Course on Two-dimensional Bose gases
Lecture 3: Rotating two-dimensional Bose gases
Seminar: Thermal melting of a vortex lattice

Hélène Perrin

Laboratoire de physique des lasers
CNRS & Université Sorbonne Paris Nord

IV School on Light and Cold Atoms
São Paulo, Oct 20–31, 2025

Outline of the course

- Lecture 1: Bose-Einstein condensation, interactions and superfluidity
- Lecture 2: Two-dimensional Bose gases at equilibrium
- Lecture 3: Rotating two-dimensional Bose gases

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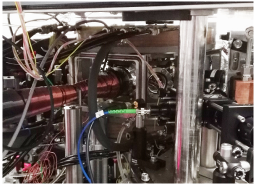
Cold atoms and molecules @LPL

4 groups, 7 setups + theory

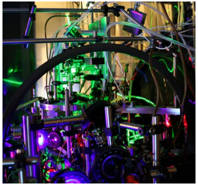
Rubidium lab

Sodium lab

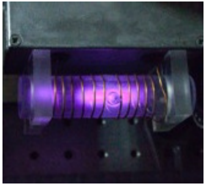
Strontium labs 1 & 2



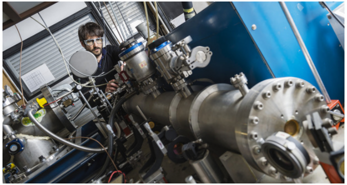
Chromium lab



Argon* lab



Cold molecules lab

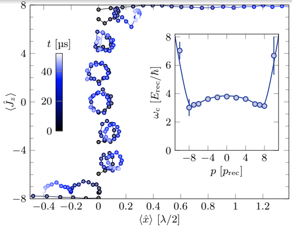


Quantum gases for hydrodynamics and quantum simulation

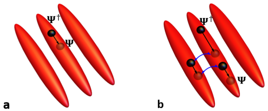
A few recent examples

Taking advantage of the great tunability of quantum gases

- Quantum Hall physics explored with synthetic dimensions [S. Nascimbene]
- phase transitions: superfluid – Mott insulator transition, Bose- and Fermi-Hubbard models [L. Tarruell]
- Hydrodynamics, e.g. in dipolar gases [J. Bohn & M. Mark]
- access to unconventional geometry (1D-2D gases, ring, bubble. . .)



Chalopin et al., Nat. Phys. **16**, 1017 (2020)



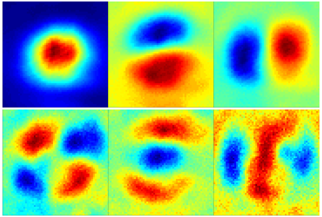
Guo et al., Nat. Phys. **20**, 934 (2024)

Out-of-equilibrium dynamics of quantum gases. includes superfluid dynamics, quantum transport, atomtronics, nonequilibrium thermodynamics. . .

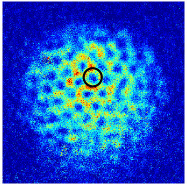
Hydrodynamics and superfluidty

The hydrodynamics equations describe

- BEC expansion after trap release
- Collective modes of trapped BEC $\longrightarrow$
- More generally the excitations: phonons, solitons, free particles, . . .
- The formation of quantized vortices in the rotating irrotational fluid

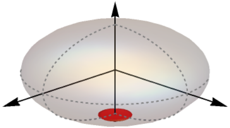


Dubessy et al., NJP **16**, 122001 (2014)



circulation h/M around each quantum vortex [LPL]

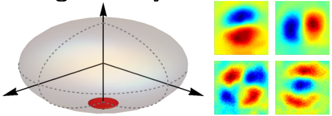
In this talk: characterize a **vortex lattice** at the bottom of a **shell trap**



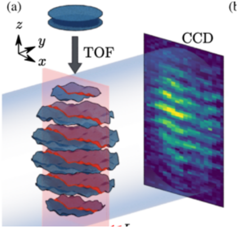
Superfluid on a shell

Gallery of recent experiments

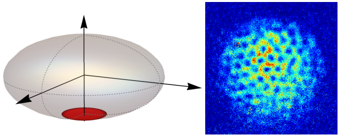
A geometry of interest to superfluid dynamics: the shell trap
[Zobay & Garraway PRL 2001, Dubessy & Perrin 2024]



