A fresh look into grand unification: What can FCC tell us about it?

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W.i.p. with:
Lukas Allwicher, Francisco Dahab, Christophe Grojean, Lucine Tabatt

- Ways to probe GUTs
  - Measurements of gauge couplings (LEP/LHC)
  - Proton life-time (Super Kamiokande)
  - Also recent ideas with gravitational waves

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- The "Future" is nearing! HL-LHC, FCC, HK, ...

- Revisit GUTs in light of the future and post-LHC era!

Outline.

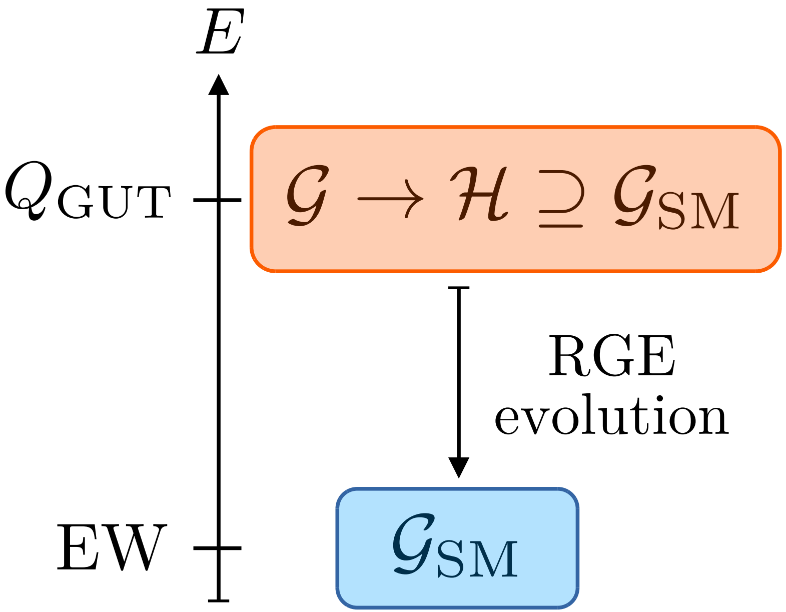
- Introduction to (SUSY) GUTs
- What do we need to unify couplings?
- Role of FCC
- Example: Minimal SUSY SU(5)

Disclaimer:
Work in progress!!!

Grand Unified Theories (GUTs).

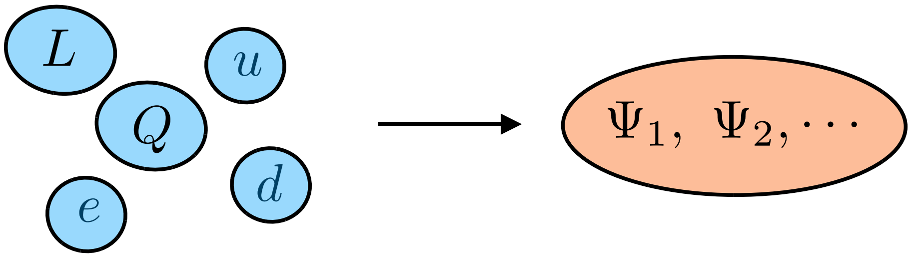
- Giving a common origin to gauge groups

- Embed the SM group into a larger group:



- Gauge couplings set by a unique coupling $g_{\mathcal{G}}$

- Fermions are put together in larger representations



[Georgi, Glashow, '74]

Grand Unified Theories (GUTs).

- Giving a common origin to gauge groups
- SU(5) example: $SU(5) \to SU(3)_c \times SU(2)_L \times U(1)_Y$

$$g_{SU(5)} = \sqrt{\frac{5}{3}}g' = g = g_s \ @Q_{\text{GUT}}$$

$$\overline{\mathbf{5}} = \begin{pmatrix} d^c_{R1} \\ d^c_{R2} \\ d^c_{R3} \\ -e_L \\ \nu_L \end{pmatrix} \quad \mathbf{10} = \begin{pmatrix} 0 & u^c_{R3} & -u^c_{R2} & u_{L1} & d_{L1} \\ -u^c_{R3} & 0 & u^c_{R1} & u_{L2} & d_{L2} \\ u^c_{R2} & -u^c_{R1} & 0 & u_{L3} & d_{L3} \\ -u_{L1} & -u_{L2} & -u_{L3} & 0 & -e_R \\ -d_{L1} & -d_{L2} & -d_{L3} & e_R & 0 \end{pmatrix}$$

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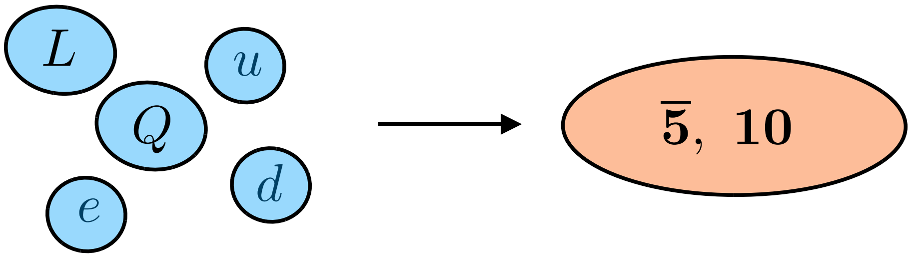

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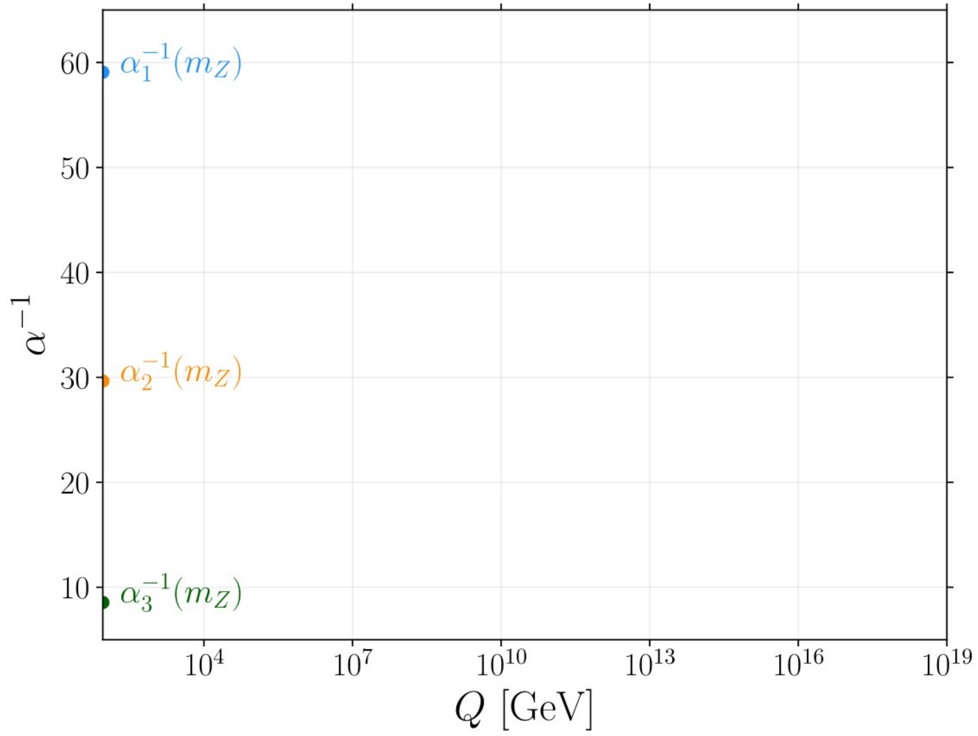

- Start with initial conditions and run!

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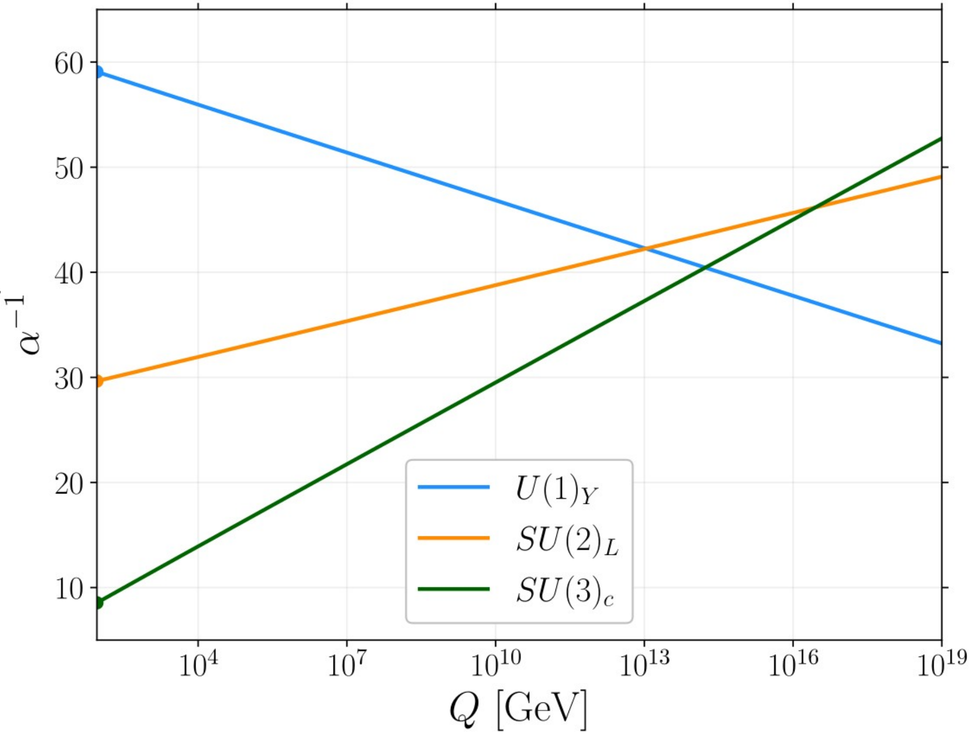

