### Twisting the Quark-Gluon Plasma: Insights into Electromagnetic Emission from a Rotating Medium

Jorge David Castaño-Yepes



Enrique Muñoz





# The one-loop photon (like) polarization tensor

Ayala, Villavicencio, and Muñoz discussed this Feynman diagram in their talks





$$\mathrm{i}\Pi^{\mu\nu} = -\frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\Big\{\mathrm{i}q_f \gamma^{\nu} \mathrm{i}S\left(k\right) \mathrm{i}q_f \gamma^{\mu} \mathrm{i}S(k-p)\Big\} + \mathrm{C.C.}$$

# Fermion Propagator in a Vortical Background

This propagator was originally derived by Ayala et al., and subsequently revised by Ayala and J.J. Medina, for a rigid cylindrical rotation

PHYSICAL REVIEW D 103, 076021 (2021)

**Fermion propagator in a rotating environment** Alejandro Ayala<sup>(b)</sup>,<sup>1,2</sup> L. A. Hernández<sup>(b)</sup>,<sup>2,3,4</sup> K. Raya<sup>(b)</sup>,<sup>1,5</sup> and R. Zamora<sup>(b)</sup>,<sup>6,7</sup>

$$S(p) = \frac{\psi_{+} + m_{f}}{p_{+}^{2} - m_{f}^{2} + i\epsilon} \mathcal{O}^{(+)} + \frac{\psi_{-} + m_{f}}{p_{-}^{2} - m_{f}^{2} + i\epsilon} \mathcal{O}^{(-)}$$
$$\mathcal{O}^{(\pm)} = \frac{1}{2} \left( \mathbb{1} \pm \frac{\mathbf{\Omega}}{\Omega} \cdot \mathbf{\Sigma} \right) = \frac{1}{2} \left( \mathbb{1} \pm i\gamma^{1}\gamma^{2} \right)$$

 $\mathbf{m}$ 

1 m

$$p_{\pm}^{\mu} \equiv \left(p^0 \pm \frac{\Omega}{2}, p^1, p^2, p^3\right)$$

This holds at all orders in perturbation theory

$$\mathcal{R}_{\gamma} \equiv \frac{d^3 R}{p_T dp_T d\phi dy} = -\frac{n_{\rm B}(\omega_{\rm ph})}{(2\pi)^3} \operatorname{Im} \left\{ g_{\mu\nu} \, \Pi_{\rm R}^{\mu\nu}(p) \right\}$$

$$g_{\mu\nu}\Pi^{\mu\nu} = -4q_f^2 \mathcal{I} + q_f^2 \sum_{\sigma=\pm 1} \left[12\mathcal{J}_{\sigma} - 4\mathcal{K}_{\sigma}\right]$$



$$\mathcal{I} \equiv \int \frac{d^4k}{(2\pi)^4} \left[ \frac{\epsilon_{ab}(k-p)^a k^b}{(k_+^2 - m^2)((k-p)_-^2 - m^2)} + \frac{\epsilon_{ab}k^a(k-p)^b}{(k_-^2 - m^2)((k-p)_+^2 - m^2)} \right]$$

$$\mathcal{J}_{\sigma} \equiv i \int \frac{d^4k}{(2\pi)^4} \frac{5[k_{\sigma} \cdot (k-p)_{\sigma}] - 2m^2}{(k_{\sigma}^2 - m^2)((k-p)_{\sigma}^2 - m^2)} \qquad \mathcal{K}_{\sigma} \equiv i \int \frac{d^4k}{(2\pi)^4} \frac{17[k_{\sigma} \cdot (k-p)_{-\sigma}] - 10m^2}{(k_{\sigma}^2 - m^2)((k-p)_{-\sigma}^2 - m^2)}$$

$$g_{\mu\nu}\Pi^{\mu\nu} = -4q_f^2 \mathcal{I} + q_f^2 \sum_{\sigma=\pm 1} \left[ 12\mathcal{J}_{\sigma} - 4\mathcal{K}_{\sigma} \right]$$
$$\mathcal{I} \equiv \int \frac{d^4k}{(2\pi)^4} \left[ \frac{\epsilon_{ab}(k-p)^a}{(k_+^2 - m^2)((k-p)^2)} + \frac{\epsilon_{ab}k^a(k-p)^b}{(k_-^2 - m^2)((k-p)_+^2 - m^2)} \right]$$

