tilde{\omega}_n, \vec{k})^{-1}] \\
V^{\mathrm{Ring}}(v, T, \mu) &= \frac{T}{2}\sum_n \int \frac{d^3k}{(2\pi)^3} \ln[1 + \Pi_{\mathrm{b}} D(\omega_n, \vec{k})]
\end{aligned}
$$

with Π the self energy

$$\Pi = \lambda\frac{T^2}{2} - N_f N_c g^2 \frac{T^2}{\pi^2}[Li_2(-e^{\mu/T}) + Li_2(-e^{-\mu/T})]$$



High Temperature

$$
\begin{aligned}
V^{(eff)} = & \boxed{-\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4} + \sum_{i=\sigma,\pi^0} \Bigg\{ \frac{m_i^4}{64\pi^2}\left[\ln\left(\frac{(4\pi T)^2}{2a^2}\right) - 2\gamma_E + 1\right] \\
& - \frac{\pi^2 T^4}{90} + \frac{m_i^2 T^2}{24} - \frac{T}{12\pi}\boxed{(m_i^2 + \Pi)^{3/2}} \Bigg\} \\
& - N_c \sum_{f=u,d} \left[\frac{m_f^4}{16\pi^2}\left[\ln\left(\frac{(4\pi T)^2}{2a^2}\right) + \psi^0\left(\frac{1}{2} + \frac{i\mu}{2\pi T}\right) \right.\right. \\
& \left. + \psi^0\left(\frac{1}{2} - \frac{i\mu}{2\pi T}\right)\right] + 8m_f^2T^2[Li_2(-e^{\mu/T}) + Li_2(-e^{-\mu/T})] \\
& \left. - 32T^4[Li_4(-e^{\mu/T}) + Li_4(-e^{-\mu/T})] \right]
\end{aligned}
$$

Model Parameters

- The parameter space consists of the λ and g coupling constants and the mass parameter a, which can be fix by LQCD data (PRL 125, 052001 (2020)).

Fixing a with:

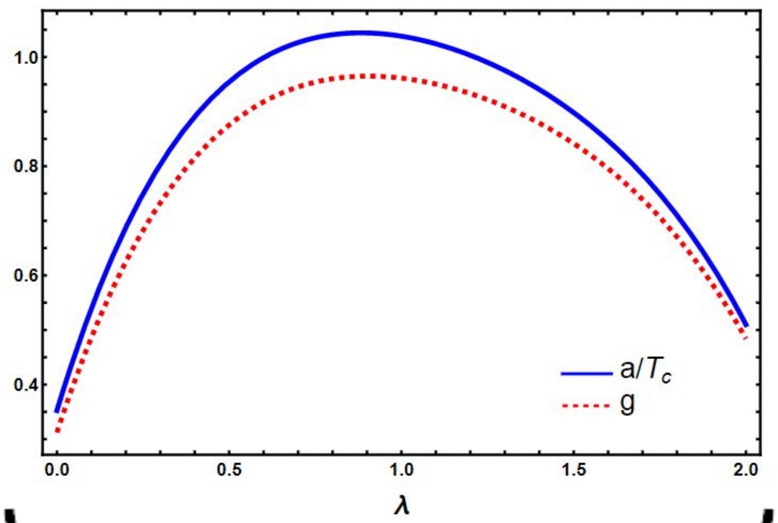
$$6\lambda\left(\frac{T_c^2}{12}-\frac{T_c}{4\pi}\left(\Pi_b(T_c,\mu_B=0)-a^2\right)^{1/2}+\frac{a^2}{16\pi^2}\left[\ln\left(\frac{\tilde{\mu}^2}{T_c^2}\right)\right]\right)+g^2T_c^2-a^2=0.$$

Fixing λ and g with the collection of curves that obey this relation:

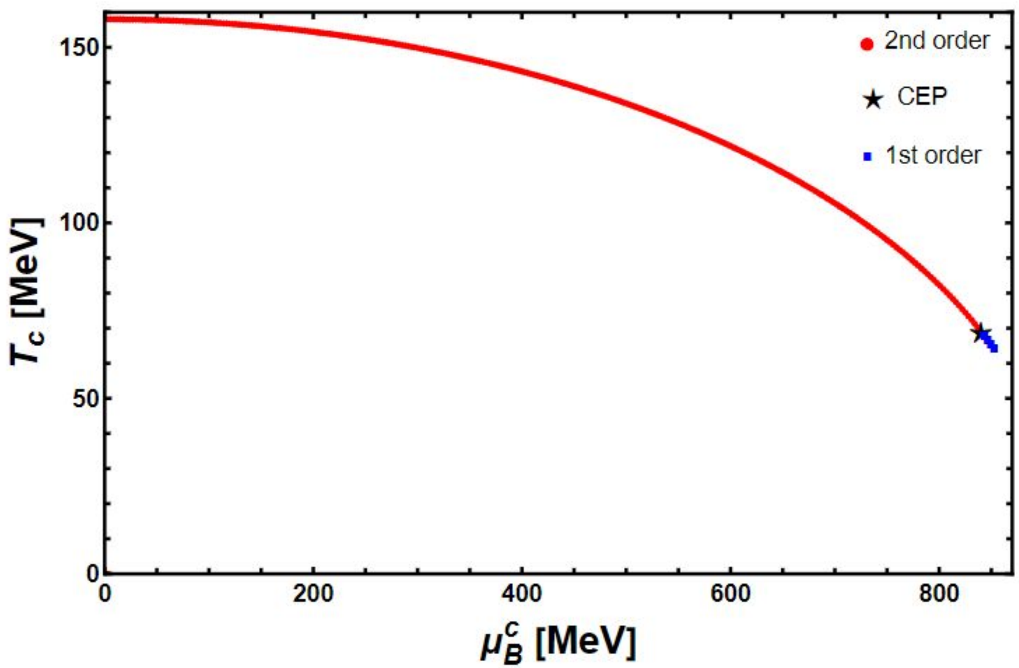
$$\frac{T_c(\mu_B)}{T_c^0}=1-\kappa_2\left(\frac{\mu_B}{T_c^0}\right)^2+\kappa_4\left(\frac{\mu_B}{T_c^0}\right)^4$$

$\kappa_2 = 0.0153$ and $\kappa_4 = 0.00032$

Results



$$\lambda = 0.4,\ \ g = 0.88\ \ \text{and}\ \ a = 141.38\ \text{MeV}$$



$$768\ \text{MeV} < \mu_B^{CEP} < 849\ \text{MeV}$$

$$69\ \text{MeV} < T^{CEP} < 70.3\ \text{MeV}$$

Ayala, A, et al, Eur. Phys. J. A 58, 87 (2022).

What now?

- Speed of sound is closely related with the thermodynamics properties of any system, including the EoS.

- For example, in neutron start researches, the c_s behavior as a function of baryon number density influences the mass-radius relationship.

- In HIC, c_s also conveys relevant information; for example, it displays a local minimum at a crossover transition.

Speed of sound

- The square of the speed of sound is usually defined as

$$c_\chi^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_\chi$$

where χ denotes the parameter fixed in the calculation of the speed of sound.

- According to the properties on the propagation medium, it may be more useful to keep one quantity fixed rather than another.

