

S Paulo Spt 2025

From cavity QED to quantum simulations with Rydberg atoms

Lecture 1

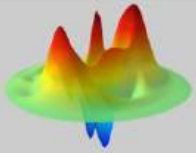
The strong coupling regime Resonant interaction



Michel Brune



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Université Pierre et Marie Curie,
Collège de France, Paris



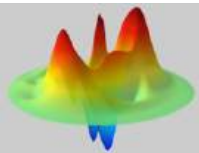
« Ridiculous » quantum phenomena

Schrödinger 1952 :

« one never experiments with just **one** electron, **one** atom or **one** molecule. In thought experiments we sometimes assume that we do, this invariably entails **ridiculous consequences**... »

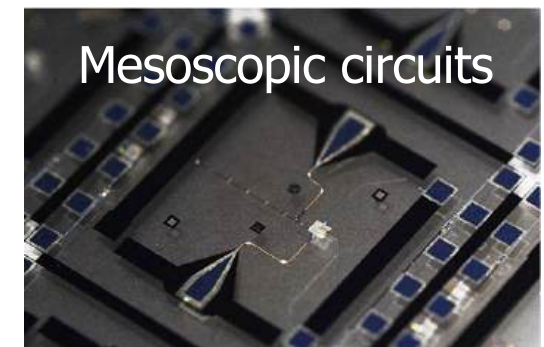
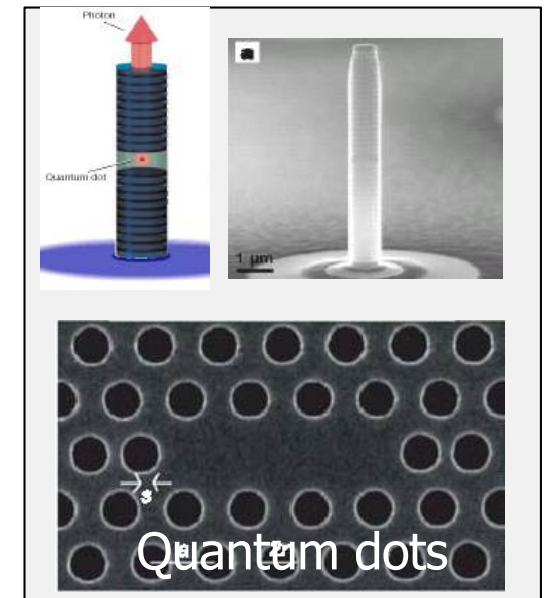
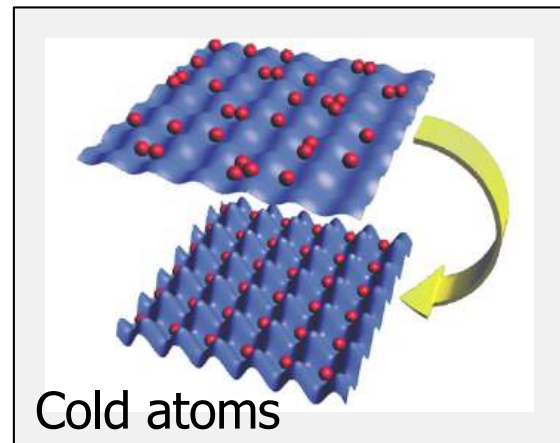
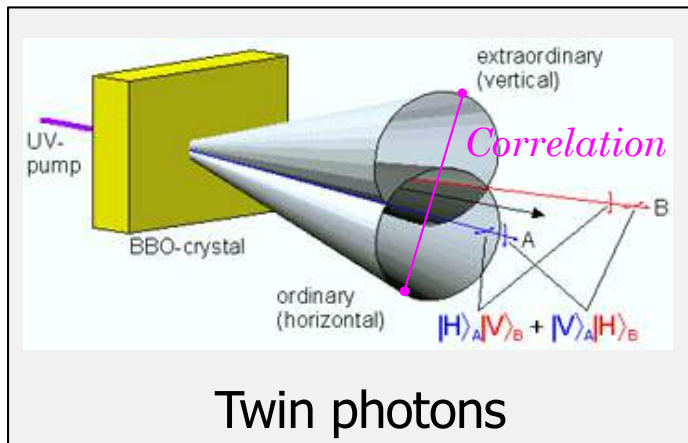
(British Journal of the Philosophy of Sciences, vol 3, 1952)

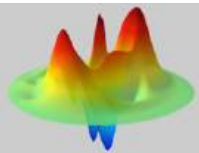




Experiment with individual quantum systems

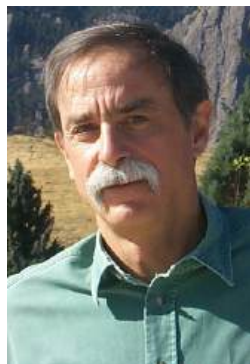
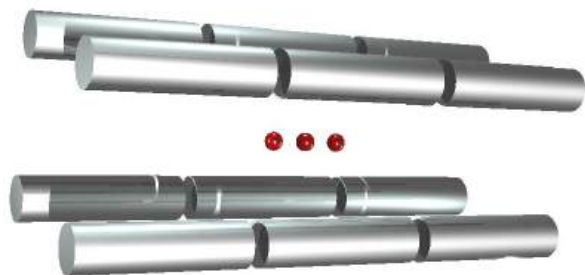
- A thriving field worldwide



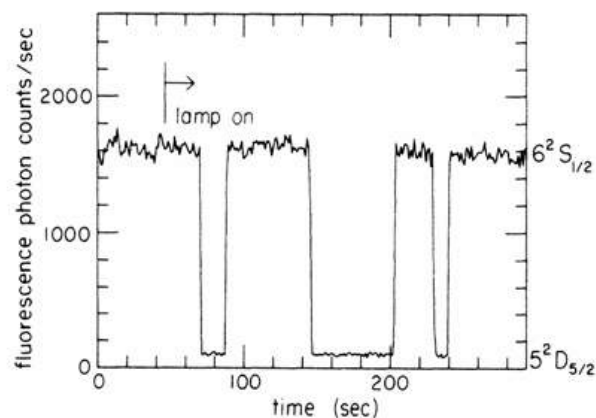


Trapped ions and photons

- Trapped ions

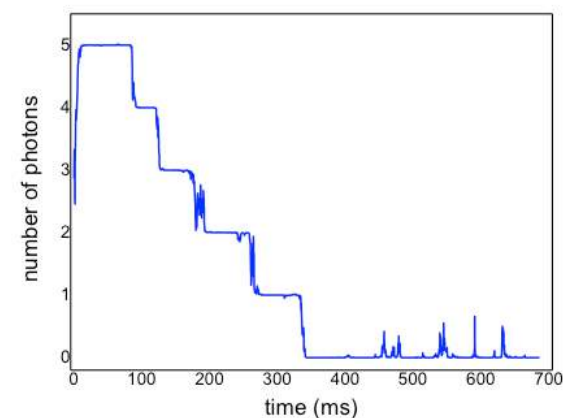
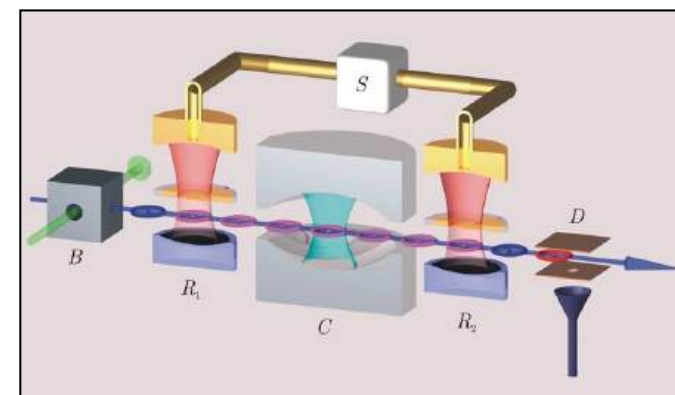


D.J. Wineland and S. Haroche



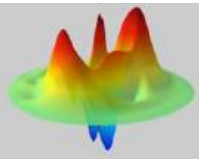
H. Dehmelt, JOSAB 1986

- Trapped photons



C. Gerlin et al. Nature 2007

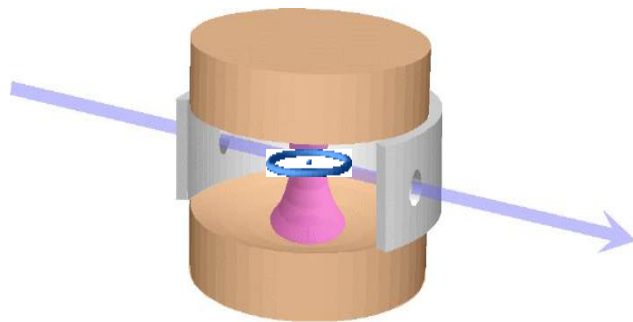
→ Observation of quantum jumps in a single realization of an experiment



Main topic of the course

Exploring the quantum with Rydberg atoms

With photons and cavities

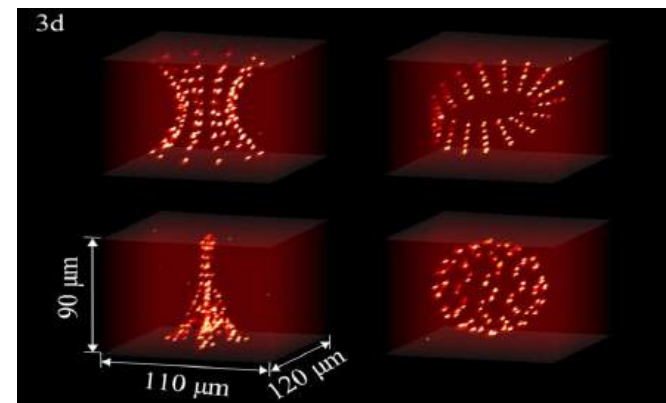


→ cavity QED exploration of the fundamental aspects of quantum measurement:

- QND photon counting:
- Schrödinger cat and decoherence

→ Topic of lectures 1-3

With neutral atoms in optical tweezers and Rydberg atoms

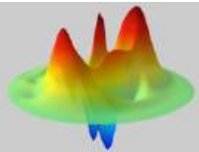


A. Browayes

→ Quantum simulation and QC

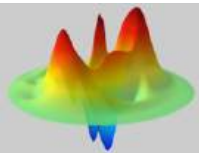
→ Topic of lectures 4-5

0. Cavity QED: a brief historical introduction



CQED, brief history

- 1870-1920: Effect of boundary conditions on dipole radiation (Maxwell, Hertz, Sommerfeld: Dipole radiating over conducting earth...).
- 1930: atom-metal surface interaction (London, Lennard Jones).
- **1946: Spins coupled with a tuned resonator (Purcel).**
- 1947: Vacuum fluctuations between two mirrors, Casimir effect.
- 1949: Boundary effects on Synchrotron radiation (Schwinger).
- 1954-60: Masers and Lasers: collective radiation of atoms in a cavity (Townes, Schalow...)
- 1974: Modification of molecular fluorescence near surfaces (Drexhage).
- 1983-87: Modification of spontaneous emission, experiments: ENS, MIT, Seattle, Yale, Rome...



A history of CQED: the origin

Proceedings of the American Physical Society

MINUTES OF THE SPRING MEETING AT CAMBRIDGE, APRIL 25–27, 1946

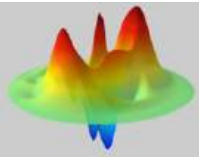
Physical Review, 69, 681, (1946)

- Purcell 1946
 - spontaneous emission rate modification for a spin in a resonant circuit
 - Definition of the ‘Purcell factor’
 - Brief but seminal paper

B10. Spontaneous Emission Probabilities at Radio Frequencies. E. M. PURCELL, *Harvard University*.—For nuclear magnetic moment transitions at radio frequencies the probability of spontaneous emission, computed from

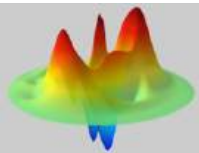
$$A_\nu = (8\pi\nu^2/c^3)h\nu(8\pi^3\mu^2/3h^2) \text{ sec.}^{-1},$$

is so small that this process is not effective in bringing a spin system into thermal equilibrium with its surroundings. At 300°K, for $\nu = 10^7 \text{ sec.}^{-1}$, $\mu = 1$ nuclear magneton, the corresponding relaxation time would be 5×10^{21} seconds. However, for a system coupled to a resonant electrical circuit, the factor $8\pi\nu^2/c^3$ no longer gives correctly the number of radiation oscillators per unit volume, in unit frequency range, there being now *one* oscillator in the frequency range ν/Q associated with the circuit. The spontaneous emission probability is thereby increased, and the relaxation time reduced, by a factor $f = 3Q\lambda^3/4\pi^2 V$, where V is the volume of the resonator. If a is a dimension characteristic of the circuit so that $V \sim a^3$, and if δ is the skin-depth at frequency ν , $f \sim \lambda^3/a^2\delta$. For a non-resonant circuit $f \sim \lambda^3/a^3$, and for $a < \delta$ it can be shown that $f \sim \lambda^3/a\delta^2$. If small metallic particles, of diameter 10^{-3} cm are mixed with a nuclear-magnetic medium at room temperature, spontaneous emission should establish thermal equilibrium in a time of the order of minutes, for $\nu = 10^7 \text{ sec.}^{-1}$.



