Testing infrared confining models beyond fundamental correlation functions

Letícia F. Palhares

Departamento de Física Teórica (UERJ, Brazil)

UNIVERSIDADE DO ESTADO DO RIO DE JANEIRO UERJ

<u>*IR-QCD Group @ UERJ:*</u>
M. Capri, M. Guimaraes,
B. Mintz, S. Sorella

C.Mena and LFP, *Phys.Rev.D* 109 (2024) 9, 094006

J.P.S.Santos and LFP, *to appear.*

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Outline

- ***Confining models*** *as an alternative approach to IR QCD*
  - Status of Refined Gribov-Zwanziger framework
- Testing IR confining models in observables and phenomenology:
  - The q-qbar-photon vertex and the anomalous magnetic moment: model contraints from an observable?
  - Color SUC via nonperturbative gluon exchange [see talk by J.P.S. Santos on Friday!]

Motivation: the confinement problem

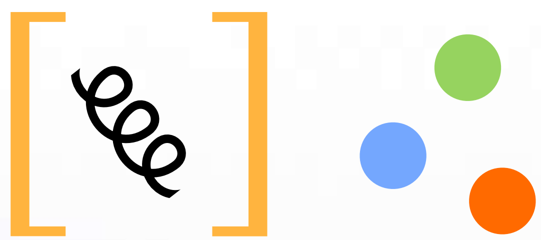
Fundamental degrees of freedom are
not part of the spectrum

?

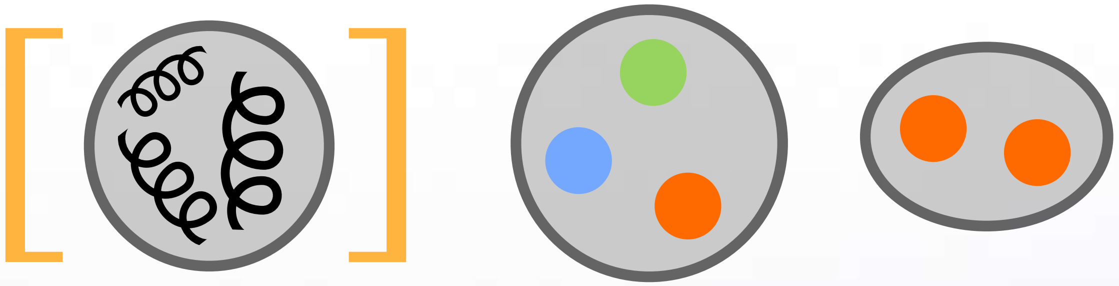
Physical spectrum of bound states
dynamically generated at low energies.

[Yang-Mills gauge theories] and QCD:

$$\mathcal{L}^{\mathrm{E}}_{\mathrm{YM}} = \left[\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}\right] + \overline{\Psi}_f[\gamma_\mu D_\mu + m_f]\Psi_f$$



?



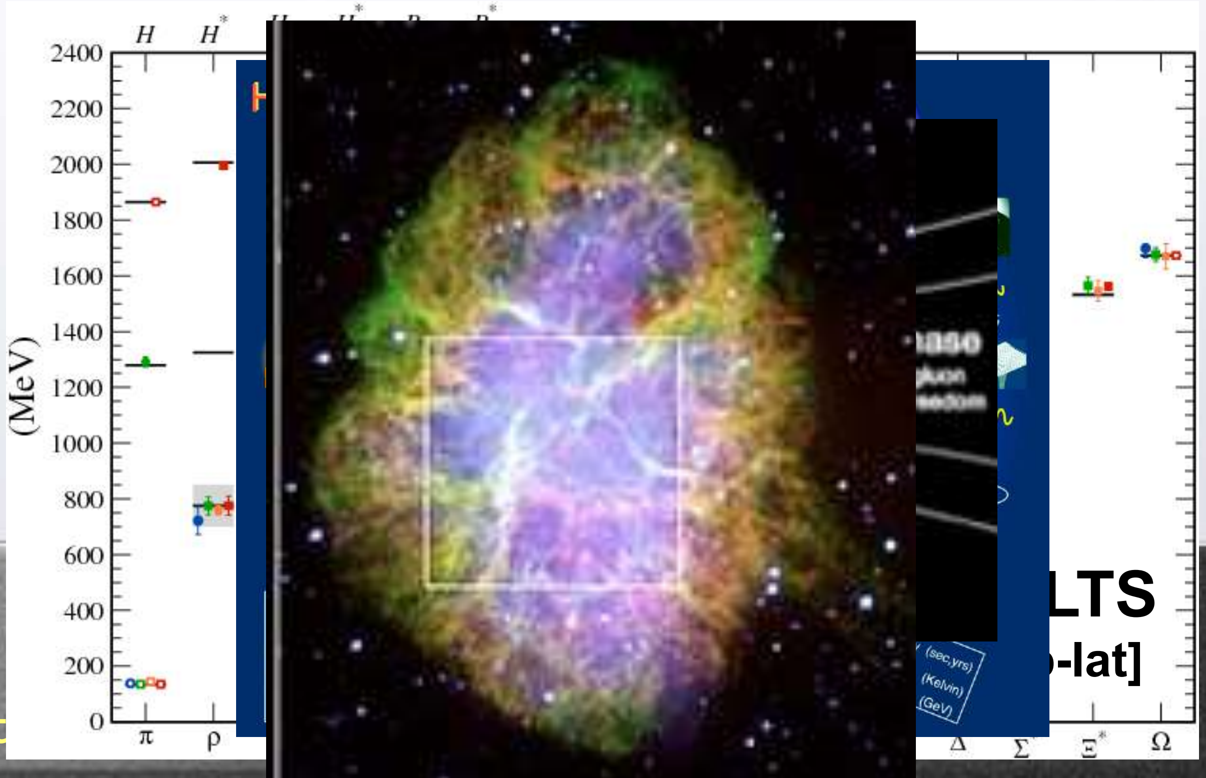
[Gluons] and quarks

[Glueballs,] baryons and mesons

What is the mechanism??

What happens to quarks and gluons in the IR??

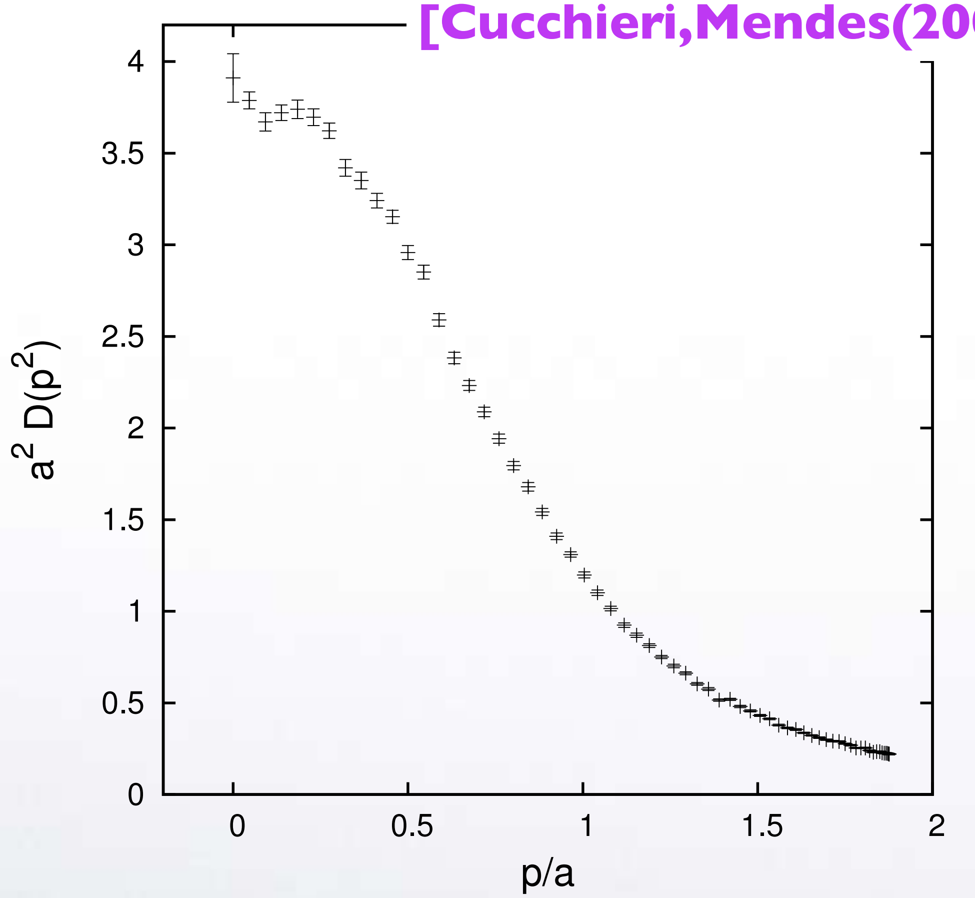
How to construct low-energy models with gluon dynamics??



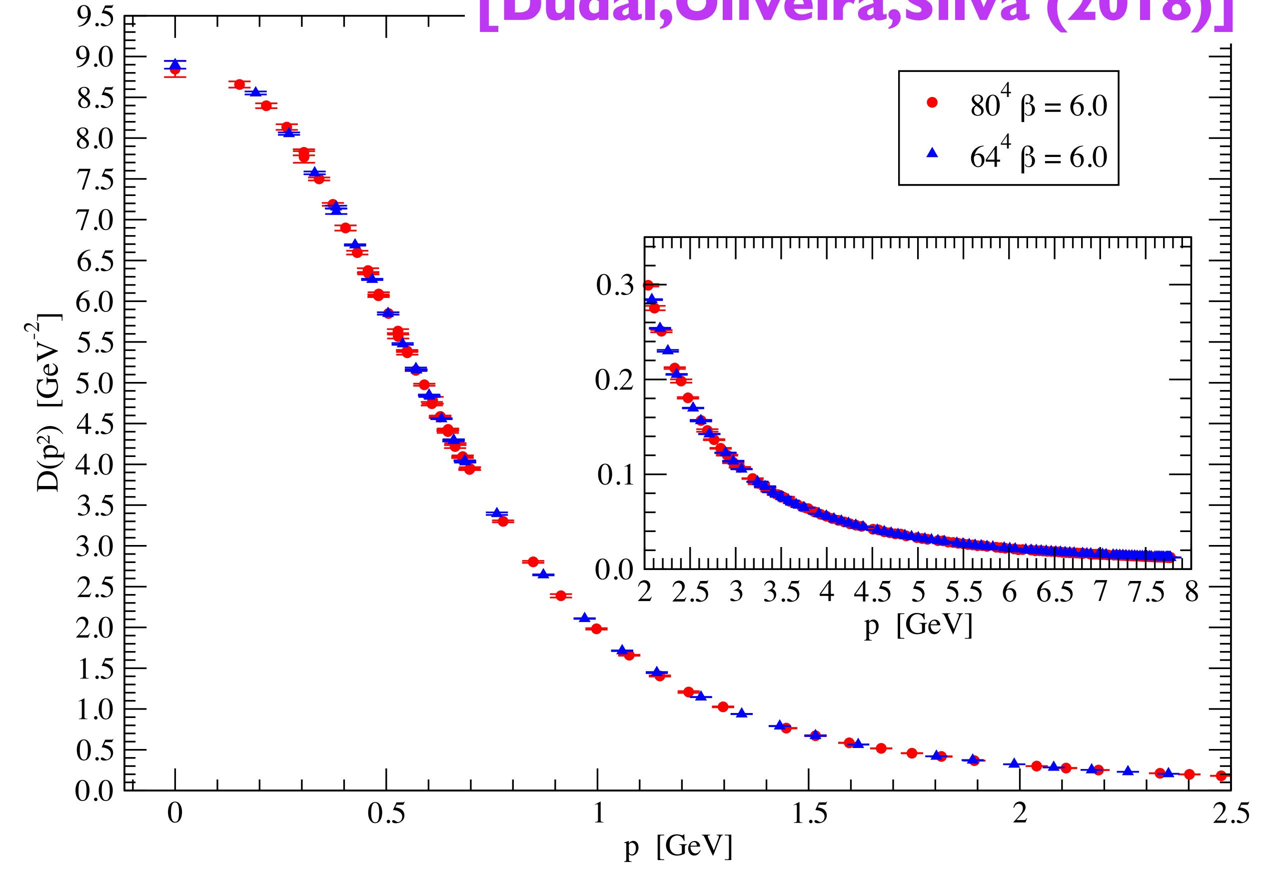
Gluon propagator in the infrared

- **Finite infrared gluon propagator in Landau gauge:**
  - early predictions in Dyson-Schwinger studies [Aguilar, Natale (2004); Frasca (2007)]
  - High-precision lattice YM results for large systems [Cucchieri, Mendes (2008)]

