# Quantum heuristics for gravitational-wave detection

## High-frequency sensitivity limits for current and near-future BSM searches

**Paolo Bilisco**

*Horizons Beyond the Standard Model: from Colliders to Dark Matter and Axions*

São Paulo, Brasil

**08/06/2026**

**Based on:** C. Beadle, PB, R.T. D'Agnolo, S.A.R. Ellis, arXiv:26XX.XXXXX

| | Monday<br>June 8 | Tuesday<br>June 9 | Wednesday<br>June 10 | Thursday<br>June 11 | Friday<br>June 12 |
|---|---|---|---|---|---|
| 9:00 - 10:00 | Registration | *Discussions* | *Discussions* | *Discussions* | *Discussions* |
| 10:00 - 10:30 | Coffee Break | Coffee Break | Coffee Break | Coffee Break | Coffee Break |
| 10:30 - 11:15 | *Discussions* | Geller<br>*Vacuum decay around black holes* | Kitano<br>*Title* | Carlos de Lima<br>*Opening the GeV Window at the LHC* | Sesma<br>*Holographic energy correlators from Wightman-Witten diagrams* |
| 11:15 - 12:00 | *Discussions* | Frigerio<br>*Monopole dark matter?* | *Discussions* | Kaplan<br>*Title* | Tesi<br>*The off-shell side of gravitational waves: particle production from inhomogeneities* |
| 12:00 - 14:00 | Lunch Break | Lunch Break | Lunch Break | Lunch Break | Lunch Break |
| 14:00 - 14:45 | Bilisco<br>*Quantum heuristics for gravitational-wave detection: high-frequency sensitivity limits for current and near-future BSM searches* | *Discussions* | **IFT Colloquium**<br>Murayama | *Discussions* | *Discussions* |
| 14:45 - 15:00 | Badziak<br>*Cosmological tests of thermal axions* | *Discussions* | | *Discussions* | *Discussions* |
| 15:00 - 15:30 | | | Coffee Break | | |
| 15:30 - 16:00 | Coffee Break | Coffee Break | *Discussions* | Coffee Break | Coffee Break |
| 16:00 - 17:00 | *Discussions* | *Discussions* | | *Discussions* | *Discussions* |

**IFT/Unesp**

# Quantum heuristics for gravitational-wave detection

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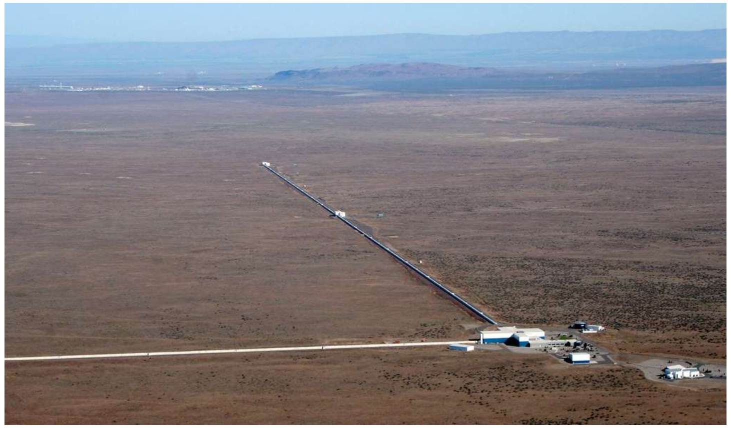
# Introduction



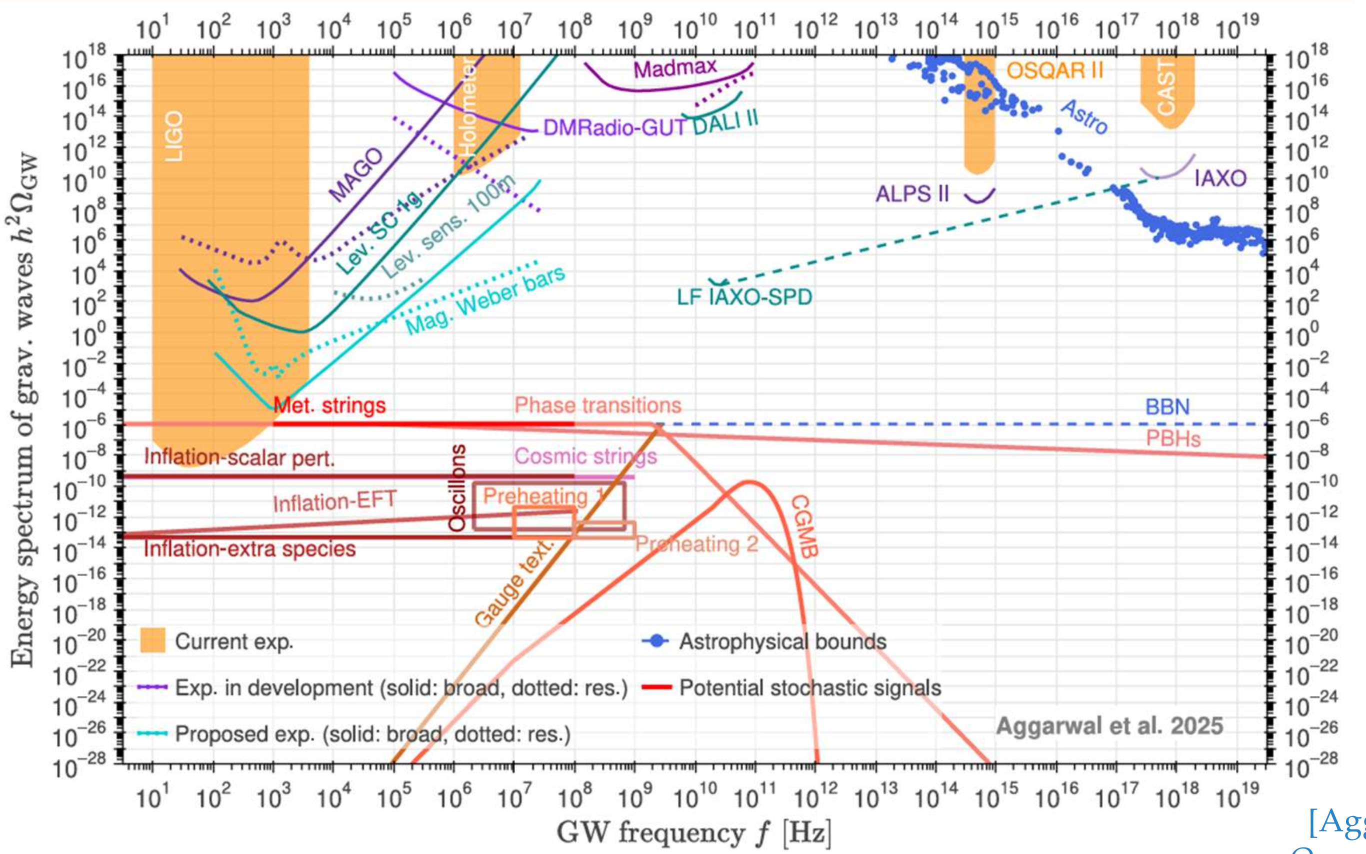
**LIGO**
(Hanford Site, Washington, USA)

