Astrophysical cosmic rays lecture II Acceleration mechanism

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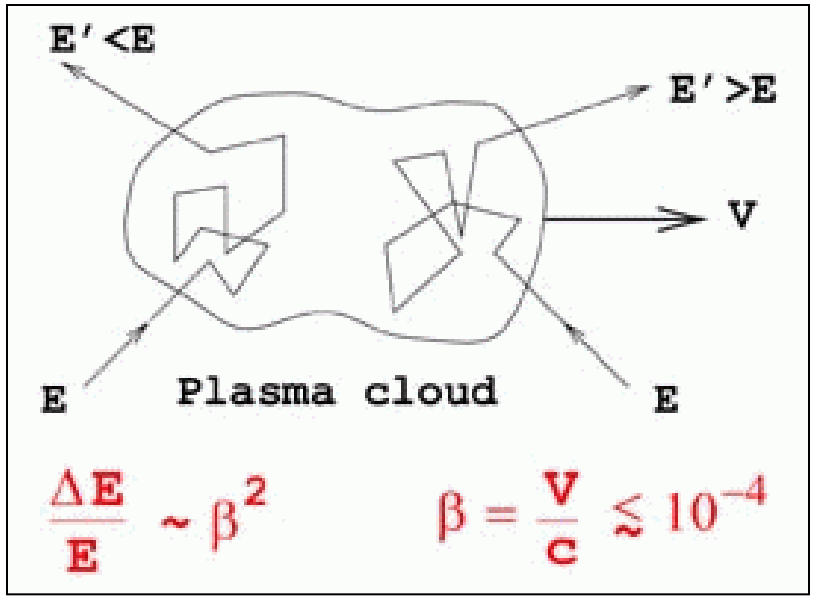
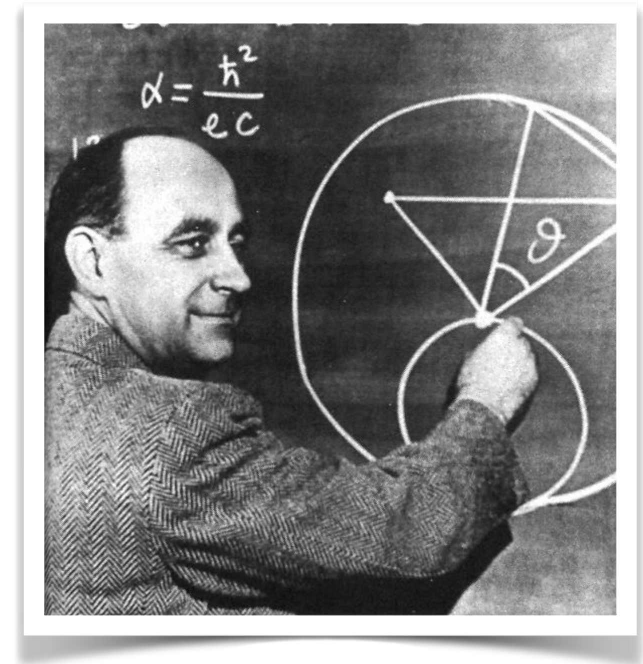
Fermi acceleration mechanisms

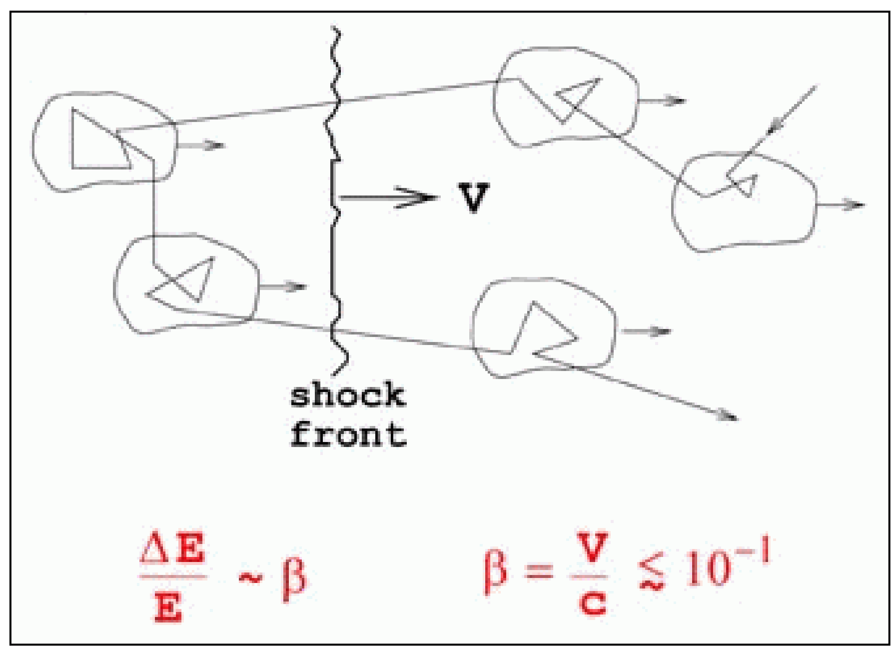
Outside the tunnels of accelerators, one needs a certain degree of randomness to accelerate particles!



2nd order mechanism proposed by Fermi in 1949.



1st order mechanism: developed along the 1970's by several astrophysicists



Both predict naturally a power law injection spectrum:

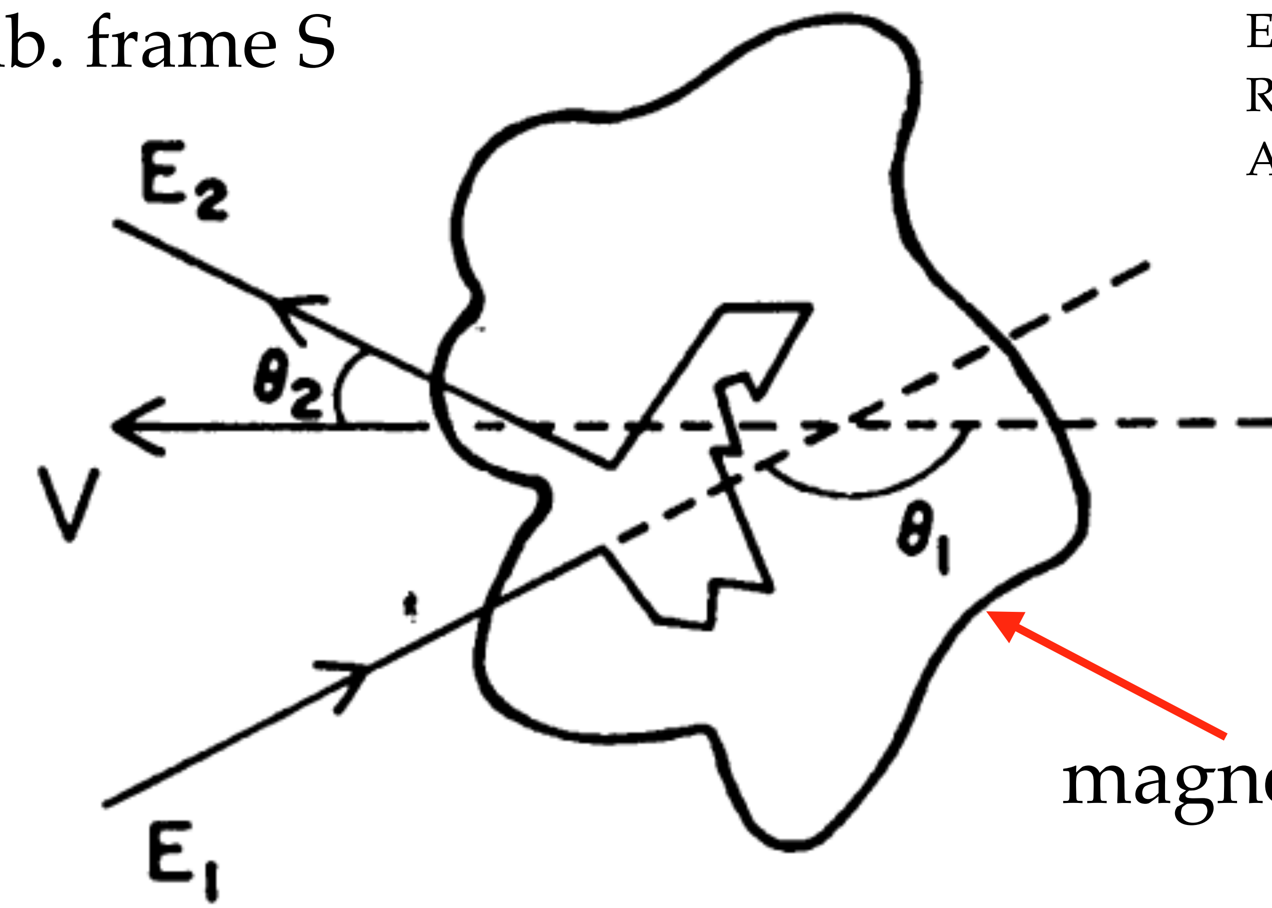
$$\Phi(E) \propto E^{-\alpha}$$

Fermi 2nd order mechanism (1949)

- In 1949, Fermi explored the random nature of collisions in the interstellar medium to calculate the effective average energy gain of test charged particles hitting plasma clouds.

lab. frame S

$$\beta \equiv \frac{V}{c}$$



magnetized cloud (plasma)

E. Fermi. On the Origin of the Cosmic Radiation. Phys. Rev., 75:1169–1174, Apr 1949.

- In the rest frame S' of the magnetized cloud:

$$E_1' = \gamma E_1 (1 - \beta \cos\theta_1)$$

- So, going back to the laboratory frame:

$$E_2 = \gamma E_2' (1 + \beta \cos\theta_2')$$

Fermi 2nd order mechanism (1949)

- In the following, Fermi neglected energy losses inside the cloud, that is, he considered the collisions as being elastic in nature.

- Energy and momentum conservation in the cloud rest frame implies:

$$E'_2 = E'_1$$

- Therefore:

$$\frac{\Delta E}{E_1} = \frac{E_2 - E_1}{E_1} = \frac{1 - \beta\cos\theta_1 + \beta\cos\theta'_2 - \beta^2\cos\theta_1\cos\theta'_2}{1-\beta^2} - 1$$

- Pay attention to the fact that the previous expression contains angles measured at different frames (S and S′).