Speed of sound

- For this work, we will focus on

$$c_{\rho_B}^2 = \frac{\partial(p, \rho_B)}{\partial(\epsilon, \rho_B)} = \frac{s\chi_{\mu\mu} - \rho_B \chi_{\mu T}}{T(\chi_{TT}\chi_{\mu\mu} - \chi_{\mu T}^2)},$$

$$c_s^2 = \frac{\partial(p, s)}{\partial(\epsilon, s)} = \frac{\rho_B \chi_{TT} - s\chi_{\mu T}}{\mu_B(\chi_{TT}\chi_{\mu\mu} - \chi_{\mu T}^2)},$$

$$c_{s/\rho_B}^2 = \frac{\partial(p, s/\rho_B)}{\partial(\epsilon, s/\rho_B)} = \frac{c_{\rho_B}^2 Ts + c_s^2 \mu_B \rho_B}{Ts + \mu_B \rho_B}.$$

Speed of sound

- The pressure, entropy and baryon number densities can be derived using the thermodynamics relations in the grand canonical ensemble as

$$p = -\Omega, \qquad \epsilon = -p + Ts + \mu_B \rho_B$$

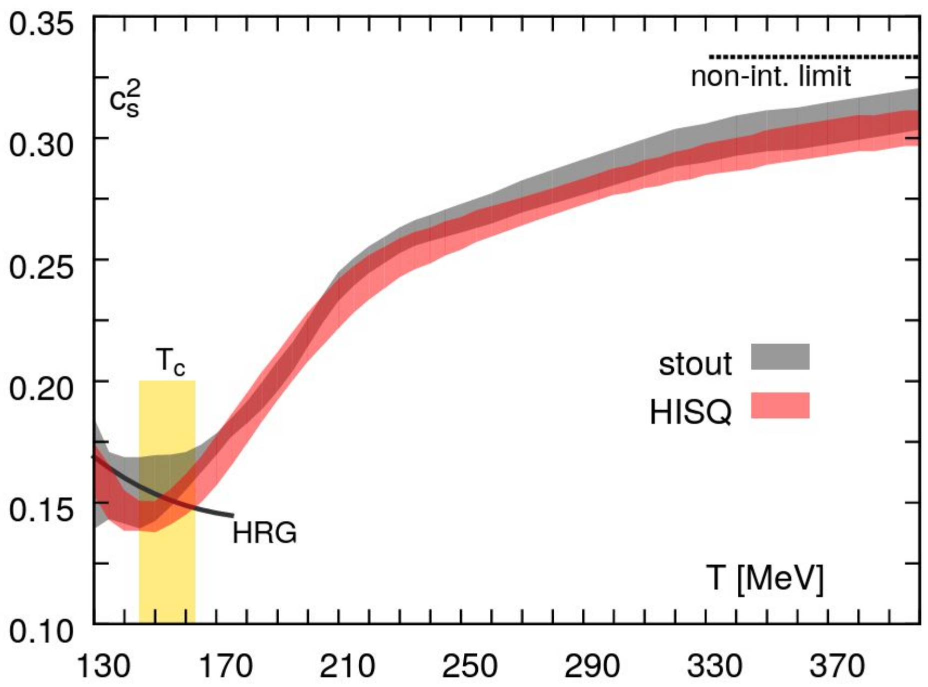
$$s = \left(\frac{\partial p}{\partial T}\right)_{\mu_B} \quad \text{and} \quad \rho_B = \left(\frac{\partial p}{\partial \mu_B}\right)_T .$$

where

$$\Omega(T, \mu) = V^{(eff)}(v = 0, T, \mu)$$

Is this useful?

This show the speed of sound squared from lattice QCD and the HRG model versus temperature, the yellow band shows the value of the critical temperature at zero chemical potential. $T_C = 154 \pm 9$



