Superconductivity in Confining Models

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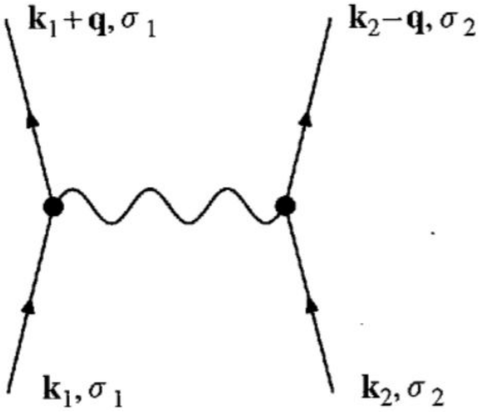
Outline

- Introduction
- Confining Propagators
- SUC Model
- Analysis
- Concluding Remarks
- Acknowledgements

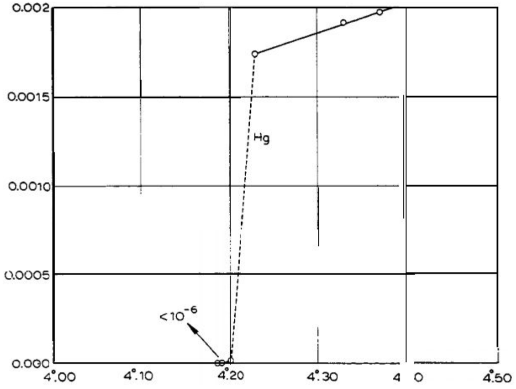
Introduction

Introduction

- The phenomenon was discovered in 1911 by ***H. Kamerlingh Onnes.***
- In 1957, ***the BCS theory*** described the formation of ***Cooper pairs***, providing a microscopic explanation for superconductivity.



(Annett, 2004)

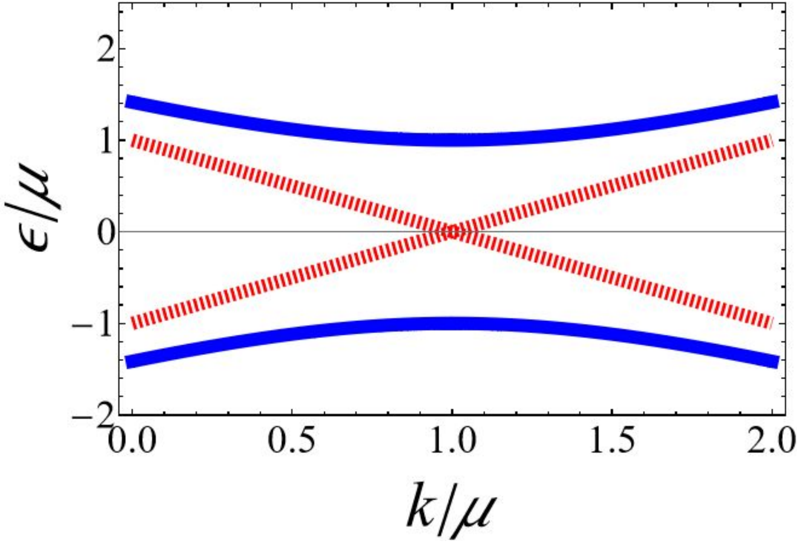


(Onnes, 1911)

Introduction

- The phenomenon was discovered in 1911 by ***H. Kamerlingh Onnes.***
- In 1957, ***the BCS theory*** described the formation of ***Cooper pairs***, providing a microscopic explanation for superconductivity.

$$\begin{cases} \epsilon = \sqrt{(k-\mu)^2 + \Delta^2} \\ \Delta \neq 0 \\ \Delta = 0 \end{cases}$$



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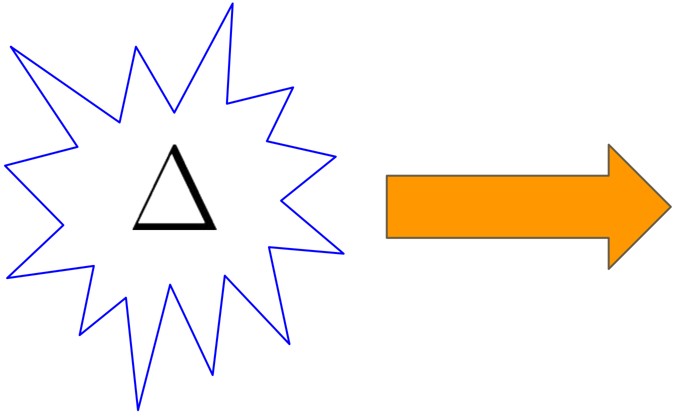
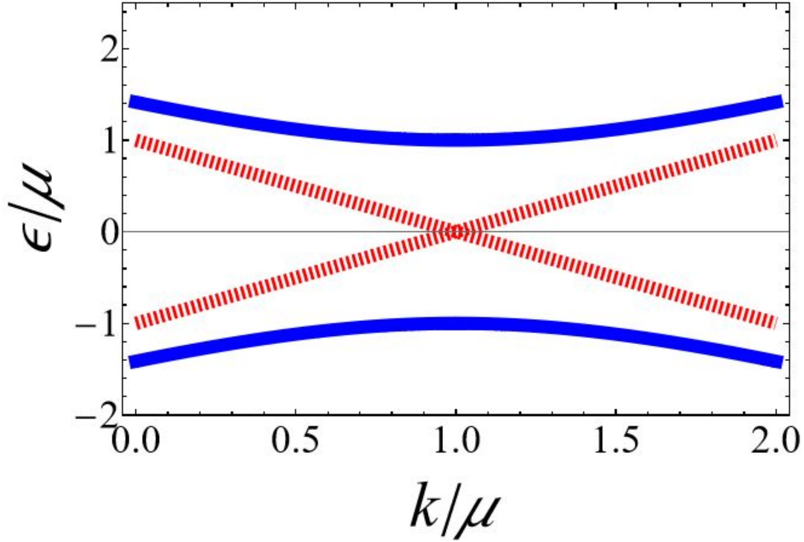
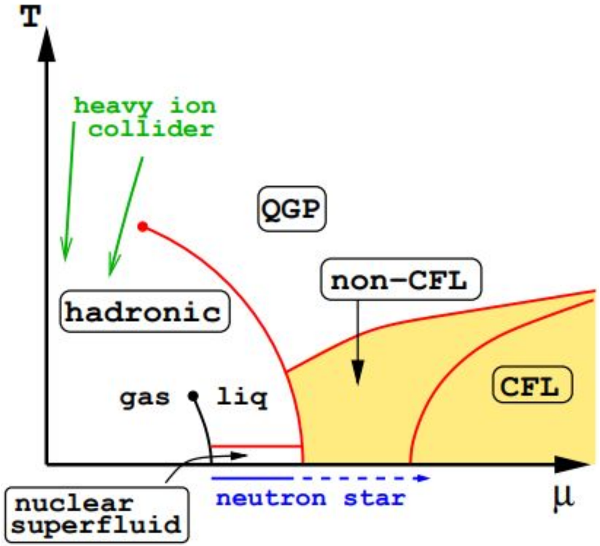




Introduction

- ***Color superconductivity (SUC)*** is currently the most well-established theoretical framework for ***superconductivity at high densities.***

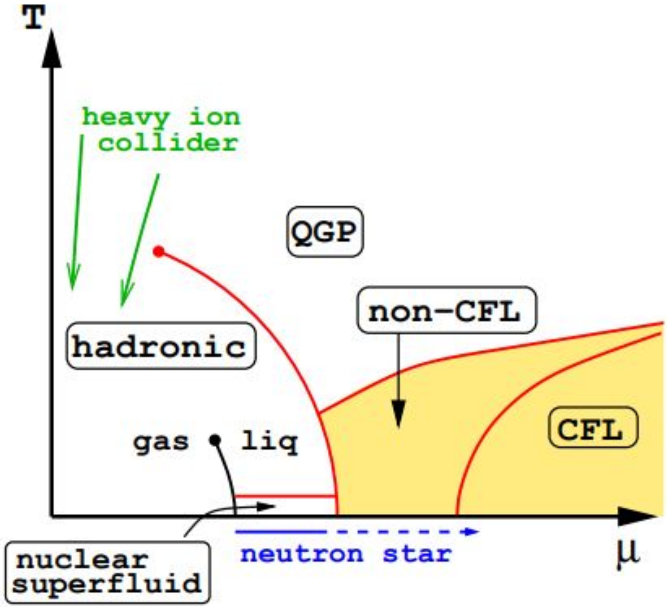
- In this phase, ***Cooper pairs*** are formed by quarks, and the attractive interaction is mediated by ***gluon exchange***.



(Alford; Schmitt; Rajagopal; Shäfer, 2008)

Introduction

$$D^{ab}_{\mu\nu}$$



In this work, we investigate **what happens when we replace the usual bosonic propagator with a confining one** and study **how this affects the superconducting gap**.

Confining Propagators

Confining propagators

Faddeev-Popov Procedure

$$Z = \int [\mathcal{D}A] e^{-S_{YM}} = \int \int [\mathcal{D}U][\mathcal{D}A] \Delta F \delta(F(A^U)) e^{-S_{YM}}$$

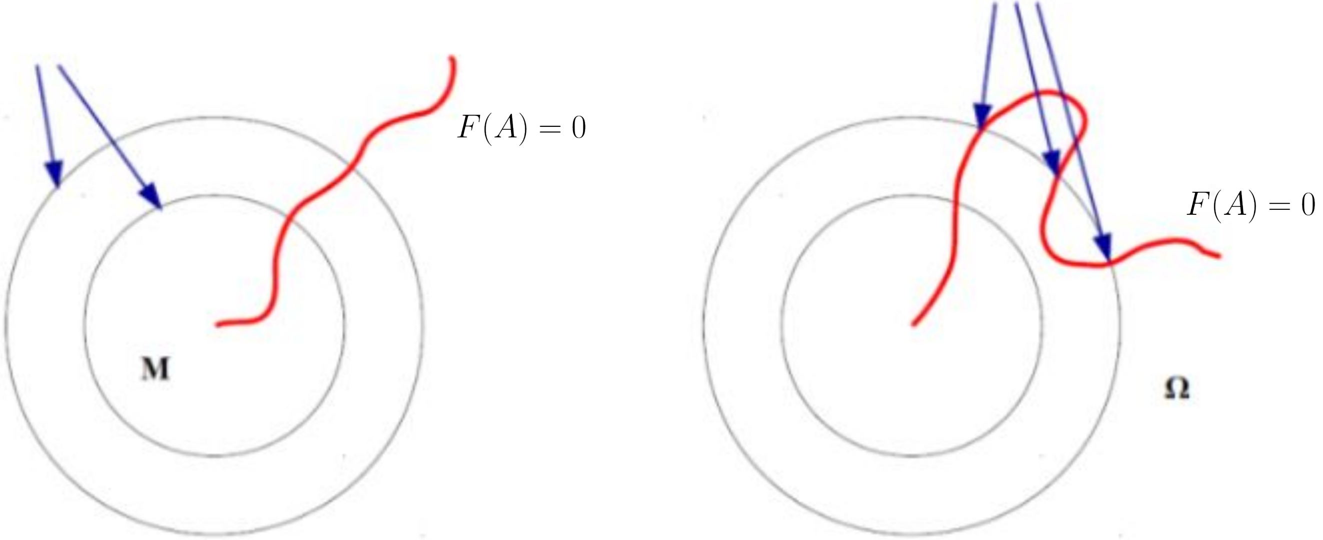
In covariant linear gauges, the ***Faddeev-Popov procedure*** **(Faddeev & Popov,1967)** introduces the Grassmann variables c and $\bar{c}$

$$\boxed{M_{ab}(x,y) = -\partial_\mu D_\mu^{ab} \delta(x-y) \Big|_{F(A)=0}}$$

$$Z = \int [\mathcal{D}A] |\det[-\partial_\mu D_\mu^{ab} \delta(x-y)]| \delta(\partial A - B) e^{-S_{YM}}$$

$$Z = \int [\mathcal{D}A][\mathcal{D}c][\mathcal{D}\bar{c}]\, e^{-S}, \quad S = S_{YM} + S_{gf} = S_{YM} + \int dx \left(\bar{c}^a M_{ab}(x,y) c^b - \frac{1}{2\alpha} (\partial_\mu A_\mu^a)^2 \right)$$

Confining propagators - Gribov Copies



(**Guimarães, 2013**)

Confining propagators - Gribov-Zwanziger (GZ)

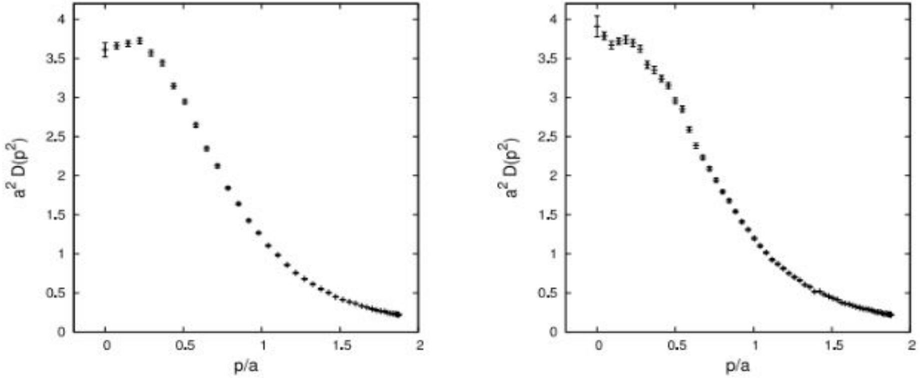
By introducing a restriction on the integration domain of the fields in the action, it is possible to compute the ***tree-level gluon propagator***, given by:

$$\boxed{D^{ab}_{\mu\nu}(p) = \frac{p^2}{p^4 + m_A^4} P^{\perp}_{\mu\nu}\delta^{ab}} \Rightarrow \frac{1}{2}\left[\frac{1}{p^2 + im_A^2} + \frac{1}{p^2 - im_A^2}\right] P^{\perp}_{\mu\nu}\delta^{ab}$$

where ***mA*** is the ***Gribov mass parameter***. This propagator exhibits ***complex-conjugate poles*** **(Gribov, 1978)**.