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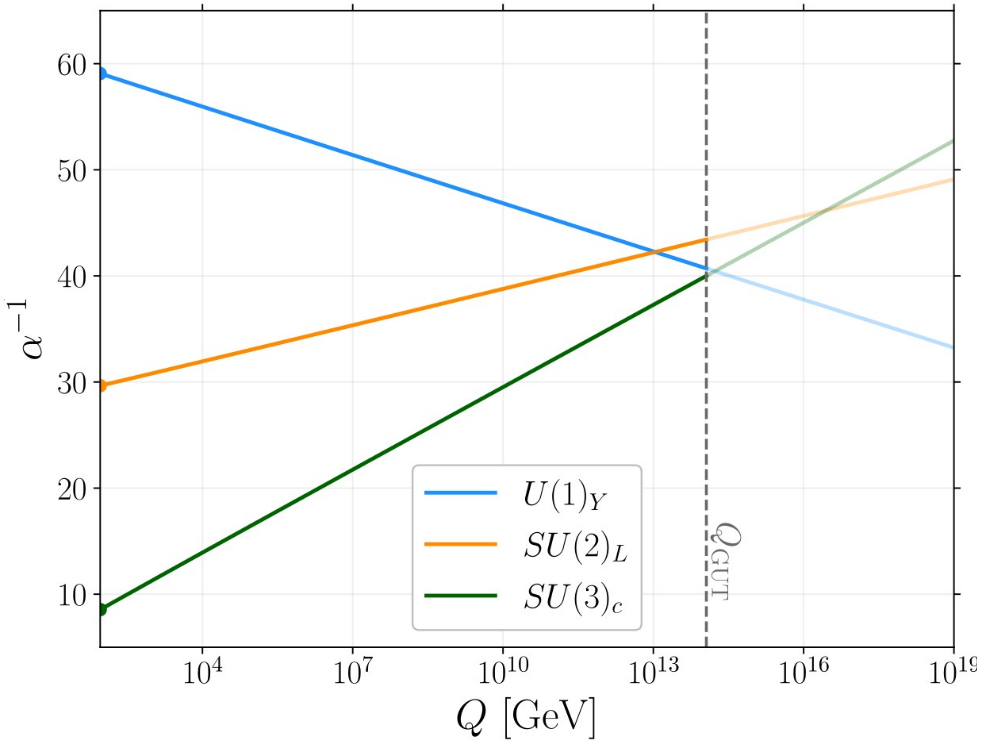

- Start with initial conditions and run!
- Define a GUT scale

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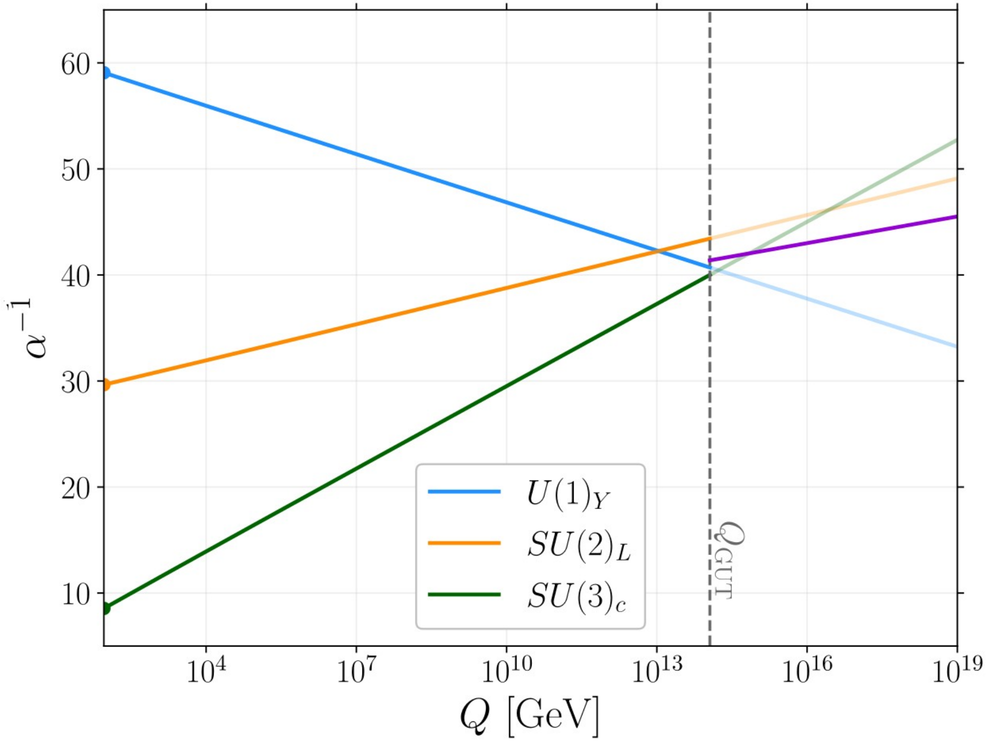

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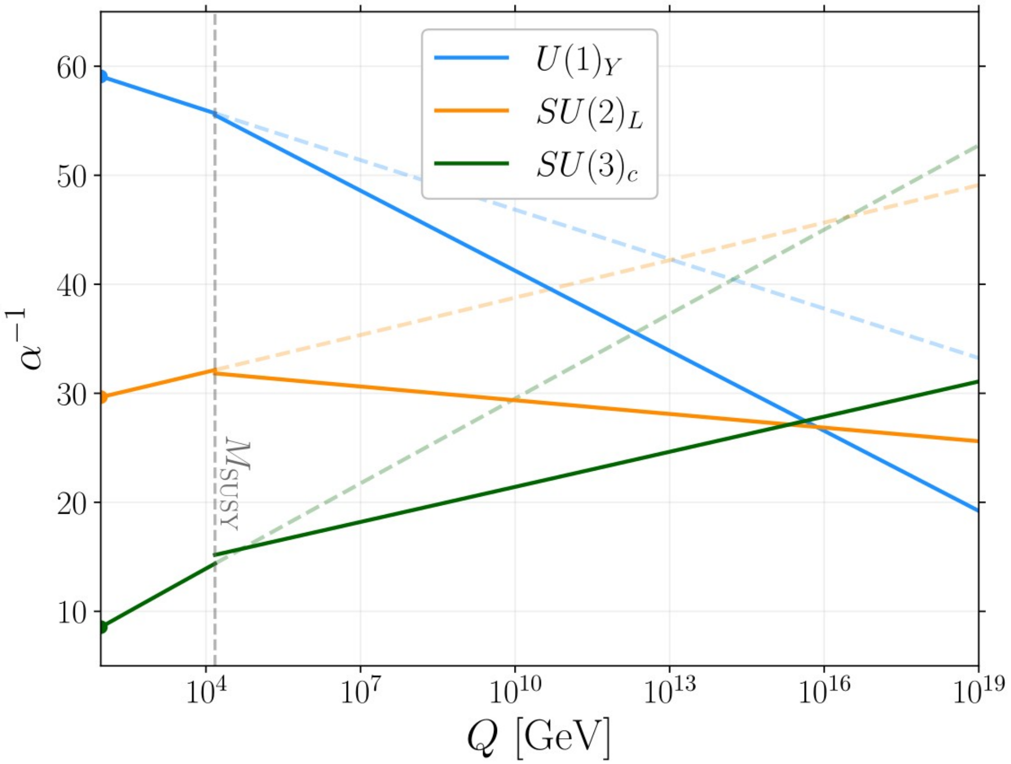

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- Define a GUT scale
- SUSY intermediate scale

$$m_S = ???$$

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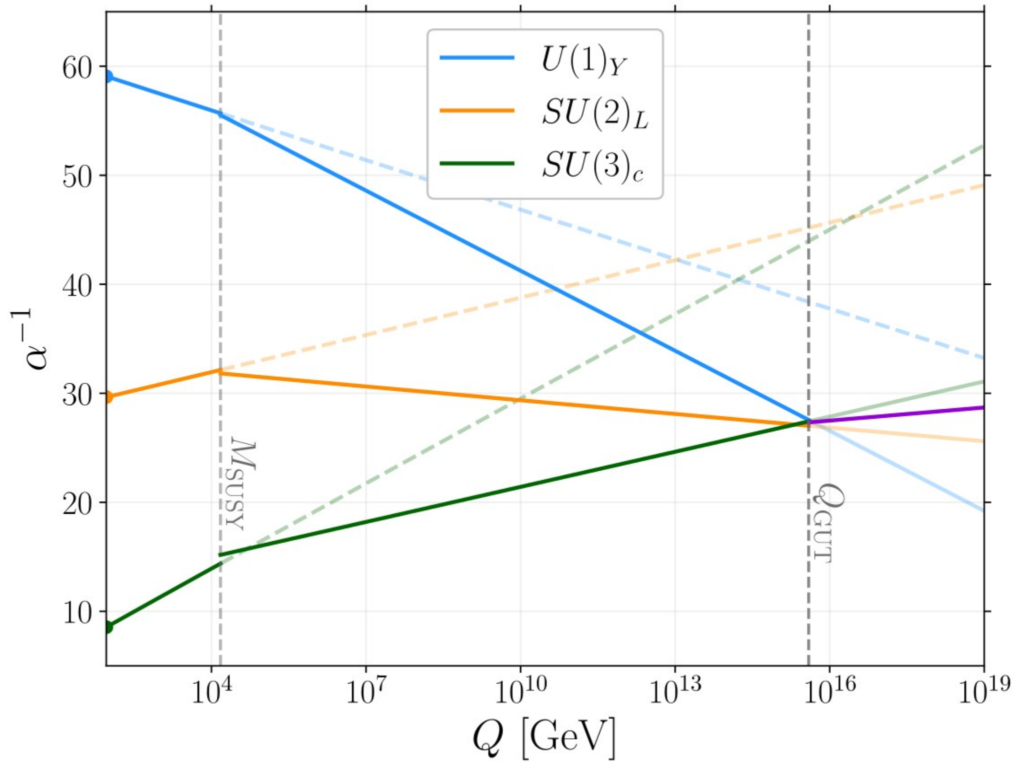

- Start with initial conditions and run!
- Define a GUT scale
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$$m_S = ???$$

- SUSY increase degree of unification!

