Medium Separation Scheme and the Cold-Magnetized Superconducting Quark Matter

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In collaboration with Francisco de Azeredo and Ricardo Farias

serrapilheira

FAPERGS

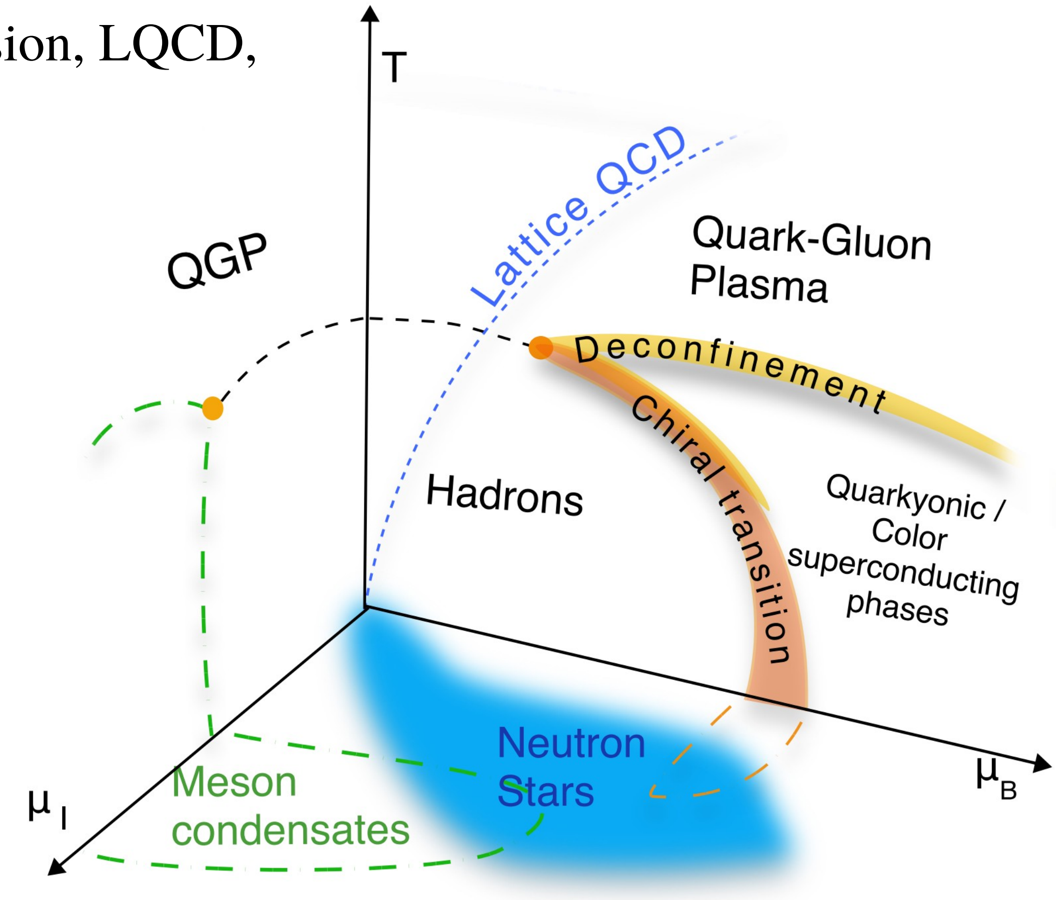
Universidade Federal de Santa Maria
SEDES SAPIENTIÆ
1960

Outline

- Motivation;
- The role of regularization: MSS;
- Finite magnetic field effects;
- Final remarks.

The QCD phase diagram

Approaches: perturbative, $1/N_c$ Expansion, LQCD, DSE, effective models...



Adam, Martín-Caro, Huidobro, Wereszczynski, Symmetry 15, 899 (2023)

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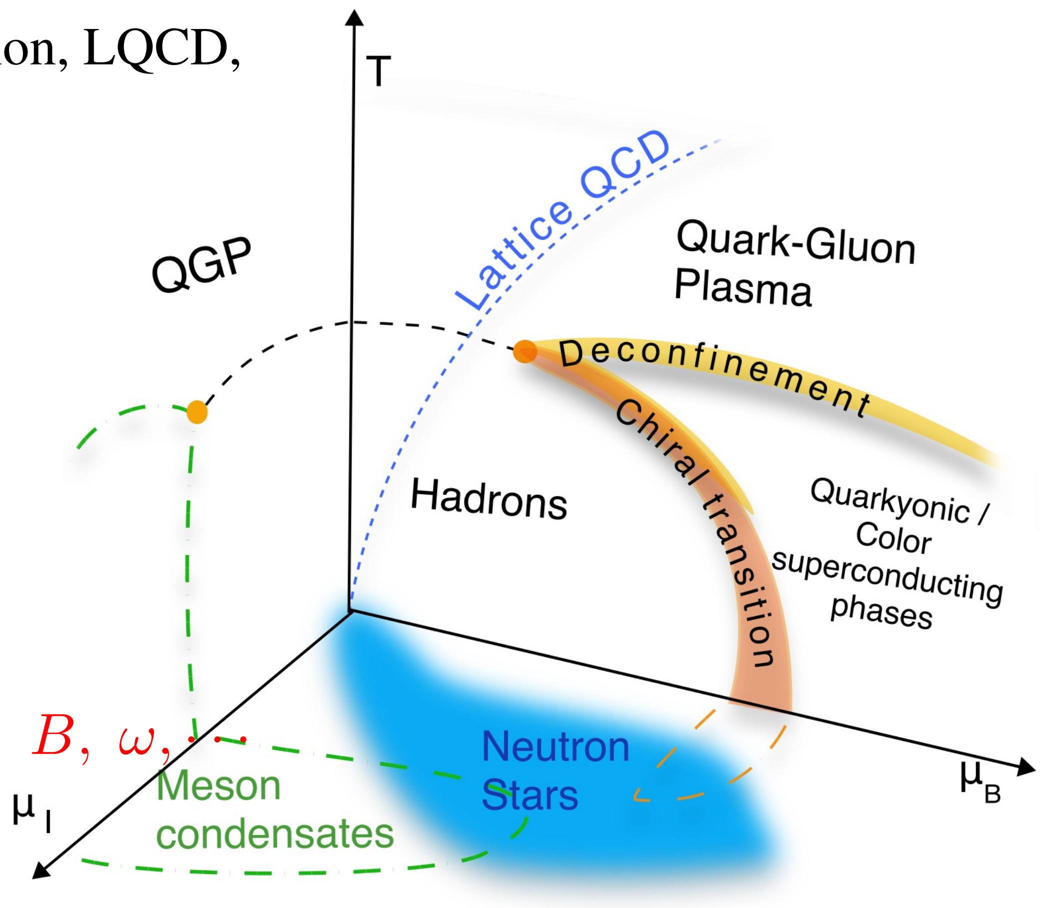

$\mu_\Sigma,\ \mu_5,\ B,\ \omega, \ldots$

Adam, Martín-Caro, Huidobro, Wereszczynski, Symmetry 15, 899 (2023)

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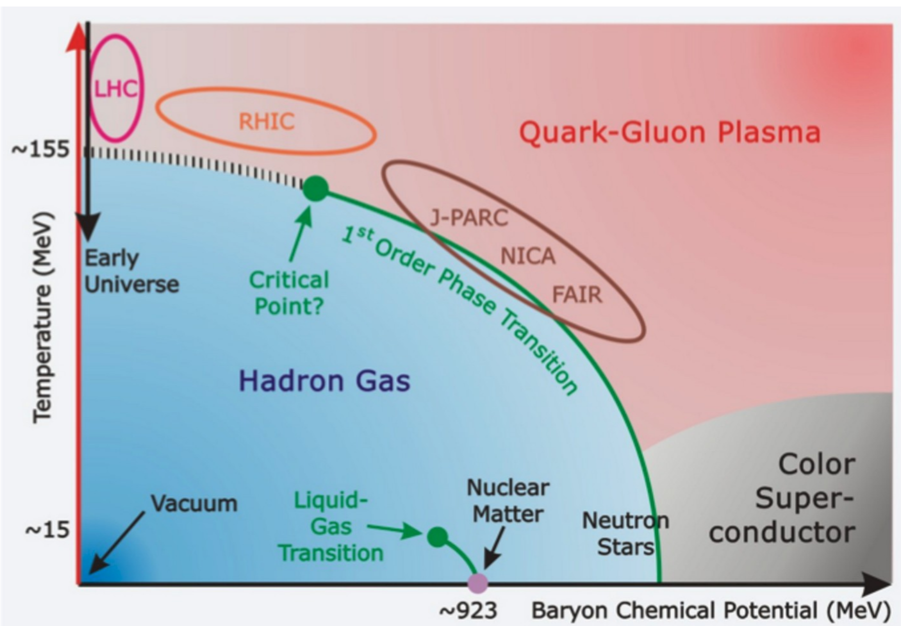

