Quark anomalous magnetic moment in the presence of an extreme magnetic field

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Outline

- Frief introduction to AAM
- Triangle diagram in terms of LL description
- structure of form factors at LLL
- AAM at LLL
- IR regularization
- Numerical results
- Conclusions and outlook

Brief introduction to AMM (QED)

$$\left[-i\not{D} - m\right]\left[i\not{D} - m\right]\psi = \left[D^2 + m^2 - g\frac{e_q}{4}F_{\mu\nu}\sigma^{\mu\nu}\right]\psi = 0$$

$g = 2$

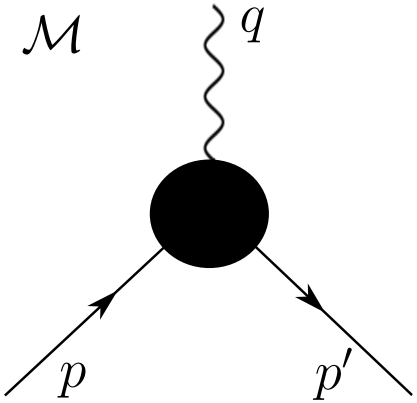
(g-factor)

$$\sigma_{\mu\nu}F^{\mu\nu} = -2\begin{pmatrix}(\boldsymbol{B} + i\boldsymbol{E})\cdot\boldsymbol{\sigma} & 0 \\ 0 & (\boldsymbol{B} - i\boldsymbol{E})\cdot\boldsymbol{\sigma}\end{pmatrix}$$

Electric field here is Lorentz-invariant completion of the magnetic moment

Radiative corrections $\Rightarrow$ $g = 2(1 + a)$ AMM

Vertex



on-shell

$$\bar{u}(p')(\not{p}' - m) = 0, \qquad (\not{p} - m)u(p) = 0$$

Gordon identity

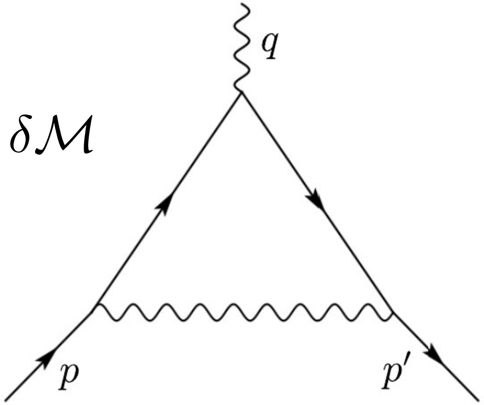
$$\bar{u}(p')(p'^{\mu} + p^{\mu})u(p) = \bar{u}(p')\left[2m\gamma^{\mu} - i\sigma^{\mu\nu}\underbrace{(p'_{\nu} - p_{\nu})}_{q_{\nu}}\right]u(p)$$

EM coupling AMM

$$\mathcal{M}^{\mu} = e_q\overline{u}(p')\left[F_1\left(\frac{q^2}{m^2}\right)\gamma^{\mu} + F_2\left(\frac{q^2}{m^2}\right)\frac{i\sigma^{\mu\nu}}{2m}q_{\nu}\right]u(p)$$

$$g = 2\left[1 + F_2\left(\frac{q^2}{m^2}\right)\right] \qquad a = F_2(0)$$

Vertex correction



$$i\delta\mathcal{M}^\mu = (ie_q)^3 \int_k D_{\mu\nu}(k)\,\overline{u}(p')\gamma^\nu S(k+p')\gamma^\mu S(k+p)\gamma^\rho\, u(p)$$

$$\Rightarrow \quad F_2(q^2) = \frac{\alpha}{\pi} m^2 \int_{xyz} \frac{z(1-z)}{(1-z)^2 m^2 - xyq^2}$$

