Unusual quark effective interactions from vacuum polarization under relatively weak magnetic field

Fabio L. Braghin

Instituto de Física - Universidade Federal de Goias, Goiânia, GO

Conference on Chirality, vorticity and magnetic fields, Principia Institute, ICTP-SAIFR, July 2025, São Paulo

Talk based on:

* FLB, PRD105 (2022)
* FLB, JPG (2020)
* THMoreira+FLBraghin, PRD105(2022)
* On going work, FLB and with C.Villavicencio, M.Loewe

Presentation Overview

Motivations/context

* Search for effects of magnetic fields in r.h.i.c./dense stars

* Associated effects in hadron structure and interactions
$\rightarrow$ to use magnetic fields as probes to hadron structure and interactions
* By comparing first principles with calculable models
$\rightarrow$ identification of relevant degrees of freedom / mechanism

* Desirable: different effects as evidence of magnetic field

One example

NJL model and quark-antiquark Mesons

* Nambu-Jona-Lasinio model:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m_f)\psi + \frac{G_0}{2}[(\bar{\psi}\lambda_i\psi)^2 + (\bar{\psi}i\gamma_5\lambda_i\psi)^2] + \mathcal{L}_{h.o.}$$

* Gluon exchange(s) and dynamics $G_0 \sim \frac{1}{M_G^2}, \frac{1}{\Lambda^2}$ (flavorless)

* Many works (from researchers present here and others)considered the model for the calculation of hadron properties under B

* As a (low energy) effective model for QCD:
its parameters should depend on QCD parameters and on thermodynamic conditions

* However, a model has its own structure/dynamics...
(hopefully as manifestation of QCD)

One loop correction to the 4 point GF

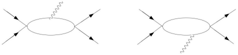


Figure: One loop correction for the NJL coupling

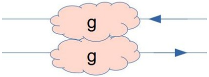


Figure: One loop Two gluon exchange (quark det)

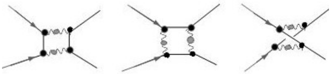


Figure: Two gluon exchange

Leading terms of 1-loop background field effective action:

$$\mathcal{L}^{(1)} = -2G_0 \sum_{f=u,d,s} tr_{DC}[iS_f^B(p)]\bar{\psi}\psi, \quad (1)$$

* Correction to the effective mass (Lagrangian) = $< \bar{q}q > (B)$ (gap equation)

$$\mathcal{L}_{1loop} = \frac{\bar{G}_s^{ij}(B)}{2}(\bar{\psi}\lambda_i\psi)(\bar{\psi}\lambda_j\psi) + \frac{\bar{G}_{ps}^{ij}(B)}{2}(\bar{\psi}i\gamma_5\lambda_i\psi)(\bar{\psi}i\gamma_5\lambda_j\psi), \quad (2)$$

Magnetic field breaks chiral/flavor symmetries:

$i, j = 0, 1, ...(N_f^2 - 1)$ defines each meson in a flavor nonet

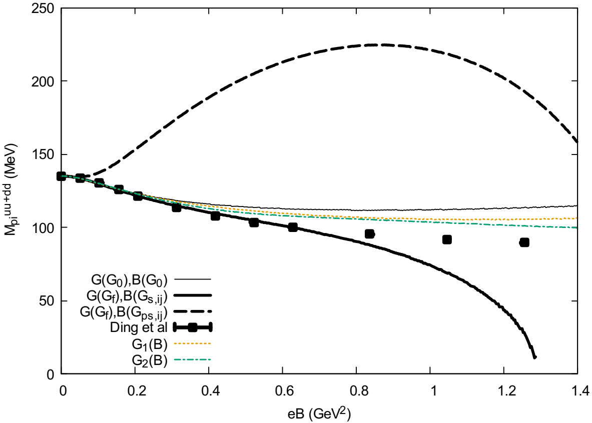


Figure: Neutral pion mass (complete state)

$$G_1(eB) \;=\; \alpha + \beta e^{-\gamma(eB)^2}, \qquad G_2(eB) = G_0 \left(\frac{1 + a\xi^2 + b\xi^3}{1 + c\xi^2 + d\xi^4} \right),$$