**Virgo**
(Santo Stefano a Macerata, Cascina, Italy)

**KAGRA**
(Hida, Gifu Prefecture, Japan)

**Interferometers are not the end of the story!**

# Introduction




[Aggarwal et al., *Challenges and Opportunities of Gravitational Wave Searches above 10 kHz*, 2025]

**… But how far can we get?**

# Scope of the talk

- **AIM: Toy models** ⟶ Analytical control

  ↓

  **Interesting features**
  - **1.** Highlight key properties of detectors (allowing for comparisons)
  - **2.** Hints on how to push forward at high frequencies

- **FOCUS: Stochastic backgrounds** (gaussian, isotropic, stationary, and unpolarized random process with zero mean)

  ↓

  **Windows on energy regimes otherwise unaccessible!**

# Heursitics break our hearts…

**Q:** Will we ever be able to detect primordial background gravitational-wave signals at very high frequencies?


In fact, **classical heuristics** tell us that [S.A.R. Ellis, R.T. D'Agnolo, arXiv:2412.17897]

**Sensitivity** ⟷ **Energy conversion efficiency**

# Energy density and 2-point correlation

$$\boldsymbol{\Omega_g(\omega) \propto \omega^3 S_{hh}(\omega)}$$

**Energy density (per logarithmic unit of frequency)**

$$\Omega_g(\omega) \equiv \frac{8\pi G_N}{3H_0^2}\frac{d\rho_g(\omega)}{d\log\omega}$$

$\tilde{h}(\omega)$

$h(t)$

**Power Spectral Density (2-pt correlation)**

$$\langle h(t)h^*(t')\rangle = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{i\omega(t-t')}S_{hh}(\omega)$$

$$\frac{\Omega_g(\omega)}{\Omega_g(\omega_{\mathrm{ref}})} \stackrel{!}{=} 1 \quad \xRightarrow{S_{hh}^{\min}\sim(U^{\mathrm{det}})^{-1}} \quad U^{\mathrm{det}} = \left(\frac{\omega}{\omega_{\mathrm{ref}}}\right)^3 U^{\mathrm{det}}_{\mathrm{ref}}$$

# The minimal detectable strain

- **Minimal detectable strain:**

$$\mathrm{SNR}_c = \left( t_{\mathrm{int}} \int \frac{\mathrm{d}\omega}{2\pi} \frac{\Omega_g(\omega)^2}{\Omega_n(\omega)^2} \right)^{\frac{1}{2}} \simeq 1 \qquad \mathrm{SNR}_q = \left( t_{\mathrm{int}} \int \frac{\mathrm{d}\omega}{2\pi} \frac{\Omega_g(\omega)}{\Omega_n(\omega)} \right)^{\frac{1}{2}} \simeq 1$$

- **Bound on minimal signal strength for detection of GWs coming from primordial backgrounds** [M. Kawasaki *et al.*, Phys. Rev. Lett. 82, 4168 (1999) & Phys. Rev. D 62, 023506 (2000), M. Maggiore, *Physics Reports* 331 (2000), 283-367]

$$\int d\log\omega \; h_{\mathrm{eff}}^2 \Omega_g(\omega) \lesssim 5 \times 10^{-6} \Delta N_{\mathrm{eff}}$$

# BSM signals' typical observed frequency

- **Which high frequencies?**

$$\begin{cases} \lambda_* \leq H_*^{-1} \\ \omega_0 = a(t_*)/a(t_0)\, \omega_* \end{cases}$$

(Process occurring at temperature $T_*$)

(Redshift of gravitons)

Signal at GUT scale:

$$\boxed{\omega_0 \gtrsim 100 \text{ MHz} \left(\frac{T_*}{10^{15} \text{ GeV}}\right) \left(\frac{g_*(T_*)}{100}\right)^{1/6}}$$

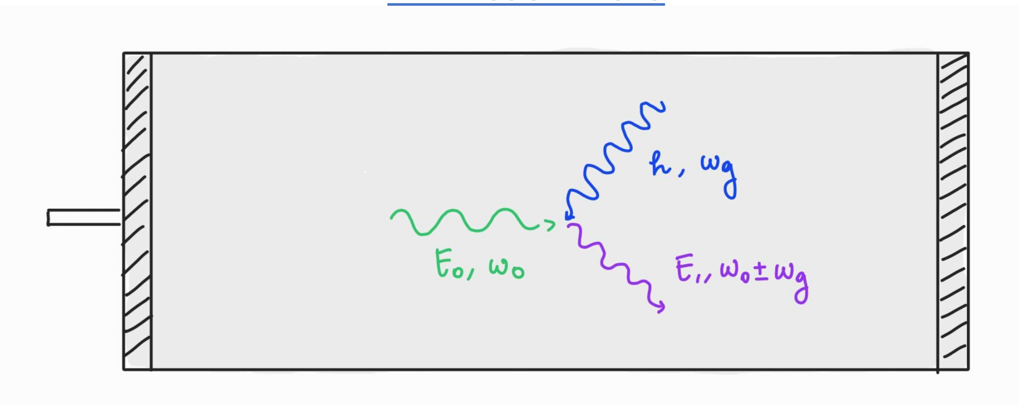
# A new question

Now, all of this was based on classical heursitics…

**Q:** Can we analytically take into account all of these factors, starting from an ab initio computation in quantum mechanics?

# Two toy models to describe (almost) any detector

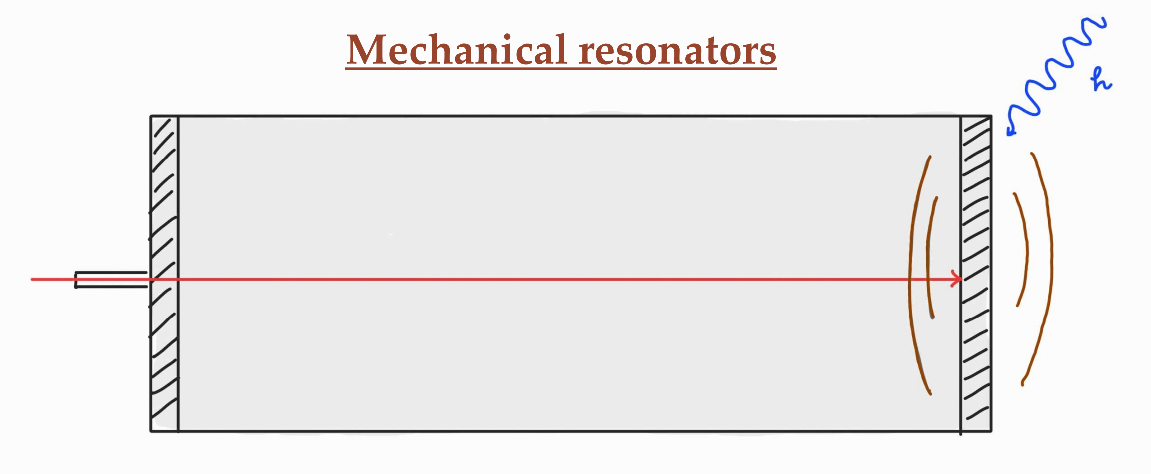
## EM resonators




- Large static magnetic field
- **Readout:** $\omega_1 \approx \omega_g$

- Transition mode 0 (loaded) → 1 (readout)
- **Readout:** $\omega_1 = \omega_0 \pm \omega_g$