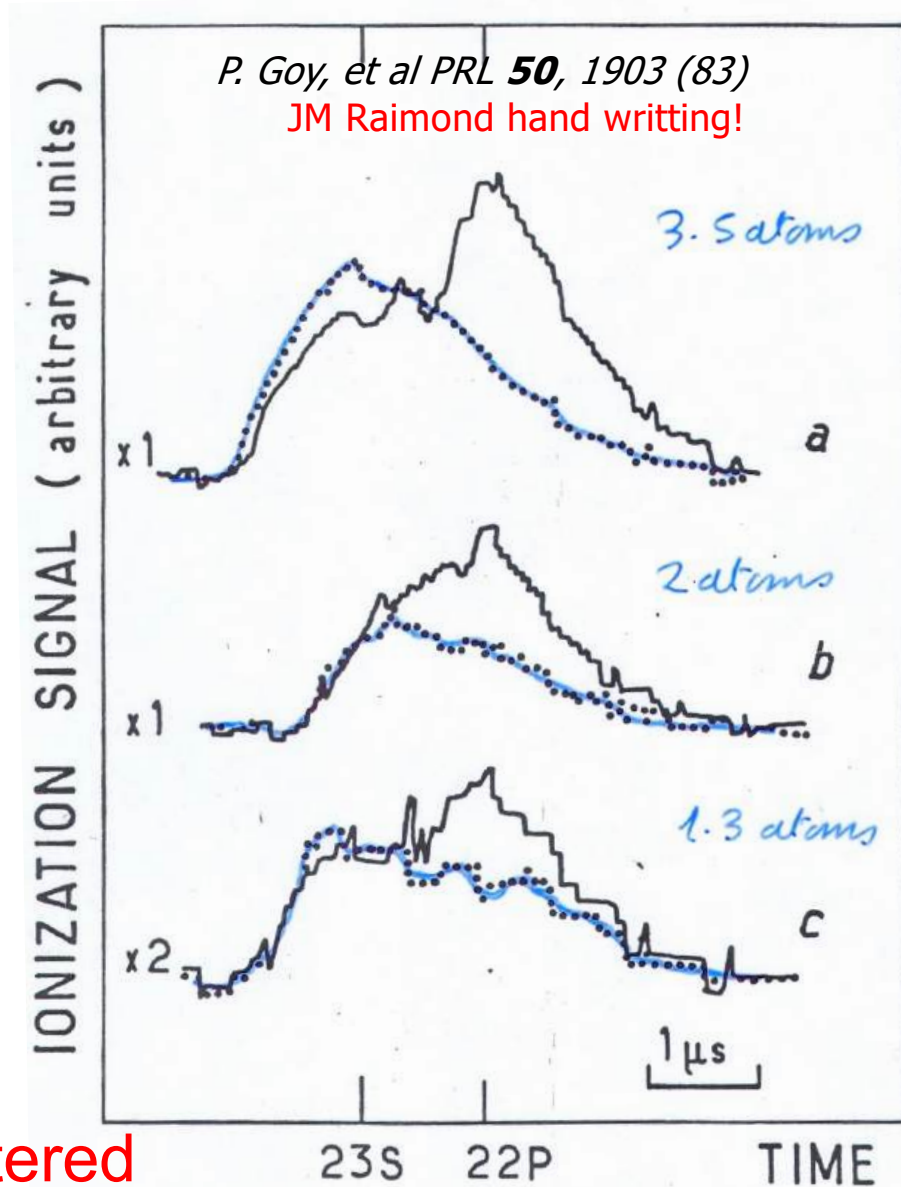
CQED, brief history

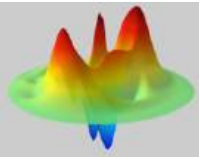
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- 1949: Boundary effects on Synchrotron radiation (Schwinger).
- 1954-60: Masers and Lasers: collective radiation of atoms in a cavity (Townes, Schalow...)
- 1974: Modification of molecular fluorescence near surfaces (Drexhage).
- **1983-87: Modification of spontaneous emission of one atom. Experiments at ENS, MIT, Seattle, Yale, Rome...**



First single-atom experiments

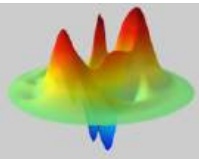
- Spontaneous emission enhancement (83)
 - Superconducting FP cavity
 - $Q \propto 10^6$
 - 340 GHz transition
 - Acceleration $\times 530$
 - First experimental evidence of Purcell effect
- Spontaneous emission inhibition
 - Gabrielse and Dehmelt (85)
 - Hulet, Hilfer and Kleppner (85)
- Spontaneous emission can be altered at will by imposing limiting conditions to the field





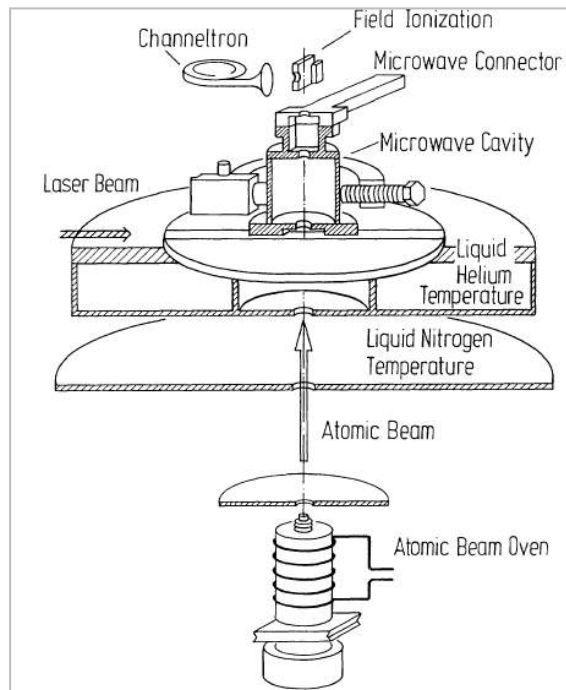
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- CQED in the PERTUBATIVE regime: Low Q cavity or effect of a single mirror: coupling strength $\ll$ dissipation rates
Perturbation of atomic radiative properties which remains qualitatively the same as in free space.
- **1985→.. : Rydberg atoms microMaser (ENS, Munich).**

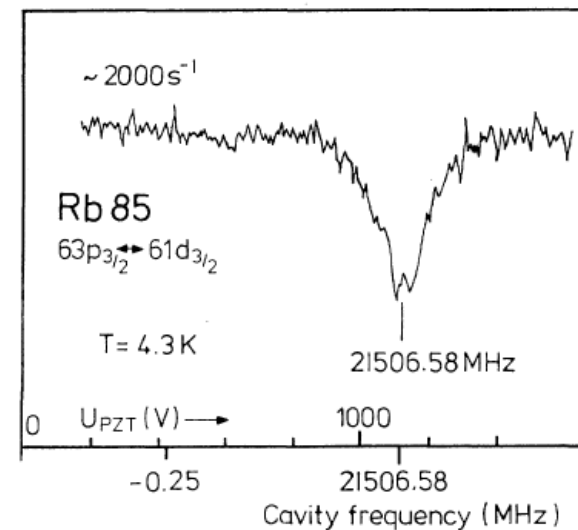


The Micromaser

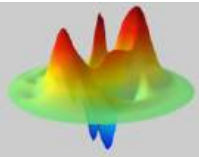
- H. Walther and D. Meschede, 85
 - Cumulative emissions in the cavity in the strong coupling regime



One-Atom Maser
D. Meschede, H. Walther, and G. Müller
Phys. Rev. Lett. 54, 551 1985



- A maser with less than one atom at a time in the cavity
- Strong coupling regime
 - Single-Atom-cavity coupling overwhelms dissipation



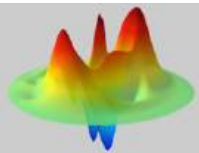
The two regimes of cavity QED

- Weak coupling regime

- Atom-field coupling small compared to dissipation
 - No qualitative modifications of the atomic radiative properties
 - Modification of the spontaneous emission rate
 - Modification of the atomic energies

- Strong coupling regime

- Atom-cavity interaction overwhelms dissipative processes
 - The simplest matter-field coupling situation
 - Radical modification of the atomic radiative properties:
Rabi oscillation replaces exponential decay
 - Creates and manipulates atom/field entangled state



The four timescales of CQED

- Atomic levels lifetime

$$T_{at} = 1 / \Gamma$$

- Cavity: photon lifetime

$$T_c = 1 / \kappa$$

- Atom-cavity coupling

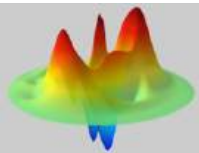
$$\Omega_0 = 2g = 1 / T_{vac}$$

- Atom-cavity interaction time

$$T_{int}$$

- Strong coupling conditions

$$T_{int} \Omega_0 \approx 1; \quad T_{vac}, T_{int} \ll T_{at}, T_c$$

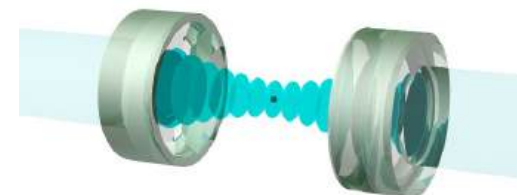


The four flavors of modern CQED

• Optical CQED

- Ordinary atomic transitions and high finesse FP cavities

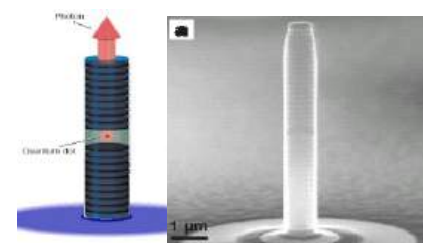
$$g \approx 50 \text{ MHz}; \kappa \approx 100 \text{ kHz}; \Gamma \approx 10 \text{ MHz}; T_{\text{int}} \approx 1 \text{ s}$$



• Solid-state CQED

- Quantum dots coupled to bragg mirrors/PBG

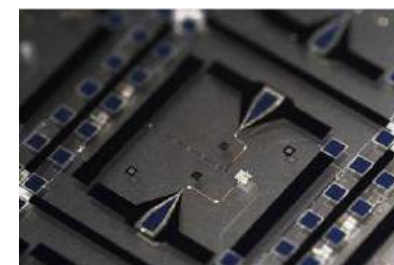
$$g \approx 10 \text{ GHz}; \kappa \approx 1 \text{ GHz}; \Gamma \approx 1 \text{ GHz}; T_{\text{int}} = \infty$$



• Circuit QED

- Solid-state qubits and stripline cavities

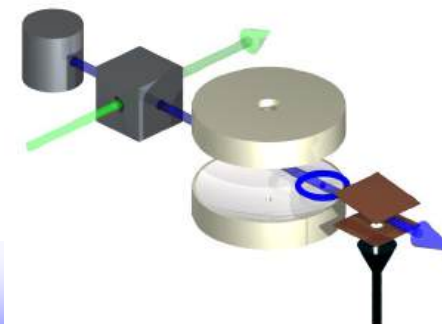
$$g \approx 100 \text{ MHz}; \Gamma \ll \kappa \approx 0.1 \text{ MHz}; T_{\text{int}} = \infty$$

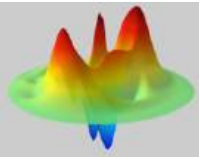


• Microwave CQED

- (Circular) Rydberg atoms and superconducting cavities

$$g \approx 10 \text{ kHz}; \kappa \approx 1 \text{ Hz}; \Gamma \approx 30 \text{ Hz}; T_{\text{int}} \approx 400 \mu\text{s}$$





Outline of the course

Topic of lectures 1-3:

CQED with Rydberg atoms

- Lecture 1: the strong coupling regime
- Lecture 2: Quantum Non Demolition photon counting

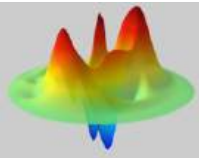
Quantum jumps

- Lecture 3: Quantum measurement Schrödinger cat and decoherence

State reconstruction

Lecture 4-5:

Rydberg atom-based quantum simulation
and quantum computation



Outline of Course 1

1. One atom, one mode, the Jaynes-Cummings model

2. Rydberg atoms in a cavity:

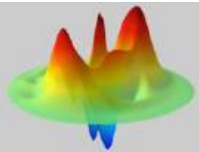
the tools achieving the strong coupling regime

- ❑ The experimental setup
- ❑ Vacuum Rabi oscillations

3. Rabi oscillation in a small coherent field

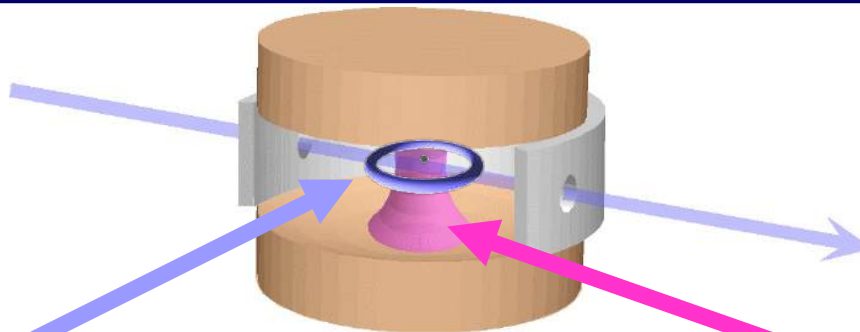
- ❑ Direct observation of field graininess
- ❑ Preparation of a "44 photons" Schrödinger cat state

1. One atom, one mode, the Jaynes-Cummings model



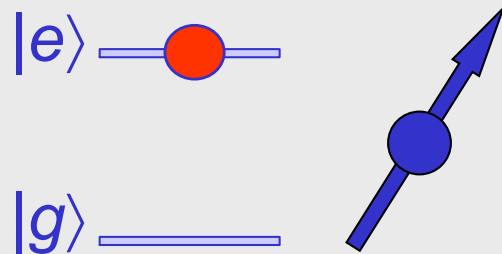
Cavity QED: spin and spring

A cavity QED experiment



The SPIN:

One atom, two levels



Electric
dipole
coupling

$$\Omega_0$$

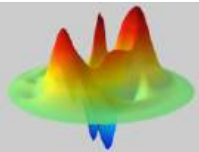


The SPRING:

One high Q cavity mode
as an harmonic oscillator

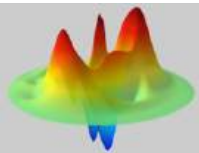


➔ Nearly ideal realization of a simple generic system



Field quantization in a cavity

- Same procedure as in free space:
 - 1- Find the classical eigenmodes of the resonator satisfying the boundary conditions.
 - Classical electric field: $\vec{E}_\alpha(\vec{r}, t) = E_\omega \cdot \vec{f}_\alpha(\vec{r}) \cdot e^{i\omega t} + cc$



Field quantization in a cavity

- Same procedure as in free space:

1- Find the classical eigenmodes of the resonator satisfying the boundary conditions.

→ Classical electric field: $\vec{E}_\alpha(\vec{r}, t) = E_\omega \cdot \vec{f}_\alpha(\vec{r}) \cdot e^{i\omega t + c.c}$ α: dimensionless complex field amplitude

2- Each mode is quantized as an **harmonic oscillator**.