Avancini et al (2017), Ferreira et al (2014)

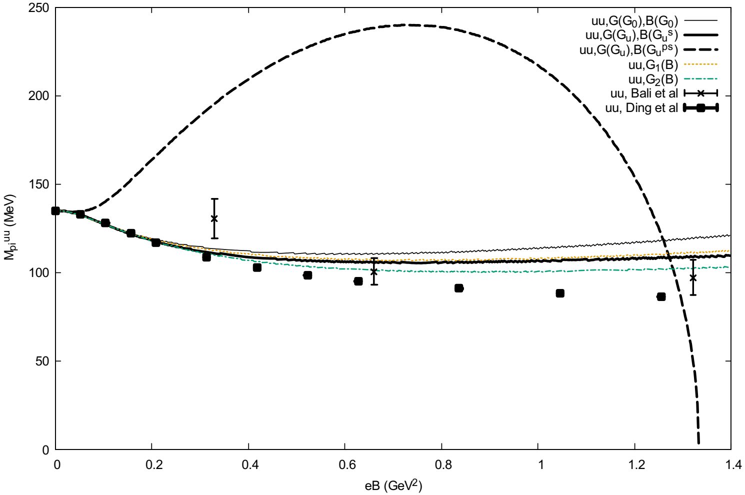


Figure: Neutral pion masses π^{uu}

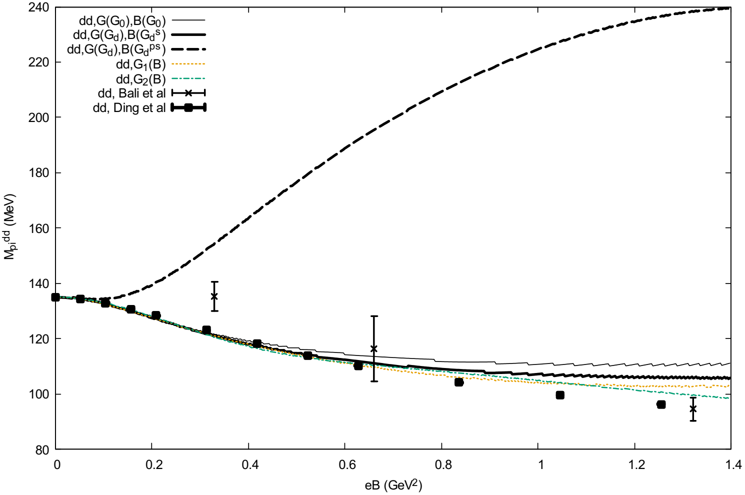


Figure: Neutral pion masses π^{dd}

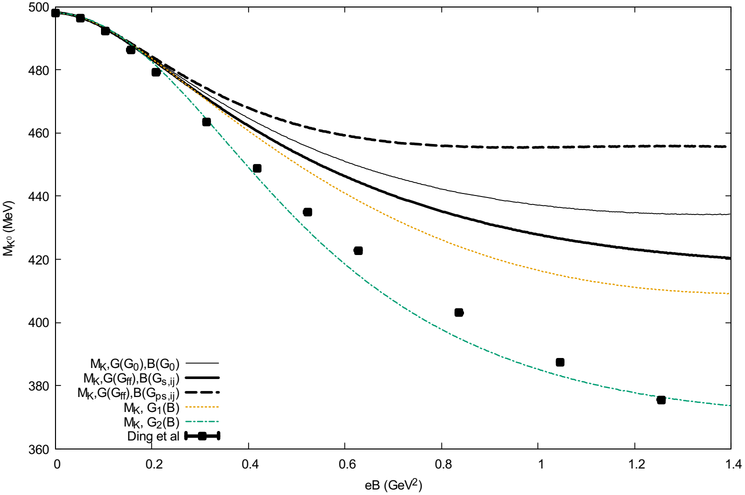
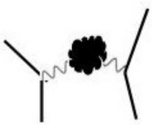


Figure: Neutral kaon masses for the following cases: gap eqs. and BSE with G_0 (thin line), and gap eqs. and BSE respectively with G^s_{ff} and G^s_{66} (thick line). The use of G^{ps}_{66} is also considered (dashed line).

