f-mode oscillations of strange stars obtained from the vector MIT bag model [arXiv: 2412.05752]

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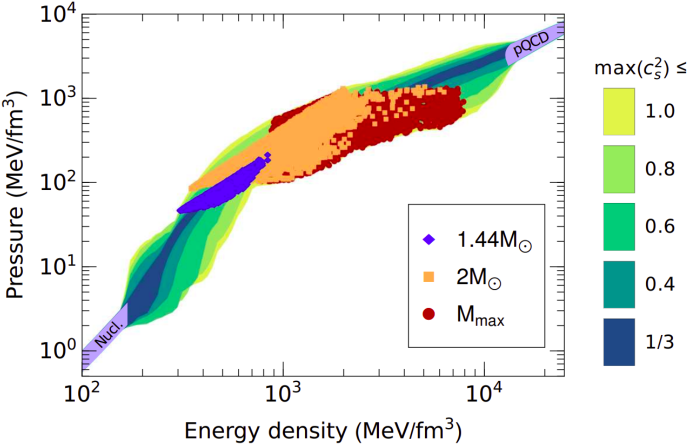
Outline

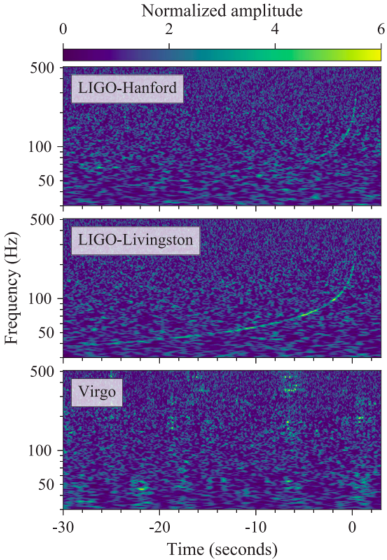
1. Motivation
2. Thermodynamics of the vMIT model
3. Structure and Radial/Non-Radial Oscillations
4. RESULTS: f-modes of vMIT stars
5. Summary and Outlook

1. Motivation A: High values of c_s^2



Equations of state for QCD matter
[Annala *et al.*, Nature Phys. 2020]

1. Motivation B: Sources of GWs in Binary Systems



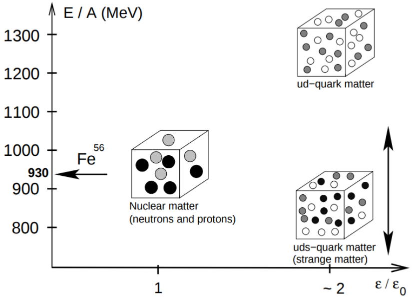
[Inspiral LIGO GW170817 event]

1. Motivation C: Bodmer-Witten Hypothesis

This hypothesis works when

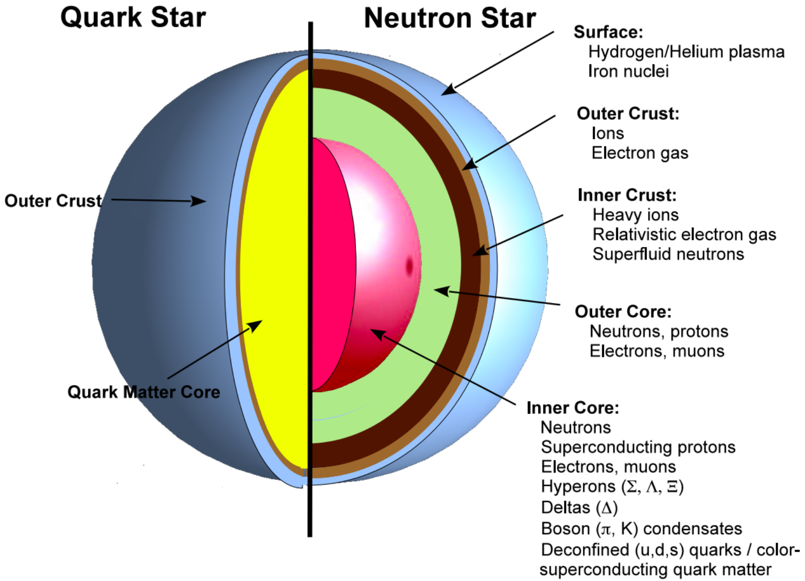
$$E_{(uds)}/A = \epsilon/n_B < 930 \text{ MeV},$$

in vacuum ($p = 0$ and $T = 0$).



