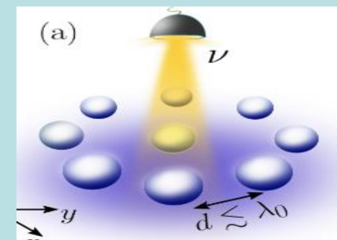
Innsbruck Physics Research Center

SAFIR School on Open Quantum Systems: theory and experiment

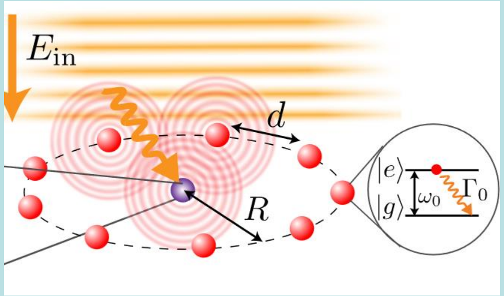


Coupled Quantum Emitter Dynamics

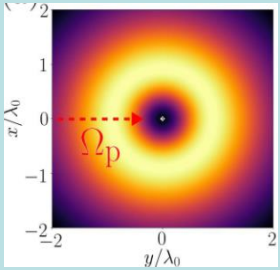
https://www.uibk.ac.at/th-physik/cqed/

Nano-optics with sub-wavelength size quantum emitter arrays

Single photon antennas

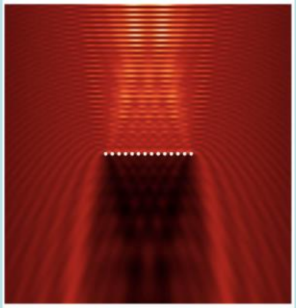


nano ring resonators

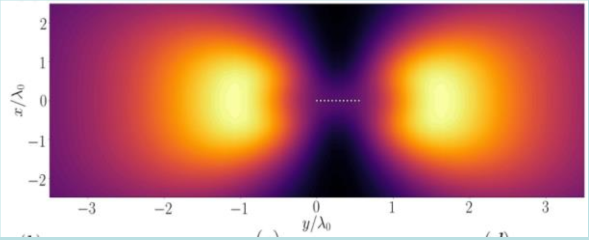


mirrorless
nano-lasers

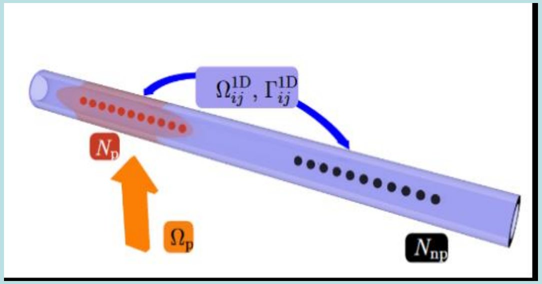
Single layer –
single photon mirrors



nano - waveguides

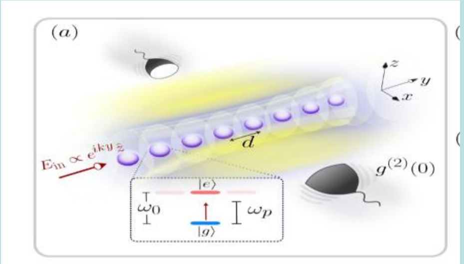


super-absorptive transport
(quantum batteries)

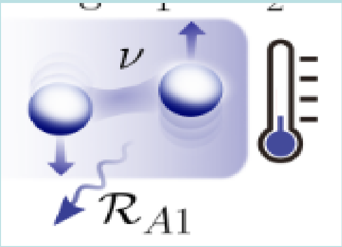


low loss quantum transport

non-classical light sources



dark state laser cooling

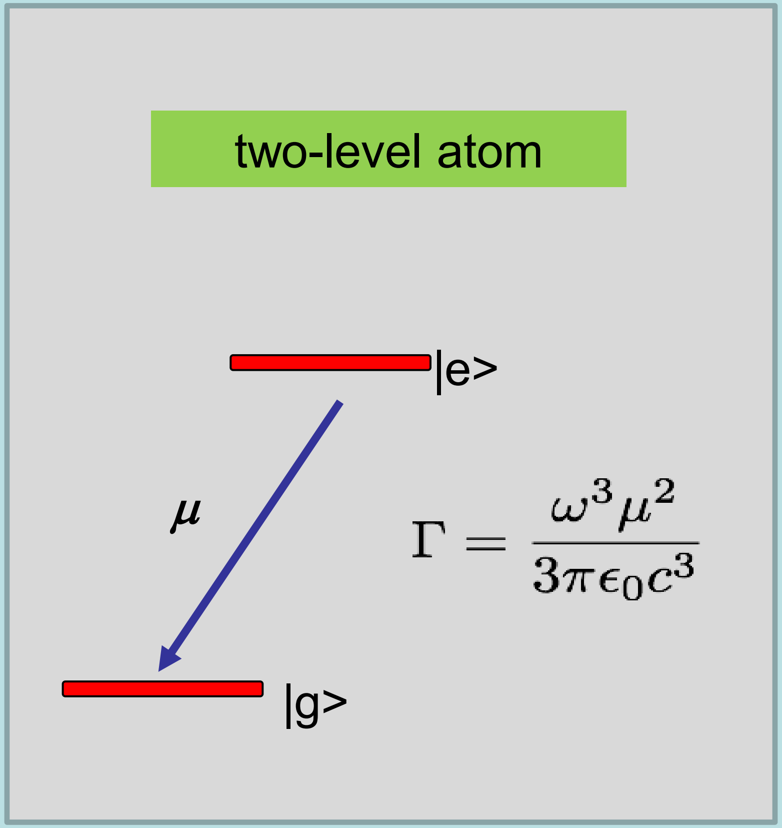


super- & sub-radiance
in
simple terms

single excited atom in free space

(„ Quantum Jump “)

spontaneous emission of a photon at rate Γ

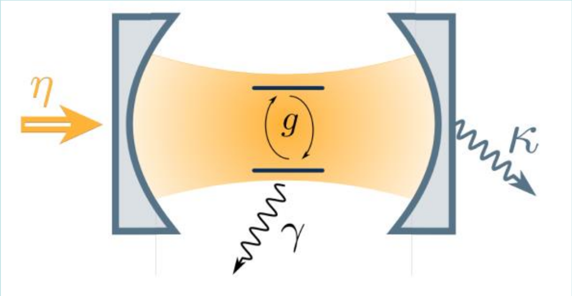


„stimulated emission“
through
quantum vacuum fluctuations
of the EM field

$$\Gamma = 2\pi \sum_{\vec{k},\lambda} \frac{\omega_k}{2\epsilon_0 V} \left| \vec{e}_{\vec{k},\lambda} \cdot \vec{\mu} \right|^2 \left(\cancel{n_{\vec{k},\lambda}} + 1 \right) \delta\left(\omega_i - \omega_f\right) = \frac{\omega_0^3 \mu^2}{8\pi^2 \epsilon_0 c^3} \frac{8\pi}{3} = \frac{\omega_0^3 \mu^2}{3\pi \epsilon_0 c^3}$$

