



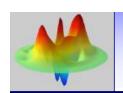
From cavity QED to quantum simulations with Rydberg atoms

Lecture 3
Quantum measurement,
Schrödinger cat
and decoherence

Michel Brune



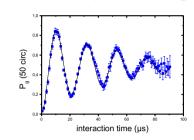
École Normale Supérieure, CNRS, Université Pierre et Marie Curie, Collège de France, Paris



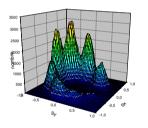
Previous lectures

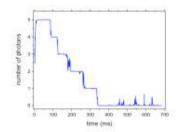
Cavity QED with microwave photons and circular Rydberg atoms:

 L1:Achieving strong coupling between single atoms and single photons



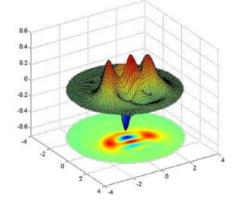
 L2: Performing QND measurement of the field state



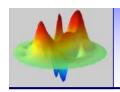


- L3: The same experiment seen from the point of view of the field:
- → Schrödinger cat preparation and monitoring of its decoherence

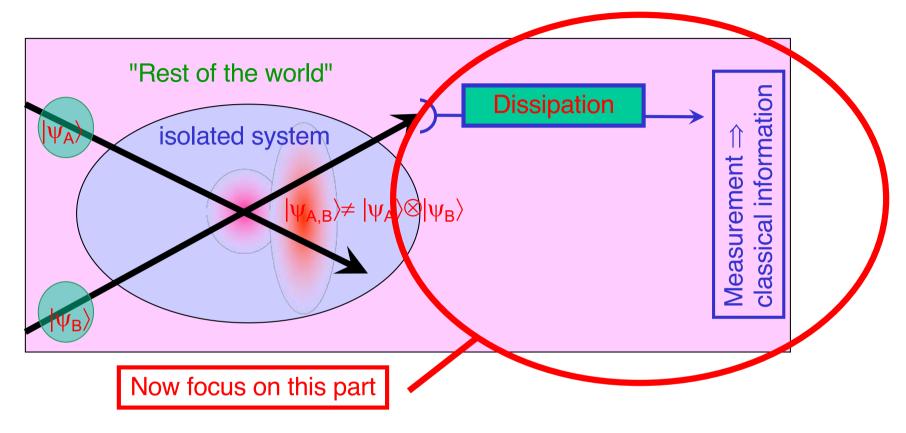




Lecture 3: Quantum measurement, Schrödinger cat and decoherence



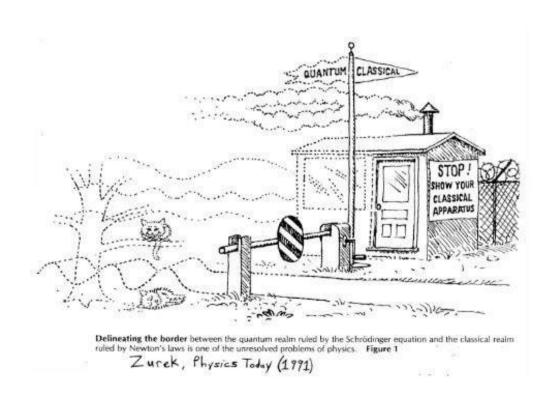
Quantum measurement: basic ingredients

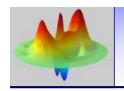


- □ We have shown how to built an ideal QND meter of the photon number
- □ This detector is based on a destructive detector of the atom energy.
- Let us now built a more complete, fully quantum, model of detector including the dissipative part

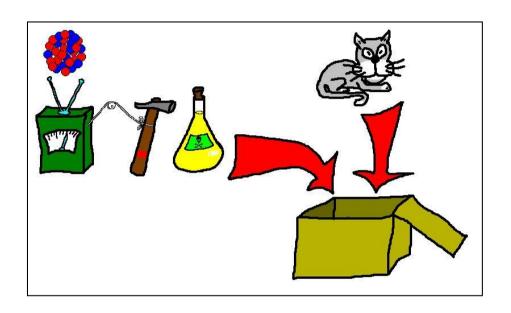
1. The "Schrödinger cat" and the quantum measurement

The border separation quantum and classical behavior

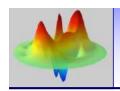




Quantum description of a meter: the "Schrödinger cat" problem

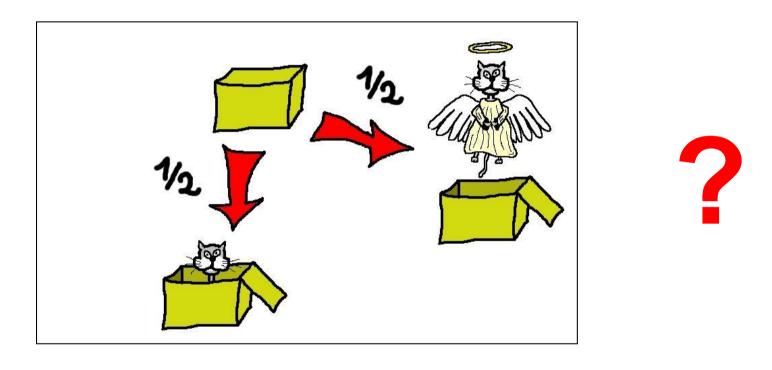


One encloses in a box a cat whose fate is linked to the evolution of a quantum system: one radioactive atom.

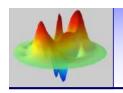


The "Schrödinger cat"

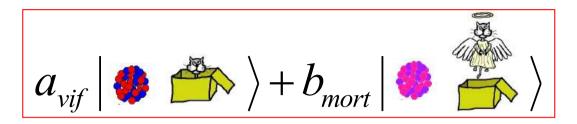
 One closes the box and wait until the atom is desintegrated with a probability 1/2



 When opening the box is the cat dead, alive or in a superposition of both?

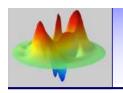


Schrödinger cat and quantum measurement



- Before opening the box, the system is isolated and unitary evolution prepares a maximally atom-meter entangled state
- Does this state "really" exists?
 - → a more relevant question: can one perform experiments demonstrating cat superposition state? Up to which limit?
- That is a fundamental question for the quantum theory of measurement: how does the unphysical entanglement of SC state vanishes at the macroscopic scale. That is the problem of the transition between quantum and classical world

$$\frac{1}{\sqrt{2}}(|a\rangle+|b\rangle) \Rightarrow \frac{1}{\sqrt{2}}(|a, \bigcirc \rangle + |b, \bigcirc \rangle)$$



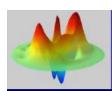
Schrödinger cat and quantum measurement

$$\frac{1}{\sqrt{2}}(|e\rangle+|g\rangle) \Rightarrow \frac{1}{\sqrt{2}}(|e, |f|) + |g, |f|)$$

- Real measurement provide one definite result and not superposition of results: SC states are unphysical?
- Schrödinger: unitary evolution should "obviously" not apply any more at "some scale".
- It seems that the atom-meter space contains to many states for describing reality
- Including dissipation due to the coupling of the meter to the environment will provide a physical mechanism "selecting" the physically acceptable states.