Grand Unified Theories (GUTs).

- Not the end of the story!

- Dimension 4 EFT matching:
  Threshold corrections!

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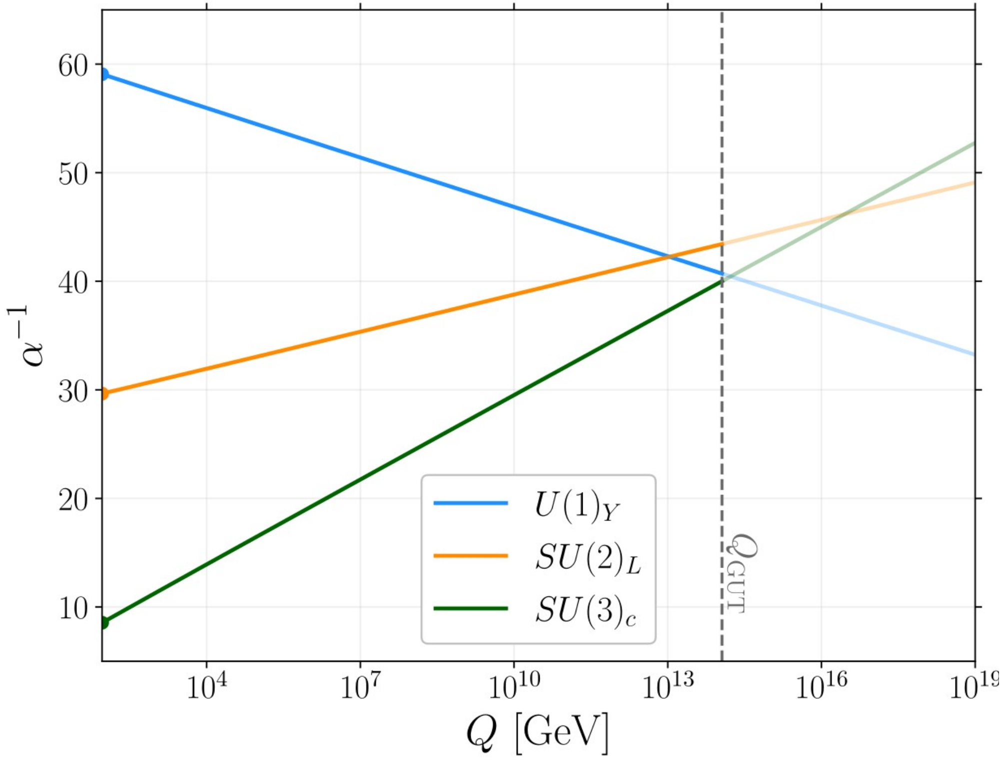

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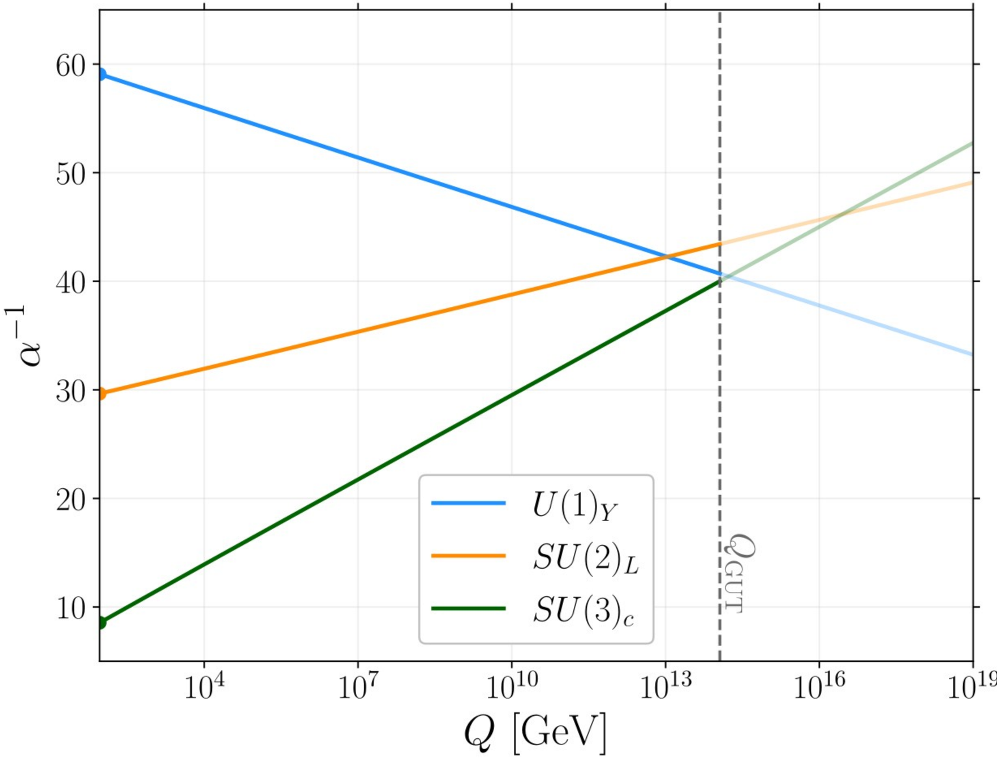

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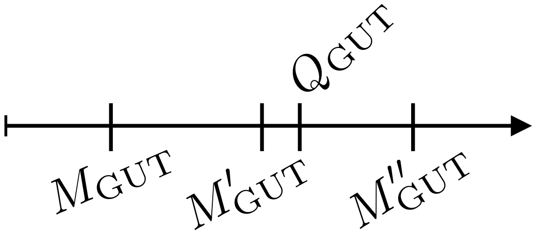

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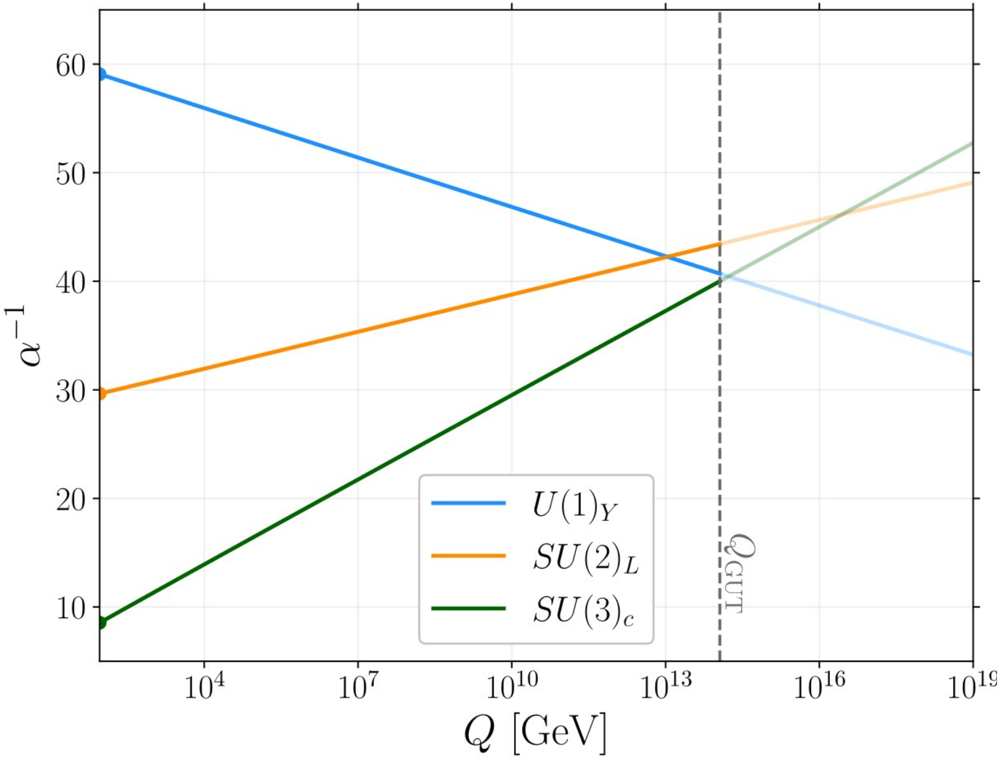

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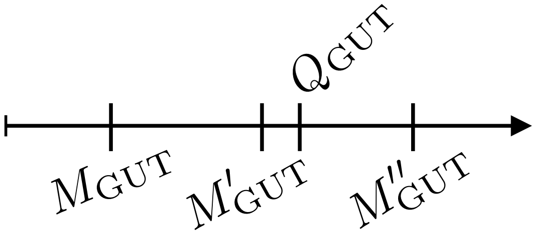