single atom in an *empty* resonator: „Purcell“ effect:

modified decay in optical resonators



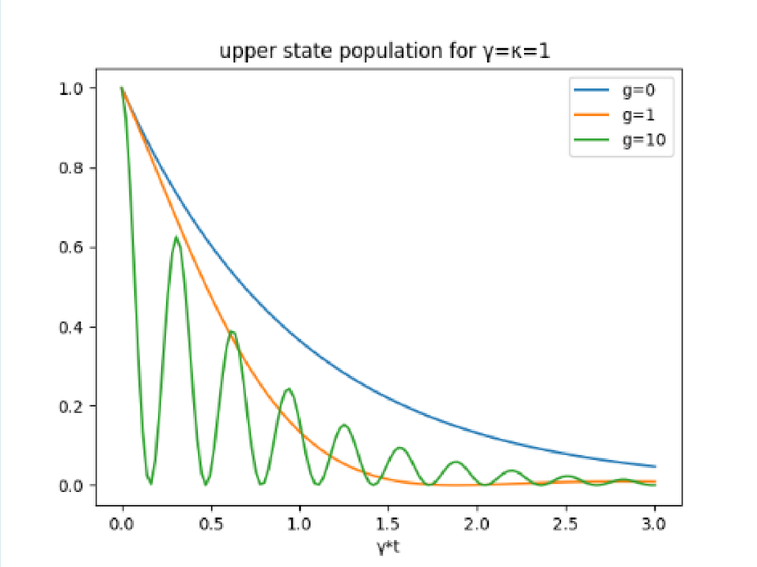
Extra: „stimulated emission“ due to locally enhanced vacuum fluctuations

Coupling strength **g** to a resonant mode

=>

Cooperativity **C = g^2 / (κ Γ)**

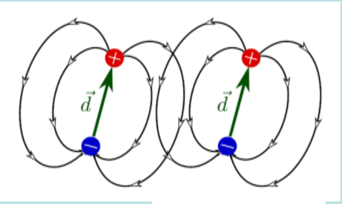
$$\Gamma \Rightarrow \Gamma * (1 + C)$$

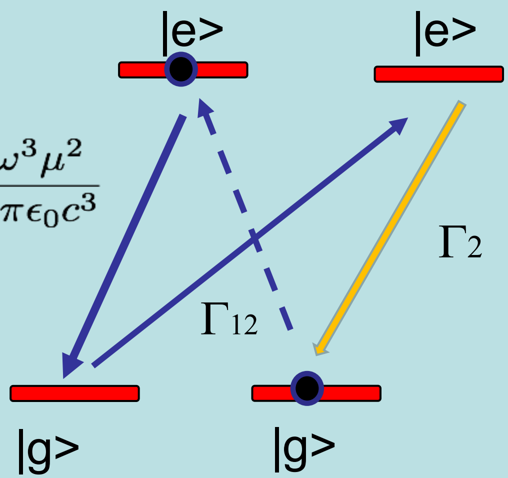


- **C** gives ratio of cavity emission **g^2/κ** versus free space emission **Γ**
- decay is limited by κ for very strong coupling

Emission in presence of second emitter: „atomic“ Purcell effect

two
two-level atoms

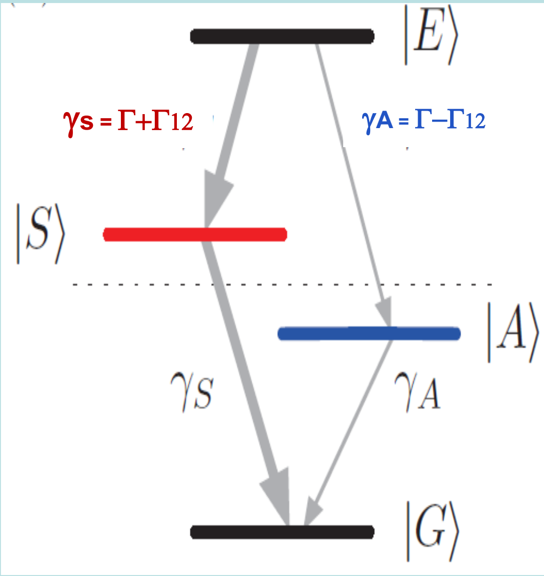




$$\Gamma = \frac{\omega^3 \mu^2}{3\pi\epsilon_0 c^3}$$

new eigenbasis:

$$|G\rangle = |gg\rangle$$
$$|S\rangle = \frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle)$$
$$|A\rangle = \frac{1}{\sqrt{2}}(|eg\rangle - |ge\rangle)$$
$$|E\rangle = |ee\rangle$$



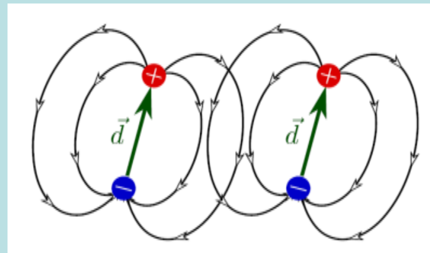
coupling strength: real + imaginary part:

$J_{12} + i * \Gamma_{12}$

$$\hat{H}_{\text{eff}} = \sum_{ij} \left(J_{ij} - i\frac{\Gamma_{ij}}{2} \right) \hat{\sigma}_i^{eg} \hat{\sigma}_j^{ge}.$$

- two 2-photon decay channels :
- super-/sub- radiant
- entangled intermediate states