[Cucchieri,Mendes(2008)]



[Dudal,Oliveira,Silva (2018)]



Also confirmed by other lattice groups: [Bogolubsky et al (2009); Oliveira & Silva (2009)]

- FRG: Cyrol, Fister, Mitter, Pawlowski, Strodthoff (PRD 2016)
- Curci-Ferrari (massive) models: Pelaez, Reinosa, Serreau, Tissier, Wschebor (2015,2016)
- Gluon condensate from lattice QCD: Boucaud, Pene, Rodriguez-Quintero et al (2001)

[Gribov (1978)]

The Gribov problem:

- In the Landau gauge, for instance, the theory assumes the form

$$\int \mathcal{D}A\mathcal{D}\bar{c}\mathcal{D}c\mathcal{D}b\, e^{-S_{YM}+S_{gf}}$$

$$S_{gf} = b^a \partial_\mu A^a_\mu - \bar{c}^a \mathcal{M}^{ab} c^b \,, \qquad \mathcal{M}^{ab} = -\partial_\mu \left(\delta^{ab}\partial_\mu + g f^{abc} A^c_\mu\right)$$

- Gribov copies $\rightarrow$ zero eigenvalues of the Faddeev-Popov operator $\mathcal{M}^{ab}$.
- Copies cannot be reached by small fluctuations around $A = 0$ (perturbative vacuum) $\rightarrow$ perturbation theory works!
- Once large enough gauge field amplitudes have to be considered (non-perturbative domain) the copies will show up enforcing the effective breakdown of the Faddeev-Popov procedure.

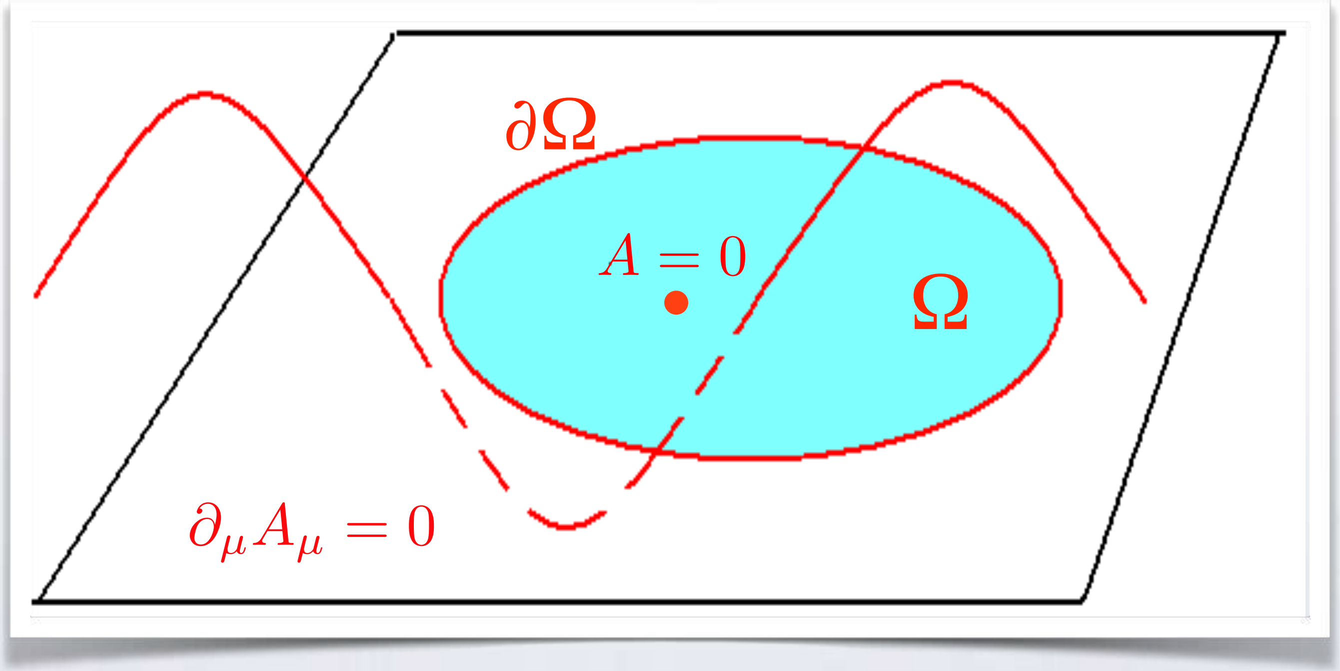
- Gribov proposed a way to eliminate (infinitesimal) Gribov copies from the integration measure over gauge fields: **the restriction to the (first) Gribov region Ω**

$$\int [DA]\delta(\partial A)\det(\mathcal{M})\mathrm{e}^{-S_{\mathrm{YM}}} \longrightarrow \int_\Omega [DA]\delta(\partial A)\det(\mathcal{M})\mathrm{e}^{-S_{\mathrm{YM}}} \qquad S_{\mathrm{YM}} = \frac{1}{4}\int_x F^2$$

with $\Omega = \left\{ A^a_\mu \;\; ; \;\; \partial A^a = 0, \mathcal{M}^{ab} > 0 \right\}$

$$\boxed{\mathcal{M}^{ab} = -\partial_\mu \left(\delta^{ab}\partial_\mu + f^{abc}A^c_\mu \right) = -\partial_\mu D^a_\mu}$$

(Faddeev-Popov operator)



- The **restriction** can be implemented as a **gap equation** for the vacuum energy obtained as:

[Zwanziger (1989,...)]

$$Z = e^{-V\mathcal{E}(\gamma)} = \int \mathcal{D}A\, \delta(\partial A)\, \det \mathcal{M}\, e^{-\left(S_{YM} \underbrace{+\gamma^4 H(A) - \gamma^4 V D(N^2-1)}_{=:\,+\gamma^4 \mathcal{H}}\right)}$$

Gap equation:

$$\frac{\partial \mathcal{E}(\gamma)}{\partial \gamma} = 0 \Rightarrow \langle H(A) \rangle_{1PI} = V D(N^2 - 1)$$

$$H(A) = \int_p \int_q A^a_\mu(-p) \left(\mathcal{M}^{ab}\right)^{-1} A^b_\mu(q)$$

- Using auxiliary fields, this can be cast in a *local* form: $Z = \int [\mathcal{D}\Phi]\, \delta(\partial A)\, \det \mathcal{M}\, e^{-S_{GZ}}$

- The GZ theory is unstable against the formation of certain dimension 2 condensates, giving rise to a refinement of the effective IR action:

[Dudal et al (2008)]

$$S_{\mathrm{YM}} \xrightarrow[\text{Gribov restriction(UV}\to\text{IR)}]{} S_{\mathrm{GZ}} = S_{\mathrm{YM}} + \gamma^4 \mathcal{H}$$

Dynamical generation of dim.2 condensates

$$\boxed{S_{\mathrm{RGZ}} = S_{\mathrm{YM}} + \gamma^4 \mathcal{H} + \frac{m^2}{2} AA - M^2 \left(\overline{\varphi}\varphi - \overline{\omega}\omega\right)}$$

Gap equation for the Gribov param.:

$$\frac{\partial \mathcal{E}(\gamma)}{\partial \gamma} = 0 \Rightarrow \langle H(A) \rangle_{1PI} = V D (N^2 - 1)$$

The parameters *M* and *m* are obtained via minimization of an effective potential for:

$$\langle \overline{\varphi}\varphi - \overline{\omega}\omega \rangle \neq 0 \qquad\qquad \langle A^2 \rangle \neq 0$$

- Non-perturbative effects included: $(\gamma, M, m) \propto \mathrm{e}^{-\frac{1}{g^2}}$

A (biased!) checklist for RGZ

- ✓ *(can be cast in a) local and renormalizable action*
- ✓ *reduces to QCD (pure gauge) at high energies?*

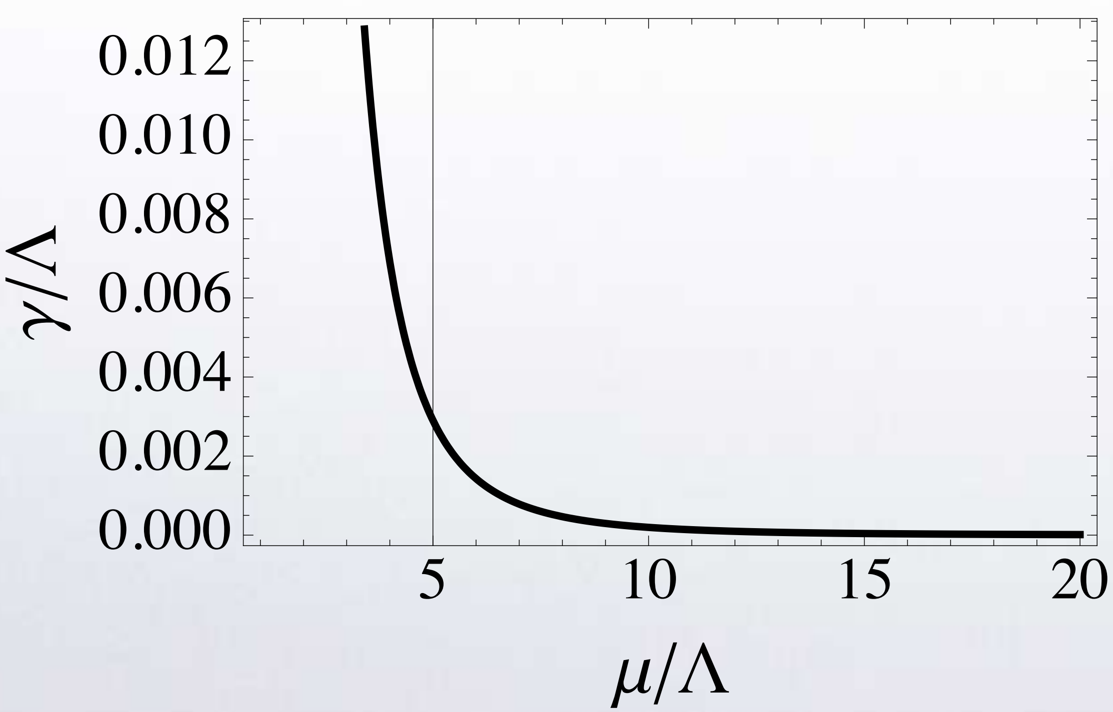
Gribov parameter in the UV

- The one-loop solution of the gap equation in the GZ theory gives:

$$2Ng^2\gamma^4 = \tilde{\gamma}^4 = \mu^4 \mathrm{e}^{\frac{5}{3} - \frac{128\pi^2}{3Ng^2}}$$