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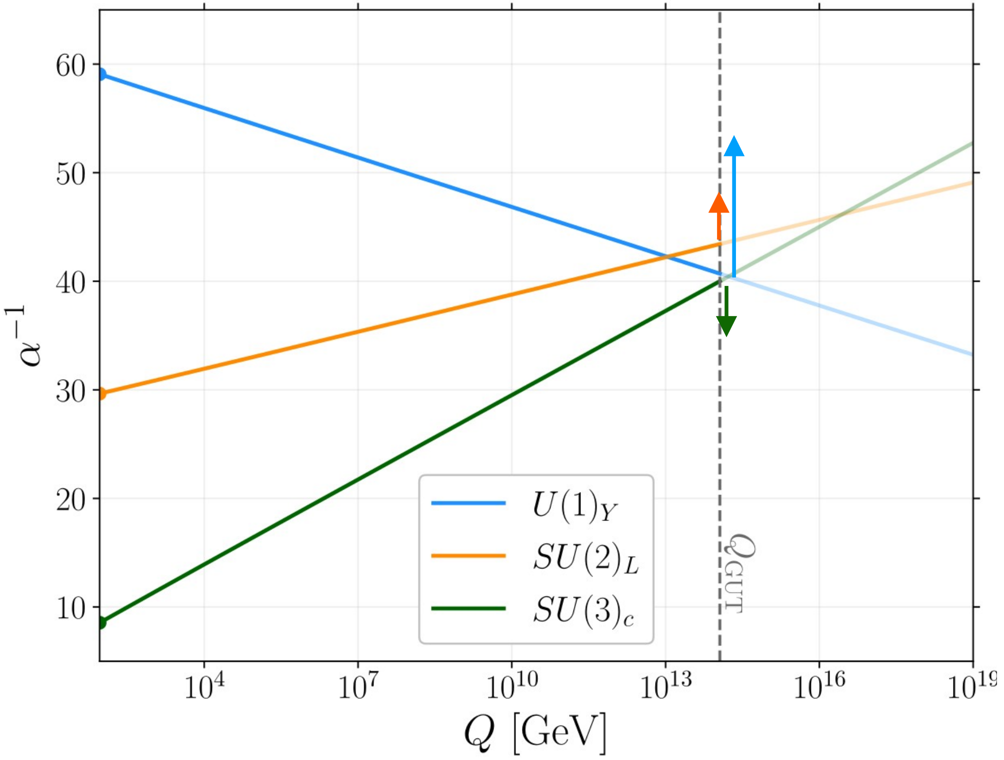

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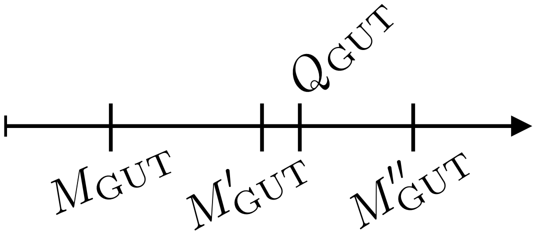

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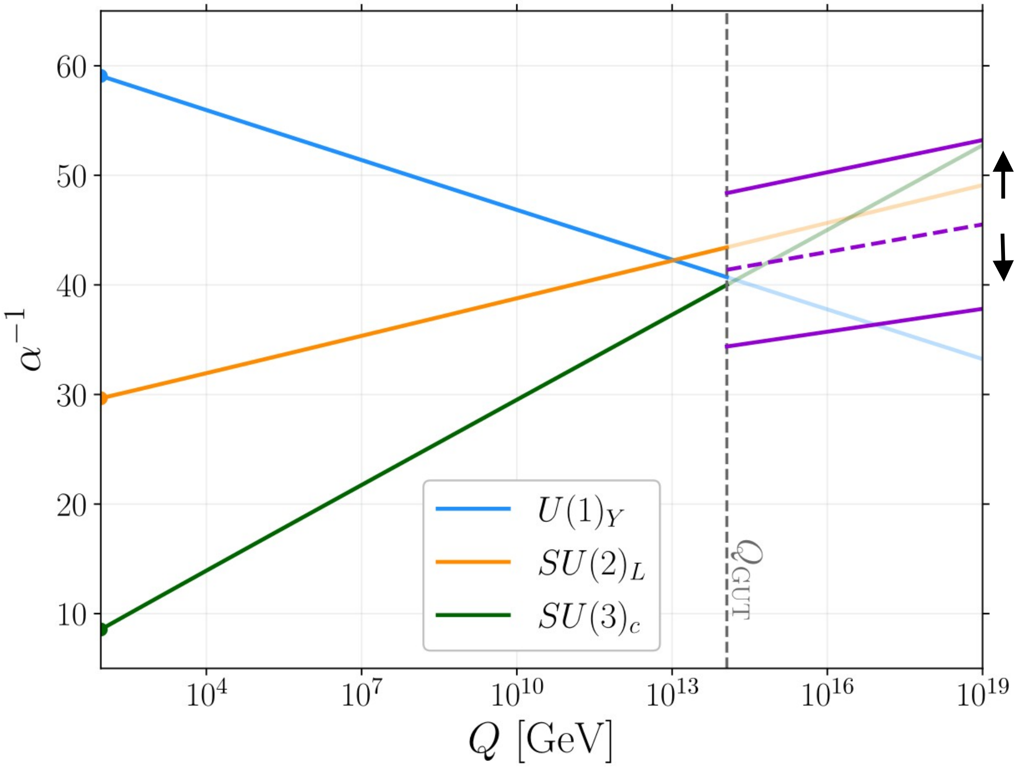

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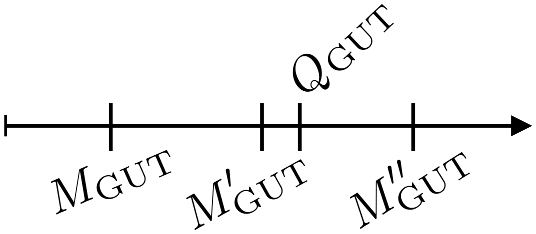

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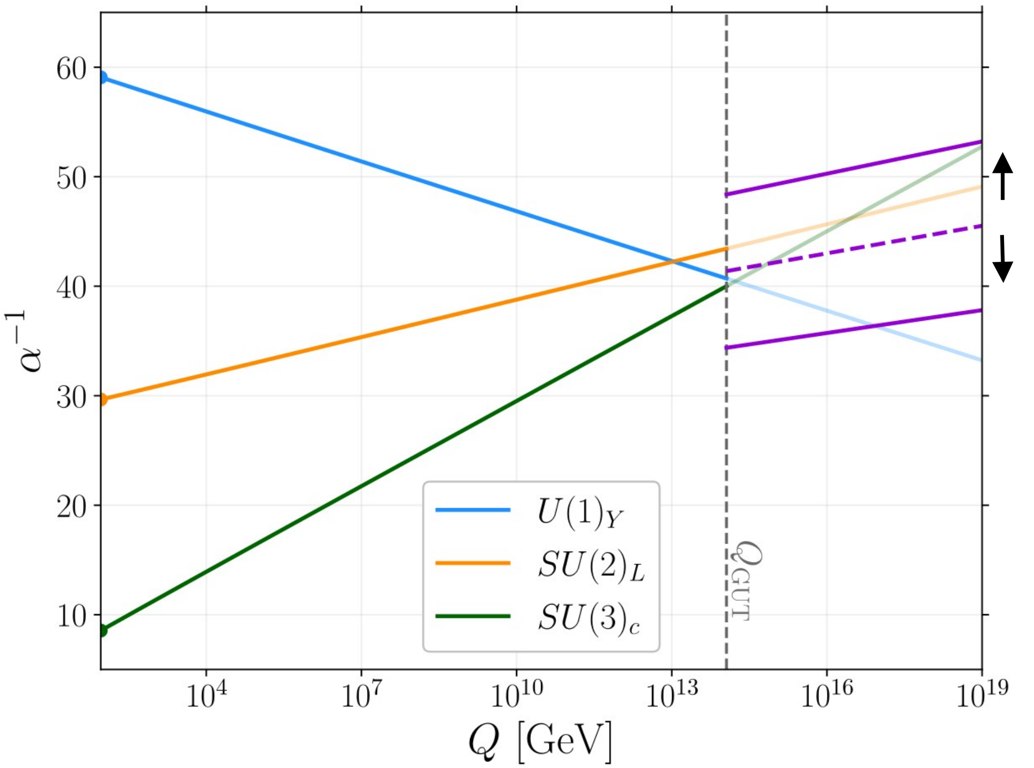
$\longrightarrow$ Choice of GUT model!



Minimal SUSY SU(5).

- New heavy particles:
  - Scalar in the adjoint $(\mathbf{24})$ Σ
  - New gauge bosons $X \sim (\mathbf{3}, \mathbf{2}, -1/2)$
  - Colour-triplet Higgs $H \rightarrow \mathbf{5}_H = \begin{pmatrix} H_c \\ H \end{pmatrix}$

(+ Super-partners)

- Threshold corrections depend on 3 masses and unified coupling:

4 parameters - 3 conditions to unification = 1 free parameter!

Minimal SUSY SU(5).