field emission patterns
of
single excitation quantum superpositions



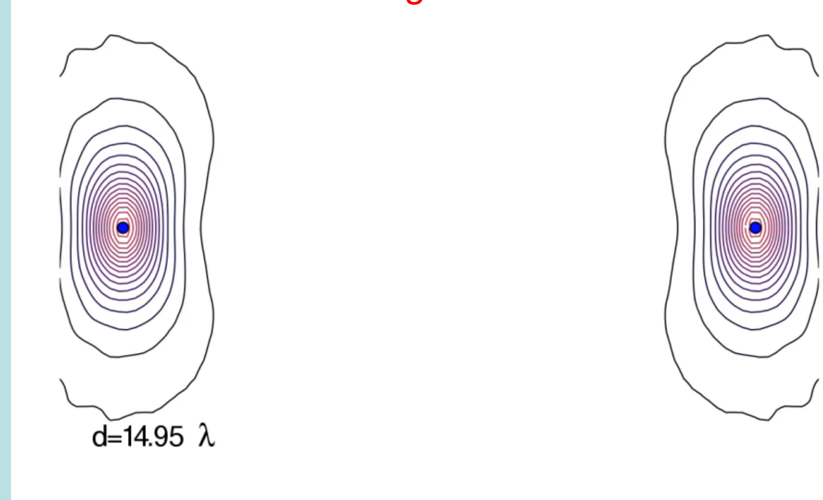
bright

|ge> **+** |eg>

dark

|ge> **-** |eg>

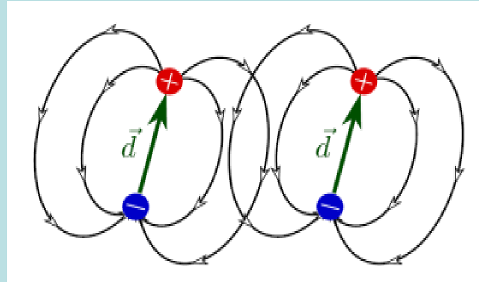
2 x - magnification



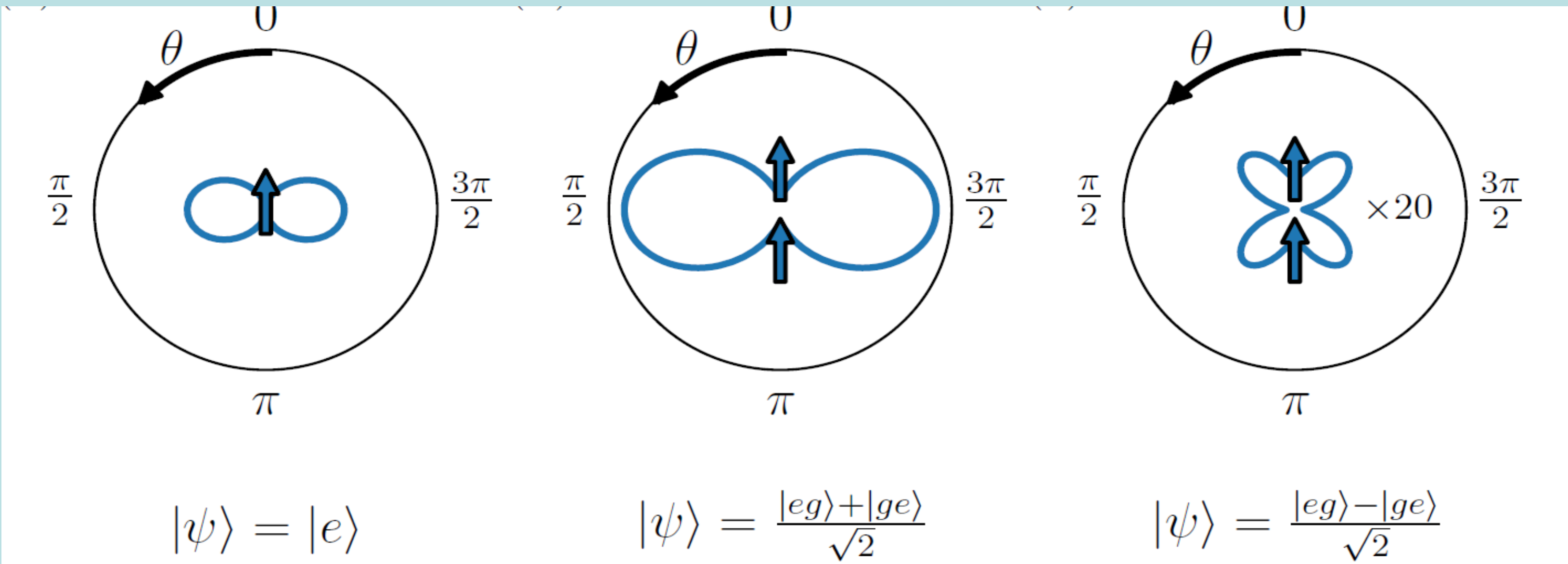
strong dipole => superradiance

weak quadropole => subradiance

D. Plankensteiner (thesis)



distribute single excitation quantum

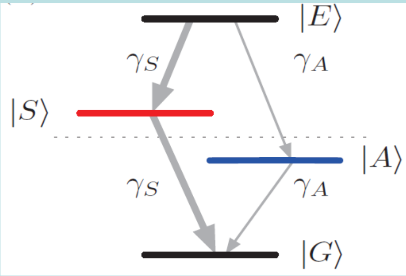


single dipole

strong dipole

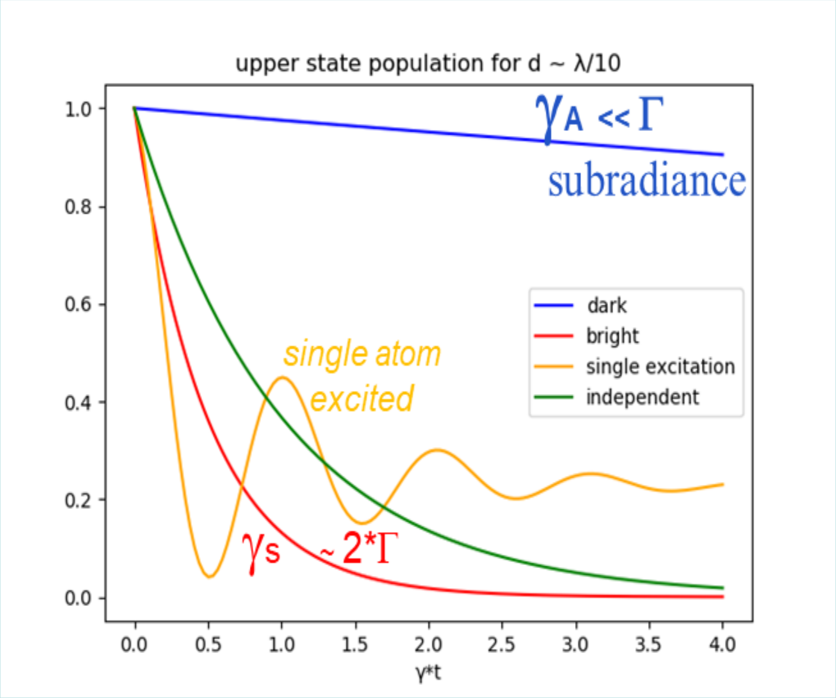
weak quadropole

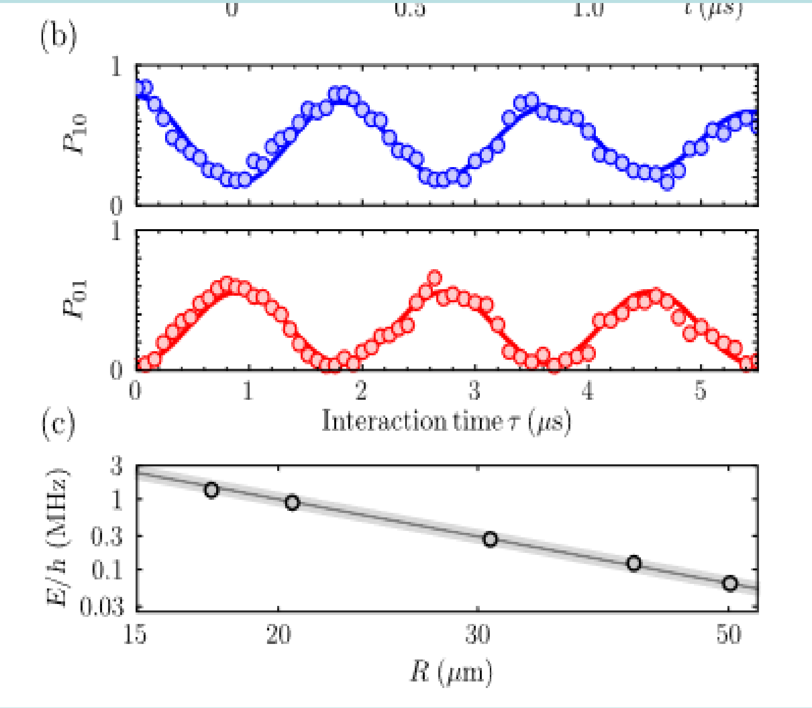
2-fold - magnification



Experiment:
Rydberg atoms
Barredo et.al., PRL 2017

Collective dynamics: tailored lifetime



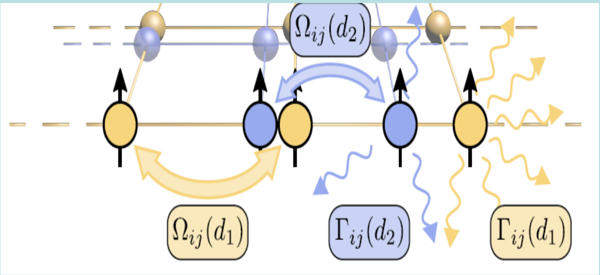


“Purcell”- or ”anti-Purcell” effect

Collective dynamics of N stationary quantum emitters

master equation for atomic excitations

array of two-level atoms
with
collective coupling to vacuum



Hamiltonian : **coupled coherent evolution:**

$$H = \sum_{i,j} -\omega_j^i \, \sigma_j^{i\,-} \sigma_j^{i\,+} + \sum_{i \neq k} \sum_j \Omega_j^{ik} \, \sigma_j^{i\,+} \sigma_j^{k\,-}$$