Confining propagators - Refined Gribov-Zwanziger (RGZ)

$$D^{ab}_{\mu\nu}(p) = \delta^{ab}\frac{p^2 + M_1^2}{p^4 + M_2^2 p^2 + M_3^4} P^{\perp}_{\mu\nu}$$



(Cucchieri and Mendes, 2007)

M1, M2 and **M3** are ***lattice datas points!***
(Dudal; Oliveira, Silva, 2019)

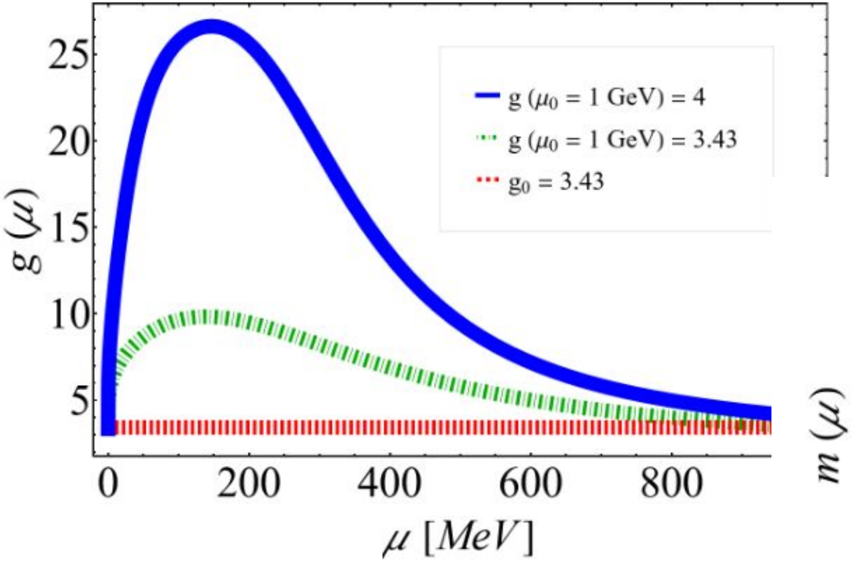
Confining propagators - Curci-Ferrari (CF)

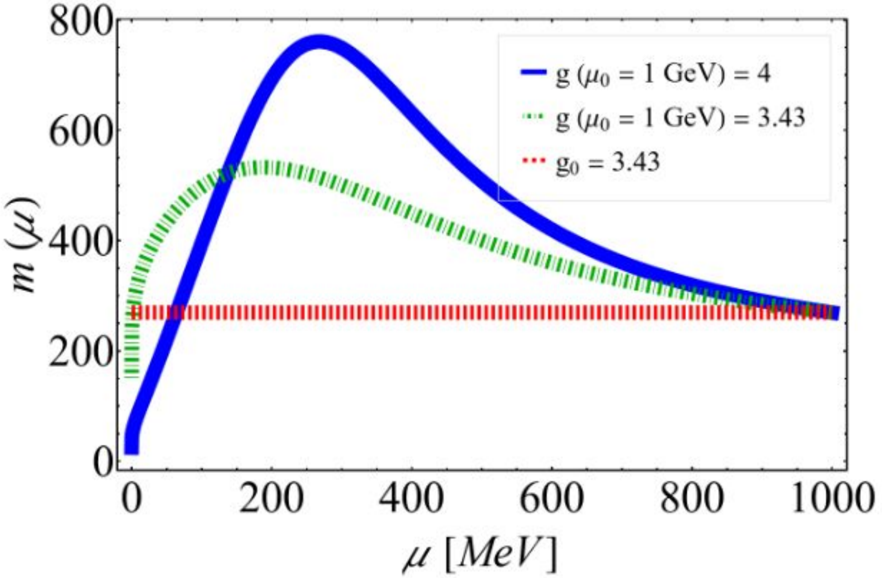
$$S = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \partial_\mu \bar{c}^a D_\mu c^a + i h^a \partial_\mu A^a_\mu + \frac{1}{2} m_A^2 A^a_\mu A^a_\mu \right\}$$

$$\boxed{D^{ab}_{\mu\nu}(p) = \delta_{ab} \frac{P^\perp_{\mu\nu}}{p^2 + m_A^2}}$$

(Curci & Ferrari,1976)

Confining propagators - Curci-Ferrari with runnings (CFr)





SUC Model

SUC Model

SUC can be described using field-theoretical methods based on a ***Yukawa-type model*** **(Pisarski & Rischke, 1999; Schmitt, 2014).**

$$\mathcal{L} = \overline{\psi}(i\gamma^{\mu}\partial_{\mu}+\gamma^{0}\mu-m_{\psi})\psi + \int_{y}\frac{1}{2}\left[\phi(x)\left(-\partial^{2}+\frac{m_{\phi}^{4}}{-\partial^{2}}\right)\delta(x-y)\phi(y)\right]-g\overline{\psi}\psi\phi$$

SUC Model

Applying a ***Hubbard-Stratonovich transformation***

$$Z = Z_0(\phi) \int [\mathcal{D}\overline{\psi}][\mathcal{D}\psi] e^{S'}$$

with

$$S' = \int \left[\overline{\psi(x)} G_0^{-1}(x,y)\psi(y) + \frac{g^2}{2}\overline{\psi}(x)\psi(x)D(x,y)\overline{\psi}(y)\psi(y) \right]$$

The ***charge conjugate Spinor*** is defined as:

$$\begin{cases} \psi_c \equiv C\overline{\psi}^T \\ C = i\gamma^2\gamma^0 \end{cases}$$

SUC Model

Mean field approximation

$$\begin{cases} \psi_c(x)\overline{\psi}(y) = \left\langle \psi_c(x)\overline{\psi}(y) \right\rangle - [\left\langle \psi_c(x)\overline{\psi}(y) \right\rangle - \psi_c(x)\overline{\psi}(y)] \\ \psi(y)\overline{\psi_c}(x) = \left\langle \psi(y)\overline{\psi_c}(x) \right\rangle - [\left\langle \psi(y)\overline{\psi_c}(x) \right\rangle - \psi(y)\overline{\psi_c}(x)] \end{cases}$$

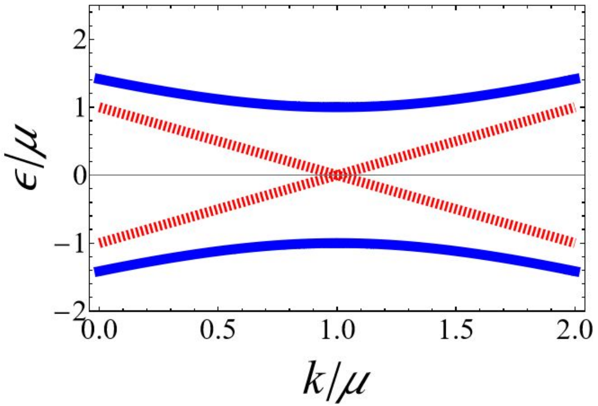
$$Z = Z(\phi)\ \ Z_0 \ \int [\mathcal{D}\overline{\psi}][\mathcal{D}\psi] e^{S''}$$

$$S'' = \int_{x,y} \left\{ \overline{\psi}(x) G_0^{-1}(x,y)\psi(y) + \frac{1}{2}[\overline{\psi_c}(x)\Phi^{+}(x,y)\psi(y) + \overline{\psi}(x)\Phi^{-}(x,y)\psi_c(y)] \right\}$$

SUC Model

Gap Function

Δ



$$\Phi^{+}(x,y) \equiv g^2 D(x,y)\left\langle \psi_c(x)\overline{\psi}(y)\right\rangle$$

$$\Phi^{-}(x,y) \equiv g^2 D(x,y)\left\langle \psi(x)\overline{\psi_c}(y)\right\rangle$$

$$S'' = \int_{x,y}\left\{\overline{\psi}(x)G_0^{-1}(x,y)\psi(y) + \frac{1}{2}[\overline{\psi_c}(x)\Phi^{+}(x,y)\psi(y) + \overline{\psi}(x)\Phi^{-}(x,y)\psi_c(y)]\right\}$$

SUC Model

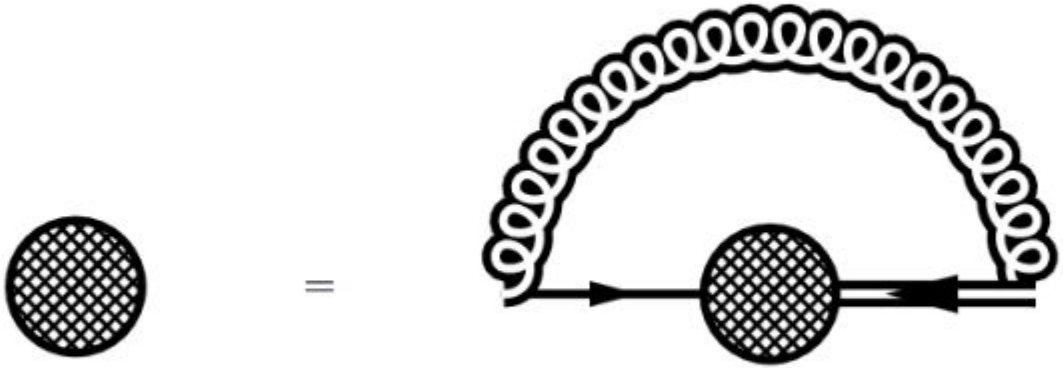
The functional can be written in terms of a new spinor basis in ***Nambu-Gorkov space*** **(Nambu, 1960; Gorkov, 1958)**:

$$\Rightarrow \quad Z = \mathcal{N}\ Z(\phi)\ Z_o \int [\mathcal{D}\overline{\Psi}][\mathcal{D}\Psi] \exp\Big[\sum_{k>0} \overline{\Psi}(k) \frac{\mathbf{S}^{-1}(k)}{T} \Psi(k)\Big]$$

$$\overline{\Psi} = \begin{pmatrix} \overline{\psi} & \overline{\psi}_c \end{pmatrix} \qquad \Psi = \begin{pmatrix} \psi \\ \psi_c \end{pmatrix}$$

$$\boxed{\mathbf{S}^{-1}(k) = \begin{pmatrix} [G_0^+(k)]^{-1} & \Phi^-(k) \\ \Phi^+(k) & [G_0^-(k)]^{-1} \end{pmatrix}}$$

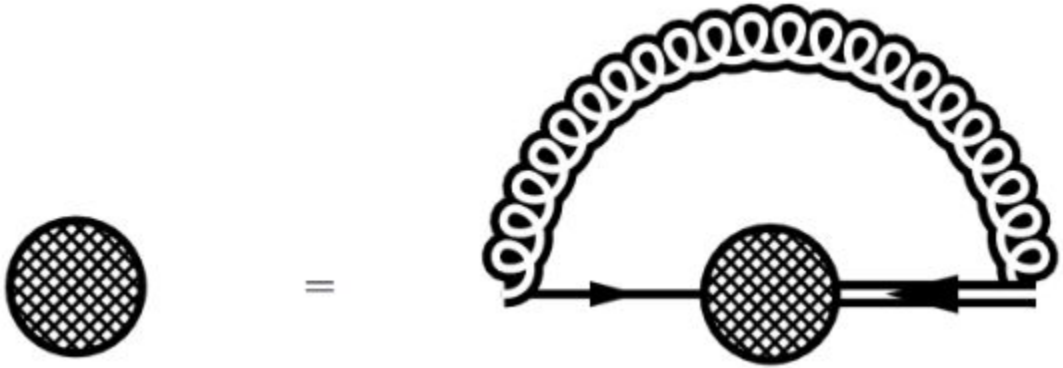
SUC Model



$$\Phi^{+}(x,y) = -g^{2} D(x,y) F^{+}(x,y) \Rightarrow \Phi^{+}(p) = -g^{2} \frac{T}{V} \sum_{k} D(p-k) F^{+}(k)$$

$$\boxed{\Phi^{+}(p) = g^{2} \frac{T}{V} \sum_{k} D(p-k) G_{0}^{-} \Phi^{+} G^{+}}$$

SUC Model



$$\Phi^+(x,y) = -g^2 D(x,y) F^+(x,y) \Rightarrow \boxed{\Phi^\pm(k) = \Delta^\pm(k)\gamma^5}$$

(Bailin & Love,1984)

SUC Model

The bosonic propagator in the theory can be replaced by that of a confining scenario, such as the ***Gribov propagator***. Therefore,

$$\Delta(p) = \frac{g^2}{4}\int \frac{d^4k}{(2\pi)^4} \frac{1}{(p_4 - k_4)^2 + (\vec{p} - \vec{k})^2 - im_\phi^2} \frac{\Delta(k)}{k_4^2 + \epsilon_k^2}$$

$$+ \frac{g^2}{4}\int \frac{d^4k}{(2\pi)^4} \frac{1}{(p_4 - k_4)^2 + (\vec{p} - \vec{k})^2 + im_\phi^2} \frac{\Delta(k)}{k_4^2 + \epsilon_k^2} ,$$

where, in low temperatures, **(Shäfer & Wilczek, 2000)**:

$$T \sum_{k_0 = i\omega_n} \to \int \frac{dk_0}{(2\pi)} \equiv \int \frac{idk_4}{(2\pi i)}$$

SUC Model

In the vicinity of the Fermi surface:

$$\Delta \cong \Delta(p_4) \quad \text{and} \quad \vec{k} = \vec{k}_F + \vec{l} \quad \text{where,} \quad |\vec{k}| \approx \mu + O(l)$$

Therefore,

$$I_{\pm} = \frac{g^2\mu^2}{4} \int \frac{dk_4}{(2\pi)^4} \int_{-1}^{1} d(\cos\theta) \int_0^{\infty} dl \int_0^{2\pi} d\phi$$

$$\left\{ \frac{1}{[q_4^2 + 2\mu^2(1-\cos\theta) \mp i m_\phi^2]} \frac{\Delta(k_4)}{[k_4^2 + l^2 + \Delta^2(k_4)]} \right\}$$

SUC Model

By applying the ***large chemical potential approximations***, as in **(Son, 1999)** and **(Shäfer & Wilczek, 2000)**;

$$\Delta(p_4) = \frac{g^2}{128\pi^2} \int \frac{dk_4 \Delta(k_4)}{\sqrt{k_4^2 + \Delta^2(k_4)}} \left\{ \log \left[\frac{(2\mu)^4}{(p_4 - k_4)^4 + m_\phi^4} \right] \right\}$$

Expanding term under $\frac{\Delta}{k_4} \frac{1}{\sqrt{1 + (\frac{\Delta}{k_4})^2}} \approx \frac{\Delta}{k_4} + O\left(\left(\frac{\Delta}{k_4}\right)^3 \right)$ with $k_4 \gg \Delta$

it is possible to compute an ***approximate gap expression*** as done by **(Son, 1999)**.