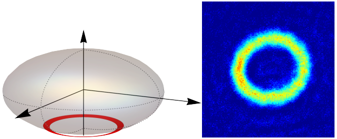
2D gas @LPL: EPL 2004,
NJP 2013, 2014, 2016



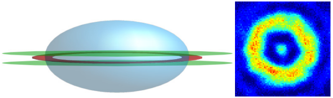
2 layers Foot's group PRA 2018,
PRL 2022, Science 2023



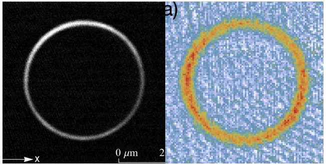
vortices PRL **133**, 143401 (2024)



giant vortex PRL **124**, 025301 (2020)



rings @LPL PRA 2006, JPB 2021,
Foot: NJP **10**, 043012 (2008)



TAAP rings von Klitzing PRL 2007,
NJP 2016, Nature 2019
Foot: PRA 2011

Outline

1. Introduction to superfluid dynamics (in a bubble)
2. Superfluids rotating in a bubble
   - Vortex lattices in a bubble trap
   - Theory of thermal melting in 2D
   - Observation of vortex lattice thermal melting
3. Summary & prospects

Rotation of a quantum gas

Hamiltonian in the rotating frame

Consider 2D gas in a potential $V(\mathbf{r})$ rotating at frequency Ω:

$$V_t(x, y, t) = V(x \cos \Omega t + y \sin \Omega t, -x \sin \Omega t + y \cos \Omega t).$$

The Hamiltonian is time-dependent: $H(t) = \frac{p^2}{2M} + V_t(x, y, t)$. We can make a unitary transform to get a **time-independent Hamiltonian in the rotating frame**:

$$H_{\rm rot} = UHU^\dagger + i\hbar\partial_t UU^\dagger, \qquad U(t) = e^{i\Omega t L_z/\hbar}.$$

$\hat{L}_z = (\hat{\mathbf{r}} \times \hat{\mathbf{p}}) \cdot \mathbf{e}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$ is the projection of the orbital angular momentum operator along z. We get for the Hamiltonian in the rotating frame:

$$H_{\rm rot} = \frac{p^2}{2M} + V(x, y) - \Omega L_z.$$

Rotation of a quantum gas

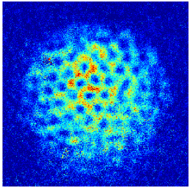
Analogy with a magnetic field for charged particles

Hamiltonian in the rotating frame:

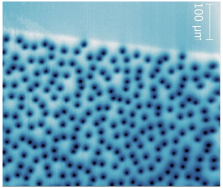
$$H_{\mathrm{rot}} = \frac{p^2}{2M} + V(r) - \Omega L_z = \frac{(p - q\mathcal{A})^2}{2M} + V(r) - \frac{1}{2}M\Omega^2 r^2$$

where $q\mathcal{A} = M\Omega(-y\mathbf{e}_x + x\mathbf{e}_y)$. Same as a particle of charge q in a magnetic field $\mathbf{B} = B\mathbf{e}_z$ with $qB = 2M\Omega$!

Vortex lattice

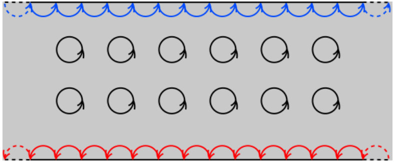


quantum gas



superconductor

Analogy: e^- in B field



lowest Laudau level

...but with an additional centrifugal potential!

Rotation in an anharmonic trap

Fighting the centrifugal force when $\Omega \sim \omega_r$

Hamiltonian in the rotating frame:

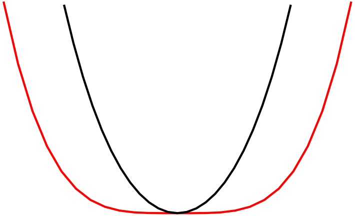
$$H_{\rm rot} = \frac{p^2}{2M} + V(r) - \Omega L_z = \frac{(p - q\mathcal{A})^2}{2M} + V(r) - \frac{1}{2}M\Omega^2 r^2$$

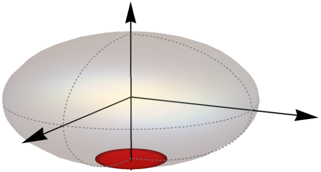
where $q\mathcal{A} = 2M\Omega(-y\mathbf{e}_x + x\mathbf{e}_y)$.