incoherent collective decay: *Liouvillian*

$$\mathcal{L}[\rho] = \frac{1}{2} \sum_{i,k,j} \Gamma_j^{ik} \left[2\sigma_j^{i\,-} \rho \, \sigma_j^{k\,+} - \sigma_j^{i\,+} \sigma_j^{k\,-} \rho - \rho \, \sigma_j^{i\,+} \sigma_j^{k\,-} \right]$$

real (Ω) & imaginary (Γ) part of pair interactions Green's-function

practical challenge:
Hilbert space grows as ~ 2^N

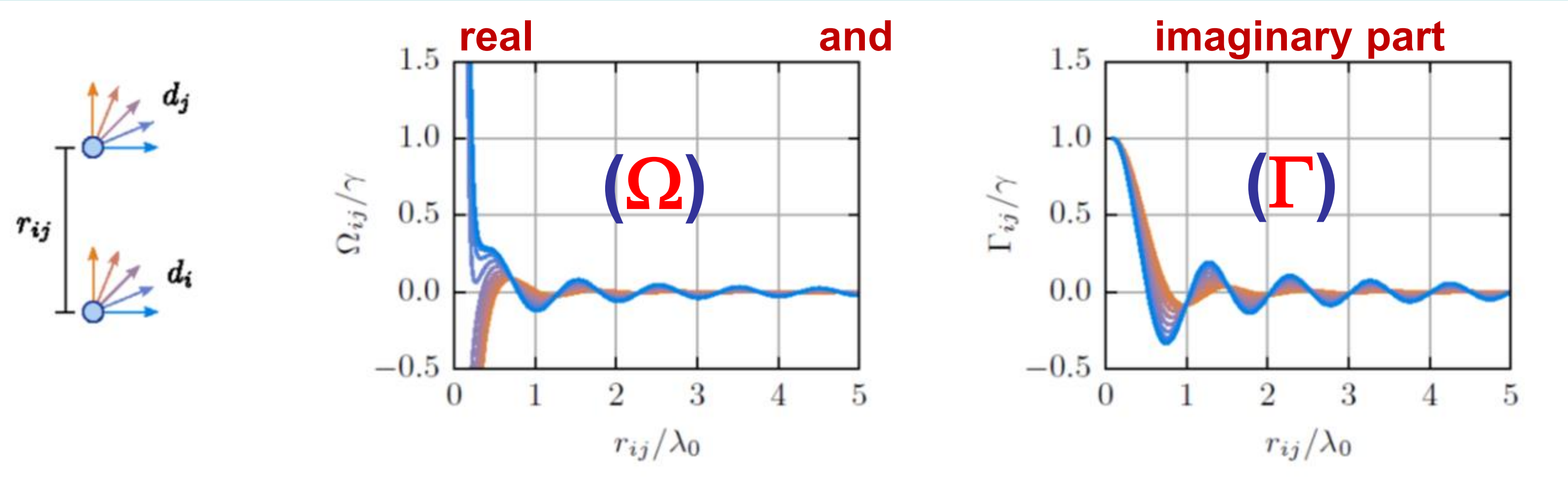
„classic" work on collective superradiance ~70's : Agarwal, Scully, Carmicheal, others

Collective interactions are pairwise and close range:

incoherent collective decay: master equation

$$H = \sum_{i,j} -\omega_j^i \, \sigma_j^{i\,-} \sigma_j^{i\,+} + \sum_{i \neq k} \sum_j \Omega_j^{ik} \, \sigma_j^{i\,+} \sigma_j^{k\,-}$$

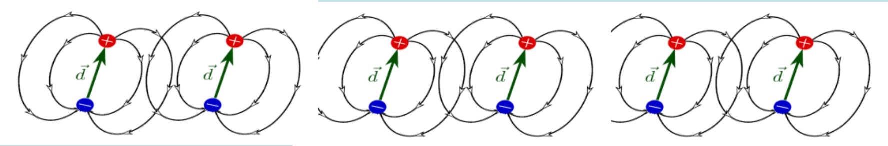
$$\mathcal{L}[\rho] = \frac{1}{2} \sum_{i,k,j} \Gamma_j^{ik} \left[2\sigma_j^{i\,-} \rho \, \sigma_j^{k\,+} - \sigma_j^{i\,+} \sigma_j^{k\,-} \rho - \rho \, \sigma_j^{i\,+} \sigma_j^{k\,-} \right]$$