## Mechanical resonators




- Like test masses. Their position is measured through an **EM readout**

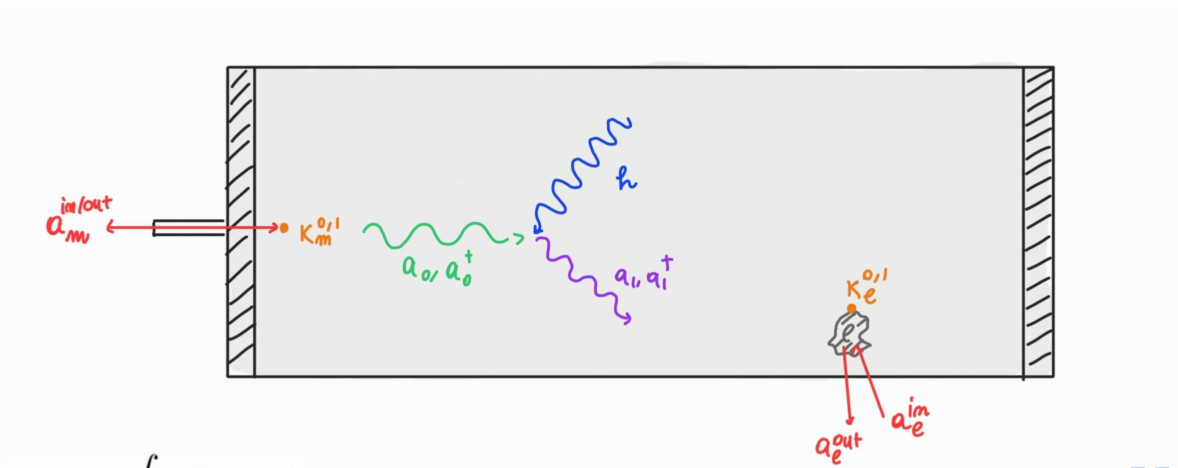
Direct interaction

Mechanical back-action

$$\boxed{H(t) = H_0(t) + H_G + H_{OM}(t) + H_R(t)}$$

# Quantum mechanical set-up

**EM resonator**




**Free** $$H_0 = \underbrace{\sum_{n,r} \Delta_n a^\dagger_{n,r} a_{n,r} + \int_V d^3x |B_0|^2}_{\text{EM}}$$

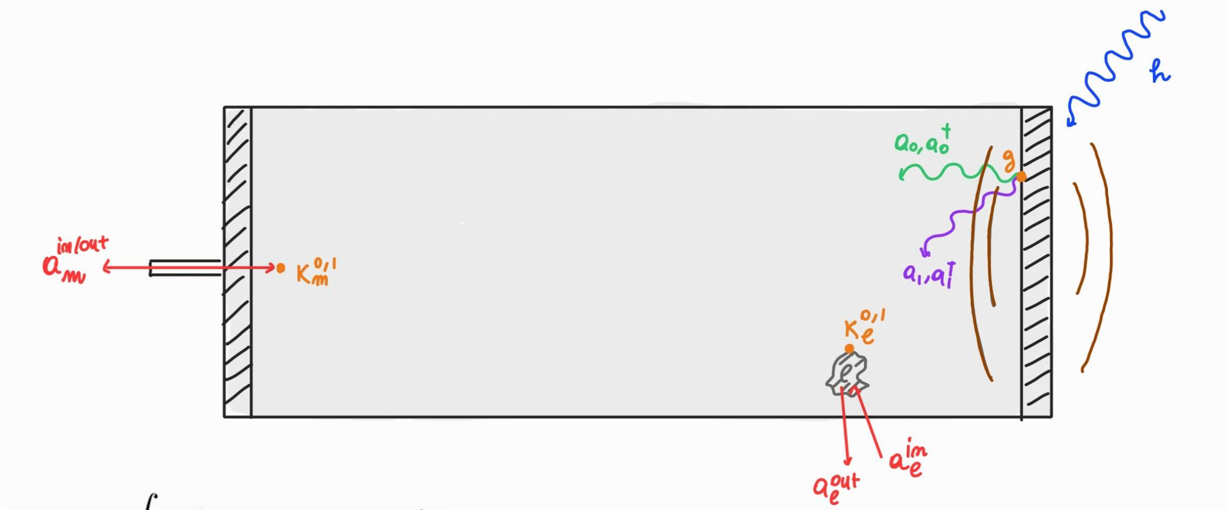
**Int.** $$H_G(t) = \int_V \mathrm{d}^3x h_{\mu\nu} T^{\mu\nu} = h(t) \left\{ \sum_{jj'} C_{jj'} a^\dagger_j(t) a_{j'}(t) + \sum_j \left[ D_j a_j(t) + D_j^* a_j^\dagger(t) \right] \right\}$$

**Readout (thermal bath)** $$H_R(t) = \sum_{j=0}^{1} \sum_l \int \mathrm{d}\omega \left\{ \omega\, b_l^\dagger(\omega) b_l(\omega) + i g_l^j \left[ b_l^\dagger(\omega) a_j(t) - b_l(\omega) a_j^\dagger(t) \right] \right\}$$

- **Resonant EM microwave cavities** [*Physical Review* D **105** 116011 (2022)], **Lumped LC resonators** [*Phys. Rev. Lett.* **129** (2022) 041101]
- **MADMAX** [arXiv:2409.06462], **CAST** [arXiv:1705.02290], **IAXO** [*Eur. Phys. J. C* **79** (2019) 1032];
- **MAGO** [*Phys. Rev. D* **108** (2023) 084058]

# Quantum mechanical set-up

<u>**Mech. resonator**</u>




**Free** $$H_0 = \underbrace{\sum_{n,r} \Delta_n a^\dagger_{n,r} a_{n,r} + \int_V d^3x |B_0|^2}_{\text{EM}} + \underbrace{\omega_m d^\dagger d}_{\text{Mech.}}$$

**Int.** $$H_{OM} = (\mathcal{C} a_1 + \text{h.c.})x$$

**Readout (thermal bath)** $$H_R(t) = \sum_{j=0}^{1} \sum_{l} \int \mathrm{d}\omega \Big\{ \omega\, b_l^\dagger(\omega) b_l(\omega) + i g_l^j \left[ b_l^\dagger(\omega) a_j(t) - b_l(\omega) a_j^\dagger(t) \right] \Big\}$$

- **Interferometers** (**LVK**, **Holometer**), **Optomechanical sensors** (**levitating sphere** [A. Arvanitaki, A. A. Gercai, *Phys. Rev. Lett.* **110** (2013) 071105]), **Weber bars** (**AURIGA** [M. Cerdonio *et al.*, *Classical and Quantum Gravity* **14** (1997) 1491]), **Magnetic Weber bars** [V. Domcke, S.A.R. Ellis, N. L. Rodd, *Phys. Rev. Lett. 134, 231401*]