SUC Model

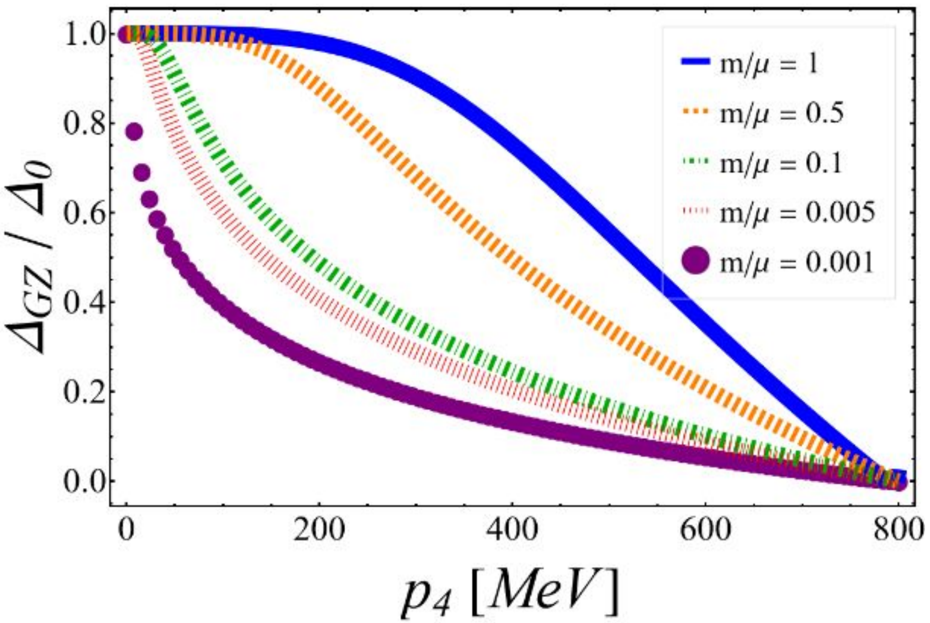
Introducing a logarithmic scale ***x*** and differentiating, we find:

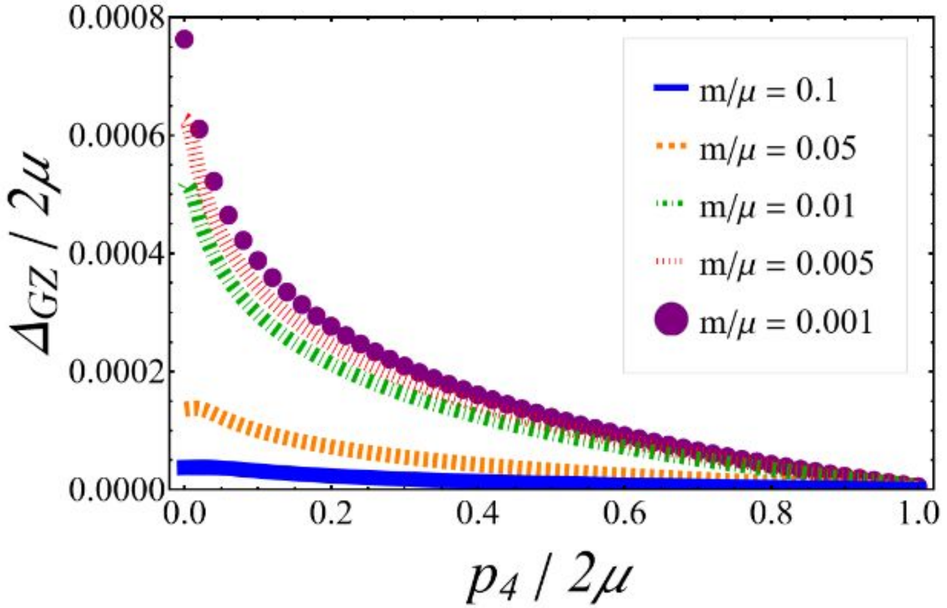
$$\Delta''(x) = -\frac{g^2}{512\pi^2}\left\{1+\left(\frac{m_\phi^4}{(2\mu)^4 e^{-x} - m_\phi^4}\right)\right\}\Delta(x)$$

approximate gap equation analogous to **(Son, 1999)**.