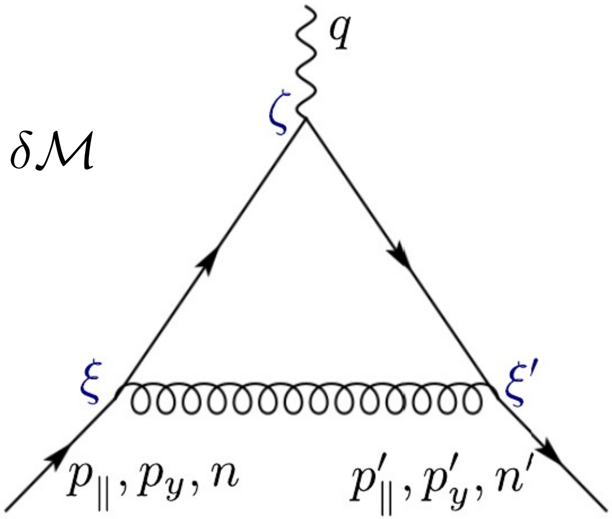
$$a = F_2(0) = \frac{\alpha}{2\pi}$$

$$\int_{xyz} = \int_0^1 dx\ dy\ dz\ \delta(x+y+z-1)$$

For quarks with a gluon interchange

$$a_q^{\text{vac}} \equiv F_2^{\text{vac}}(0) = \frac{\alpha_s}{2\pi}\frac{N_c^2-1}{2N_c}$$

AMM under extreme magnetic field



$$\boldsymbol{B}_{\text{ext}} = B_{\text{ext}}\hat{\boldsymbol{z}}$$

Landau Gauge

$$\mathcal{T} = (2\pi)^2 \delta^{(2)}(q_\parallel - p'_\parallel + p_\parallel)\, \varepsilon_\mu\, \delta\mathcal{M}^\mu_{nn'}$$

on-shell

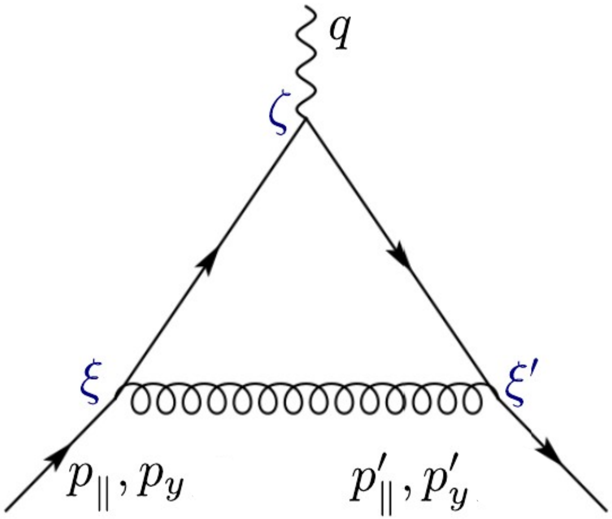
$$\ell_q = |e_q B_{\text{ext}}|^{1/2}$$

$$p_0 = \sqrt{p_z^2 + 2n\ell_q^{-2} + m^2},$$
$$p'_0 = \sqrt{p_z'^2 + 2n'\ell_q^{-2} + m^2},$$
$$q_0 = |\boldsymbol{q}|,$$

Matrix elements

$$\langle 0|\mathcal{A}_\mu(\zeta)|q\rangle = \varepsilon_\mu(q)\, e^{-iq\cdot\zeta}$$

$$\langle 0|\psi(\xi)|p_\parallel, p_y, n\rangle = f_n(p_y, \xi_\perp) u_n(p_\parallel)\, e^{-ip\cdot\xi_\parallel}$$



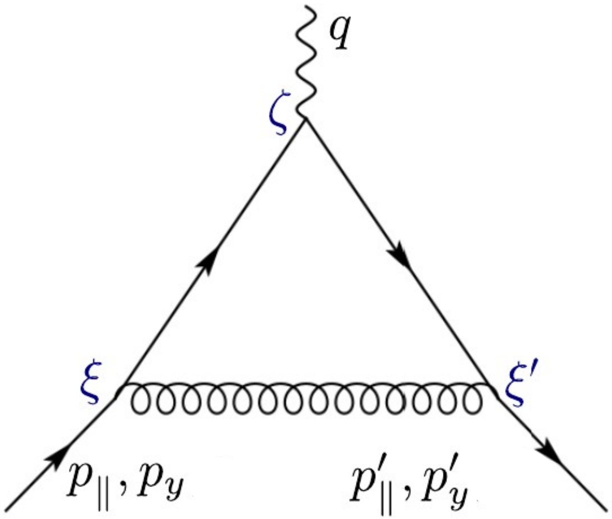
Lowest Landau Level

$$\delta\mathcal{M}^{\mu}_{\rm LLL} = \int_{k_\perp} \mathcal{F}(p_y, p'_y, k_\perp)\, \mathcal{G}^{\mu}(p_\parallel, p'_\parallel, k_\perp^2)$$