Use anharmonicity of the shell trap to warrant trapping for all Ω.

$\Omega = 0$

$\Omega = \omega_r$





$\Omega = \omega_r$

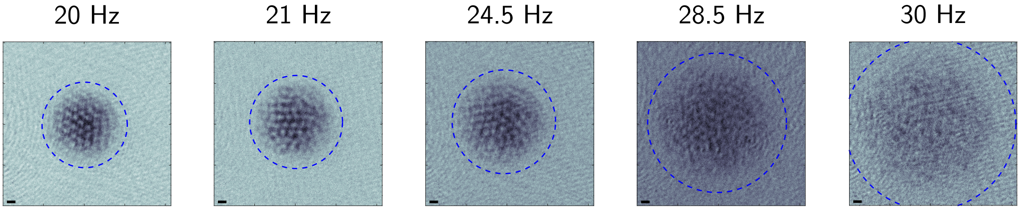
[See also Cornell & Dalibard groups, PRL **92**, 040404 & 050403 (2004)]

Vortex lattice in fast rotating trap

A first quick look

- Start from a degenerate cloud at rest at the bottom of the bubble, $\omega_r = 2\pi \times 34$ Hz
- Induce an in-plane elliptic deformation $V(r) = M\omega_r^2/2 \times \left[(1-\varepsilon)x'^2 + (1+\varepsilon)y'^2\right] + \dots$
- Rotate the trap main axes x', y' at frequency Ω_{rot}
- Back to rotationally invariant trap

Increasing Ω_{rot} . . . vortex density $n_v \sim M\Omega/(\pi\hbar)$. . . disordered lattice. . . melting!



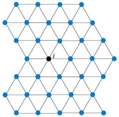
Blue dashed circle: Thomas-Fermi radius. $\varepsilon = 0.18$

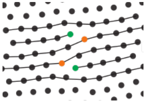
Thermal melting phase transition in 2D

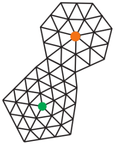
Kosterlitz Thouless Halperin Nelson Young theory

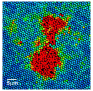
Thermal melting transition in 2D is very general.
It is described by KTHNY theory and involves 3 phases:

- $T < T_1$: crystalline phase: translational and orientational orders are preseved: triangular lattice, each particle has 6 neighbors
- $T_1 < T < T_2$: hexatic phase: break down of translational order, apparition of dislocations (5-7 pairs)
- $T > T_2$: isotropic liquid phase: break down of both orders, disclinations (breaking of 5-7 pairs)









Relevant for classical systems (colloids) and quantum systems: vortices in superconductors or quantum fluids

©Peter Keim

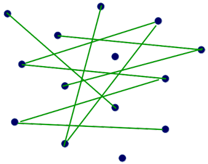
Our approach for a vortex lattice in a rotating superfluid:
Effective temperature $\tau = T/T_c(\Omega)$ at fixed T and increasing Ω.

Characterizing the lattice order

Correlation functions

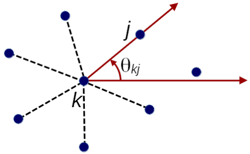
Ordering characterized by relevant correlation functions:

- Translational order: normalized pair correlation function (easier):



$$g(r) = \frac{\sharp \text{ vortex pairs split by } r \text{ in sample}}{\sharp \text{ pairs split by } r \text{ for uniform vortex density}}$$

- Orientational order: orientational correlation function



$$G_6(r) = \langle \Psi_k^{(6)*} \Psi_p^{(6)} \rangle_{|\mathbf{r}_k - \mathbf{r}_p| \sim r} \quad \text{with} \quad \Psi_k^{(6)} = \frac{1}{N_k} \sum_{j=1}^{N_k} e^{6i\theta_{kj}}$$

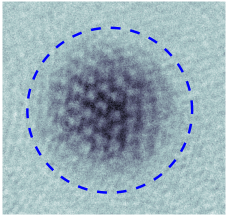
$g(r)$, $G_6(r)$: $r^{-\nu_g}$, cst e^{-r/ℓ_g}, $r^{-\nu}$ e^{-r/ℓ_g}, e^{-r/ℓ_G}