Fermi 2nd order mechanism (1949)

- Taking the average over all possible output angles (in S') and using the fact that the internal collisions inside the cloud can be treated as random ones, we have:

$$\langle \cos\theta_2' \rangle = 0 \Longrightarrow \frac{\langle \Delta E \rangle_2}{E_1} = \frac{1-\beta\cos\theta_1}{1-\beta^2} - 1$$

- On the other hand, the distribution of incoming angles (in S) should be proportional to the relative particle-cloud velocity: in S, head-on collisions are more likely than rear-end ones!

- Defining $\cos\theta_1 \equiv \mu$, we can write

$$\begin{aligned}
P(\mu) = N v_{\rm rel} \quad &= \quad N\frac{v_1 - V\mu}{1 - v_1 V\mu/c^2} \\
&= \quad N(v_1 - V\mu)\left(1 + \frac{v_1}{c}\frac{V\mu}{c} + \cdots\right) \\
&\simeq \quad Nc\left(\frac{v_1}{c} - \frac{V\mu}{c}\right) \\
&\overset{v_1=c}{\simeq} \quad Nc(1-\beta\mu)
\end{aligned}$$

- Normalization condition for P $\longrightarrow$ $\displaystyle\int_{-1}^{1} P(\mu)d\mu = 1 \Longrightarrow N = \frac{1}{2c}$

Fermi 2nd order mechanism (1949)

- **Task:** show that after taking the average over all possible values of $\cos\theta_1$, we can write:

$$\xi = \left\langle \frac{\Delta E}{E} \right\rangle = \frac{1 + \frac{1}{3}\beta^2}{1 - \beta^2} - 1 \simeq \frac{4}{3}\beta^2$$

- In other words, the relative average energy gain per collision is quadratic in β

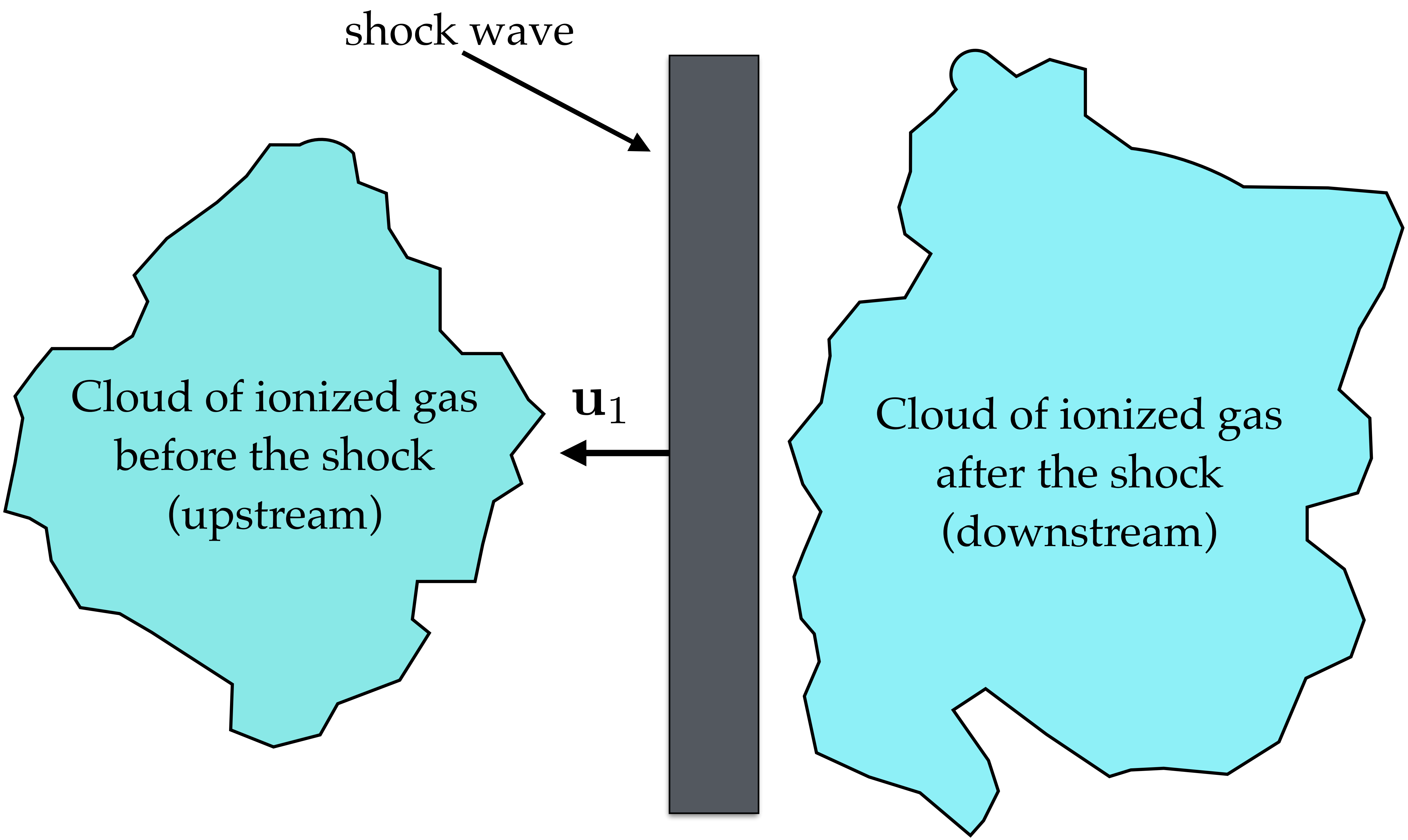
- That's a plausible acceleration mechanism, but certainly inefficient, since typical velocities of magnetized clouds in the interestelar medium satisfy:

$$\beta \lesssim 10^{-4}$$

- It became clear soon to astrophysicists, that a more efficient mechanism would be necessary to explain the fluxes of UHECRs

- **QUESTION:** How is it possible to gain energy on average, if there will be collisions in which the particle gain energy and others in which it loses energy?