- Using the definition of the MSbar YM scale Λ (RG-invariant scale):

$$\frac{\tilde{\gamma}^4}{\Lambda} = \mathrm{e}^{5/12}\left[\frac{\Lambda}{\mu}\right]^{\frac{ab_0\pi}{2N}} \qquad \frac{ab_0\pi}{2N} \sim 3.9$$



$$\Lambda = 300\mathrm{MeV}$$

$$\tilde{\gamma}(\mu = 1\,\mathrm{GeV}) \sim 4\,\mathrm{MeV}$$
$$\tilde{\gamma}(\mu = 5\,\mathrm{GeV}) \sim 0.008\,\mathrm{MeV}$$

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✓ ***(can be cast in a) local and renormalizable action***

✓ ***reduces to QCD (pure gauge) at high energies***

✓ ***consistent with gluon 'confinement'? Confining propagator (no physical propagation; violation of reflection positivity)***

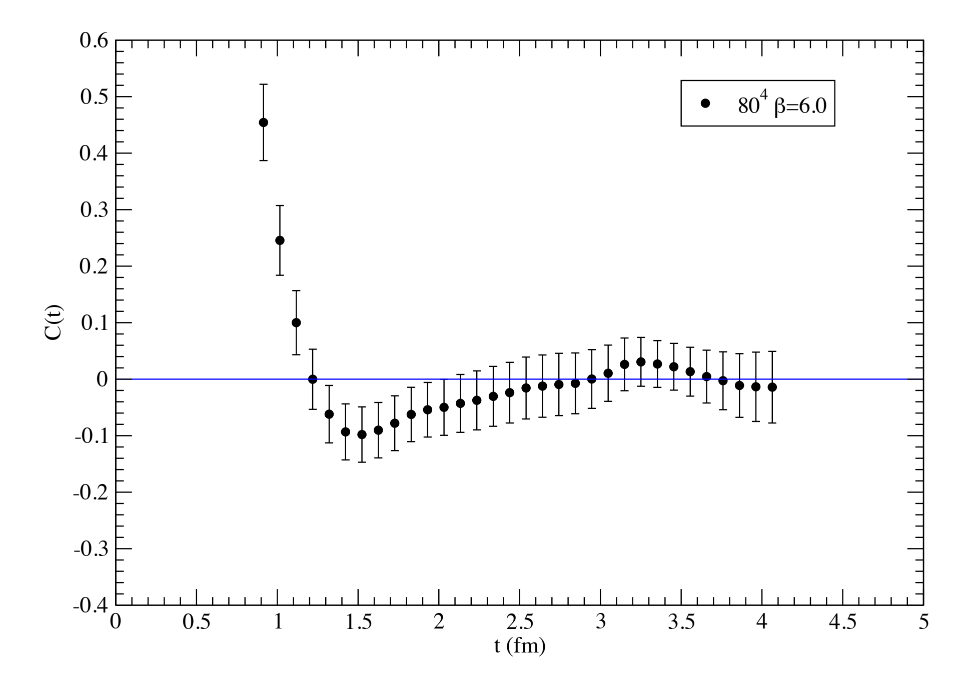
Schwinger function (computed directly from the gluon propagator):

$$C(t) = \int_{-\infty}^{\infty} \frac{\mathrm{d}p}{2\pi} D(p^2) \exp(-ipt).$$

Strictly positive if the gluon spectral function is physical:

$$C(t) = \int_0^{\infty} \mathrm{d}\omega \rho(\omega^2) e^{-\omega t}, \qquad D(p^2) = \int_0^{\infty} \mathrm{d}\mu \frac{\rho(\mu)}{\mu + p^2}.$$

Positivity violation for the gluon in lattice data:



SU(3) latt.: [Silva et al (2014)]

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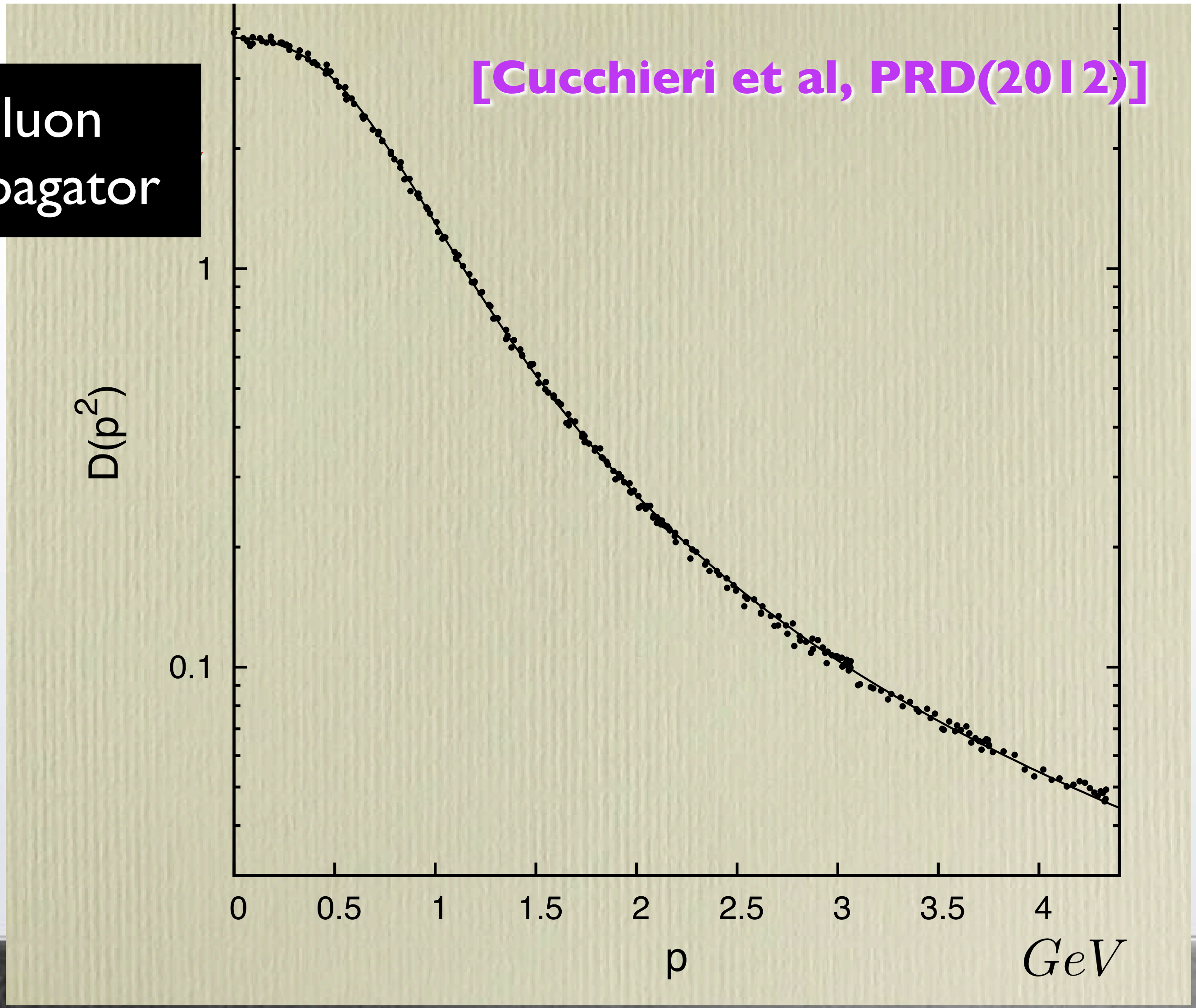
✓ *consistent with lattice IR results ?*

Gluon Propagator



$$\langle A^a_\mu A^b_\nu \rangle_p = \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) D(p^2)$$

$$D_{\rm fit}(p^2) = C \frac{p^2 + s}{p^4 + u^2 p^2 + t^2}$$

$$C = 0.56(0.01)\,,\ u = 0.53(0.04)\ \mathrm{GeV}\,,$$

$$t = 0.62(0.01)\ \mathrm{GeV}^2\,,\ u = 2.6(0.2)\ \mathrm{GeV}^2$$

poles: $m^2_\pm \ = \ (0.352 \pm 0.522i)\mathrm{GeV}^2$

$$D_{\rm RGZ}(p^2) = \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + 2g^2 N \gamma^4}$$

NB.: Complex conjugated poles!

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- ✓ *physical spectrum of bound states? Glueballs w/ masses compatible w/ lattice*
  [Dudal,Guimaraes,Sorella, PRL(2011), PLB(2014)]
- ✓ *other applications…*
- ✓ *Exact modified BRST invariance => gauge-parameter independence*
  [Capri *et al* (2016,2017)]

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[Dudal, Felix, LFP, Rondeau, Vercauteren, EPJC (2019)]

✗ *no quantitative prediction without fitting lattice data for propagators*

✗ *no general definition of physical operators, unitarity*

✗ *quark confinement properties: linear potential, etc…*

✗ *Minkowski space*

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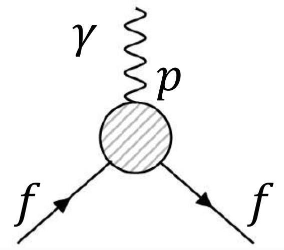
✗ ***other observables and phenomenological tests?***

Does a theory constructed with positivity-violating fundamental DOFs produce physical phenomenology?

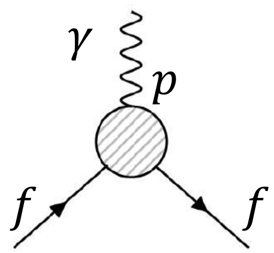
The quark-photon vertex and anomalous mag. mom.

- A different correlation function that is accessible by lattice simulations.
- The soft limit of the F_2 form factor is ***gauge independent*** and, in non-confined fermions, directly related to an ***observable***.