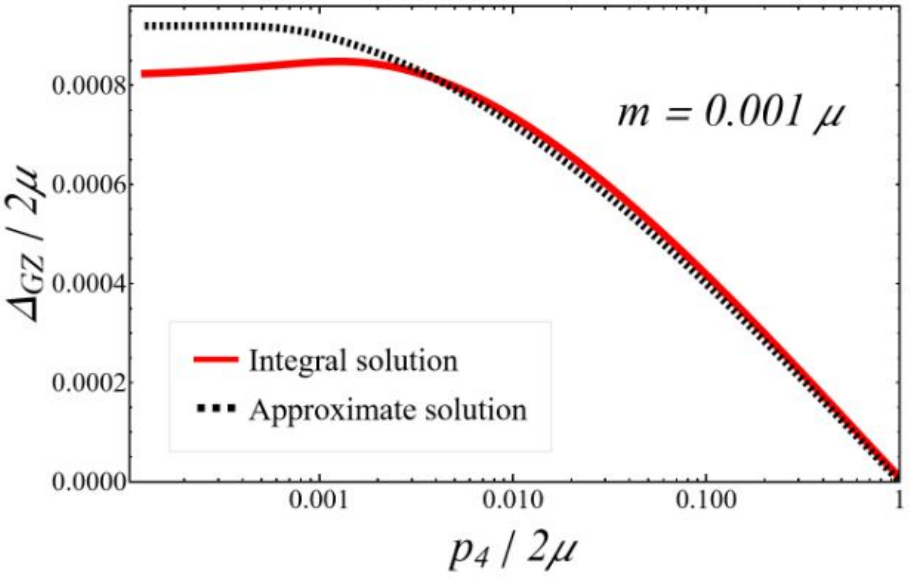
Analysis

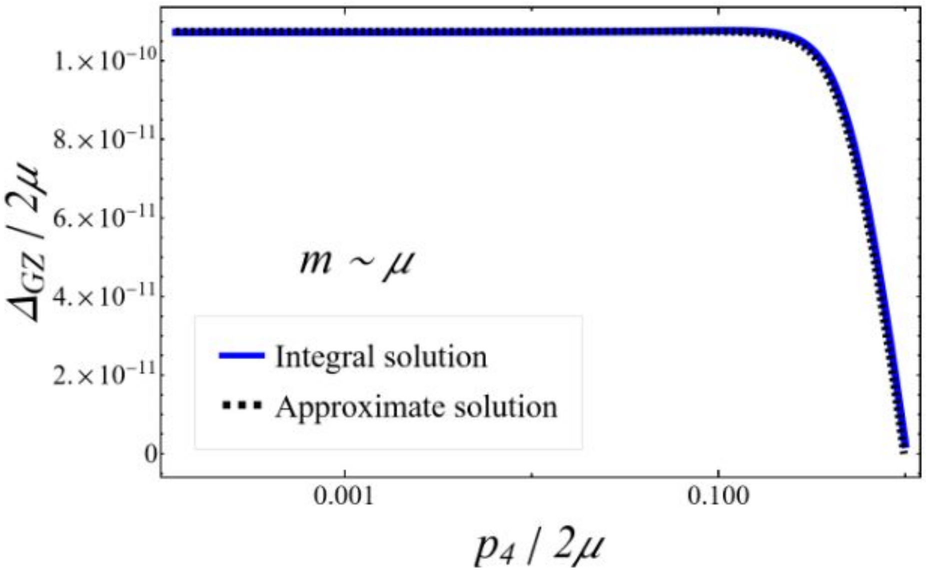
ANALYSIS - GZ type



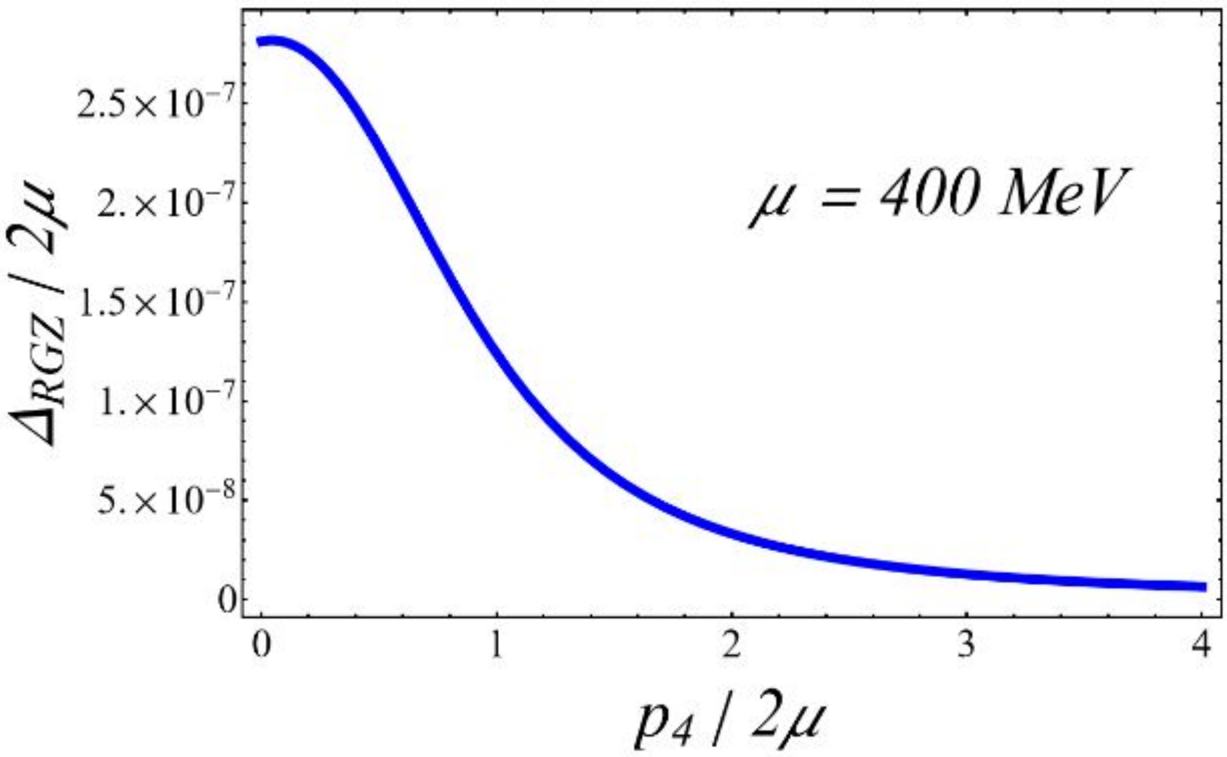


ANALYSIS - GZ type

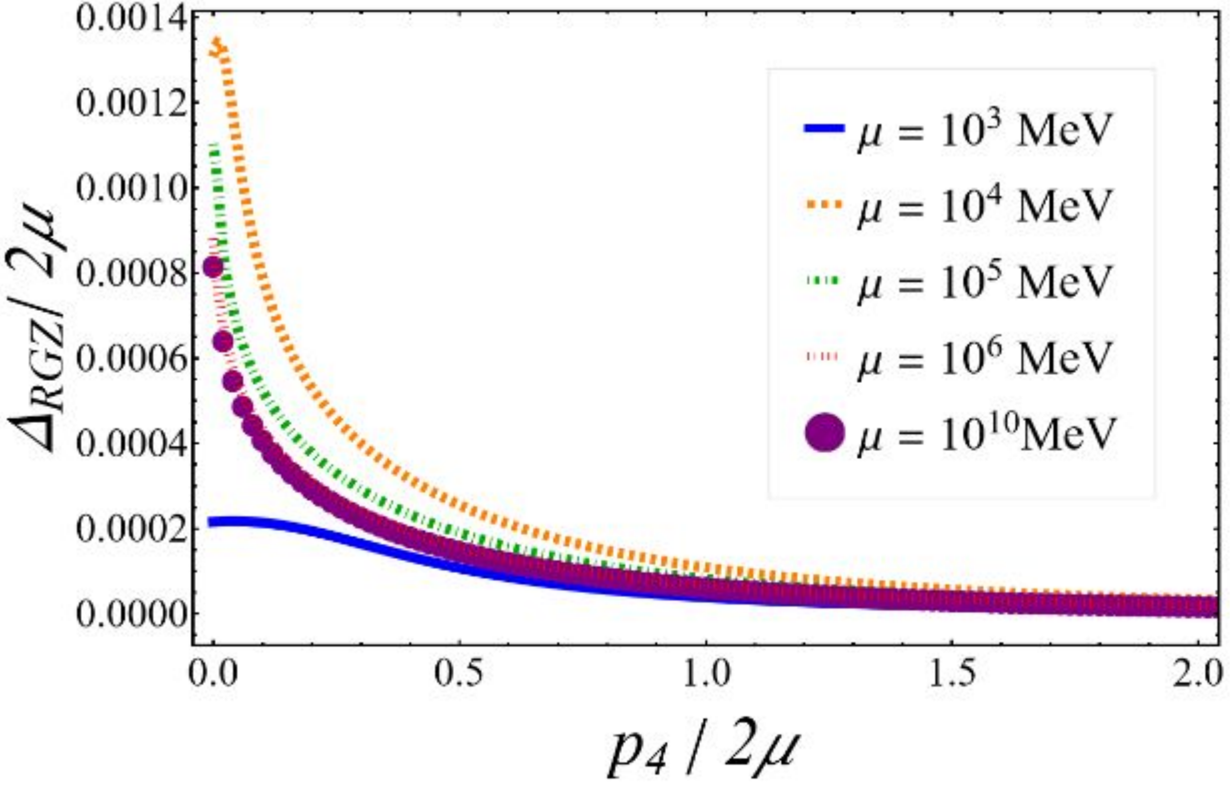




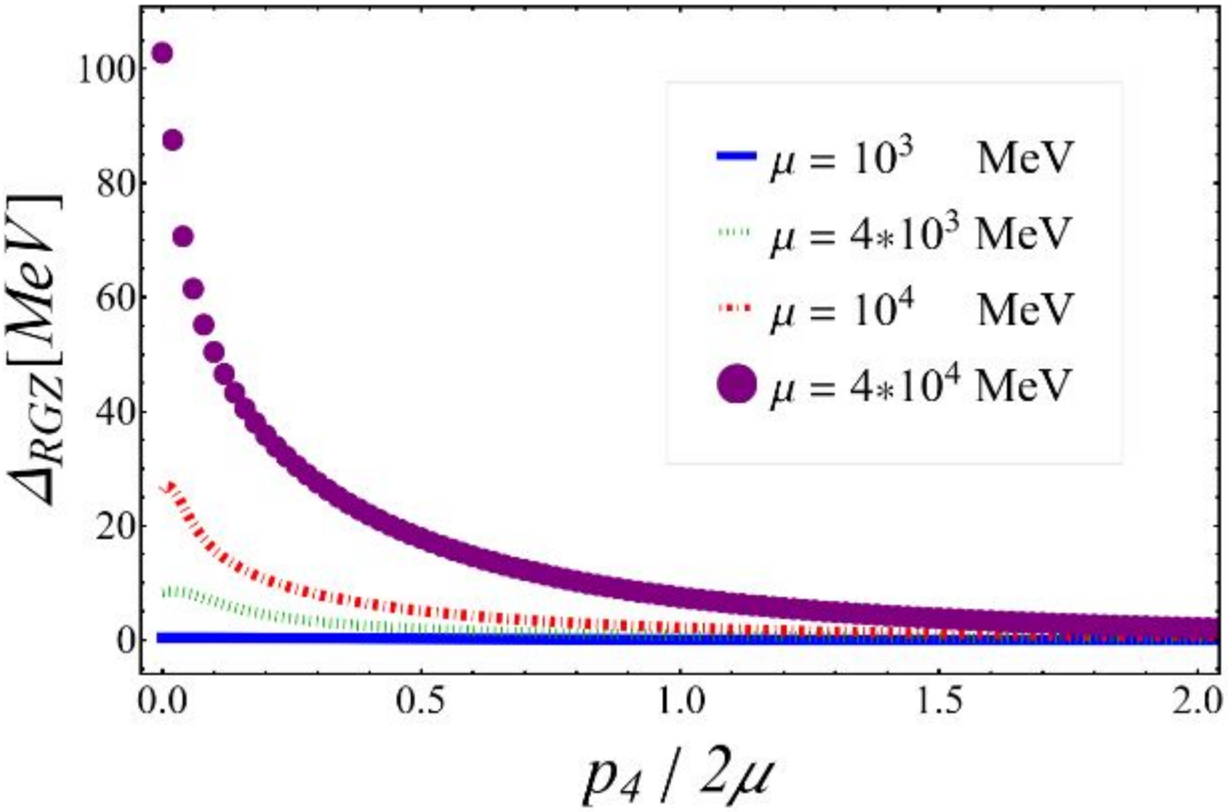
ANALYSIS - RGZ type



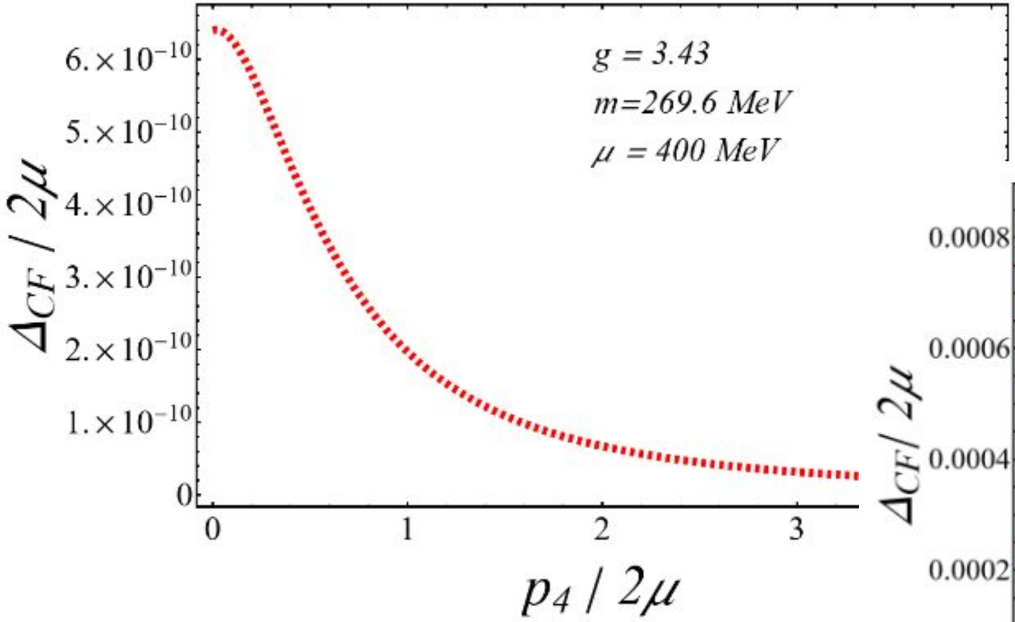
ANALYSIS - RGZ type

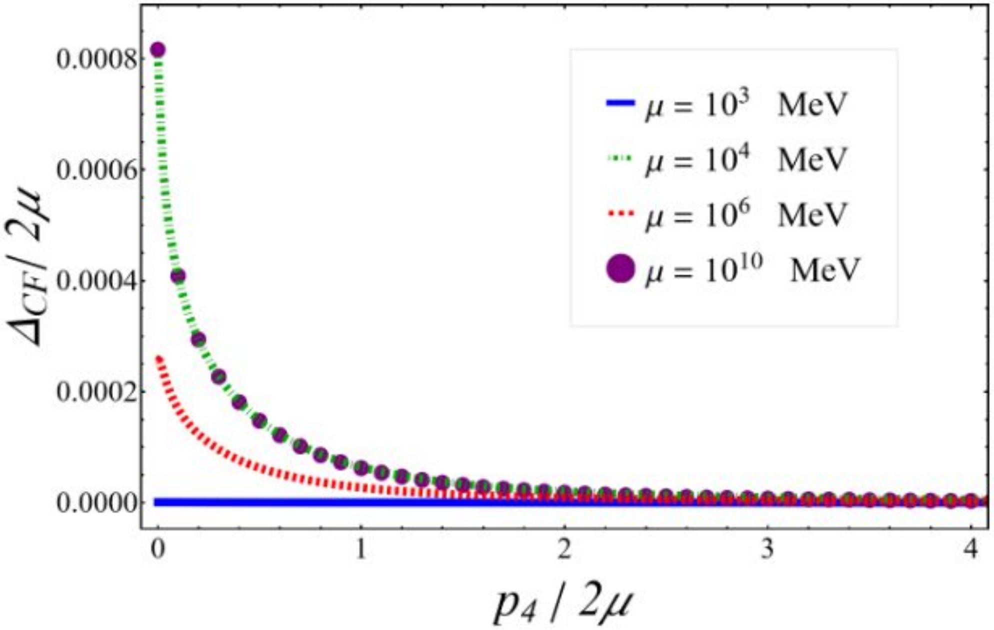


ANALYSIS - RGZ type

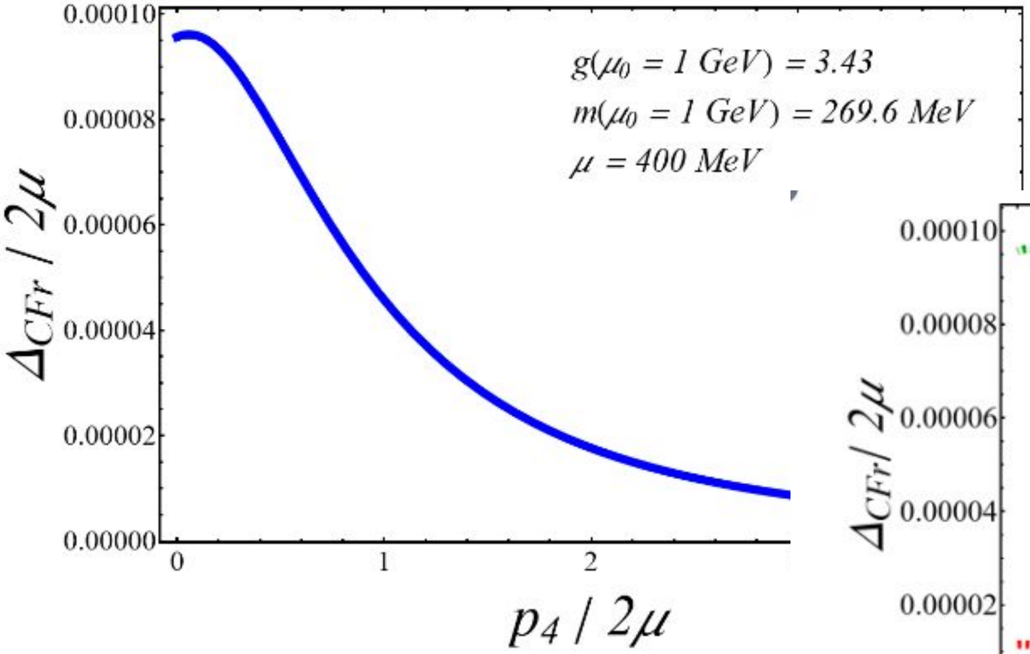


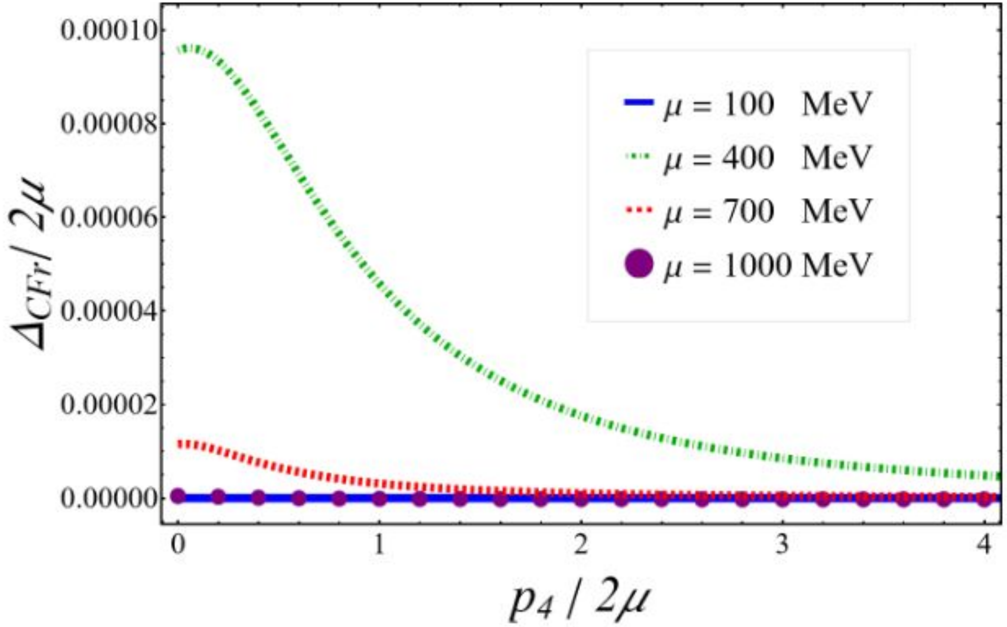
ANALYSIS - CF type



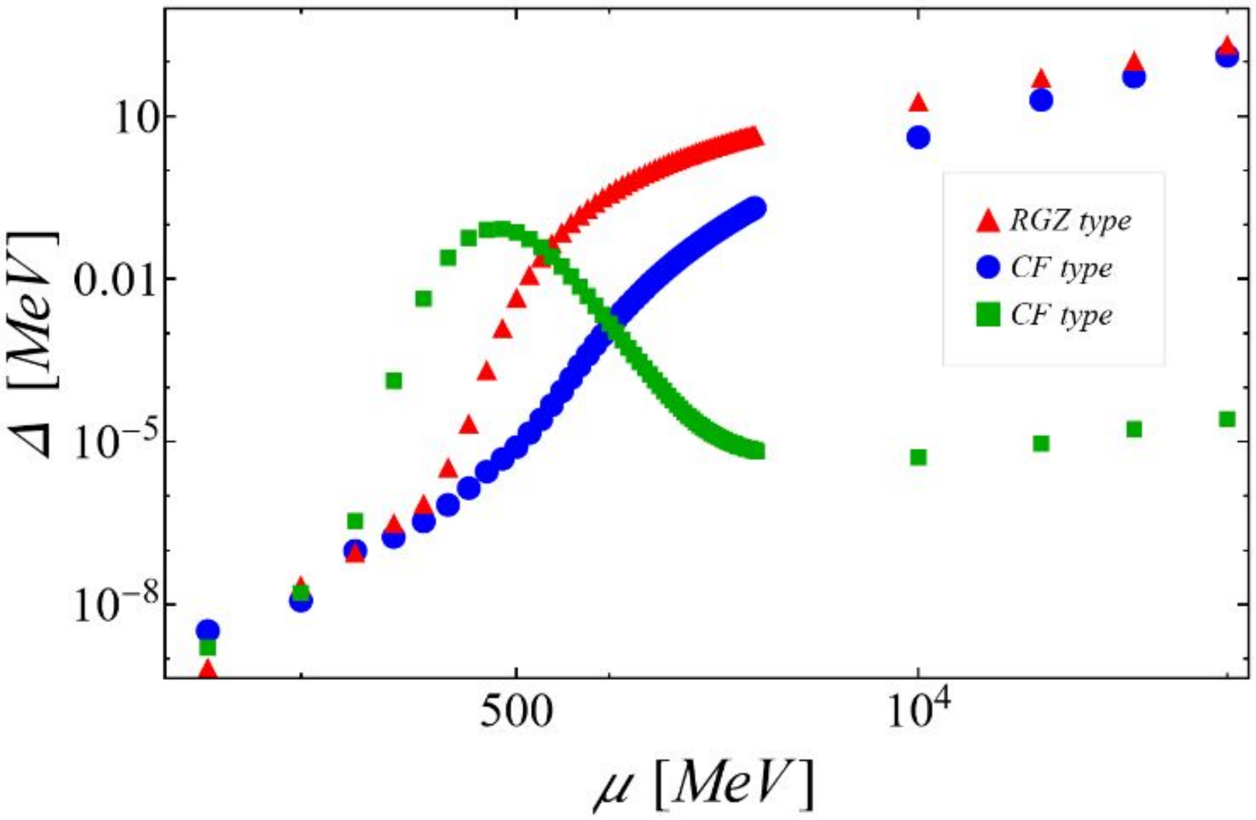


ANALYSIS - CFr type





ANALYSIS - SUC Model



Remarks

- The ***color superconductivity is sensitive to nonperturbative gluon mass.*** This behavior remains valid across both moderate and very high chemical potentials.