Emergence of Constituent quarks And meson-quark interactions



quark-gluon interaction

Leading term for quark - QCD effective action

$$Z[\eta,\bar\eta] = N \int \mathcal{D}[\bar\psi,\psi]$$

$$\exp i \int d^4x \left[\bar\psi\left(i\not{D} - m\right)\psi - \frac{g^2}{2}\int_y j^\beta_\mu(x)\tilde{R}^{\mu\nu}_{\beta\alpha}(x-y)j^\alpha_\nu(y) + \bar\psi\eta + \bar\eta\psi\right],$$

color quark current $j^\mu_\alpha = \bar\psi\lambda_\alpha\gamma^\mu\psi$,

Fierz transformation $\rightarrow$ all flavor-Dirac channels (mesons)

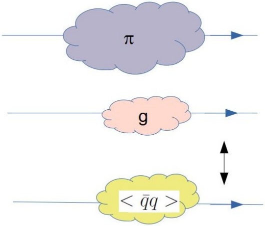
By introducing quark background currents J_ϕ

By integrating out quark field $\rightarrow$ Quark determinant

Quark determinant → constituent quark' Gluon cloud

* Gluon cloud:
It dresses background quark currents → constituent quarks

* By including meson states → Pion cloud
from *Goldstone boson* couplings



(FLB, EPJA-2016/2018, PRD-2018/2019)

Expansion of the quark determinant: constituent quark and mesons interactions

LARGE quark and gluon effective masses
$\rightarrow$ longwavelength limit (low energies) limit

In the presence of weak magnetic fields:

$$\begin{aligned} S(k) &= S_0(k) + S_1(k)(eB_0) \\ &= \frac{\not{k} + M^*}{k^2 - {M^*}^2} + i\gamma_1\gamma_2 \frac{{M^*}^2(\gamma_0 k^0 - \gamma_3 k^3 + M^*)}{(k^2 - {M^*}^2)^2} \hat{Q} \frac{eB_0}{{M^*}^2} (3) \end{aligned}$$

Leading magnetic field corrections to hadron couplings/masses
Analytically calculated from a dynamical approach

Quark sector-polarization 4-point GF

Resulting quark (antiquark) interactions can be written and intepreted as:

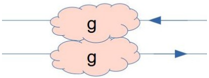


Figure: One loop Two gluon exchange (quark det)

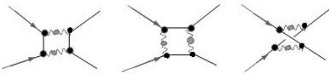
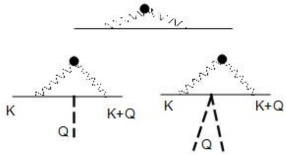


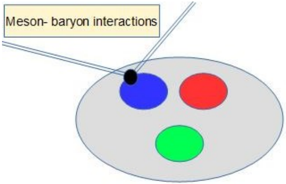
Figure: CQM and QCD-Two gluon exchange

Coupling constants resolved at zero external momenta

Leading couplings: meson- constituent quarks

Expansion of quark determinant (some ambiguities-symmetries)





Examples of leading meson-constituent quark couplings (form factors)

$$\mathcal{L}_{j_A} = \left[G_A^{B,T}(Q,K) Q_\mu \pi^i(Q) + G_{\bar{A}}^{B,T}(Q,K) \bar{A}^i_\mu(Q) \right] j^\mu_{A,i}(K,Q) \quad (4)$$

$G_A(Q,K), G_{\bar{A}}(Q,K)$ are one loop integrals
Coupling constants ($K = Q = 0$) or ($Q^2 = M_\pi^2$) ..
Numerically: correct order of magnitude (renormalization=1-fit)

Leading couplings: under weak magnetic field

* Constituent quark-pion quark interactions under B field
And leading terms of ChPT under B field
$\rightarrow$ FLB, Eur. Phys. Joun. A (2018)
$\rightarrow$ FLB+WFS, Journ. Phys. G (2020)

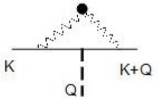
* Constituent quark-vector and axila mesons under B field
$\rightarrow$ FLB, Phys. Rev. D (2018)

* Vector meson dominance and vector/axial meson mixings in B
$\rightarrow$ FLB, Journ. Phys. G (2020).