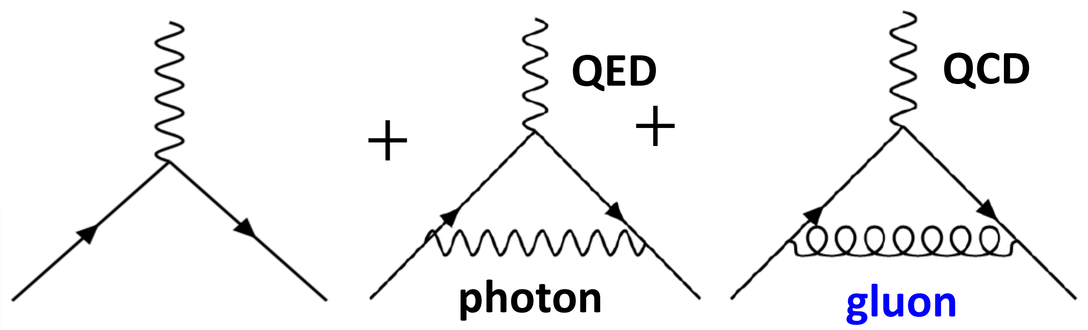
$$= e\, Q_f\, \bar{\boldsymbol{U}}(q_2)\left[\gamma_\mu\, \boldsymbol{F_1}(p^2) + \frac{p_\nu \sigma_{\mu\nu}}{2m}\, \boldsymbol{F_2}(p^2)\right] \boldsymbol{U}(q_1) \Rightarrow g = 2\left[\overset{1}{\cancel{F_1(0)}} + F_2(0)\right]$$

The quark-photon vertex and anomalous mag. mom.



$$= e\, Q_f\, \bar{\boldsymbol{U}}(q_2)\left[\gamma_\mu\, \boldsymbol{F_1}(p^2) + \frac{p_\nu \sigma_{\mu\nu}}{2m}\, \boldsymbol{F_2}(p^2)\right] \boldsymbol{U}(q_1) \Rightarrow g = 2\,[\overset{1}{\cancel{F_1(0)}} + F_2(0)]$$

- At one-loop:



…yielding a quark anomalous magnetic moment:

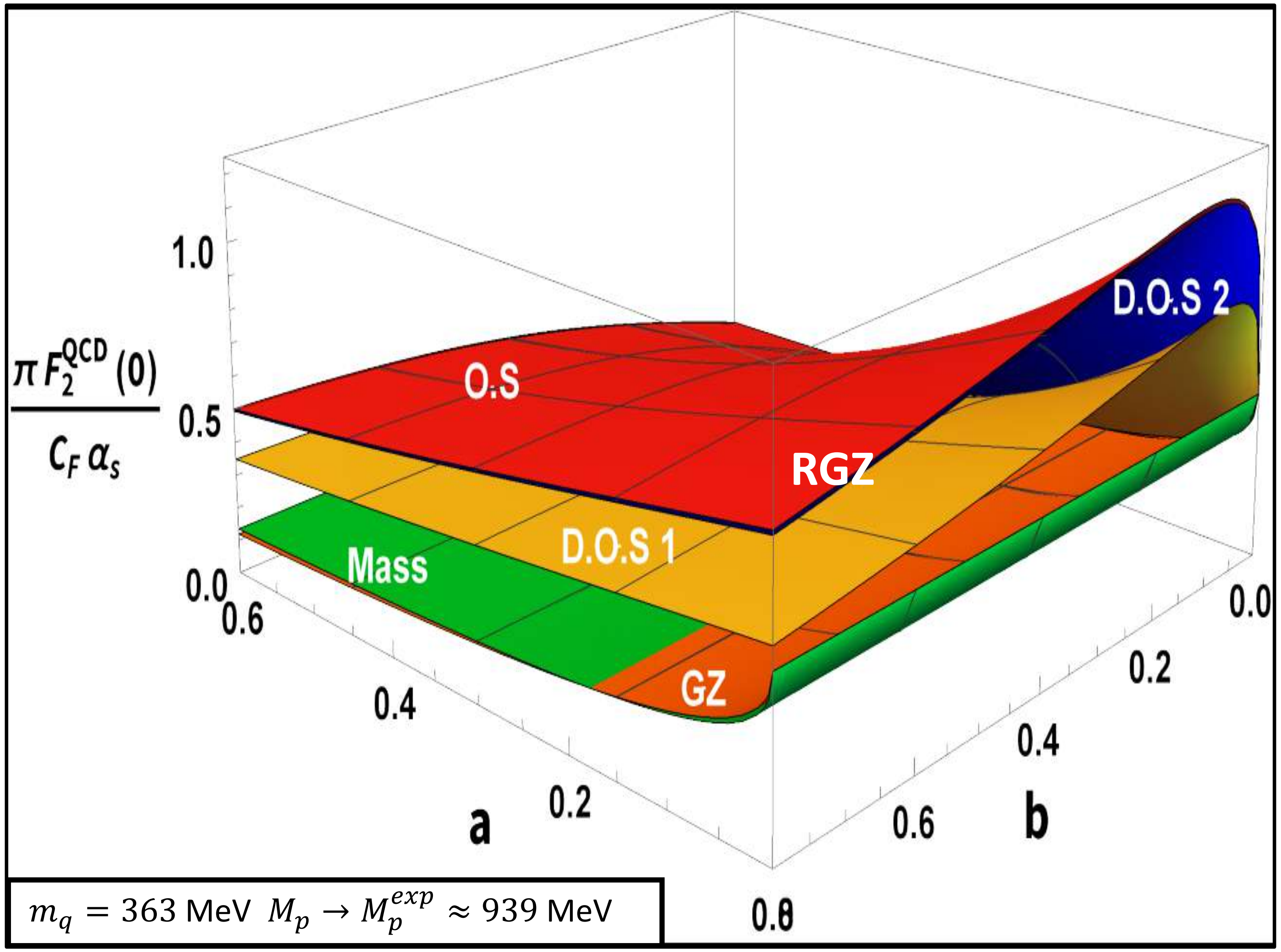
$$\mu_q = Q_q\left(\frac{e}{2m_q}\right)\left(1 + Q_q^2\left(\frac{\alpha}{2\pi}\right) + C_F\left(\frac{\alpha_s}{\pi}\right)\overline{F}_2(0)\right)$$

<u>**Gluon Confining models**</u>	**Massive**	**GZ**	**RGZ**
	$\dfrac{1}{l^2 + m_g^2}$	$\dfrac{l^2}{l^4 + \lambda^4}$	$\dfrac{l^2 + M_1^2}{l^4 + l^2 M_2^2 + M_3^4}$

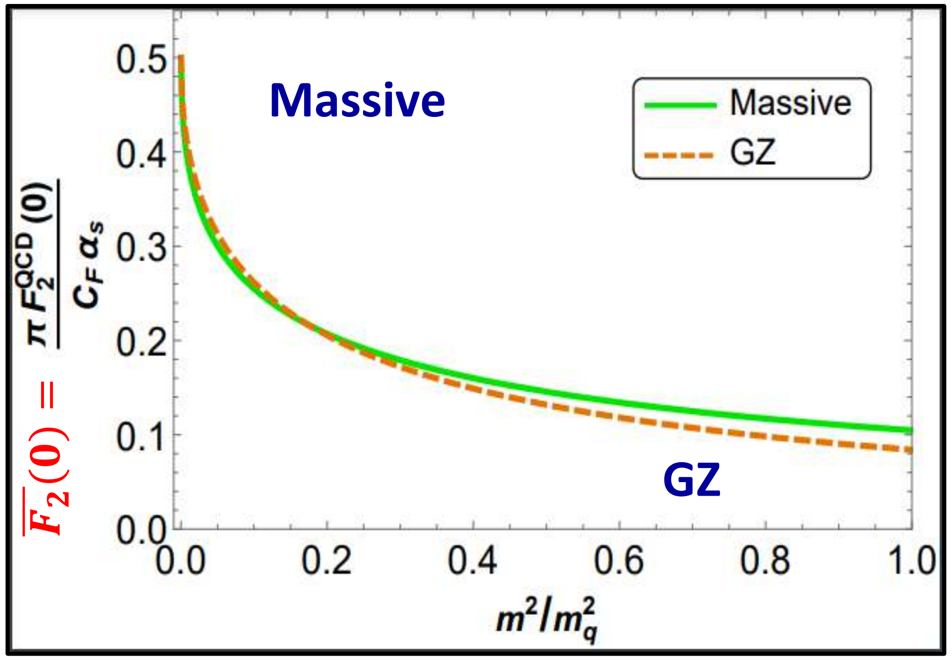
[Mena & LFP, PRD (2024)]

[Mena & LFP, PRD (2024)]

$\overline{F_2}(0)$ RESULTS



*a and b are ratios of the gluon mass parameters and the quark mass.



RGZ:

DATA	M [GeV]	m [GeV]	λ [GeV]	$\overline{F_2}(0)$
O.S	2.1150	1.9414 i	2.0381	0.2456
D.O.S 2	2.0389	1.8881 i	1.9730	0.2658

Massive parameters fit from lattice data:
Oliveira & Silva, PRD (2012)
Dudal, Oliveira & Silva, Ann. Phys. (2018)

Estimating the proton AMM from confining models...

We adopt the simplest Constituent Quark Model to estimate the effect on the proton AMM:

$$\boxed{\mu_p^{CQM} = \left[\frac{4}{3}\mu_u - \frac{1}{3}\mu_d\right]} \quad \text{with} \quad \mu_q = Q_q\left(\frac{e}{2m_q}\right)\left(1 + Q_q^2\left(\frac{\alpha}{2\pi}\right) + C_F\left(\frac{\alpha_s}{\pi}\right)\overline{F}_2(0)\right)$$

- CQM parameters:
  constituent quark mass fixed to proton mass

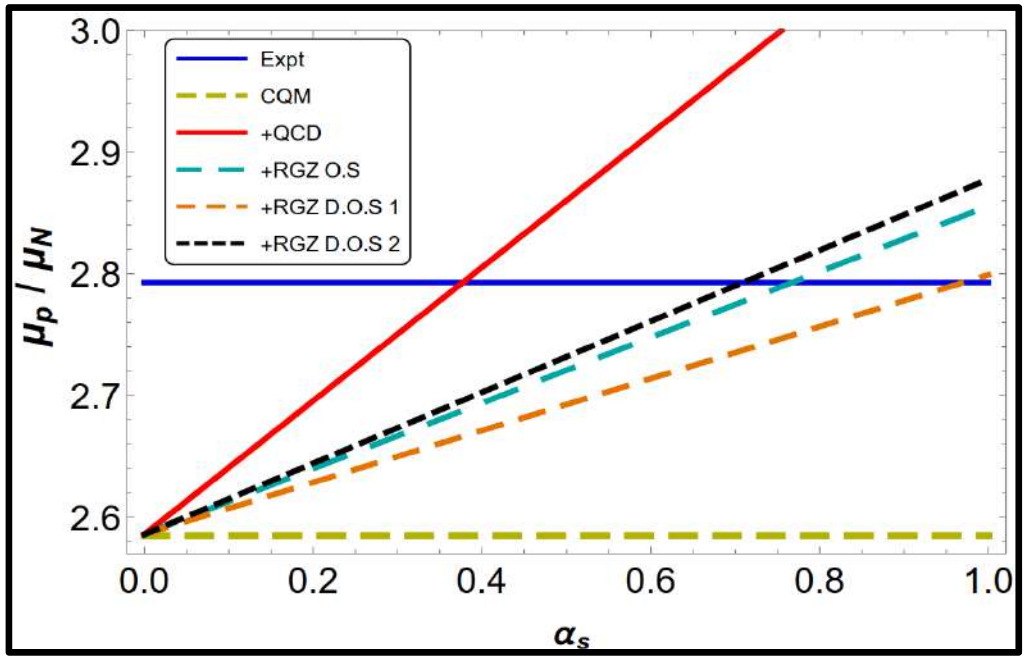
$$m_q = 363 \text{ MeV} \;\; M_p \to M_p^{exp} \approx 939 \text{ MeV}$$

[Mena & LFP, PRD (2024)]

- Confining model parameters:
  dynamically generated gluon mass(es)
  +
  strong coupling in the deep IR

m_g	0 MeV	140 MeV	185.64 MeV	600 MeV
α_s ‖ $\lambda_{CF} = 3\alpha_s/4$	0.38 ‖ 0.091	0.83 ‖ 0.198	1.00 ‖ 0.239	3.24 ‖ 0.773

RGZ model



Estimating the proton AMM from confining models…

1. Confining models — even with complex conjugated poles — yield reasonable results;

2. Dynamically generated gluon masses can be accomodated
   if the strong coupling is large enough in the IR (or changing other CQM parameters…)