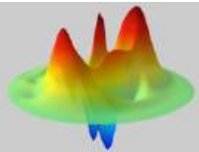
→ **Electric field operator**: $\hat{\vec{E}}_\alpha(\vec{r}) = E_\omega \cdot \left(\vec{f}_\alpha(\vec{r}) \cdot \hat{a}_\alpha + \vec{f}_\alpha^*(\vec{r}) \cdot \hat{a}_\alpha^\dagger \right)$ $[a, a^\dagger] = 1$

Where:

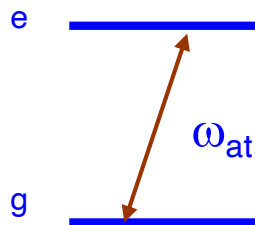
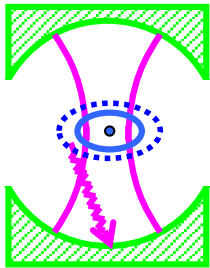
- $E_\omega = \sqrt{\frac{\hbar \omega}{2\epsilon_0 V_{cav}}}$ "vacuum electric field".
- $V_{cav} = \int_{Cavity} |\vec{f}_\alpha(\vec{r})|^2 d^3\vec{r}$ volume of the mode. V_{cav} is really a physical volume.
- $\vec{f}_\alpha(\vec{r})$ complex function of Normalization: $Max |\vec{f}_\alpha(\vec{r})| = 1$
(real functions will be enough for us)

- Here the quantized object is a **collective excitation of the field and all the electric charges** at the surface of the mirror.

- We now consider a single mode and drop the index α.



The Jaynes Cummings model:



- + a single two level atom, frequency ω_{at}
- + a single field mode, frequency ω_c
- + dipole coupling
- + negligible damping

- Atom-field Hamiltonian: $H = H_{at} + H_{cav} + V_{at-cav}$

$$H_{at} = \frac{\hbar\omega_{at}}{2} [|e\rangle\langle e| - |g\rangle\langle g|]$$

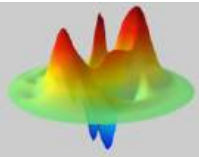
$$H_{cav} = \hbar\omega_c \left[a^\dagger a + 1/2 \right]$$

$$V_{at-cav} = -\vec{d} \cdot \hat{\vec{E}}(\vec{r})$$

$$\vec{d} = \vec{d}_{eg} [|e\rangle\langle g| + |g\rangle\langle e|]$$

Condition of validity:

- ω_c close to a single atomic transition: $|\delta| = |\omega_c - \omega_{at}| \ll \omega_c, \omega_{at}$
- small cavity: only one mode close to resonance $FSR \gg \delta$



The Jaynes Cummings hamiltonian

- Rotating wave approximation (RWA):

$$V_{at-cav} = \hbar \Omega(\vec{r})/2 \left[a|e\rangle\langle g| + a|g\rangle\langle e| + a^+|g\rangle\langle e| + a^+|e\rangle\langle g| \right]$$

Non-resonant terms are neglected

$$V_{at-cav} \approx \hbar \Omega(\vec{r})/2 \left[a|e\rangle\langle g| + a^+|g\rangle\langle e| \right]$$

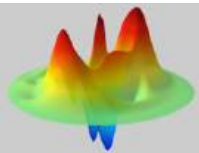
Jaynes-Cummings
Hamiltonian

- Vacuum Rabi frequency:

$$\Omega(\vec{r}) = -2d_{eg} \cdot \vec{f}(\vec{r}) \cdot E_\omega = \Omega_0 \cdot |\vec{f}(\vec{r})|$$

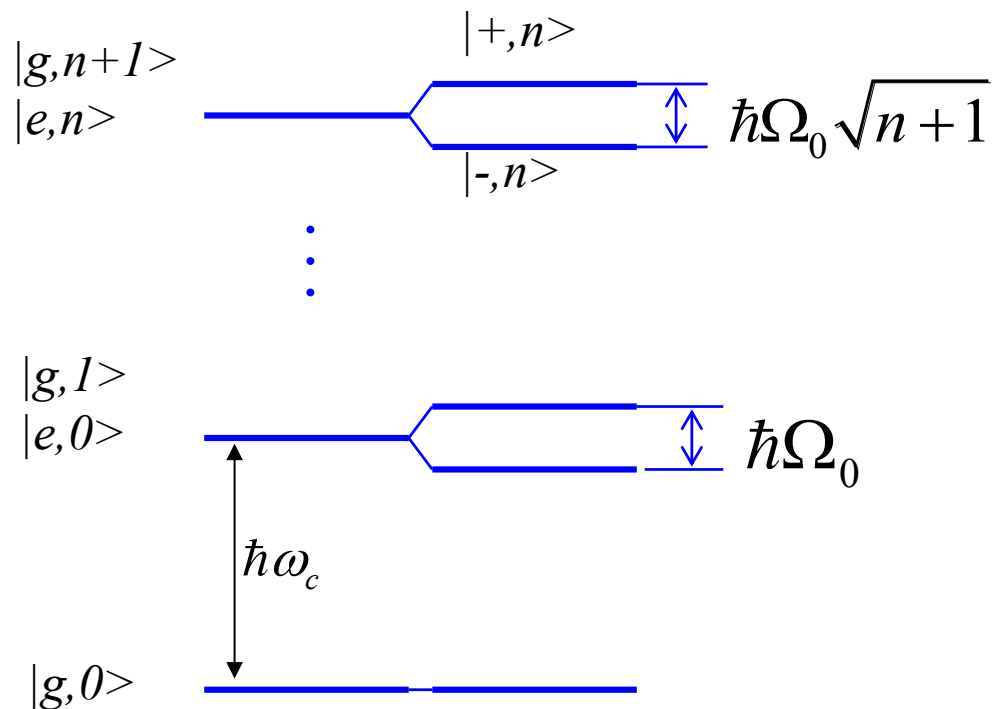
$$\Omega_0 = 2d_{eg} \cdot \sqrt{\frac{\hbar \omega}{2\epsilon_0 V_{cav}}}$$

- Validity of RWA: $\Omega_0 \ll \omega_{at}, \omega_c$

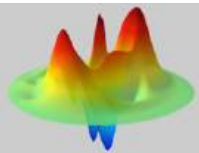


Dressed energy levels at resonance ($\omega_{at} = \omega_c$)

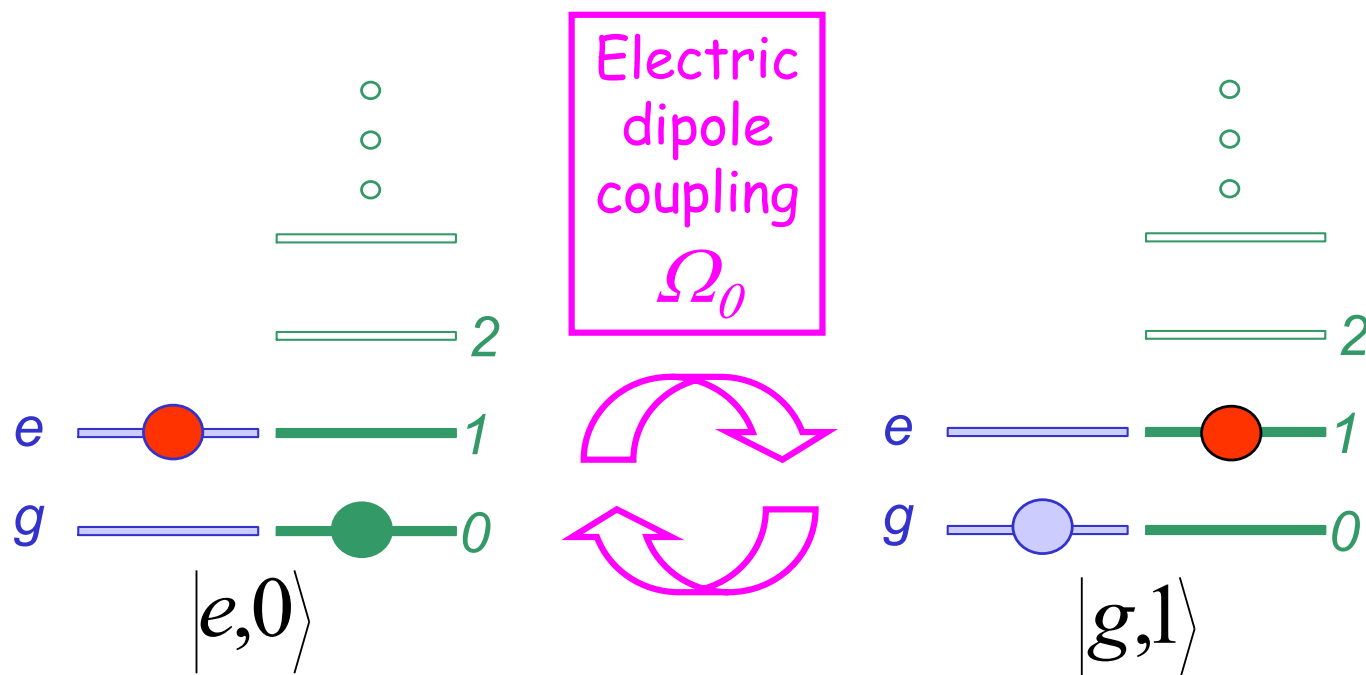
- Eigenvalues: $E_{\pm n} = \hbar\omega_c(n + 1/2) + \hbar\omega_{at} \pm \hbar\Omega_0/2\sqrt{n+1}$
- Eigenstates: $|\pm n\rangle = 1/\sqrt{2} [|e, n\rangle \pm |g, n+1\rangle]$



- Levels just couple by pairs (except the ground state)
- level splitting scales as the Field amplitude



Resonant atom-field coupling: dynamic point of view



$$\Omega_0/2\pi = 50 \text{ kHz}$$

$$T_{\text{rabi}} = 20 \text{ } \mu\text{s}$$

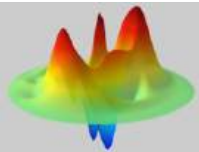
$$|e,0\rangle \rightarrow \cos\left(\frac{\Omega_0 t}{2}\right) \cdot |e,0\rangle - i \sin\left(\frac{\Omega_0 t}{2}\right) \cdot |g,1\rangle$$

→ Coherent Rabi oscillation

2. Rydberg atoms in a cavity: achieving the strong coupling regime

One photon and one atom
in a box:

- ❑ Photon box: superconducting microwave cavity
- ❑ “circular” Rydberg atoms



Microwave Rydberg atom CQED

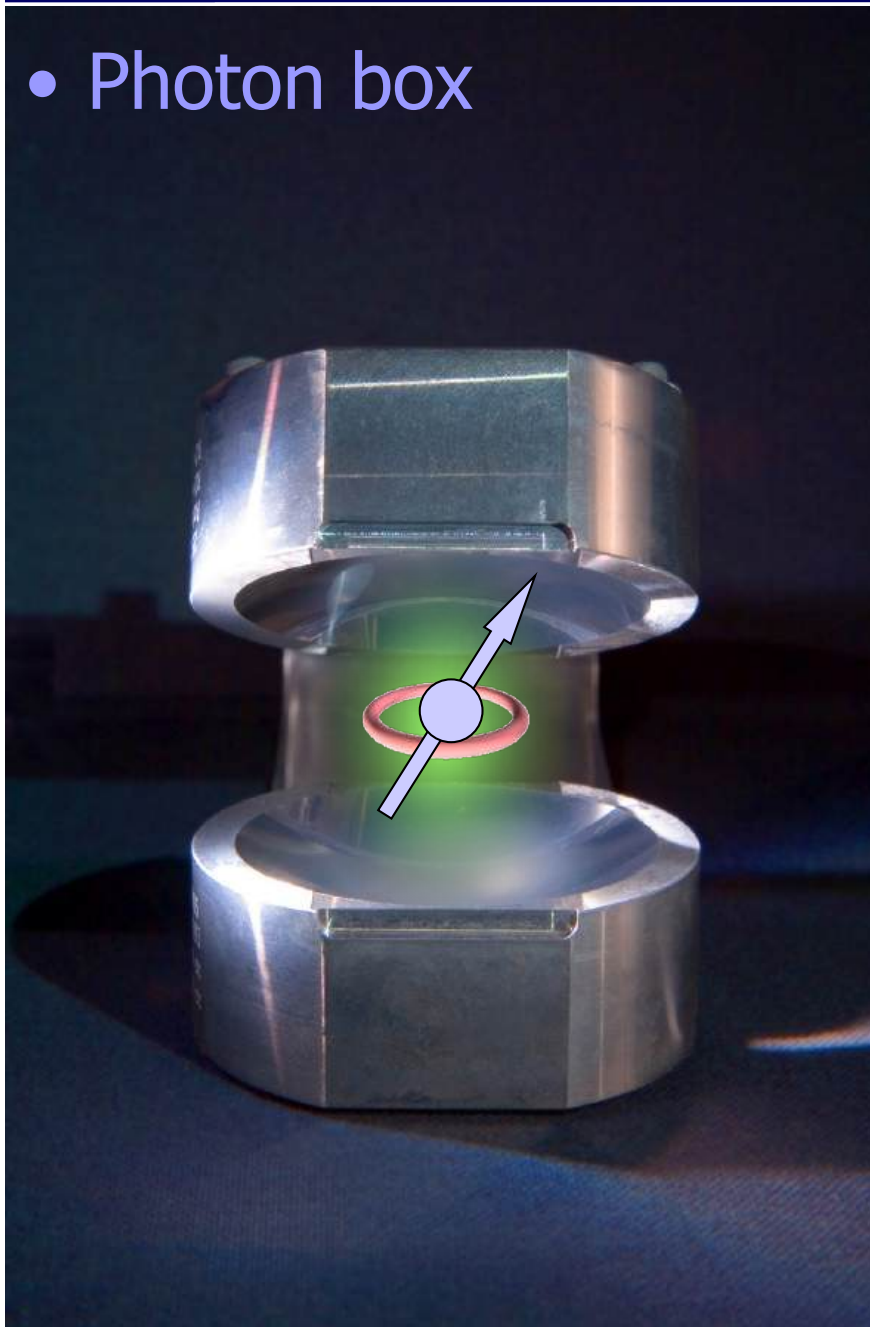
- Photon box

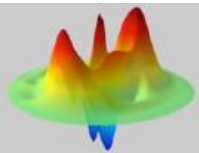
Two essential ingredients:

- Photon trap:
the "spring"

