Nonequilibrium Dynamics of the Chiral Quark Condensate

Magnetic Field Effects, Inhomogeneous condensates

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Motivation

1. Time dependence is a common feature of strongly interacting phenomena in the early universe, magnetars, and heavy-ion collisions
2. The quark condensate is a prominent QCD property affected by temperature, baryon density, and strong magnetic fields
3. Time scales of the condensate dynamics are important for hadron production in heavy-ion collisions
4. How is the quark condensate dynamics affected by temperature, density and magnetic fields?
5. Here: studies of quark condensate nonequilibrium time evolution (magnetic field, inhomogeneos chiral condensate)

Work with Arthur Frazon + Carlisson Miller + Juan Pablo Carlomagno + Theo Motta

Phase change - time dependence

Typical situation:

- A system is forced to change from a thermodynamic equilibrium phase to another, out-of-equilibrium phase
- Evolution to new equilibrium through spatial fluctuations that take the system (initially homogeneous) through a sequence of highly (not in equilibrium) inhomogeneous states

Theory: coarse-graining

Rational:

- It is difficult/impossible to describe the system with microscopic d.o.f.
- Focus on a small number of semi-macroscopic variables; order parameters φ
- Dynamics of φ is slower than that of the microscopic degrees of freedom; described by Ginzburg-Landau-Langevin type of equations

From A. Zee book

Dynamical equations

Landau: system's state characterized by a macroscopic free energy $F[\varphi]$

Example: $F[\varphi] = \int d^3x \, \left[\kappa(\nabla\varphi)^2 + V(\varphi)\right]$, $\quad V(\varphi) = \frac{1}{2}\, r\, \varphi^2 + \frac{1}{4}\, u\, \varphi^4$

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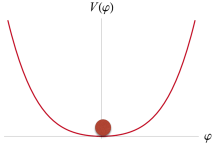
Equilibrium $(r > 0)$:



$$\frac{\delta F}{\delta \varphi} = 0$$

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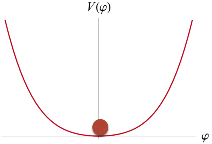
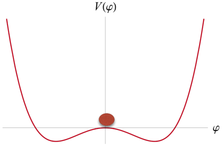


$$\frac{\delta F}{\delta \varphi} = 0$$

Out-of-equilibrium ($r = -|r| < 0$):



$$\varphi(x) \to \varphi(x,t): \qquad \eta\, \frac{\partial \varphi}{\partial t} = -\frac{\delta F}{\delta \varphi}$$

Near equilibrium: $\widetilde{\varphi}(k,t) \approx e^{(|r|-k^2)\, t/\eta}$

Dynamical equations

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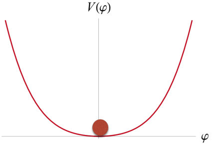

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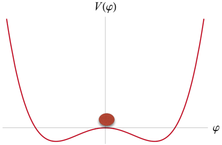

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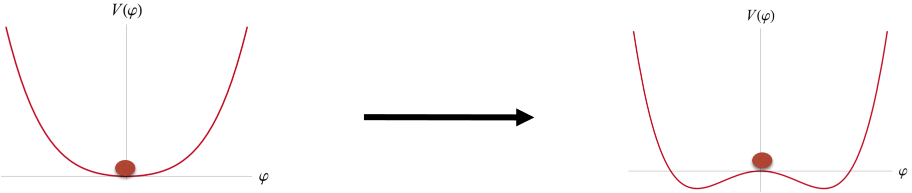
Purely diffusive dynamics - no (thermal) fluctuations

Phase change, fluctuations



$T > T_c$ $\qquad$ $T < T_c$

$$\eta \frac{\partial \varphi}{\partial t} = -\frac{\delta F}{\delta \varphi} + \xi(x,t) \quad \leftarrow$$ Ginzburg-Landau-Langevin (GLL) equation

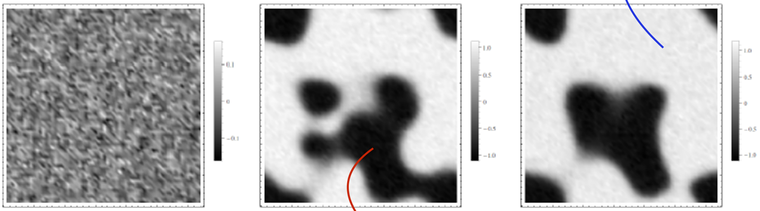
$$\langle \xi(x,t)\xi(x',t') \rangle = \sqrt{2\eta T}\, \delta(x-x')\delta(t-t')$$

Fluctuation-dissipation theorem

Phase change

Symmetry broken phase

Time increases



Symmetry broken phase

Dynamical universality classes

Dynamical phase changes/transitions
can be classified in universality classes according
to the nature and couplings of the order parameters*

- Model A: nonconserved order parameter
- Model B: conserved order parameter
- Model C: nonconserved and conserved order parameters
- . . . . . .