Fermi 1st order mechanism

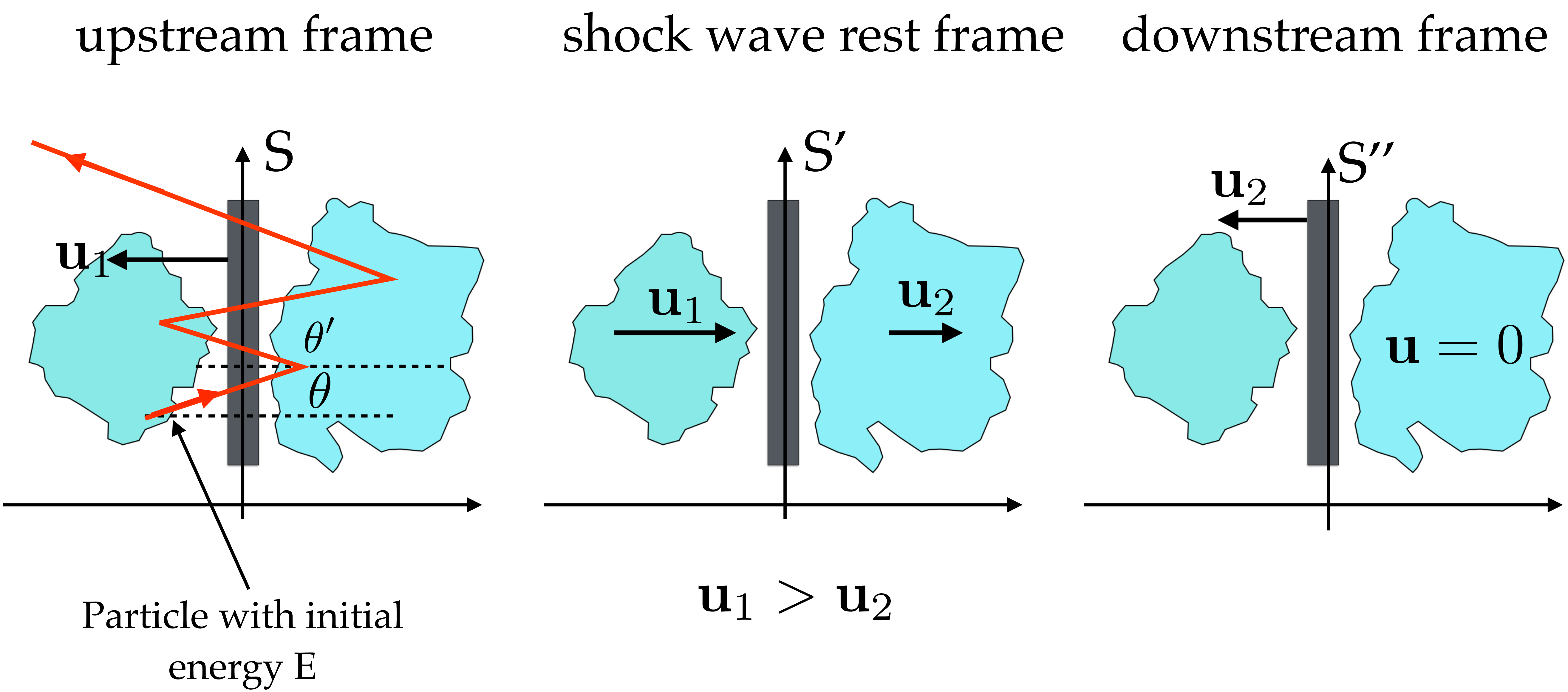


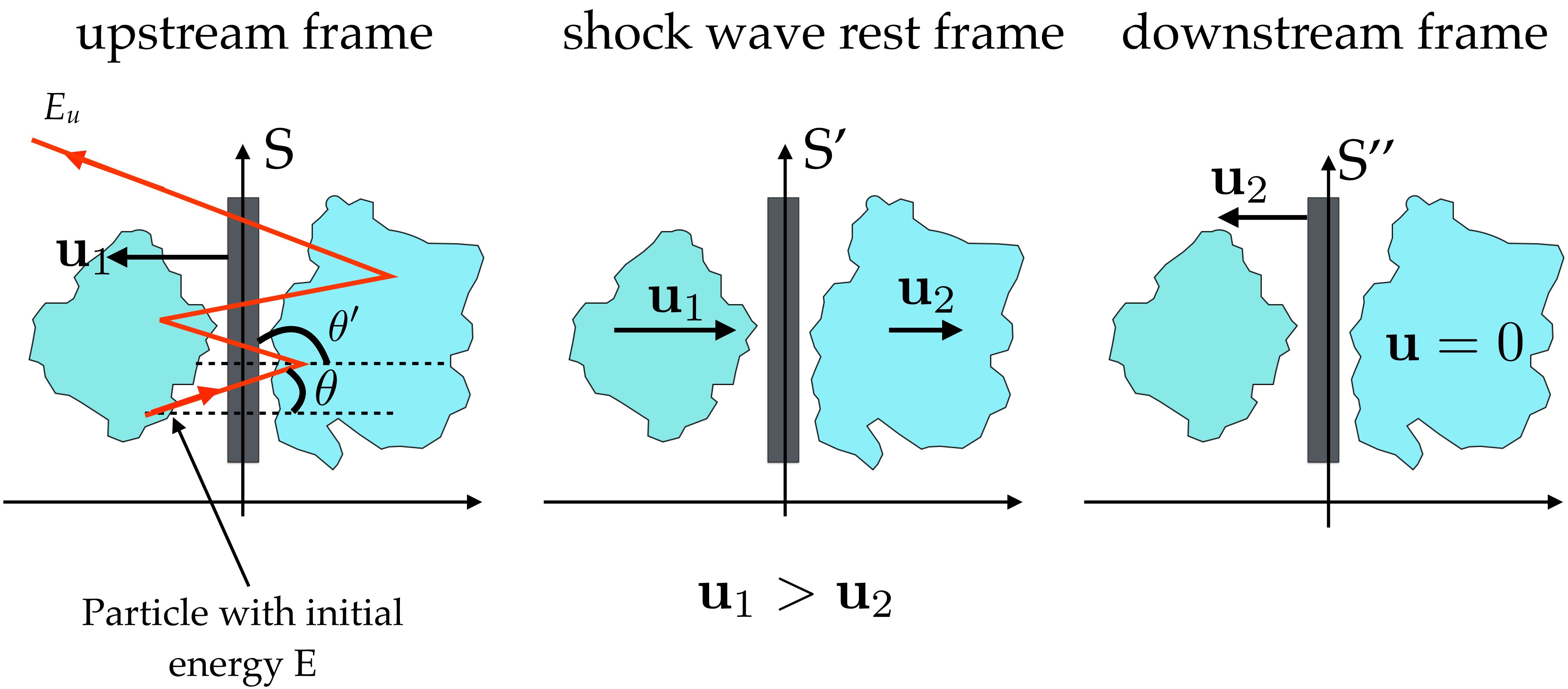
Laboratory frame

- **QUESTION:** how does the previous analysis can be modified to accommodate the scattering by a plane shock wave?

Fermi 1st order mechanism

- Here, we are going to need 3 auxiliary inertial frames in addition to the lab frame





In the downstream frame S'': $$E_d^{(i)} = \gamma E(1+\beta\mu) \quad (0 \le \mu \le 1)$$

Energy-momentum conservation in S'': $$E_d^{(f)} = E_d^{(i)}$$

Back to S (upstream): $$E_u = \gamma E_d^{(f)}(1-\beta\mu') \quad (-1 \le \mu' \le 0)$$

Fermi 1st order mechanism

- **Task:** from the previous scattering geometry, write the boost velocity β as a function of the shock wave velocities in the upstream and downstream frames u_1 and u_2.

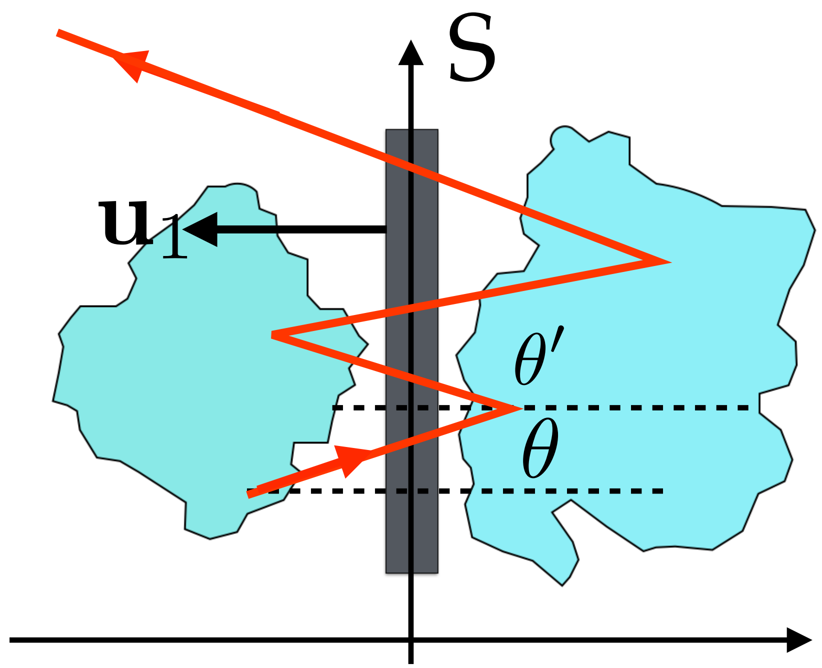
- We can therefore write:

$$E_u = \gamma E_d^{(f)}(1-\beta\mu') = \gamma^2 E(1+\beta\mu)(1-\beta\mu')$$

- So, the average energy gain per collision is:

$$\xi = \left\langle \frac{E_u - E}{E} \right\rangle = \int_0^1 d\mu \int_{-1}^0 d\mu' \left(\frac{E_u - E}{E} \right) P(\mu) P(\mu') \simeq \frac{4}{3}\beta$$

Fermi 1st order mechanism



- In the Fermi first order mechanism, the relative average energy gain per collision is linear in β.
- Moreover, for shock waves in the interestelar medium:

$$\beta \lesssim 10^{-1}$$

- So, 1st order is much more efficient that 2nd order!
- The first order mechanism is also known as **Diffusive Shock Acceleration (DSA).**

Fermi 1st order mechanism

- Pay attention to the fundamental role played in the DSA by the shock wave: it establishes a velocity gradient between the upstream and downstream regions. This gradient was essential in the previous derivation.

- To understand that, a very useful frame is the center of momentum of the clouds. In that frame, the relative velocity is of approximation, so they act as a macroscopic magnetic walls onto which the test particles reflect several times before escaping preferentially along direction parallel to the shock front.

DSA original references

- Different from the 2nd order mechanism which is a creation of Fermi alone, DSA is a mechanism to which several astrophysicists contributed.

- Here is a list of papers from the 1970's showing the development of what became known as the Fermi first order mechanism:

1. Krymsky, G. F. Dokl. Akad. Nauk SSSR 234, 1306 (1977).
2. Bell, A. R. M.N.R.A.S. 182, 147 (1978).
3. Axford, W. I., Leer, E., and Skadron, G. Proc. 15th ICRC (Plovdiv) XI, 132 (1977).
4. Blandford, R. D. & Ostriker, J. P. Astrophys. J. Lett. 221, L29 (1978).

Injection spectrum

- The UHECR spectrum, as we saw before, can be approximated by a series of power laws with different regions of the spectrum having their appropriate value of spectral index.

- Usually power laws indicate the existence of a small amount of particles in the very high energy tail of the distribution. Therefore, they are signatures of non-thermal processes.

- Here, we are assuming such non-thermal processes through which a test particle gains its kinetic energy from a macroscopic magnetized medium.

- According to our previous derivation, after n scatterings in the acceleration region, a test particle gaining energy through the 1st order Fermi mechanism, after n such scatterings will reach an energy

$$E_n = (1+\xi)^n E_0$$

Injection spectrum

- As a first approximation, we should assume that in a certain energy range, the escape probability from the acceleration region P_{esc} is energy independent, so that the probability of remaining in the acceleration region after n scatterings is:

$$P_{rem} = (1 - P_{esc})^n$$

- So, the number of scatterings necessary to reach a certain energy E is given by

$$n = \frac{\ln\left(\frac{E}{E_0}\right)}{\ln(1+\xi)}$$

- Therefore, the fraction of particles accelerated above a certain energy threshold E can be written as

$$N(\geq E) \propto \sum_{m=n}^{\infty} (1 - P_{esc})^m$$

- **TASK:** show that this fraction can be put into the form

$$N(\geq E) \propto \frac{(1-P_{esc})^n}{P_{esc}} = \frac{1}{P_{esc}}\left(\frac{E}{E_0}\right)^{-\gamma}, \quad \text{with} \quad \gamma = -\frac{\ln(1-P_{esc})}{\ln(1+\xi)}$$

Injection spectrum

- We conclude that the previous mechanism naturally leads to an injection spectrum in the form of a power law

$$\frac{dN}{dE} \propto E^{-\alpha} \quad \text{with} \quad \alpha = \gamma + 1$$

- See that in this model the spectral index is determined by the escape probability and the fractional energy gain per collision.

- The success of the Fermi mechanisms in predicting a power law injection spectrum made them very popular in the astrophysics community.

- Of course this is a simplified model, since we have, for example, considered the escape probability as energy independent. We know however that as the test particle gain energy inside the acceleration region, its escape probability should increase.

- More detailed simulations, however, indicate that the power law behavior in the Fermi mechanisms is a more or less general feature of stochastic acceleration and typical values of α are around 2.0.