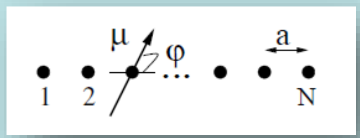
can attain positive and negative values

=> super- & subradiance

Collective decay of a 1D chain



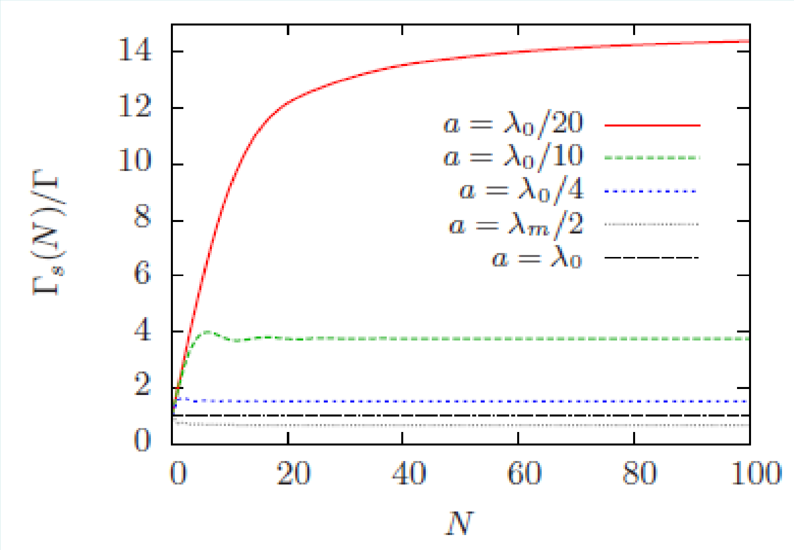
N-particle chain:

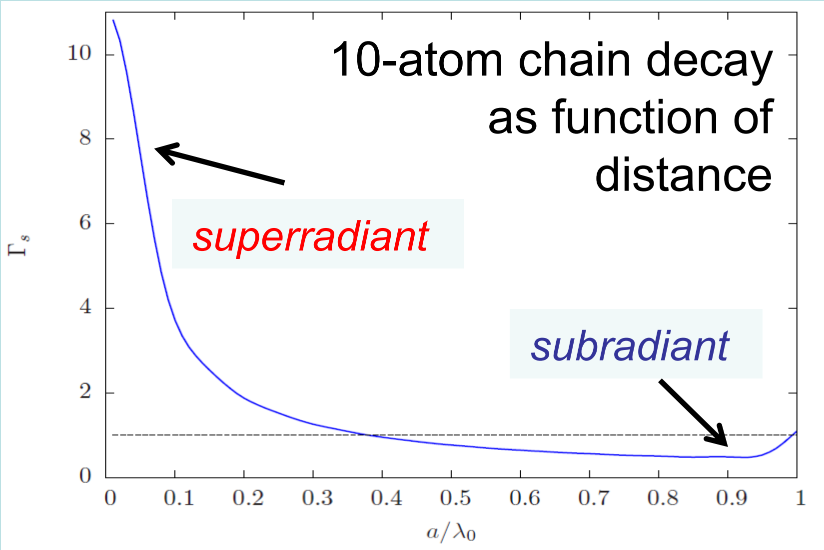


Symmetric excited state:

$$|i\rangle_s = \frac{1}{\sqrt{N}} \sum_i |g_1, \cdots, e_i, \cdots, g_N\rangle.$$

$$\Gamma_s(N) = \Gamma_A \left\{ 1 + 2 \sum_{n=1}^{N-1} \frac{(N-n)}{N} F(q_A a n) \right\}.$$





Super-radiance grows with number N

"Super"-radiance depends on distance

⇒ collective effects depend on state and geometry

⇒ analytic result: dense infinite chain supports perfectly dark excitons

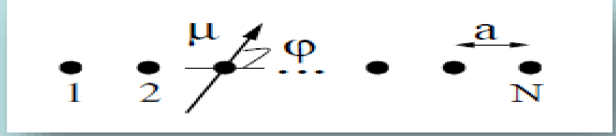
Advances in Atomic, Molecular, and Optical Physics

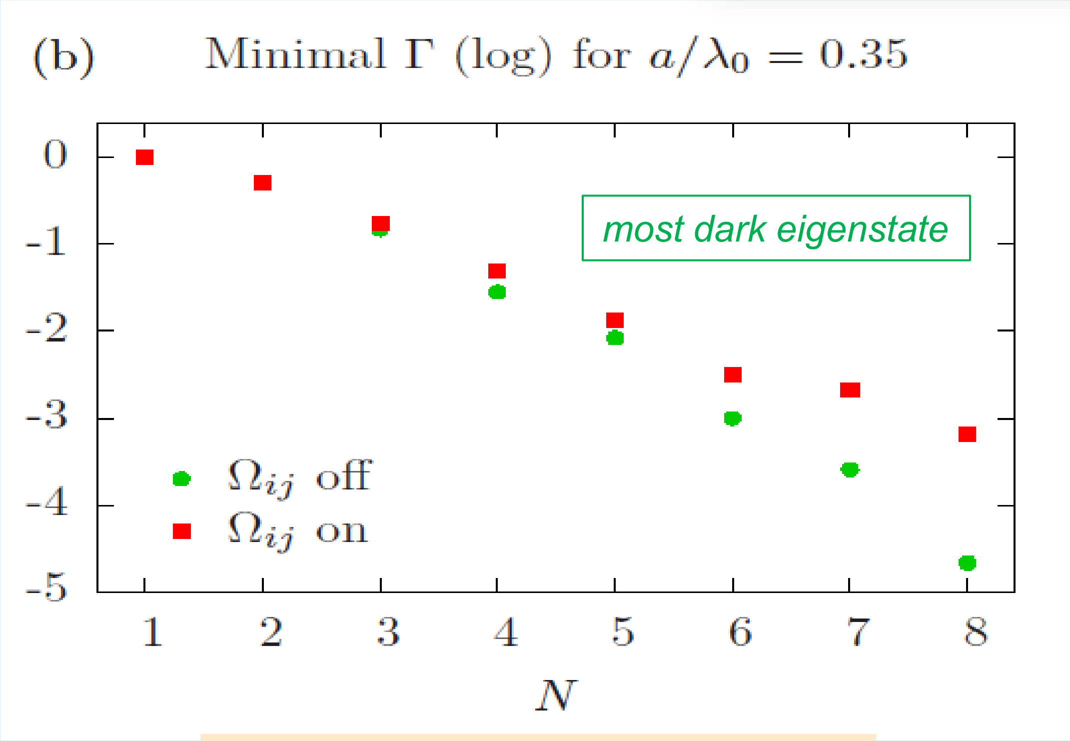
H. Zoubi, H. R., *Excitons and Cavity Polaritons for Optical Lattice Ultracold Atoms* (2013)

* Which is the most sub-radiant state of a finite chain ?