[Martens et al, **1008.3070**]

- 1-loop threshold corrections:

$$\Rightarrow -\frac{1}{2\pi}\begin{pmatrix} -10 & \frac{2}{5} & 0 \\ 6 & 0 & -2 \\ 4 & -1 & -3 \end{pmatrix}\begin{pmatrix} \log\frac{Q}{M_X} \\ \log\frac{Q}{M_{H_c}} \\ \log\frac{Q}{M_\Sigma} \end{pmatrix} = \begin{pmatrix} \alpha_1^{-1}(Q) - \alpha_5^{-1}(Q) \\ \alpha_2^{-1}(Q) - \alpha_5^{-1}(Q) \\ \alpha_3^{-1}(Q) - \alpha_5^{-1}(Q) \end{pmatrix} \quad @Q = Q_{\mathrm{GUT}}$$

$$\Rightarrow M_X \sim Q_{\mathrm{GUT}} e^{\alpha_5^{-1}}, \quad M_{H_c} \sim Q_{\mathrm{GUT}}, \quad M_\Sigma \sim Q_{\mathrm{GUT}} e^{-\alpha_5^{-1}}$$

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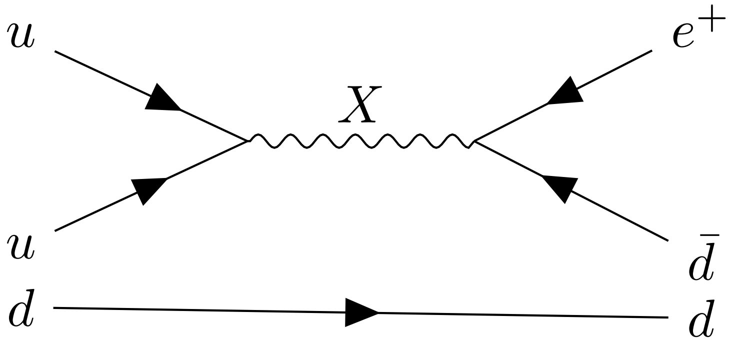
- But what about proton decay?

Proton decay – Gauge boson exchange.

[Super-Kamiokande, '20]
[Hisano, '00]
[Ellis et al, '20]

- $p^+ \to \pi^0 e^+$ Decay channel: $\tau(p^+ \to \pi^0 e^+)\big|_{\mathrm{SK}} = 2.4 \times 10^{34}$ years



- Relevant for SUSY and non-SUSY models

$$\frac{\tau(p^+ \to \pi^0 e^+)}{\tau(p^+ \to \pi^0 e^+)|_{\mathrm{SK}}} \simeq \left(\frac{\alpha_5^{-1}}{25}\right)^2 \left(\frac{M_X}{10^{16}\ \mathrm{GeV}}\right)^4$$

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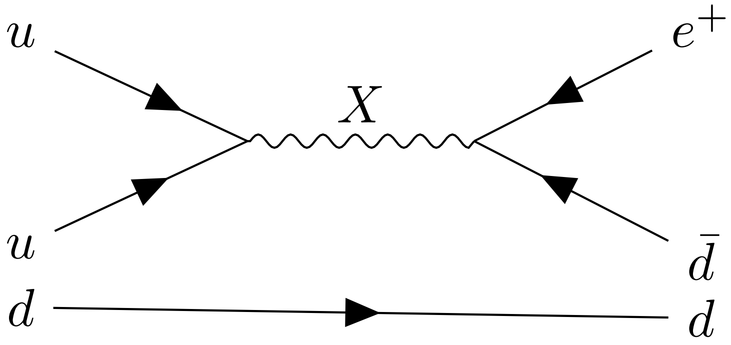

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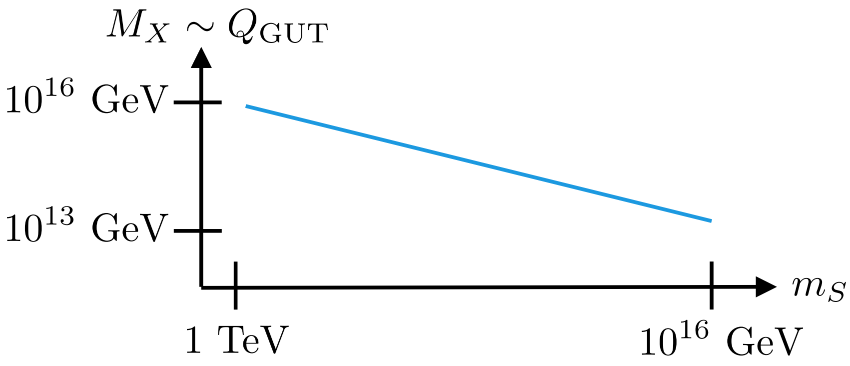

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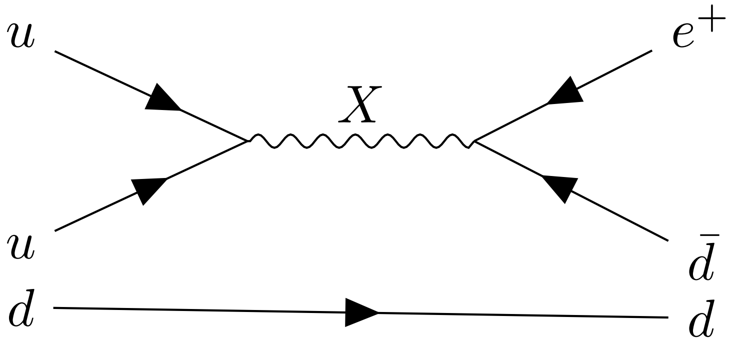

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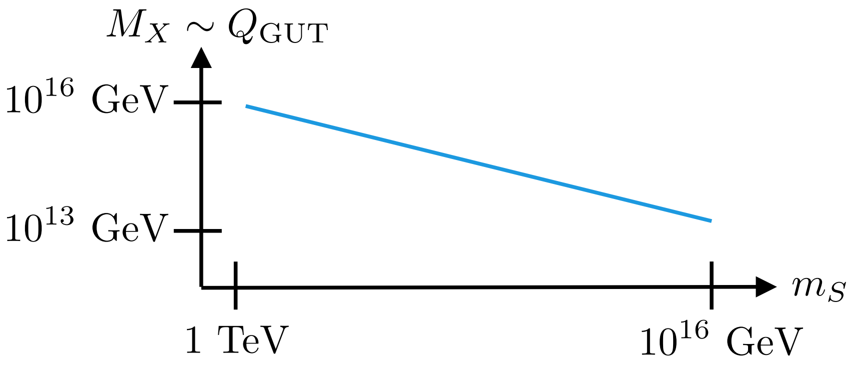


- Higher m_S $\to$ Lower $M_{\rm GUT}$

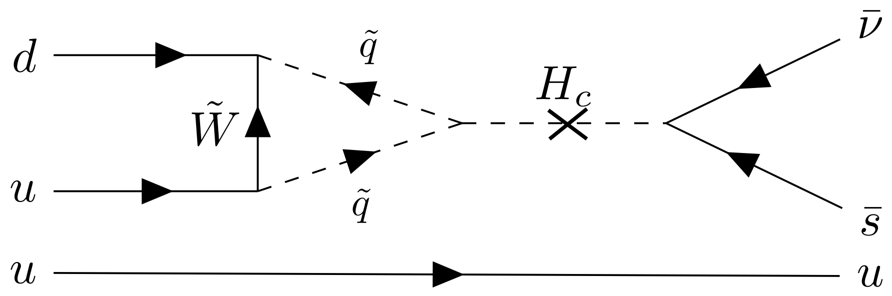
$\Rightarrow$ Excludes higher SUSY scales!

Proton decay – Higgs colour triplet.

[Super-Kamiokande, '14]
[Hisano, '00]
[Ellis et al, '20]

- $p^+ \to K^+\bar{\nu}$ Decay channel: $\tau(p^+ \to K^+\bar{\nu})\big|_{\text{SK}} = 5.9 \times 10^{33}$ years

- Higgs colour triplet exchange + loop of sparticles



$$\frac{\tau(p^+ \to K^+\bar{\nu})}{\tau(p^+ \to K^+\bar{\nu})|_{\text{SK}}}$$

$$\simeq \left(\frac{M_{H_c}}{10^{17}\text{ GeV}}\right)^2 \left(\frac{m_S}{40\text{ TeV}}\right)^2$$

[Murayama, Pierce, **hep-ph/0108104**]

[Hisano et al, **1304.3651**]

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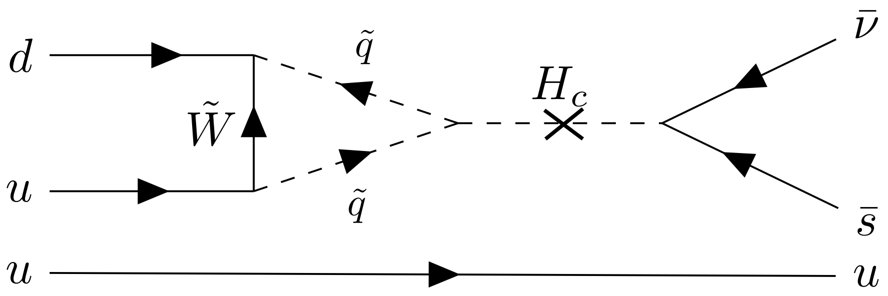

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[Murayama, Pierce, hep-ph/0108104]
[Hisano et al, 1304.3651]

- Need heavier SUSY scales!