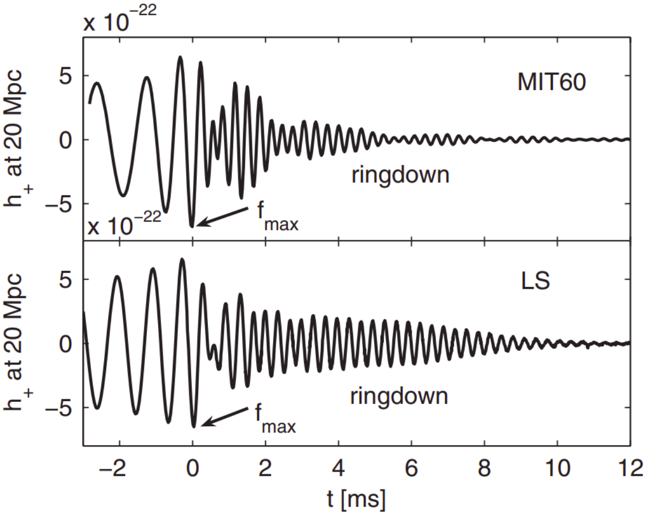
[F. Weber, 2005]

1. Motivation D: Strange vs Neutron Stars



[F. Weber, 2012]

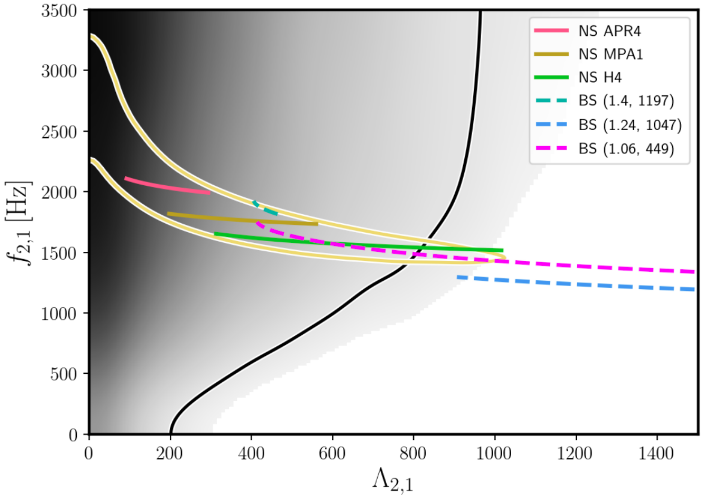
1. Motivation E: Strange vs Neutron Star Signals



[Bauswein et al., arXiv: 0910.5169.]

See also 1408.0929, 2107.13997, 2204.11034, 2306.12326

1. Motivation F: Non-radial f mode



[Pratten et al., arXiv: 1905.00817]

2. Thermodynamics of the vMIT model

The corresponding Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\mathrm{MIT}} + \mathcal{L}_V,$$

where

$$\mathcal{L}_{\mathrm{MIT}} = \sum_i \{\bar{\psi}_i[i\gamma^\mu\partial_\mu - m_i]\psi_i - B\}\Theta(\bar{\psi}_i\psi_i),$$

$$\mathcal{L}_V = \sum_i \{\bar{\psi}_i g_{iV}(\gamma^\mu V_\mu)\psi_i - \frac{1}{2}m_V^2 V^\mu V_\mu\}\Theta(\bar{\psi}_i\psi_i),$$

Notice that

- $\Theta(\bar{\psi}_i\psi_i)$ assures that the quarks exist only confined to the bag.
- Unlike the ω meson of quantum hadrodynamics, the V^μ field does not necessarily correspond to any real meson field.

2. Thermodynamics of the vMIT model

- After applying the mean-field approximation, one obtains (with $V_0 = \langle V_\mu \rangle \delta_0^\mu$):

$$E_i = \sqrt{m_i^2 + k_i^2} + g_{iV} V_0$$

$$m_V^2 V_0 = \sum_i g_{iV} n_i,$$

$$n_i = \gamma_i \frac{k_f^3}{3\pi^2}$$

where $\gamma_i = 6 = (3 \times 2)$.