P. C. Hohenberg & M.I Halperin, Rev. Mod. Phys. 49, 435 (1977).

Derivation of GLLeq - Linear σ model*

$$\mathcal{L} = \bar{q}[i\partial\!\!\!/ - g(\sigma + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi})]q + \frac{1}{2}\left[\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}\right] - U(\sigma,\boldsymbol{\pi})$$

$$U(\sigma,\boldsymbol{\pi}) = \frac{\lambda}{4}(\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2 - h_q\sigma - U_0$$

Parameters: $v^2 = f_\pi^2 - \frac{m_\pi^2}{\lambda^2},\ m_\sigma^2 = 2\lambda^2 f_\pi^2 + m_\pi^2,\ h_q = f_\pi m_\pi^2,\ m_q = g\langle\sigma\rangle,\ U_0 :\ U(0,0) = 0$

Crossover at $T \simeq 150\ \mathrm{MeV}$: $g = 3.3$

Quark condensate: $m\langle\bar{q}q\rangle_{\mathrm{QCD}} = -h_q\langle\sigma\rangle$

Coupling magnetic field: $\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu$ (for the charged fields)

M. Nahrgang et al. Phys. Rev. C 84, 024912 (2011).

The scenario, approximations

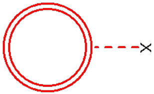
- QGP scales for temperature (T) and magnetic field (B)
- Perturbation around local equilibrium (medium at local T and B)
- No expansion & energy transfer field and medium
- Source of dissipation η: $\sigma \to \bar{q}q$ (no pions)
- Look at qualitative changes due to strong B
- Analytical understanding

Closed time path formalism

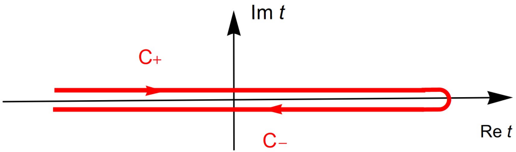
Semiclassical effective action:

$$\Gamma[\sigma, S] = \Gamma_{\mathrm{cl}}[\sigma] + i\,\mathrm{Tr}\ln S - i\,\mathrm{Tr}\left(i\not{D} - m_0\right) S + \Gamma_2[\sigma, S]$$

$$\Gamma_2[\sigma, S] = g \int_{\mathcal{C}} d^4x\, \mathrm{tr}\left[S^{++}(x,x)\sigma^{+}(x) + S^{--}(x,x)\sigma^{-}(x)\right]$$



- σ and S defined on the Schwinger-Keldysh contour (CTP contour), $\sigma^{\pm}$, $S^{\pm}$
- $\sigma^{\pm}$, $S^{\pm}$ are not independent, they are equal at some large time (CTP b.c.)



Integrate out the quarks

$$\frac{\delta\Gamma[\sigma,S]}{\delta S^{ab}(x,y)} = 0$$

$$\Downarrow$$

$$\left(i\not{D} - g\sigma_0(x)\right) S^{ab}(x,y) - \int_{\mathcal{C}} d^4z\, \frac{\delta\Gamma_2[\sigma,S]}{\delta S^{ac}(x,z)}\, S^{cb}(z,y) = i\delta^{ab}\delta^{(4)}(x-y)$$

Very difficult to solve (even numerically): expand around local equilibrium

$$\sigma^a(x) = \sigma_0^a(x) + \delta\sigma^a(x)$$

$$S^{ab}(x,y) = S^{ab}_{\rm thm}(x,y) + \delta S^{ab}(x,y) + \delta^2 S^{ab}(x,y) + \cdots$$

where $\frac{\delta\Gamma_{\rm cl}}{\delta\sigma_0^a(x)} = -g{\rm Tr}S^{aa}(x,x)$ and $\left[i\not{D} - m_0 - g\,\sigma_0(x)\right] S^{ab}_{\rm thm}(x,y) = -i\delta^{ab}\delta^{(4)}(x-y)$