Exponential terms including Schwinger phase

$$\mathcal{F} = \frac{e_q g_s^2}{3\ell_q^5 \pi^{5/2}} \int_{\xi_\perp \xi'_\perp \zeta_\perp} \exp\left\{ -ik\cdot(\xi-\xi')_\perp + ip_y\xi_y - ip'_y\xi'_y + \frac{(\zeta-\xi)_\perp^2}{4\ell_q^2} + \frac{(\xi'-\zeta)_\perp^2}{4\ell_q^2} \right.$$
$$\left. -\frac{1}{2}(\xi_x/\ell_q + sp_y\ell_q)^2 - \frac{1}{2}(\xi'_x/\ell_q + sp'_y\ell_q)^2 - \frac{is}{2\ell_q^2}(\xi_x+\zeta_x)(\xi_y-\zeta_y) - \frac{is}{2\ell_q^2}(\zeta_x+\xi'_x)(\zeta_y-\xi'_y) \right\}$$

$$s = \mathrm{sgn}(e_q B)$$



Lowest Landau Level

$$\delta\mathcal{M}^{\mu}_{\mathrm{LLL}} = \frac{4}{3} e_q g_s^2 \int_{k_\perp} e^{\frac{1}{2}k_\perp^2 \ell_q^2} \mathcal{G}^{\mu}(p_{\parallel}, p'_{\parallel}, k_\perp)$$

$$\mathcal{G}^{\mu} = -\int_{k_{\parallel}} D(k)\, \bar{u}_0(p'_{\parallel})\, \gamma^{\nu} S(k_{\parallel} + p'_{\parallel})\, \mathcal{P}_+ \gamma^{\mu} S(k_{\parallel} + p_{\parallel})\, \mathcal{P}_+ \gamma_{\nu}\, u_0(p_{\parallel})$$

$$\mathcal{P}_{\pm} = \frac{1}{2}\left[1 \pm s i \gamma_1 \gamma_2\right]$$

$$s = \mathrm{sgn}(e_q B)$$

Structure of form factors in LLL

on-shell

$$\bar{u}_0(p'_\parallel)(\not{p}'_\parallel - m) = 0, \qquad (\not{p}_\parallel - m)u_0(p_\parallel) = 0$$

Gordon identity

$$\bar{u}_0(p')(p'^\mu + p^\mu)u_0(p) = \bar{u}_0(p'_\parallel)\left[2m\gamma^\mu_\parallel - i\sigma^{\mu\nu}_\parallel q^\parallel_\nu\right]u_0(p_\parallel)$$

Separated in polarization projections $\delta\mathcal{M}^\mu_{\mathrm{LLL}} = \delta\mathcal{M}^\mu_+ + \delta\mathcal{M}^\mu_-$

$$\delta\mathcal{M}^\mu_\pm = e_q\bar{u}_0(p'_\parallel)\left[\delta F_1^\pm(q_\parallel^2)\,\gamma^\mu_\parallel + F_2^\pm(q_\parallel^2)\,\sigma^{\mu\nu}_\parallel\frac{iq^\parallel_\nu}{2m} + F_3^\pm(q_\parallel^2)\,q^\mu_\parallel\right]\mathcal{P}_\pm\,u_0(p_\parallel)$$

contributtes $\partial\cdot\mathcal{A}_\parallel$

Structure of form factors in LLL

$$(1+a)\sigma_{\mu\nu}F^{\mu\nu} \to \sum_{s=\pm}(\sigma_{\mu\nu}F^{\mu\nu} + a^s\sigma^{\parallel}_{\mu\nu}F^{\mu\nu}_{\parallel})\mathcal{P}_s$$

$$\sum_{s=\pm} a^s\sigma^{\parallel}_{\mu\nu}F^{\mu\nu}_{\parallel}\mathcal{P}_s = -2iE_z\,\mathrm{diag}(a^+, -a^-, -a^+, a^-)$$

$$= -(a^+ + a^-)\begin{pmatrix} iE_z\sigma_z & 0 \\ 0 & -iE_z\sigma_z \end{pmatrix} + (a^+ - a^-)E_z i\gamma_5$$