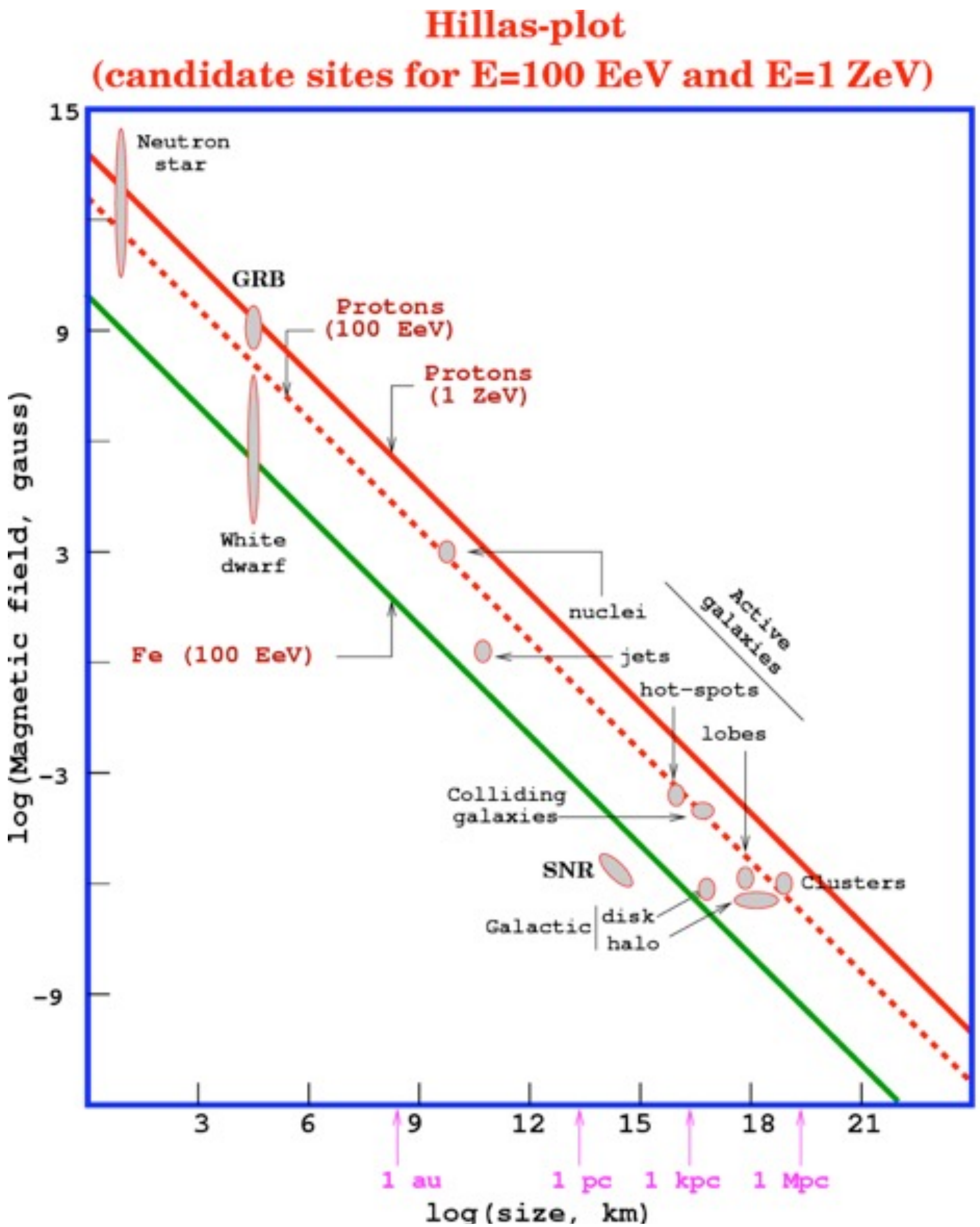
The Hillas plot

- Studying the limits of the confinement process (the energy dependence of the escape probability) inside the acceleration region that Hillas arrived at a plot that became known as the **Hillas plot.**

- This plot is particularly useful to identify candidate sites of acceleration for cosmic rays with energies above EeV (1 EeV = 10^{18} eV).

A. M. Hillas. The Origin of Ultra-High-Energy Cosmic Rays. Annual review of astronomy and astrophysics, 22:425–444, 1984.

The Hillas plot



1 EeV = 10^{18} eV

1 ZeV = 10^{21} eV

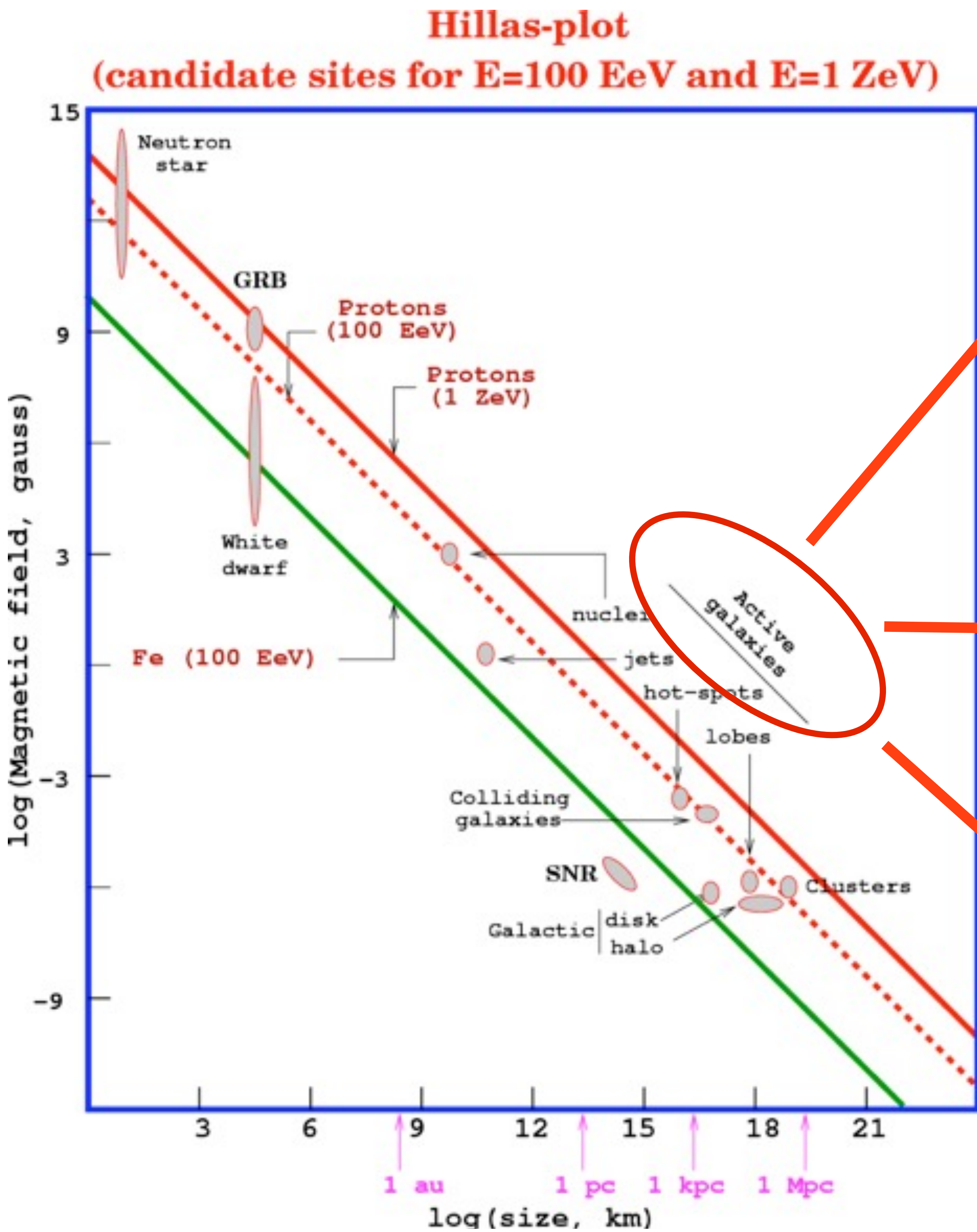
There should be a compromise between the source size and its magnetic field

Larmor radius has to adjust to the source size

$$R_L = \frac{\gamma m v_\perp}{qB}$$

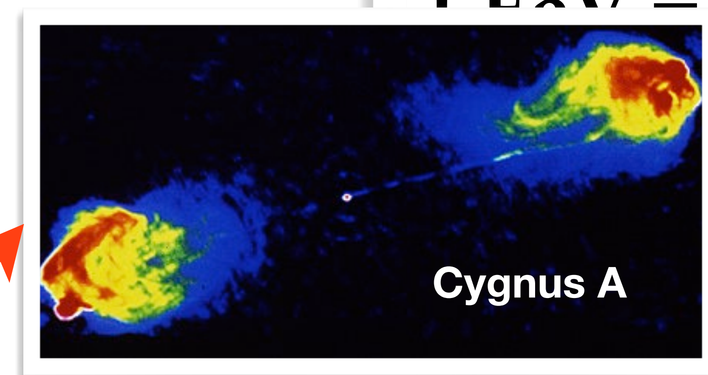
WARNING: Energy losses impose additional constraints to the source candidates!