- Since we work at $T = 0$, the energies 'E_i' are also the chemical potentials (μ_f) for each flavor.

2. Thermodynamics of the vMIT model

- One can construct the equation of state (EoS) for our quark system in the mean field approximation by constructing the Hamiltonian as

$$\mathcal{H} = -\langle \mathcal{L} \rangle .$$

- One obtains

$$\varepsilon_i = \frac{\gamma_i}{2\pi^2} \int_0^{k_f} E_i \; k^2 dk,$$

$$\varepsilon = \sum_i \varepsilon_i + B - \frac{1}{2} m_V^2 V_0^2 \, ,$$

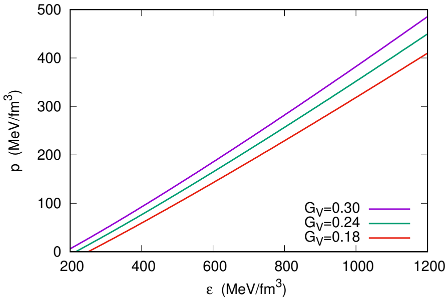
wherethe last term of equation is absent in current literature being crucial to kept the thermodynamic consistency.

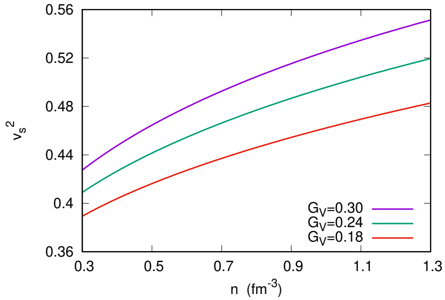
2. Thermodynamics of the vMIT model

We fix the vMIT free parameters as follows:

- We redefine $G_V \equiv (g_{uV}/m_V)^2$ and define $X_V \equiv (g_{sV}/g_{uV})$.
- We use a universal coupling for $X_V = 1.0$, producing more massive stars for the same value of G_V.
- Besides $m_u = m_d = 4\,\text{MeV}$, $m_s = 95\,\text{MeV}$, and the values of G_V and the bag are not fully independent.

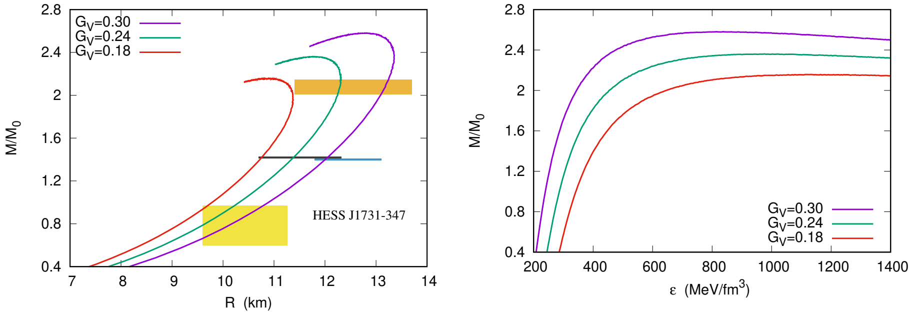
2. Thermodynamics of the vMIT model





EoS (left) and the square of the speed of sound (right).

3. Hydrostatic Structure of vMIT stars



M-R diagram (left) and M-central density plane (right) for vMIT stars.

3. Oscillation equations: The radial case

We use the formalism of [Gondek *et al.*, 1997] given by

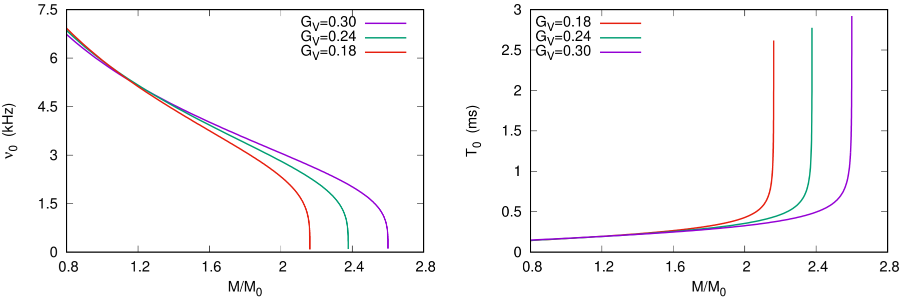
$$\frac{d\xi}{dr} = -\frac{1}{r}\left(3\xi + \frac{\Delta P}{\Gamma P}\right) - \frac{dP}{dr}\frac{\xi}{(P+\epsilon)} ,$$