GLL equation

- Dissipation ($\sigma \to \bar{q}q$): imaginary part of $\Gamma[\sigma, S]$
- Variation of $\Gamma[\sigma, S]$ w.r.t. σ to obtain e.o.m. not possible
- Solution: use Feynman-Vernon trick, identify the imaginary part with a noise source coupling linearly to the field
- Obtain real action, variation w.r.t. σ leads to GLL equation

$$\partial_\mu \partial^\mu \sigma(x) + \frac{\delta U[\sigma]}{\delta \sigma(x)} + g\rho_s(\sigma_0) - D_\sigma(x) = \xi_\sigma(x)$$

GLL equation

$$\partial_\mu \partial^\mu \sigma(x) + \frac{\delta U[\sigma]}{\delta \sigma(x)} + g\rho_s(\sigma_0) - D_\sigma(x) = \xi_\sigma(x)$$

Scalar Density: $\rho_s(\sigma_0) = \mathrm{tr}\, S^{++}_{thm}(x,x)$

Dissipation kernel: $D_\sigma(x) = ig^2 \int d^4y\, \theta(x^0 - y^0)\, M(x,y)\, \delta\sigma(y)$ $\leftarrow$ Memory

$$M(x,y) = \mathrm{tr}\left[S^{+-}_{thm}(x,y)S^{-+}_{thm}(y,x) - S^{-+}_{thm}(x,y)S^{+-}_{thm}(y,x)\right]$$

Colored noise: $\langle \xi_\sigma(x) \rangle_\xi = 0$ and $\langle \xi_\sigma(x)\xi_\sigma(y) \rangle_\xi = N(x,y)$

$$N(x,y) = -\frac{1}{2} g^2 \mathrm{tr}\left[S^{+-}_{thm}(x,y)S^{-+}_{thm}(y,x) + S^{-+}_{thm}(x,y)S^{+-}_{thm}(y,x)\right]$$

$\langle \cdots \rangle_\xi$: functional average with prob. distr. $P[\xi] = \exp\left[-\frac{1}{2}\int d^4x d^4y\, \xi(x) N^{-1}(x,y)\, \xi(y)\right]$

Quark propagator $S^{ab}_{\text{thm}}(x,y)$

Structure of $\rho_s(x)$, $M(x,y)$ and $N(x,y)$

— Schwinger phase drops out, use Fourier transform

In the lowest Landau level (LLL) approximation:

$$S^{++}_{thm}(p) = e^{-\boldsymbol{p}_\perp^2/|q_f B|}\, A(p) \left[\frac{i}{p_\parallel^2 - m_q^2 + i\epsilon} - 2\pi n_F(p_0) \delta(p_\parallel^2 - m_q^2) \right]$$

$$S^{+-}_{thm}(p) = e^{-\boldsymbol{p}_\perp^2/|q_f B|}\, A(p)\, 2\pi\delta(p_\parallel^2 - m_q^2)\, [\theta(-p_0) - n_F(p_0)]\,,$$

$$S^{-+}_{thm}(p) = e^{-\boldsymbol{p}_\perp^2/|q_f B|}\, A(p)\, 2\pi\delta(p_\parallel^2 - m_q^2)\, [\theta(p_0) - n_F(p_0)]$$

$$S^{--}_{thm}(p) = e^{-\boldsymbol{p}_\perp^2/|q_f B|}\, A(p) \left[\frac{-i}{p_\parallel^2 - m_q^2 - i\epsilon} - 2\pi n_F(p_0) \delta(p_\parallel^2 - m_q^2) \right]$$

where $A(p) = (\not{p}_\parallel + m_q)\left[1 + i\gamma^1\gamma^2 \text{sign}(qB)\right]$ and $n_F(p_0) = \dfrac{1}{e^{|p_0|}/T + 1}$