The Hillas plot



10^{18} eV

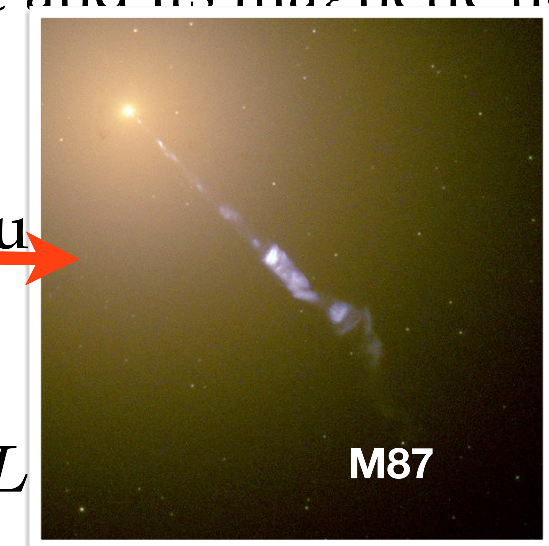
10^{21} eV



There should be a compromise between the source size and its magnetic field

Larmor radiu ... to the source size

R_L



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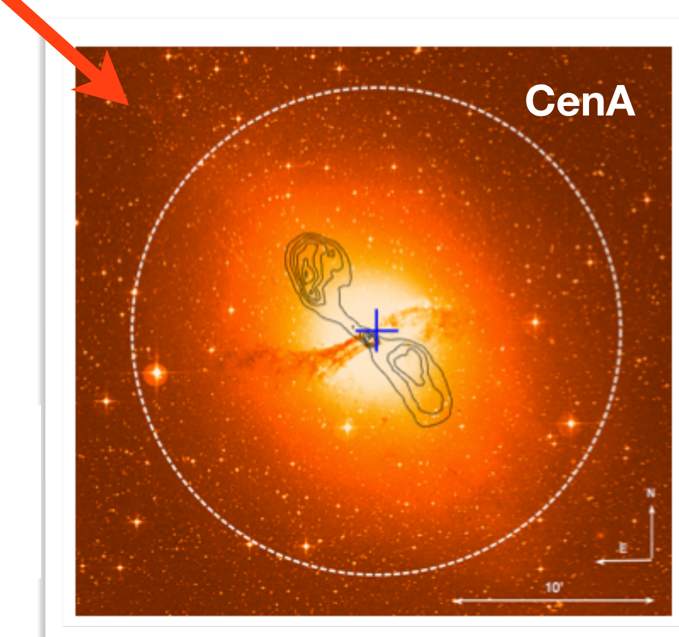
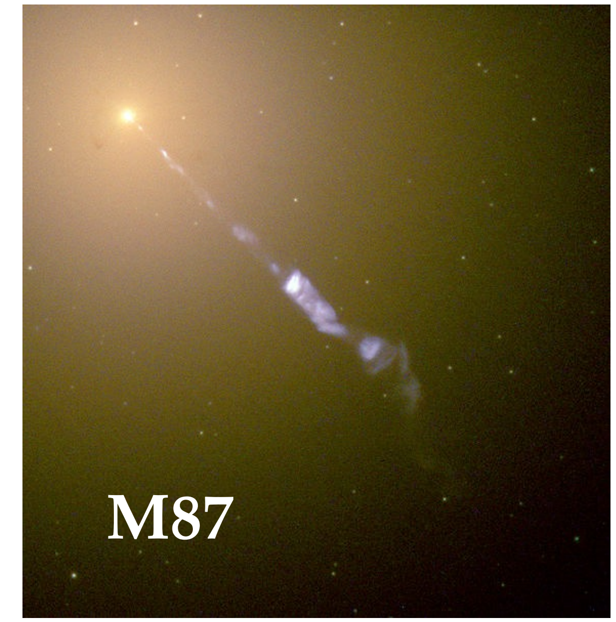


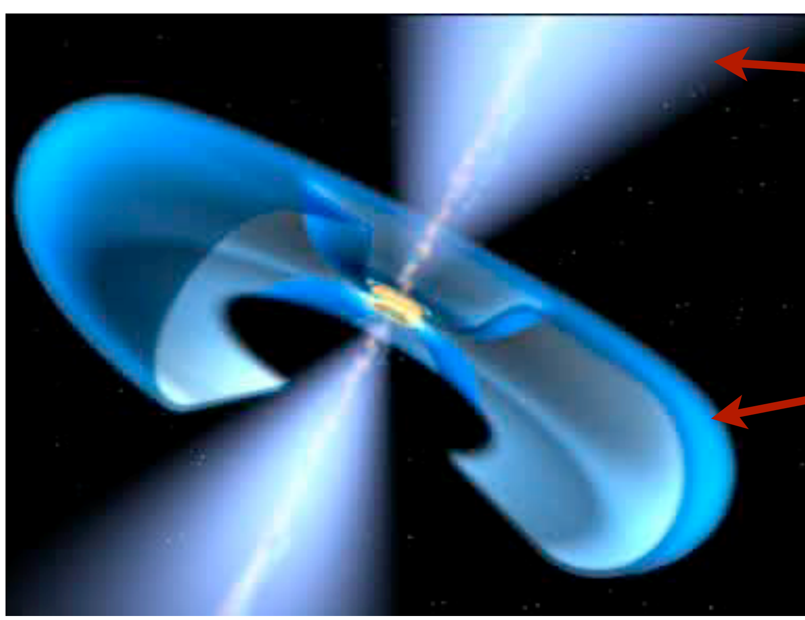
Active Galactic Nuclei (AGN)

Galaxies with anomalous high electromagnetic emission, specially at radio

Their usually associated to the accretion process of a supermassive black hole (millions or even billions of solar mass)

In many of them, relativistic jets are observed emanating from the their galactic center

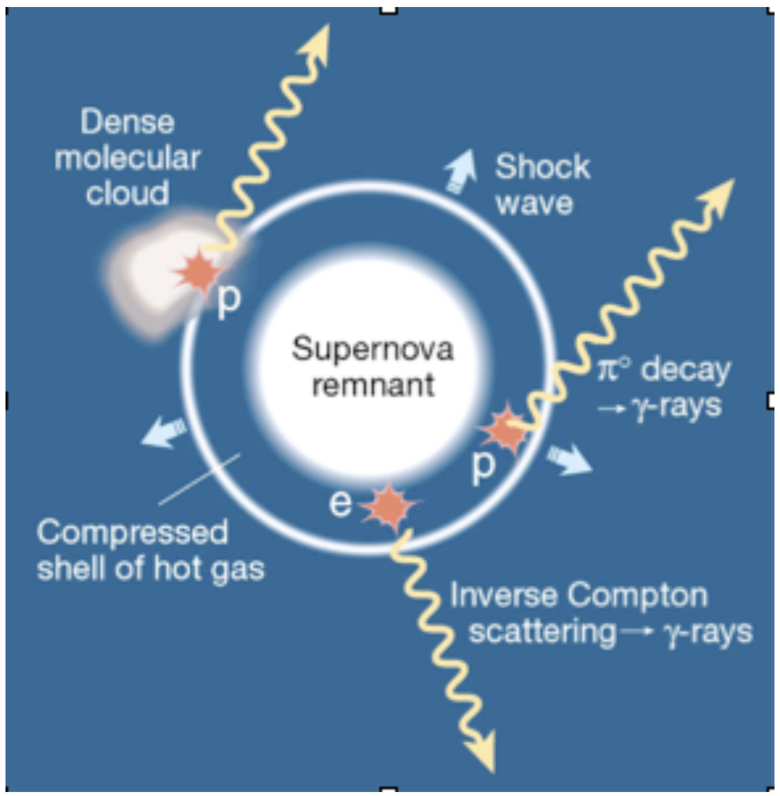


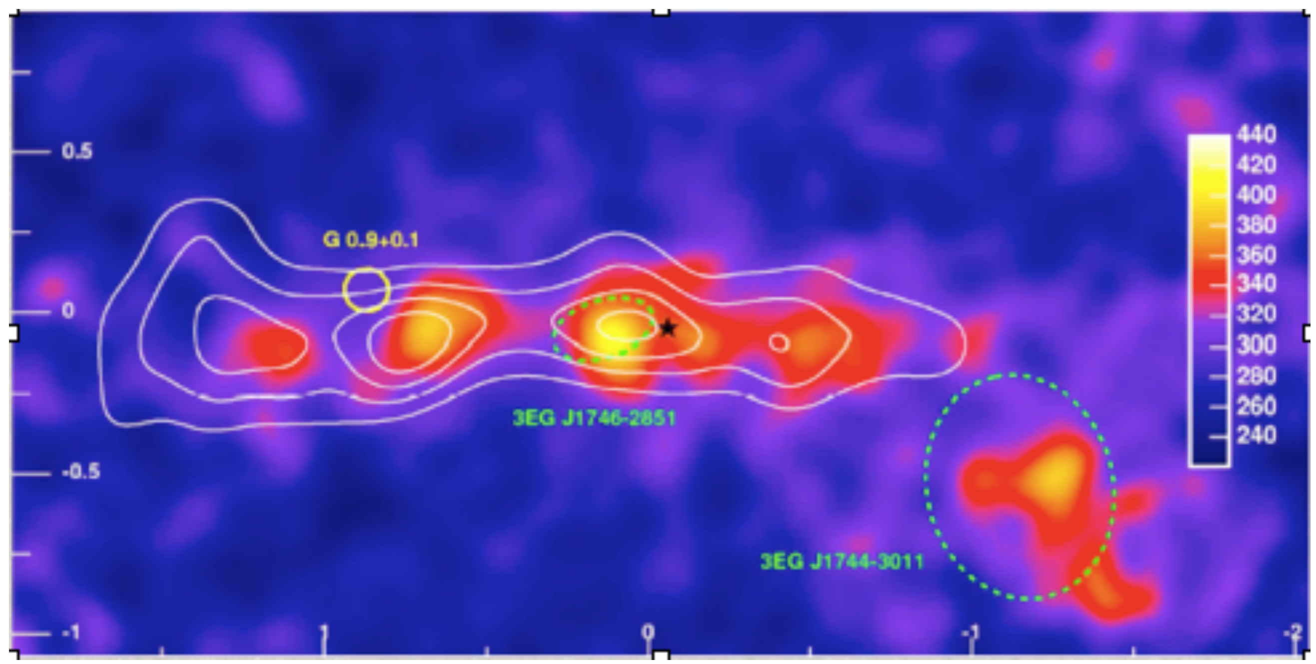


jet

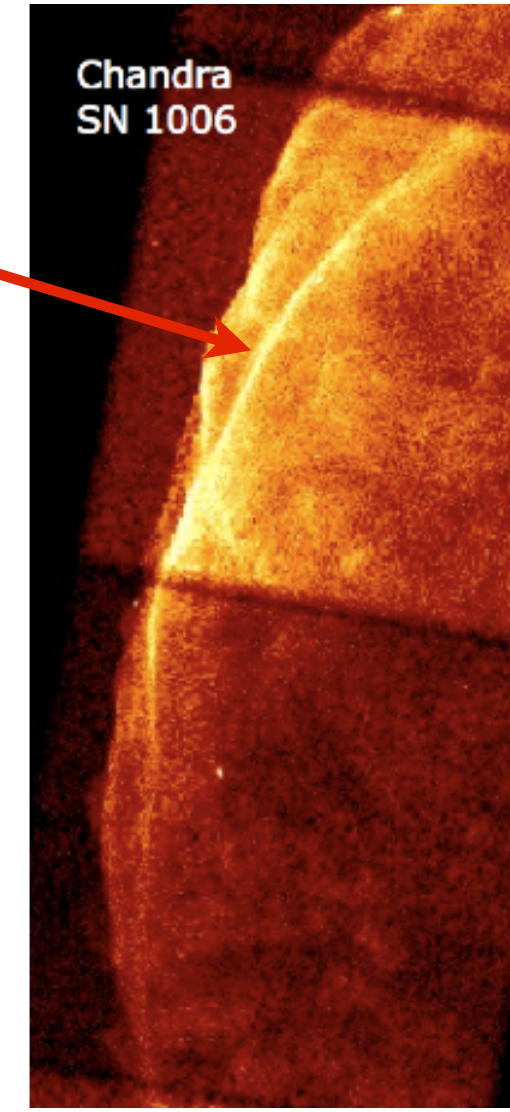
Torus + accretion disk (ionized gas + dust)