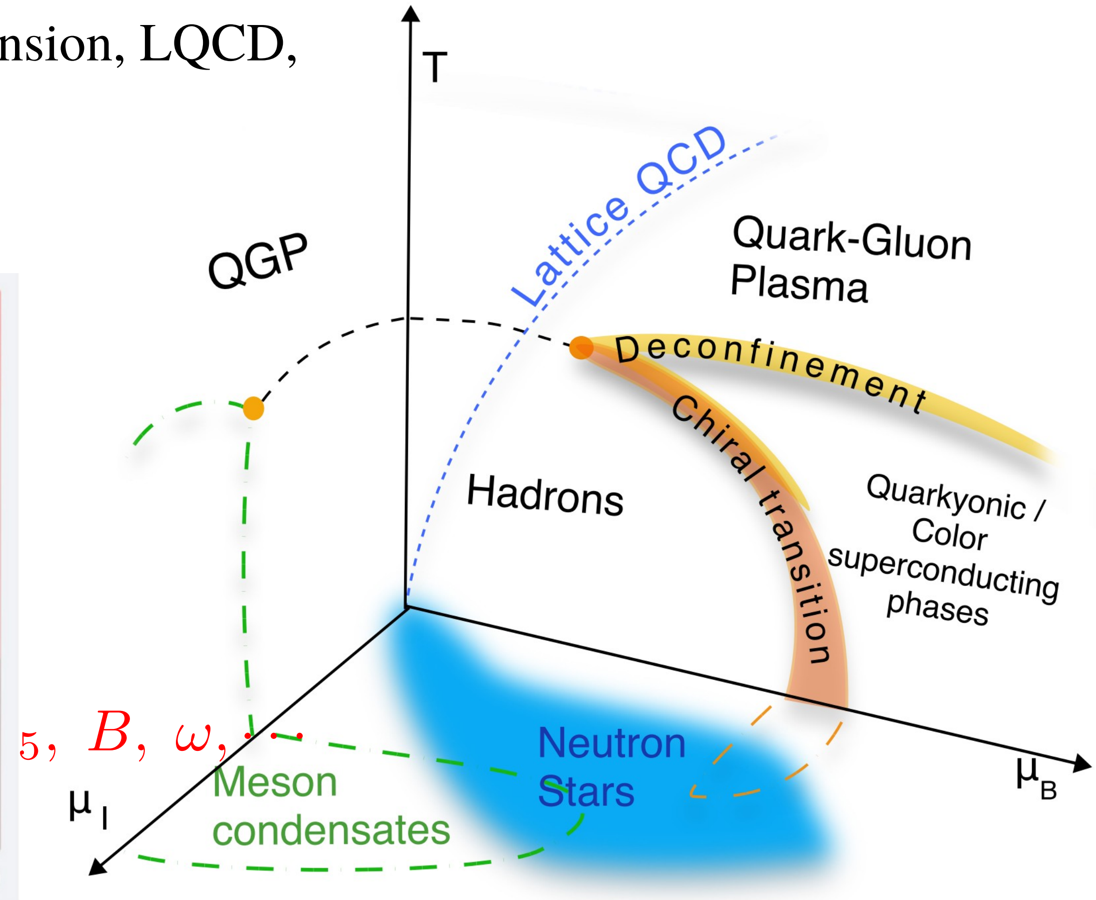

Adam, Martín-Caro, Huidobro, Wereszczynski, Symmetry 15, 899 (2023)

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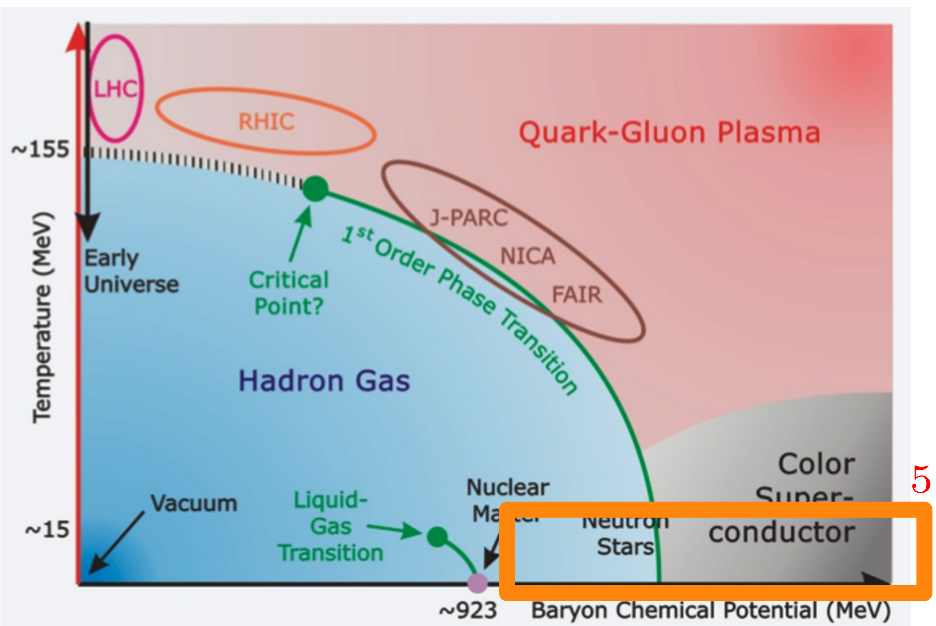
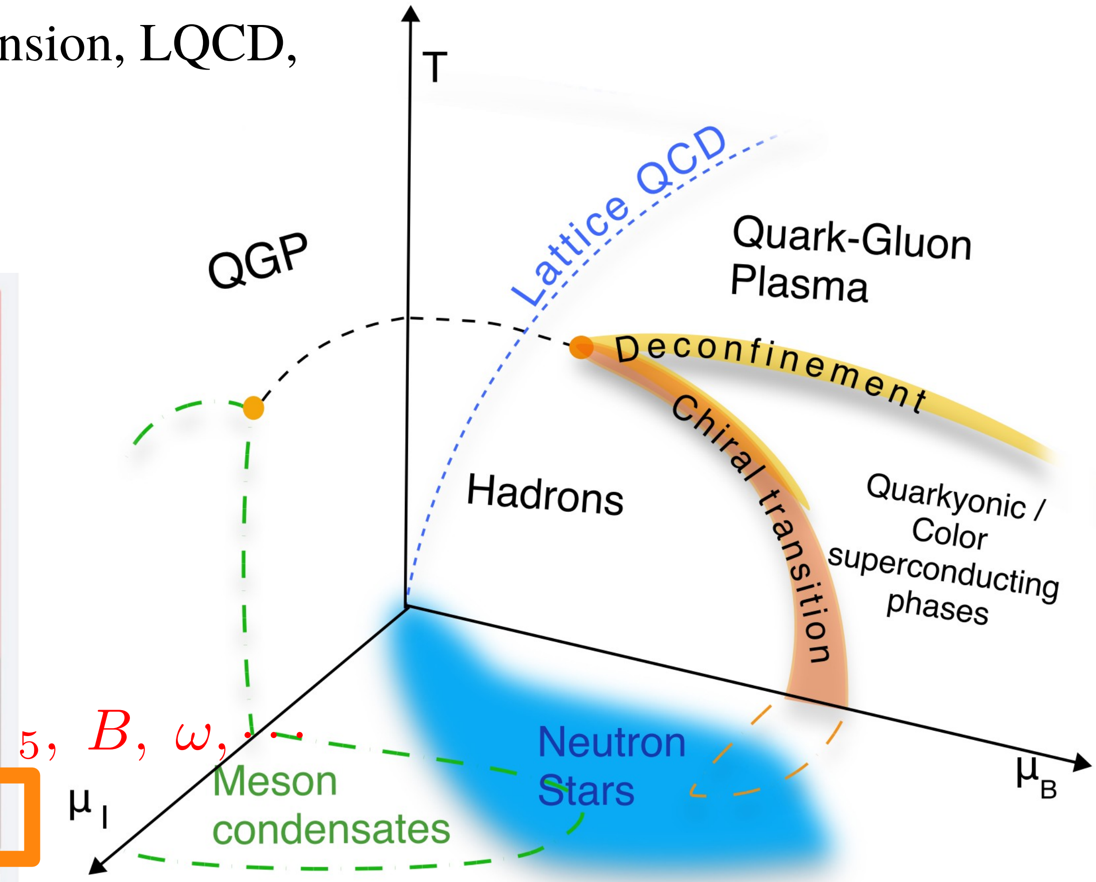


Effects of B here!



Adam, Martín-Caro, Huidobro, Wereszczynski, Symmetry 15, 899 (2023)

Magnetic field effects in the compact objects

- Self-consistent modulation via many-body forces in a RMF approach: gravitational mass, deformation, internal density profiles of neutron stars and particle population. Gomes, Franzon, Dexheimer, Schramm, APJ 850:20 (2017)
- Changes in the microscopic structure of the neutron star crust: equilibrium composition, melting temperature, and moment of inertia. Changes also in the nuclear pasta geometry (~10–15%), even though global star properties remain largely unaffected. Parmar, Das, Sharma, Patra, PRD 107, 043022 (2023)
- A mechanism involving charged gluon vortices in neutral 2SC matter may amplify moderate the magnetic fields ($\sim 10^{8} \rightarrow 10^{17}$) G: microscopic explanation for the intense magnetic fields in magnetar core. Yuan, Feng, Ferrer, Pinero, PRD 110, 114038 (2024)
- Induction of frequency shifts in the postmerger GW spectrum similar to those caused by EoS features like phase transitions, introducing a degeneracy that complicates the interpretation of GW observations and demands magnetic effects be consistently included in the analysis. Tsokaros, Bamber, Ruiz, Shapiro, PRL 134 121401 (2025).
- ….