$T = 0$ crystal T_1 hexatic T_2 liquid

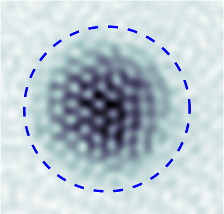
Automatic vortex detection

Locate and count the vortices with an automatic procedure

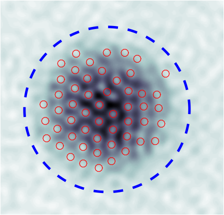
raw data

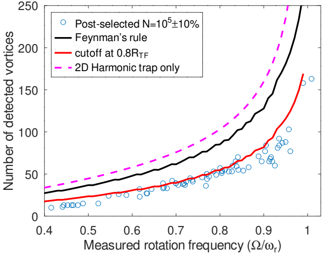


Gauss filtering



vortex detection

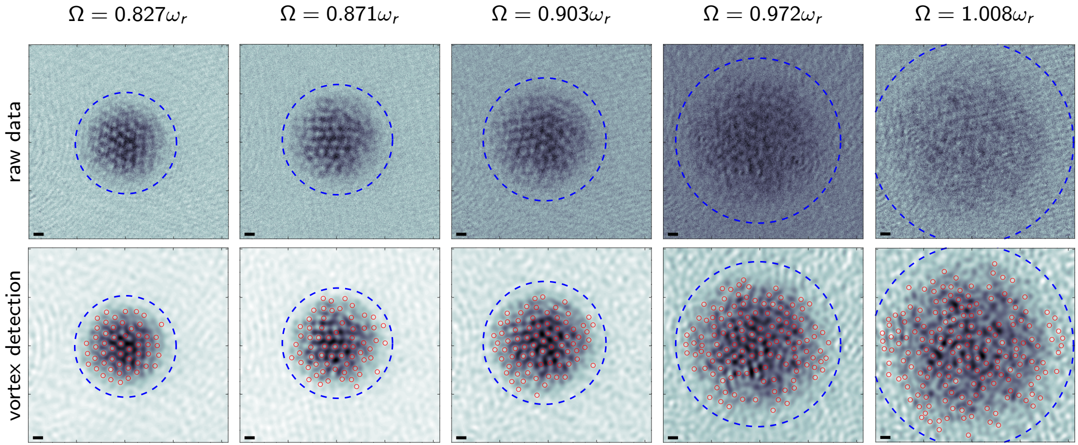




Vortex density $n_v = N_v/\pi R_e^2$ agrees with Feynman's formula $n_v = M\Omega/\pi\hbar$ taking an effective TF radius $R_e = 0.8R_{\mathrm{TF}}$.

Automatic vortex detection

Locate and count the vortices with an automatic procedure



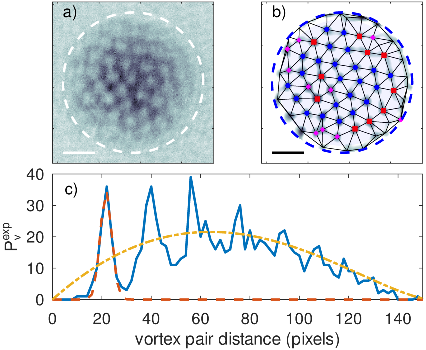
Field of view $\sim 380 \times 390\ \mu\mathrm{m}^2$

Experimental results: pair correlations

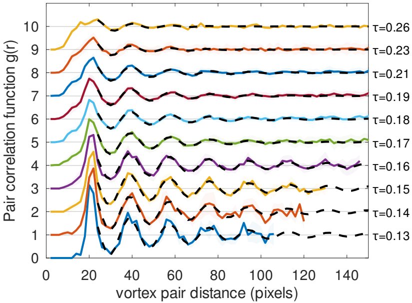
Correlations in vortex position: distances

Evolution of the pair correlation function $g(r)$ across the transition.

Delaunay triangulation



decay of $g(r)$ for various Ω



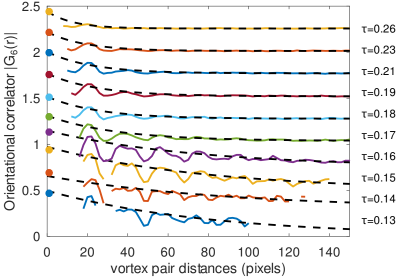
Typical behavior of a transition to a liquid state: extract decay length ℓ_g of the pair correlation function.