and

$$\frac{d\Delta P}{dr} = \xi\left\{\omega^2 e^{\lambda-\nu}(P+\epsilon)r - 4\frac{dP}{dr}\right\} + \xi\left\{\left(\frac{dP}{dr}\right)^2\frac{r}{(P+\epsilon)} - 8\pi e^{\lambda}(P+\epsilon)Pr\right\} + \Delta P\left\{\frac{dP}{dr}\frac{1}{P+\epsilon} - 4\pi(P+\epsilon)re^{\lambda}\right\} ,$$

where ω is the oscillation frequency.

Results: Radial f-mode frequencies



Radial f-mode frequencies (left) and period τ_0 (right) for different G_V.

3. Oscillation equations: The non-radial case

*Somewhat similar to the radial case, one can start by applying polar non-radial perturbations to a non-rotating perfect fluid star giving a set of coupled equations where the perturbed metric tensor reads

$$
\begin{aligned}
ds^2 &= -e^{\nu}(1 + r^{\ell} H_0 Y^{\ell}_m e^{i\omega t})dt^2 - 2i\omega r^{\ell+1} H_1 Y^{\ell}_m e^{i\omega t} dtdr \\
&\quad + e^{\lambda}(1 - r^{\ell} H_0 Y^{\ell}_m e^{i\omega t})dr^2 \\
&\quad + r^2(1 - r^{\ell} K Y^{\ell}_m e^{i\omega t})(d\theta^2 + \sin^2\theta d\phi^2),
\end{aligned}
$$

where H_0, H_1, K are metric functions and ω is the angular frequency.
*The polar perturbations in the position of the fluid elements are given by the following Lagrangian displacements

$$
\begin{aligned}
\xi^r &= r^{\ell-1} e^{-\lambda/2} W Y^{\ell}_m e^{i\omega t}, \\
\xi^{\theta} &= -r^{\ell-2} V \partial_{\theta} Y^{\ell}_m e^{i\omega t}, \\
\xi^{\phi} &= -r^{\ell}(r\sin\theta)^{-2} V \partial_{\phi} Y^{\ell}_m e^{i\omega t},
\end{aligned}
$$

where W and V are fluid perturbation functions.

Non-radial oscillation equations: Inside

$$
\begin{aligned}
H_1' &= -r^{-1}[\ell + 1 + 2me^{\lambda}/r + 4\pi r^2 e^{\lambda}(p-\epsilon)]H_1 \\
&\quad + e^{\lambda} r^{-1}\left[H_0 + K - 16\pi(\epsilon + p)V\right], \\
K' &= r^{-1}H_0 + \frac{\ell(\ell+1)}{2r}H_1 - \left[\frac{(\ell+1)}{r} - \frac{\nu'}{2}\right]K \\
&\quad -8\pi(\epsilon + p)e^{\lambda/2}r^{-1}W\,, \\
W' &= -(\ell+1)r^{-1}W + re^{\lambda/2}[e^{-\nu/2}\gamma^{-1}p^{-1}X \\
&\quad -\ell(\ell+1)r^{-2}V + \tfrac{1}{2}H_0 + K]\,,
\end{aligned}
$$

$$
X' = \ldots .
$$

Non-radial oscillation equations: Outside

Outside the star, i.e. the vacuum, $m = M$, the perturbations equations on the fluid are null and the differential equations reduce to the Zerilli equations:

$$\frac{d^2 Z}{dr^{*2}} = [V_Z(r^*) - \omega^2]Z,$$

where $Z(r^*)$ and $dZ(r^*)/dr^*$ are related to the metric perturbations $H_0(r)$ and $K(r)$. We can also note the "tortoise" coordinate given by