https://doi.org/10.1103/PhysRevD.90.094503

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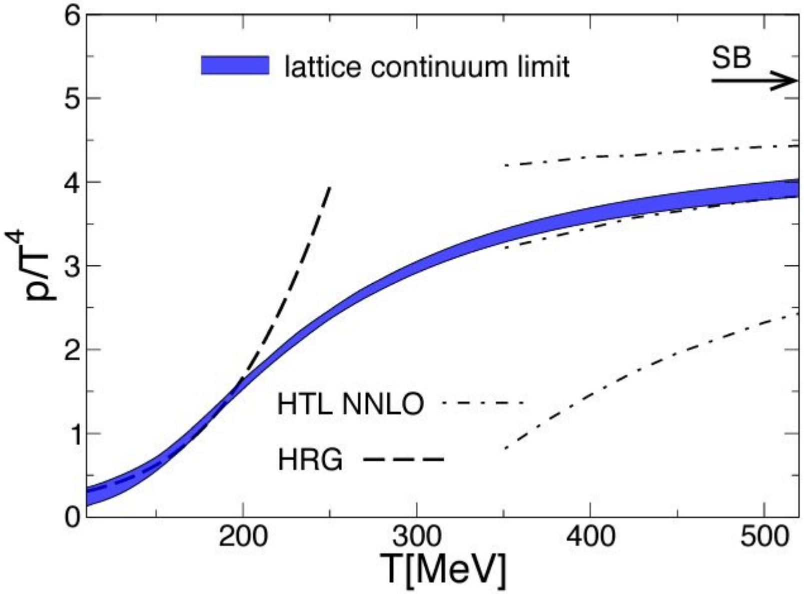
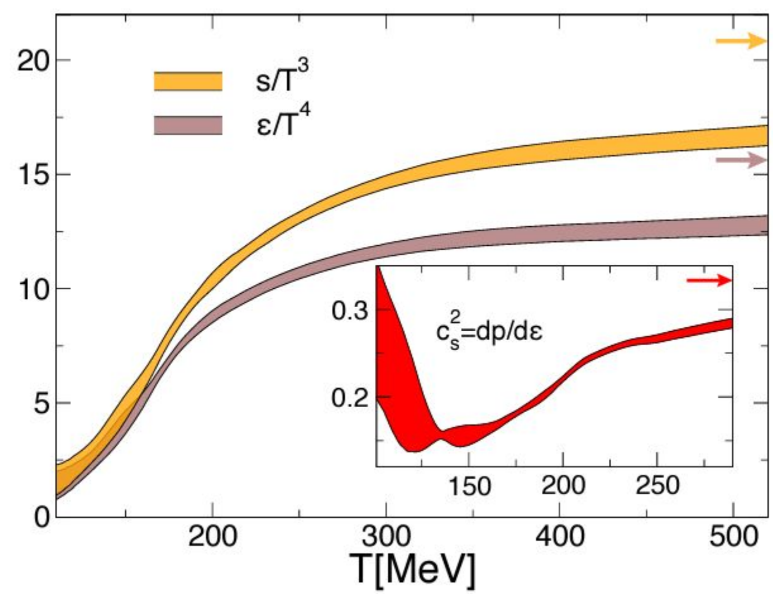




https://doi.org/10.1016/j.physletb.2014.01.007

Model Parameters

- The parameter space consists of the λ and g coupling constants and the mass parameter a, which can be fix by LQCD data (PRD 102, 034027).

$N_f = 2$; $T_C = 166$ MeV

$N_f = 2 + 1$; $T_C = 158$ MeV

Fixing a with:

$$6\lambda\left(\frac{T_c^2}{12} - \frac{T_c}{4\pi}\left(\Pi_b(T_c,\mu_B=0) - a^2\right)^{1/2} + \frac{a^2}{16\pi^2}\left[\ln\left(\frac{\tilde{\mu}^2}{T_c^2}\right)\right]\right) + g^2 T_c^2 - a^2 = 0.$$

Fixing λ and g with the collection of curves that obey this relation:

$$\frac{T_c(\mu_B)}{T_c^0} = 1 - \kappa_2\left(\frac{\mu_B}{T_c^0}\right)^2 + \kappa_4\left(\frac{\mu_B}{T_c^0}\right)^4$$