Supernova explosions



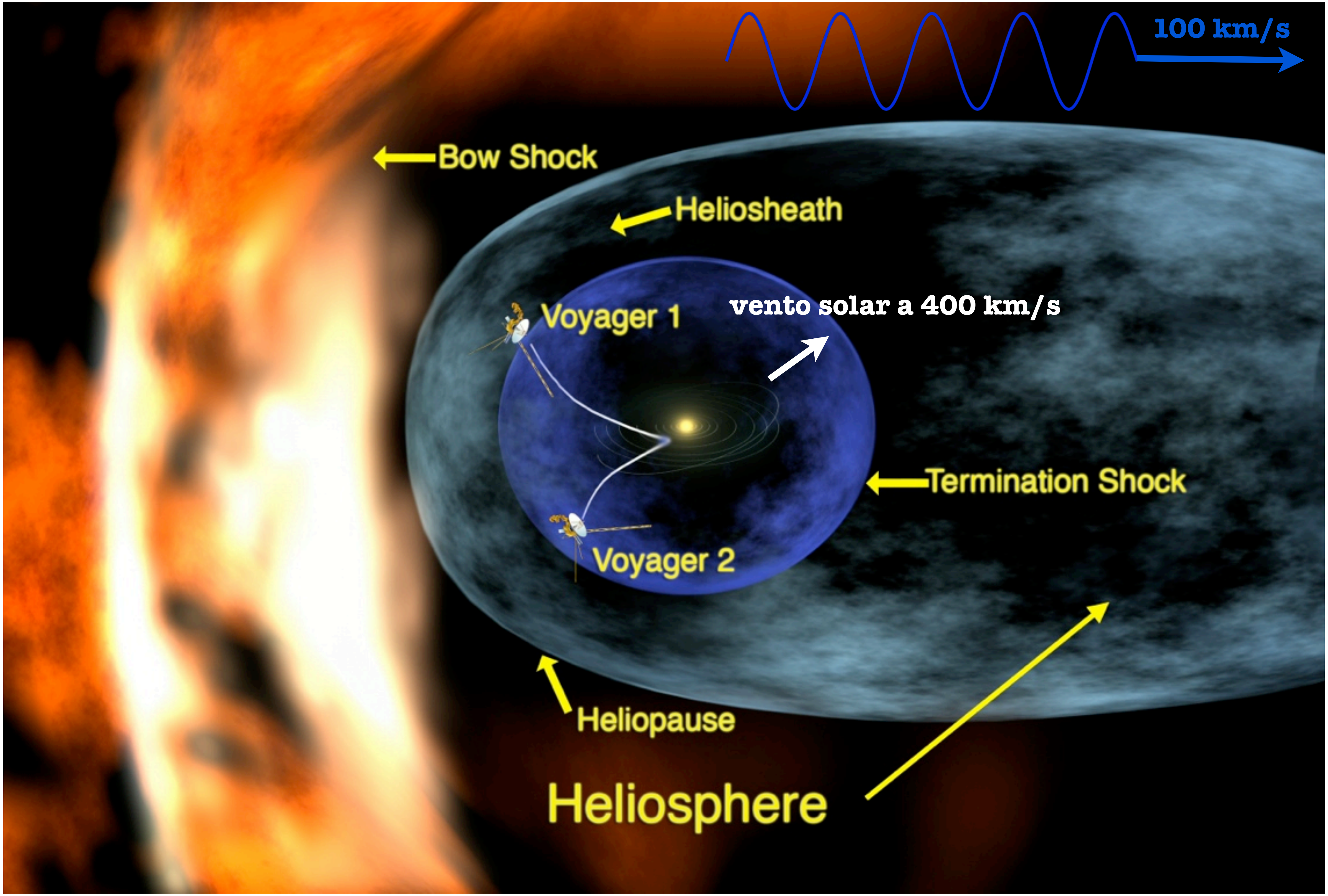


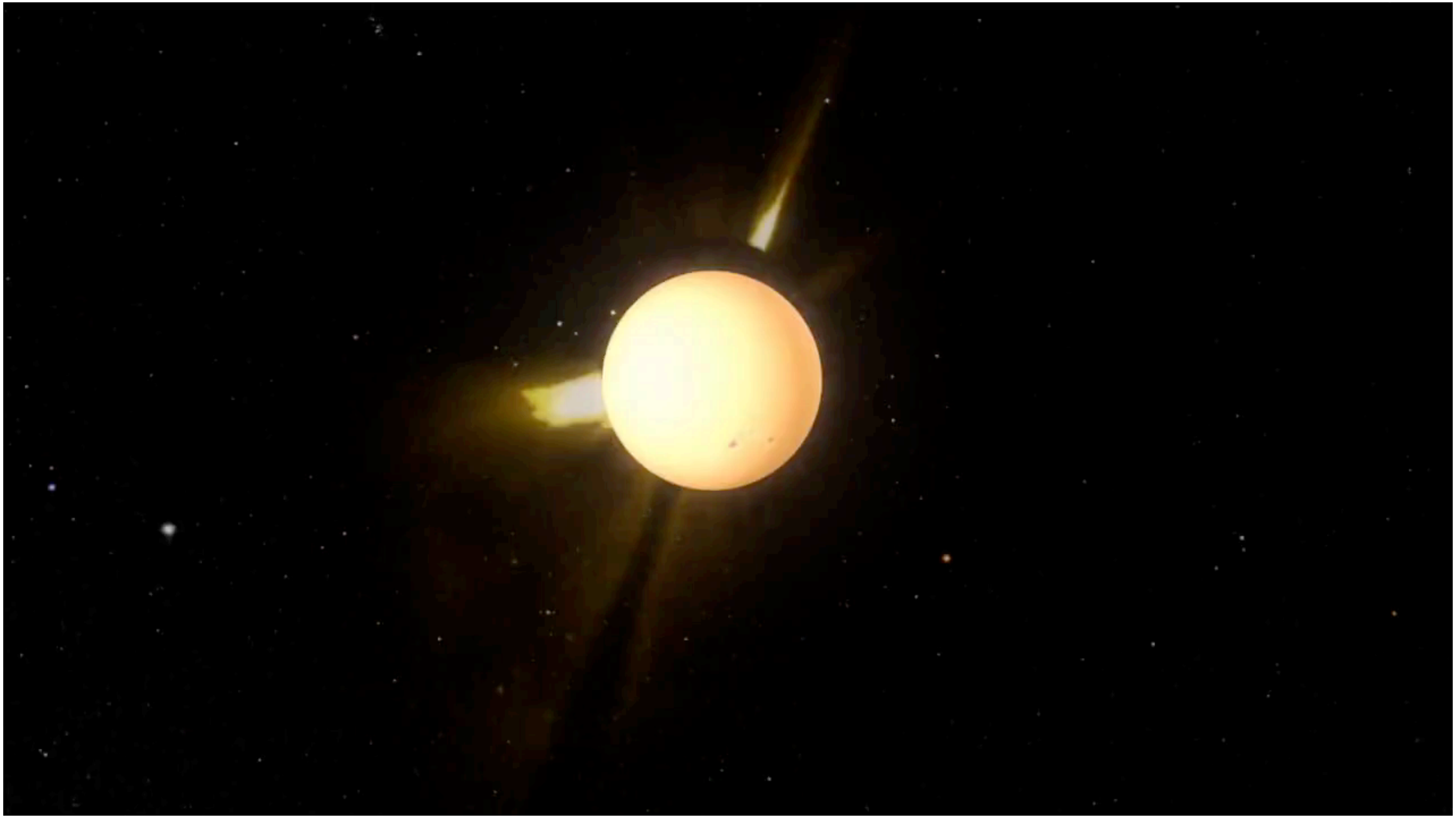
acceleration site



- We believe that shock waves generated by supernova explosions can accelerate protons up to ~10^{15} eV.