Let's look at this in a real experiment using a meter whose size can be varied continuously from microscopic to macroscopic world.

2. A mesoscopic field as atomic state measurement aparatus



A mesoscopic "meter": coherent field states

- Number state: $|N\rangle$
- Quasi-classical state:

$$\left|lpha
ight>=e^{-\left|lpha
ight|^{2}/2}\sum_{N}rac{lpha^{N}}{\sqrt{N\,!}}\left|N
ight> \;\;\;;\;\;\;lpha=\left|lpha
ight|e^{i\Phi}$$

Photon number distribution

$$P(N) = e^{-\overline{N}} \frac{\overline{N}^{N}}{N!} ; \overline{N} = |\alpha|^{2}$$

$$0.4 \quad \text{Poisson distribution}$$

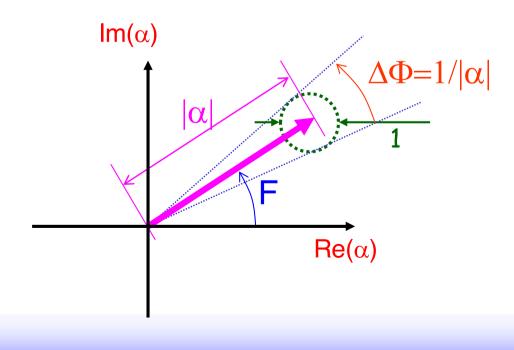
$$0.3 \quad \overline{N} = 2$$

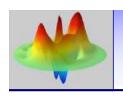
$$0.1 \quad \overline{N} = 2$$

 $\Delta N = 1/|\alpha|$

Phase space representation

$$\Delta N \cdot \Delta \Phi > 1$$





Displacement operator

• Definition:

$$\hat{D}(lpha) = \exp\Bigl(lpha \hat{a}^\dagger - lpha^* \hat{a}\Bigr)$$

Where α is a C-number

Properties

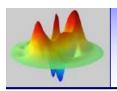
$$|lpha^-
angle \equiv \hat{D}^-(lpha)|0
angle$$
 Easily demonstrated using $\hat{D}(lpha) = e^{-rac{1}{2}|lpha|^2}e^{+lpha\hat{a}^\dagger}e^{-lpha^*\hat{a}}$

$${\hat D}^{\dagger}(lpha){\hat a}{\hat D}(lpha)={\hat a}+lpha$$

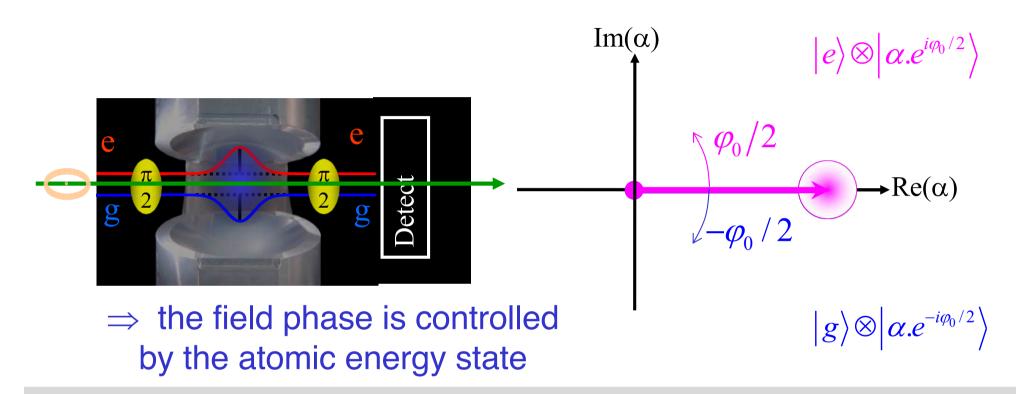
$$\hat{D}(lpha)\hat{D}(eta)=e^{(lphaeta^*-lpha^*eta)/2}\hat{D}(lpha+eta)$$

· Coherent states are generated in a cavity by a classical source:

 $\hat{D}(\alpha)$ is the corresponding evolution operator



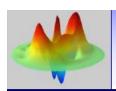
Preparing a phase Schödinger cat state



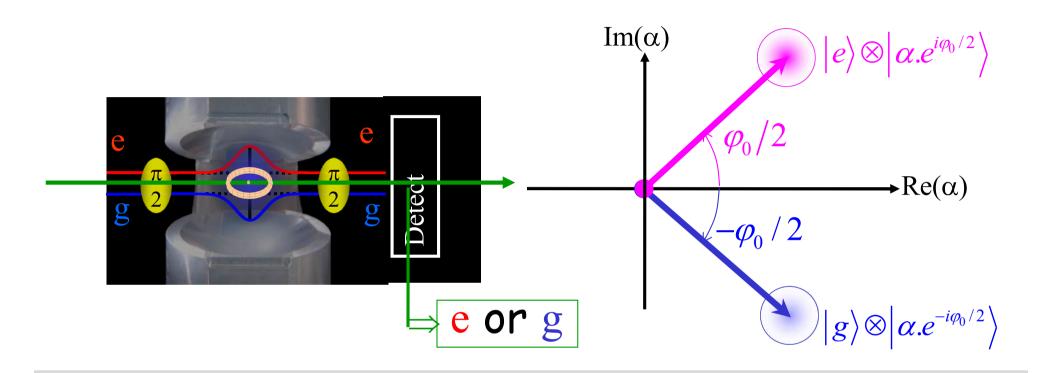


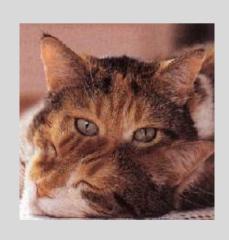
$$\frac{1}{\sqrt{2}} \left(\left| e, \alpha. e^{i\varphi_0/2} \right\rangle \pm \left| g, \alpha. e^{-i\varphi_0/2} \right\rangle \right)$$

→ Entangled atom-field cat state



Preparing a phase Schödinger cat state





Detection of the atom projects the field on:

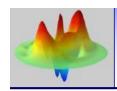
$$\frac{1}{\sqrt{2}} \left(\left| e, \alpha. e^{i\varphi_0/2} \right\rangle \pm \left| g, \alpha. e^{-i\varphi_0/2} \right\rangle \right)$$

±: depends on detected state e or g

Field changed by a quantum interference process

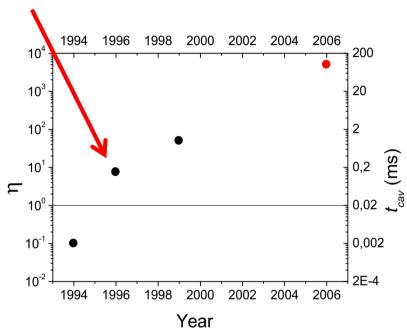
3. Experimental characterization of the cat state