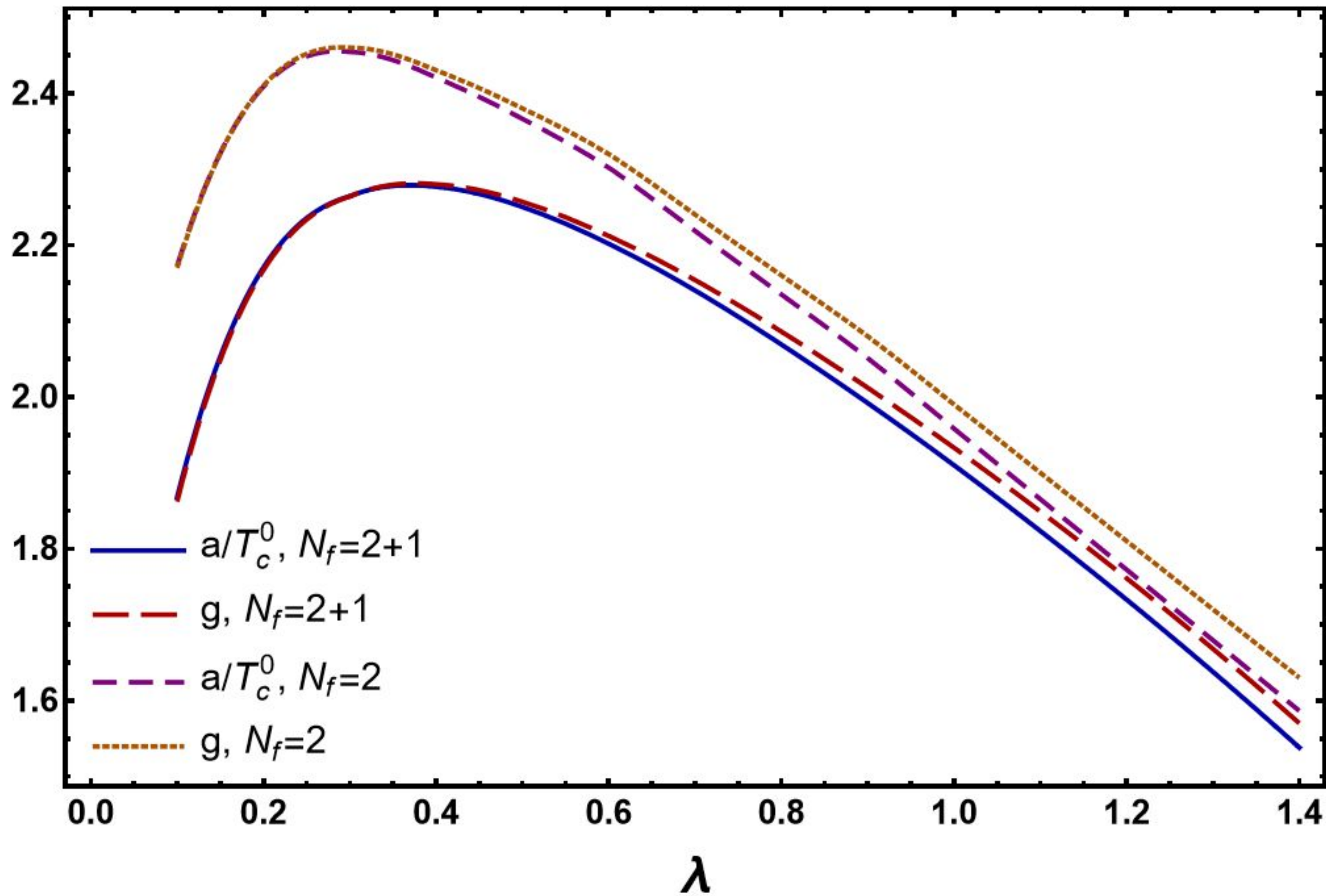
$N_f = 2$

$\kappa_2 = 0.0176$, and $\kappa_4 = 0.0$