# Pipeline

- **Input-output formalism** [Beckey *et al.*, arXiv:2311.07270]

**QUADRATURES**

$$X_A^{\binom{\text{in}}{\text{out}}} = \frac{a_A^{\binom{\text{in}}{\text{out}}} + a_A^{\binom{\text{in}}{\text{out}}\dagger}}{\sqrt{2}}$$

$$Y_A^{\binom{\text{in}}{\text{out}}} = \frac{a_A^{\binom{\text{in}}{\text{out}}} - a_A^{\binom{\text{in}}{\text{out}}\dagger}}{i\sqrt{2}}$$

$$(A = j, \Lambda)$$

$$a_\Lambda^{\frac{\text{in}}{\text{out}}} = -\frac{1}{\sqrt{2\pi}} \int \mathrm{d}\omega \, e^{-i\omega(t-t_0)} b_\Lambda(\omega, t_0) \,, \; t \gtrless t_0$$

$$\begin{cases} \dot{X}_n(t) = \Delta_n Y_n(t) \mp \frac{1}{2} \sum_j \tilde{\kappa}_j^n X_j(t) + \sum_\Lambda \sqrt{\kappa_\Lambda^n} X_\Lambda^{\frac{\text{in}}{\text{out}}}(t) + F_X^n(t) \\ \dot{Y}_n(t) = -\Delta_n X_n(t) \mp \frac{1}{2} \sum_j \tilde{\kappa}_j^n Y_j(t) + \sum_\Lambda \sqrt{\kappa_\Lambda^n} Y_\Lambda^{\frac{\text{in}}{\text{out}}}(t) + F_Y^n(t) \\ \dot{x}(t) = \frac{p(t)}{M} \\ \dot{p}(t) = -M\omega_m^2 x(t) - \gamma_m p(t) - g X_1(t) + F_m(t) \end{cases}$$

EM

Mech.

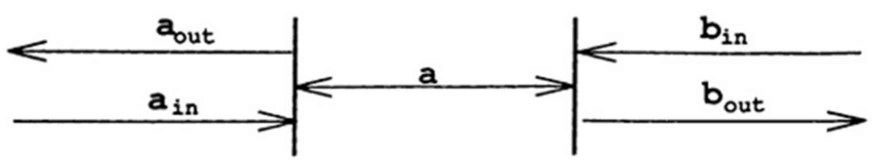
Ex: $\begin{cases} F_X^W = -\sqrt{2} E_0 \mathrm{Im} C_W \\ F_Y^W = -\sqrt{2} E_0 \mathrm{Re} C_W \end{cases}$

**SOLVE**
(in $\mathcal{F}$-space)

**Loss rates**

$$\kappa_\Lambda^n = 2\pi (g_\Lambda^n)^2$$

$$\tilde{\kappa}_j^n = \sum_\Lambda \sqrt{\kappa_\Lambda^n \kappa_\Lambda^j}$$



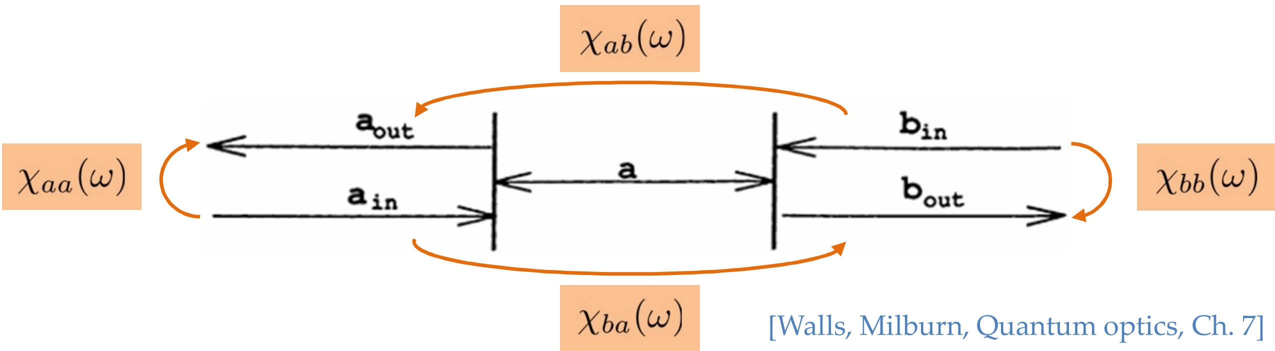

[Walls, Milburn, Quantum optics, Ch. 7]

# Transfer functions

- <u>Input-output relations</u>

$$
\begin{cases}
X_\Lambda^{\text{out}} = X_\Lambda^{\text{in}} - \sum_j \sqrt{\kappa_\Lambda^j} X_j \\
Y_\Lambda^{\text{out}} = Y_\Lambda^{\text{in}} - \sum_j \sqrt{\kappa_\Lambda^j} Y_j
\end{cases}
\xrightarrow[\text{sols. to the EoMs}]{\text{Substituting the}}
\begin{cases}
\tilde{X}_m^{\text{out}}(\omega) = \sum_\Lambda \left[\chi_{X_m X_\Lambda}(\omega)\tilde{X}_\Lambda^{\text{in}}(\omega) + \chi_{X_m Y_\Lambda}(\omega)\tilde{Y}_\Lambda^{\text{in}}(\omega)\right] + \sum_I \chi_{X_m F_I}(\omega)\tilde{F}_I(\omega) \\
\tilde{Y}_m^{\text{out}}(\omega) = \sum_\Lambda \left[\chi_{Y_m X_\Lambda}(\omega)\tilde{X}_\Lambda^{\text{in}}(\omega) + \chi_{Y_m Y_\Lambda}(\omega)\tilde{Y}_\Lambda^{\text{in}}(\omega)\right] + \sum_I \chi_{Y_m F_I}(\omega)\tilde{F}_I(\omega)
\end{cases}
$$

$$(\Lambda = m, \ell)$$




[Walls, Milburn, Quantum optics, Ch. 7]

# Power Spectral Density and Minimal Detectable Strain

- Quadrature PSD:

$$
\left.
\begin{aligned}
S^{\mathrm{out}}_{Y_m Y_m}(\omega) = &\sum_{\Lambda}\left[|\chi_{Y_m Y_\Lambda}(\omega)|^2 S^{\mathrm{in}}_{Y_\Lambda Y_\Lambda}(\omega) + |\chi_{Y_m X_\Lambda}(\omega)|^2 S^{\mathrm{in}}_{X_\Lambda X_\Lambda}(\omega)\right] \\
&+ \sum_{\Lambda}\left[\chi_{Y_m X_\Lambda}(\omega)\chi_{Y_m Y_\Lambda}(\omega)^* S^{\mathrm{in}}_{Y_\Lambda X_\Lambda}(\omega) + \chi_{Y_m Y_\Lambda}(\omega)\chi_{Y_m X_\Lambda}(\omega)^* S^{\mathrm{in}}_{X_\Lambda Y_\Lambda}(\omega)\right]
\end{aligned}
\right\} \text{Noise}
$$

$$
\left. + \sum_{I} |\chi_{Y_m F_I}(\omega)|^2 S_{F_I F_I}(\omega) \supset S_{hh}(\omega) \right\} \text{Signal}
$$