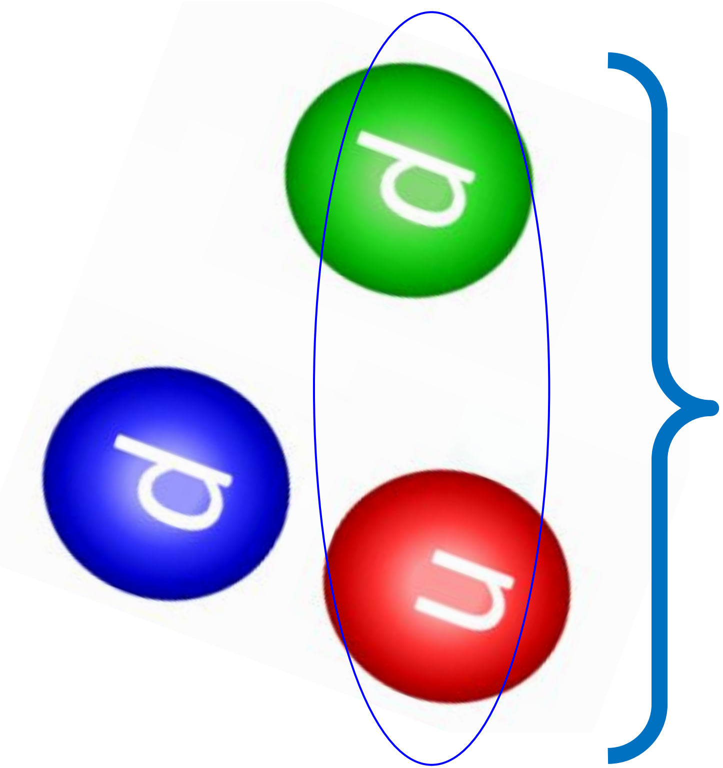
- The ***CFr*** gap is larger than in the ***RGZ*** and ***CF*** cases due to the ***strong running coupling effects in the infrared (IR) limit***.

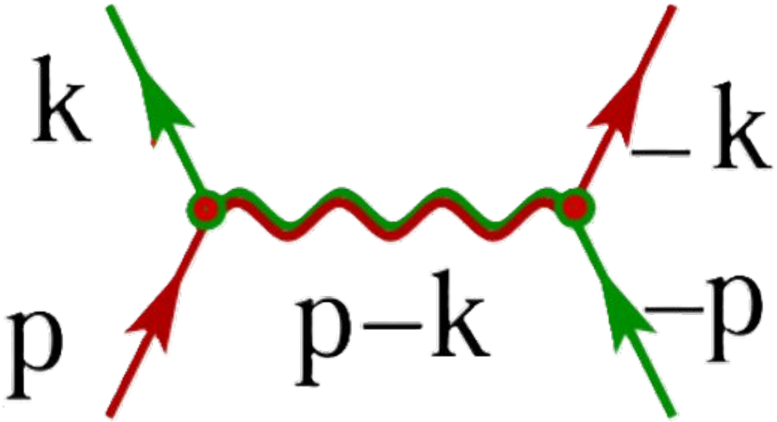
- ***For large chemical potentials***, we observe that the gap behaves as $\Delta \propto \mu$.

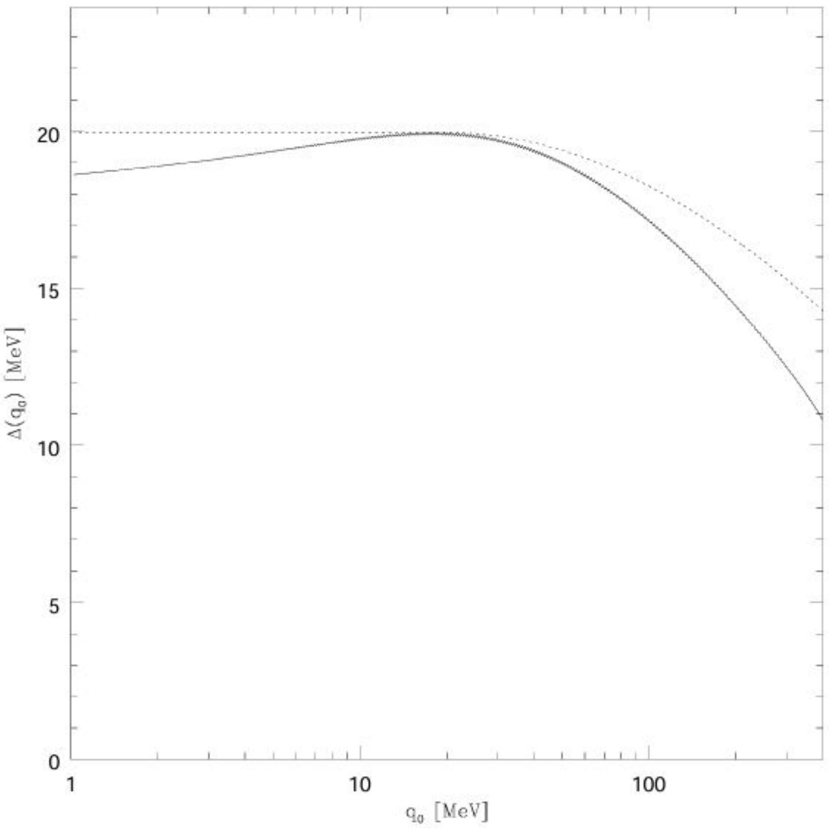
- The ***presence of complex poles does not lead to unphysical results!*** As a natural next step, we propose investigating an ***Abelian gauge extension of the QCD-inspired model*** **[Santos, J.P.S., Palhares, L.F.P., and Dudal, D., in preparation]**.

Thank you !

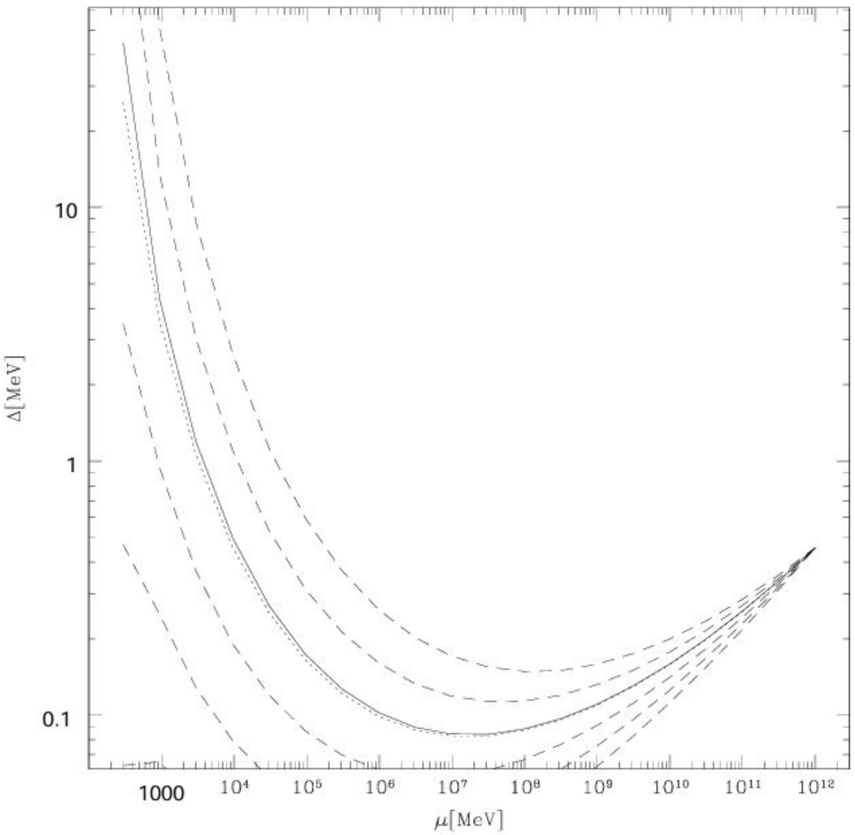
Introduction - QCD & Confinement



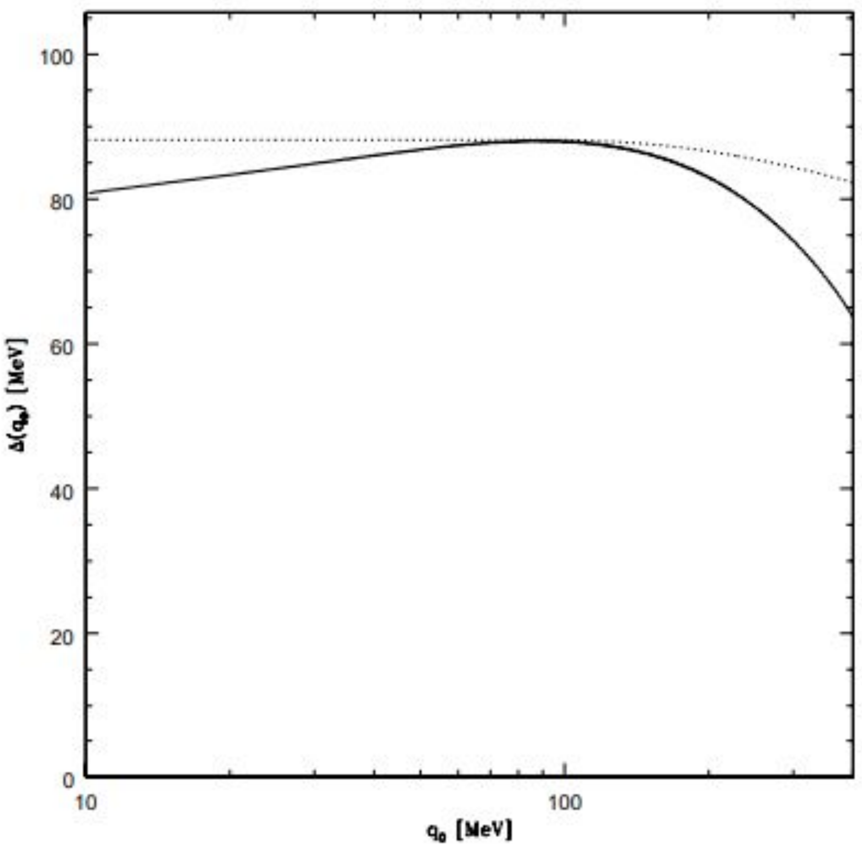




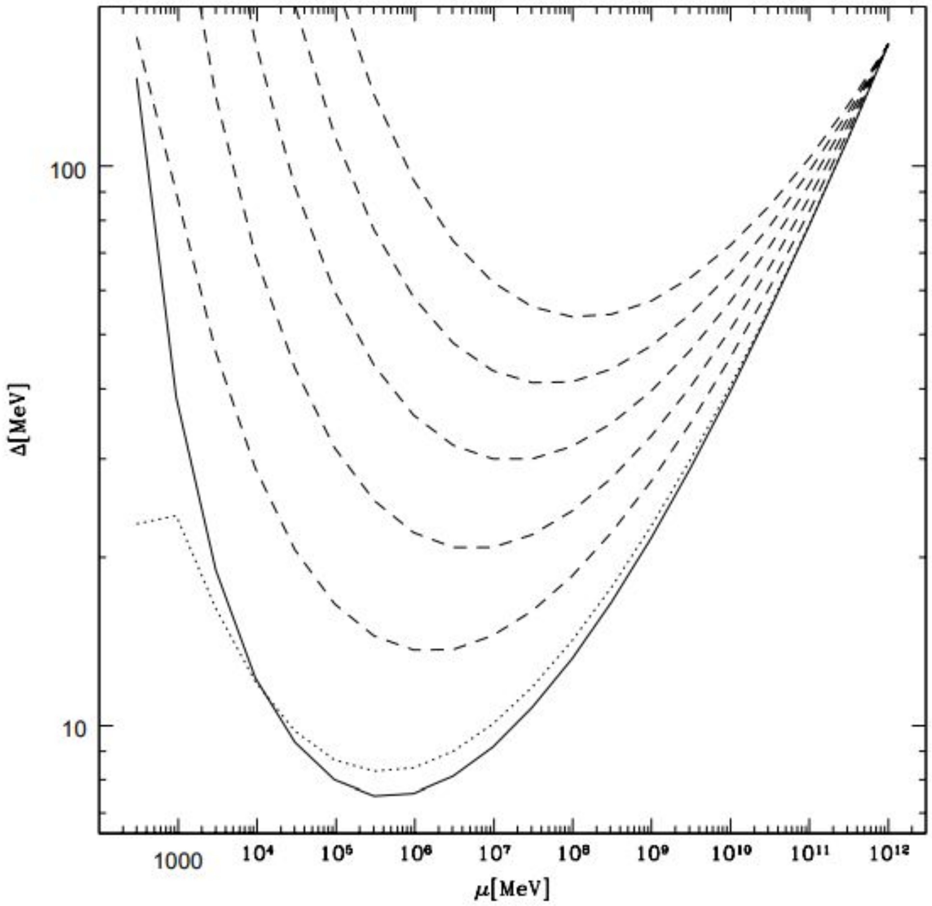
20
15
10
5
0
1
10
100
Δ(q₀) [MeV]
q₀ [MeV]



10
1
0.1
1000
10⁴
10⁵
10⁶
10⁷
10⁸
10⁹
10¹⁰
10¹¹
10¹²
Δ[MeV]
μ[MeV]



100
80
60
40
20
0
10
100
Δ(q₀) [MeV]
q₀ [MeV]

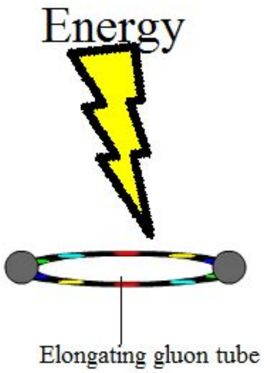


100
10
1000
10⁴
10⁵
10⁶
10⁷
10⁸
10⁹
10¹⁰
10¹¹
10¹²
Δ[MeV]
μ[MeV]

BACKUP SLIDES

- **Asymptotic Freedom and Confinement**

  Perturbation theory is successful at high energies, where the strong coupling becomes small. However, at low energies, the coupling grows large, and **perturbative methods break down**.