* Renormalization scheme : A.Nogueira, FLB, IJMPA (2022)

Wess Zumino Witten type coupling

Next leading for isosinglet V_μ and isotriplet V^i_μ mesons



Note that:
Meson-quark momenta
Transversal to each other and
Transversal meson polarization

$$\mathcal{L}_{vja} = i\delta_{ij}\epsilon^{\sigma\rho\mu\nu}\frac{F^{vja}}{M^{*2}}\,\partial_\sigma\mathcal{F}^i_{\rho\mu}\,j^{A,j}_\nu + i\epsilon^{\sigma\rho\mu\nu}\frac{F^{vja}}{M^{*2}}\,\partial_\sigma\mathcal{F}_{\rho\mu}\,j^A_\nu,$$

$$\mathcal{F}^i_{\rho\mu} = \partial_\rho V^i_\mu - \partial_\mu V^i_\rho, \qquad \mathcal{F}_{\rho\mu} = \partial_\rho V_\mu - \partial_\mu V_\rho.$$

* Polarized vector meson, transversal directions in $\epsilon^{\sigma\rho\mu\nu}$

$$\mathcal{L}_{vjaB} = \frac{(eB_0)}{M^{*2}}\epsilon_{ij3}\frac{F^B_{vja}}{M^{*2}}\left[\epsilon^{12\rho\mu}\partial_\rho\partial_\nu\cdot V^\nu_i + 2\epsilon_{12\rho\nu}\partial^\rho\mathcal{F}^{\mu\nu}_i\right]\,j^{A,j}_\mu$$
$$+ \frac{(eB_0)}{M^{*2}}\frac{F^B_{vja}}{3M^{*2}}\left[\epsilon^{12\rho\mu}\partial_\rho\partial_\nu\cdot V^\nu + 2\epsilon_{12\rho\nu}\partial^\rho\mathcal{F}^{\mu\nu}\right]\,j^{A,3}_\mu \quad (5)$$

Photon probes the axial current (Vector Meson Dominance)

FLB, PRD (2022)

Chiral partners: axial mesons-vector current

For isosinglet $\bar{A}_\mu$ and isotriplet $\bar{A}^i_\mu$ mesons

$$\mathcal{L}_{vja-A} = i\epsilon^{\sigma\rho\mu\nu} F^{vja} \partial_\sigma \mathcal{G}^i_{\rho\mu} j^{V,i}_\nu + i\epsilon^{\sigma\rho\mu\nu} F^{vja} \partial_\sigma \mathcal{G}_{\rho\mu} j^V_\nu , \quad (6)$$

$j^{V,i}_\mu = \bar{\psi}\gamma_\mu\sigma^i\psi$
$j^V_\mu = \bar{\psi}\gamma_\mu\psi$.

$$\mathcal{G}^i_{\mu\nu} = \partial_\mu \bar{A}^i_\nu - \partial_\nu \bar{A}^i_\mu, \qquad \mathcal{G}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu. \quad (7)$$

Axial pion coupling and axial ρ coupling

Couplings to the axial current

$$\mathcal{L}_{j_A} = \left[\frac{G_A^B}{f_\pi} \partial_\mu \pi^i + G_A^B \bar{A}_\mu^i + i \frac{F_{vja}^B}{M^{*2}} \epsilon_{\mu\nu\rho\sigma} \partial^\nu \partial^\rho V_i^\sigma \right] j_{A,i}^\mu, \quad (8)$$

From the same method:

$$\frac{F_{vja}(K,Q)/M^{*2}}{G_A(K,Q)} = \frac{1}{4M^* f_\pi} \sim \text{constant.} \quad (9)$$