$N_f = 2 + 1$

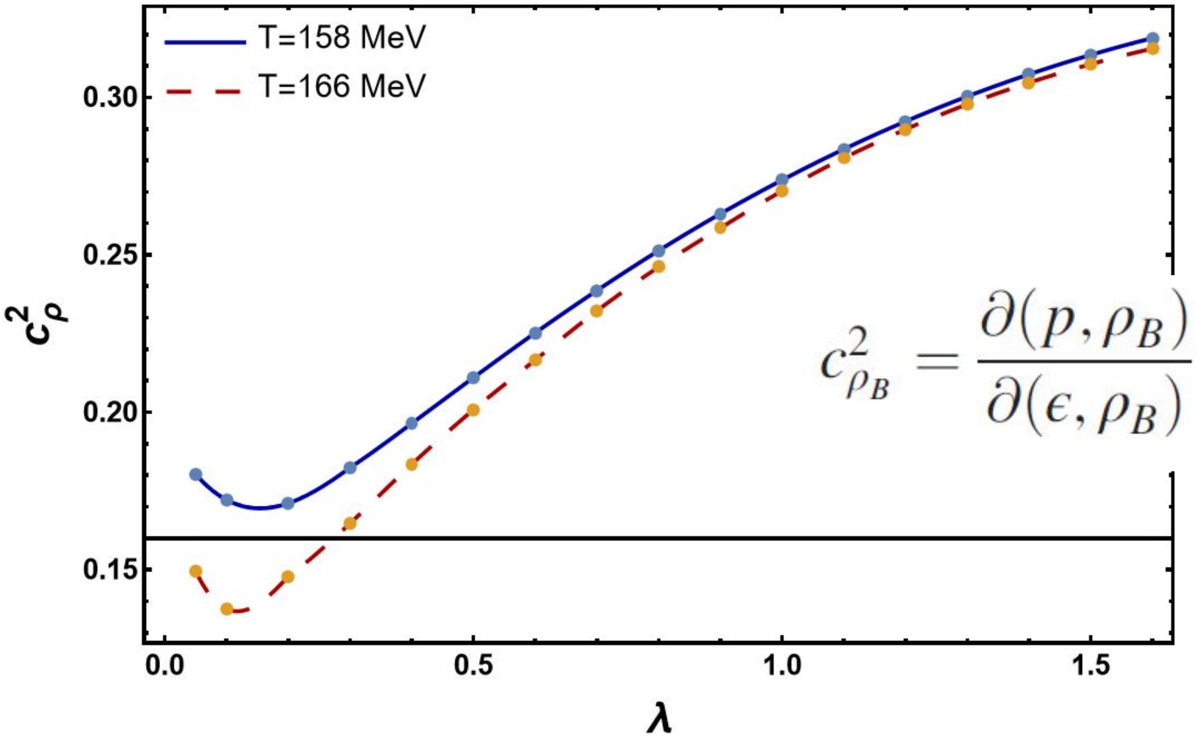
$\kappa_2 = 0.0153$, and $\kappa_4 = 0.0$