BACKUP SLIDES

The **Yang-Mills Theory** is a non-abelian gauge theory that describes the gluon sector in QCD.

$$S_{YM} = \int d^4x \, \mathrm{Tr} F_{\mu\nu} F_{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \quad , \quad A_\mu^U = U A_\mu U^\dagger - \frac{i}{g}(\partial_\mu U) U^\dagger$$

$$U = e^{-ig\omega^a T^a} \Rightarrow [T^a, T^b] = i f_{abc} T^c, \quad f^{abc} \in \Re$$

BACKUP SLIDES

To quantize the gauge fields, the functional integral should select each configurations of equivalent fields just one time! To make this procedure we should use the **Faddeev-Popov procedure**

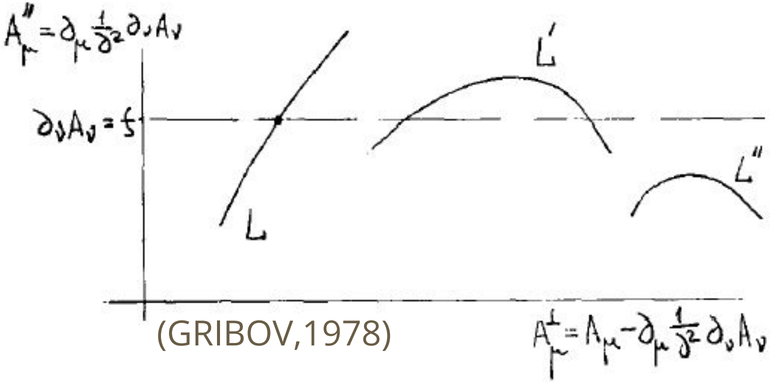
$$\int [\mathcal{D}U] \Delta F \delta(F(A^U)) = 1$$

with

$$\delta(F(A^U)) = \Pi_x \Pi_a \delta(F^a(A^U_\mu(x))) \,, \qquad [\mathcal{D}U] \approx \Pi_x \Pi_a d\omega^a(x)$$

$$\Delta F = |\det M_{ab}(x,y)| \,, \qquad M_{ab}(x,y) = \left.\frac{\delta F^a(A^U_\mu(x))}{\delta \omega^b(y)}\right|_{F(A^U)=0}$$

BACKUP SLIDES



(GRIBOV,1978)

However, using the Landau gauge $\partial_\mu A_\mu = 0$ making infinitesimals transformations

$$\partial_\mu A^U_\mu = 0 \quad \Rightarrow \quad \boxed{-\partial_\mu(\partial_\mu \alpha + ig[\alpha, A_\mu]) = -\partial_\mu D_\mu \alpha = 0}$$

Gribov (GRIBOV, 1978) realized that there is a non trivial solution and, based on the **gauge orbit** concept, more than one field configuration is being counted!

Backup Slides

Teorias de calibre que apresentam **modos zero** não podem ser simplesmente quantizadas pela formulação das integrais de trajetória.

$$S_{YM} = \int d^4x F^a_{\mu\nu} F^a_{\mu\nu} = \int d^4x d^4y A^a_\mu (\partial^2 g_{\mu\nu} - \partial_\mu \partial_\nu) \delta(x-y) A^a_\nu$$

Dessa forma, para $A^a_\mu = k^a_\mu \Gamma$, o funcional fica mal definido!

$$(-k^2 g_{\mu\nu} + k_\mu k_\nu) k^\mu \Gamma = (-k^2 k_\nu + k_\mu k^\mu k_\nu) \Gamma = 0$$

BACKUP SLIDES

- In gauge theories, given a gauge transformation, gauge fields present the possibility of a field configuration being counted more than once.
- To avoid this problem Gribov (GRIBOV, 1978) restricted the limit of integration of the fields in an integration region that would improve the problems of fixing the gauge, making the Faddeev-Popov operator only present positive eigenvalues. This region became known as the **Gribov region**.

BACKUP SLIDES

Thus, Gribov, in the landau gauge, supposes a restriction, limiting the integration only in the **Gribov region** Ω

$$Z = \int [\mathcal{D}A] V(\Omega) \delta(\partial A) e^{-S_{YM}} \det[\partial_\mu D_\mu^{ab}]$$

$$\Downarrow$$

$$Z = \int [\mathcal{D}A] \theta(1 - \sigma(0; A)) \delta(\partial A) e^{-S_{YM}} \det[\partial_\mu D_\mu^{ab}]$$

BACKUP SLIDES

$$
\begin{aligned}
\left\langle \bar{c}^a(x) c^b(y) \right\rangle_c &= \mathcal{N} \int [\mathcal{D}A][\mathcal{D}\bar{c}][\mathcal{D}c] V(\Omega) \delta(\partial A) \bar{c}^a(x) c^b(y) \exp\left[-S_{YM} - \int dx \bar{c}^a \partial_\mu D_\mu^{ab} c^b\right] \\
&= \mathcal{N} \int [\mathcal{D}A] V(\Omega) \delta(\partial A) e^{-S_{YM}} \det[\partial_\mu D_\mu^{ab}] [(\partial_\mu D_\mu^{ab})^{-1}]^{ab}(x,y)
\end{aligned}
$$

$$
M_{ab} = -\partial_\mu D_\mu^{ab} = -\partial_\mu(\partial_\mu \delta^{ab} - f^{abc} A_\mu^c)
$$

BACKUP SLIDES

$$\left\langle \bar{c}^a(x) c^b(y) \right\rangle_c = \delta^{ab} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{\left(1 - \frac{11 g^2 N}{48\pi^2} \log \frac{\Gamma^2}{k^2}\right)^{9/44}} e^{ik(x-y)}$$

Backup Slides

Relação entre o **Operador de F-P** e o **propagador do *ghost***

$$\left\langle \bar{c}^a(x) c^b(y) \right\rangle_c = \delta^{ab} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{\left(1 - \frac{11 g^2 N}{48\pi^2} \log \frac{\Gamma^2}{k^2}\right)^{9/44}} e^{ik(x-y)}$$

- Pólos : $k_1^2 = 0$ e $k_2^2 = \Gamma^2 \exp\left[-\frac{1}{g^2} \frac{48\pi^2}{11} N\right]$
- Singularidade em $k_1^2 = 0$
- Para altos valores de k^2 na região de Gribov se obtém os valores da T.P.

BACKUP SLIDES

Como o fator de forma é decrescente

$$\sigma(k, A) \leq \sigma(0, A) < 1$$

o operador de FP se restringe a somente autovalores positivos! Substituindo no funcional, com algumas aproximações, pode-se introduzir a **massa de Gribov** $\gamma^4 = \frac{\beta_0 N}{N^2 - 1} \frac{2}{dV} g^2$ e com isso o **propagador de Gribov-Zwanziger**, (VANDERSICKEL, 2011), fica:

$$\left\langle A_\mu^a(p) A_\nu^b(k) \right\rangle = \delta(p+k)\delta^{ab} \frac{k^2}{k^4 + \gamma^4} P_{\mu\nu}(k) \Rightarrow \frac{k^2}{k^4 + \gamma^4} = \frac{1}{2}\left(\frac{1}{k^2 + i\gamma^2} + \frac{1}{k^2 - i\gamma^2}\right)$$

BACKUP SLIDES

Complete propagator of the theory

$$\mathbf{S}(k) = \begin{pmatrix} G^+ & F^- \\ F^+ & G^- \end{pmatrix} = -\begin{pmatrix} \left\langle \psi(x)\overline{\psi}(y) \right\rangle & \left\langle \psi(x)\bar{\psi}_c(y) \right\rangle \\ \left\langle \psi_c(x)\overline{\psi}(y) \right\rangle & \left\langle \psi_c(x)\overline{\psi}_c(y) \right\rangle \end{pmatrix}$$

Where

$$G^{\pm} \equiv \left([G_0^{\pm}]^{-1} - \Phi^{\mp} G_0^{\mp} \Phi^{\pm} \right)^{-1}$$

$$F^{\pm} \equiv -G_0^{\mp} \Phi^{\pm} G^{\pm}$$

$$\tau = \frac{m_\phi}{2\mu}$$

BACKUP SLIDES

Differents models and gap equations:

1. PL

(Point-like approx.)

$$\Delta = G \int \frac{d^3k}{(2\pi)^3} \frac{\Delta}{2\epsilon_k} \tanh \frac{\epsilon_k}{2T} \quad \Rightarrow \quad \begin{cases} \Delta = \Delta_0 = Cte \\ \Delta_{PL} = 1 \end{cases}$$

2. CER

(Real Scalar Field)

$$\Delta''_{CER}(x) = -k^2_{CER} \left\{ 1 + \left(\frac{\tau^2}{e^{-x} - \tau^2} \right) \right\} \Delta_{CER}(x) \quad \Rightarrow \quad \begin{cases} \Delta(0) = 0 \text{ and } \Delta(x_0) = \Delta_0 \\ k^2_{CER} = \frac{g^2}{128\pi^2} \end{cases}$$

3. GZ

(Gribov-Zwanziger)

$$\Delta''_{GZ}(x) = -k^2_{GZ} \left\{ 1 + \left(\frac{\tau^4}{e^{-x} - \tau^4} \right) \right\} \Delta_{GZ}(x) \quad \Rightarrow \quad \begin{cases} \Delta(0) = 0 \text{ and } \Delta(x_0) = \Delta_0 \\ k^2_{GZ} = \frac{g^2}{512\pi^2} \end{cases}$$

4. QCD

(Color Superconductivity)

$$\Delta''(x) = -k^2_{QCD} \Delta(x) \quad \Rightarrow \quad \begin{cases} \Delta(0) = 0 \text{ e } \Delta'(x_0) = 0 \\ k^2_{QCD} = \frac{g^2}{18\pi^2} \end{cases}$$

BACKUP SLIDES

Differents models and gap equations:

1. ***PL***

(Point-like approx.)

$$\Delta = G \int \frac{d^3k}{(2\pi)^3} \frac{\Delta}{2\epsilon_k} \tanh \frac{\epsilon_k}{2T}$$

2. **CER**

(Real Scalar Field)

$$\Delta(p_4) = \frac{g^2}{64\pi^2} \int \frac{dk_4 \Delta(k_4)}{\sqrt{k_4^2 + \Delta^2(k_4)}} \times \left\{ \log \left[1 + \frac{4\mu^2}{(p_4 - k_4)^2 + m_\phi^2} \right] \right\}$$

3. **GZ**

(Gribov-Zwanziger)

$$\Delta(p_4) = \frac{g^2}{128\pi^2} \int \frac{dk_4 \Delta(k_4)}{\sqrt{k_4^2 + \Delta^2(k_4)}} \times \left\{ \log \left[\frac{(2\mu)^4}{(p_4 - k_4)^4 + m_\phi^4} \right] \right\}$$

4. **QCD**

(Color Superconductivity)

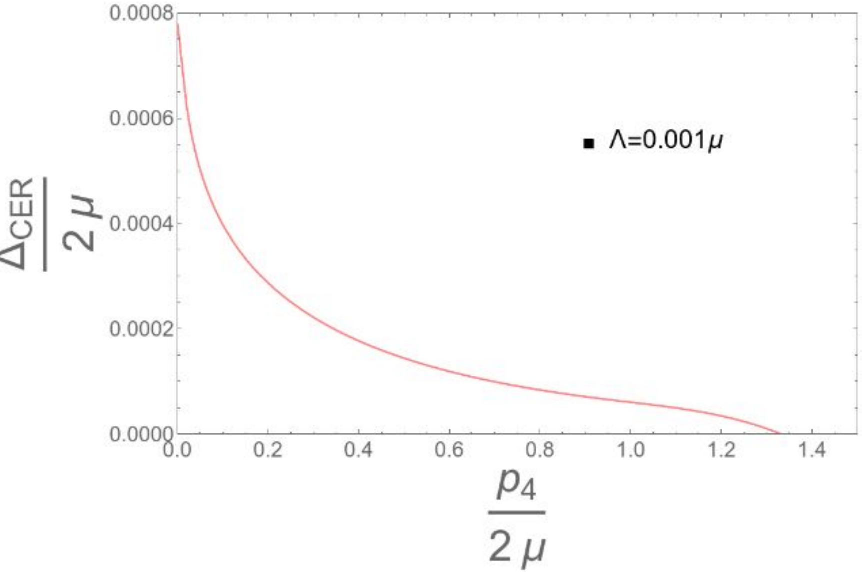
$$\Delta(p_4) = \frac{g^2}{18\pi^2} \int \frac{dk_4 \Delta(k_4)}{\sqrt{k_4^2 + \Delta^2(k_4)}} \times \left\{ \log \left[\frac{\mu}{|p_4 - k_4|} \right] \right\}$$

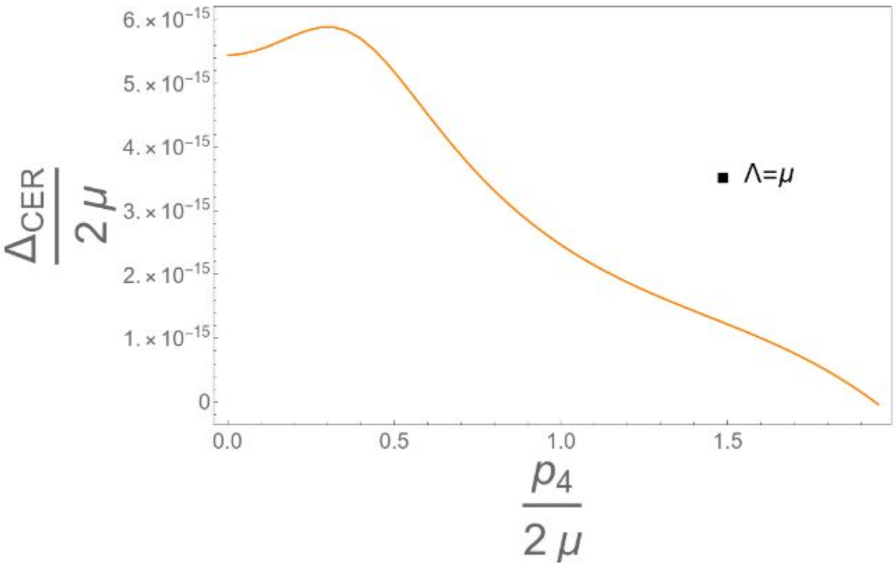
BACKUP SLIDES

$$\Delta(p_4) = \frac{g^2}{64\pi^2}\int \frac{dk_4\Delta(k_4)}{\sqrt{k_4^2+\Delta^2(k_4)}}\left\{\log\left[1+\frac{(2\mu)^2}{(p_4-k_4)^2+m(k_4)^2}\right]\right\}$$

$Data:$
$g = 3.43$
$\mu = 400MeV$

$m(q_4) = \Lambda\exp\left\{-\left(\frac{q_4}{\Lambda}\right)^2\right\}$





BACKUP SLIDES

The **bosonic propagator of the theory can be replaced by a propagator of a confining theory,** such as the Gribov propagator.