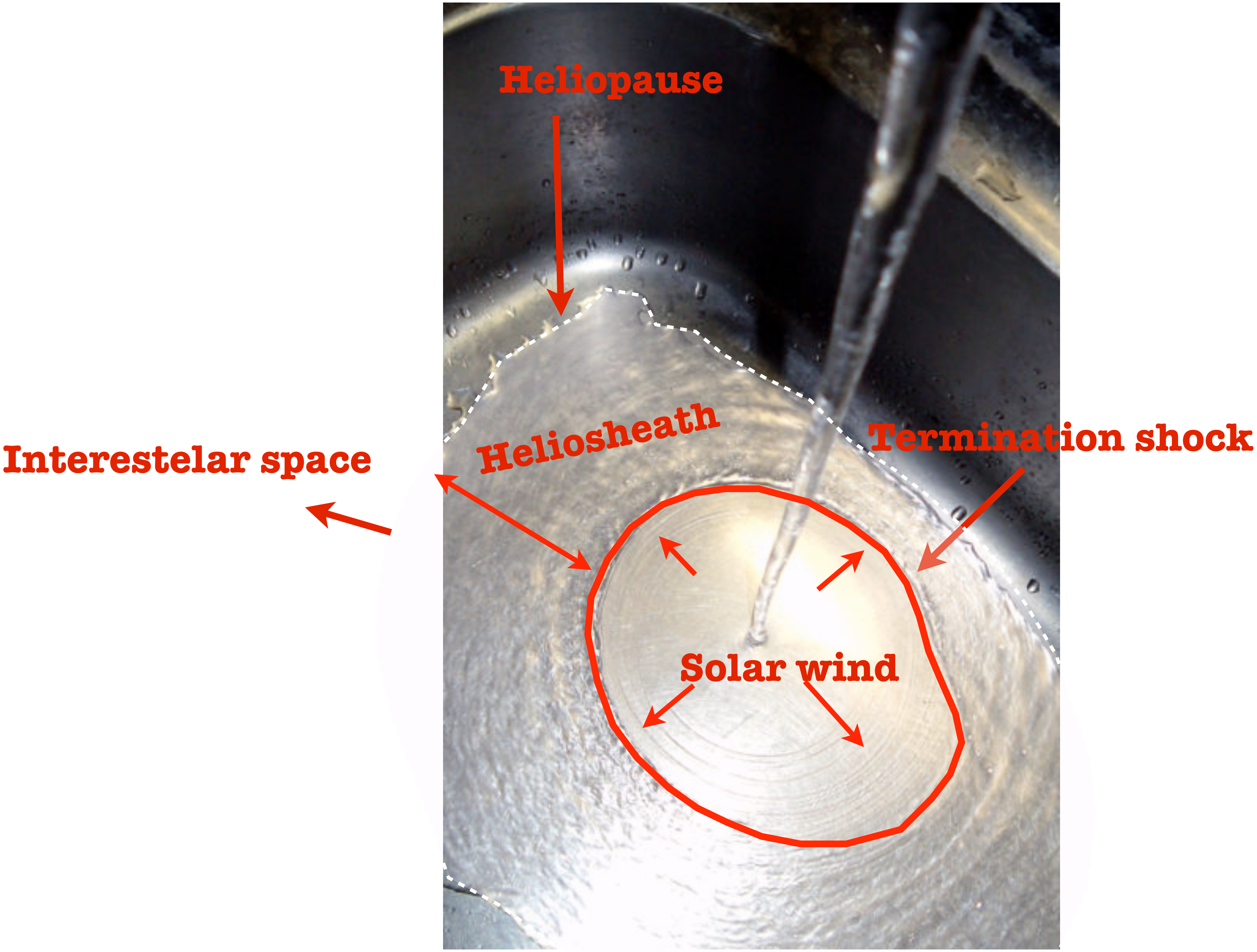
Heliosphere





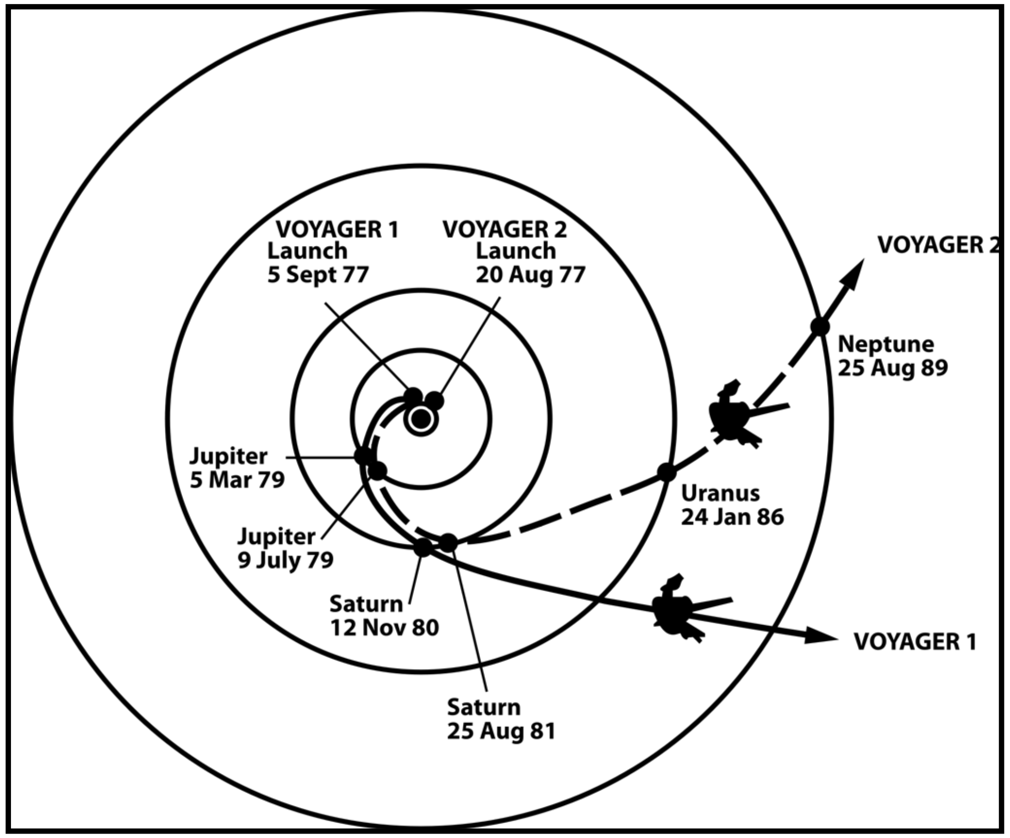
Homemade heliosphere

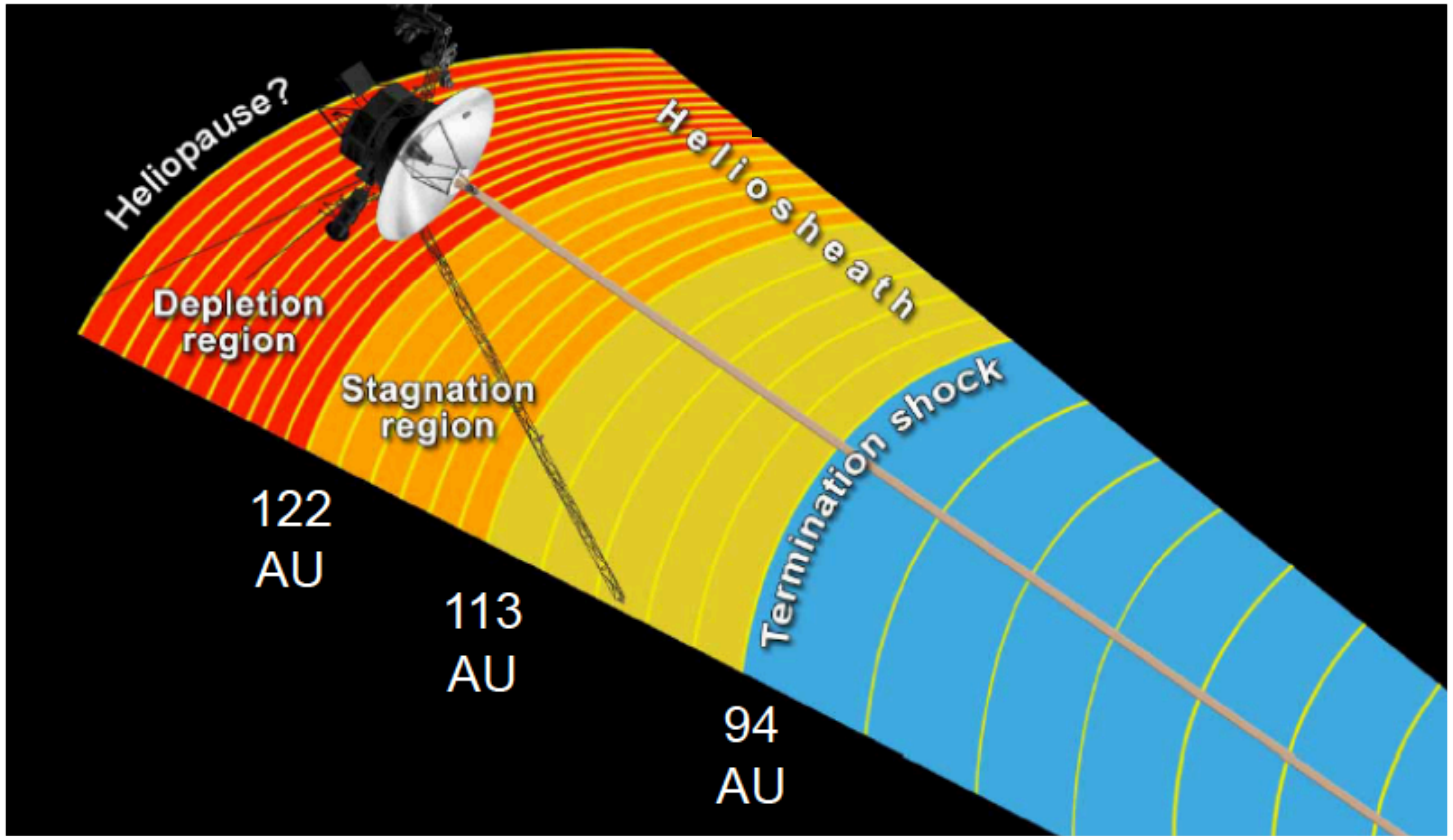
Homemade heliosphere



Voyagers 1 and 2

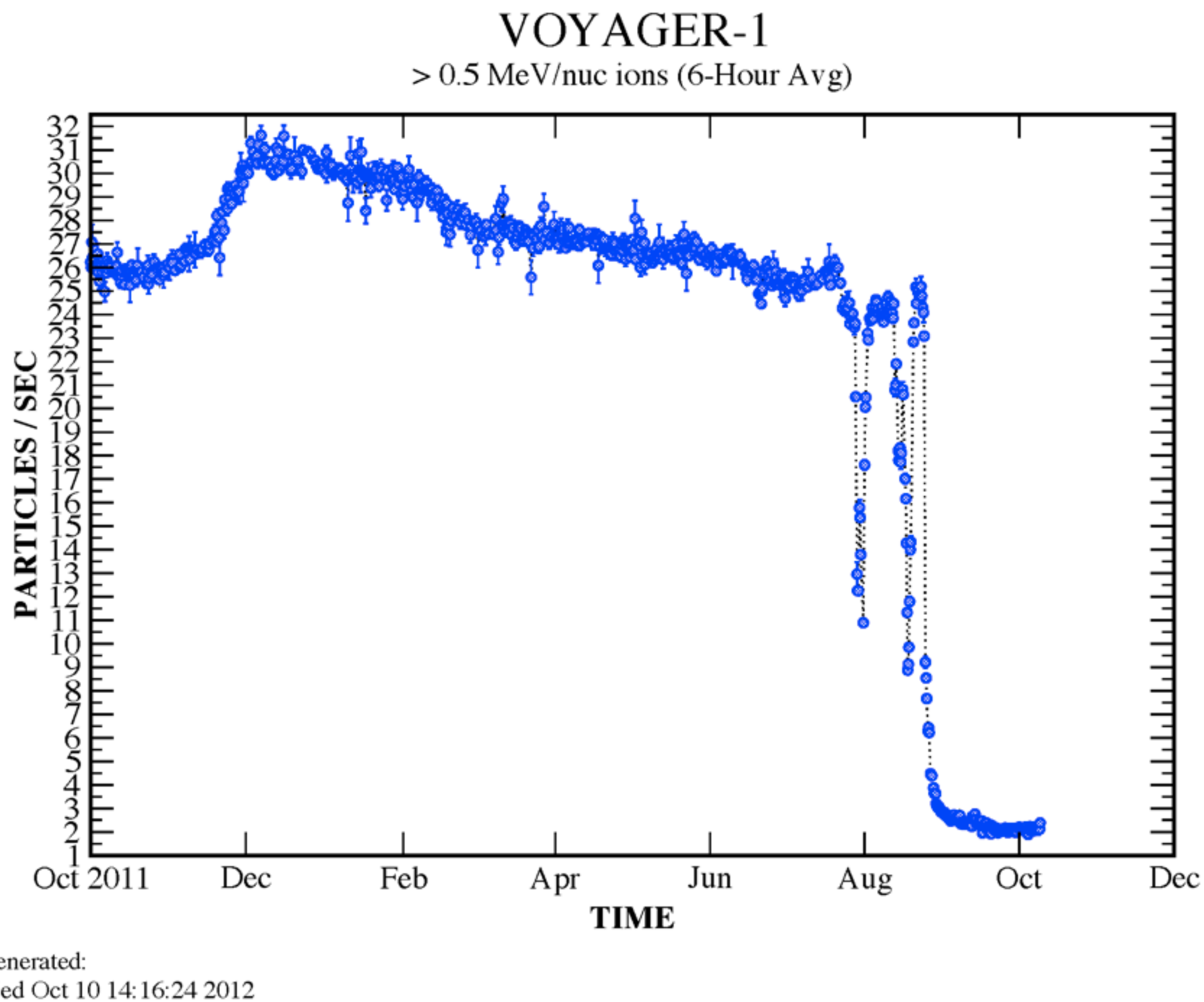
- Launched 1977 to study Jupiter and Saturn
- Voyager 1 is the human made instrument most distant from Earth





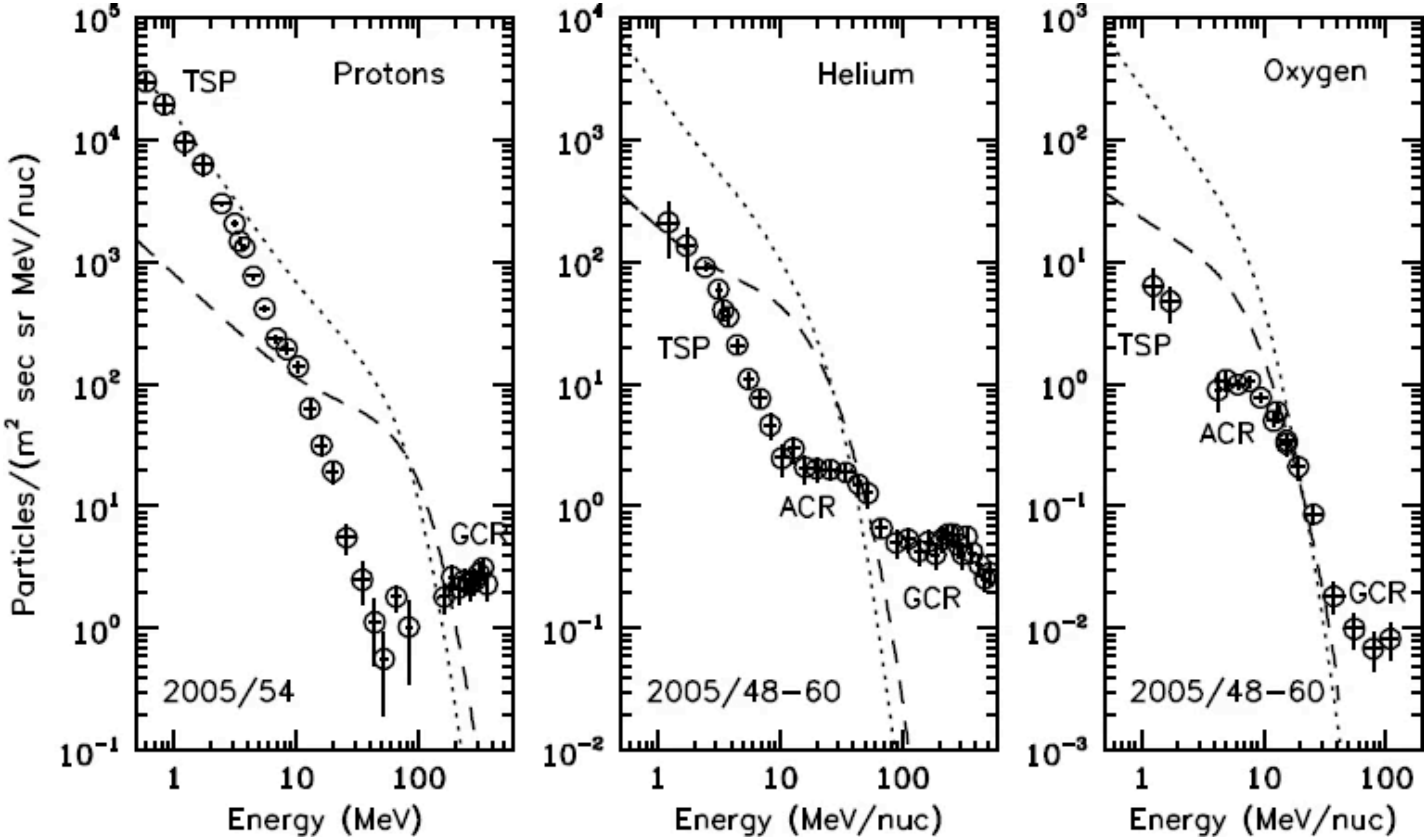