AMM at LLL

$$F_2^{\pm}(q_{\|}^2) = \int_{xyz} \int_{k_\perp} \frac{\frac{4}{3} g_s^2\, e^{-\frac{1}{2}k_\perp^2 \ell_q^2}\, m^2 f^{\pm}(z)/4\pi}{\left[m^2(1-z)^2 - q_{\|}^2 xy + (k_\perp^2 + m_g^2)z\right]^2}$$

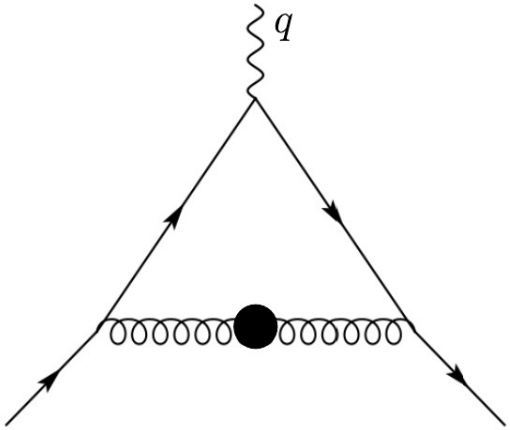
$$f^+(z) = \tfrac{2}{3}(1+6z), \qquad f^-(z) = z(4-z) - \tfrac{2}{3}$$

We can set $q_{\|}^2 = 0$ if $p_z' = p_z$ or $p' \gg m$ and $p \gg m$

$$\boxed{a_q^{\pm} \equiv F_2^{\pm}(0) = a_q^{\text{vac}} \int_0^1 dz \int_0^\infty d\eta\, \frac{e^{-(m\ell_q)^2\eta}\, f^{\pm}(z)(1-z)}{[(1-z)^2 + (m_g/m)^2 z + 2\eta z]^2}}$$

IR regularization

Infrared divergence when $m_g \to 0$



1. fixed effective masses $m_g = 0.3\ \mathrm{GeV}$
   $m_q = 0.35\ \mathrm{GeV}$

2. $m_q(\mu)$ and magnetically-dressed gluon self-energy in LLL

$$\Pi_g^{\mu\nu}(k) = \left(g_\parallel^{\mu\nu} - \frac{k_\parallel^\mu k_\parallel^\nu}{k_\parallel^2}\right)\Pi_g(k_\parallel^2, k_\perp^2)$$

$$\Pi_g(k_\parallel^2, k_\perp^2) = \frac{\alpha_s}{\pi} N_c \sum_q \frac{e^{-\frac{1}{2}\ell_q^2 k_\perp^2}}{\pi^2 \ell_q^2} g(m_q^2/k_\parallel^2)$$