 $\operatorname{Im}(z)$ 

$$\mathcal{J}_{\sigma} = \mathbf{i} \int \frac{d^3k}{(2\pi)^3} \mathbf{i} \oint_C \frac{dz}{2\pi \mathbf{i}} \frac{1}{e^{\beta z} + 1} \frac{5\left(z + \sigma\Omega/2\right)\left(z - \mathbf{i}\nu_l + \sigma\Omega/2\right) - 5\mathbf{k}\cdot(\mathbf{k} - \mathbf{p}) - 2m^2}{\left[\left(z + \sigma\Omega/2\right)^2 - E_k^2\right] \left[\left(z - \mathbf{i}\nu_l + \sigma\Omega/2\right)^2 - E_{kp}^2\right]}$$

$$\mathcal{K}_{\sigma} = \mathbf{i} \int \frac{d^3k}{(2\pi)^3} \mathbf{i} \oint_C \frac{dz}{2\pi \mathbf{i}} \frac{1}{e^{\beta z} + 1} \frac{17\left(z + \sigma\Omega/2\right)\left(z - \mathbf{i}\nu_l - \sigma\Omega/2\right) - 17\mathbf{k}\cdot(\mathbf{k} - \mathbf{p}) - 10m^2}{\left[\left(z + \sigma\Omega/2\right)^2 - E_k^2\right] \left[\left(z - \mathbf{i}\nu_l - \sigma\Omega/2\right)^2 - E_{kp}^2\right]}$$

$$\mathcal{J}_{\sigma} = i \int \frac{d^{3}k}{(2\pi)^{3}} i \oint_{C} \frac{dz}{2\pi i} \frac{1}{e^{\beta z} + 1} \frac{5(z + \sigma\Omega/2)(z - i\nu_{l} + \sigma\Omega/2) - 5\mathbf{k} \cdot (\mathbf{k} - \mathbf{p}) - 2m^{2}}{\left[(z + \sigma\Omega/2)^{2} - E_{k}^{2}\right] \left[(z - i\nu_{l} + \sigma\Omega/2)^{2} - E_{kp}^{2}\right]} \mathcal{K}_{\sigma} = i \int \frac{d^{3}k}{(2\pi)^{3}} i \oint_{C} \frac{dz}{2\pi i} \frac{1}{e^{\beta z} + 1} \frac{17(z + \sigma\Omega/2)(z - i\nu_{l} - \sigma\Omega/2) - 1}{\left[(z + \sigma\Omega/2)^{2} - E_{k}^{2}\right] \left[(z - i\nu_{l} - \sigma\Omega/2) - \frac{1}{\sigma\Omega/2} + \frac{10m^{2}}{\sigma\Omega/2}\right]} \mathcal{K}_{\sigma}$$

Although the angular velocity defines a preferred direction in space, it does not break Lorentz symmetry in the sense of separating spatial momenta, as happens in the case of a magnetic field.

This is because the fermion propagator only experiences a shift in energy.

$$S(p) = \frac{\not{p}_{+} + m_{f}}{p_{+}^{2} - m_{f}^{2} + i\epsilon} \mathcal{O}^{(+)} + \frac{\not{p}_{-} + m_{f}}{p_{-}^{2} - m_{f}^{2} + i\epsilon} \mathcal{O}^{(-)}$$

$$p_{\pm}^{\mu} \equiv \left(p^0 \pm \frac{\Omega}{2}, p^1, p^2, p^3\right)$$

$$\mathcal{J}_{\sigma} = i \int \frac{d^{3}k}{(2\pi)^{3}} i \oint_{C} \frac{dz}{2\pi i} \frac{1}{e^{\beta z} + 1} \frac{5(z + \sigma\Omega/2)(z - i\nu_{l} + \sigma\Omega/2) - 5\mathbf{k} \cdot (\mathbf{k} - \mathbf{p}) - 2m^{2}}{\left[(z + \sigma\Omega/2)^{2} - E_{k}^{2}\right] \left[(z - i\nu_{l} + \sigma\Omega/2)^{2} - E_{kp}^{2}\right]} \mathcal{K}_{\sigma} = i \int \frac{d^{3}k}{(2\pi)^{3}} i \oint_{C} \frac{dz}{2\pi i} \frac{1}{e^{\beta z} + 1} \frac{17(z + \sigma\Omega/2)(z - i\nu_{l} - \sigma\Omega/2) - 17\mathbf{k} \cdot (\mathbf{k} - \mathbf{p}) - 10m^{2}}{\left[(z + \sigma\Omega/2)^{2} - E_{k}^{2}\right] \left[(z - i\nu_{l} - \sigma\Omega/2) - 2m^{2}\right]}$$

Although the angular velocity defines a preferred direction in space, it does not break Lorentz symmetry in the sense of separating spatial momenta, as happens in the case of a magnetic field.

This is because the fermion propagator only experiences a shift in energy.

$$S(p) = \frac{\not{p}_{+} + m_{f}}{p_{+}^{2} - m_{f}^{2} + i\epsilon} \mathcal{O}^{(+)} + \frac{\not{p}_{-} + m_{f}}{p_{-}^{2} - m_{f}^{2} + i\epsilon} \mathcal{O}^{(-)}$$



As a result, the yield remains angle-independent, and the flow coefficients are unaffected by this mechanism.