- Photon probe:
the spin

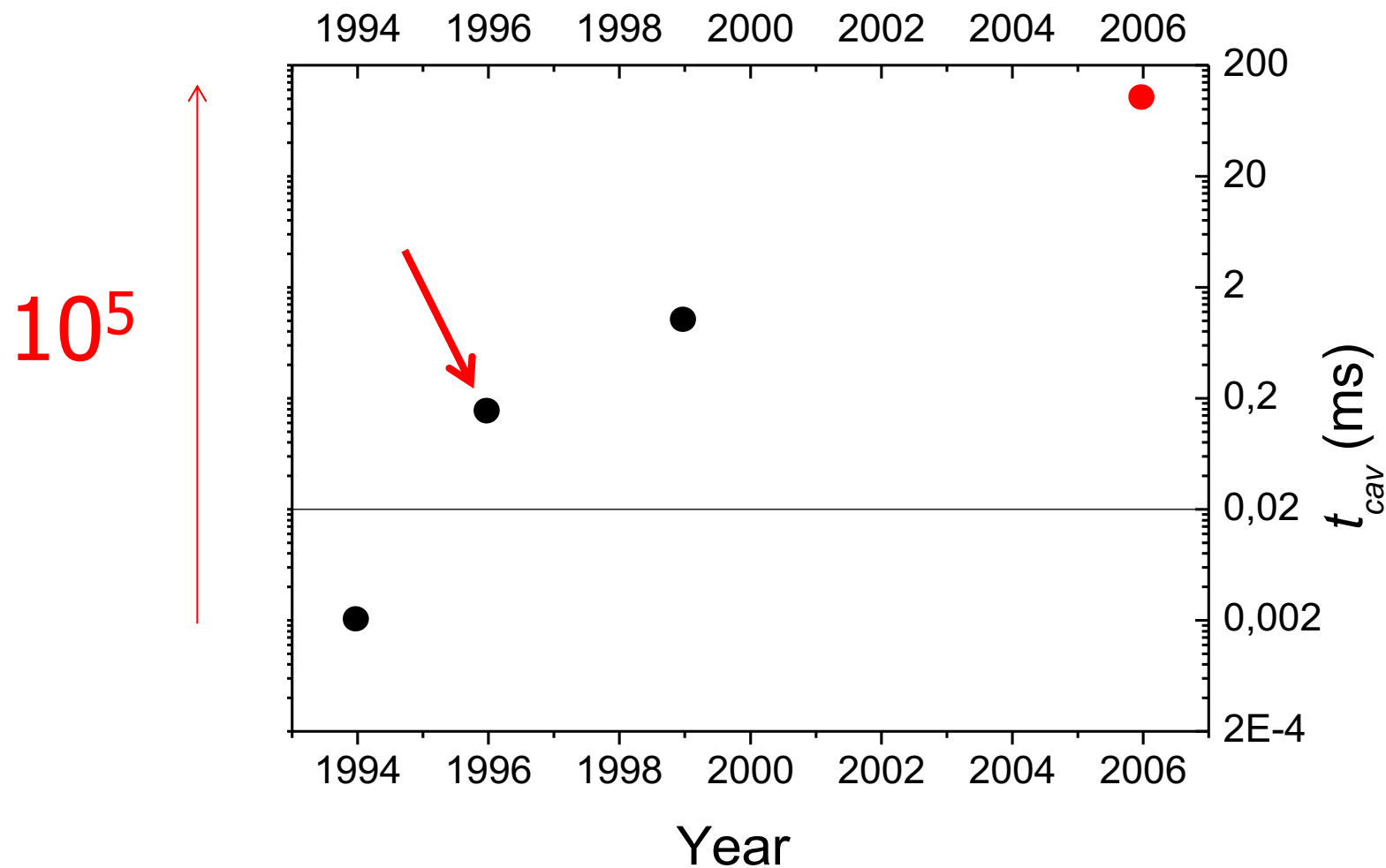
Single Rydberg atoms

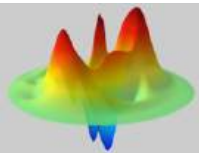




Superconducting cavity technology

- Our version of Moore's law:

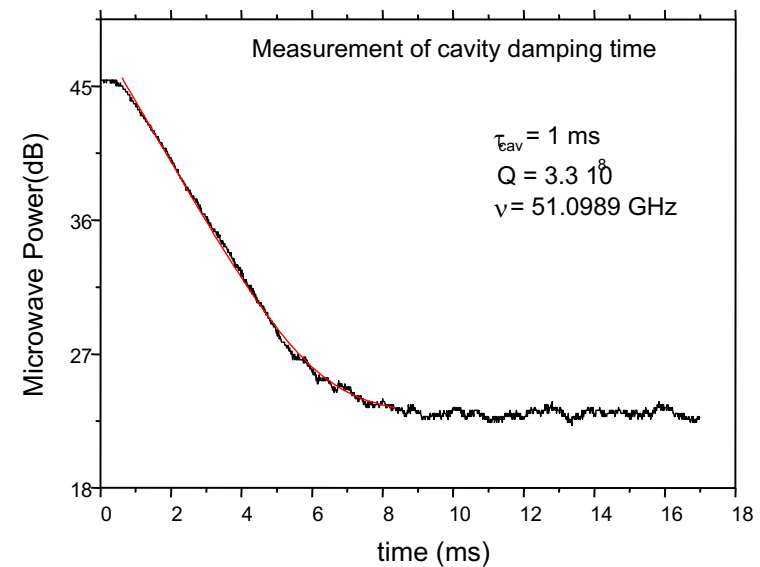
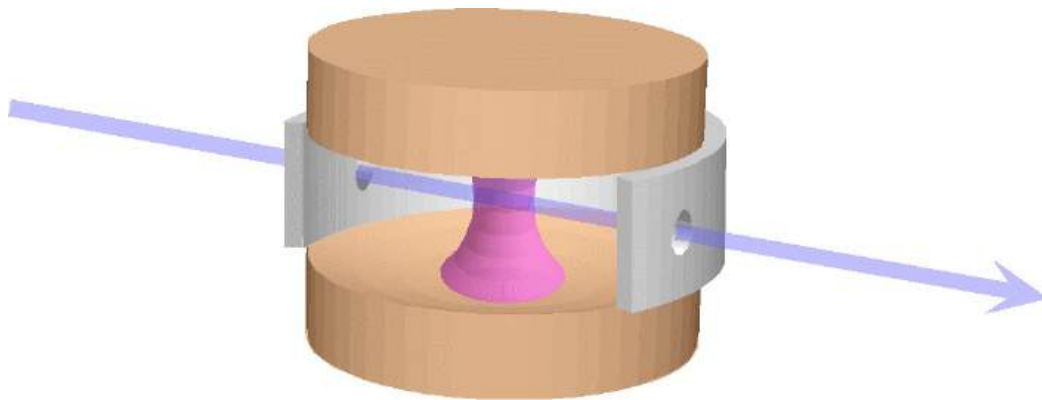




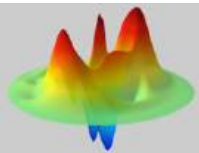
The cavity (first generation)

Superconducting cavity

- resonance: $\nu_{\text{cav}} = 51 \text{ GHz}$
- Massive Niobium mirrors



- Q factor = $2 \cdot 10^8$
- $T_{\text{cav}} = 1 \text{ ms}$



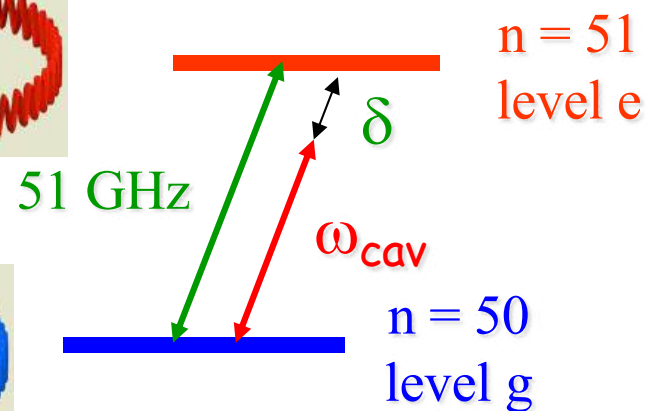
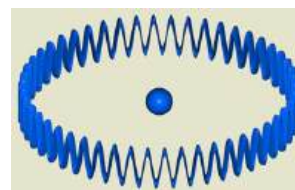
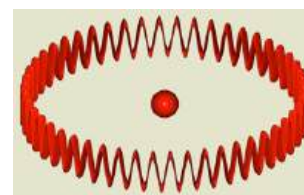
The "Spin": Circular Rydberg atoms

- Photon box

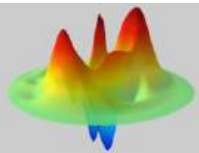
- Photon probes

Circular Rydberg atoms:

$$l=|m|=n-1$$



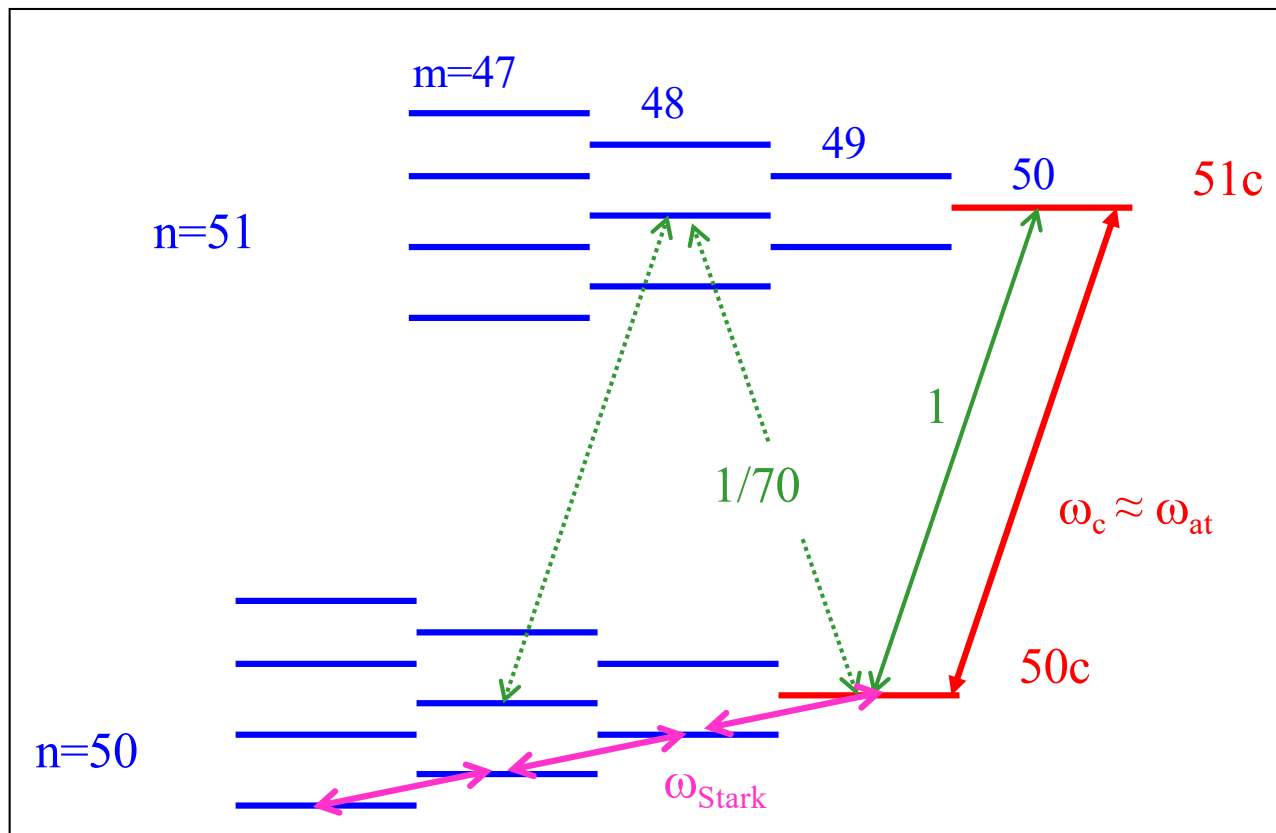
- Large dipole 1500 au
- Long lifetime: 30ms
- detected one by one



"Circular" atoms as two level atoms

- Stark diagram of Rydberg levels:

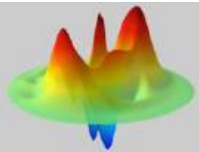
Good quantum number: m



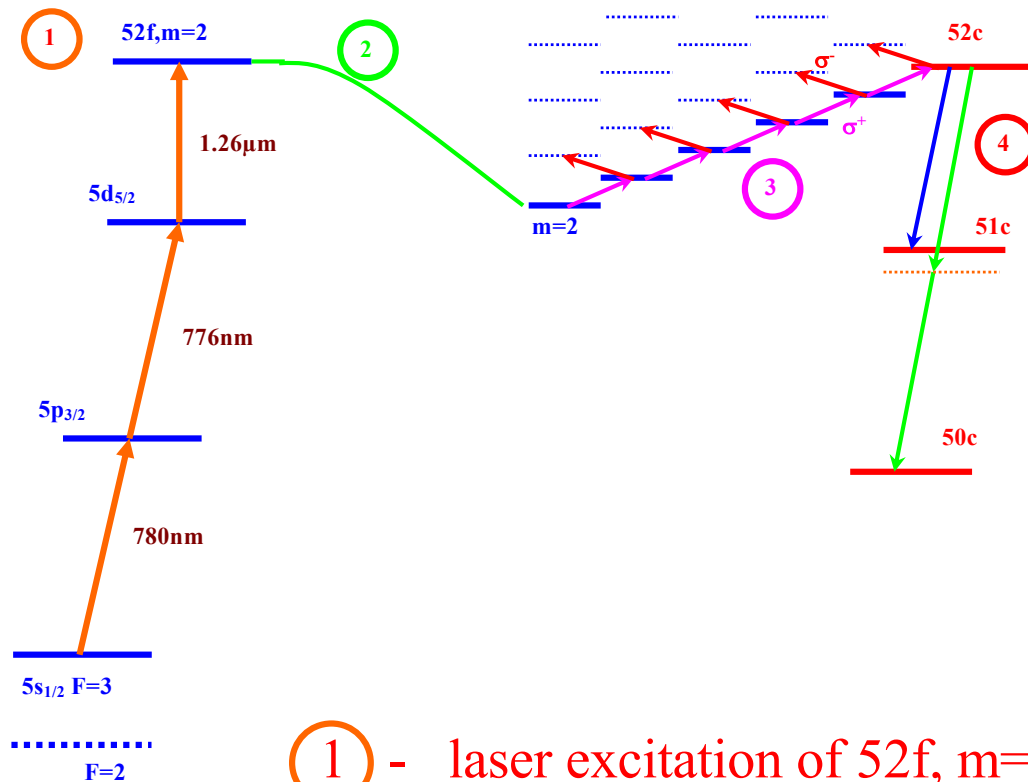
The electric field removes the degeneracy between transitions:

good isolation of the 51c to 50c transition

- Linear Stark effect: $\omega_{\text{Stark}}/2\pi = 100 \text{ MHz}/(V/\text{cm})$
- Quadratic Stark shift of the 51c-50c transition: $255 \text{ kHz}/(V/\text{cm})^2$
used for fast tuning of the atom in or out of resonance