**h-PSD ↔ GW energy density**

$$
\boxed{\Omega_g(\omega) = \frac{\omega^3 \, S_{hh}(\omega)}{3\pi H_0^2}} \xrightarrow{\mathrm{SNR} \simeq 1} \boxed{(\Omega_g(\omega))_{\mathrm{min}} = \frac{\omega^3}{3\pi H_0^2}\left[t_{\mathrm{int}} \int_{\omega-\Delta\omega/2}^{\omega+\Delta\omega/2} \frac{\mathrm{d}\omega'}{2\pi} \frac{1}{(S_{hh}(\omega')|_{\mathrm{NOISE}})^2}\right]^{-\frac{1}{2}}}
$$

$$
S_{hh}(\omega)|_{\mathrm{NOISE}} = \frac{S^{\mathrm{out}}_{Y_m Y_m}(\omega')|_{\mathrm{NOISE}}}{|\chi_{Y_m h}(\omega')|^2}
$$

# Assumptions and regimes

- **Assumptions**
  - Mechanical detectors have infinite mass: $M \to \infty$
  - No intrinsic losses: $\kappa_m^1 = \kappa_1$
- **Regimes**

**Thermal-limited**

$$S_{YY}^{\text{in}} = \frac{1}{2}\coth\left(\frac{\hbar\omega}{2k_B T}\right)$$

**Quantum-limited**

$$S_{YY}^{\text{in}} \leq \frac{1}{2}$$

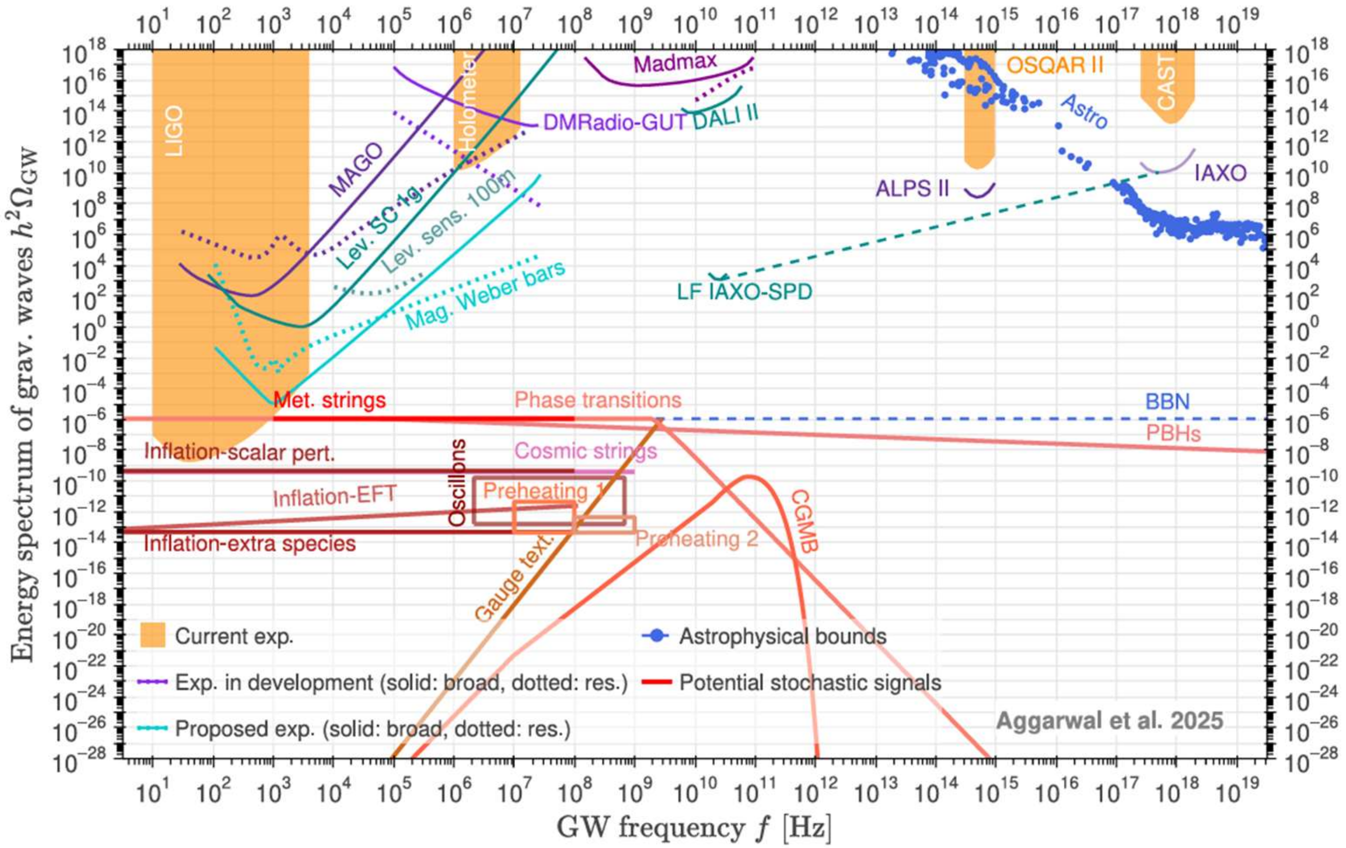
**Standard Quantum Limit (SQL)**

$$S_{YY}^{\text{in}} = \frac{1}{2}$$

**Heisenberg Limit (HL)**

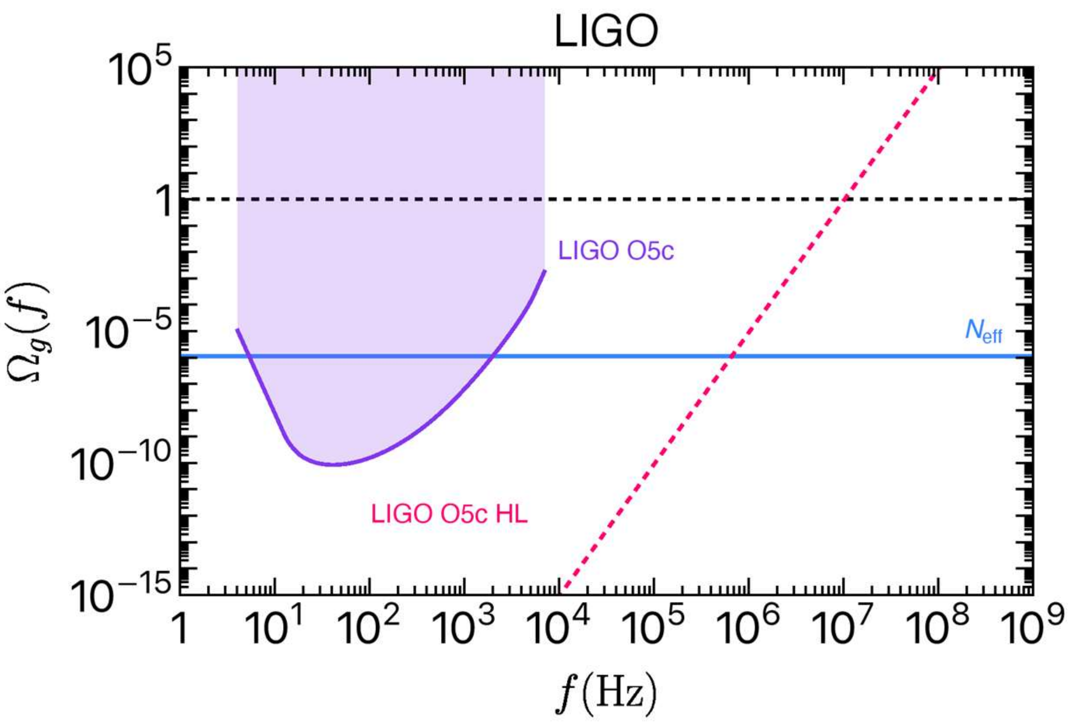
$$\text{“ } S_{YY}^{\text{in}} = 0 \text{ ”}$$