This is Goldberber Treiman-type relation

In the limit of large quark /gluon effective masses

$$\frac{F_{vja}^B}{F_{vja}} \sim \frac{eB}{M^{*2}} \quad (10)$$

Witten's procedure: quantization

$\mathcal{L}_{vja}$ as a 5dim closed surface (Stoke's theorem)

$$n\,\Gamma = -\epsilon^{\sigma\rho\mu\nu}\frac{i}{240\pi^2}\int d^4K\, d^4Q\, F^{vja}(K,Q)K_\sigma \mathcal{F}^i_{\rho\mu}(Q) j^{A,i}_\nu(K,K+Q), (1$$

n is an integer: $\Gamma = \epsilon_{\sigma\rho\mu\nu}\Gamma^{\sigma\rho\mu\nu}$

Quantized integrals (Sum over $\mu\nu\rho\sigma$) contain integrals of the type

$$\Gamma_{(xyz0)} = -\frac{i}{240\pi^2}\int d^4K\, d^4Q\, F^{vja}(K,Q)K_x Q_y$$
$$\times \left[\rho_z^-(Q)\bar{u}(K+Q)\gamma_0\gamma_5 d(K) + \rho_z^+(Q)\bar{d}(K+Q)\gamma_0\gamma_5 u(K)\right],$$

* $\rho_z^\pm(Q)$ = z-polarization component

PRD (2022) Still missing to complete

Some mixings due to magnetic fields

A large number of mixings can arise

The present approach leads to results akin to the NJL-model

How to test or probe them?

Vector -axial mesons mixing

$$\mathcal{L}_{mix,B} = \frac{(eB_0)}{M^{*2}} G^{B,1}_{v-a} i\epsilon_{12\mu\nu} M^{*2} i\epsilon_{ij3} V^{\mu}_i(Q)\bar{A}^{\nu}_j(K)\delta(Q+K)$$

where $G^{B,1}_{v-a}$ is given by a momentum integral
(can depend on external Q or K)

How to test strength of $G^{B,1}_{v-a}$?

FLB, PRD 2022

Mixings from coupling to background photon/eB

Some examples: VMD and mesons mixings

$$\mathcal{L}_{VMD} = -g_{F\rho}\mathcal{F}^3_{\mu\nu}F^{\mu\nu} - g_{F\omega}\mathcal{F}_{\mu\nu}F^{\mu\nu}, \quad (12)$$

$$\mathcal{L}_F = g_{F\rho\omega}(F^{\mu\nu}\mathcal{F}^3_{\nu\rho}\mathcal{F}^\rho_\mu + F^{\mu\nu}\mathcal{G}^3_{\nu\rho}\mathcal{G}^\rho_\mu) - g_{FF\omega}F_{\mu\nu}F^{\nu\rho}\mathcal{F}^\mu_\rho - g_{FF\rho}F_{\mu\nu}F^{\nu\rho}\mathcal{F}^{3,\mu}_\rho, \quad (13)$$

where the following Abelian strength tensors have been defined:

$$\mathcal{F}^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu, \qquad \mathcal{F}^{\mu\nu}_i = \partial^\mu V^\nu_i - \partial^\nu V^\mu_i, \quad (14)$$

$$\mathcal{G}^{\mu\nu} = \partial^\mu \bar{A}^\nu - \partial^\nu \bar{A}^\mu \qquad \mathcal{G}^{\mu\nu}_i = \partial^\mu \bar{A}^\nu_i - \partial^\nu \bar{A}^\mu_i. \quad (15)$$

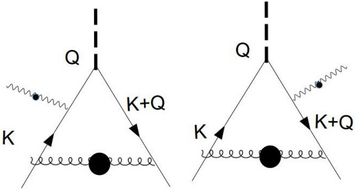
And corresponding magnetic field insertions...

* FLB, JPG 2020
Test of VMD at finite B/photon in pion form factor

* In this work the background field method is applied in a gauge invariant way

Anomalous Pion couplings to quark currents

On going with M. Loewe and C. Villavicencio



* (Relatively) weak magnetic field $\rightarrow \sim$ analytical equations

* Pion coupling to vector - quark current

Anomalous pion coupling to gluons Three processes
* Pion absorption or emission
* Gluon fusion into pion
* Pion decay into gluons

Summary

- Magnetic fields lead to a large variety of effects in hadron interactions
- These may be used to probe hadron dynamics or search evidence of magnetic fields
- Different types of meson mixings induced by B

* Looking for ways to test (effects of) these couplings

Thank you for your attention!