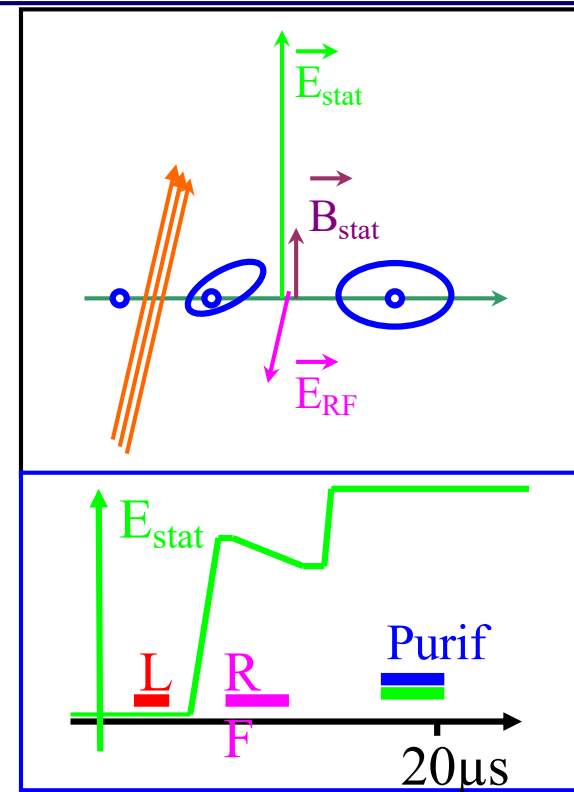


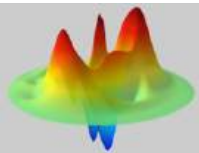
Preparation of circular atoms



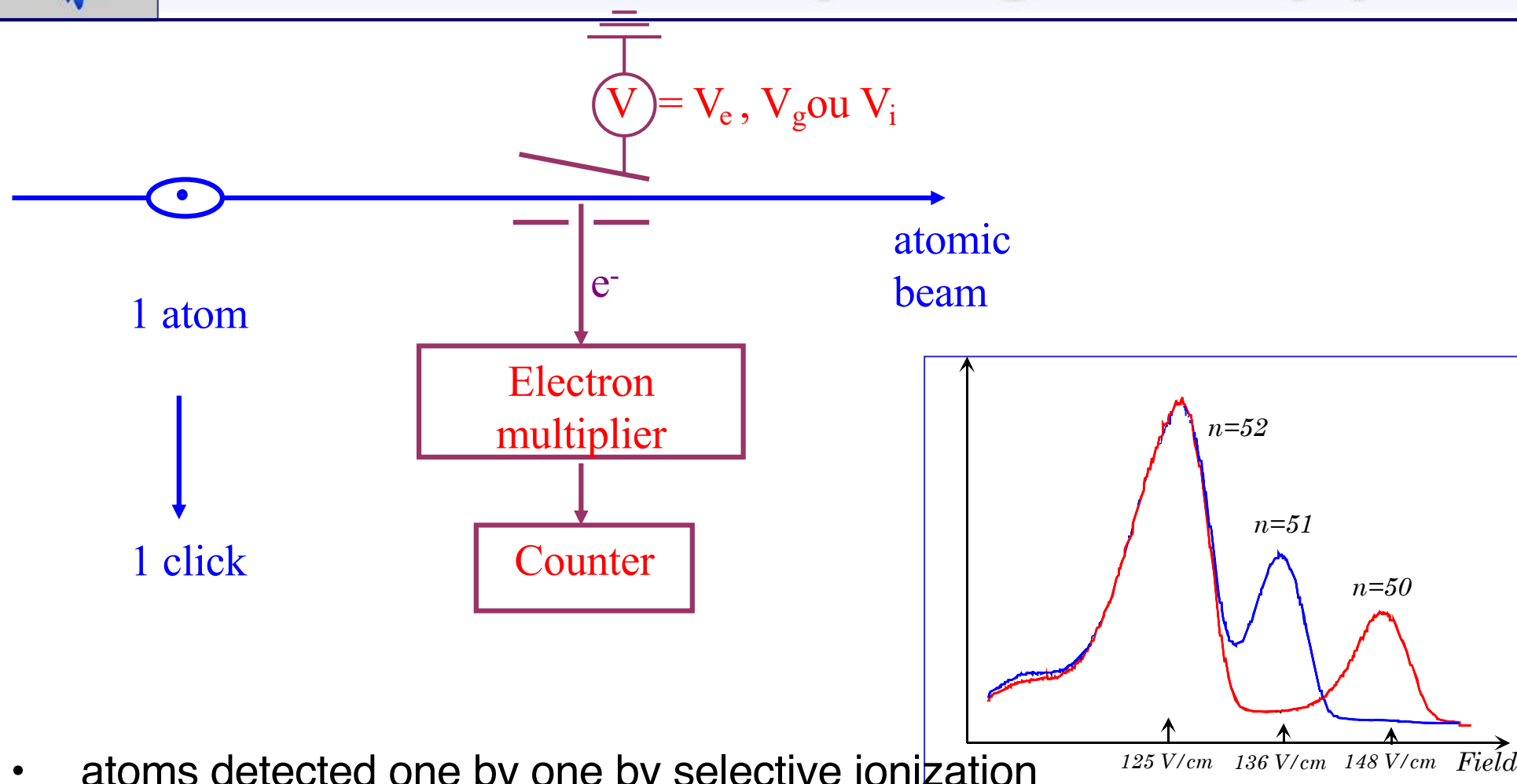
^{85}Rb

- ① - laser excitation of $52f, m=2$.
- ② - "Stark switching": $E_{\text{stat}}=0 \rightarrow 2.5\text{V/cm}$.
- ③ - 49 photons adiabatic transfer to $52c$ induced by $E_{\text{RF}}(\nu=250\text{MHz})$. B_{stat} removes degeneracy between σ^+ and σ^- .
- ④ - "Purification": selective transfer to $51c$ or $50c$.
 $\rightarrow$ 300 circular atom/laser pulse. Purity $> 99\%$



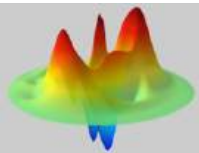


Detection of Rydberg atoms (1)



- atoms detected one by one by selective ionization in an electric field
- measurement of internal energy state of the atom after interaction with C

CW detection in a field gradient: efficiency 80-40%



Experimental set-up

^{85}Rb

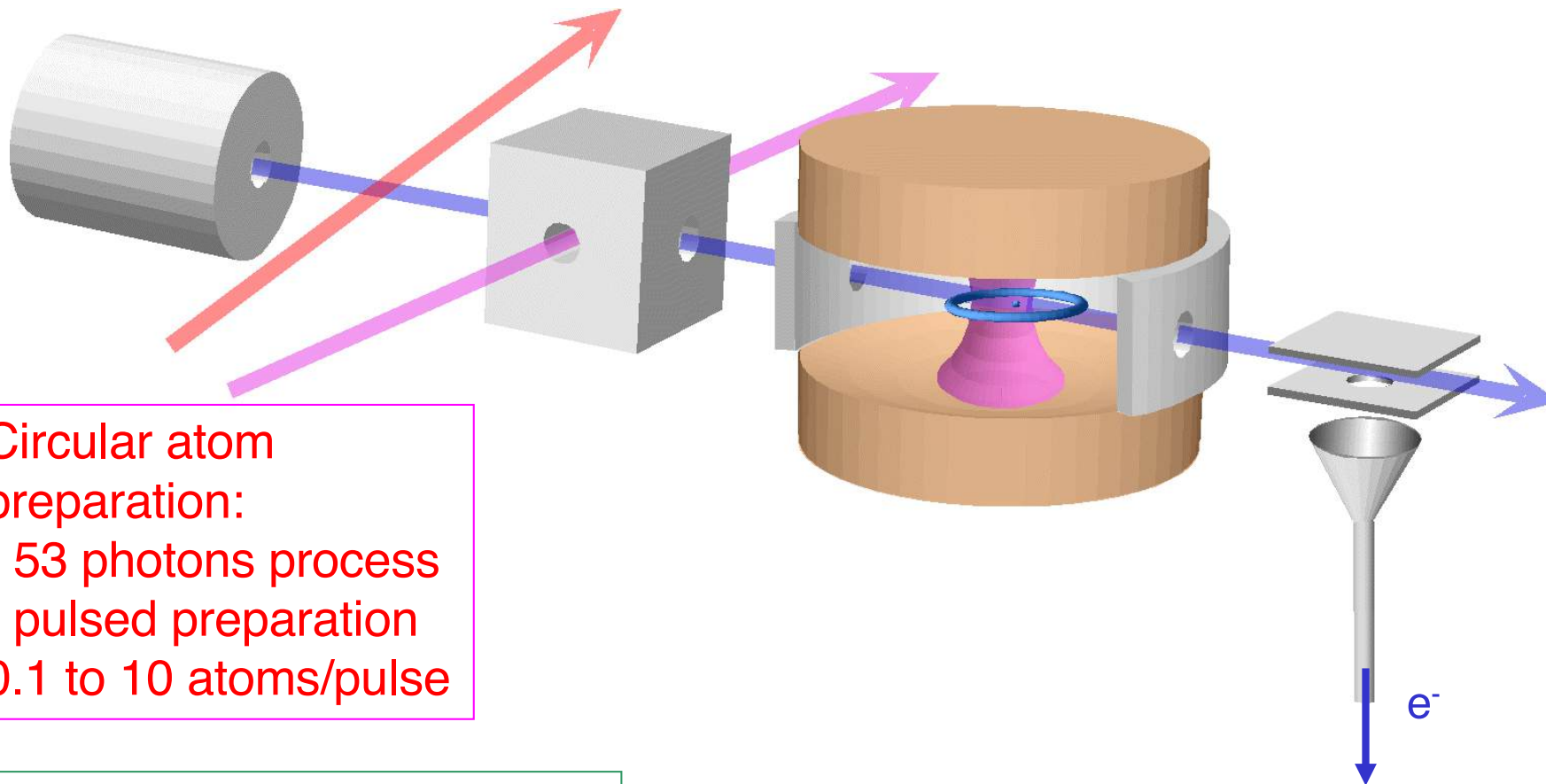
Laser velocity selection

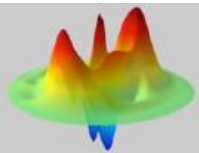
Circular atom
preparation:

- 53 photons process
 - pulsed preparation
- 0.1 to 10 atoms/pulse

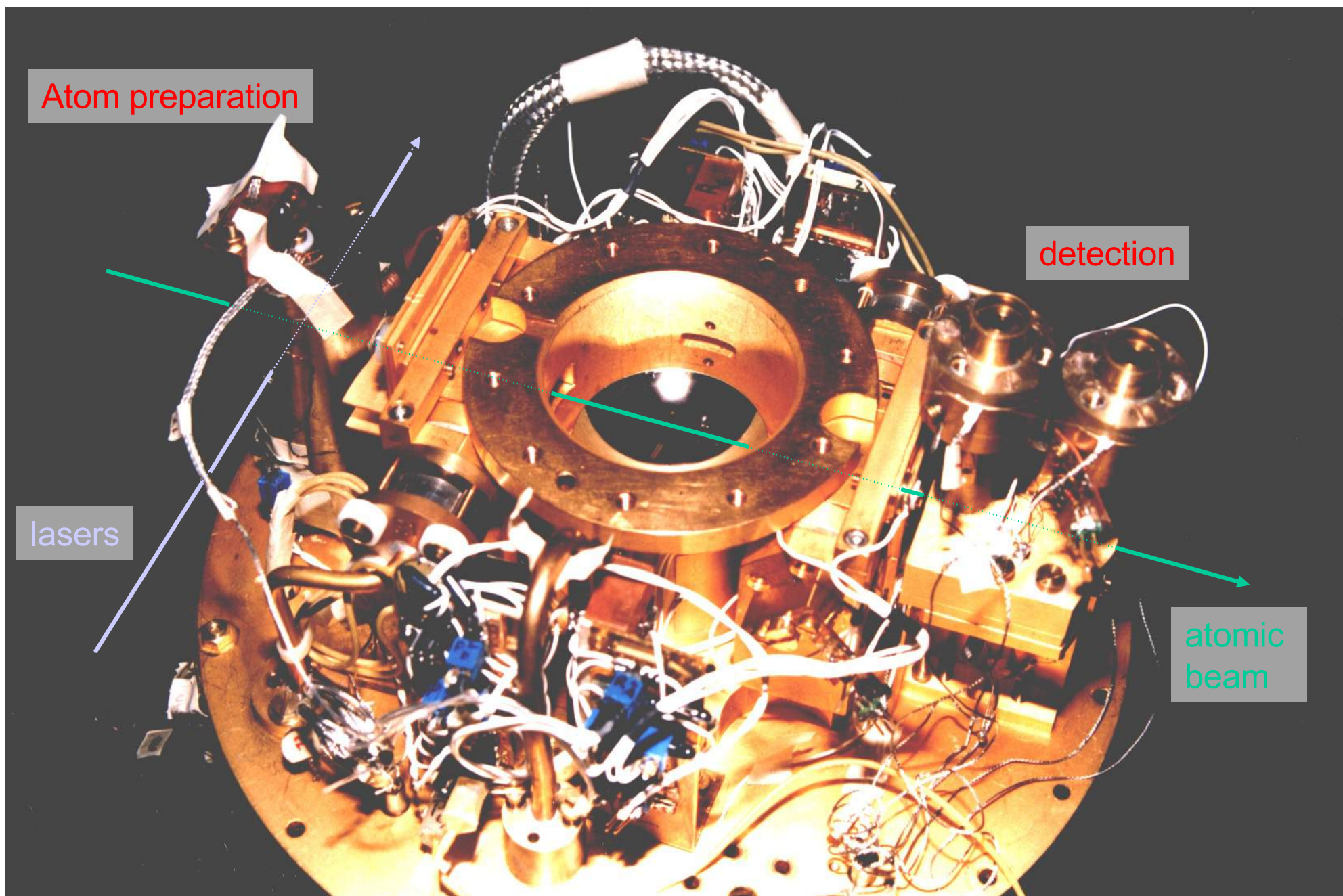
Cryogenic environment
 $T=0.6$ to 1.3 K
→ weak blackbody radiation