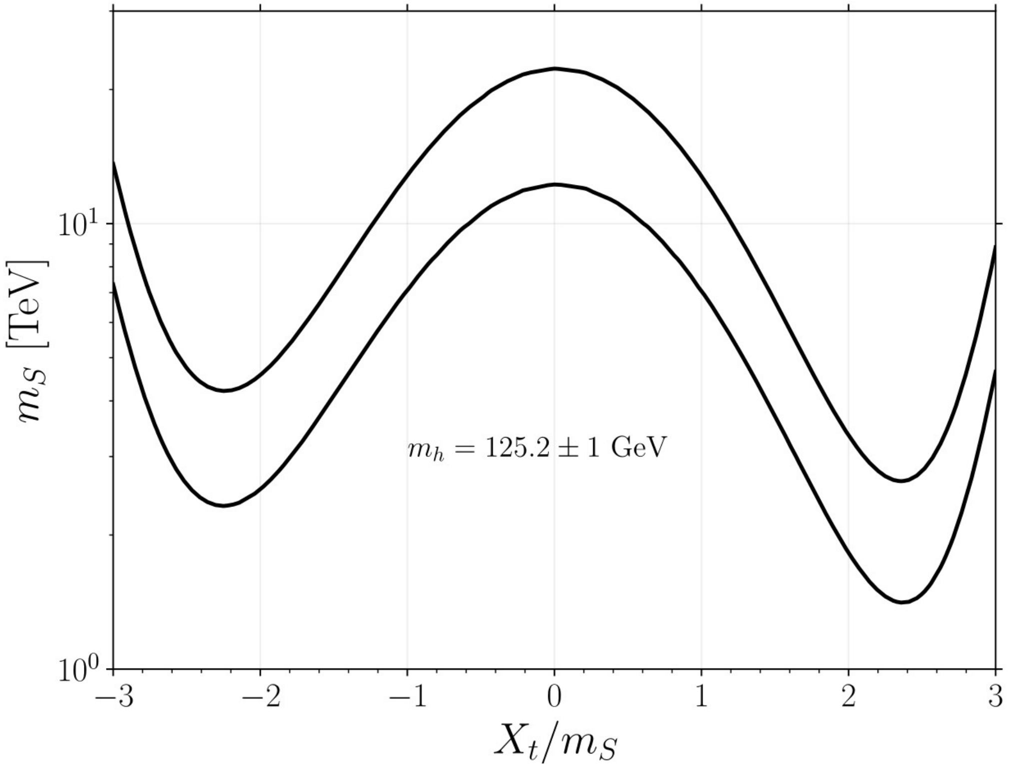
Procedure.

- Fix initial conditions for couplings
- Scan over SUSY parameters (we use CMSSM)
- Determine running of parameters [FlexibleSUSY, **1406.2319, 1710.03760**] [SOFTSUSY, **hep-ph/0104145**]
  (3-loop in MSSM)
- Compute Higgs mass & determine GUT spectrum [Martens et al, **1008.3070**] [Vega, Villadoro, **1504.05200**] [FlexibleSUSY, **1406.2319, 1710.03760**]
  (2-loop + partial 3-loop)
- Obtain proton decay bounds [Hisano, **'00**] [Hisano et al, **1304.365100**] [Ellis et al, **2003.03285**]

Results.

[Vega, Villadoro, **1504.05200**]
[FlexibleSUSY, **1406.2319, 1710.03760**]

- SUSY: Region where Higgs mass is reproduced



$m_S =$ SUSY scale, $X_t =$ Stop mixing

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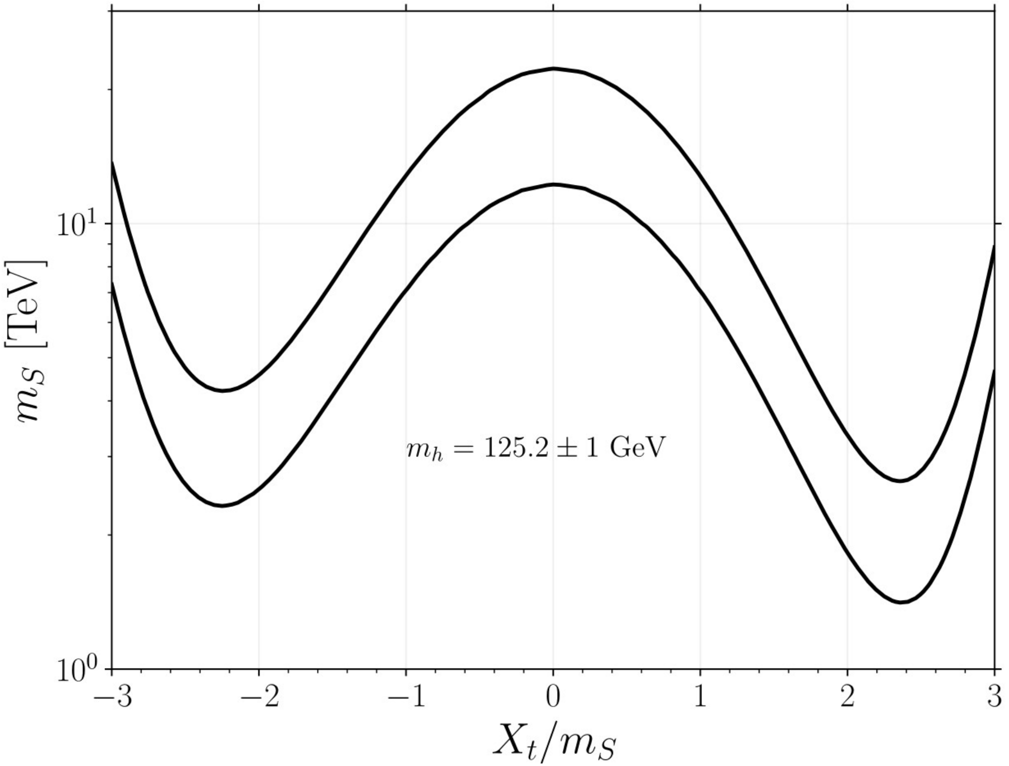

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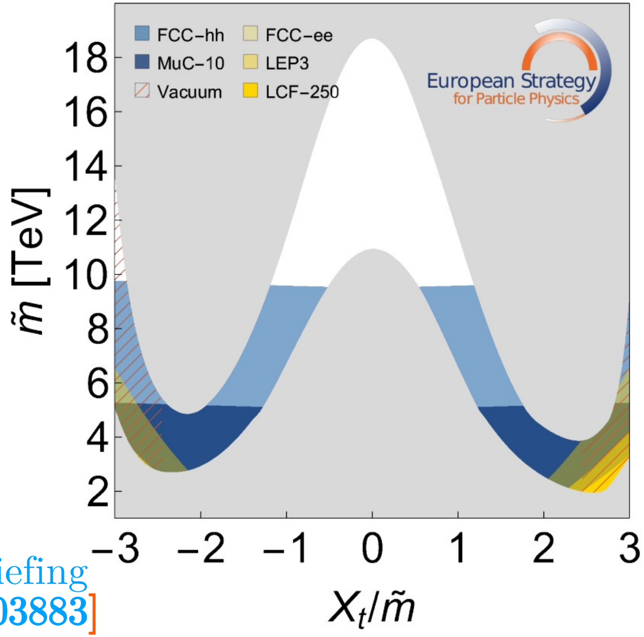

[Physics Briefing Book, **2511.03883**]

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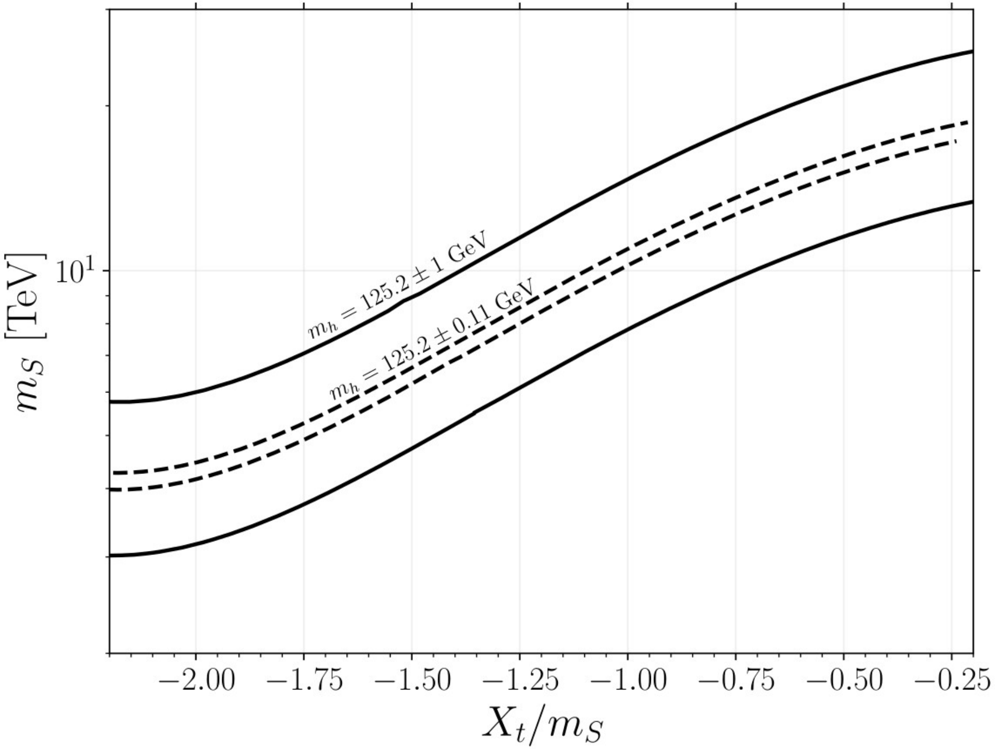