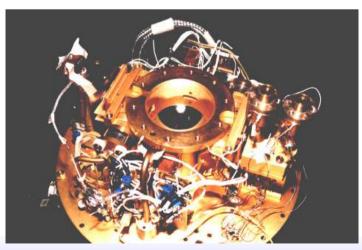
- Full tomography of the quantum state

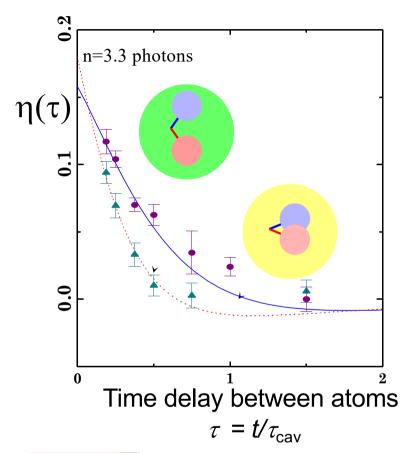


First cat state generation

Back to 1996

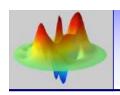




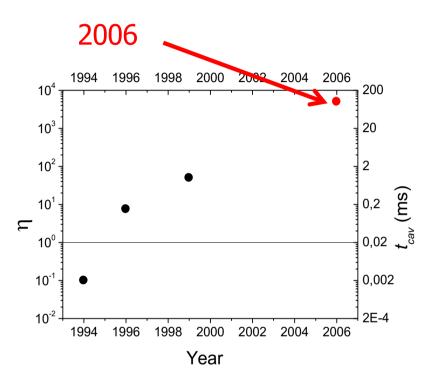


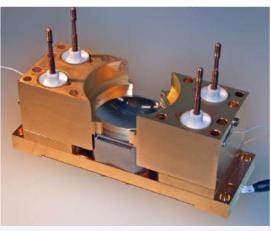


3 photons kitten



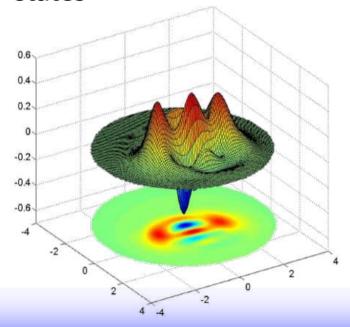
Second cat state generation

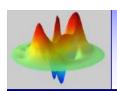




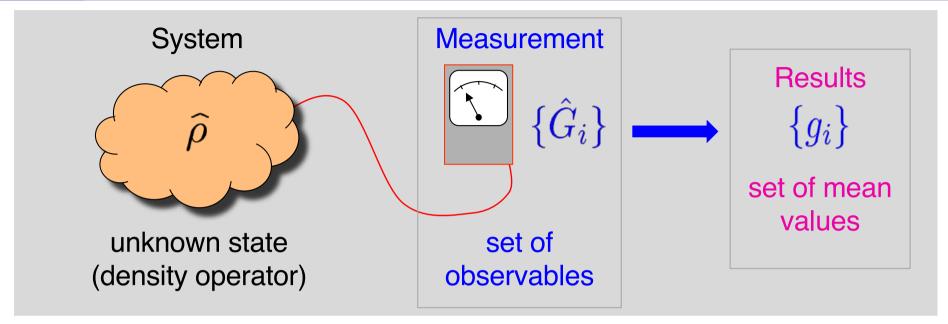
0.1 ms photon lifetime longenough to

- perform full quantum tomography of the cat state a
- monitor the decoherence of the reconstructed quantum states





Principle of state reconstruction

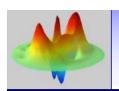


→ Each measurement sets a constrain to the density operator

$$\operatorname{Tr}(\hat{\rho}\,\hat{G}_i) = g_i$$

Problems to face:

- Having a complete set of observable $\{\hat{G}_i\}$ fully determining $\hat{oldsymbol{
 ho}}$
- Statistical noise on $\{g_i\}$ may lead to unphysical/very noisy density operators



Measuring the field density operator?

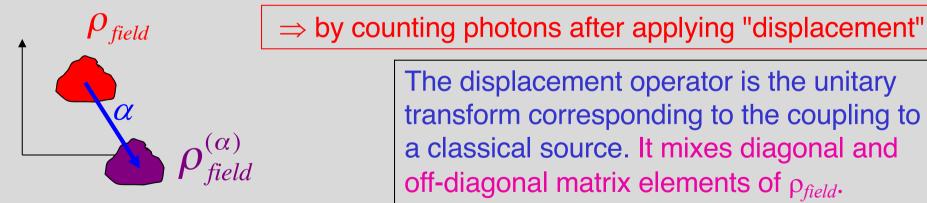
General field state description: density operator

$$\rho_{field} = \begin{bmatrix} \rho_{00} & \rho_{01} & \rho_{02} \\ \rho_{10} & \rho_{11} & \rho_{12} \\ \rho_{20} & \rho_{21} & \rho_{22} \\ \vdots & \vdots & \ddots \end{bmatrix}$$
• Previous lectures:

QND counting of photons
$$\Rightarrow \text{measurement of diagonal elements } \rho_{nn}$$

$$\Rightarrow \text{How to measure the off-diagonal elements of } \rho_{field} ?$$

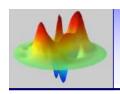
- diagonal elements of ρ_{field} ?



$$\rho_{field}^{(\alpha)} = \hat{D}(\alpha)\rho_{field}\hat{D}^{+}(\alpha)$$

$$\hat{D}(\alpha) = e^{\alpha a^{+} - \alpha^{*}a} \quad \text{Displacement operator}$$

The displacement operator is the unitary transform corresponding to the coupling to a classical source. It mixes diagonal and off-diagonal matrix elements of ρ_{field} . Measuring the photon number after displacement for a large number of different a gives information about all matrix elements of ρ_{field} .



Choice of reconstruction method

Various possibilities:

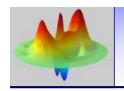
- \Box Direct fit of ρ_{field} on the measured data $g_i = tr[\rho_{field}.\cos(\phi(\hat{n}) + \varphi)]$
- □ Maximum likelihood: find ρ_{field} which maximizes the probability of finding the actually measured results g_i .
- \Box Maximum entropy principle: find ρ_{field} which fits the measurements and additionally maximizes entropy

$$S = \rho_{field} \log(\rho_{field})$$
.

V. Bužek and G. Drobný, *Quantum tomography via the MaxEnt principle*,
Journal of Modern Optics **47**, 2823 (2000)

Estimates the state only on the basis of measured information: in case of incomplete set of measurements, gives a "worse estimate" of ρ_{field} .