Voyager 1

124 AU = 18,6 bilhoes de km



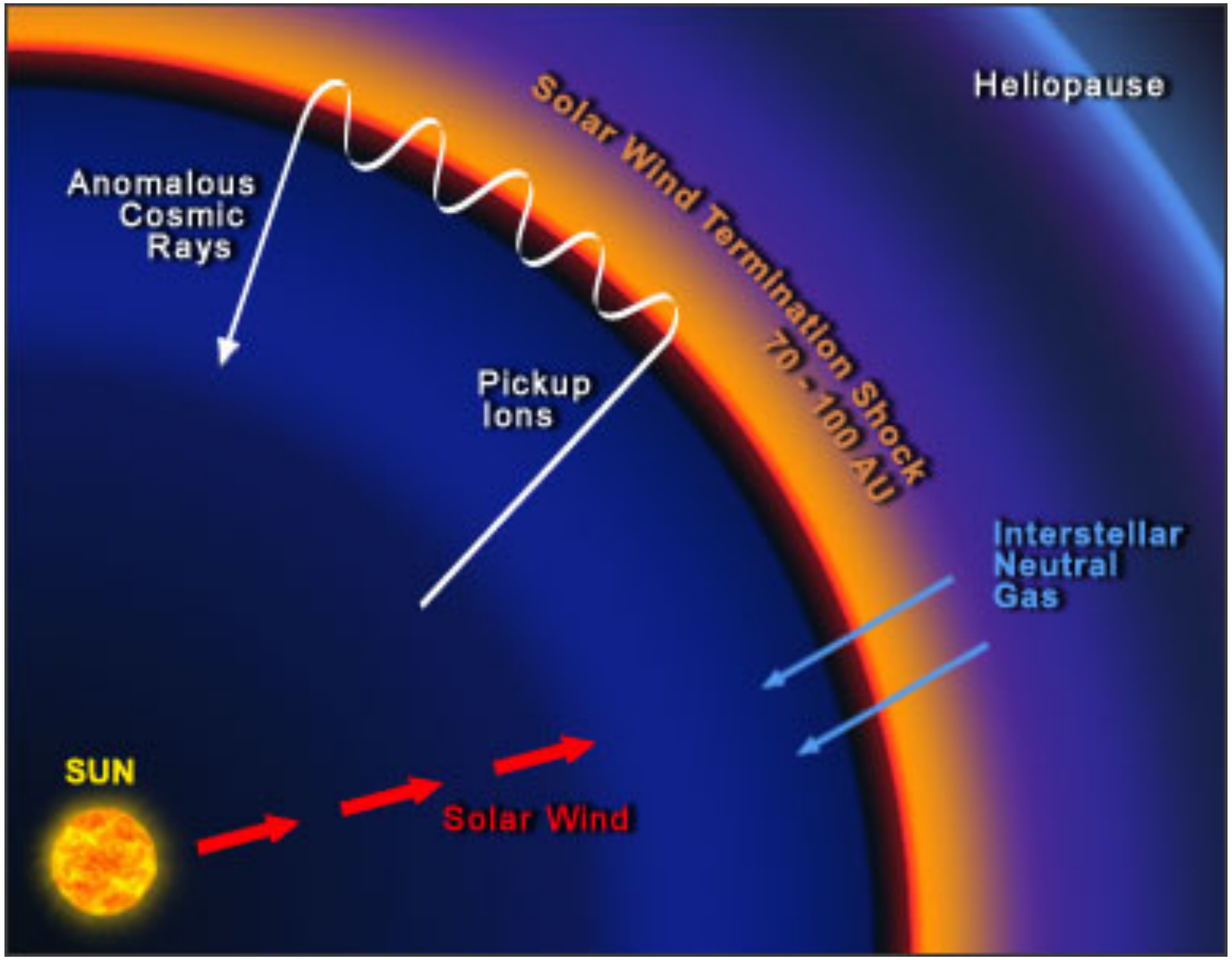
Anomalous cosmic rays

Voyager 1 data



Stone, E.C., Cummings, A.C., McDonald, F.B., Heikkila, B.C., Lal, N. and Webber, W.R., 2005, "Voyager 1 explores the termination shock region and the heliosheath beyond", *Science*, **309**, 2017–2020.

Anomalous cosmic rays



Fermi 1st order mechanism is believed to be at work here!