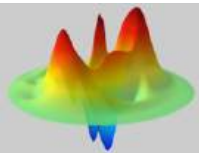
State selective
detector
One atom = one click



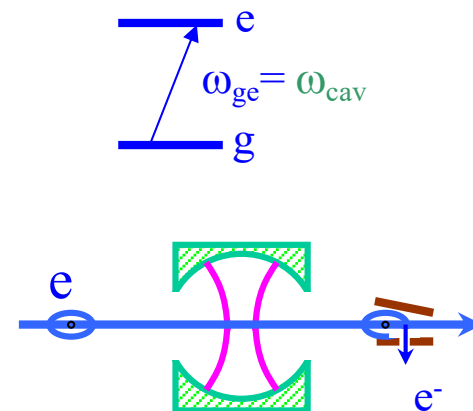
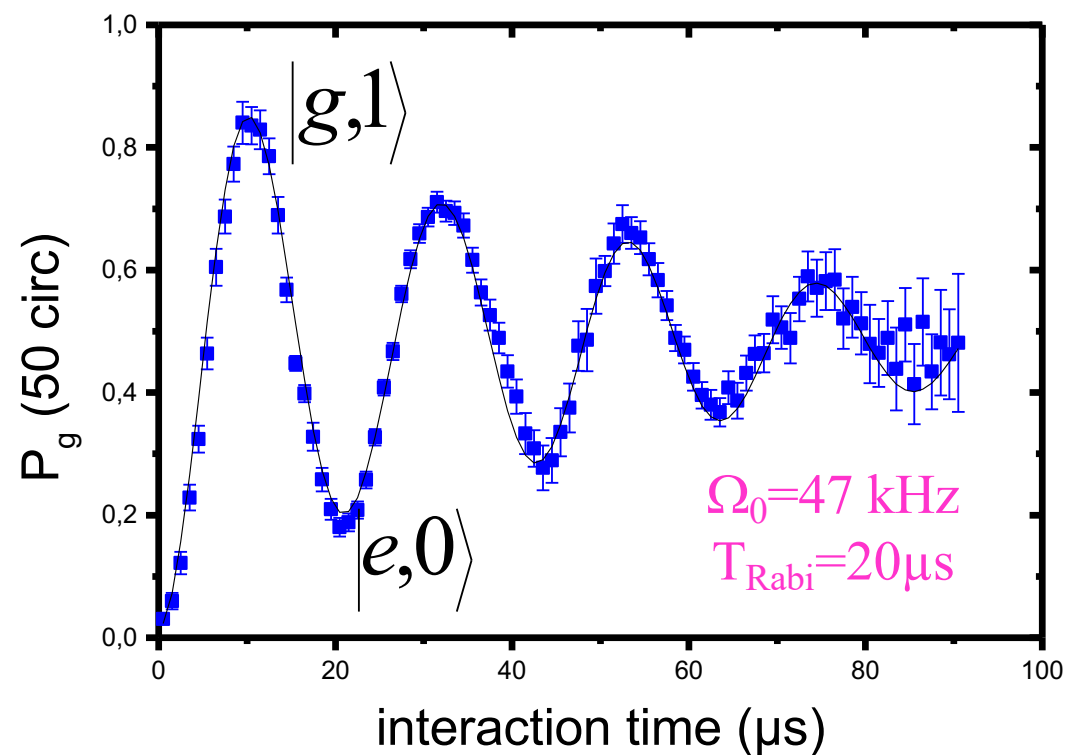


Experimental setup

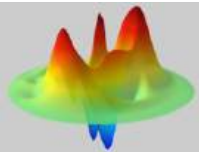




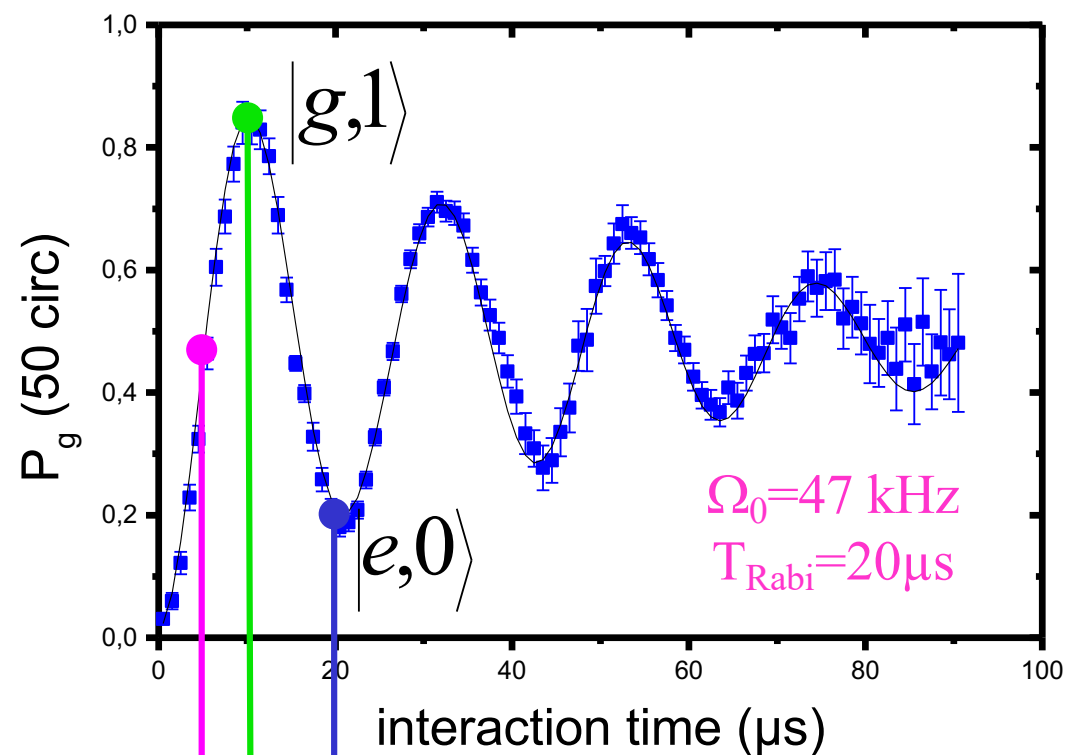
Single photon induced Rabi oscillation



Coherent Rabi oscillation
replaces irreversible damping
by spontaneous emission



Vacuum Rabi oscillation and quantum gates



2π

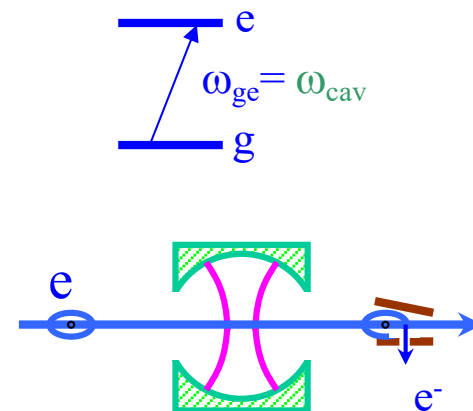
- Phase gate, QND detection of a single photon

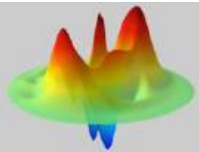
π

- Atom-field state exchange

$\pi/2$

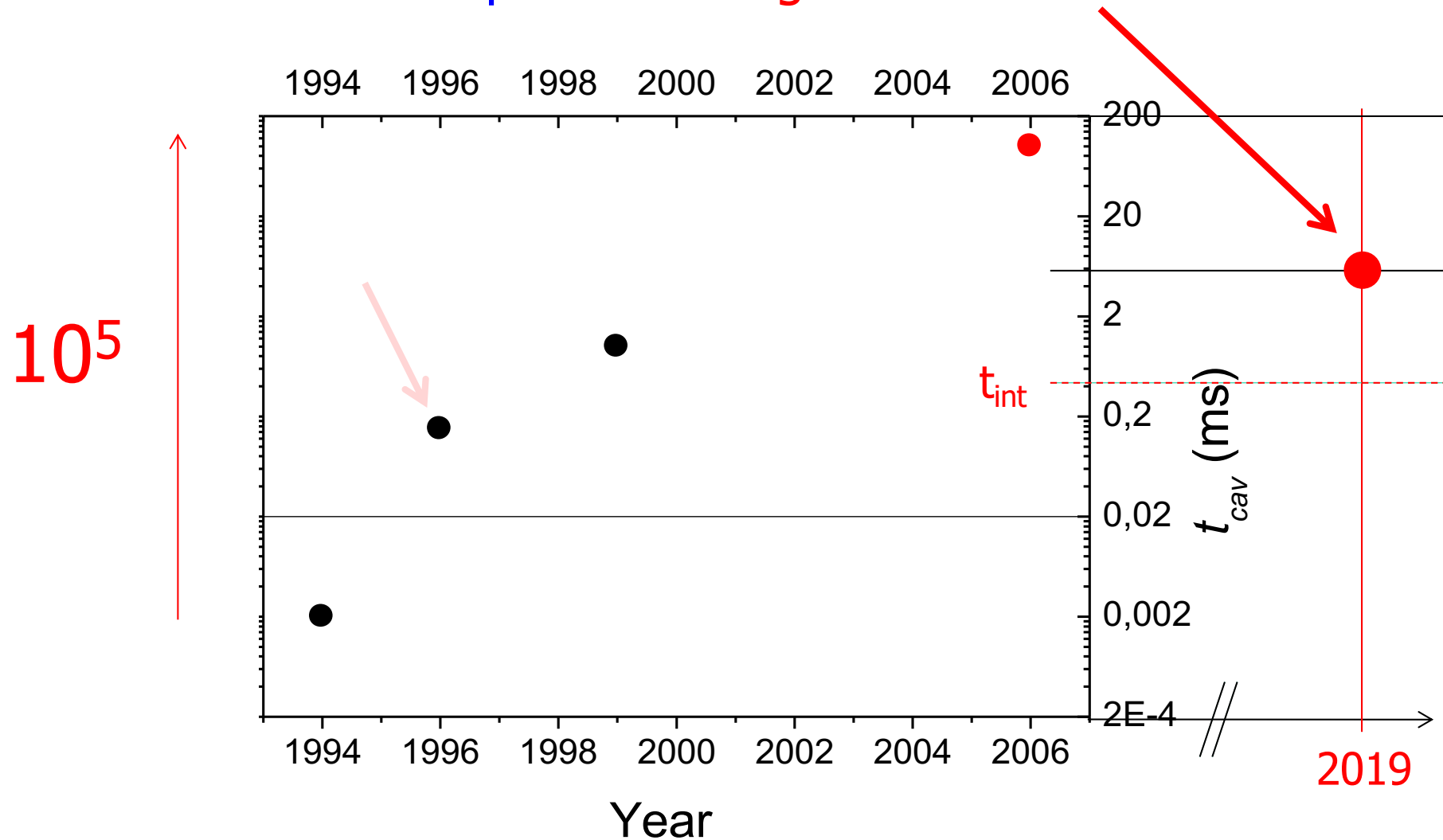
- EPR pair preparation



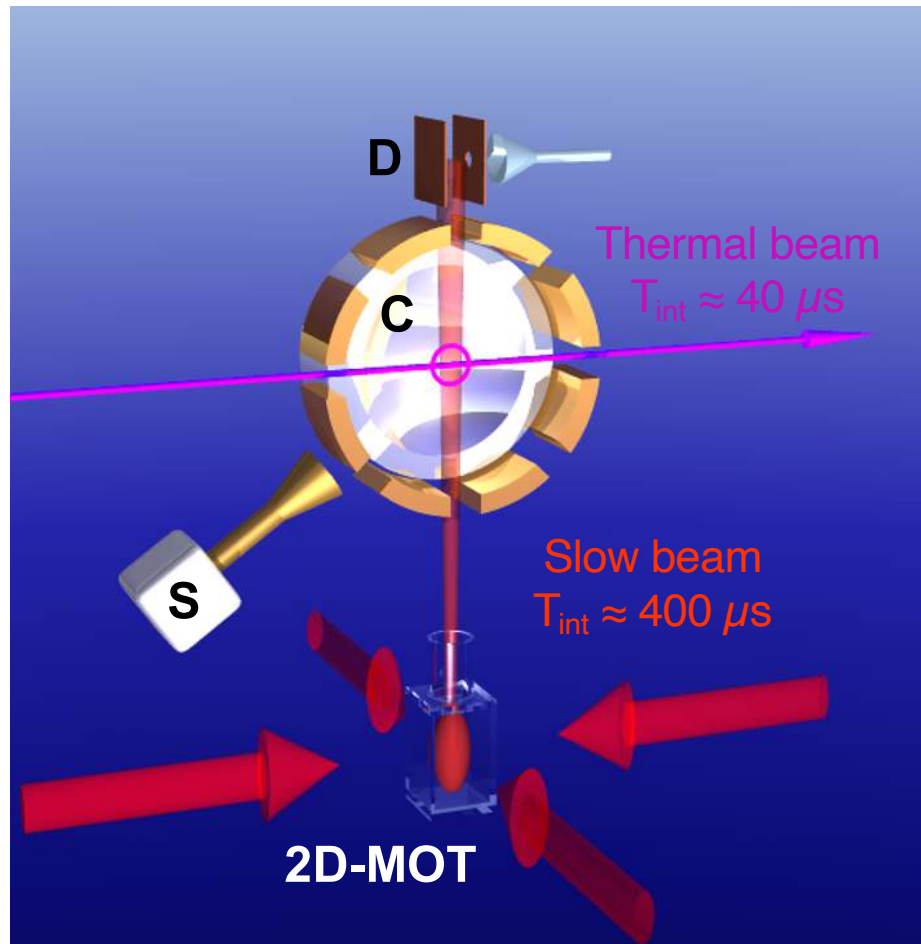


Superconducting cavity technology

- Last version of the experiment using slow atoms



A new, slow-atoms cavity QED setup



- Technical challenges

- preparation of Circular Rydberg atoms inside the cavity

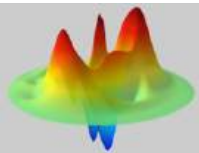
- fabrication of a new superconducting cavity setup