Momentum space GLL equation

$$\frac{\partial^2 \sigma(t,\boldsymbol{p})}{\partial t^2} + \boldsymbol{p}^2\, \sigma(t,\boldsymbol{p}) + \eta(\boldsymbol{p})\, \frac{\partial \sigma(t,\boldsymbol{p})}{\partial t} + F_\sigma(t,\boldsymbol{p}) = \xi_\sigma(t,\boldsymbol{p})$$

$$\eta(\boldsymbol{p}) = g^2 \frac{1}{2E_\sigma(\boldsymbol{p})} M(\boldsymbol{p}) \quad \leftarrow M(\boldsymbol{p}) = M(E_\sigma, \boldsymbol{p}),\; E_\sigma = \sqrt{\boldsymbol{p}^2 + m_\sigma^2}$$

$$F_\sigma(t,\boldsymbol{p}) = \int d^3x\, e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \left[\frac{\delta U[\sigma]}{\delta\sigma(t,\boldsymbol{x})} + g\, \rho_s(\sigma_0) \right]$$

$$\langle \xi_\sigma(t,\boldsymbol{p}) \xi_\sigma(t',\boldsymbol{p}) \rangle_\xi = (2\pi)^3 \delta(\boldsymbol{p} + \boldsymbol{p}') N(t - t', \boldsymbol{p})$$

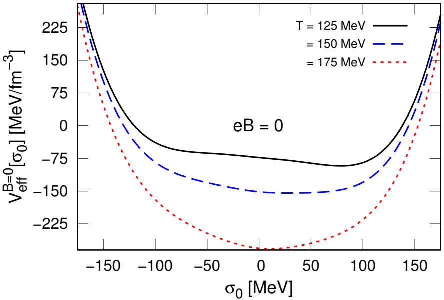
$\eta = 0$: "classical" equation of motion

$\eta \neq 0$: slows the dynamics

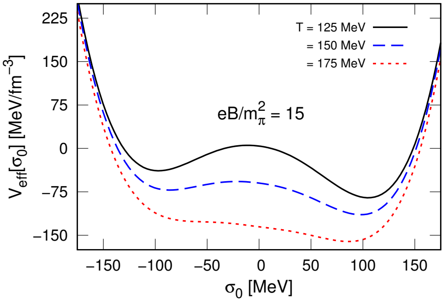
GK and C. Miller, Symmetry 13, 551 (2021).

Equilibrium*

$B = 0$



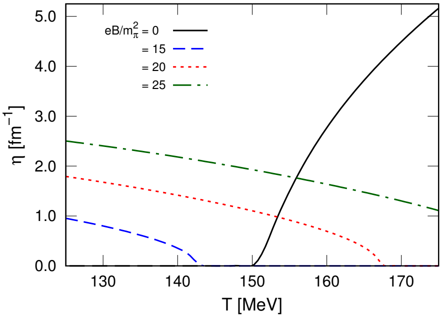
$B \neq 0$



E.S. Fraga & A.J. Mizher, Phys. Rev. D 78, 025016 (2008).

Zero-mode η

Recall, source of dissipation is $\sigma \rightarrow \bar{q}q$



$$\underline{B=0} \quad \eta_0 = g^2 \frac{2N_c}{\pi} \left[1 - 2n_F(m_\sigma/2)\right] \frac{1}{m_\sigma^2} \left(m_\sigma^2 - 4m_q^2\right)^{3/2}$$

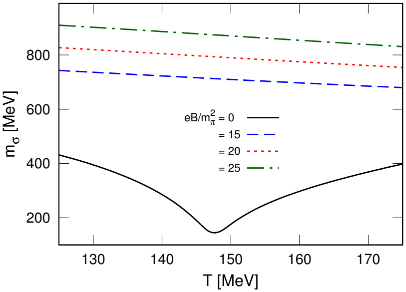
$$\underline{B\neq 0} \quad \eta_B = g^2 \frac{N_c}{4\pi} \left[1 - 2n_F(m_\sigma/2)\right] (eB) \frac{1}{m_\sigma^2} \sqrt{m_\sigma^2 - 4m_q^2} \quad (LLL)$$

σ and quark masses

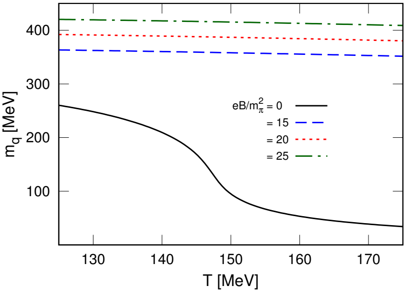
$\eta \neq 0$ when $m_\sigma > 2m_q$

B modifies minimum of V_{eff} $(\leftarrow m_q)$ and its curvature $(\leftarrow m_\sigma)$