3. Still hard to constrain models, but lattice data may help.

- CQM parameters:
  constituent quark mass fixed to proton mass

  $$m_q = 363\text{ MeV}\ \ M_p \to M_p^{exp} \approx 939\text{ MeV}$$

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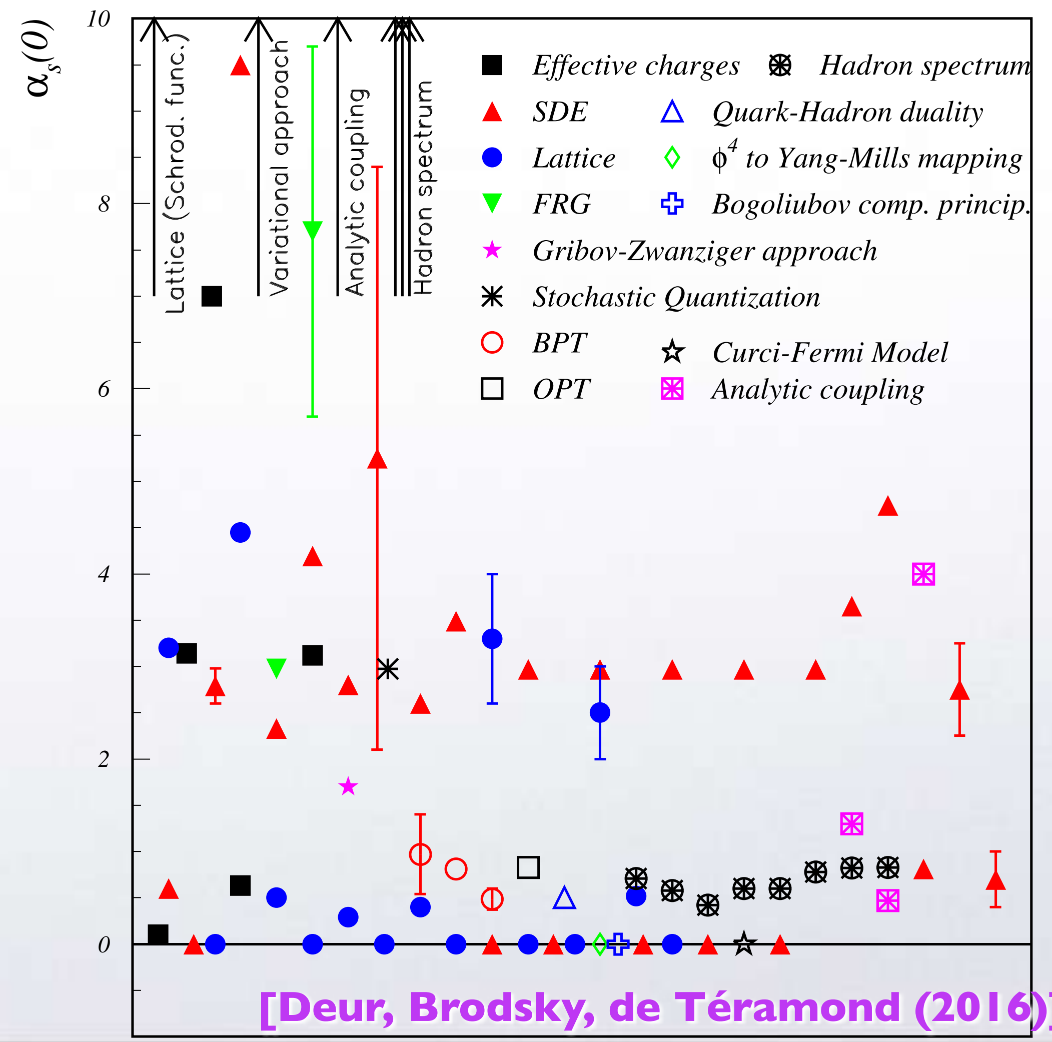


[Deur, Brodsky, de Téramond (2016)]

Testing IR models with color SUC phenomenology

- Color superconductivity mediated by gluons at intermediate to high densities should probably be affected by nonperturbative modifications of the gluon propagator;

- **<u>Aims here:</u>** **[see talk by J.P.S. Santos on Friday!]**

  Do complex-conjugated poles generate non-physical features in SUC?

  How is the SUC gap influence by the presence of massive gluon parameters?

  Could one discriminate between the predictions of different IR models?

[Santos & LFP, to appear; Dudal, LFP & Santos, *in progress*]

Toy model for color SUC wiht confining propagators

- For a simple testing ground, we choose a Yukawa theory with the (scalar) bosons displaying propagators with IR modifications (Curci-Ferrari, GZ or RGZ-inspired):

IR-modified boson propagators

$$S = \int_x \mathscr{L} = \int_{x,y} \left[\overline{\psi}(x) G_0^{-1}(x,y)\psi(y) - \frac{1}{2}\varphi(x) D^{-1}(x,y)\varphi(y) \right] - g\int_x \overline{\psi}(x)\psi(x)\varphi(x)\,,$$

Dirac (free), finite density => $G_0^{-1}(x,y) = \delta(x-y)(i\gamma^\mu \partial_\mu + \gamma^0 \mu - m)$

Massive	GZ	RGZ
$\frac{1}{l^2 + m_g^2}$	$\frac{l^2}{l^4 + \lambda^4}$	$\frac{l^2 + M_1^2}{l^4 + l^2 M_2^2 + M_3^4}$

Toy model for color SUC with confining propagators

- Integrating out the boson, one arrives at a 4-fermion theory that is convenient to study SUC:

IR-modified boson propagators

$$S' = \int_{x,y} \left[\overline{\psi}(x) G_0^{-1}(x,y) \psi(y) + \frac{g^2}{2} \overline{\psi}(x)\psi(x) D(x,y) \overline{\psi}(y)\psi(y) \right]$$

- To describe the di-quark condensate, one can transform to Nambu Gorkov space, with a charge-conjugate spinor defined by:

$$\psi_C \equiv C\overline{\psi}^T \ , \qquad\qquad C = i\gamma^2\gamma^0$$

$$\overline{\psi}(x)\psi(x)\overline{\psi}(y)\psi(y) = \frac{1}{2}[\overline{\psi}_C(x)\psi_C(x)\overline{\psi}(y)\psi(y) + \overline{\psi}(x)\psi(x)\overline{\psi}_C(y)\psi_C(y)]$$

$$= -\frac{1}{2}\mathrm{Tr}[\psi_C(x)\overline{\psi}(y)\psi(y)\overline{\psi}_C(x) + \psi(x)\overline{\psi}_C(y)\psi_C(y)\overline{\psi}(x)]$$

Toy model for color SUC with confining propagators

- Introducing the di-quark condensate in mean-field approximation:

$$S'' = \int_{x,y} \left\{ \overline{\psi}(x) G_0^{-1}(x,y) \psi(y) + \frac{1}{2} [\overline{\psi}_C(x) \Phi^+(x,y) \psi(y) + \overline{\psi}(x) \Phi^-(x,y) \psi_C(y)] \right\}$$

$$\boxed{\begin{aligned} \Phi^+(x,y) &\equiv g^2 D(x,y) \langle \psi_C(x) \overline{\psi}(y) \rangle \,, \\ \Phi^-(x,y) &\equiv g^2 D(x,y) \langle \psi(x) \overline{\psi}_C(y) \rangle \,. \end{aligned}}$$

- or, in terms of the inverse propagator matrix in Nambu-Gorkov space:

$$Z = \mathscr{N} Z_{\mathrm{bosons}} Z_0 \int \mathscr{D}\overline{\Psi}\, \mathscr{D}\Psi \exp\left[\sum_{k>0} \overline{\Psi}(k) \frac{\mathscr{S}^{-1}(k)}{T} \Psi(k) \right]$$

$$\boxed{\mathscr{S}^{-1}(k) = \begin{pmatrix} [G_0^+(k)]^{-1} & \Phi^-(k) \\ \Phi^+(k) & [G_0^-(k)]^{-1} \end{pmatrix}} \qquad \boxed{\Psi \equiv \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}}$$

Toy model gap equation

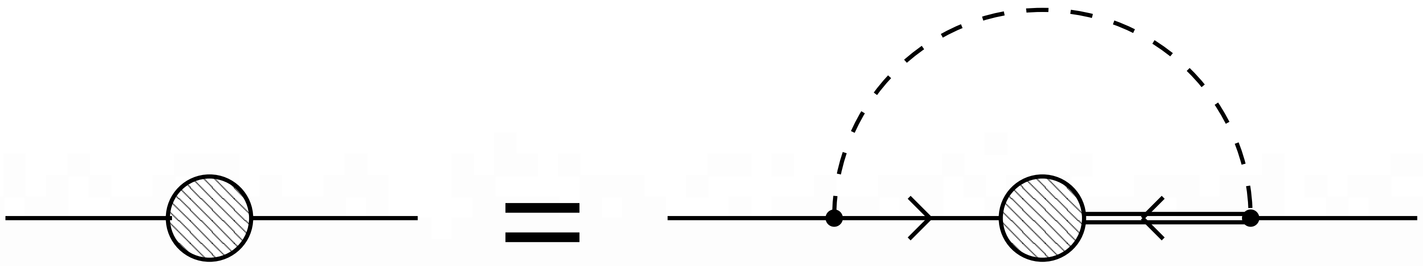
- Implementing a Dyson resummation, the full propagator matrix is written as:

$$\mathbf{S}(k) = \begin{pmatrix} G^+ & F^- \\ F^+ & G^- \end{pmatrix}$$

$$G^\pm \equiv \left([G_0^\pm]^{-1} - \Phi^\mp G_0^\mp \Phi^\pm\right)^{-1}$$

$$F^\pm \equiv -G_0^\mp \Phi^\pm G^\pm$$

$$\boxed{\Phi^+(x,y) = -g^2 D(x,y) F^+(x,y)}$$



$$\boxed{\Phi^+(p) = g^2 \frac{T}{V} \sum_k D(p-k) G_0^- \Phi^+ G^+}$$

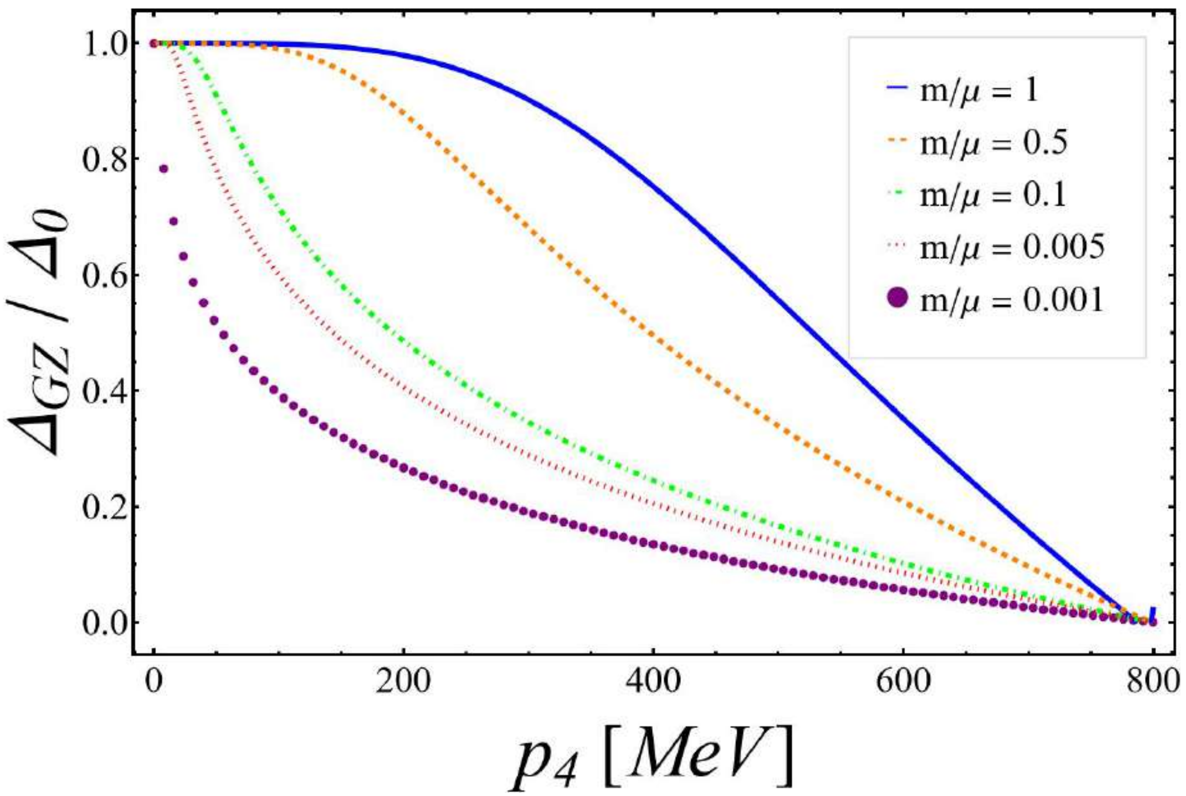
IR-modified boson propagators

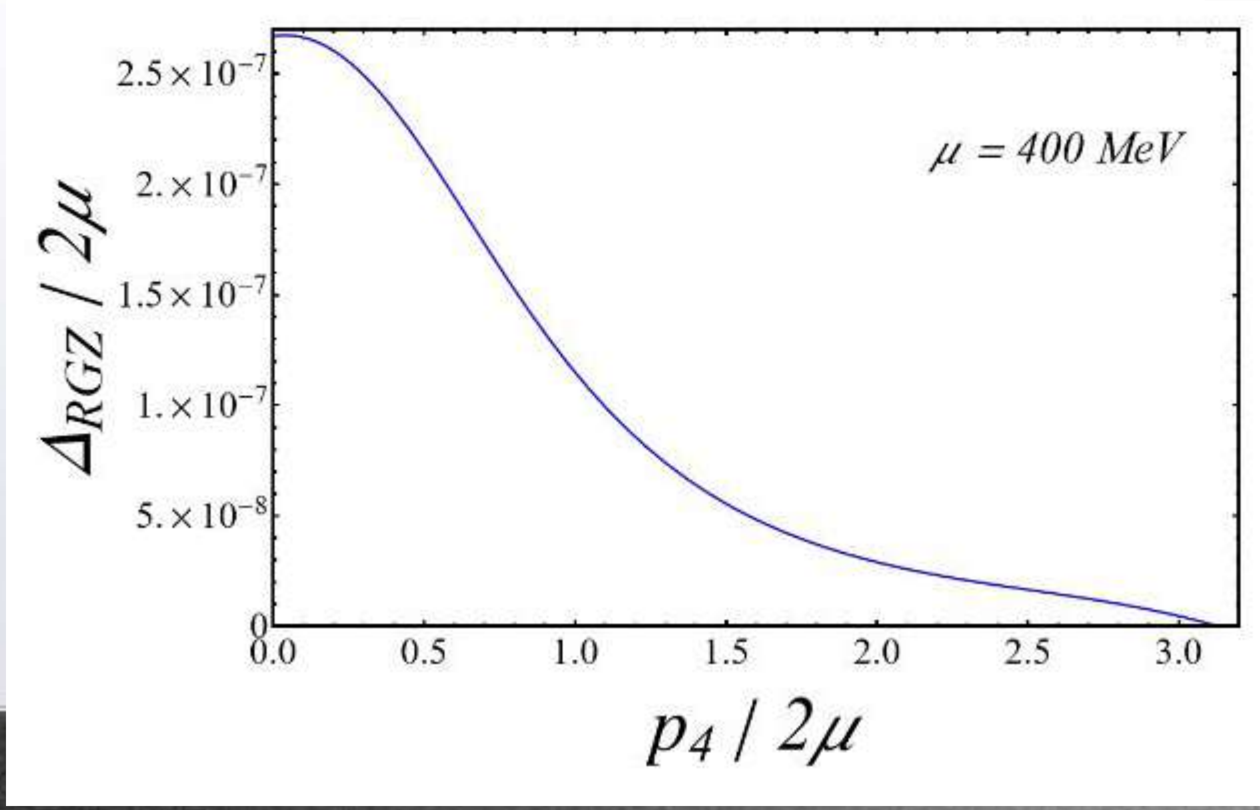
$$\boxed{\Phi^\pm(K) = \pm\Delta(K)\gamma^5 ,}$$

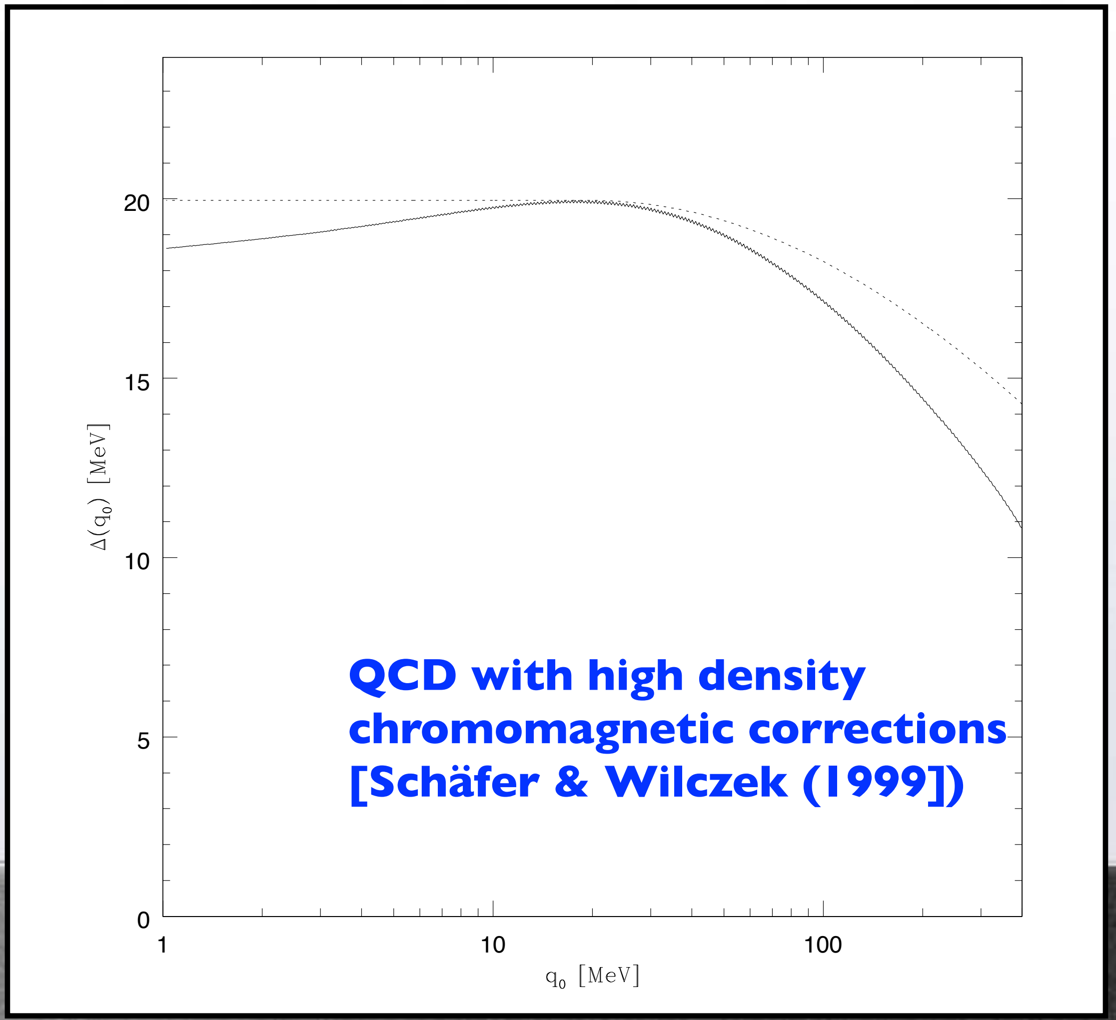
(even parity, spin-singlet pairing)

D. Bailin, A. Love, Phys.Rept. **107**, 325 (1984)
R.D. Pisarski, D.H. Rischke, Phys.Rev. **D60**, 094013 (1999)

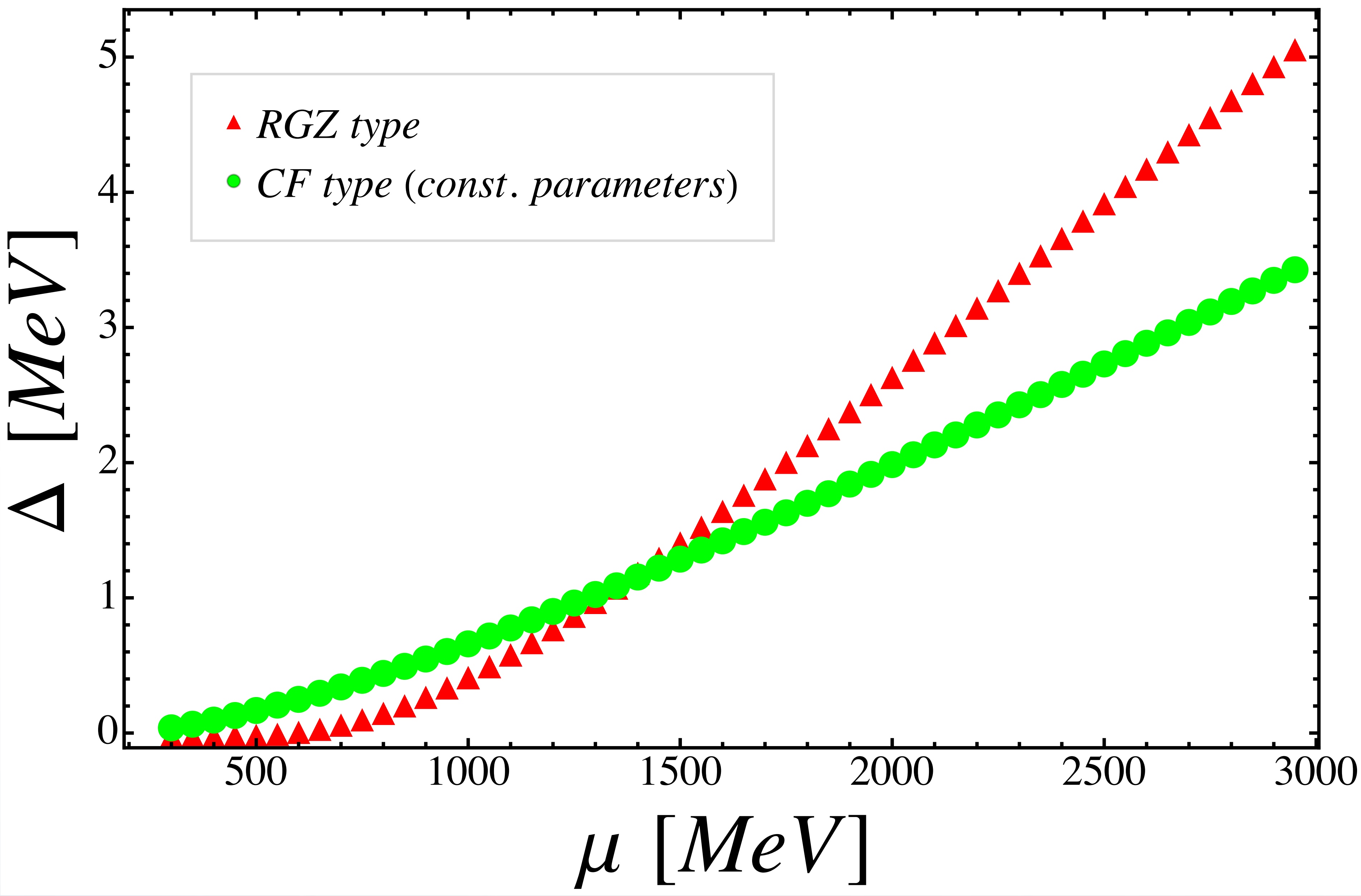
[Santos & LFP, *appear*]