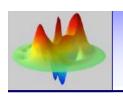
In practice the two last methods give the same result provided one measures enough data completely determining the state.



State reconstruction: experimental method

- 1- prepare the state to be measured $|\psi_{field}\rangle$
- 2- measure $\hat{G}(\alpha)$ for a large number of different values of α (400 to 600 points).
- 3- reconstruct ρ_{field} by maximum entropy method
- 4- Represent the field stae using Wigner function calculated from from ρ_{field} .

What is the Wigner function?



Wigner function representation of the quantum state

Definition

- Position proba. density
- Joint x,p quasi-proba density

$$P(x) = Tr(\rho|x\rangle\langle x|) = Tr(\rho\,\delta(\hat{x} - x))$$

- Momentum proba. Density
$$P(p) = Tr(
ho|p\rangle\langle p|) = Tr(
ho\,\delta(\hat{p}-p))$$

$$W(x,p) = Tr(\rho \, \delta(\hat{x} - x, \hat{p} - p))$$
 "Wigner function"

- Looks like a joined probability **BUT can be negative**.
- Any observable can be expressed as a function of W, which thus contains all the information on the quantum state.
- Another equivalent definition:

$$W(x,p) = rac{1}{\pi \hbar} \int_{-\infty}^{\infty} \langle x-y|\hat{
ho}|x+y
angle e^{2ipy/\hbar}\,dy,$$

Quantum optics version: field quadratures plays the role of x and p

$$\hat{X} = \frac{1}{\sqrt{2}} (\hat{a} + \hat{a}^{\dagger})$$

$$\hat{P} = \frac{1}{i\sqrt{2}} (\hat{a} - \hat{a}^{\dagger})$$

$$\hat{P} = \frac{1}{i\sqrt{2}} \left(\hat{a} - \hat{a}^{\dagger} \right)$$

Diplacement operator $\hat{D}(\alpha)$

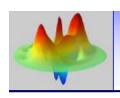
Parity operator

$$e^{i\pi\hat{a}^{\dagger}\hat{a}}$$

$$W(\alpha) = \operatorname{Tr} \left[\hat{\rho} \, \hat{D}(-\alpha) e^{i\pi \hat{a}^{\dagger} \hat{a}} \hat{D}(\alpha) \right]$$

$$\alpha = x + i p$$

W is an observable

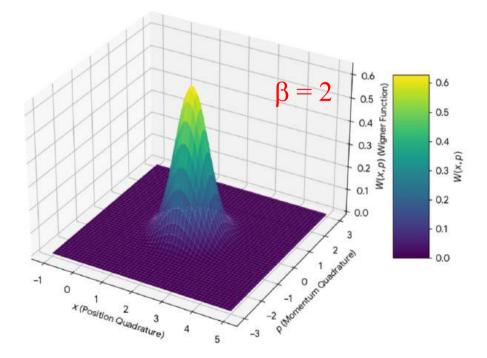


Exemple of Wigner Function

$$W(\alpha) = \operatorname{Tr}\left[\hat{\rho}\,\hat{D}(-\alpha)e^{i\pi\hat{a}^{\dagger}\hat{a}}\hat{D}(\alpha)\right]$$

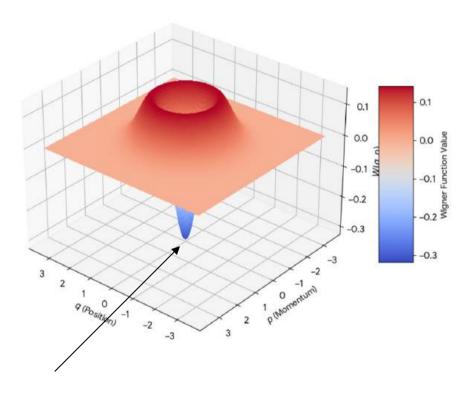
Wigner function for a coherent state $|\beta\rangle$

$$W(lpha)=rac{2}{\pi}e^{-2|lpha-eta|^2}$$

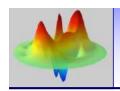


Gaussian quasi-probability distribution

Number state N=1

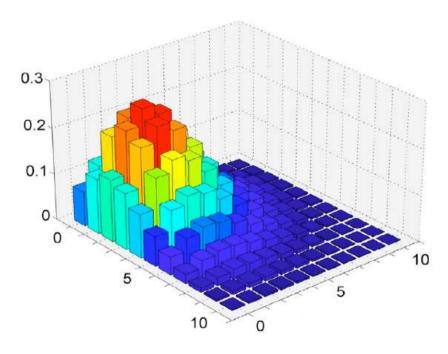


 $W(\alpha)$ is negative at some points

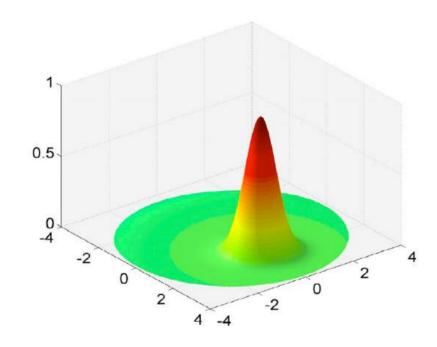


reconstruction of a coherent field

- Measurement for 161 values of α (<1 hour measurement)
- 7000 detected atoms in 600 repetition of the experimental sequence for each α .



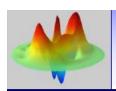
Density matrix



Wigner function (measurement)

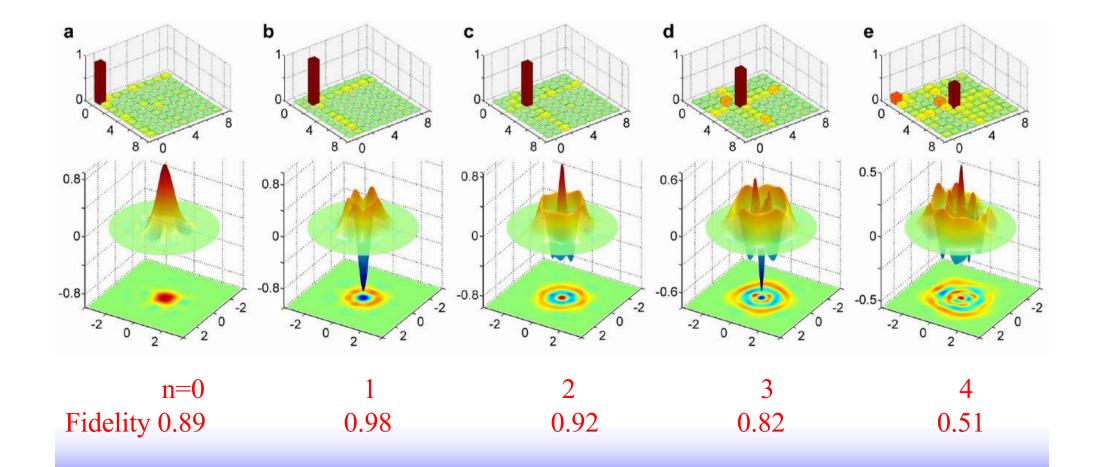
• State fidelity:
$$F = tr(|\beta\rangle\langle\beta|\rho_{field})$$