Model Parameters



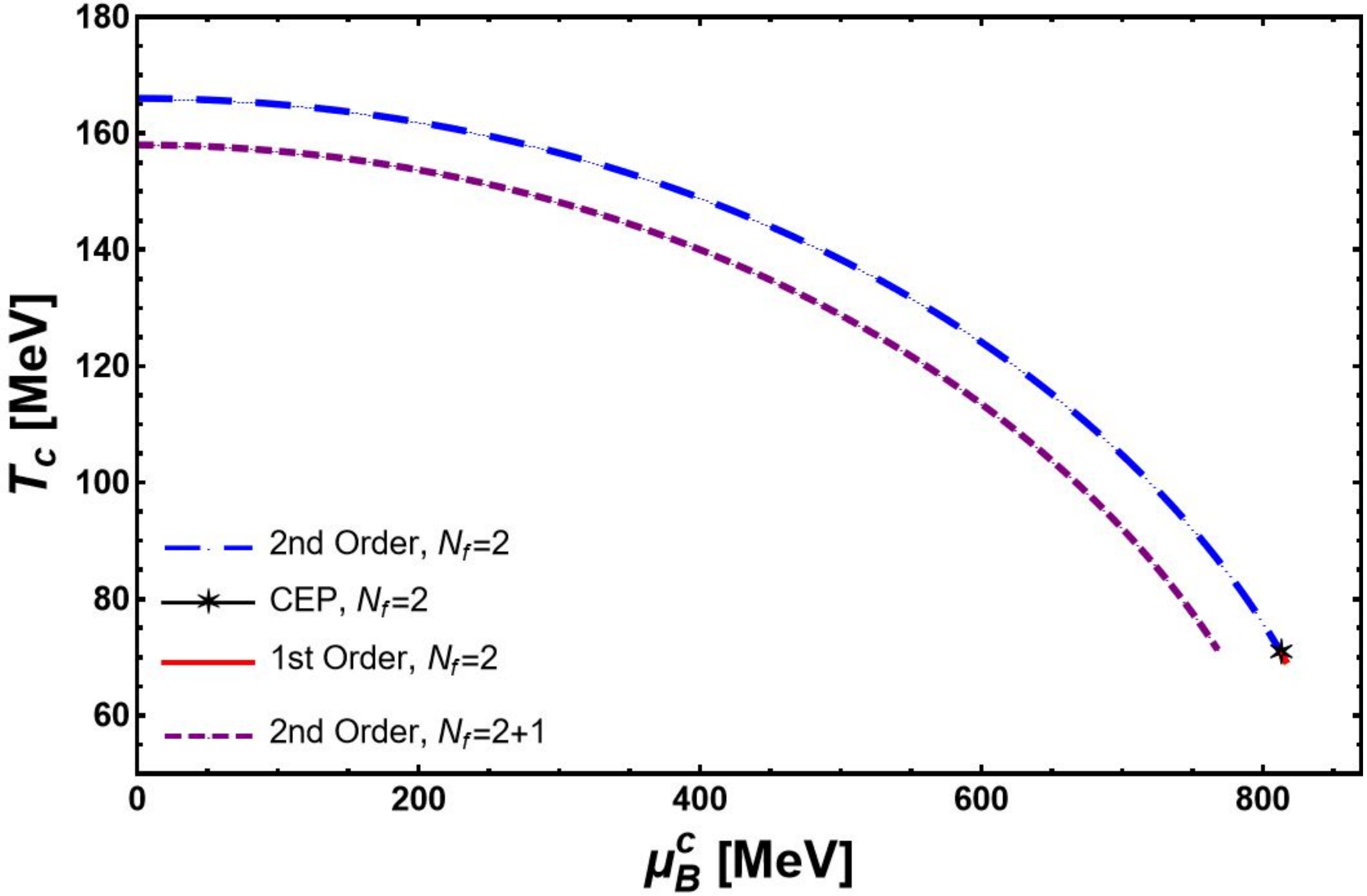
Model Parameters

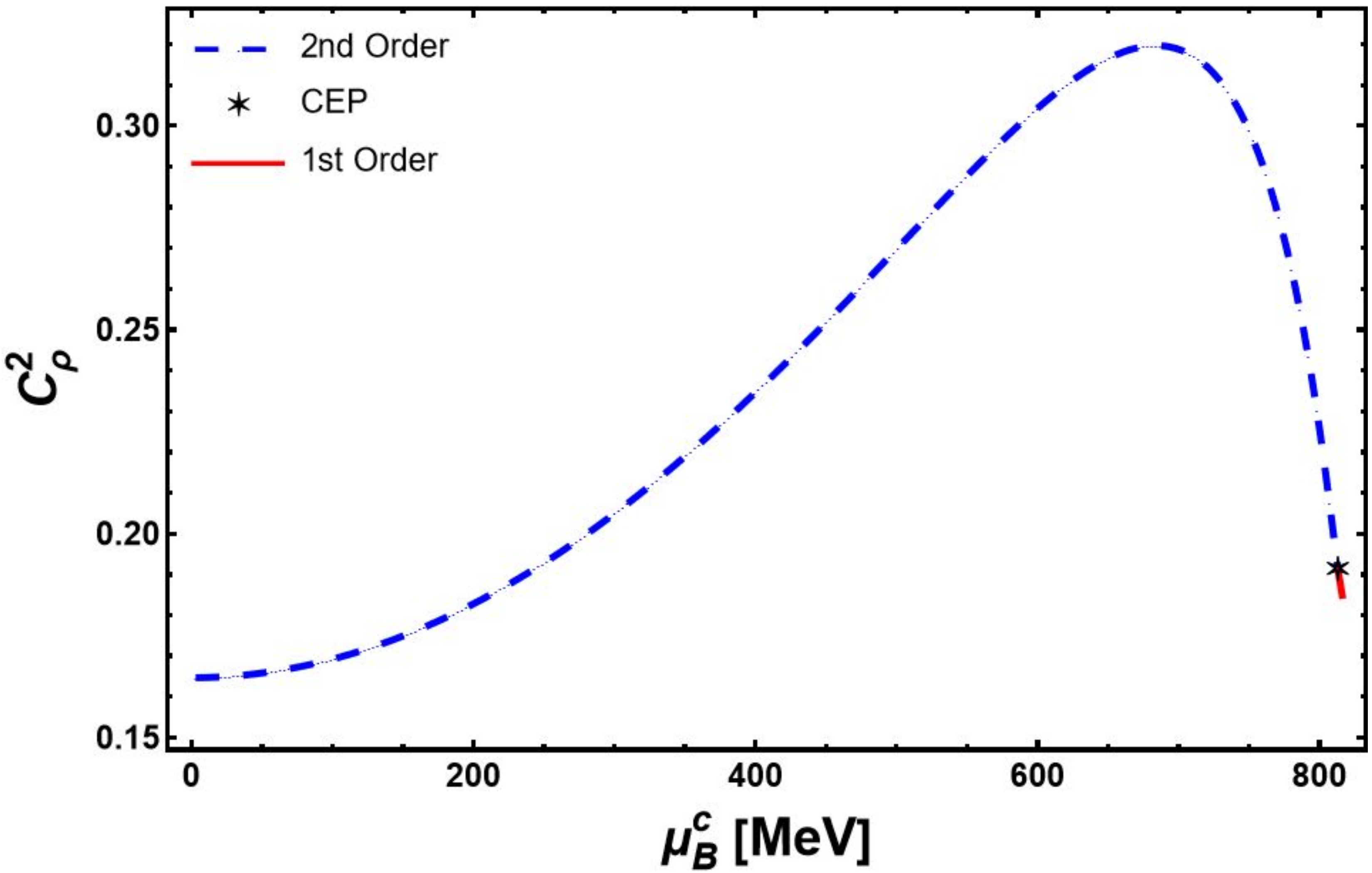
- Now, let us use the value of the speed of sound at μ = 0, meaning $C_S \approxeq 0.16$ and try to fix lambda.



$$c_{\rho_B}^2 = \frac{\partial(p,\rho_B)}{\partial(\epsilon,\rho_B)} = \frac{s\chi_{\mu\mu} - \rho_B\chi_{\mu T}}{T(\chi_{TT}\chi_{\mu\mu} - \chi_{\mu T}^2)},$$

Nf = 2+1:   a = 325.744, λ = 0.15, g = 2.057
Nf = 2   :   a = 407.441, λ = 0.30, g = 2.461




2nd Order
CEP
1st Order
0.30
0.25
0.20
0.15
0
200
400
600
800
C_ρ^2
μ_B^c [MeV]

For the future…

- We can build upon previous studies that incorporate a magnetic field into the LSMq to investigate the behavior of the speed of sound both along and away from the transition line.

QCD phase diagram in a magnetized medium from the chiral symmetry perspective: The linear sigma model with quarks and the Nambu–Jona-Lasinio model effective descriptions

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THANKS FOR WATCHING!