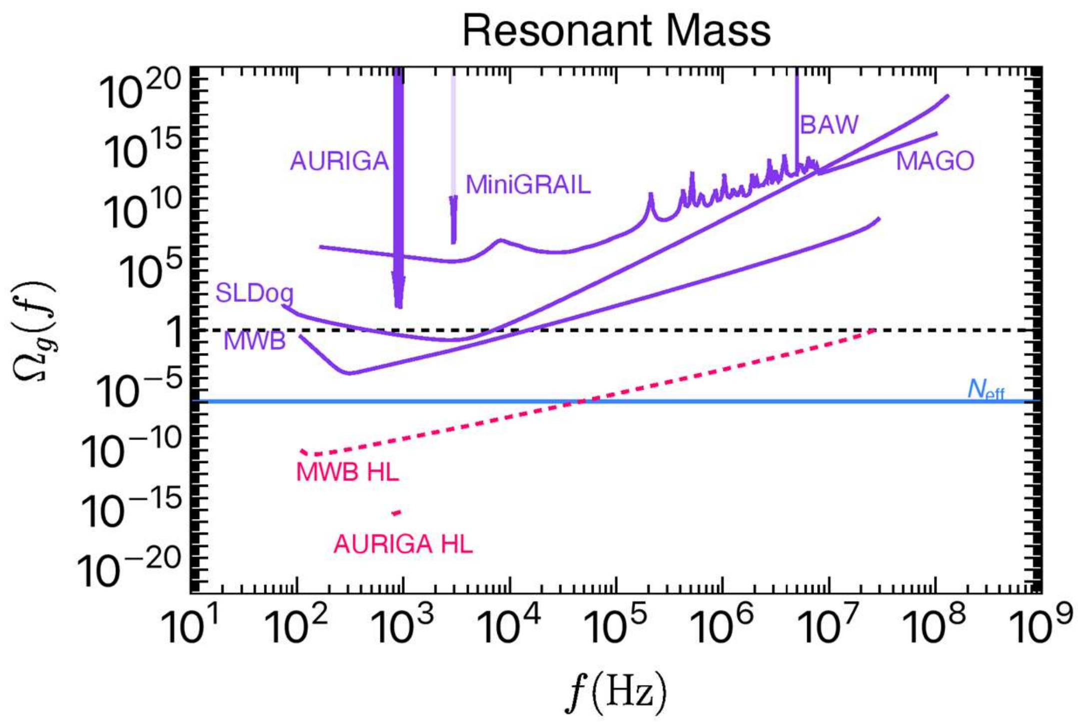
$$(S_{hh}(\omega)|_{\text{NOISE}} = S_{1\gamma})$$



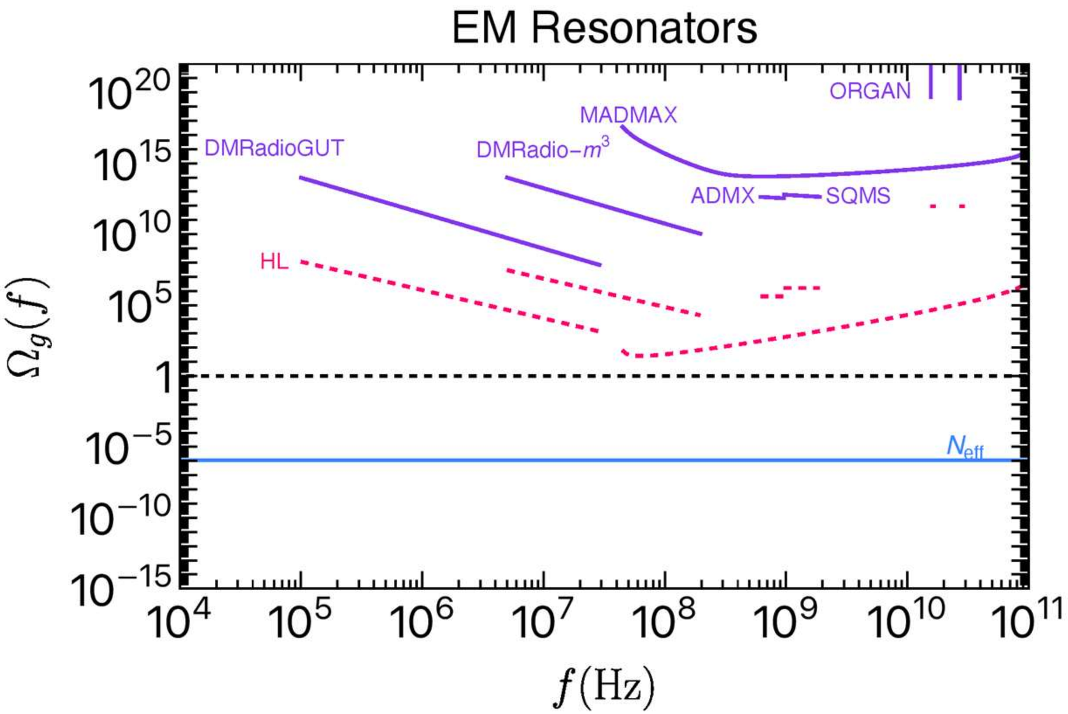

[Aggarwal et al., *Challenges and Opportunities of Gravitational Wave Searches above 10 kHz*, 2025]