PHYSICAL REVIEW C **73**, 018201 (2006)

Cutoff-independent regularization of four-fermion interactions for color superconductivity

R. L. S. Farias,[1] G. Dallabona,[1] G. Krein,[1] and O. A. Battistel[2]
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[2]*Departamento de Física, Universidade Federal de Santa Maria, 97119-900 Santa Maria, RS, Brazil*
(Received 31 October 2005; published 27 January 2006)

$$1 = \lambda G i \int \frac{d^4k}{(2\pi)^4}\left[\frac{1}{k_0^2-(k+\mu)^2-\Delta^2} + (\mu \to -\mu)\right], \quad (2)$$

- Algebraic manipulations leads to

$$1 = 8\lambda G\{2[i\, I_{\rm quad}(\Delta^2)] - 4\mu^2[i\, I_{\rm log}(\Delta^2)] + I_{\rm fin}(\Delta^2,\mu) + I_{\rm fin}(\Delta^2,-\mu)\},$$

where

$$I_{\rm quad}(\Delta^2) = \int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{k_0^2-k^2-\Delta^2},$$

$$I_{\rm log}(\Delta^2) = \int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{(k_0^2-k^2-\Delta^2)^2},$$

$$I_{\rm fin}(\Delta^2,\mu) = i\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k_0^2-k^2-\Delta^2)^3} \times\left[2\mu^2(\mu^2-4\Delta^2) + \frac{(\mu^2+2k\mu)^3}{(k_0^2-(k+\mu)^2-\Delta^2)}\right]$$

William Tavares talk yesterday

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Related to physical quantities in vacuum

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Integrated without restrictions

William Tavares talk yesterday

The original “Cutoff independent regularization”

- Problem: $\Delta = 0$ in the vacuum $\longrightarrow$ Not a good scale parameter.

$$I_{\text{quad}}(\Delta^2) = I_{\text{quad}}(M^2) + (\Delta^2 - M^2) I_{\log}(M^2) + \frac{i}{(4\pi)^2}\left[\Delta^2 - M^2 - \Delta^2 \ln\left(\frac{\Delta^2}{M^2}\right)\right]$$

$$I_{\log}(\Delta^2) = I_{\log}(M^2) - \frac{i}{(4\pi)^2} \ln\left(\frac{\Delta^2}{M^2}\right).$$

- In chiral models I_{quad} and $I_{\log}$ are related to the chiral condensate and pion decay constant as

$$iI_{\text{quad}}(M^2) = -\frac{\langle \bar{q} q \rangle}{12M}$$

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Divergent integrals in terms of the quark mass in the vacuum

Finite terms

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- Back to the gap equation: No divergent integrals!

$$1 = \lambda G \left\{ -\frac{\langle \bar{q}q \rangle}{6M} - (\Delta^2 - M^2)\frac{f_\pi^2}{6M^2} - \frac{1}{8\pi^2}\left[\Delta^2 - M^2 - \Delta^2 \ln\left(\frac{\Delta^2}{M^2}\right)\right] - \mu^2 \left[\frac{f_\pi^2}{3M^2} - \frac{1}{4\pi^2} 2\ln\left(\frac{\Delta^2}{M^2}\right)\right] + I_{\text{fin}}(\Delta^2, \mu) + I_{\text{fin}}(\Delta^2, -\mu)\right.$$

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$$\left. -\mu^2 \left[\frac{f_\pi^2}{3M^2} - \frac{1}{4\pi^2} 2\ln\left(\frac{\Delta^2}{M^2}\right)\right] + I_{\rm fin}(\Delta^2, \mu) + I_{\rm fin}(\Delta^2, -\mu) \right.$$

Modern implementation: Medium Separation Scheme (MSS)

$$I = \sum_{j=\pm 1} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{\left(\sqrt{\vec{k}^2 + M^2} + j\mu\right)^2 + \Delta^2}} = 2I_{\text{quad}}(M_0) - (M^2 - M_0^2 + \Delta^2 - 2\mu^2) I_{\log}(M_0)$$

$$+ \left[\frac{3(A^2 + 4\mu^2 M^2)}{4} - 3\mu^2 M_0^2\right] I_1 + 2I_2$$

with the definitions

$$I_{\text{quad}}(M_0) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{\vec{k}^2 + M_0^2}}$$

$$I_{\log}(M_0) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{(\vec{k}^2 + M_0^2)^{\frac{3}{2}}}$$

Related to the quark condensate and f_π^2

Finite

$$I_1 = \int \frac{d^3k}{(2\pi)^3} \frac{1}{(\vec{k}^2 + M_0^2)^{\frac{5}{2}}}$$

$$I_2 = \frac{15}{32} \sum_{j=\pm 1} \int \frac{d^3k}{(2\pi)^3} \int_0^1 dt (1-t)^2 \frac{(A - 2j\mu\sqrt{\vec{k}^2 + M^2})^3}{\left[(2j\mu\sqrt{\vec{k}^2 + M^2} - A)t + \vec{k}^2 + M_0^2\right]^{\frac{7}{2}}}$$

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Effective quark mass $M \equiv M(T, \mu, eB, \ldots)$

Vacuum quark mass $M_0 \equiv M(T = \mu = eB, \ldots = 0)$

with the definitions

$$I_{\rm quad}(M_0) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{\vec{k}^2 + M_0^2}}$$

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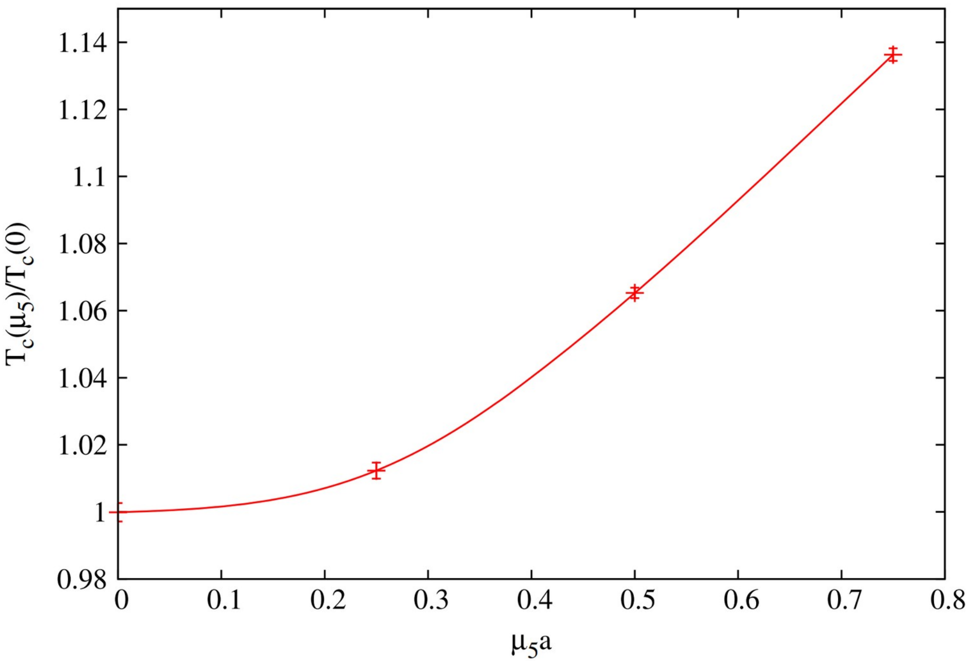
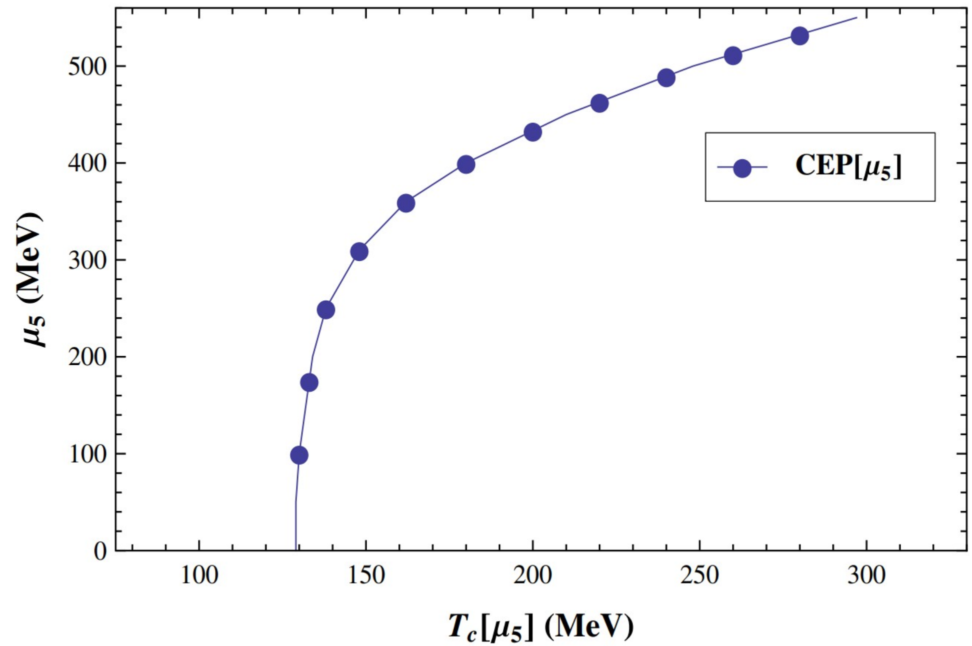
Successful applications: Phase diagram with a chiral chemical potential

- Universality arguments of large N_c, DSE and lattice simulations predicts an increasing T_{pc} with μ_5, with NO CEP...