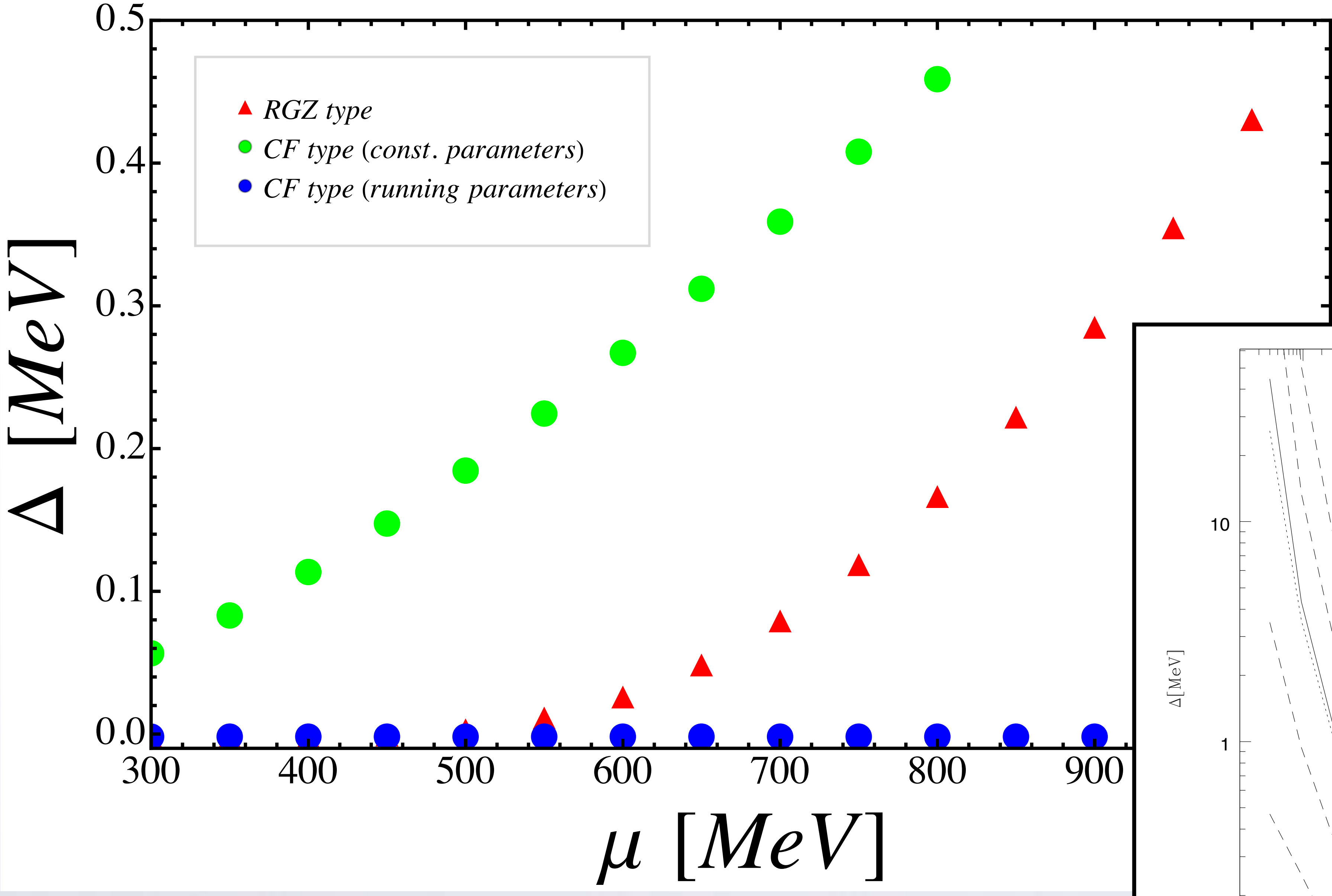


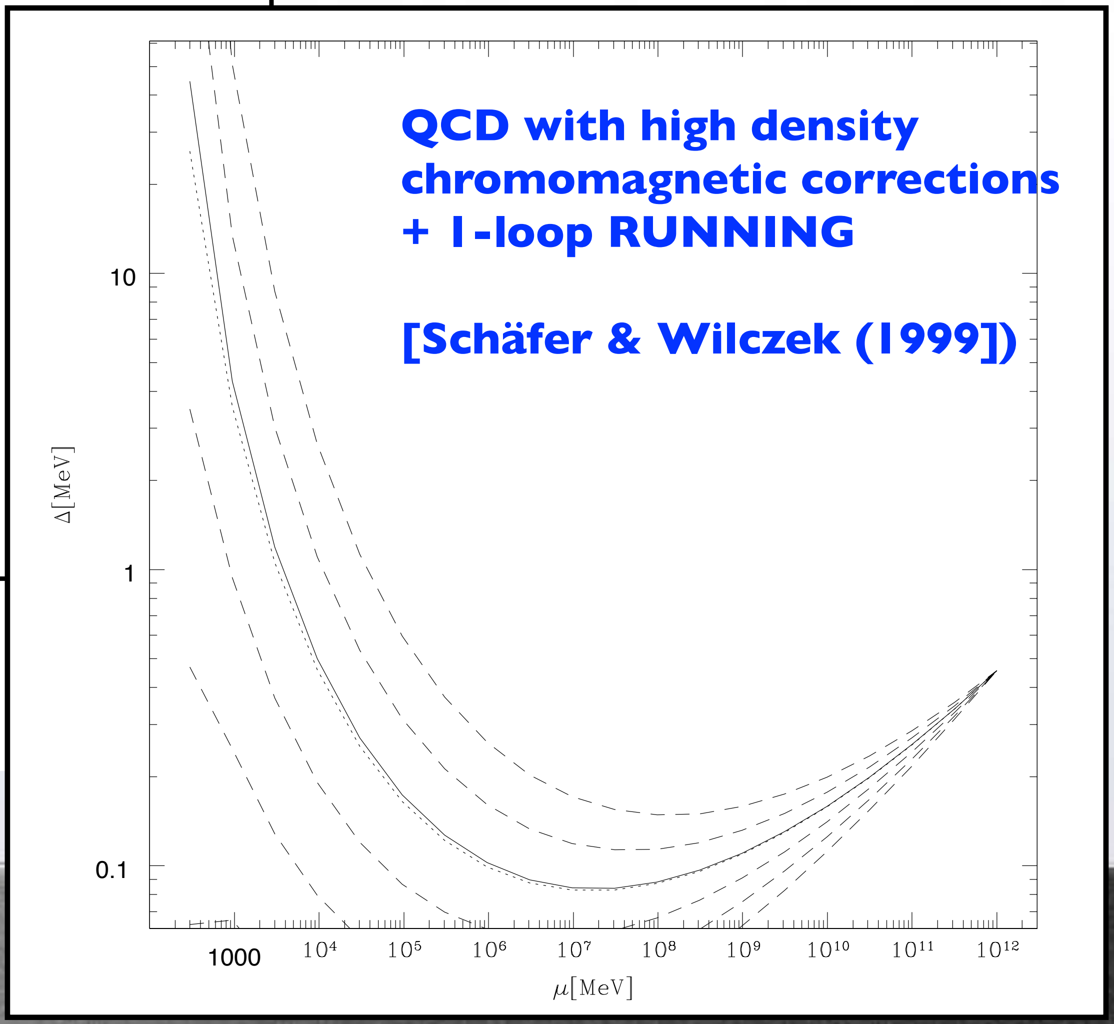


Results for the SUC gap



- Infrared-safe inspired running suppresses the SUC gap in the Yukawa toy model:





Final comments

- ***Dynamical gluon mass generation should occur in IR YM theories.***
- The ***Gribov problem*** is present and should profoundly affect the IR regime of gauge-fixed non-Abelian gauge theories.
- The ***RGZ framework*** represents a consistent scenario to study the non-perturbative IR physics and has provided ***interesting results for correlation functions in the gluon sector fitting lattice propagators***.
- The q-qbar-photon may be calculated on the lattice and offers a window to observables like the anomalous magnetic moment (possibility of parameter and/or model constraining)
- Color SUC is also sensitive to the nonperturbative gluon mass and IR models yield physical results, with in general a suppression of the value of the gap in the toy model studied.
- *Construct low energy effective models with nonperturbative gluons and quarks!*

Thank you for your attention!

BACKUP SLIDES

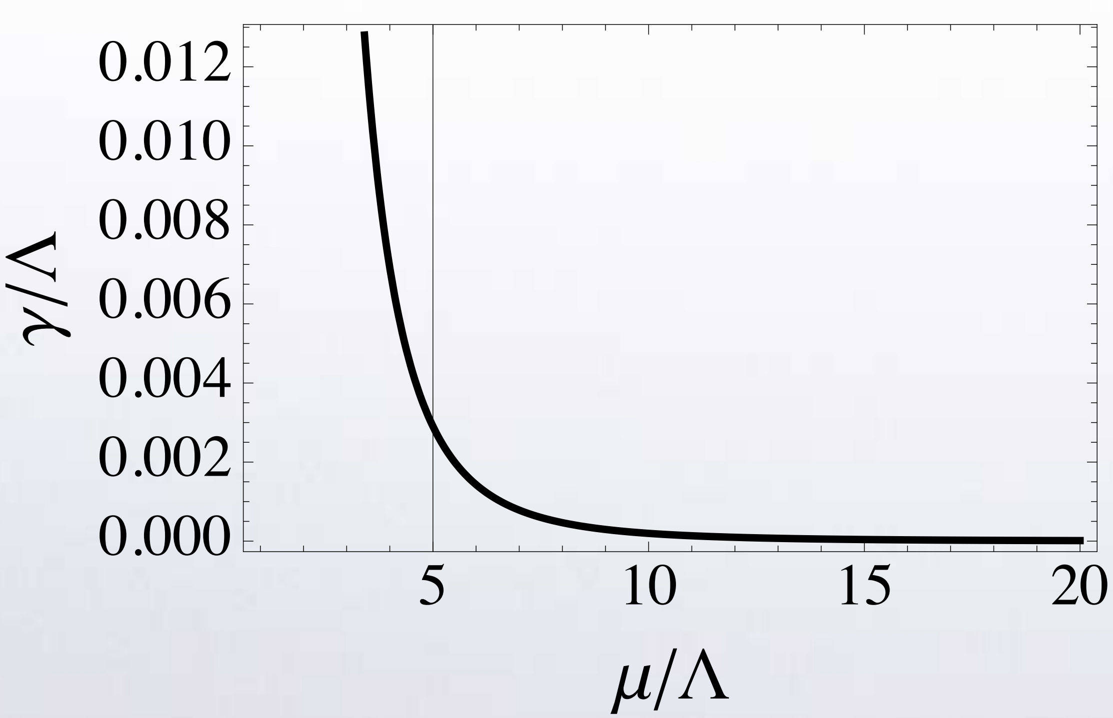
Gribov parameter in the UV

- The one-loop solution of the gap equation in the GZ theory gives:

$$2Ng^2\gamma^4 = \tilde{\gamma}^4 = \mu^4 \mathrm{e}^{\frac{5}{3} - \frac{128\pi^2}{3Ng^2}}$$