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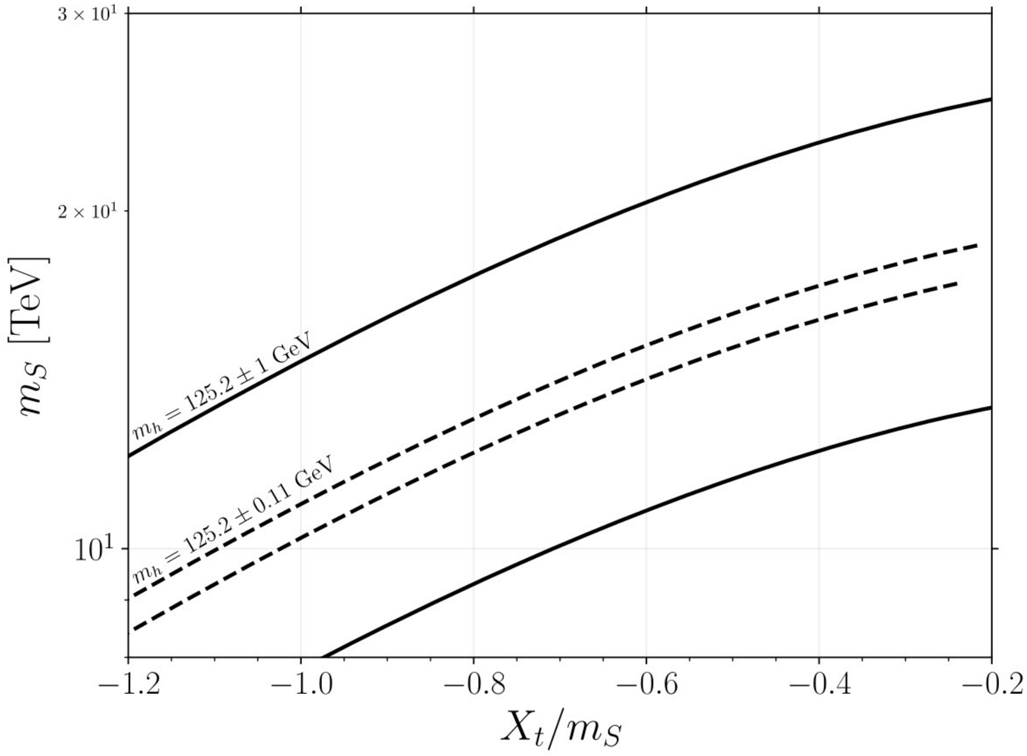

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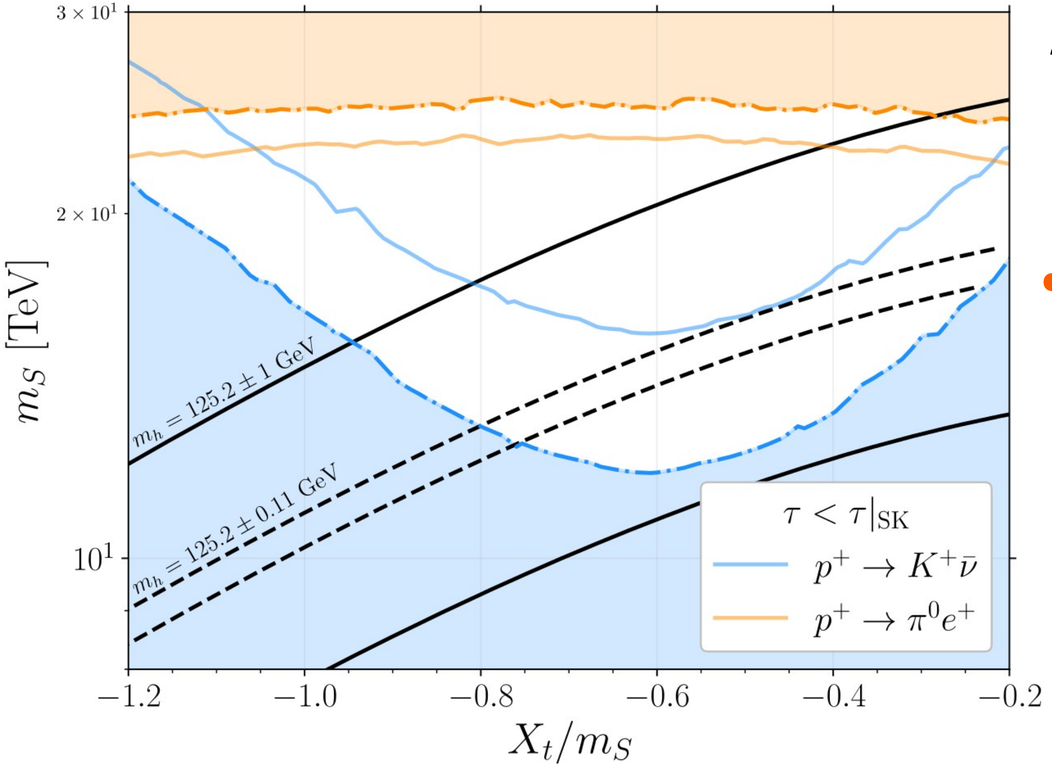

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CMSSM @ GUT $\to$ Low-energy @ m_S

- Present uncertainty on $\alpha_i(m_Z)$

$$\tau(p^+ \to \pi^0 e^+) \propto M_X^4 \alpha_5^{-2}$$

$$\tau(p^+ \to K^+ \bar{\nu}) \propto M_{H_c}^2 m_S^2$$

Central values

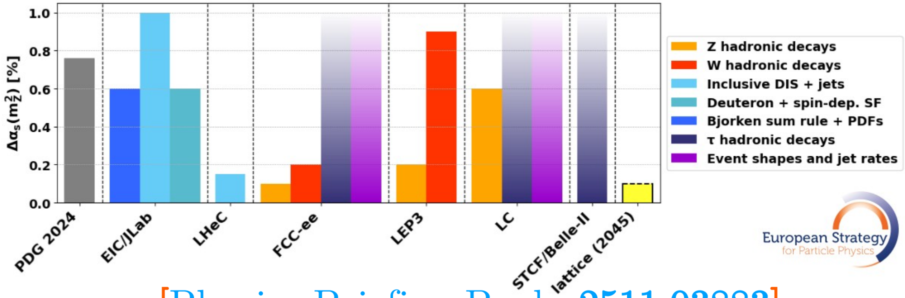
W/ current uncertainty

Where FCC comes into play.

- Increase precision on couplings at the Z pole

[FCC, 2505.00272]

Observable	present value	±	uncertainty	FCC-ee Stat.	FCC-ee Syst.
$\sin^2\theta_{\mathrm{W}}^{\mathrm{eff}}$ ($\times 10^6$)	231,480	±	160	**1.2**	1.2
$1/\alpha_{\mathrm{QED}}(m_{\mathrm{Z}}^2)$ ($\times 10^3$)	128 952	±	14	**3.9** **0.8**	small tbc
$\alpha_{\mathrm{S}}(m_{\mathrm{Z}}^2)$ ($\times 10^4$)	1 196	±	30	**0.1**	1



[Physics Briefing Book, 2511.03883]

- At least factor 10 improvement!

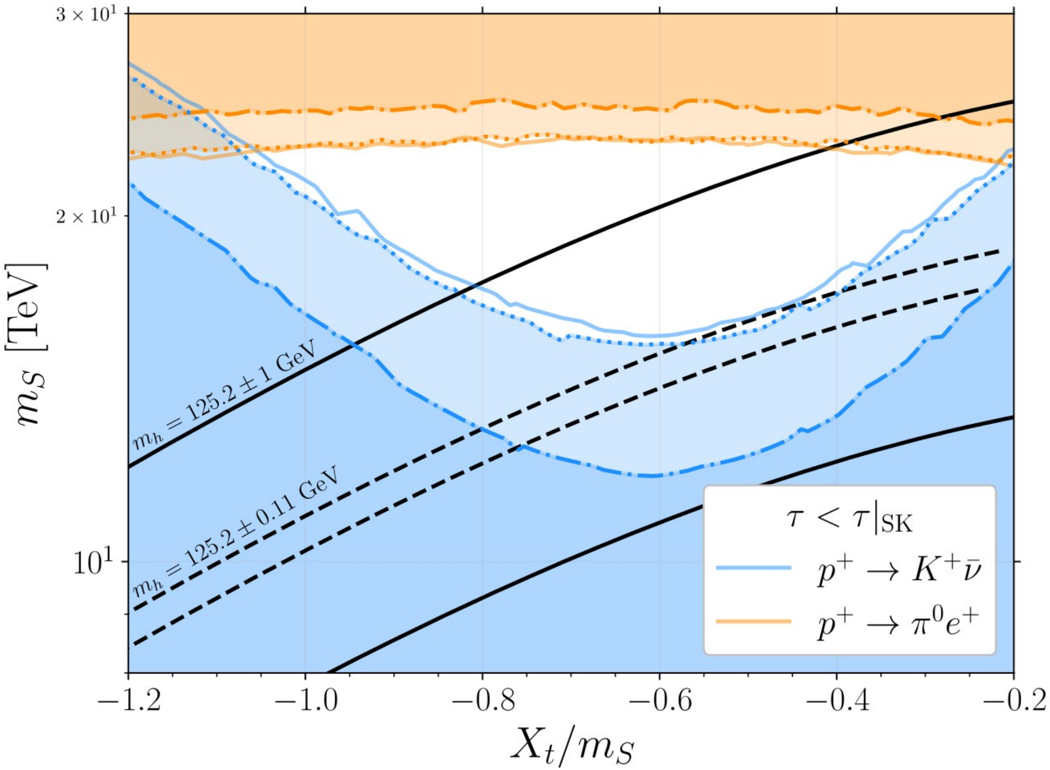
- Dominant uncertainty from $\alpha_3(m_Z)$

More precision on $\alpha_i(m_Z)$ $\Rightarrow$ More precise determination of $M_X,\ M_{H_c},\ M_\Sigma$ $\Rightarrow$ Better determination of bounds from proton decay!