Braguta, Ilgenfritz, Kotov, Petersson, Skinderev, PRD93, 034509 (2016)

(Lattice)

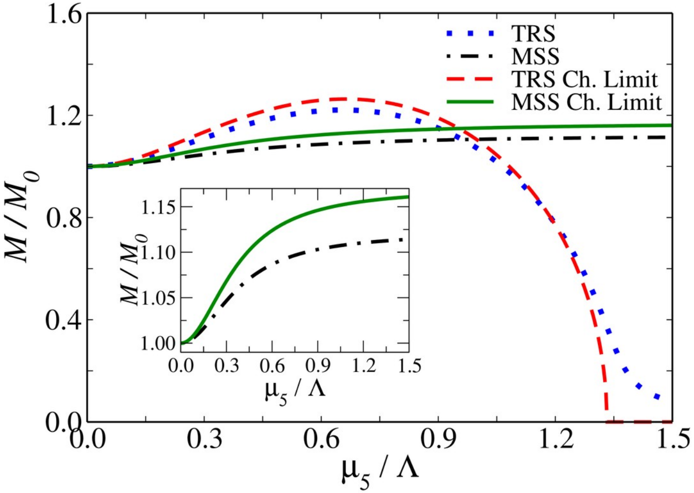


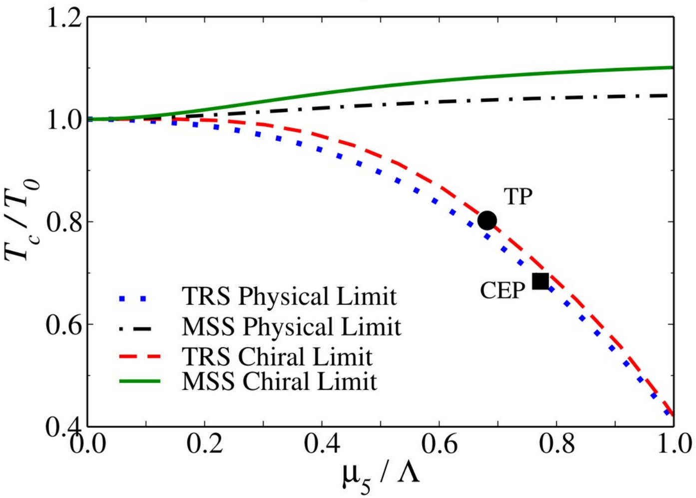
Xu, Cui, Wang, Shi, Yang, Zong, PRD 91, 056003 (2015)

(DSE)

Successful applications: Phase diagram with a chiral chemical potential

- MSS cure the problem, and more!



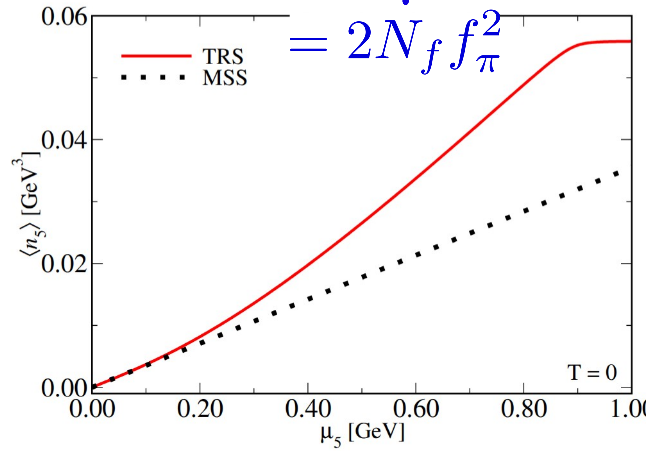


Farias, DD, Krein,Ramos, PRD 91, 056003 (2015)

- ChPT at finite μ_5: $\langle n_5 \rangle = \dfrac{1}{\beta V}\dfrac{\partial \log[Z(\mu_5)]}{\partial \mu_5} = 2N_f f_\pi^2 \mu_5$

$$\langle n_5 \rangle^{\rm TRS} = 2N_c \sum_{s=\pm 1} \int_0^\Lambda \frac{dp\, p^2}{2\pi^2} \frac{s(|\mathbf{p}| + s\mu_5)}{\sqrt{(|\mathbf{p}| + s\mu_5)^2 + M^2}}$$

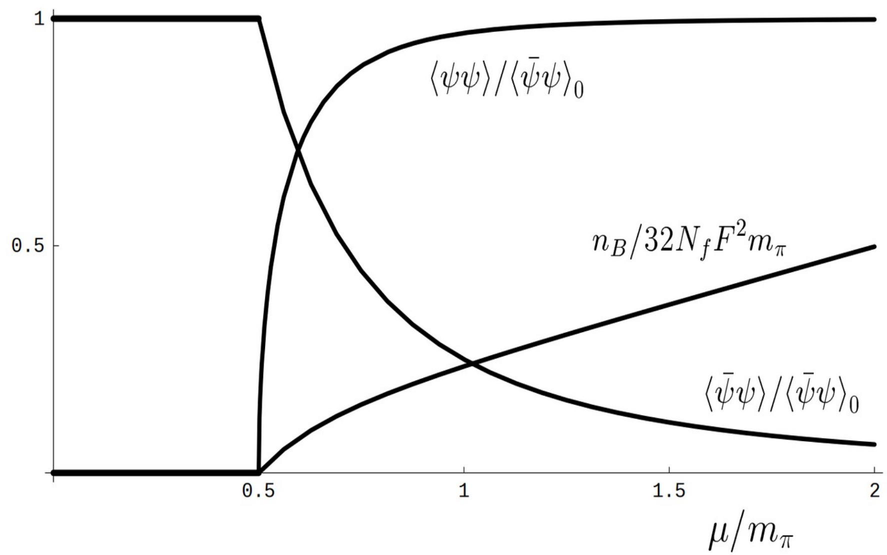
$$\langle n_5 \rangle^{\rm MSS} = \underbrace{4N_c \left[M^2 I_{\log}(M_0) - \frac{M^2}{4\pi^2} \ln\left(\frac{M^2}{M_0^2}\right)\right]}_{= 2N_f f_\pi^2} \mu_5$$



Azeredo, DD, Farias, Krein,Ramos, PRD110 076007 (2024)

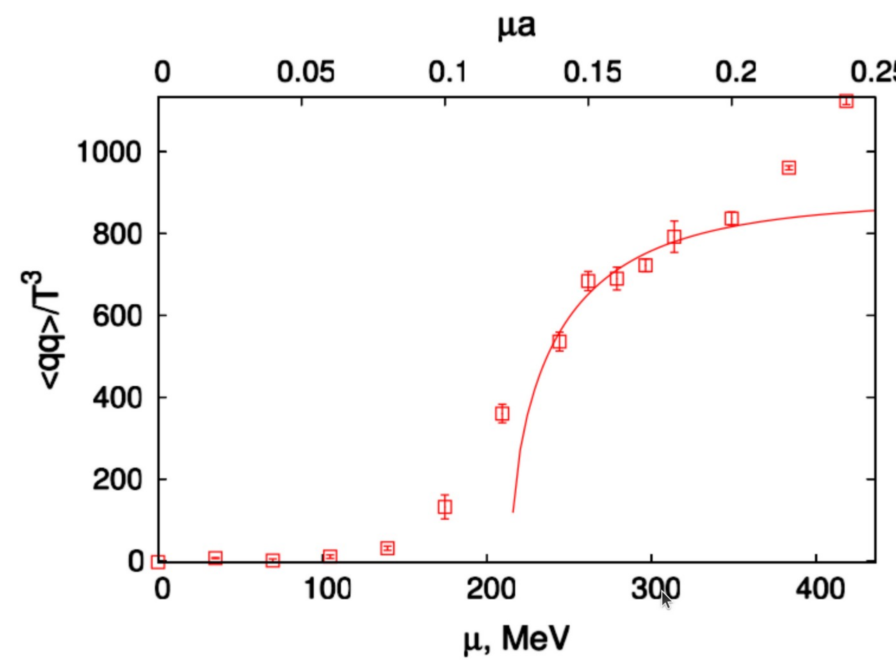
Successful applications: 2-color QCD

- $\Delta \longrightarrow 0$ for high values of μ_B $(= N_c\mu)$is an artifact of the incorrect regularization also in the physical limit.



phase	$\langle\bar\psi\psi\rangle$	$\langle\psi\psi\rangle$	n_B
$\mu < m_\pi/2$	$\langle\bar\psi\psi\rangle_0$	0	0
$\mu > m_\pi/2$	$\langle\bar\psi\psi\rangle_0 \left(\frac{m_\pi}{2\mu}\right)^2$	$\langle\bar\psi\psi\rangle_0\sqrt{1-\left(\frac{m_\pi}{2\mu}\right)^4}$	$8\mu N_f F^2\left(1-\left(\frac{m_\pi}{2\mu}\right)^4\right)$

Kogut, Stephanov, Toublan, Verbaarschot, Zhitnitsky,
NPB 582 477 (2000)



Braguta, Ilgenfritz, Kotov, Molochko, Nikolaev,
PRD 94, 114510 (2016)