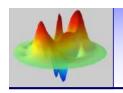
$$F=98\%$$
 for $\beta^2=2.5$ photons



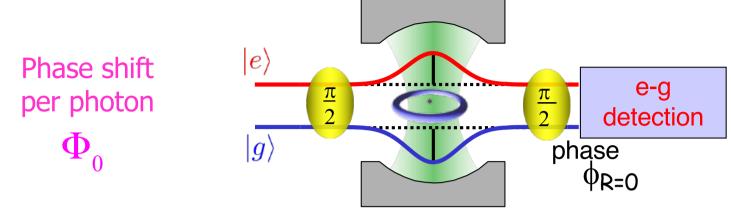
Reconstruction of number states

- Prepare a coherent state $\beta^2 = 1.3$ or 5.5 photons.
- Select pure number state by QND measurement of n. Phase shift per photon $\phi_0 \approx \pi/2$: measurement of n modulo 4.
- Measurements of $G(\alpha)$ for 2 different values of φ and \sim 400 values of α .

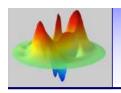




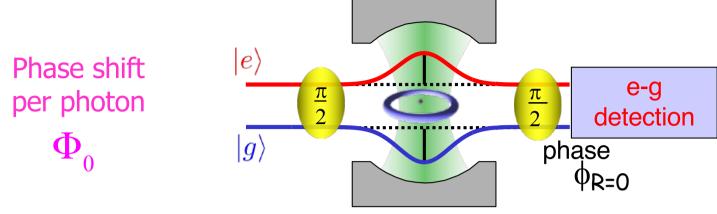
Preparation of the cavity cat state



$$\frac{1}{\sqrt{2}} \left(\left| e \right\rangle + \left| g \right\rangle \right) \otimes \left| \alpha \right\rangle \quad \Rightarrow \quad \frac{1}{\sqrt{2}} \left(\left| e \right\rangle \otimes \left| \alpha . e^{i\Phi_0/2} \right\rangle + \left| g \right\rangle \otimes \left| \alpha . e^{-i\Phi_0/2} \right\rangle \right)$$



Preparation of the cavity cat state



$$\frac{1}{\sqrt{2}} (|e\rangle + |g\rangle) \otimes |\alpha\rangle \implies \frac{1}{\sqrt{2}} (|e\rangle \otimes |\alpha \cdot e^{i\Phi_0/2}\rangle + |g\rangle \otimes |\alpha \cdot e^{-i\Phi_0/2}\rangle)$$

Field state after detection:

$$\Rightarrow \frac{1}{\sqrt{2}} \left(\left| \alpha . e^{i\Phi_0/2} \right\rangle + \left| \alpha . e^{-i\Phi_0/2} \right\rangle \right) \text{ if "e" detected}$$

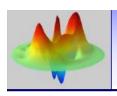
$$+ \frac{1}{\sqrt{2}} \left(\left| \alpha . e^{i\Phi_0/2} \right\rangle - \left| \alpha . e^{-i\Phi_0/2} \right\rangle \right) \text{ if "g" detected}$$

$$\Phi_0 = \pi$$

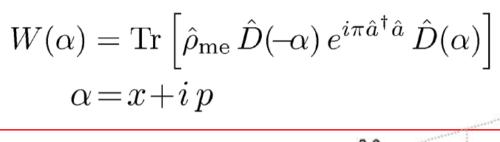
g → odd cat state

Depending on the detected atomic state the cat has a well defined photon number parity.

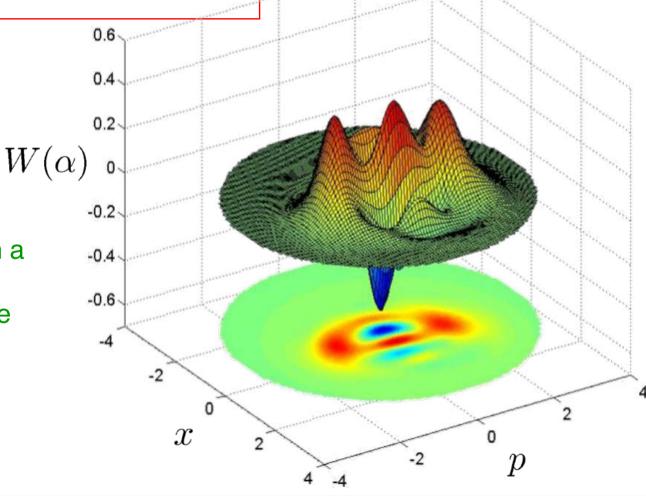
For π per photon phase shift, one atom measures just the field parity. Projection on a cat state is the "back-action" of parity measurement.

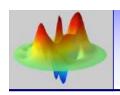


Reconstructed Wigner function



No a priori knowledge on a prepared state except for the size of the Hilbert space of $N_{Hilbert} = 9$





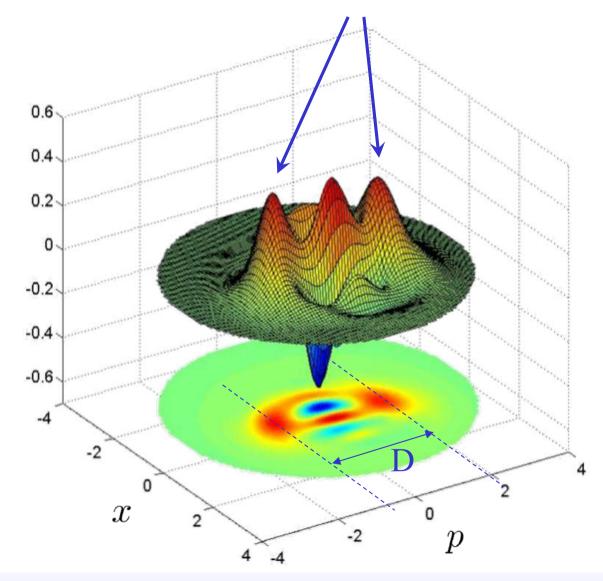
Reconstructed Wigner function

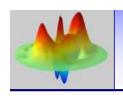
Classical components

≈ 2.1 photons in each classical component (amplitude of the initial coherent field)

cat size $D^2 \approx 7.5$ photons

coherent components are completely separated (D > 1)