Liftetime growth of optimal dark state in a chain with atom number N

a < λ/2



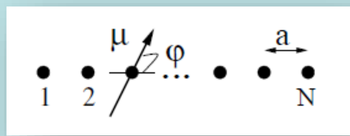


most dark eigenstate decay:
Γ ~ 1/N^3

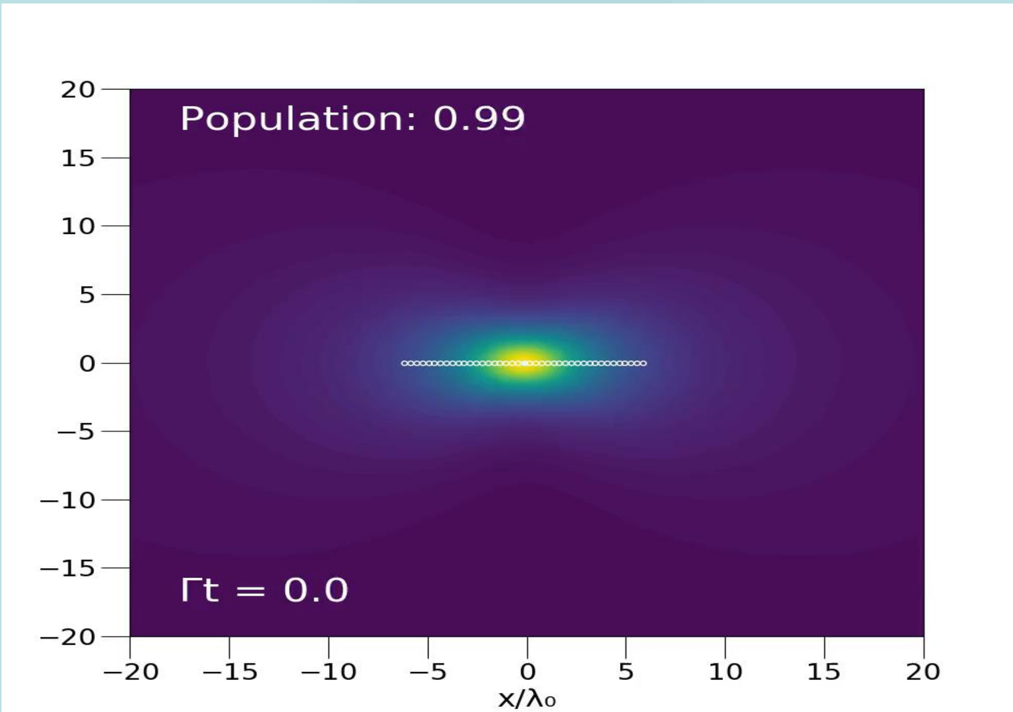
D. Plankensteiner, et. al. , *Scientific reports 5 (2015)*

Collective decay of a 1D chain

N-particle chain:



$$\hat{\mathbf{E}}_{\text{out}}(\mathbf{r}) = \hat{\mathbf{E}}_{\text{in}}(\mathbf{r}) + \mu_0 \omega_{eg}^2 \wp \sum_i \mathbf{G}(\mathbf{r}, \mathbf{r}_i, \omega_{eg}) \hat{\sigma}_i^{ge}$$



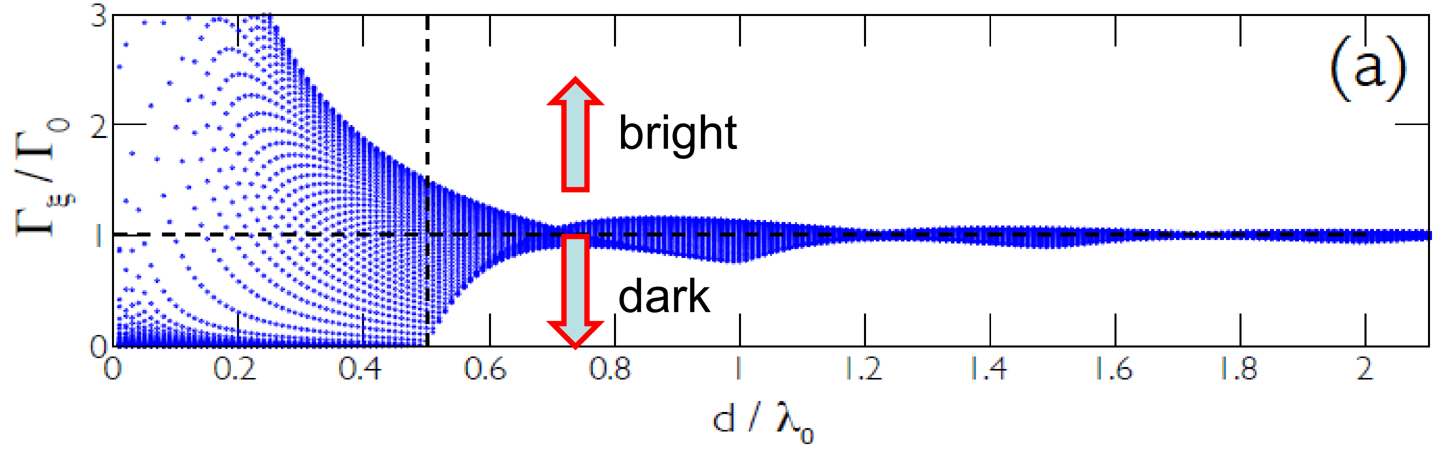
N^3 scaling
of
most dark state lifetime
limited
by end emission

⇒ Single atom chain is a waveguide with end emission only
⇒ Sub-radiant states allow high fidelity exciton transport

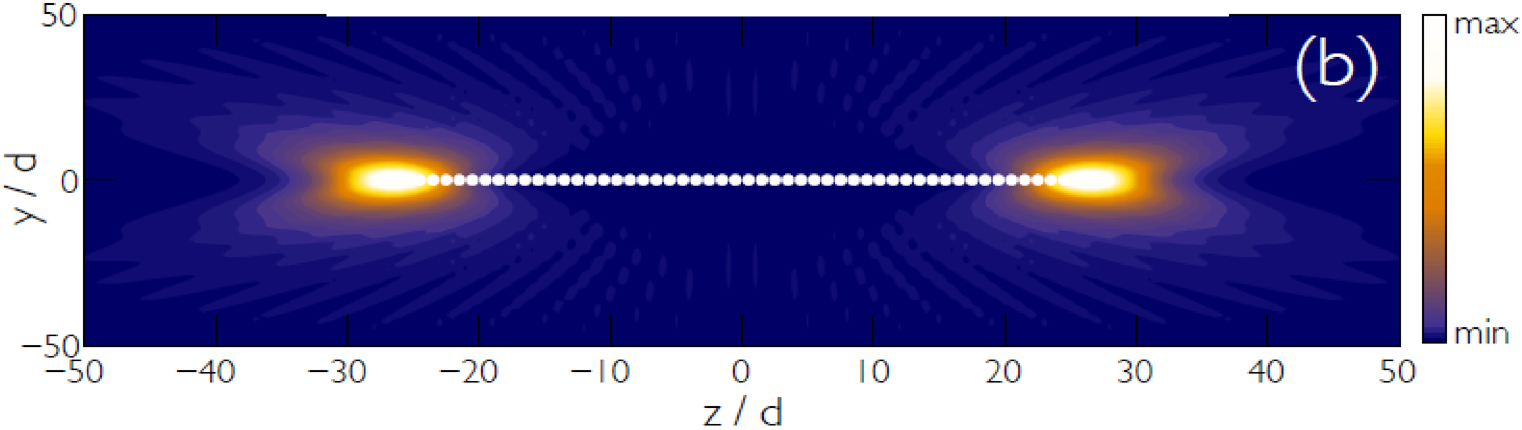
H. Zoubi, H.R. , **Advances in Atomic, Molecular, and Optical Physics, 2013**
A. Asenjo-Garcia et,al. , **Phys. Rev. X 7, 031024 (2017)**