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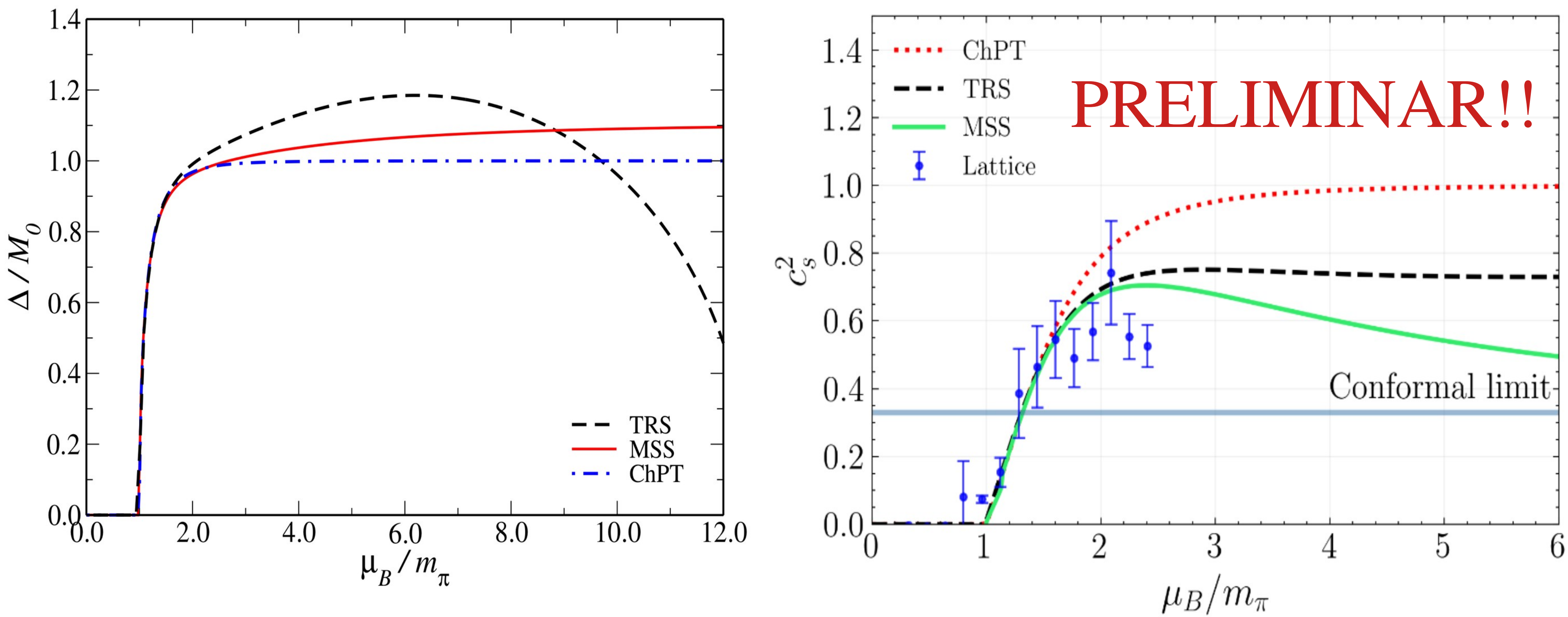


DD, Farias, Pasqualotto, Ramos

Successful applications: Nonvanishing Δ at high μ

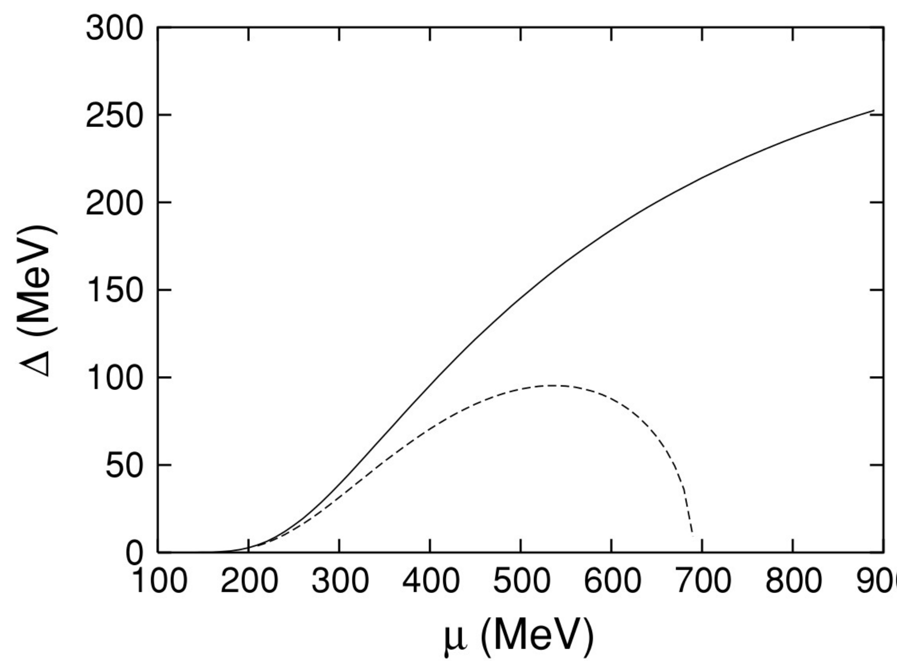
- Two-flavor spin-0 superconducting gap prediction using the weak-coupling renormalization group techniques:

$$\Delta \sim \frac{\mu}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$

D. T. Son, PRD59, 094019 (1999)

- Δ is an increasing function of μ, and corrections to this formula does not seem to change this behavior.

Hong, Miransky,Shovkovy, Wijewardhana, PRD61 056001 (2000)
Hsu, Schwetz, Nucl. Phys. B572, 211(2000).



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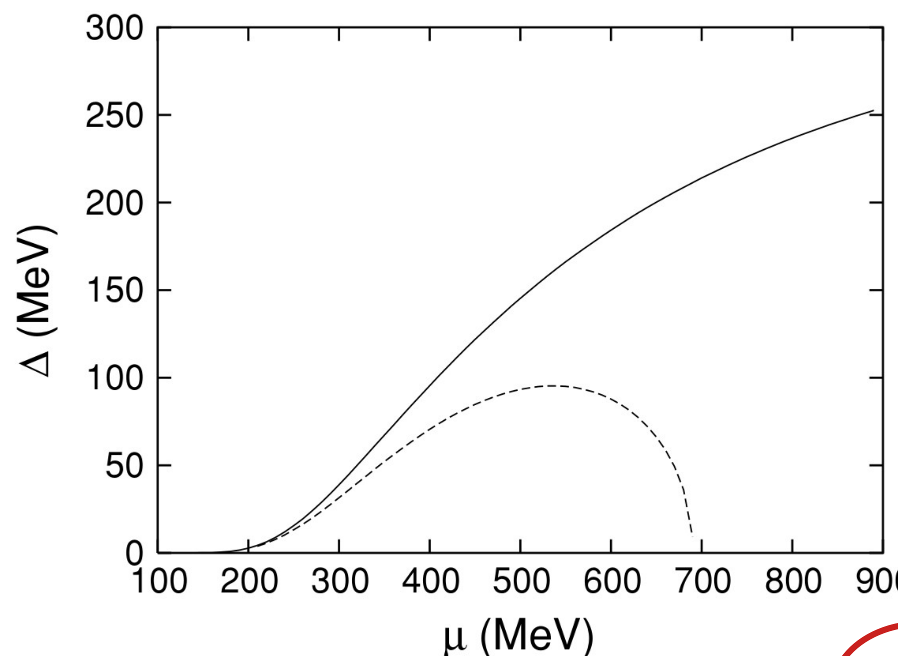

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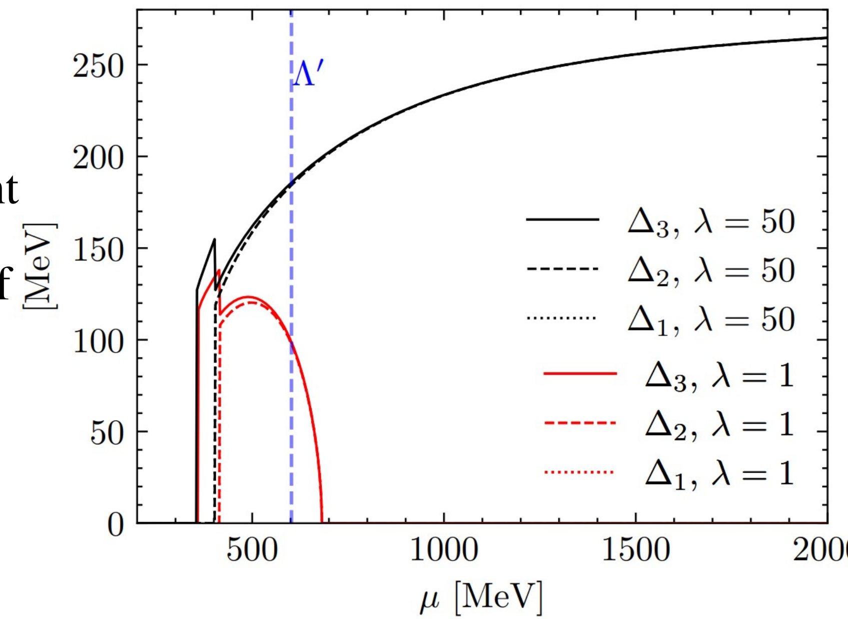
Gholami, Hofmann, Buballa, PRD 111 014006 (2025)

RG -consistent treatment remove cutoff artifacts!