Experimental results: orientation correlations

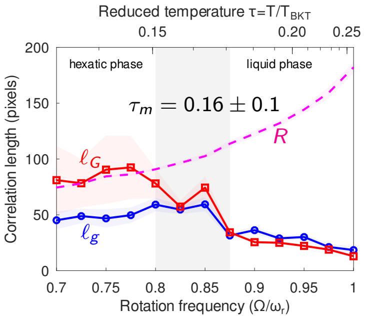
Correlations in vortex position: angles

Evolution of the orientational correlation function $|G_6(r)|$ across the transition $\Rightarrow$ extract decay length ℓ_G.

decay of $G_6(r)$



evolution of decay lengths



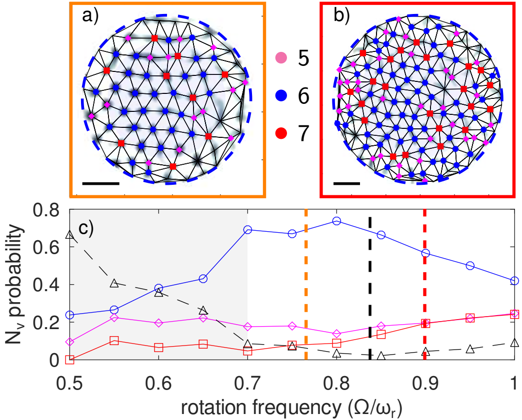
Two different behaviors:

$\tau < 0.16$: $\ell_g < R$, $\ell_G = R$: hexatic phase?

$\tau > 0.16$: $\ell_g = \ell_G < R$: liquid phase?

From hexatic to liquid phase

Proliferation of defects



- low Ω: finite size effects
- moderate Ω, $\tau < \tau_m$: majority of 6 neighbors
- 5-neighbor defects almost constant
- increase of 7-neighbor defects with τ
- consistent with KTHNY scenario

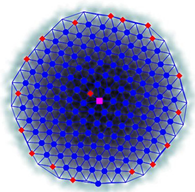
R. Sharma et al. PRL **133** 143401 (2024)

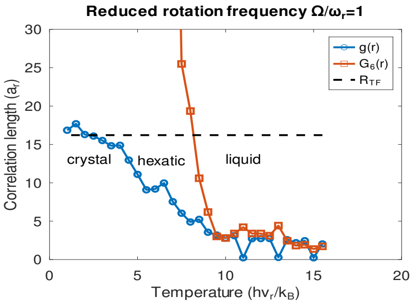
Preliminary: comparison to numerical simulations

Finite temperature SPGPE simulations

NB: $\tau_m = 0.16$ consistent with the **upper bound** 0.23 derived by Gifford & Baym [GPRA **78**, 043607 (2008)] $\Rightarrow$ can we give a better estimate of τ_m?

Collaboration with USP



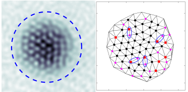


SGPE simulations by
Sálvio J. Bereta (USP/USPN)

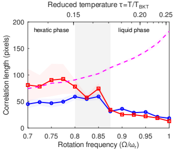
Preliminary finding: **3 phases** observed in the simulations. Quantitative agreement with experimental data yet to be checked.

Summary & prospects

Fast rotation: a very smooth and tunable bubble trap to study the collective modes and fast rotations

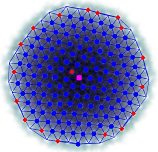


- Preparation of fast rotating clouds $\Omega \sim \omega_r$
- Correlations between vortex positions and orientations as indicator of vortex ordering
- Vortex lattice melting observed at large rotations and fixed T



Outlook:

- look for defect pairing
- quantitative comparaison to simulations at finite T
- explore the effect of bubble curvature



BEC group at Sorbonne Paris Nord

Y. Guo*

D. Rey

M. Cassus M.Ballu **R. Dubessy*** **S. Bereta**

B. Mirmand **L. Longchambon** **R. Sharma**

Collaborations

M. Olshanii
A. Minguzzi
B. Garraway
S. Nazarenko

R. Dubessy*
V. S. Bagnato
L. Madeira
M. Caracanhas

T. Schumm
D. Papoular
P. Vignolo & M. Albert
Q. Glorieux

bec.lpl.univ-paris13.fr