Decay and field for a long chain:

Mode decay rates



spatial field distribution of „most dark mode"

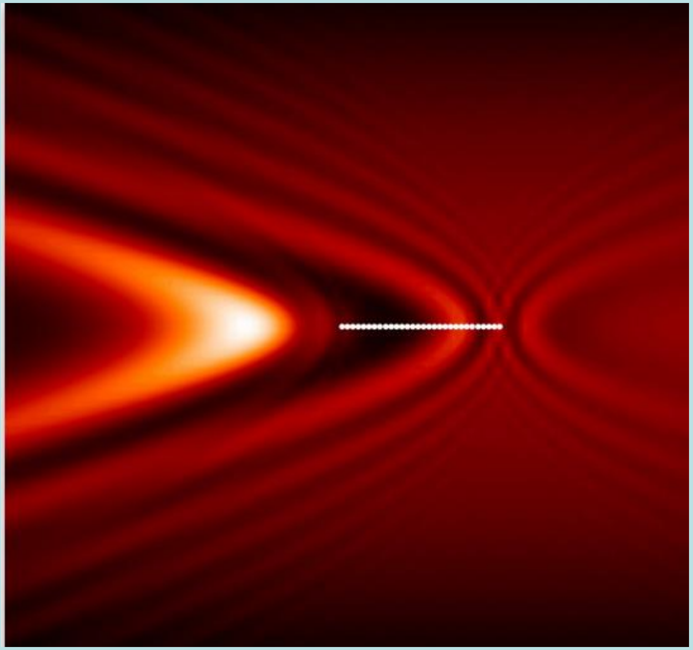


emission only at ends

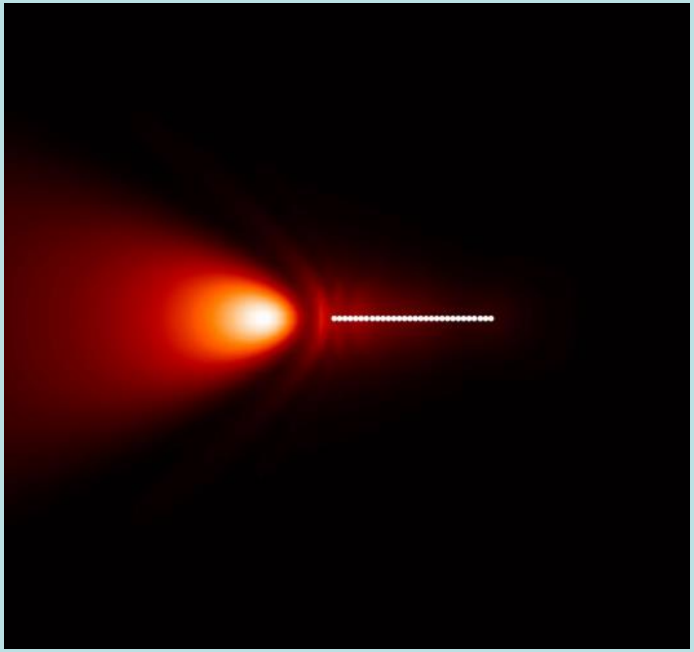
A. Asenjo-Garcia, M. Moreno-Cardoner, A. Albrecht, H. J. Kimble, and D. E. Chang, Phys. Rev. X 7, 031024 (2017) -

1D – dense string: minimalistic nonlinear reflector for Gaussian beam

total field



reflected / scattered field



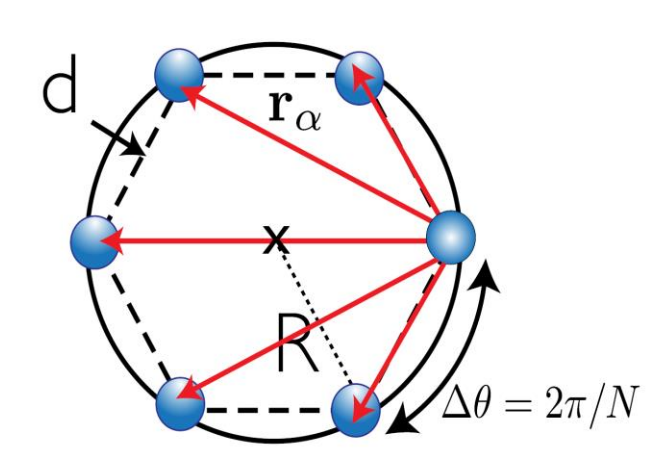
in-coming single photon pulse is almost perfectly reflected

(=> ideal basis for Cavity QED system)

What is the optimal sub-radiant geometry of a finite ensemble ?

1D chain without ends
=> Ring

$$\hat{H}_{\text{eff}} = \sum_{ij} \left(J_{ij} - i\frac{\Gamma_{ij}}{2}\right) \hat{\sigma}_i^{eg} \hat{\sigma}_j^{ge}.$$

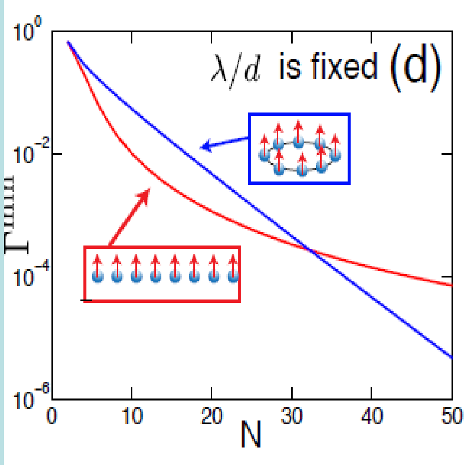


collective eigenstates :

$$|\psi_m\rangle = N^{-1/2} \sum_{j=1}^{N} e^{im\theta_j} |e_j\rangle$$

complex eigenvalues:

$$\lambda_m = -\frac{3\pi\Gamma_0}{k_0} \sum_{j\ell} e^{im(\theta_\ell - \theta_j)} G_{j\ell}.$$