λ = Λ/Λ’

$$\Delta_{ur,dg} = \Delta_{ug,dr} = \Delta_3$$
$$\Delta_{ur,sb} = \Delta_{ub,sr} = \Delta_2$$
$$\Delta_{dg,sb} = \Delta_{db,sg} = \Delta_1$$

SU(2) NJL model in a magnetic field

$$\mathcal{L} = \bar{\psi}\left[i\gamma^{\mu}\left(\partial_{\mu} - i\tilde{e}\tilde{Q}\tilde{A}_{\mu}\right) + \hat{\mu}\gamma^{0} - \hat{m}\right]\psi + G_S\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right]$$
$$+ G_D\left[\left(i\bar{\psi}^C\epsilon_f\epsilon_c^3\gamma_5\psi\right)\left(i\bar{\psi}\epsilon_f\epsilon_c^3\gamma_5\psi^C\right)\right]$$

Allen, Grunfeld, Scoccola have a very complete discussion on this context! See PRD 92, 074041 (2015)

Thermodynamic potential:

$$\Omega_T = \frac{(M - m_c)^2}{4G_S} + \frac{\Delta^2}{4G_D} - \int \frac{d^3p}{(2\pi)^3}\left[f\left(E^{+}_{p,0}\right) + f\left(E^{-}_{p,0}\right)\right]$$
$$-\frac{eB}{8\pi^2}\sum_{n=0}^{\infty}\alpha_n\int_{-\infty}^{+\infty}dp_z\Bigg[f\left(E^{+}_{p,1}\right) + f\left(E^{-}_{p,1}\right)$$
$$+2f\left(E^{+}_{p,\frac{1}{2}}\right) + 2f\left(E^{+}_{p,\frac{1}{2}}\right)\Bigg],$$

$$f(x) = x + 2T\ln\left(1 + e^{-x/T}\right)$$
$$\alpha_n = 2 - \delta_{n,0}$$

Zero temperature thermodynamic potential in a final magnetic field

$$\Omega_{T=0} = \frac{(M-m_c)^2}{4G_S} + \frac{\Delta^2}{4G_D} - \sum_{|q|=0,1,\frac{1}{2}} \Omega_{|q|}$$

with

$$\Omega_0 = 2\int \frac{d^3p}{(2\pi)^3}\left[E_{p,0} + (\mu - E_{p,0})\theta(\mu - E_{p,0})\right]$$

$$\Omega_1 = \frac{eB}{8\pi^2}\sum_{n=0}^{\infty}\alpha_n \int_{-\infty}^{+\infty} dp_z\left[2E_{p,1} + (\mu - E_{p,1})\theta(\mu - E_{p,1})\right]$$

$$\Omega_{\frac{1}{2}} = \frac{eB}{4\pi^2}\sum_{n=0}^{\infty}\alpha_n \int_{-\infty}^{+\infty} dp_z\left[E^+_{p,\frac{1}{2}} + E^-_{p,\frac{1}{2}}\right]$$

$$E_{p,a} = \sqrt{\mathbf{p}^2_{\perp,a} + p_z^2 + M^2}$$

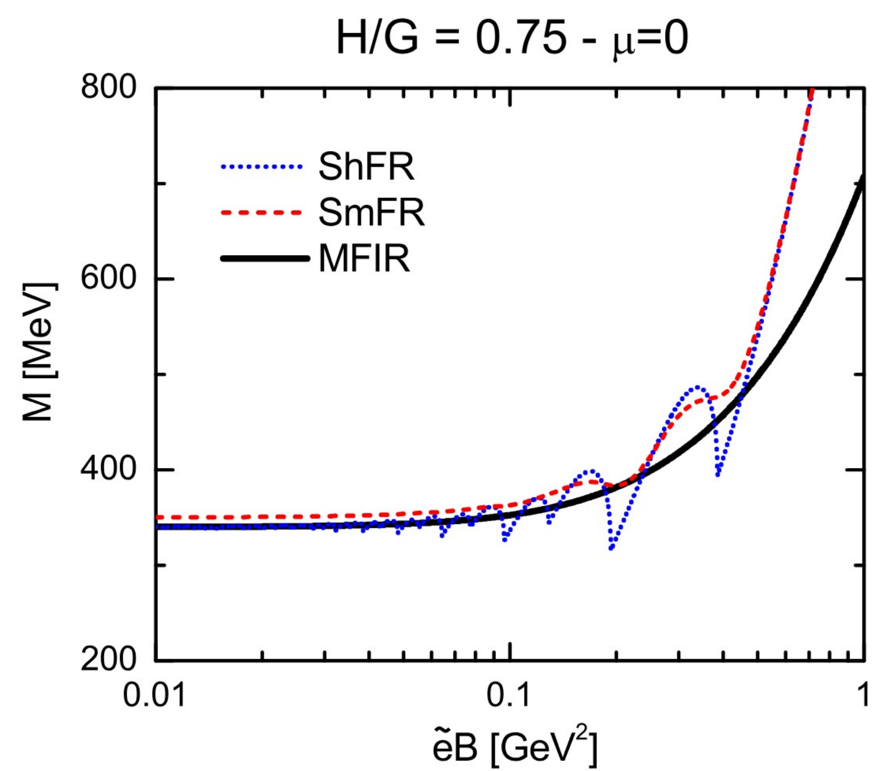
$$\mathbf{p}^2_{\perp,a} = \begin{cases} p_x^2 + p_y^2 & \text{if} \quad |a| = 0 \\ 2\,|a|\,eBn & \text{if} \quad |a| = 1, \frac{1}{2} \end{cases}$$

Regularization: $\Lambda(n)$ and smooth regulators

1) $\int_0^{\Lambda} \frac{dk_z}{2\pi} \longrightarrow \int_0^{\Lambda(n)} \frac{dk_z}{2\pi}$, with $\Lambda(n) = \sqrt{k_z^2 + 2k|q|n}$

2) $\sum_{n=0}^{\infty} \alpha_n \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi} \to \sum_{n=0}^{\infty} \alpha_n \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi} U_\Lambda\left(\sqrt{p_z^2 + 2|a|eBn}\right)$,

- Fermi-Dirac: $U(x) = \frac{1}{2}\left[1 - \tanh\left(\frac{x/\Lambda - 1}{\alpha}\right)\right]$
- Wood-Saxon: $U(x) = \frac{1}{\left[1 + \exp\left(\frac{x/\Lambda - 1}{\alpha}\right)\right]}$
- Lorentzian: $\left[1 + \left(\frac{x^2}{\Lambda^2}\right)\right]^{-1}$
- $\cdots$



Allen, Grunfeld, Scoccola PRD 92, 074041 (2015)

Zero temperature thermodynamic potential in MFIR

$$\Omega_0 = 4\int_\Lambda \frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^2+M^2} + 2\int \frac{d^3p}{(2\pi)^3}\Big[(\mu - E_{p,0})\theta(\mu - E_{p,0})\Big]$$

$$\begin{aligned}\Omega_1 &= \frac{(eB)^2}{2\pi^2}\left\{\zeta'(-1,\chi) + \frac{\chi-\chi^2}{2}\ln(\chi) + \frac{\chi^2}{4}\right\} \\ &+ \frac{eB}{4\pi^2}\sum_{n=0}^{p_{B,\max}} \alpha_n \left\{\mu\sqrt{\mu^2-p_B^2} - p_B^2 \ln\left(\frac{\mu+\sqrt{\mu^2-p_B^2}}{p_B}\right)\right\}\end{aligned}$$

$$\begin{aligned}\Omega_{\frac{1}{2}} &= 4\int_\Lambda \frac{d^3p}{(2\pi)^3}\left[\sqrt{\left(\sqrt{\vec{p}^2+M^2}+\mu\right)^2+\Delta^2} + \sqrt{\left(\sqrt{\vec{p}^2+M^2}-\mu\right)^2+\Delta^2}\right] \\ &+ \frac{(eB)^2}{2\pi^2}\left\{\zeta'\left(-1,x\right) + \frac{(x-x^2)}{2}\ln(x) + \frac{x^2}{4}\right\} + \frac{eB}{4\pi^2}\int_{-\infty}^{+\infty} dp_z F\left(p_z^2\right) \\ &+ \frac{eB}{2\pi^2}\int_{-\infty}^{+\infty} dp_z \left\{\left[\sum_{n=1}^{\infty} F\left(p_z^2+neB\right)\right] - \int_0^\infty dy\ F\left(p_z^2+eBy\right)\right\}\end{aligned}$$

$$\chi = \frac{M^2}{2eB}$$

$$p_B = \sqrt{M^2+2eBn}$$

$$p_{B,max} = \frac{\mu^2-M^2}{2eB}$$

$$x = \frac{M^2+\Delta^2}{eB}$$

$$F(z) = \sqrt{\left(\sqrt{z+M^2}+\mu\right)^2+\Delta^2} + \sqrt{\left(\sqrt{z+M^2}-\mu\right)^2+\Delta^2} - 2\sqrt{z+M^2+\Delta^2}$$

Zero temperature thermodynamic potential in MFIR

$$\Omega_0 = 4\int_\Lambda \frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^2+M^2} + 2\int \frac{d^3p}{(2\pi)^3}\left[(\mu - E_{p,0})\theta(\mu - E_{p,0})\right]$$

$$\Omega_1 = \frac{(eB)^2}{2\pi^2}\left\{\zeta'(-1,\chi) + \frac{\chi-\chi^2}{2}\ln(\chi) + \frac{\chi^2}{4}\right\} + \frac{eB}{4\pi^2}\sum_{n=0}^{p_{B,\max}} \alpha_n \left\{\mu\sqrt{\mu^2 - p_B^2} - p_B^2 \ln\left(\frac{\mu + \sqrt{\mu^2 - p_B^2}}{p_B}\right)\right\}$$

$$\Omega_{\frac{1}{2}} = 4\int_\Lambda \frac{d^3p}{(2\pi)^3}\left[\sqrt{\left(\sqrt{\vec{p}^2+M^2}+\mu\right)^2+\Delta^2} + \sqrt{\left(\sqrt{\vec{p}^2+M^2}-\mu\right)^2+\Delta^2}\right] + \frac{(eB)^2}{2\pi^2}\left\{\zeta'\left(-1,x\right) + \frac{(x-x^2)}{2}\ln(x) + \frac{x^2}{4}\right\} + \frac{eB}{4\pi^2}\int_{-\infty}^{+\infty} dp_z F\left(p_z^2\right) + \frac{eB}{2\pi^2}\int_{-\infty}^{+\infty} dp_z \left\{\left[\sum_{n=1}^{\infty} F\left(p_z^2 + neB\right)\right] - \int_0^\infty dy\ F\left(p_z^2 + eBy\right)\right\}$$

$$\chi = \frac{M^2}{2eB}$$

$$p_B = \sqrt{M^2 + 2eBn}$$

$$p_{B,max} = \frac{\mu^2 - M^2}{2eB}$$

$$x = \frac{M^2+\Delta^2}{eB}$$

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Zero temperature thermodynamic potential in MFIR

$$\Omega_0 = \boxed{4\int_\Lambda \frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^2+M^2}} + 2\int \frac{d^3p}{(2\pi)^3}\left[(\mu - E_{p,0})\theta(\mu - E_{p,0})\right]$$

$$\Omega_1 = \frac{(eB)^2}{2\pi^2}\left\{\zeta'(-1,\chi) + \frac{\chi-\chi^2}{2}\ln(\chi) + \frac{\chi^2}{4}\right\} + \frac{eB}{4\pi^2}\sum_{n=0}^{p_{B,\max}} \alpha_n \left\{\mu\sqrt{\mu^2 - p_B^2} - p_B^2 \ln\left(\frac{\mu + \sqrt{\mu^2 - p_B^2}}{p_B}\right)\right\}$$