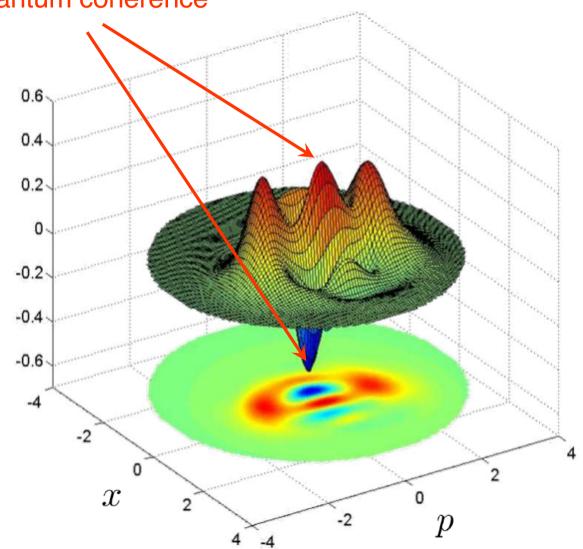


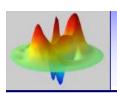
Reconstructed Wigner function

Quantum coherence

quantum superposition of two classical fields (interference fringes)

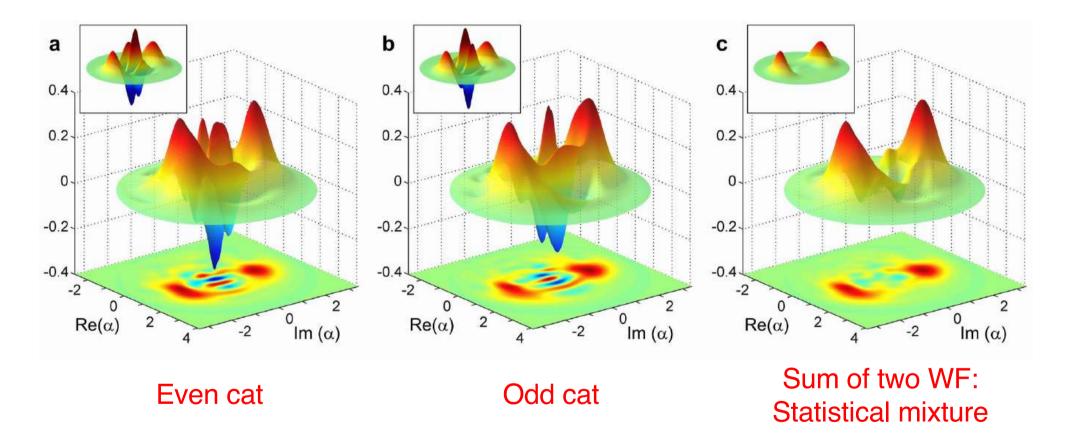
quantum signature of the prepared state (negative values of Wigner function)





A larger cat for observing decoherence

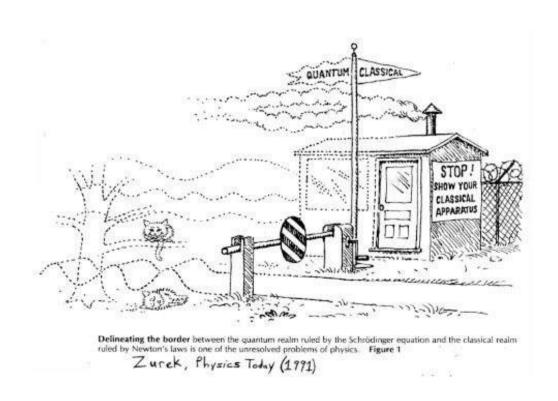
- Initial coherent field $\beta^2 = 3.5$ photons
- Measurement for 400 values of α .



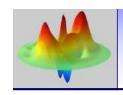
State fidelity with respect to the expected state including phase shift non-linearity (insets)

F = 0.72

4. Quantum measurement and cat decoherence



Going across the border between the classical and the quantum world



The role of the "environment":

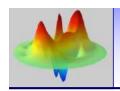
- For long atom-cavity interaction time field damping couples the system to the outside world
- → a complete description of the system must take into account the state of the field energy "leaking" in the environment.
- General method for describing the role of the environment:

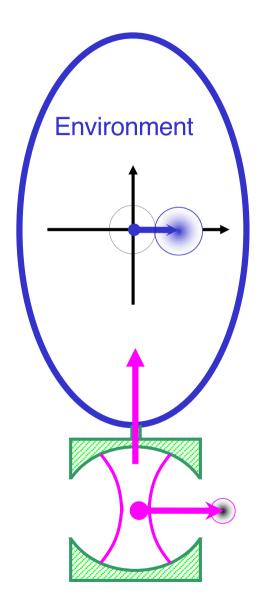
$$\frac{d\rho^{\text{field}}}{dt} = -\frac{1}{2T_{cav}} \left[a^+ a, \rho^{\text{field}} \right]_+ + \frac{1}{T_{cav}} a \rho^{\text{field}} a^+$$

master equation of the field density matrix

Physical result: decoherence

$$au_{dec} pprox rac{ au_{cav}}{\overline{N}}$$





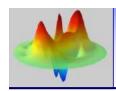
Decay of a coherent field:

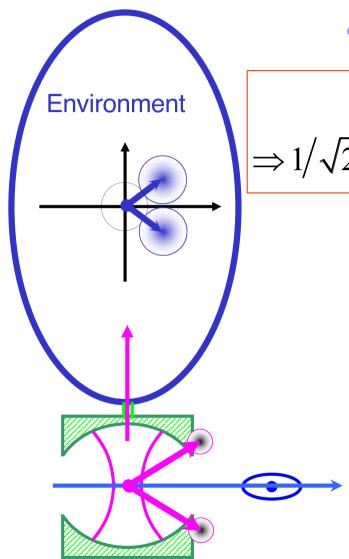
$$|\alpha(0)\rangle \otimes |vacuum\rangle_{env} \rightarrow |\alpha(t)\rangle \otimes |\beta(t)\rangle_{env}$$

$$\alpha(t) = \alpha(0).e^{-t/2\tau_{cav}}$$

- □ the cavity field remains coherent
- $\hfill \square$ the leaking field has the same phase as α
- □ no entanglement during decay:

That is a property defining coherent states: coherent state are the only one which do not get entangled while decaying

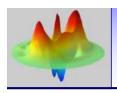


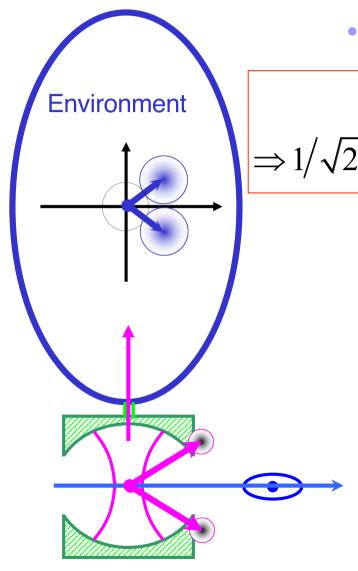


Decay of a "cat" state:

$$|\Psi_{cat}\rangle \otimes |vacuum\rangle_{env}$$

$$\Rightarrow 1/\sqrt{2} (|\alpha_{+}(t)\rangle \otimes |\beta_{+}(t)\rangle_{env} + |\alpha_{-}(t)\rangle \otimes |\beta_{-}(t)\rangle_{env})$$