- Using the definition of the MSbar YM scale Λ (RG-invariant scale):

$$\frac{\tilde{\gamma}^4}{\Lambda} = \mathrm{e}^{5/12}\left[\frac{\Lambda}{\mu}\right]^{\frac{ab_0\pi}{2N}} \qquad\qquad \frac{ab_0\pi}{2N} \sim 3.9$$



$$\Lambda = 300\mathrm{MeV}$$

$$\tilde{\gamma}(\mu = 1\,\mathrm{GeV}) \sim 4\,\mathrm{MeV}$$
$$\tilde{\gamma}(\mu = 5\,\mathrm{GeV}) \sim 0.008\,\mathrm{MeV}$$

BRST-invariant (R)GZ framework in a nutshell

- A gauge-invariant gluon field: [Dell'Antonio & Zwanziger ('89), van Ball ('92), Lavelle & McMullan ('96)]

$$f_A[u] \equiv \min_{\{u\}} \mathrm{Tr} \int d^dx\, A^u_\mu A^u_\mu \qquad A^h_\mu = \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2}\right)\phi_\nu$$

$$A^u_\mu = u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u \qquad \phi_\nu = A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu \frac{1}{\partial^2}\partial A\right] + \mathcal{O}(A^3)\,.$$

- Localization is possible through the introduction of a Stueckelberg field ξ^a:

$$A^h_\mu = (A^h)^a_\mu T^a = h^\dagger A^a_\mu T^a h + \frac{i}{g} h^\dagger \partial_\mu h\,, \qquad h = e^{ig\,\xi^a T^a}$$

- The BRST-invariant Gribov region and condensates: **[Capri *et al* (2016,2017)]**

$$\Omega = \{A^a_\mu;\ \partial_\mu A^a_\mu = i\alpha b^a, \qquad \mathcal{M}^{ab}(A^h) = -\partial_\mu D^{ab}_\mu(A^h) > 0\}$$ (ex. in Linear Cov. Gauges)

$$\langle A^a_\mu(p) A^b_\nu(-p)\rangle = \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + M^2 m^2 + \lambda^4}\mathcal{P}_{\mu\nu}(p)\delta^{ab} + \frac{\alpha_g}{p^2} L_{\mu\nu}\delta^{ab}$$

$\langle \bar\varphi^{ab}_\mu \varphi^{ab}_\mu \rangle$

$\langle A^{h,a}_\mu A^{h,a}_\mu \rangle$

$$\boxed{\begin{aligned} & sA^a_\mu = -D^{ab}_\mu c^b\,, \\ & sc^a = \frac{g}{2} f^{abc} c^b c^c\,, \qquad s\bar c^a = ib^a \\ & sb^a = 0\,, \\ & s\varphi^{ab}_\mu = 0\,, \qquad s\omega^{ab}_\mu = 0\,, \\ & s\bar\omega^{ab}_\mu = 0\,, \qquad s\bar\varphi^{ab}_\mu = 0\,, \\ & s\varepsilon^a = 0\,, \qquad s(A^h)^a_\mu = 0\,, \\ & sh^{ij} = -igc^a (T^a)^{ik} h^{kj}\,. \end{aligned}}$$

- To explicitly calculate the values of the condensates in RGZ, one should construct an effective potential for the composite operators:

$$\Sigma[\cdots,\tau,Q] = S + \tau \; \underbrace{\boxed{\frac{1}{2}A^{h,a}_{\mu}A^{h,a}_{\mu}}}_{O_A} + Q \; \underbrace{\boxed{\bar{\varphi}^{ac}_{\mu}\varphi^{ac}_{\mu}}}_{O_\varphi} \xrightarrow{\text{Legendre transf.}} \Gamma[O_A, O_\varphi] \xrightarrow{\text{Minimize}} \langle O_I \rangle$$

- For composite operators (mass dimension 2 or higher) a lot of complications appear!

- In the non-BRST-invariant formulation of RGZ, there could be many more condensates and the full effective potential calculation was never achieved.

[cf. Dudal, Sorella & Vandersickel (2011)]

Quantizing Yang-Mills theory

$$S_{YM} = \frac{1}{4}\int dx\, F^a_{\mu\nu}F^a_{\mu\nu},$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

- The quantum theory may be formulated in a path-integral approach:

$$\int \mathcal{D}A\, e^{-S_{YM}}$$

- Gauge reduncancy must be properly fixed to work with these dofs:

$$A_\mu \to U A_\mu U^\dagger - U\partial_\mu U^\dagger$$

- We only know how to do it perturbatively!

Faddeev-Popov procedure:

- The procedure amounts to disentangle the gauge redundancy from the integral measure

$$\int \mathcal{D}A\, e^{-S_{YM}} \to \int \mathcal{D}\Omega \int \mathcal{D}A\, \delta[G(A)]\, \det \mathcal{M} e^{-S_{YM}}$$

supposing we can write

$$\int \mathcal{D}\Omega\, \delta[G(A)]\, \det \mathcal{M} = \mathbb{1}$$

with the Faddeev-Popov operator

$$\mathcal{M}^{ab}(A) = \left.\frac{\delta G^a[A^{(g)}]}{\delta \Omega^b}\right|_{\Omega=0}$$

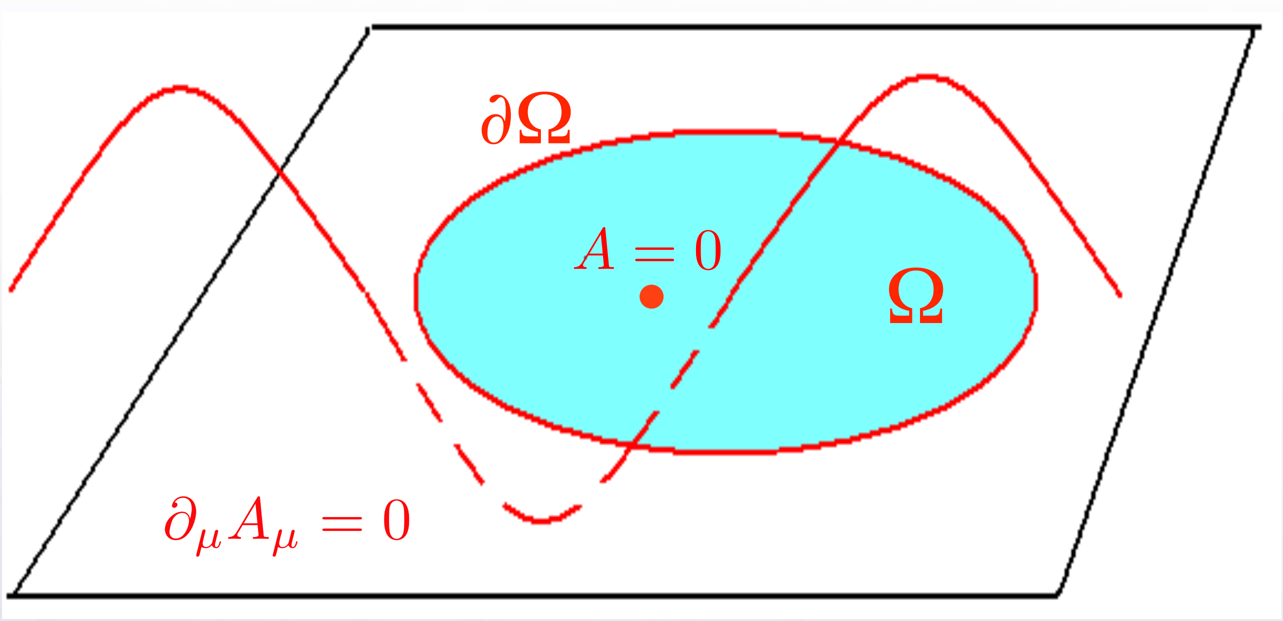
- Gribov proposed a way to eliminate (infinitesimal) Gribov copies from the integration measure over gauge fields: **the restriction to the (first) Gribov region Ω**

$$\int [DA]\delta(\partial A)\det(\mathcal{M})\mathrm{e}^{-S_{\rm YM}} \longrightarrow \int_\Omega [DA]\delta(\partial A)\det(\mathcal{M})\mathrm{e}^{-S_{\rm YM}} \qquad S_{\rm YM}=\frac{1}{4}\int_x F^2$$

with $\Omega = \left\{A^a_\mu \;\; ; \;\; \partial A^a = 0, \mathcal{M}^{ab} > 0\right\}$

$$\boxed{\mathcal{M}^{ab} = -\partial_\mu\left(\delta^{ab}\partial_\mu + f^{abc}A^c_\mu\right) = -\partial_\mu D^a_\mu}$$

(Faddeev-Popov operator)



- The FP operator is related to the ghost 2-point funtion:

$$\mathcal{G}^{ab}(k;A) = \langle k|c^a\bar{c}^b|k\rangle = \langle k|\left(\mathcal{M}^{ab}\right)^{-1}|k\rangle$$

positivity of $\mathcal{M}^{ab}$ $\longleftrightarrow$ ***No-pole condition*** for the ghost prop.

[Gribov (1978)]

A checklist for RGZ

✓ *(can be cast in a) local and renormalizable action*

✓ *reduces to QCD at high energies*

✓ *gluon confinement: confining propagator (no physical propagation; violation of reflection positivity)*

✓ *consistent with lattice IR results*

✓ *physical spectrum of bound states ??*

Glueball masses are obtained by computing two-point correlation functions of composite operators with the appropriate quantum numbers and casting them in the form of a Källén-Lehmann spectral representation.

J^{PC}	confining gluon propagator
0^{++}	2.27
2^{++}	2.34
0^{-+}	2.51
2^{-+}	2.64

[Dudal,Guimaraes,Sorella, PRL(2011), PLB(2014)]

-Lattice: (1) Y. Chen *et al.* PRD **73**, 014516 (2006)
-Flux tube model: M. Iwasaki *et al.* PRD **68**, 074007 (2003).
-Hamiltonian QCD: A. P. Szczepaniak and E. S. Swanson, PLB **577**, 61 (2003).
-AdS/CFT: K. Ghoroku, K. Kubo, T. Taminato and F. Toyoda, arXiv:1111.7032.
-AdS/CFT: H. Boschi-Filho, N. R. F. Braga JHEP 0305, 009 (2003)

J^{PC}	Lattice	Flux tube model	Hamiltonian QCD	ADS/CFT
0^{++}	1.71	1.68	1.98	1.21
2^{++}	2.39	2.69	2.42	2.18
0^{-+}	2.56	2.57	2.22	3.05
2^{-+}	3.04	–	–	–

RGZ: Correct hierarchy of masses