σ mass



Quark mass



m_σ increases faster than m_q as the temperature decreases:

– η_B increases at low temperatures

– increase in η_B delays evolution of σ

Short-time dynamics

Linearized GLL equation:

$$\eta(\boldsymbol{p}_\perp)\,\frac{\partial\overline{\sigma}(t,\boldsymbol{p}_\perp)}{\partial t} - \left(\mu^2 - \boldsymbol{p}_\perp^2\right)\overline{\sigma}(t,\boldsymbol{p}_\perp) + g\rho_s(\sigma_0) - f_\pi m_\pi^2 = \overline{\xi}_\sigma(t,\boldsymbol{p}_\perp)$$

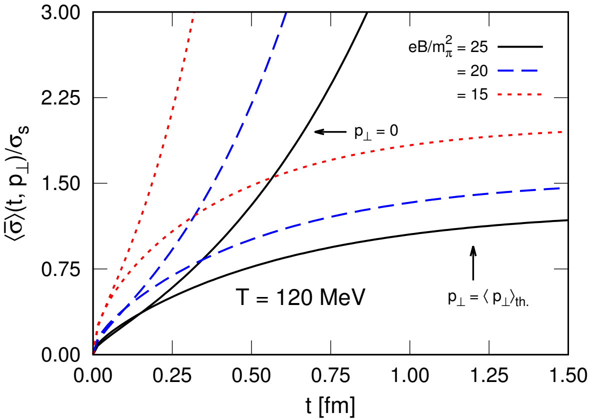
$$\mu^2 = \lambda\left(f_\pi^2 - \frac{m_\pi^2}{\lambda}\right), \qquad \overline{\sigma} = \sigma/L^3, \qquad \overline{\xi} = \xi/L^3$$

Quench from high to low T:

$$\langle\,\overline{\sigma}^2(t,\boldsymbol{p}_\perp^2)\,\rangle_\xi = \frac{\left[g\rho_s(\sigma_0) - f_\pi m_\pi^2\right]^2}{(\mu^2 - \boldsymbol{p}_\perp)^2}\left(e^{\,\lambda(\boldsymbol{p}_\perp)\,t/\tau_s} - 1\right)^2 + \frac{E(\boldsymbol{p}_\perp)\coth(E(\boldsymbol{p}_\perp))}{L^3(\mu^2 - \boldsymbol{p}_\perp^2)}\left(e^{\,2\,\lambda(\boldsymbol{p}_\perp)\,t/\tau_s} - 1\right)$$

$$\tau_s = \frac{\eta_B}{\mu^2} \qquad \text{and} \qquad \lambda(\boldsymbol{p}_\perp) = \frac{1 - \boldsymbol{p}_\perp^2/\mu^2}{\eta(\boldsymbol{p}_\perp)/\eta_B},$$

Short-time dynamics



$$\sigma_s = (g\rho_s - f_\pi m_\pi^2)/\mu^2$$

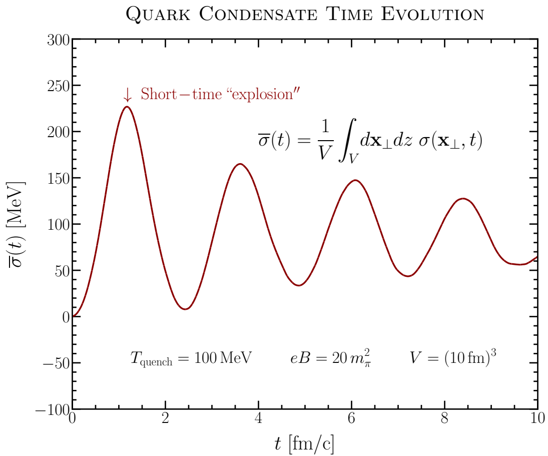
$$\langle \boldsymbol{p}_\perp^2 \rangle_{\text{th.}} = \frac{\int d^2p_\perp\, p_\perp^2\, n_B(\boldsymbol{p}_\perp)}{\int d^2p_\perp\, n_B(p_\perp)}$$