$$v_n = \frac{1}{\mathcal{R}_0} \int_0^{2\pi} d\phi \cos(n\phi) \frac{d^3 R}{p_T dp_T d\phi dy}$$

$$\lim_{\epsilon \to 0} \frac{1}{(A + i\epsilon)(B + i\epsilon)} = P.V.\left(\frac{1}{AB}\right) - i\pi \frac{\delta(A)}{B - A} + i\pi \frac{\delta(B)}{B - A}$$

$$\lim_{\epsilon \to 0} \frac{1}{(A + i\epsilon)(B + i\epsilon)} = P.V. \qquad R \end{pmatrix} - i\pi \frac{\delta(A)}{B - A} + i\pi \frac{\delta(B)}{B - A}$$
$$\operatorname{Im} \left[\mathcal{J}_{\sigma}\right] = -\frac{3m^{2}\pi}{4\omega_{\mathrm{ph}}(2\pi)^{2}} \sum_{s=\pm 1} \int_{m}^{\infty} dE \left\{ \left( n_{\mathrm{F}} \left[ \beta \left( sE - \frac{\sigma\Omega}{2} \right) \right] - n_{\mathrm{F}} \left[ \beta \left( sE - \omega_{\mathrm{ph}} - \frac{\sigma\Omega}{2} \right) \right] \right) \Theta \left[ E - s\omega_{\mathrm{ph}} - m \right] - \left( n_{\mathrm{F}} \left[ \beta \left( sE - \frac{\sigma\Omega}{2} \right) \right] - n_{\mathrm{F}} \left[ \beta \left( -sE + \omega_{\mathrm{ph}} - \frac{\sigma\Omega}{2} \right) \right] \right) \Theta \left[ -E + s\omega_{\mathrm{ph}} - m \right] \right\}$$

$$\operatorname{Im}\left[\mathcal{K}_{\sigma}\right] = \frac{\pi}{4\omega_{\rm ph}(2\pi)^{2}} \left(\omega_{\rm ph}^{2} - (\omega_{\rm ph} + \sigma\Omega)^{2} + 7m^{2}\right)$$

$$\times \sum_{s=\pm 1} \int_{m}^{\infty} dE \left\{ \left( n_{\rm F} \left[ \beta \left( sE - \sigma \frac{\Omega}{2} \right) \right] + n_{\rm F} \left[ \beta (sE - \omega_{\rm ph} - \sigma \frac{\Omega}{2}) \right] \right) \Theta \left[ E - s \left( \omega_{\rm ph} + \sigma\Omega \right) - m \right] - \left( n_{\rm F} \left[ \beta \left( -sE - \sigma \frac{\Omega}{2} \right) \right] + n_{\rm F} \left[ \beta \left( -sE - \omega_{\rm ph} - \sigma \frac{\Omega}{2} \right) \right] \right) \Theta \left[ -E - s \left( \omega_{\rm ph} + \sigma\Omega \right) - m \right] \right\}$$

$$\Pi_R^{\mu\nu}(p^0 = \omega_{\rm ph}, \mathbf{p}) = \Pi^{\mu\nu}(i\nu_n \to \omega_{\rm ph} + i\epsilon, \mathbf{p})$$



PHYSICAL REVIEW C 92, 014906 (2015)

Vorticity and hydrodynamic helicity in heavy-ion collisions in the hadron-string dynamics model

Oleg Teryaev<sup>\*</sup> Joint Institute for Nuclear Research, 141980 Dubna (Moscow region), Russia and National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe Shosse 31, 115409 Moscow, Russia

> Rahim Usubov<sup>†</sup> Joint Institute for Nuclear Research, 141980 Dubna (Moscow region), Russia (Received 28 January 2015; revised manuscript received 23 April 2015; published 13 July 2015)

Vorticity in heavy-ion collisions at the JINR Nuclotron-based Ion Collider fAcility

PHYSICAL REVIEW C 95, 054915 (2017)

Yu. B. Ivanov<sup>1,2,\*</sup> and A. A. Soldatov<sup>2,†</sup> <sup>1</sup>National Research Centre "Kurchatov Institute", Moscow 123182, Russia <sup>2</sup>National Research Nuclear University "MEPhI" (Moscow Engineering Physics Institute), Moscow 115409, Russia (Received 10 January 2017; published 31 May 2017)

$$\Pi_R^{\mu\nu}(p^0 = \omega_{\rm ph}, \mathbf{p}) = \Pi^{\mu\nu}(\mathrm{i}\nu_n \to \omega_{\rm ph} + \mathrm{i}\epsilon, \mathbf{p})$$



$$\Pi_R^{\mu\nu}(p^0 = \omega_{\rm ph}, \mathbf{p}) = \Pi^{\mu\nu}(\mathrm{i}\nu_n \to \omega_{\rm ph} + \mathrm{i}\epsilon, \mathbf{p})$$





The yield is affected by the parent particle's mass, and the threshold shifts accordingly



The yield is affected by the parent particle's mass, and the threshold shifts accordingly



## **Conclusions and future** work



Under the rigid rotation approximation, vorticity enters as an effective chemical potential.



Within the same approximation, rapid rotation can suppress low-energy photon production.



For phenomenological values of angular vorticity, the deviations from the non-rotating case are almost negligible.



The rigid rotation approximation should be relaxed to capture more realistic dynamics.



The system's geometry should be generalized beyond the cylindrical configuration.