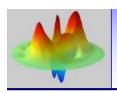
Decay of a "cat" state:

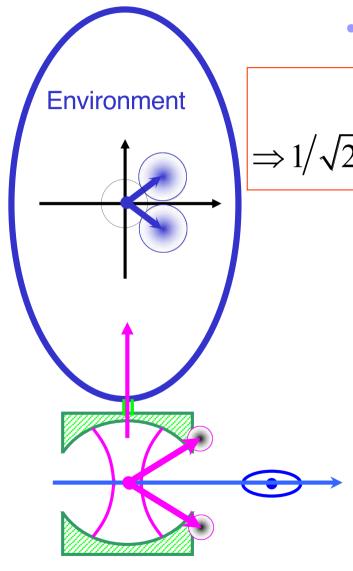
$$|\Psi_{cat}\rangle \otimes |vacuum\rangle_{env}$$

$$\Rightarrow 1/\sqrt{2} (|\alpha_{+}(t)\rangle \otimes |\beta_{+}(t)\rangle_{env} + |\alpha_{-}(t)\rangle \otimes |\beta_{-}(t)\rangle_{env})$$

- cavity-environment entanglement:
 the leaking field "broadcasts" phase information
- trace over the environment
- ⇒ decoherence (=diagonal field reduced density matrix) as soon as:

$$\langle \beta_{-}(t) | \beta_{+}(t) \rangle_{env} \approx 0$$





Decay of a "cat" state:

$$|\Psi_{cat}\rangle \otimes |vacuum\rangle_{env}$$

$$\Rightarrow 1/\sqrt{2} (|\alpha_{+}(t)\rangle \otimes |\beta_{+}(t)\rangle_{env} + |\alpha_{-}(t)\rangle \otimes |\beta_{-}(t)\rangle_{env})$$

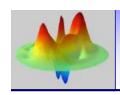
$$\langle \beta_{+}(t) | \beta_{-}(t) \rangle = e^{-|\beta|^{2}(1-e^{2i\Phi_{0}})}$$

Energy
$$|\alpha(t)|^2 + |\beta(t)|^2 = |\alpha_0|^2$$

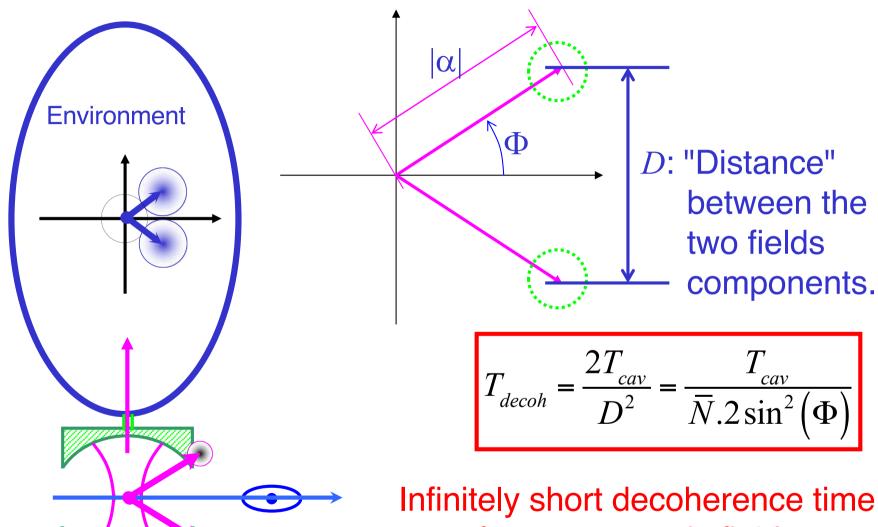
$$\Rightarrow \left| \beta(t) \right|^2 = \left| \alpha_0 \right|^2 \left(1 - e^{-t/T_{cav}} \right) \approx \left| \alpha_0 \right|^2 .t/T_{cav}$$

The two states of the environment become orthogonal as soon as

$$\left\|\beta\left(t\right)\right\|^{2} \approx 1 \Rightarrow t > \frac{T_{cav}}{\overline{N}} \approx T_{dec}$$



The decoherence time

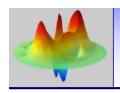


Detailed calculation in PHYSICA SCRIPTA T78, 29 (1998)

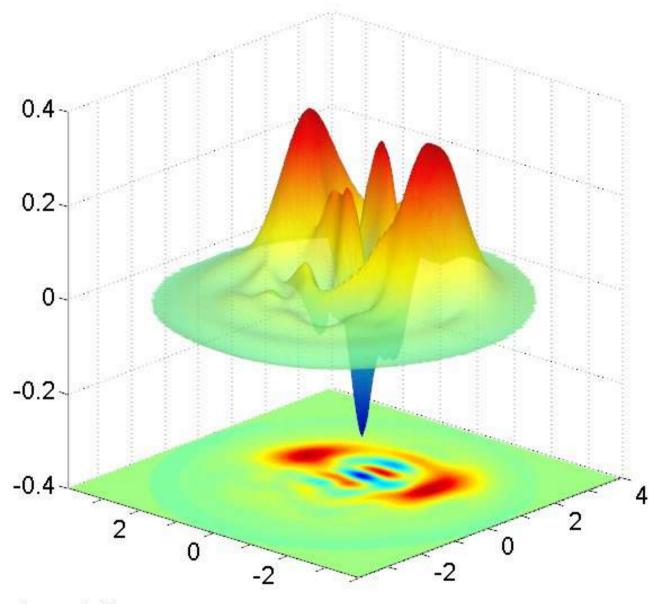
Infinitely short decoherence time for macroscopic fields

→ The Schrödinger cat does not exist for

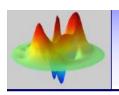
"long" time



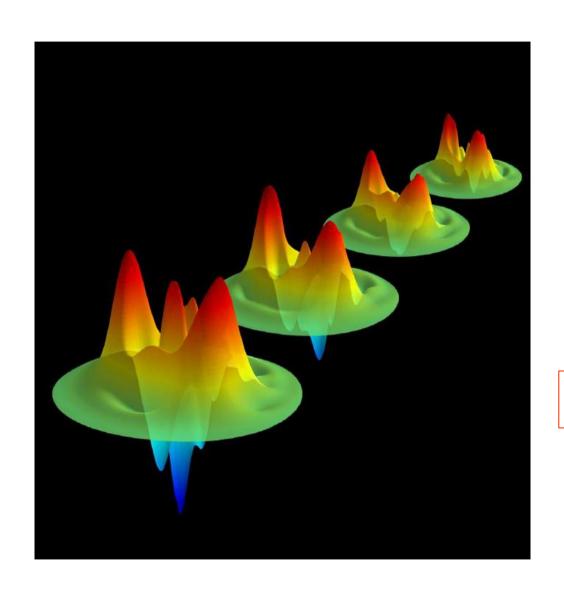
Movie of decoherence

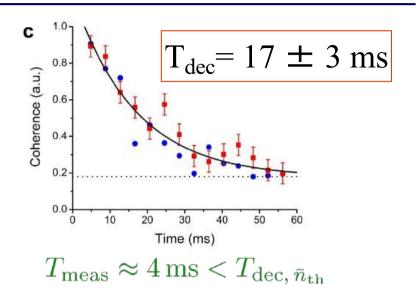


 $t = 1.3 \, \text{ms}$



Decoherence of a D²=11.8 photon cat state



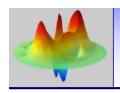


Theory:

$$T_{dec} = 2T_{cav}/D^2 = 22 \text{ ms}$$

+ small blackbody contribution @ 0.8 K

$$T_{dec} = 19.5 \text{ ms}$$



Quantum measurement: the role of the environment 1

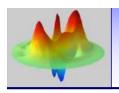
- ⇒ Physical origin of decoherence: leak of information into the environment.
- ⇒ The experimentalist does not kill the cat when opening the box: the environment "knows" whether the cat is dead or alive well before one opens the box.
- ⇒ The environment performs continuously unread repeated measurement of the cat state

The "collapse" of the quantum state can be considered as a shortcut to describe this complex physical process

Does it solve "the measurement problem"?

No: if the problem consists in telling how or why nature chooses randomly one classical state.

Yes: once one a priori accepts the statistical nature of quantum theory, dechoherence is the mechanism providing classical probabilities



Quantum measurement: the role of the environment 2

⇒ Definition of "pointer basis" of a meter: (Zurek)

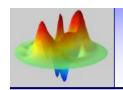
- □ the pointer state of the meter is a classical state
- once decoherence occurs, the physical state of a meter is described by a diagonal density matrix in the pointer basis:

$$|e, \bigcirc\rangle |g, \bigcirc\rangle$$

$$\rho_{dec} = \begin{pmatrix} P_e & 0 \\ 0 & P_g \end{pmatrix}$$

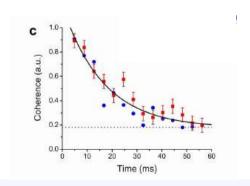
⇒ at this level, quantum description only involves classical probabilities and no macroscopic superposition states.

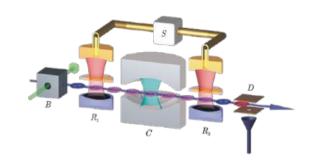
The decoherence approach shows that quantum theory is consistent with classical logic at macroscopic scale: it only provides classical statistics at the macroscopic scale.

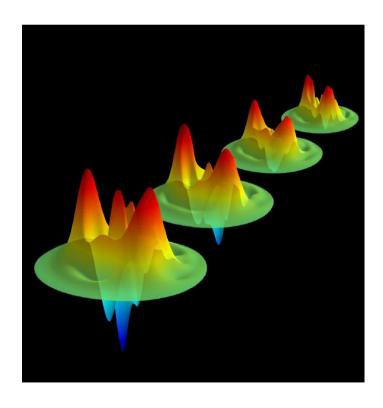


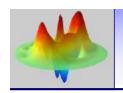
Summary

- Generation of cat states in a cavity
- State measurement (QND) and reconstruction (MaxEnt)
- Wigner function of the cat states
- Time evolution and decoherence of the cats



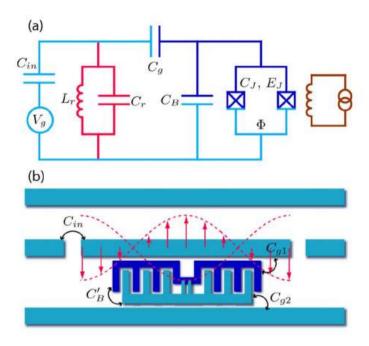






Continuation of cavity QED

Artificial atoms eventually became as good as real one: comparable factor of merit T_{coh}/T_{op}



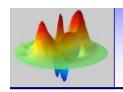
J. Koch et al. PRA 2007

Transmon qubit + stripline cavity





The Nobel Prize in Physics 2025 was awarded jointly to John Clarke, Michel H. Devoret and John M. Martinis "for the discovery of macroscopic quantum mechanical tunnelling and energy quantisation in an electric circuit"



A work starting in 1991



Jean-Michel Raimond Serge Haroche Michel Brune

The LKB-ENS cavity QED team

Staring, in order of apparition

	Serge Haroche	Peter Domokos		Ulrich Busk-Hoff
	Michel Gross	Ferdinand Schmidt-		Andreas Emmert
	Claude Fabre	Kaler		Adrian Lupascu
	Philippe Goy	Ed Hagley		Jonas Mlynek
	Pierre Pillet	Xavier Maître		Igor Dotsenko
	Jean-Michel Raimond	Christoph Wunderlich		Samuel Deléglise
	Guy Vitrant	Gilles Nogues		Clément Sayrin
	Yves Kaluzny	Vladimir Ilchenko	Alo	Xingxing Zhou
	Jun Liang	Jean-François Roch	10	Bruno Peaudecerf
	Michel Brune	Stefano Osnaghi		Raul Teixeira
	Valérie Lefèvre-Seguin	Arno Rauschenbeutel		Sha Liu
	Jean Hare	Wolf von Klitzing		Theo Rybarczyk
	Jacques Lepape	Erwan Ja <mark>hie</mark> r		Carla Hermann
	A <mark>ephraim</mark> Steinberg	Patrice Bertet		Adrien Signolles
	An <mark>dre Nu</mark> ssenzveig	Alexia Auffèves		Adrien Facon
	Fré <mark>déric Bern</mark> ardot	Romain Long		Stefan Gerlich
	Pau <mark>l Nuss</mark> enzveig	Sébastien Steiner		Than Long Nguyen
	Laurent Collot	Paolo Maioli		
	Matthias Weidemuller	Philippe Hyafil		Eva Dietsche
	François Treussart	Tristan Meunier		Dorian Grosso
	Abde <mark>lamid</mark> Maali	Perola Milman		Frédéric Assémat
	Davi <mark>d Weis</mark> s	Jack Mozley		Athur Larrouy
	Vahi <mark>d Sand</mark> oghdar	Stefan Kuhr		Valentin Métillon
	Jona <mark>than Kni</mark> ght	Sébastien Gleyzes		Tigrane Cantat-
	Nicolas Dubreuil	Christine Guerlin		Moltrecht
	Peter Domokos	Thomas Nirrengarten		
	Ferdinand Schmidt-Kaler	Cédric Roux		
	Jochen Dreyer	Julien Bernu		

Collaboration: L davidovich, N. Zaguri, P. Rouchon, A. Sarlette, S Pascazio, K. Mölmer ...

Cavity technilogy: CEA Saclay, Pierre Bosland