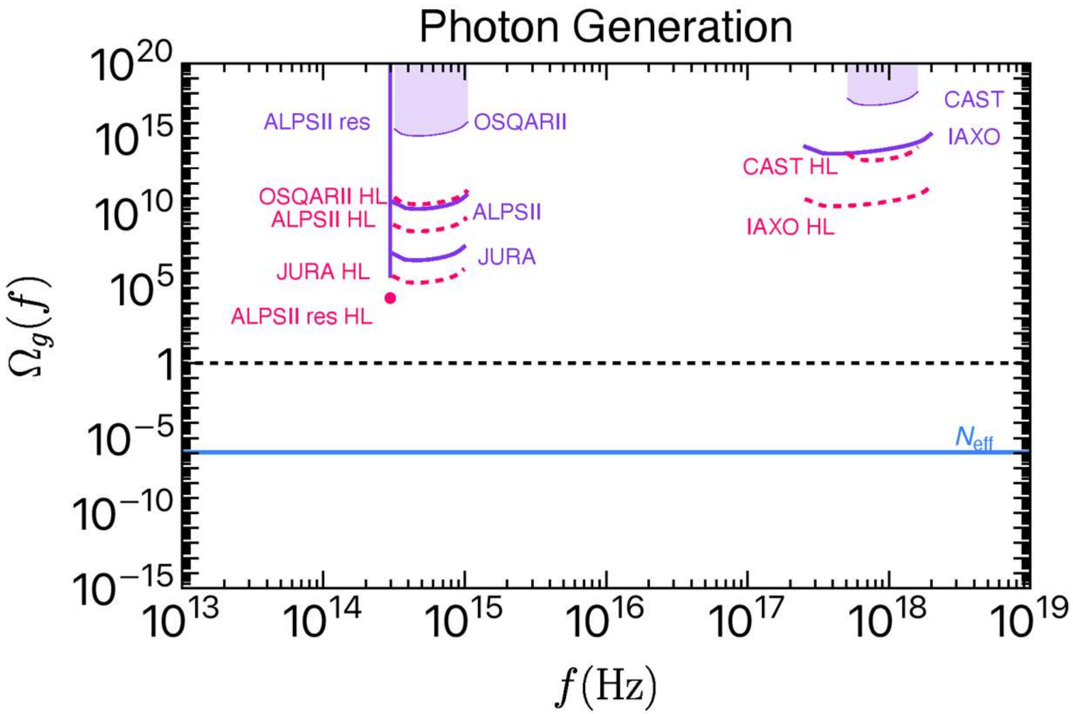
# Quantum heuristics for mechanical resonators



# Quantum heuristics for EM resonators



# Summary & conclusion

- With the aid of the **input-output formalism** applied to **two elementary quantum mechanical models**, we were able to study the response of most operating and proposed gravitational-wave detectors under unrealistically generous assumptions on their efficiency.

- The two quantum-mechanical toy models we considered allowed us to **reproduce the overall features of all the different detectors**' sensitivity curves.

- By taking the **SQL** and **HL** of these models, we were able to put **analytic bounds** on the sensitivity GW detectors can ever hope to achieve. The analytic bounds confirm the validity of classical heuristics: **most existing detectors cannot probe energy densities below the critical density line, even at their HL. Improving over the BBN bound much above a kHz does not seem feasible.**

**High-frequency gravitational waves coming from primordial backgrounds may remain out of the experimental reach of current and near-future detectors**

# Heurísticas quânticas para detecção de ondas gravitacionais

Limites de sensibilidade em alta frequência para buscas atuais e no futuro próximo por BSM

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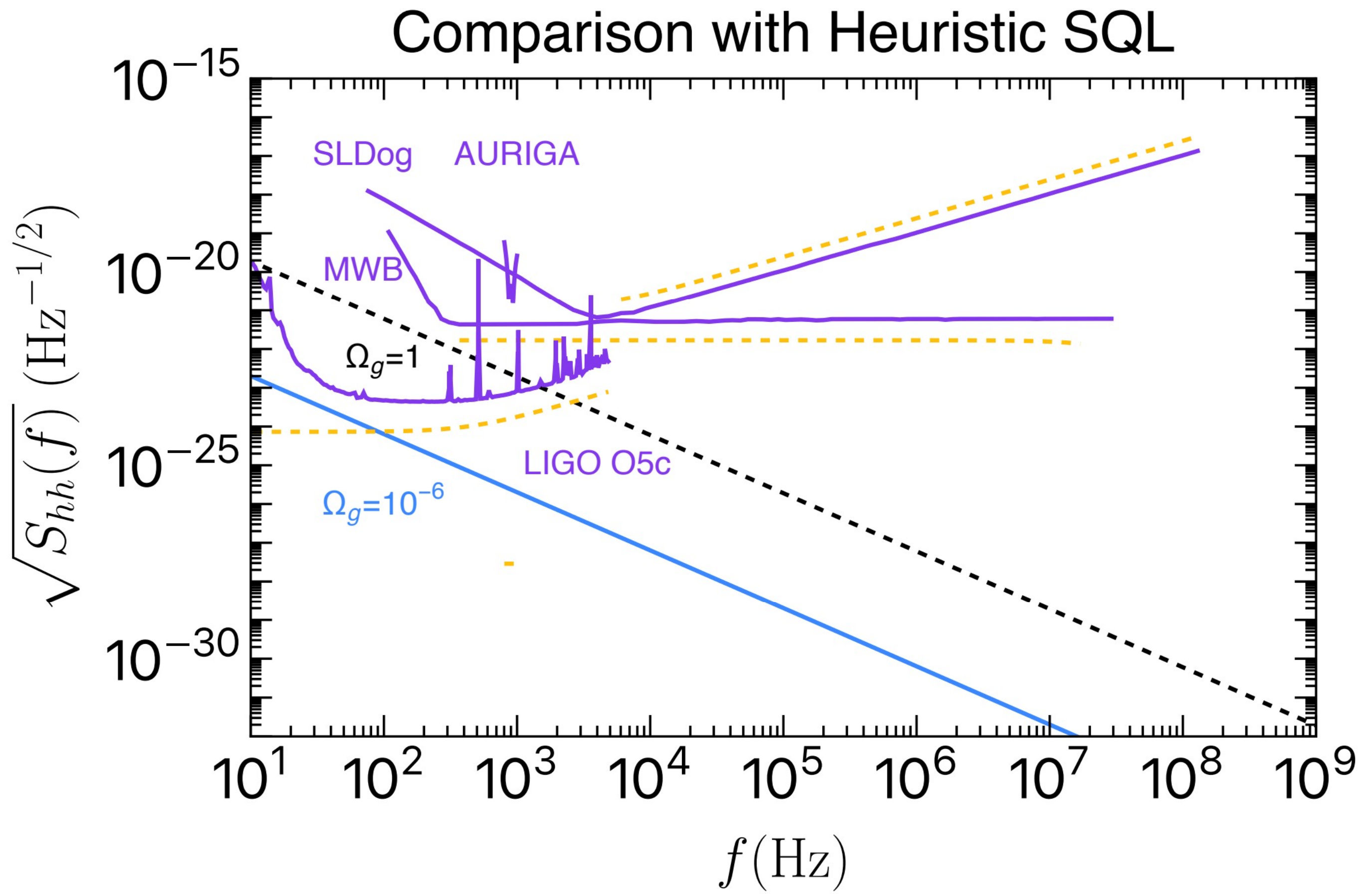
# BACK-UP