$$T_{cav} = 8 \text{ ms}$$

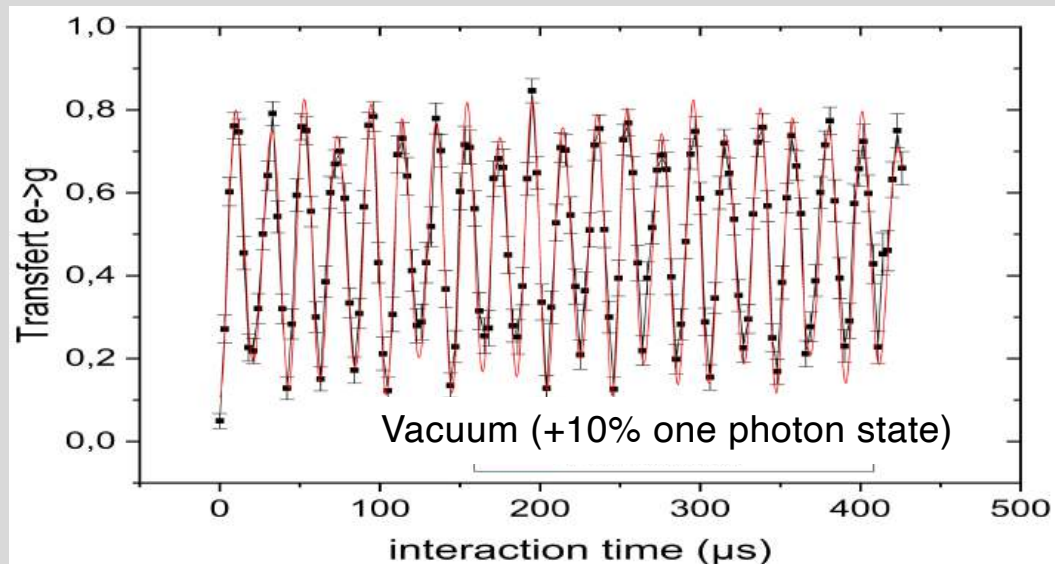
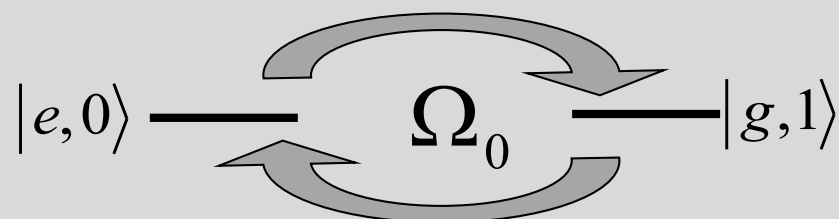
Not the best ever ... but good-enough for what follows

⇒ Cavity QED experiment in a new regime with 10 m/s atoms:

- Resolution of atom-cavity **dressed states by microwave spectroscopy** using the classical source S
- Observation of resonant interaction over unprecedented timescale



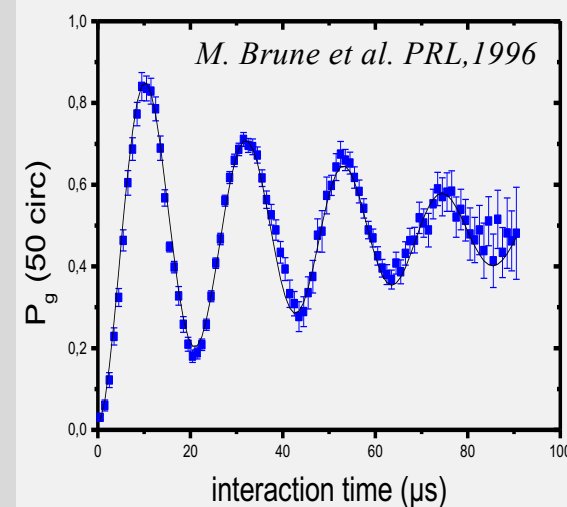
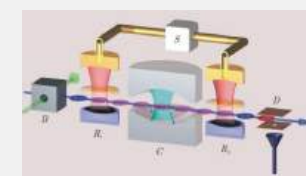
Rabi oscillation in vacuum



- No damping visible
- ($T_{\text{cav}} = 8 \text{ ms}$)
- 0.1 photon residual blackbody field

Very pure coherent evolution

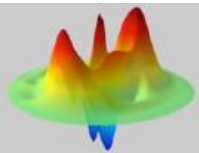
F. Assémat et al., Arxiv, accepted PRL(2018)



Our 1996 initial experiment

3. Rabi oscillation in a small coherent field

Direct observation
of discrete Rabi frequencies



Coherent field states

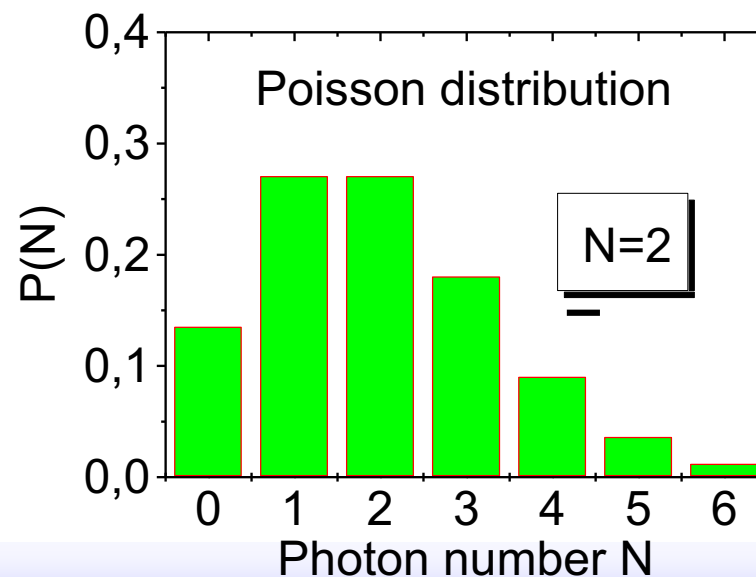
- Number state: $|N\rangle$
- Quasi-classical state: defined as eigenvectors of $\hat{a}$
Field radiated by a classical source

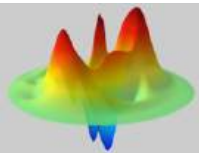
$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \text{ and } \langle\alpha|\hat{a}^\dagger = \alpha^*\langle\alpha|$$

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_N \frac{\alpha^N}{\sqrt{N!}} |N\rangle \quad ; \quad \alpha = |\alpha| e^{i\Phi}$$

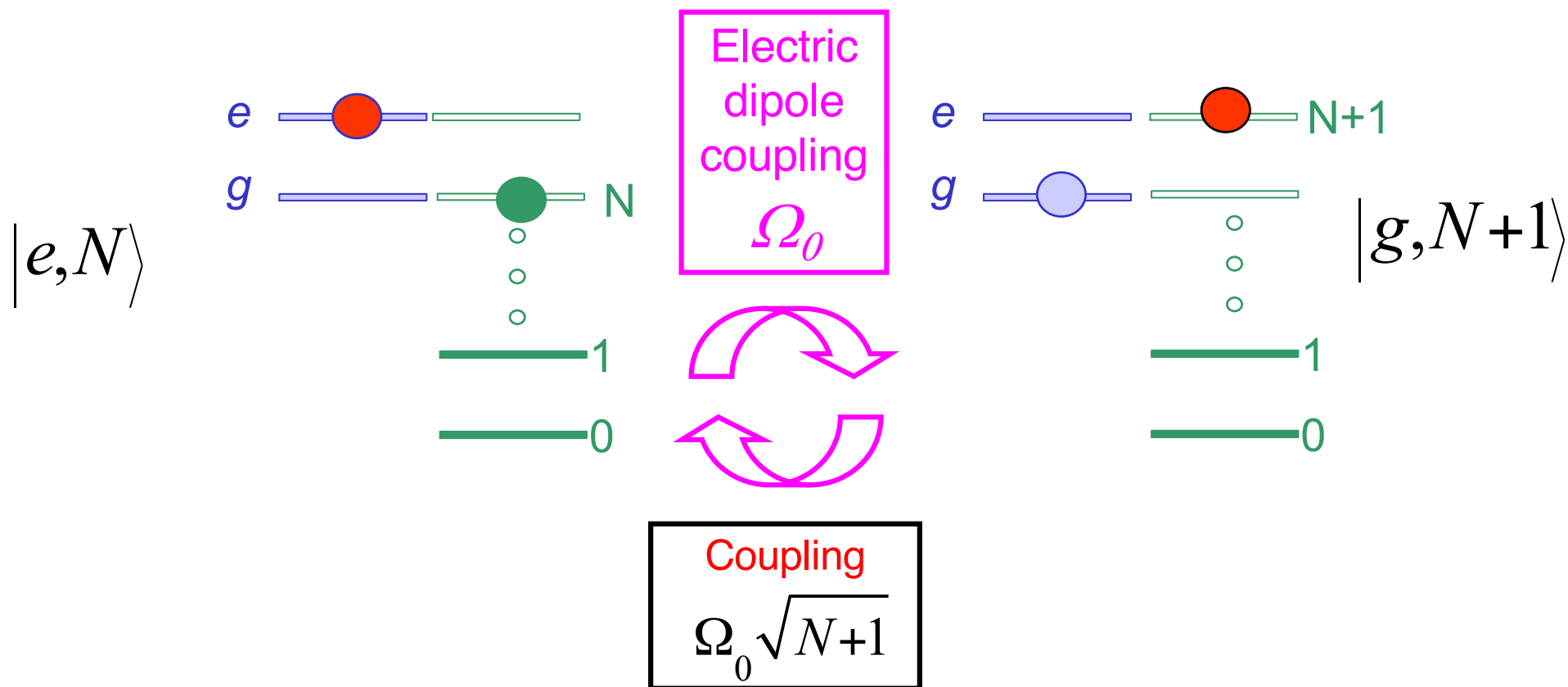
Photon number distribution:

$$P(N) = e^{-\bar{N}} \frac{\bar{N}^N}{N!} \quad ; \quad \bar{N} = |\alpha|^2$$



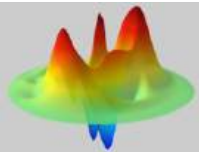


Resonant atom-field coupling: Cavity containing N photons

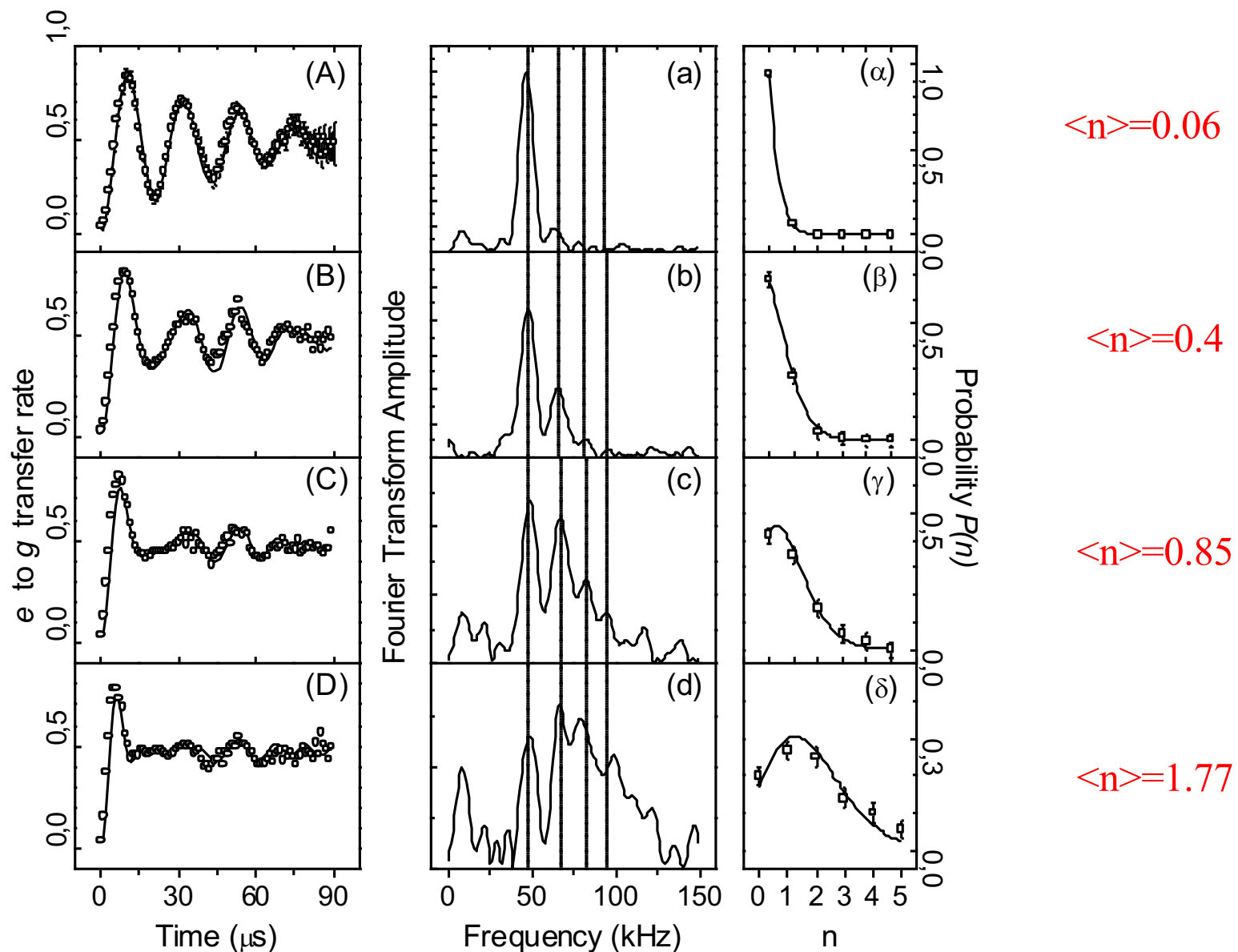


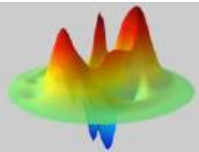
$$|e, 0\rangle \rightarrow \cos\left(\frac{\Omega_0 \sqrt{N+1} t}{2}\right) \cdot |e, 0\rangle - i \sin\left(\frac{\Omega_0 \sqrt{N+1} t}{2}\right) \cdot |g, 1\rangle$$

$$P_g(t) = \sum_N P(N) \frac{1}{2} \left(1 - \cos\left(\Omega_0 t \sqrt{N+1}\right) \right)$$