Results.

[Vega, Villadoro, 1504.05200]
[FlexibleSUSY, 1406.2319, 1710.03760]

- Higgs mass + Proton decay



$m_S =$ SUSY scale, $X_t =$ Stop mixing

CMSSM @ GUT $\to$ Low-energy @ m_S

- FCC uncertainty on $\alpha_i(m_Z)$

$$\tau(p^+ \to \pi^0 e^+) \propto M_X^4 \alpha_5^{-2}$$

$$\tau(p^+ \to K^+\bar{\nu}) \propto M_{H_c}^2 m_S^2$$

Central values

W/ current uncertainty

W/ FCC uncertainty

Results.

[Vega, Villadoro, **1504.05200**]
[FlexibleSUSY, **1406.2319, 1710.03760**]

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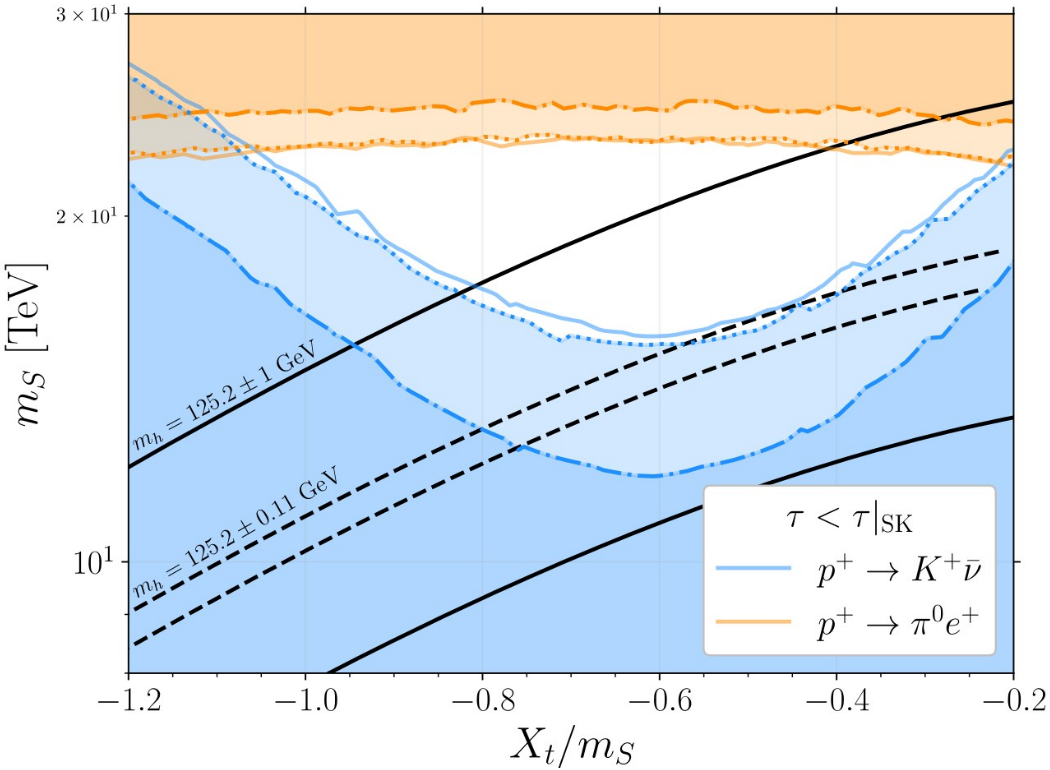

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- Closing the parameter space compatible with m_h!

Conclusions so far.

- Re-investigation of *precision* unification
- Connect low- to high-energy via unification
- FCC measurements are a way to better test UV scenarios!
- To do - Theory uncertainties, scan of SUSY parameters, more observables, other models ...

Conclusions so far.

- Re-investigation of *precision* unification
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Thank you!

Backup

Constrained MSSM (CMSSM).

- Simplified choice of soft-parameters

E

$$m^2_{\tilde{q},\tilde{\ell}} = m_0^2\mathbf{1},\ m^2_{H_u,H_d} = m_0^2$$

$Q_{\rm GUT}$

$$m_{\tilde{G},\tilde{W},\tilde{B}} = m_{1/2}$$

$$A_{u,d,e} = A_0 Y_{u,d,e}$$

EW

$$\tan\beta$$

$$\begin{aligned}\mathscr{L}_{\rm soft} = &-\frac{1}{2}\left(m_{\tilde{G}}\tilde{G}\tilde{G} + m_{\tilde{W}}\tilde{W}\tilde{W} + m_{\tilde{B}}\tilde{B}\tilde{B}\right)\\ &-\left(\tilde{u}^c A_u \tilde{Q} H_u - \tilde{d}^c A_d \tilde{Q} H_d - \tilde{e}^c A_e \tilde{L} H_d + h.c.\right)\\ &-\tilde{Q}^\dagger m^2_{\tilde{Q}}\tilde{Q} - \tilde{L}^\dagger m^2_{\tilde{L}}\tilde{L} - \tilde{u}^{c\dagger} m^2_{\tilde{u}}\tilde{u}^c - \tilde{d}^{c\dagger} m^2_{\tilde{d}}\tilde{d}^c - \tilde{e}^{c\dagger} m^2_{\tilde{e}}\tilde{e}^c\\ &-m^2_{H_u}H_u^* H_u - m^2_{H_d}H_d^* H_d - (bH_u H_d + h.c.)\end{aligned}$$

Unification scale.

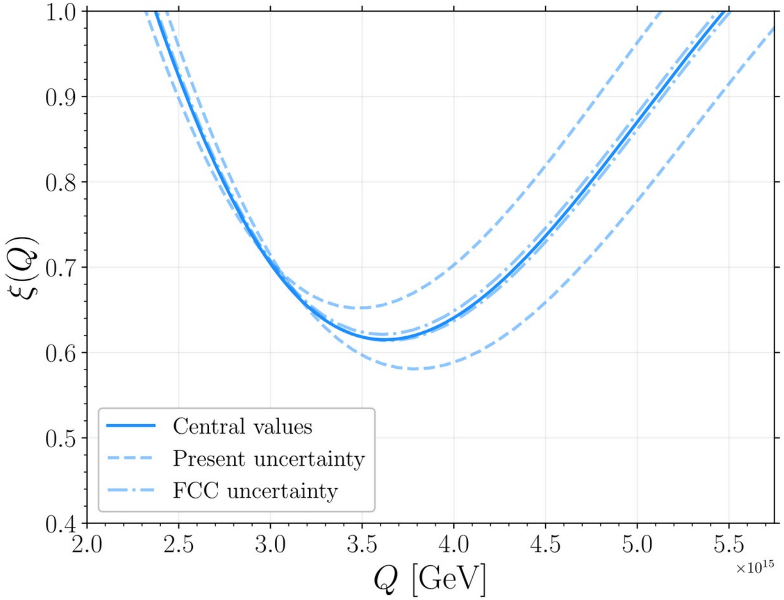
[Bhattiprolu, Wells, **2309.12954**]

- Systematic prescription to determine GUT scale

$$\xi(Q)^2 \equiv \left(\alpha_1^{-1} - \alpha_2^{-1}\right)^2 + \left(\alpha_1^{-1} - \alpha_3^{-1}\right)^2 + \left(\alpha_3^{-1} - \alpha_2^{-1}\right)^2$$

$$\xi^2 = 0 \Leftrightarrow \alpha_1 = \alpha_2 = \alpha_3$$

- Unique minimum around $\sim 10^{15} - 10^{16}$ GeV



Meson decays.

[Babu, Kolda, **hep-ph/9909476**]

[Haish, Mahmoudi, **1210.7806**]

- $B_s \to \mu^+\mu^-$

$$BR(B_s \to \mu^+\mu^-) = \frac{1}{128\pi} f_{B_s}^2 \tau_{B_s} m_{B_s}^3 \sqrt{1 - 4\frac{m_\mu^2}{m_{B_s}}}$$

$$\times \left[\frac{m_{B_s}^2}{(m_b + m_s)^2} \left(1 - 4\frac{m_\mu^2}{m_{B_s}} \right) C_{SP}^2 + \left| \frac{2m_\mu}{m_{B_s}} C_{AA} + \frac{m_{B_s}}{(m_b + m_s)} C_{PP} \right|^2 \right]$$

$$C_{SP} \approx -C_{PP} \approx \frac{-\sqrt{2} G_F \alpha}{\pi} V_{tb} V_{ts}^* \, \mu A_t \frac{\tan^3\beta}{(1 + \epsilon_b \tan\beta)^2} \, \frac{m_t^2}{m_{\tilde{t}^2}} \, \frac{m_b m_\mu}{4\sin^2\theta_W M_W^2 M_A^2} f(m_{\tilde{t}}/\mu)$$

$$f(x) = -\frac{x}{1-x} - \frac{x}{(1-x)^2} \ln x \qquad \epsilon_b = \frac{\mu A_t}{16\pi^2} \frac{y_t^2}{m_{\tilde{t}^2}} f(m_{\tilde{t}}/\mu) + \frac{2\alpha_s}{3\pi} \frac{\mu M_3}{m_{\tilde{b}^2}} f(m_{\tilde{b}}/\mu)$$

Meson decays.

- $B_s \to \mu^+\mu^-$