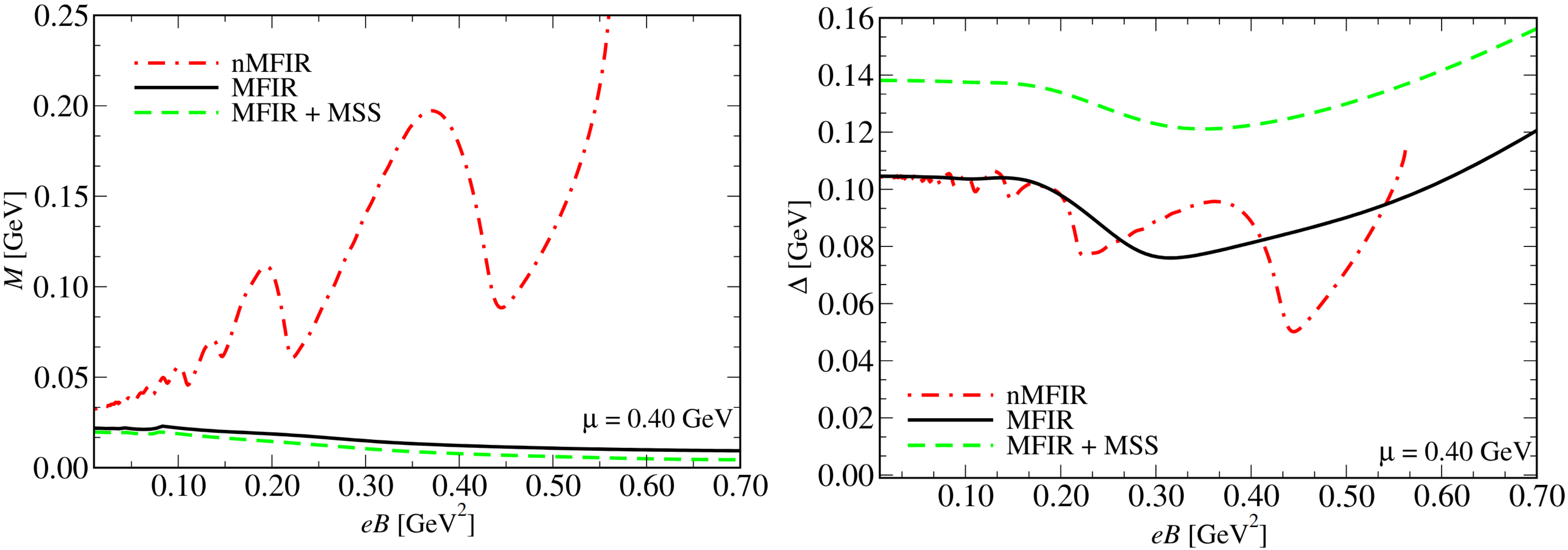
$$\Omega_{\frac{1}{2}} = \boxed{4\int_\Lambda \frac{d^3p}{(2\pi)^3}\left[\sqrt{\left(\sqrt{\vec{p}^2+M^2}+\mu\right)^2+\Delta^2} + \sqrt{\left(\sqrt{\vec{p}^2+M^2}-\mu\right)^2+\Delta^2}\right]}$$
$$+\frac{(eB)^2}{2\pi^2}\left\{\zeta'\left(-1,x\right) + \frac{(x-x^2)}{2}\ln(x) + \frac{x^2}{4}\right\} + \frac{eB}{4\pi^2}\int_{-\infty}^{+\infty} dp_z F\left(p_z^2\right)$$
$$+\frac{eB}{2\pi^2}\int_{-\infty}^{+\infty} dp_z \left\{\left[\sum_{n=1}^{\infty} F\left(p_z^2 + neB\right)\right] - \int_0^\infty dy\ F\left(p_z^2 + eBy\right)\right\}$$

$$\chi = \frac{M^2}{2eB}$$
$$p_B = \sqrt{M^2 + 2eBn}$$
$$p_{B,max} = \frac{\mu^2 - M^2}{2eB}$$
$$x = \frac{M^2+\Delta^2}{eB}$$

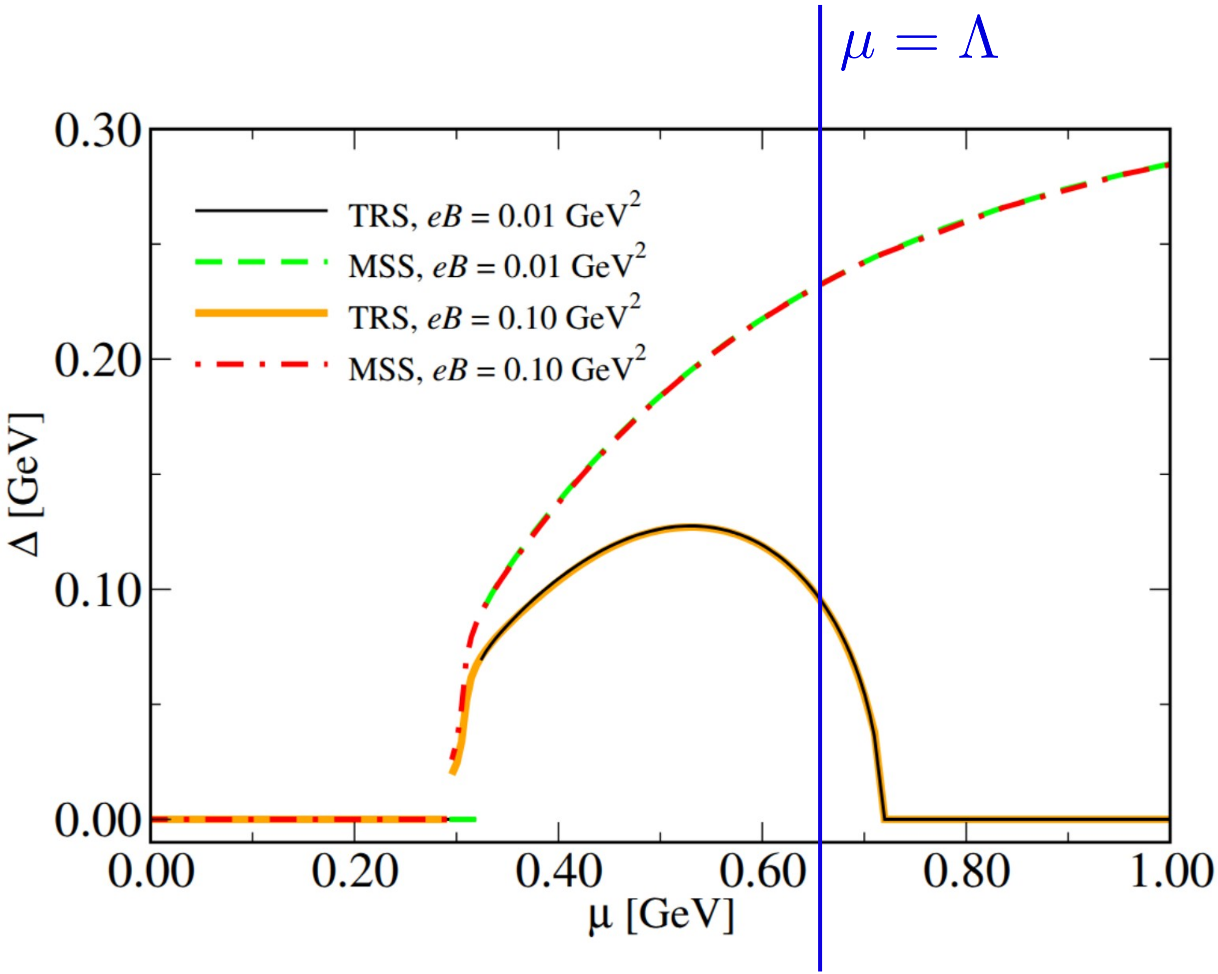
$$F(z) = \sqrt{\left(\sqrt{z+M^2}+\mu\right)^2+\Delta^2} + \sqrt{\left(\sqrt{z+M^2}-\mu\right)^2+\Delta^2} - 2\sqrt{z+M^2+\Delta^2}$$

Order parameters at finite μ

$$\frac{\partial \Omega}{\partial M} = \frac{\partial \Omega}{\partial \Delta} = 0$$