$$r^* = r + 2M \ln(r/(2M) - 1),$$

and the effective potential $V_Z(r^*)$ is given by

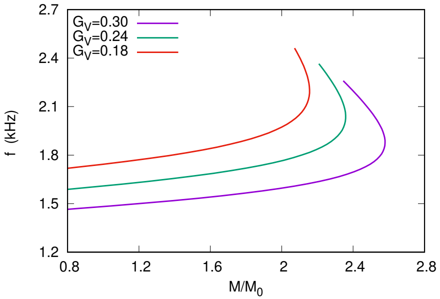
$$V_Z(r^*) = \frac{(1 - 2M/r)}{r^3(nr + 3M)^2} f(r),$$

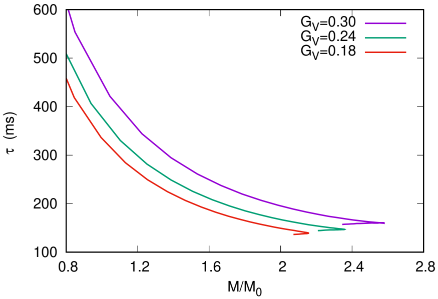
where

$$f(r) = [2n^2(n+1)r^3 + 6n^2 M r^2 + 18nM^2 r + 18M^3]$$

with $n = (l-1)(l+2)/2$.

Results: Non-radial f frequencies





Non-radial f-mode frequencies (left) and damping time (right).

Results: Proposed Universal Relation

- Inspired by (Benhar 2004), we realize that our non-radial f-mode can be fitted by the following linear expression:

$$f = a + b \cdot (M/R^3)^{1/2},$$

  where a is given in kHz and b in km $\times$ kHz for masses above 0.7 $M_\odot$.

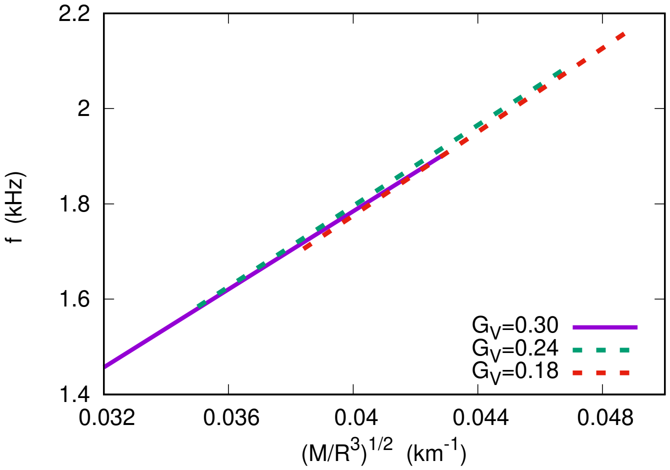
- We obtain $a = +0.142$, $+0.107$, and $+0.009$ kHz, $b = 41.1$, 42.3, and 44.2 km $\times$ kHz for $G_V = 0.30$, 0.24 and 0.18 fm^2 respectively.

- We notice that 'b' is in agreement with those found in the literature.

- Besides, 'b' seems very similar for hadron and quark stars.

- However, 'a' for vMIT stars is much closer to zero than those of hadron stars.

Proposed Universal Relation



Fundamental-mode frequency versus the square root of the average density. The linear relation is satisfied for all three EoSs.

5. Summary and Outlook

- We used the vector MIT model to explore the effect of the vector field exchange (repulsive) term, on the mass and radius of strange quark stars satisfyig data from HESS J1731-347 and PSR J0437-4715.
- The fundamental mode of the radial oscillations show that an increasing G_V has a stabilizing property for stars in the range (1.2 - 2.0)M$\odot$ but for stars below 1.2 $M_\odot$ we have the opposite effect.
- The non-radial f mode frequencies were also calculated for which its gravitational wave frequency is restricted to (1.6 - 1.8) kHz for high mass stars and to (1.5 - 1.6) kHz for low mass stars.
- A linear universal relation between the f-mode and the square root of the average density expressed was found for all EoS studied.
- We are currently exploring the effects of strong magnetic fields on these universal relations for vMIT stars, until now only explored for NSs.

Acknowledgements

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