$$n_B(p) = \frac{1}{e^{E_\sigma(p)/T} - 1}$$

Large B slows considerably the short-time dynamics

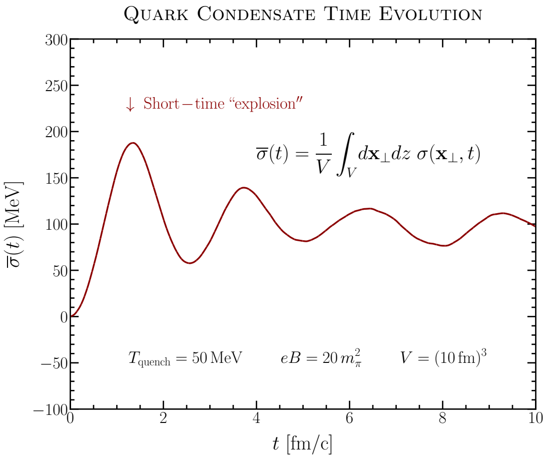
Present estimate: delays of $\simeq 1$ fm/c $\leftarrow 1/10$ of QGP lifetime

Long-time dynamics



Condensate not thermalized within QGP lifetime

Long-time dynamics



Condensate thermilized within QGP lifetime

Inhomogeneous chiral condensates

— Low T and high μ: stable phase of quark matter might break translational invariance (condensates spatially modulated)

— Different kinds of modulations, depending on dimensionality of space
For example:

- D = 1: kinks
- D = 2: Checkerboard crystals
- D = 3: Domain wall networks

— Next: even when not strictly stable, have remarkably long lifetimes

Review: M. Buballa and S. Carignano, Prog. Part. Nucl. Phys. 81, 39 (2015)

Time evolution

Phenomenological GLL equation:

$$\eta \frac{\partial \phi^a(\boldsymbol{x},t)}{\partial t} = -\frac{\delta \Omega_{GL}}{\delta \phi^a(\boldsymbol{x},t)} + \xi(\boldsymbol{x},t), \qquad a = \sigma, \pi$$

$$\Omega_{GL}[\phi^a] = \int d^4x \left[\frac{\alpha_2}{2}\phi^2 + \frac{\alpha_4}{4}\left(\phi^2\right)^2 + \frac{\alpha_{4b}}{4}(\nabla\phi)^2 + \cdots\right]$$

$$\langle \xi(\boldsymbol{x},t)\rangle = 0, \qquad \langle \xi(\boldsymbol{x},t)\xi(\boldsymbol{x}',t')\rangle = 2\eta T\delta(t-t')\delta^{(3)}(\boldsymbol{x}-\boldsymbol{x}')$$

Ω_{GL}: for a bosonized nonlocal NJL model Carlomagno, Dumm, and Scoccola PLB 745, 1 (2015).

$$S_E = \int d^4x \left[-i\bar\psi(x)\partial\!\!\!/\psi(x) - \frac{G}{2}j_a(x)j_a(x)\right], \qquad j_a(x) = \int d^4x' \mathcal{G}(x')\bar\psi\left(x + \frac{x'}{2}\right)\Gamma_a\psi\left(x - \frac{x'}{2}\right)$$

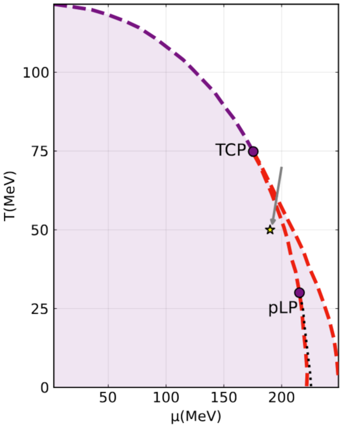
$$\alpha_2 = \frac{1}{G} - 8N_c \sum\!\!\!\!\!\!\int \frac{g^2}{p_n^2}, \qquad \alpha_4 = 8N_c \sum\!\!\!\!\!\!\int \frac{g^4}{p_n^4}, \qquad \alpha_{4b} = 8N_c \sum\!\!\!\!\!\!\int \frac{g^2}{p_n^4}\left(1 - \frac{2}{3}\frac{g'}{g}\vec{p}^2\right), \cdots$$