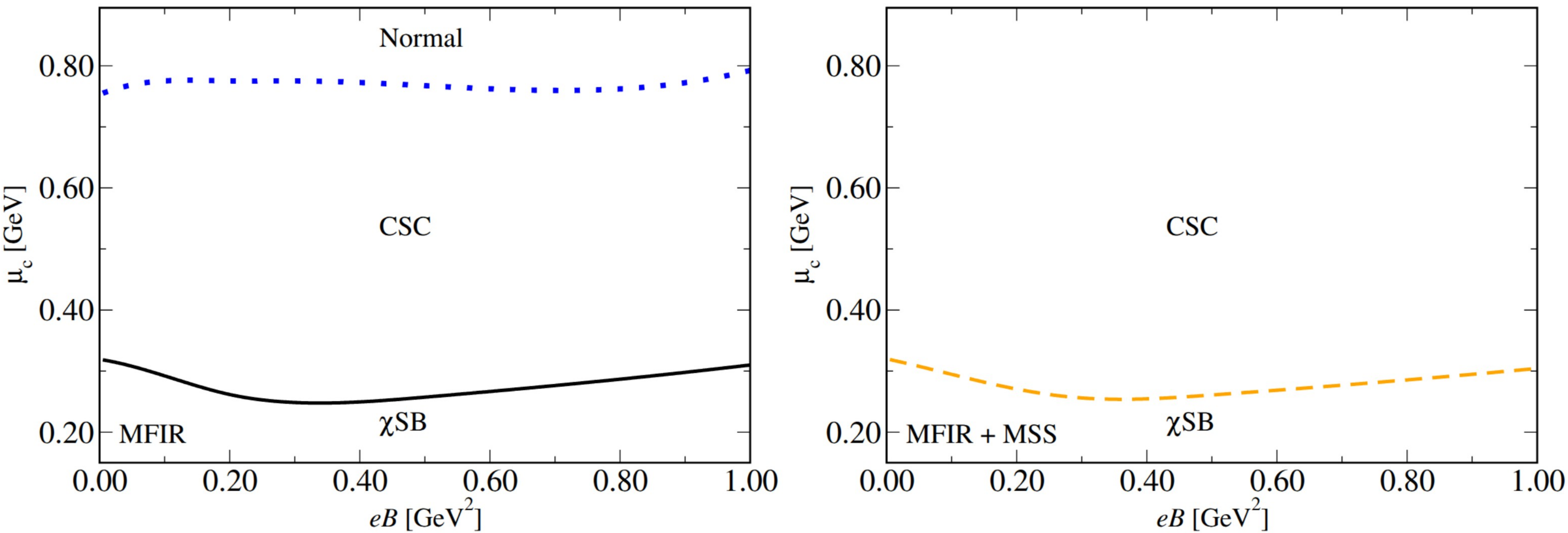


Δ at fixed magnetic field

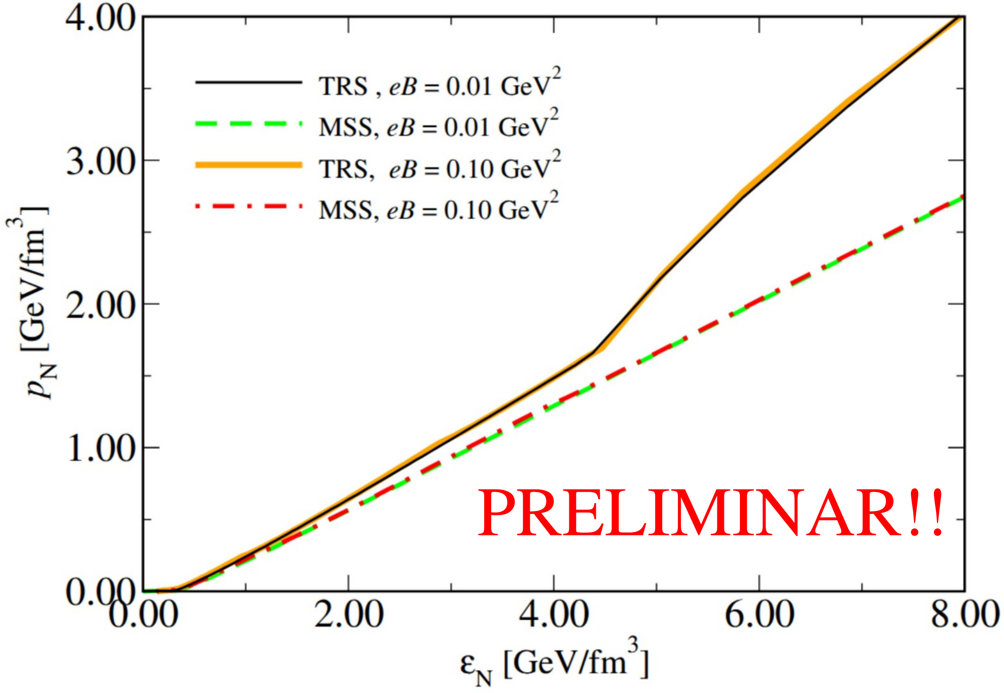


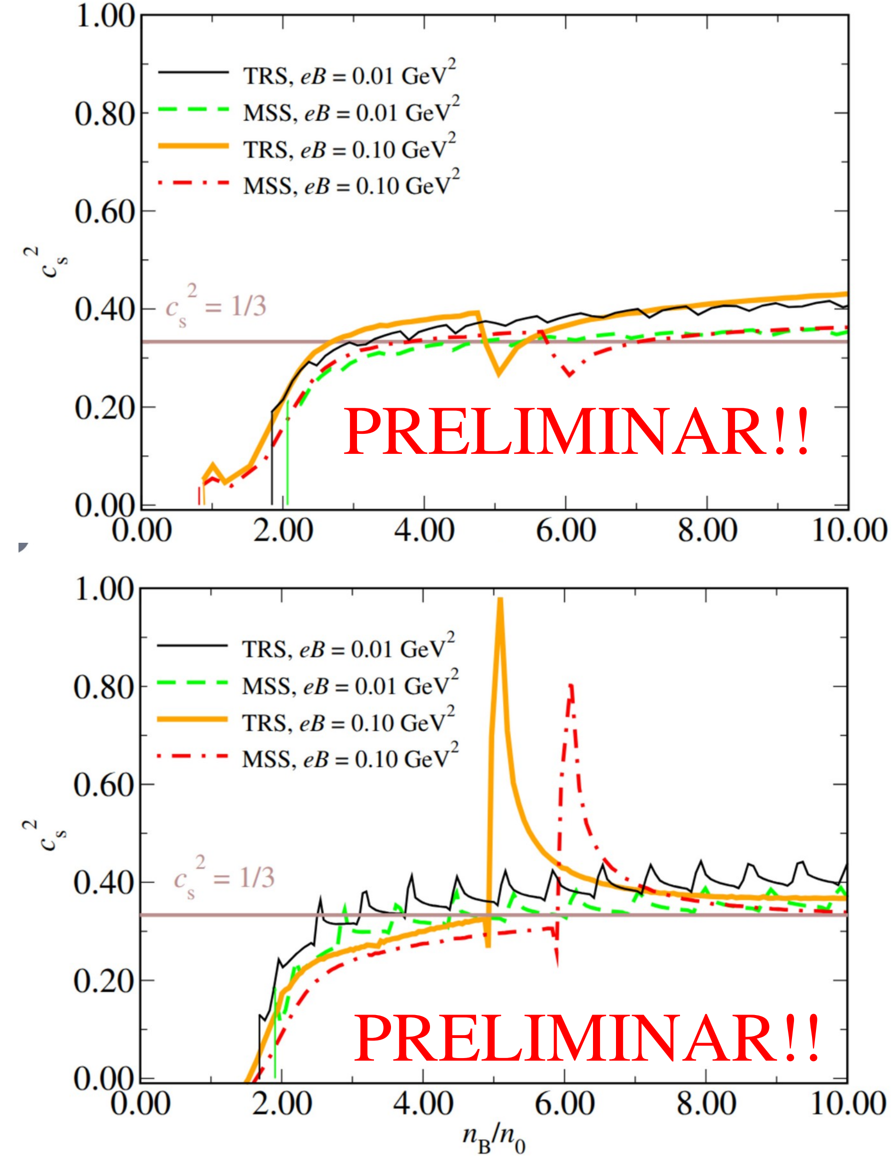
In other words...

There is no normal phase at high chemical potential!

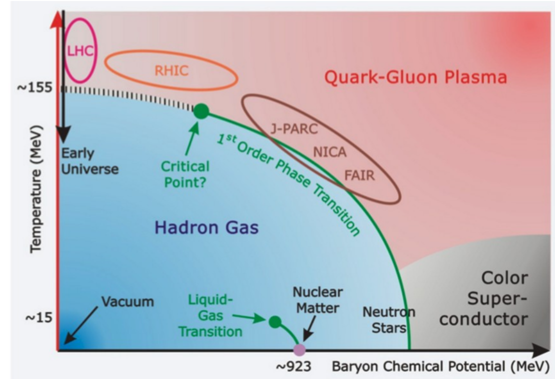


$\Delta \neq 0$ soften the EoS at high densities





Final Remarks



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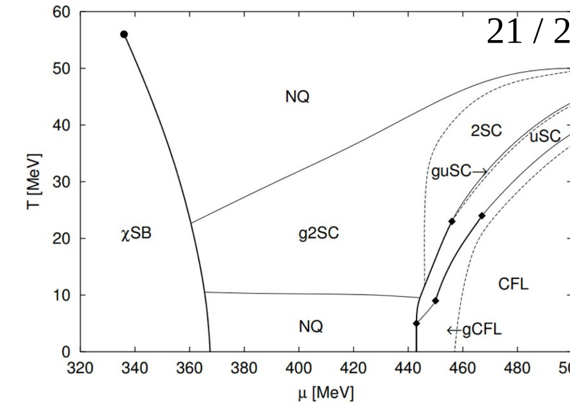

- Significant progress has been made in recent years toward describing the QCD phase diagram, but magnetic field effects are important.

- **The careful treatment of divergences through the separation of medium-dependent terms in non-renormalizable models is essential for an accurate description of the physical quantities of interest;**

- MSS proves to be a powerful tool for this task when applied to NJL/PNJL. Extension to other models/contexts?

Thanks for your attention!