Following some **approximations at the Fermi surface**:

$\Delta \cong \Delta(p_4)$ and $\vec{k} = \vec{k}_F + \vec{l}$ where, $|\vec{k}| \approx \mu + O(l)$

Therefore,

$$I_{\pm} = \frac{g^2\mu^2}{4} \int \frac{dk_4}{(2\pi)^4} \int_{-1}^{1} d(\cos\theta) \int_0^{\infty} dl \int_0^{2\pi} d\phi \left\{ \frac{1}{[q_4^2 + 2\mu^2(1-\cos\theta) \mp i m_\phi^2]} \frac{\Delta(k_4)}{[k_4^2 + l^2 + \Delta^2(k_4)]} \right\}$$

BACKUP SLIDES

The bosonic propagator of the theory can be replaced by a propagator of a confining theory, such as the Gribov propagator. Therefore

$$\Delta(p) = -\frac{g^2}{2}\frac{T}{V}\frac{1}{2}\sum_k \frac{1}{(p_0-k_0)^2-(\vec{p}-\vec{k})^2-im_\phi^2}\frac{\Delta(k)}{k_0^2-\epsilon_k^2}$$
$$-\frac{g^2}{2}\frac{T}{V}\frac{1}{2}\sum_k \frac{1}{(p_0-k_0)^2-(\vec{p}-\vec{k})^2+im_\phi^2}\frac{\Delta(k)}{k_0^2-\epsilon_k^2}\,.$$

We will work with an integral in Euclidean space

$$T\sum_{k_0=i\omega_n} \to \int\frac{dk_0}{(2\pi)} \equiv \int\frac{idk_4}{(2\pi i)}$$

BACKUP SLIDES

Introducing **logarithmic scales** for each region, then

$$x = \log\left(\frac{(2\mu)^4}{p_4^4 + m_\phi^4}\right) \quad y = \log\left(\frac{(2\mu)^4}{k_4^4 + m_\phi^4}\right) \quad y\Big|_{k_4=0} \equiv x_0 = \log\left(\frac{(2\mu)^4}{\lambda^4 + m_\phi^4}\right)$$

Considering $k_4 = [(2\mu)^4 e^{-y} - m_\phi^4]^{1/4}$ and $p_4 = [(2\mu)^4 e^{-x} - m_\phi^4]^{1/4}$, then

$$\Delta(x) = \frac{C}{4}\left\{\left[x\int_x^{x_0} dy\Delta(y)\right] + \left[\int_0^x dy\Delta(y)y\right] + \left[x\int_x^{x_0} dy\Delta(y)\left(\frac{m_\phi}{k_4}\right)^4\right] + \left[\int_0^x dy\Delta(y)y\left(\frac{m_\phi}{k_4}\right)^4\right]\right\}$$

BACKUP SLIDES

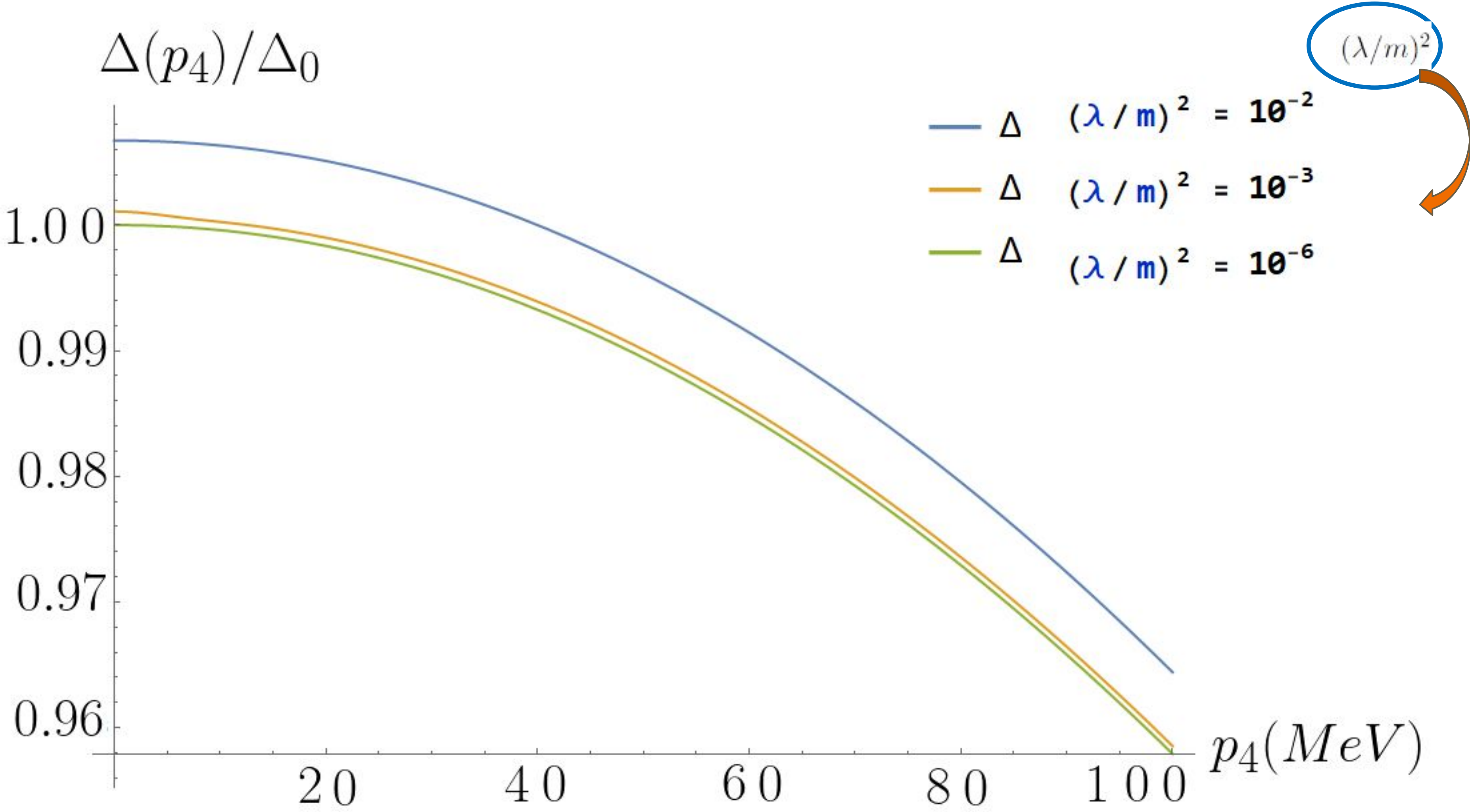
Parameters : $(\lambda/m)^2$ and $\tau = \frac{2\mu}{m}$

- Singularities

$$x_s = 4\log\left[\frac{2\mu}{m}\right] \qquad x_s = 2\log\left[\frac{2\mu}{m}\right]$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$x_0 = \log\left[\frac{\tau^4}{1+\lambda^4/m^4}\right] \qquad x_0 = \log\left[\frac{\tau^2}{1+\lambda^2/m^2}\right]$$

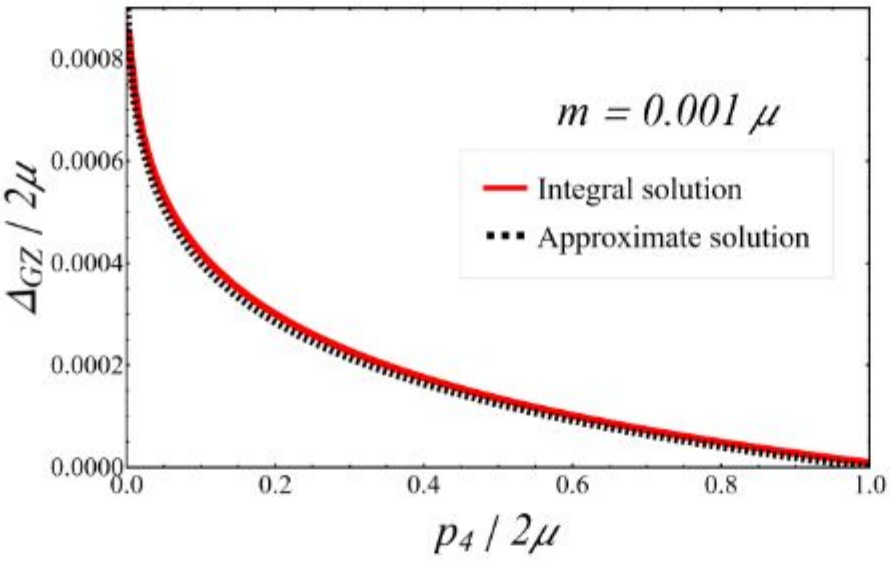


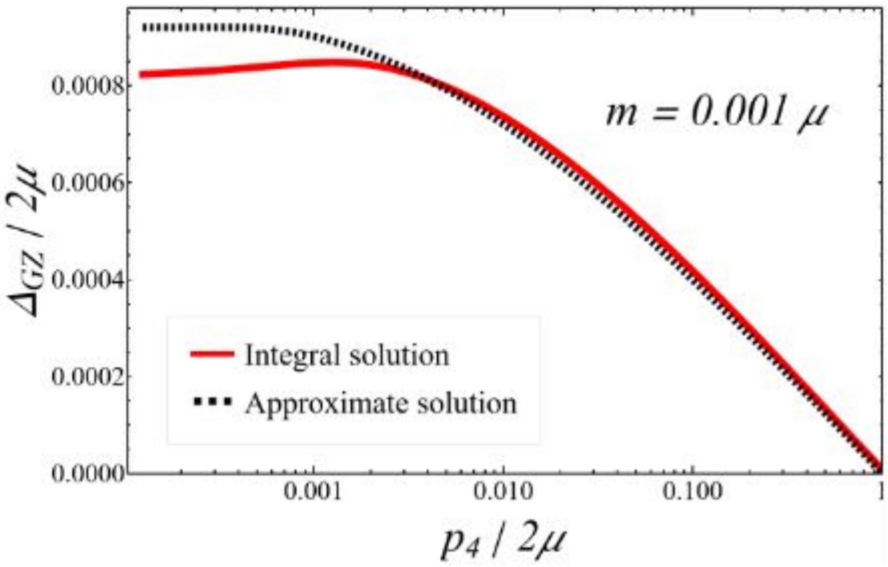
$\Delta(p_4)/\Delta_0$
1.00
0.99
0.98
0.97
0.96
20
40
60
80
100
$p_4(MeV)$
$(\lambda/m)^2$
Δ $(\lambda/m)^2 = 10^{-2}$
Δ $(\lambda/m)^2 = 10^{-3}$
Δ $(\lambda/m)^2 = 10^{-6}$

BACKUP SLIDES

$$\Delta(p_4) = \frac{g^2}{128\pi^2} \int \frac{dk_4 \Delta(k_4)}{\sqrt{k_4^2 + \Delta^2(k_4)}} \times \left\{ \log\left[\frac{(2\mu)^4}{(p_4 - k_4)^4 + m_\phi^4} \right] \right\}$$

$Data:$
$g = 3.43$
$\mu = 400 MeV$

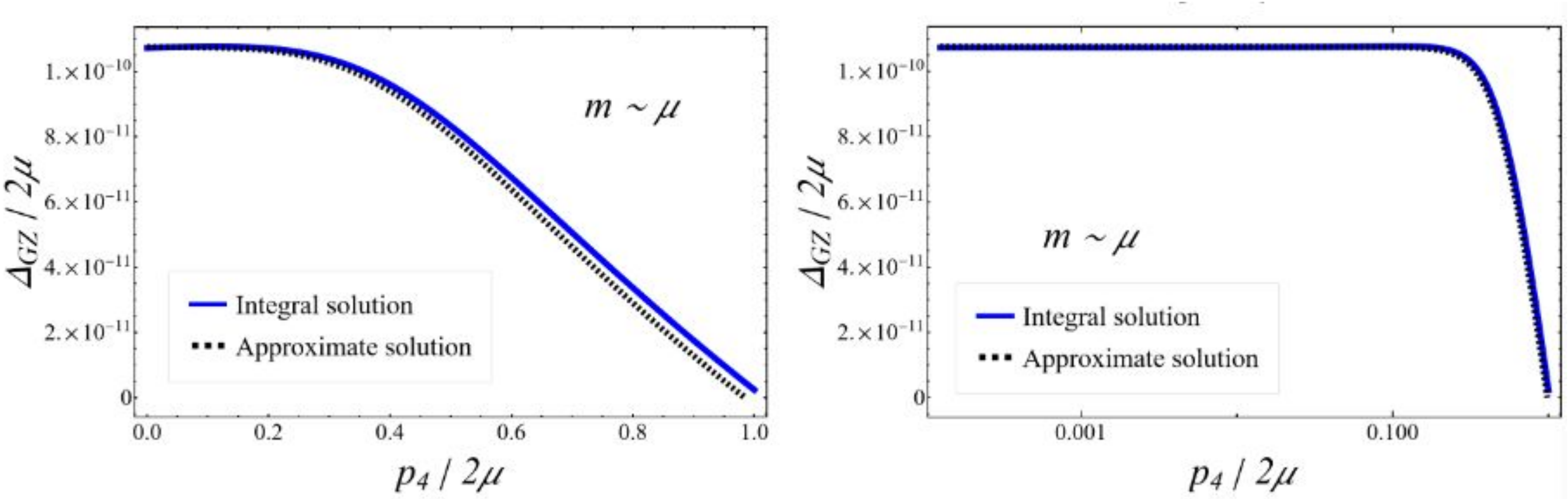




BACKUP SLIDES

$$\Delta(p_4) = \frac{g^2}{128\pi^2}\int \frac{dk_4 \Delta(k_4)}{\sqrt{k_4^2+\Delta^2(k_4)}} \times \left\{ \log\left[\frac{(2\mu)^4}{(p_4-k_4)^4+m_\phi^4}\right]\right\}$$