$$g(m^2/k_\parallel^2) \approx g(0) = 1$$

Scales $\alpha_s(\mu)$ $m_q(\mu)$

$a.$ $\mu = 1\ \mathrm{GeV}$

$b.$ $\mu = \sqrt{|eB_{\mathrm{ext}}|}$

Numerical results

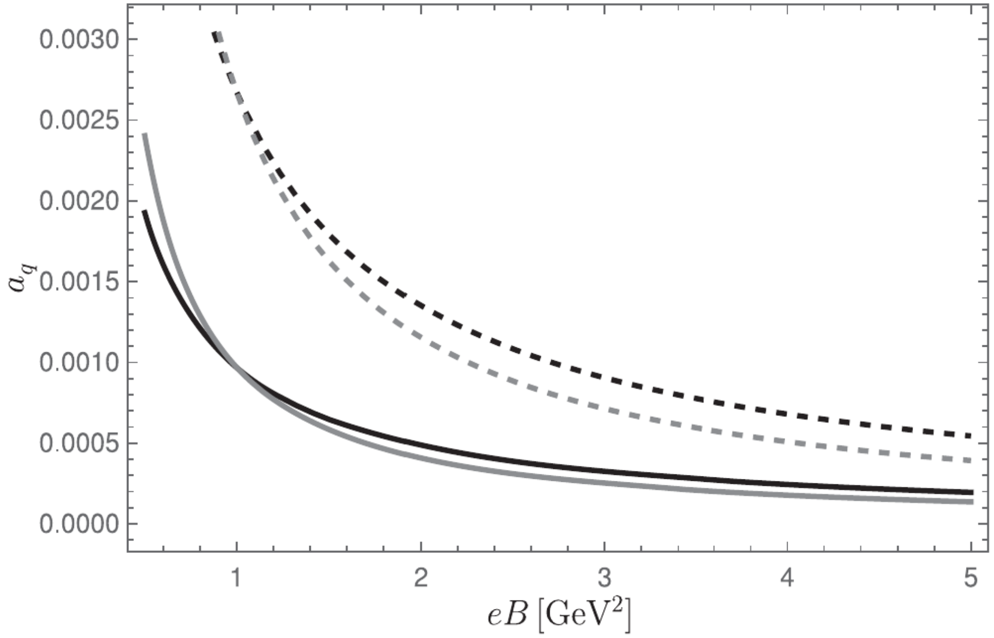
$$m_g^2 = \Pi_g, \quad m_q(\mu)$$

$u-\text{quark} \rightarrow$

$d-\text{quark} \rightarrow$

$\mu = 1GeV \rightarrow$ **black**

$\mu = \sqrt{|eB_{\text{ext}}|} \rightarrow$ **gray**



$a_u^+ \rightarrow$

$a_u^- \rightarrow$

$a_d^+ \rightarrow$

$a_d^- \rightarrow$

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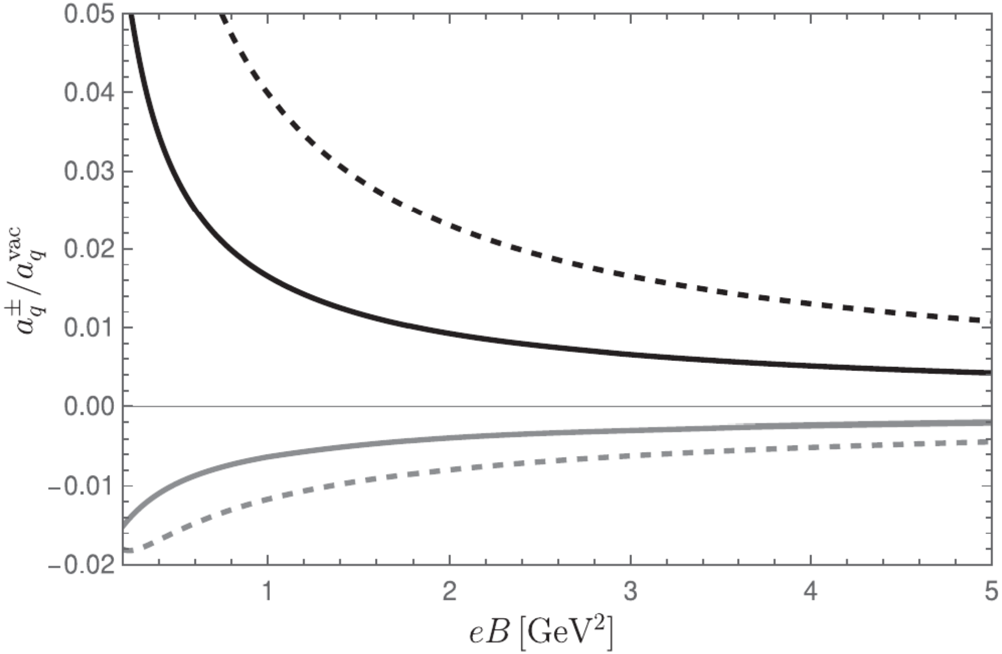