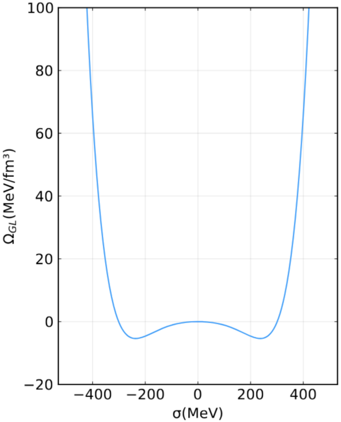
$$\sum\!\!\!\!\!\!\int (\cdots) = T\sum_{\omega_n}\int d^3p\,(\cdots)$$

Phase diagram

Equilibrium: at the star mark, $T = 50$ MeV and $\mu = 150$ MeV,

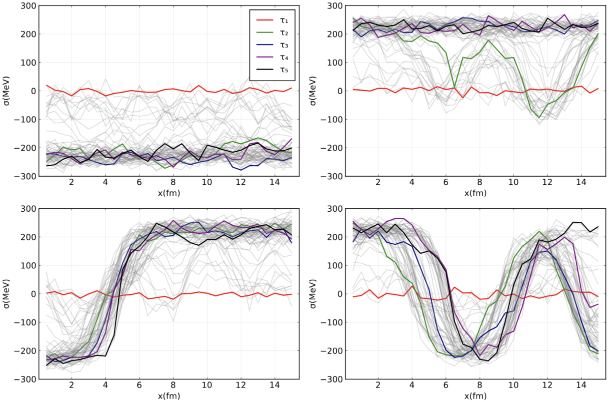
stable homogeneous condensates





Time evolution

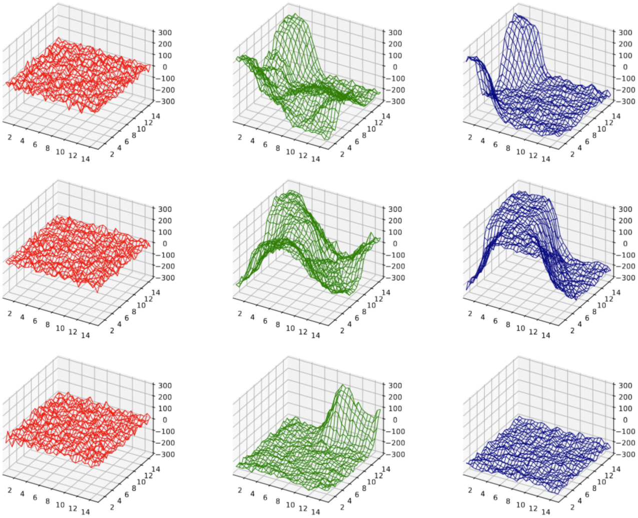
1 + 1 dimensions



Different realizations of the noise field (fluctuations)

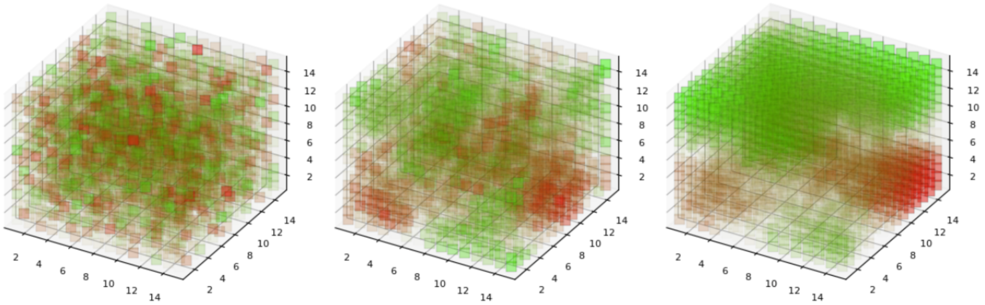
Time evolution

2 + 1 dimensions



Time evolution

3 + 1 dimensions



Conclusions & Perspectives

1. Presented a nonequilibrium QFT setup to tackle temperture, baryon density, and magnetic field effects on chiral dynamics

2. Used Ginzburg-Landau-Langevin framework, made numerical estimates of the short- and long-time dynamics

3. More realistic applications to HIC: beyond LLL approximation (weak fields), include pion dynamics, couple to hydrodynamics

4. Dense matter, patches of nonhomogeneous condensates: although not strictly stable, have remarkably long lifetimes $\leftarrow$ might produce deviations in the relation between postmerger gravitational wave frequency f_{peak} and tidal deformability λ akin to Bauswein et l. PRL 122, 061102 (2019)

Thank you

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