Rabi oscillation in small coherent fields

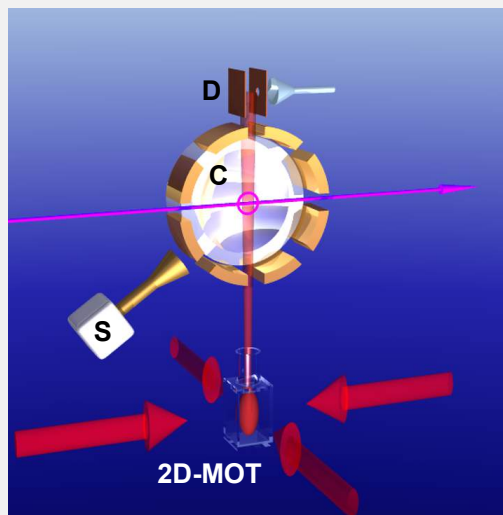
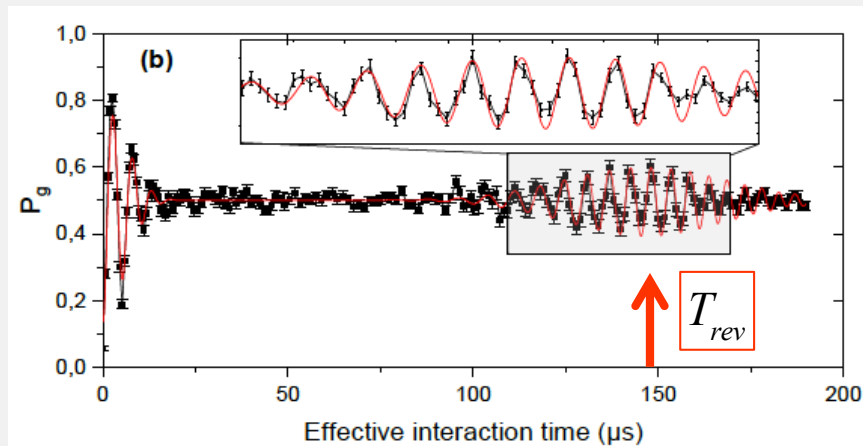


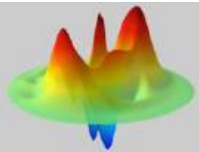


Rabi oscillation in a 13 photon coherent field

- Rabi oscillation in a **coherent** state

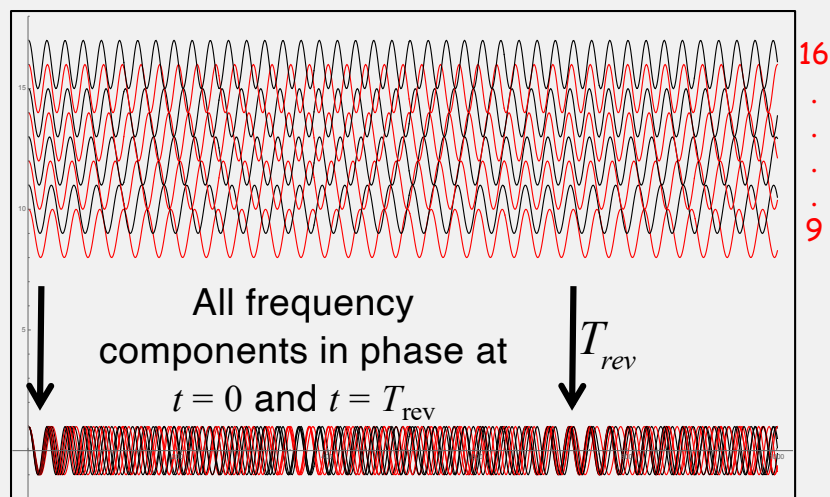
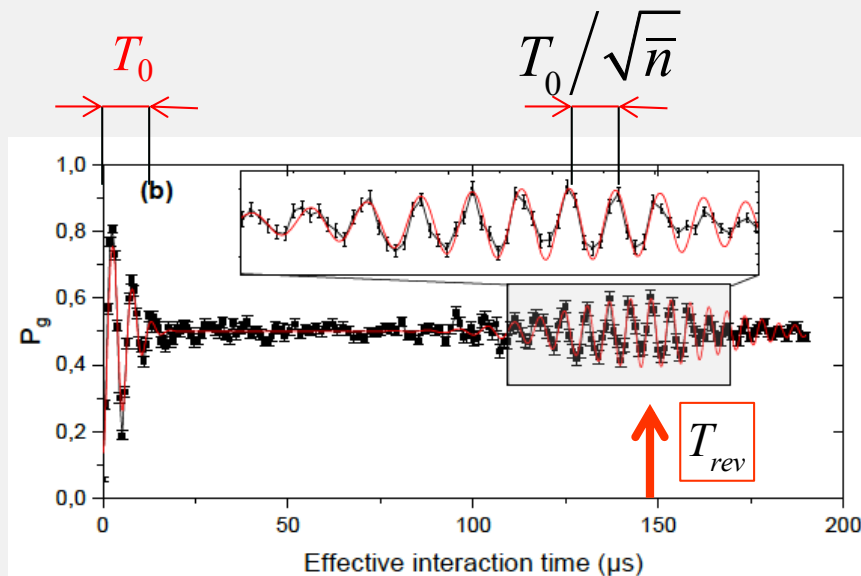
Eberly et al. PRL 44, 1323 (1980)





Rabi oscillation in a 13 photon coherent field

- Rabi oscillation in a **coherent** state



Eberly et al. PRL **44**, 1323 (1980)

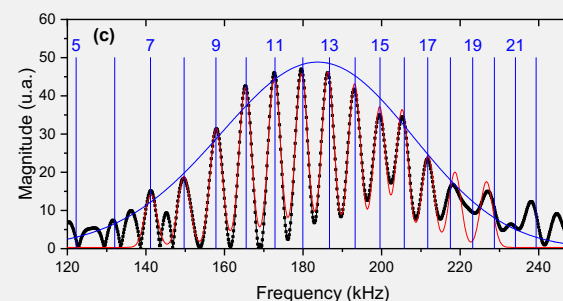
Revival time

$$T_{rev} = 2T_0\sqrt{n}$$

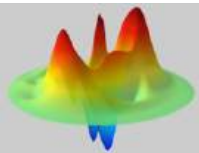
Usual interpretation :

Collapse – revival is due to beat note between different Rabi frequencies

Fourier transform of the signal

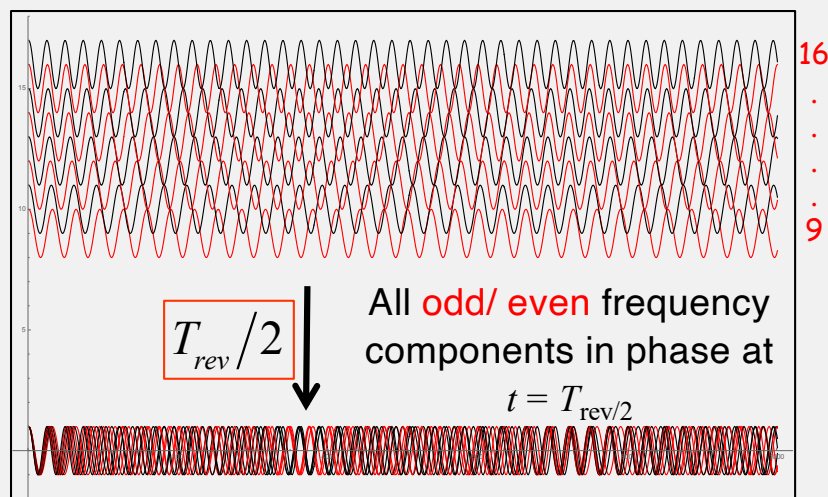
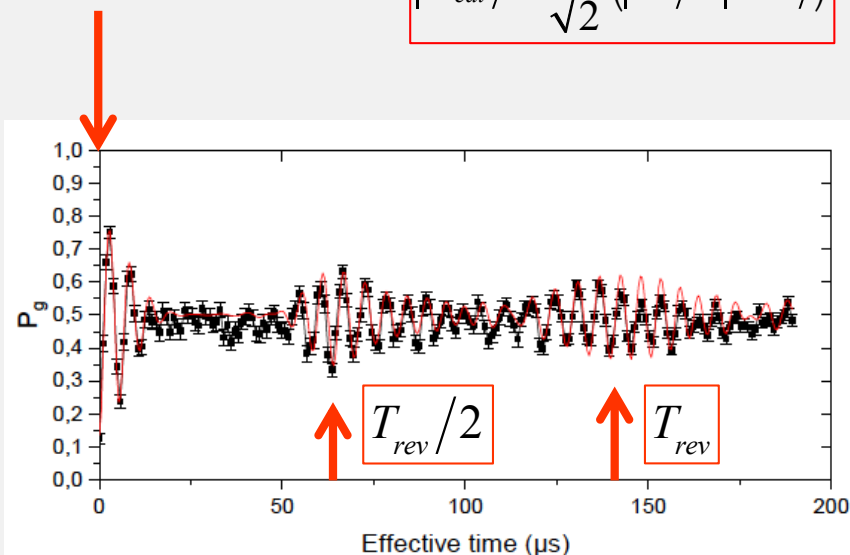


→ a direct manifestation of field quantization



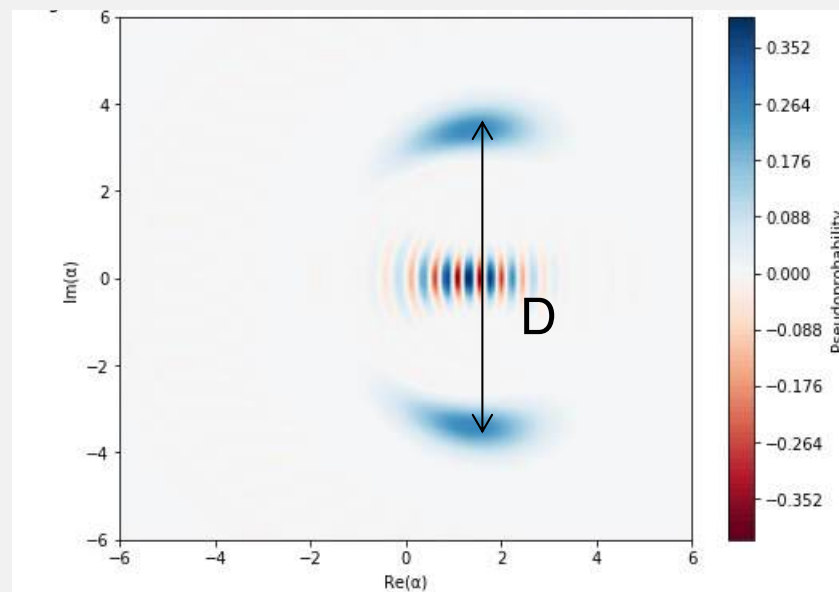
Revealing the cat state coherence

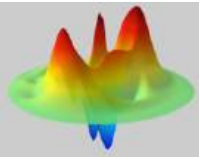
- Initial state: $|\psi_{cat}\rangle = \frac{1}{\sqrt{2}}(|i\beta\rangle - |-i\beta\rangle)$



"Odd" cat state: the "half revival" of Rabi oscillation spectrum reveals the **cat parity**
 → signature of quantum interference between the two classical components

Cat size: $D^2 = 46$ photons





Conclusion of lecture 1:

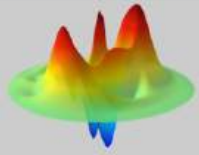
Cavity QED with microwave photons and circular Rydberg atoms:

.... a powerfull tool for:

- Achieving strong coupling between single atoms and single photons
- Demonstrating quantum features: collapse and revival of Rabi oscillations, Schrödinger cat state preparation

.... next lecture:

- CQED in the dispersive regime
 - Non-destructive photon counting



References

- **Strong coupling regime in Rydberg atom CQED experiments:**
 - ❑ F. Bernardot, P. Nussenzveig, M. Brune, J.M. Raimond and S. Haroche. "Vacuum Rabi Splitting Observed on a Microscopic atomic sample in a Microwave cavity". *Europhys. Lett.* **17**, 33-38 (1992).
 - ❑ P. Nussenzveig, F. Bernardot, M. Brune, J. Hare, J.M. Raimond, S. Haroche and W. Gawlik. "Preparation of high principal quantum number "circular" states of rubidium". *Phys. Rev. A* **48**, 3991 (1993).
 - ❑ M. Brune, F. Schmidt-Kaler, A. Maali, J. Dreyer, E. Hagley, J. M. Raimond and S. Haroche: "Quantum Rabi oscillation: a direct test of field quantization in a cavity". *Phys. Rev. Lett.* **76**, 1800 (1996).
 - ❑ J.M. Raimond, M. Brune and S. Haroche : "Manipulating quantum entanglement with atoms and photons in a cavity", *Rev. Mod. Phys.* vol.73, p.565-82 (2001).
 - ❑ P. Bertet, S. Osnaghi, A. Rauschenbeutel, G. Nogues, A. Auffeves, M. Brune, J.M. Raimond and S. Haroche : "Interference with beam splitters evolving from quantum to classical : a complementarity experiment". *Nature* 411, 166 (2001).
 - ❑ E. Hagley, X. Maître, G. Nogues, C. Wunderlich, M. Brune, J.M. Raimond and S. Haroche: "Generation of Einstein-Podolsky-Rosen pairs of atoms", *PRL* 79,1 (1997).
 - ❑ F. Assemat et al., accepted PRL, arXiv:1905.05247