$Data:$
$g = 3.43$
$\mu = 400MeV$



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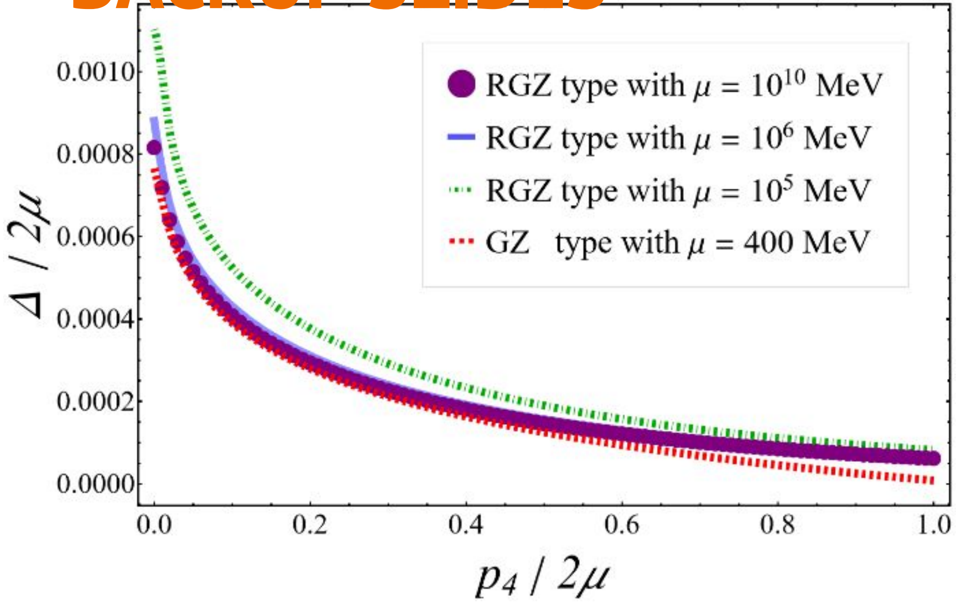


$Data:$

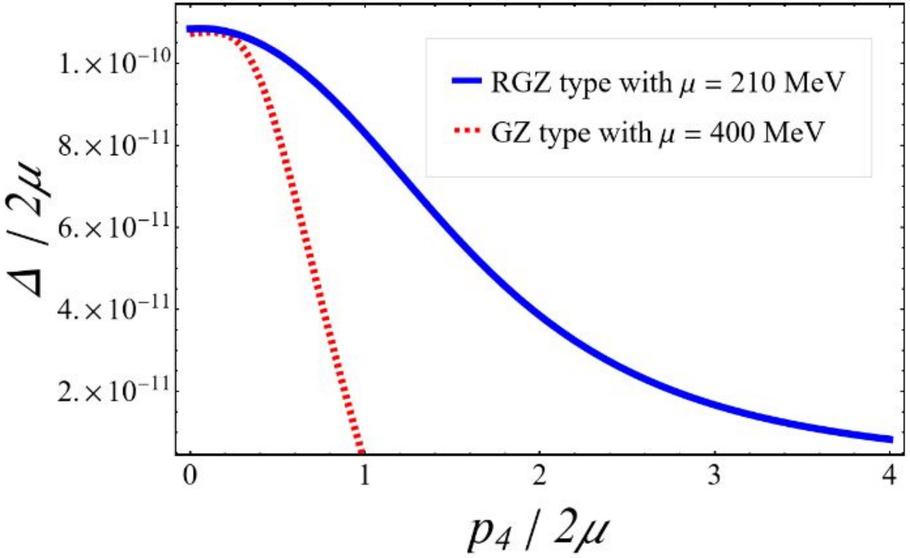
$g = 3.43$

μ

$M_1^2 = 2.525(36) GeV^2$

$M_2^2 = 0.510(11) GeV^2$

$M_3^4 = 0.2803(34) GeV^4$



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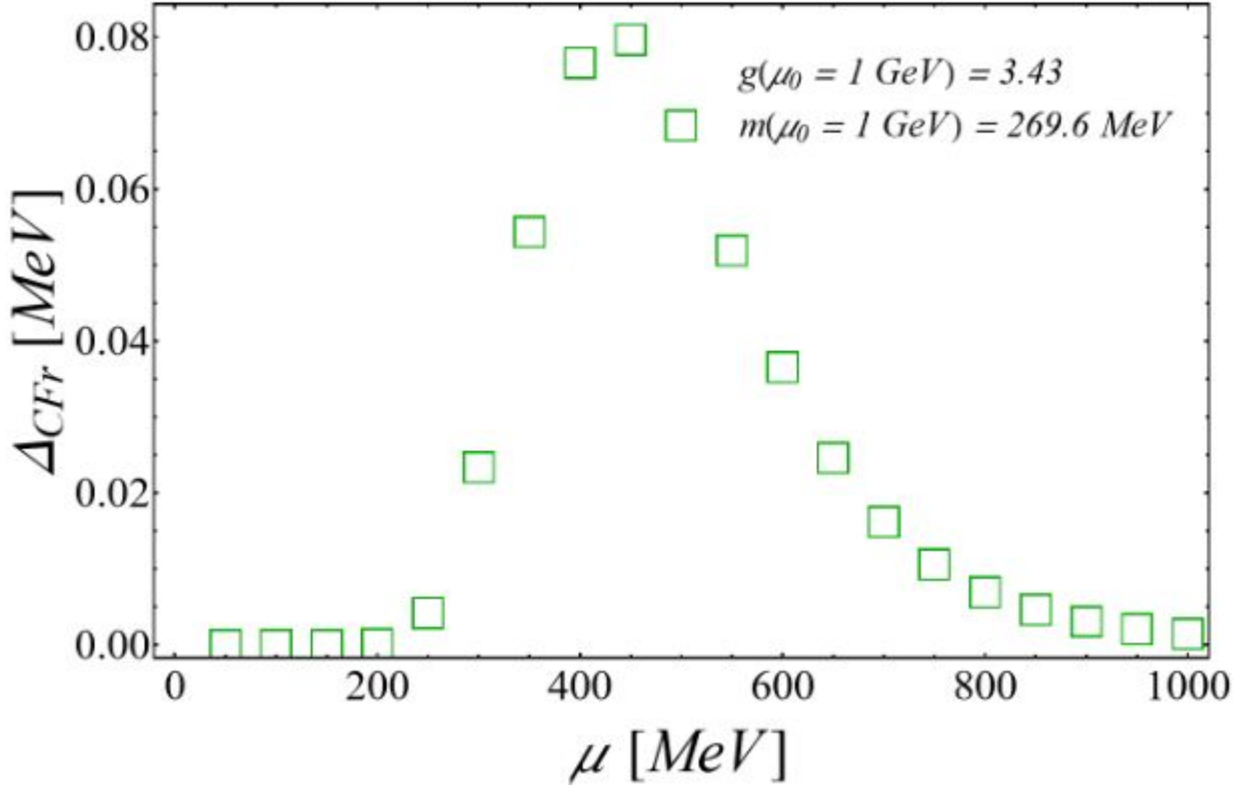

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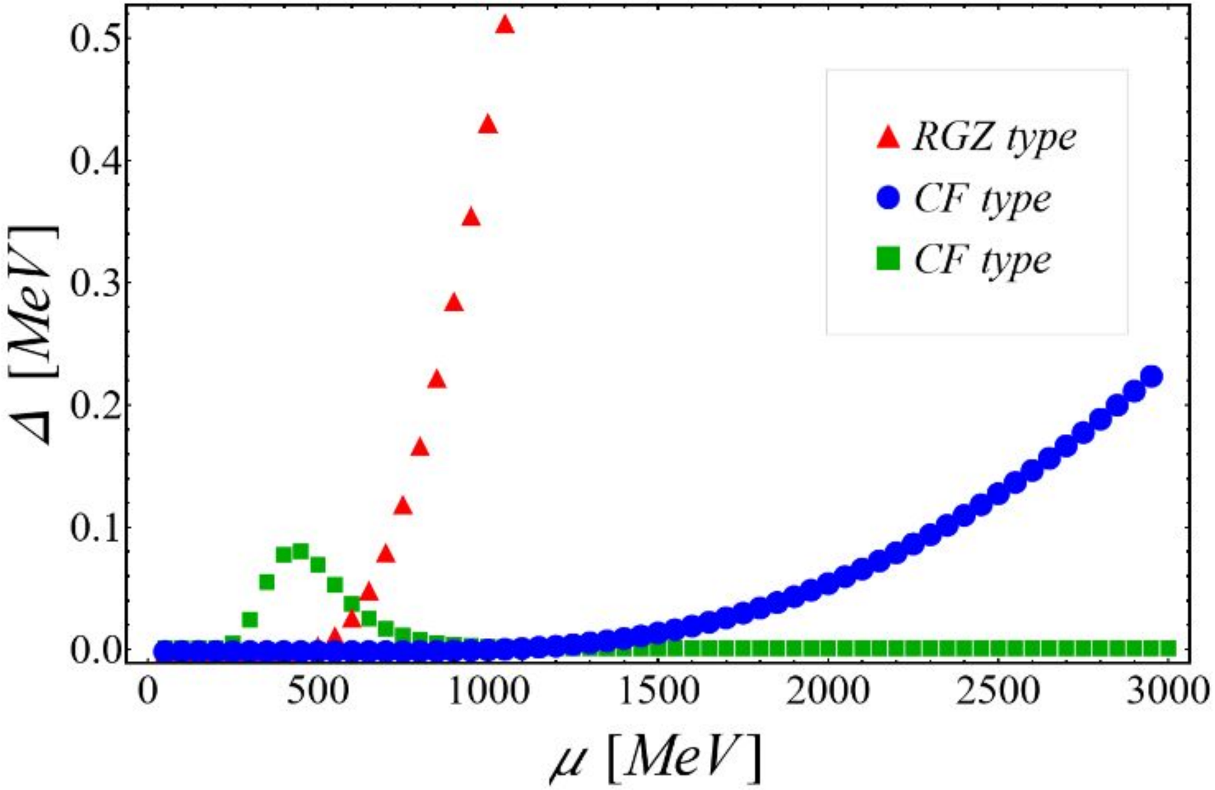

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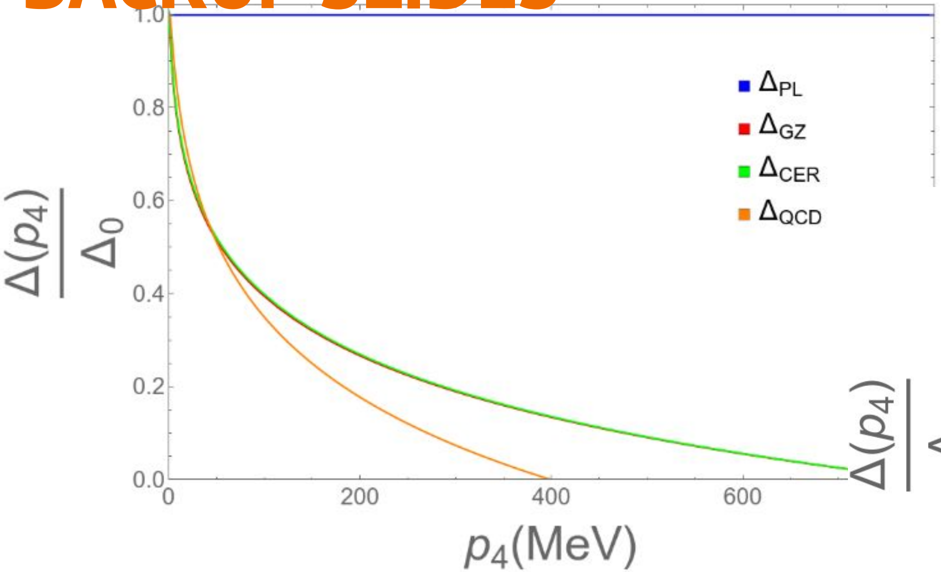
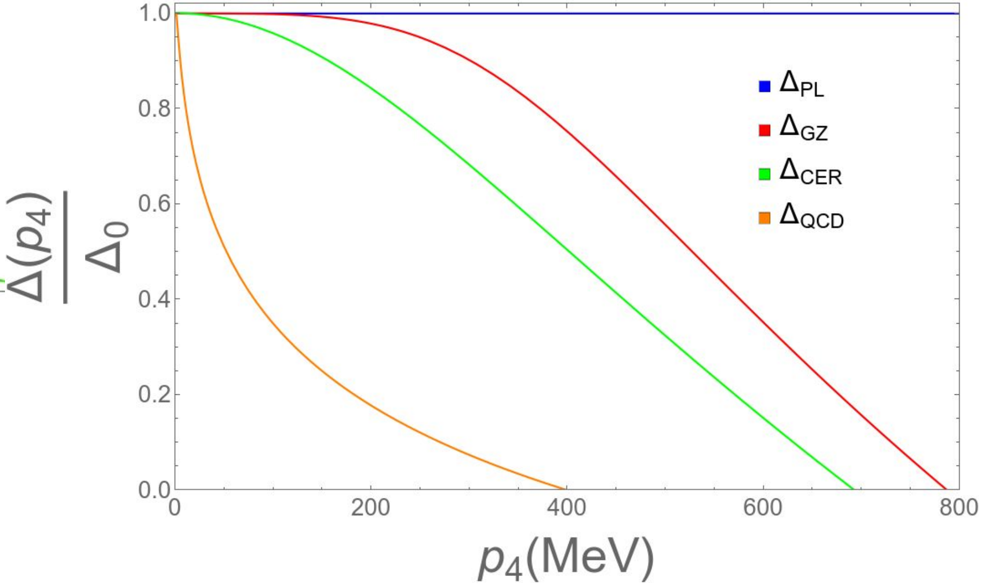


Low mass limit :
(m = 0.001 μ)



High mass limit :
(m = μ)

BACKUP SLIDES

- **Gluon propagator** in a medium

$$D_{\mu\nu}(q) = \frac{P^T_{\mu\nu}}{q^2 - G} + \frac{P^L_{\mu\nu}}{q^2 - F} - \xi \frac{q_\mu q_\nu}{q^4}$$

G, and **F** are functions of the momentum and **P** are the projectors