Comparison with Heuristic SQL
SLDog
AURIGA
MWB
$\Omega_g$=1
LIGO O5c
$\Omega_g$=$10^{-6}$
$10^{-15}$
$10^{-20}$
$10^{-25}$
$10^{-30}$
$10^1$ $10^2$ $10^3$ $10^4$ $10^5$ $10^6$ $10^7$ $10^8$ $10^9$
$\sqrt{S_{hh}(f)}\ (\mathrm{Hz}^{-1/2})$
$f\,(\mathrm{Hz})$

# Some PSDs

- **Interferometers:**

$$S_{hh}(\omega) = 2\pi \frac{(\kappa^2 + 4\omega^2)[\omega^2\gamma_m^2 + (\omega^2 - \omega_m^2)^2]}{32 U_{\text{in}} \omega_1^2 \omega_g^4 |I|^2} S^{\text{out}}_{Y_m Y_m}(\omega)$$

- **Weber bars:**

$$S_{hh}(\Omega) = 2\pi \frac{[(\kappa_1^2/4 + \omega_1^2 - \Omega^2)^2 + \kappa_1^2\Omega^2][(\omega_m^2 - \Omega^2)^2 + \gamma_m^2\Omega^2]}{2\kappa_1 L^2 \omega_g^4 |I|^2 E_0^2 \left\{[\omega_1 \text{Im}(C_{\text{W}}) - \kappa_1/2\,\text{Re}(C_{\text{W}})]^2 + \Omega^2 \text{Re}(C_{\text{W}})^2\right\}} S^{\text{out}}_{Y_m Y_m}(\Omega)$$

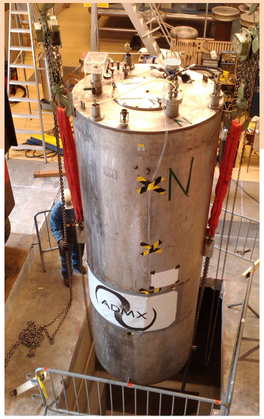
$$C_W = \sqrt{\frac{\omega_1 S}{L}} \tilde{C}_{\text{W}}$$

- **Photon (re)generation:**

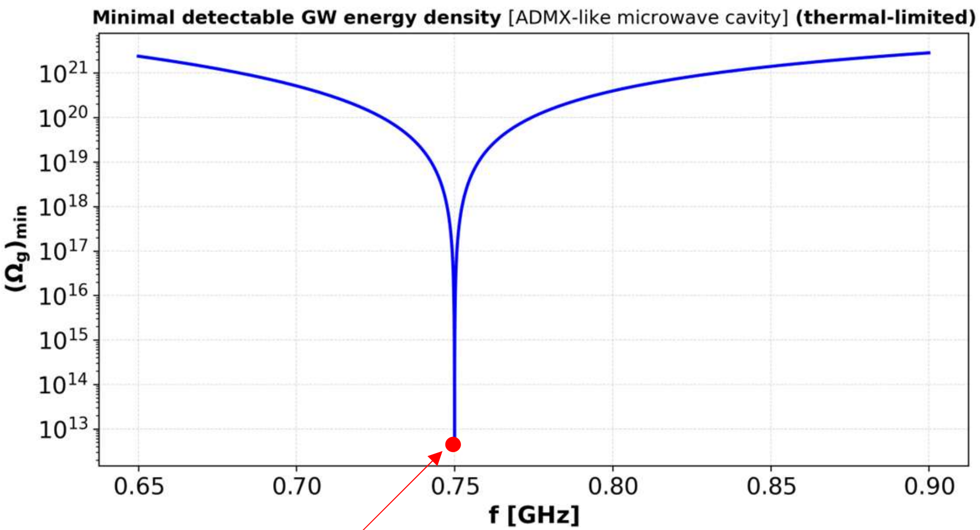
$$S_{hh}(\Omega) = 2\pi \frac{[(\kappa_1^2/4 + \omega_1^2 - \Omega^2)^2 + \kappa_1^2\Omega^2]}{2\kappa_1 \left\{[\omega_1 \text{Im}\,(D_1) - \kappa_1/2\,\text{Re}\,(D_1)]^2 + \Omega^2 \text{Re}\,(D_1)^2\right\}} S^{\text{out}}_{Y_m Y_m}(\Omega)$$

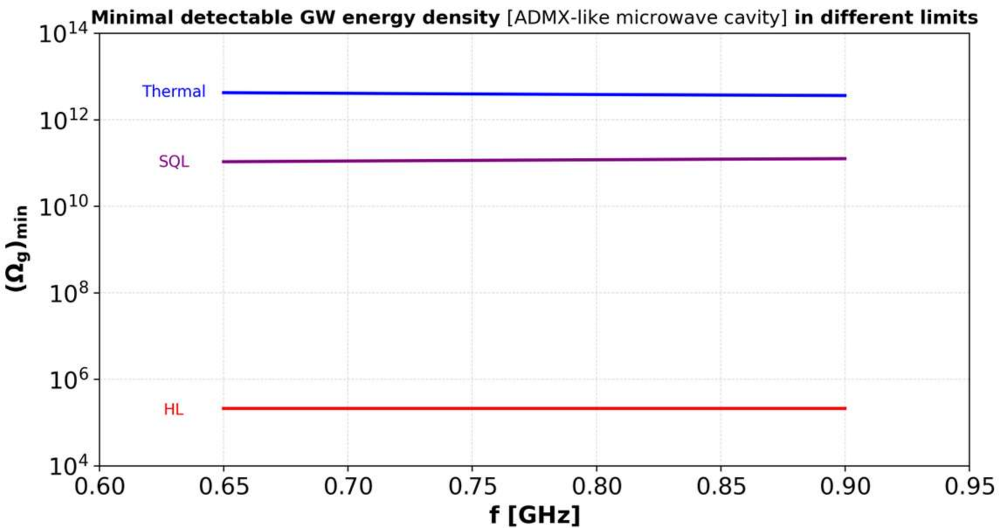
$$D_j = \tilde{D}_j \sqrt{V\omega_j}\, B_0\, \omega_g L\, \min[1, \omega_g L]$$

# Example: EM resonator w/ ext. static magnetic field




$$S_{hh}(\omega)|_{\text{NOISE}} = \# \frac{[(\kappa_1^2/4 + \omega_1^2 - \omega^2)^2 + \kappa_1^2\omega^2][(\omega_m^2 - \omega^2)^2 + \gamma_m^2\omega^2]}{\kappa_1 L^2 \omega^4 B_0^2 \left\{ [\omega_1 \mathrm{Im} C_{\text{MC}} - \kappa_1/2\, \mathrm{Re} C_{\text{MC}}]^2 + \omega^2 \mathrm{Re} C_{\text{MC}}{}^2 \right\}} S^{\text{out}}_{Y_m Y_m}(\omega)|_{\text{NOISE}}$$




**Minimal detectable strain (on res. $\omega_1 = \omega_g$):** $h_{\text{min}}^{\text{DC, res}} \gtrsim \# \sqrt{\frac{\hbar}{U_{\text{in}}}} \left(\frac{2\pi\,\omega_g}{t_{\text{int}}}\right)^{1/4} \frac{1}{Q^{3/4}} \left(\frac{2\pi k_B T}{\hbar\omega}\right)$