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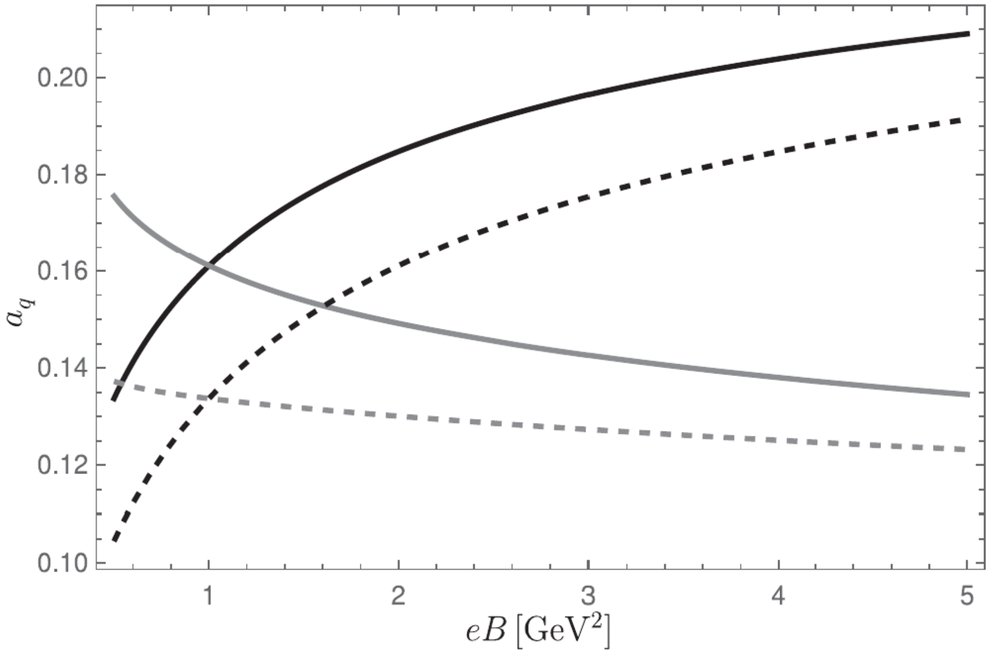

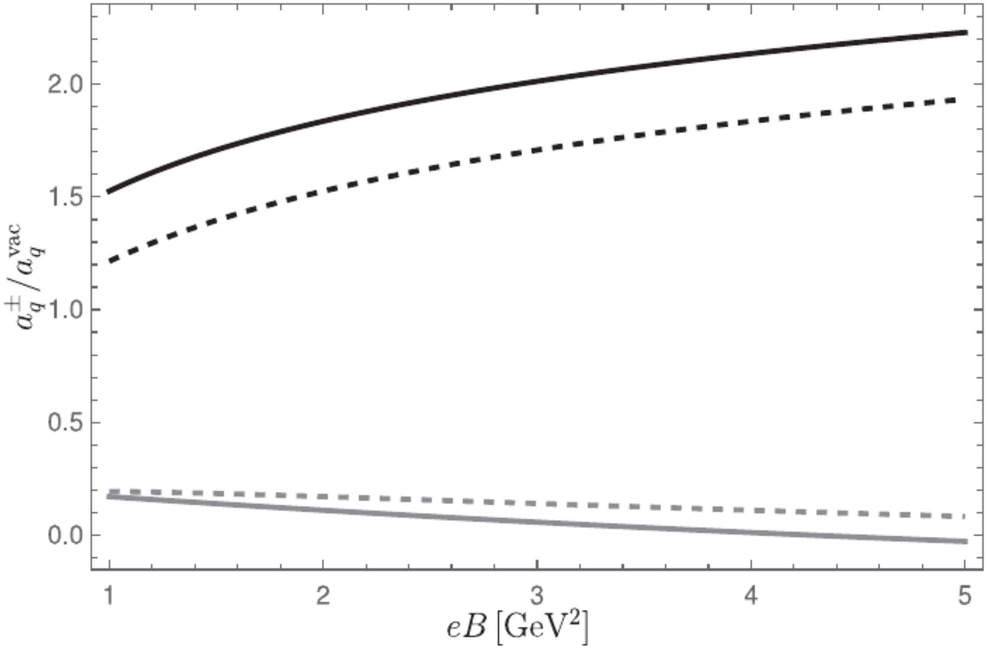

Conclusions and outlook

- AMM at LLL contributes to the electric part
  The other contributions are completely suppressed $\sim iE_z\sigma_z$

- IR divergences controled with gluon mass

- Results with gluon SE more closer to a physical description

- Results with effective fixed masses are of the order of the ones obtained recently in arXiv: 2506.20246

→ Full LL description

→ consequences of pseudoscalar contribution $\sim Ei\gamma_5$

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OBRIGADO!