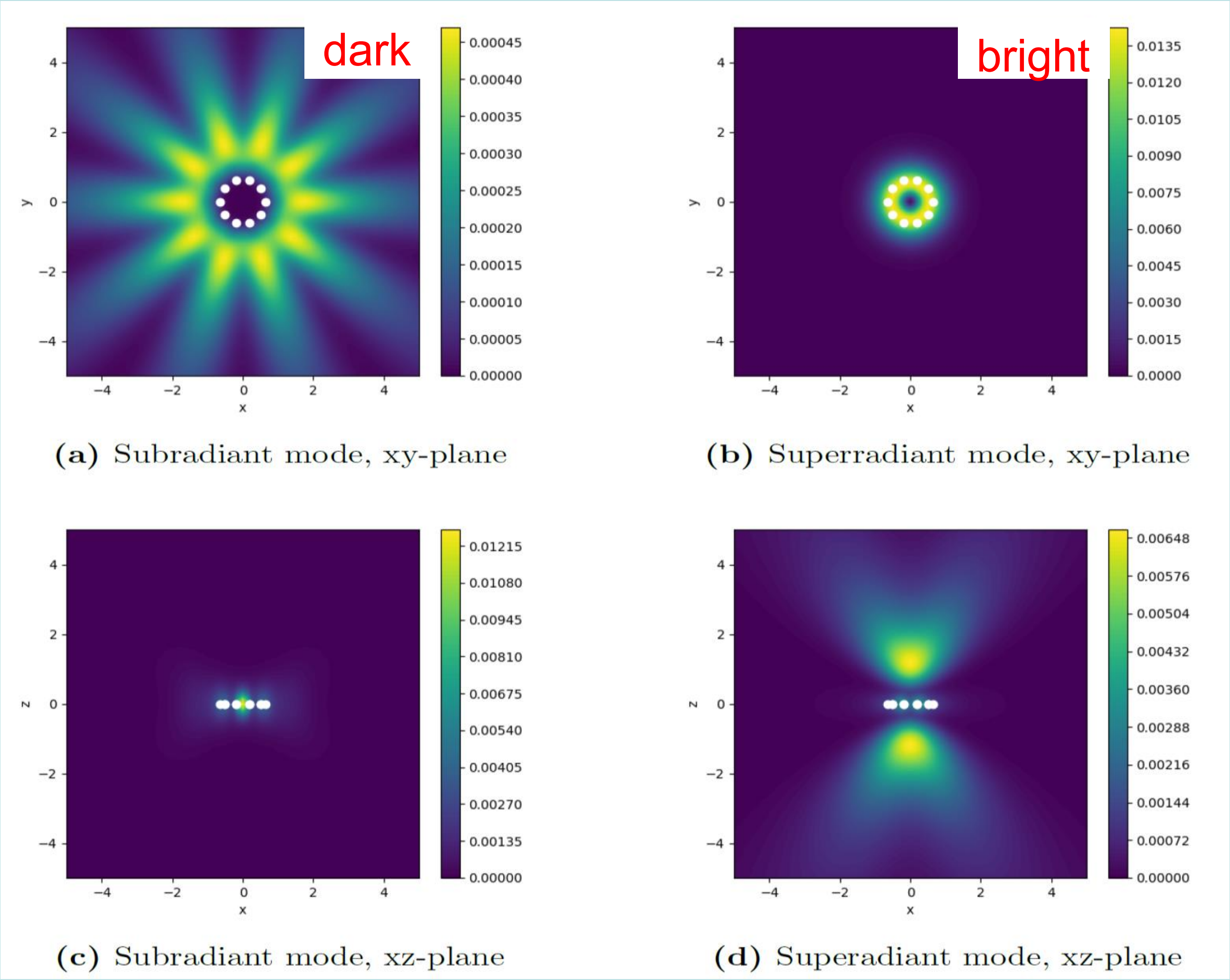
Lifetime of darkest eigenstate increases exponentially with N

=> sub-wavelength ring is a high Q nano optical ring resonator !

M. Moreono-Cardoner et.al, PRA 2019

field distributions of collective states in dipole rings

sub - λ confined excitation modes



(a) Subradiant mode, xy-plane

(b) Superradiant mode, xy-plane

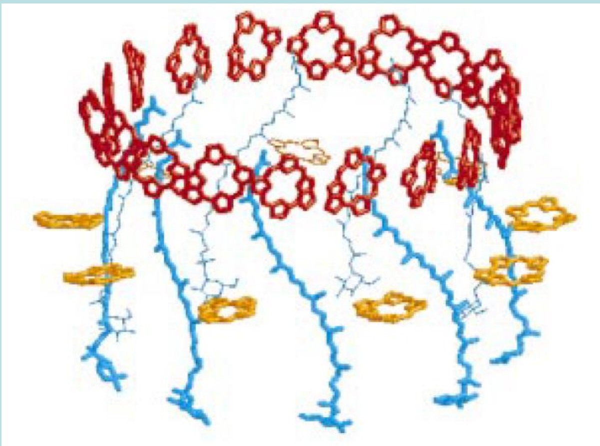
(c) Subradiant mode, xz-plane

(d) Superadiant mode, xz-plane

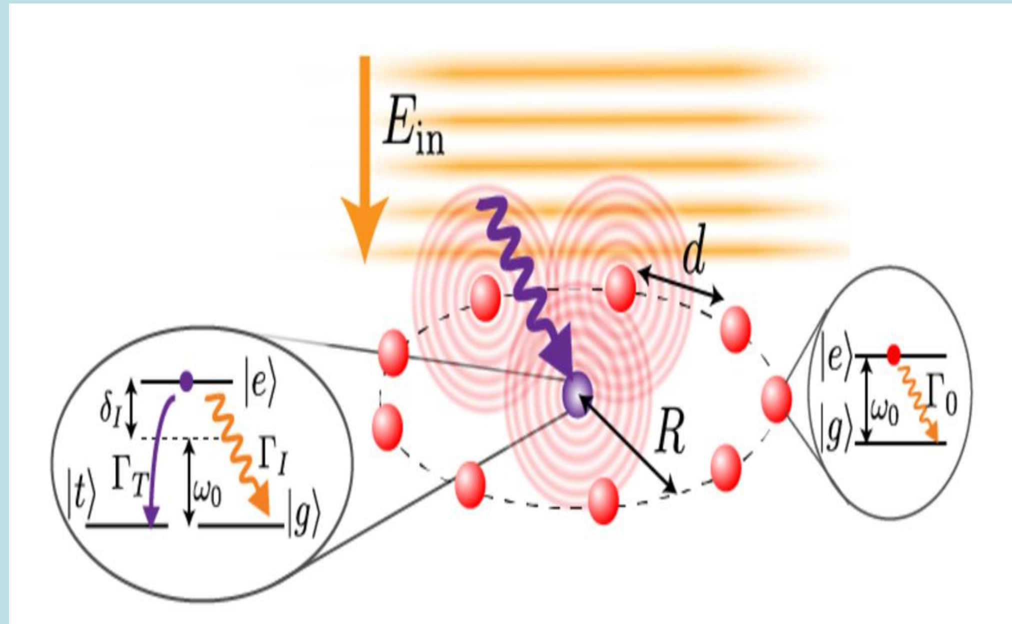
dark excitations in nanorings => sub-λ localized high-Q resonators

Bio-complex
as inspiration for a
effective nano-antenna for single photons

d << λ



LH2
light harvesting antenna
=
deep sub-λ dipole structure with ring symmetry

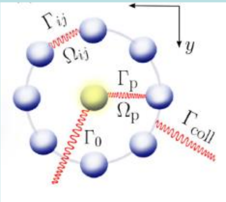


study atom ring antenna
with
absorbing center dipole

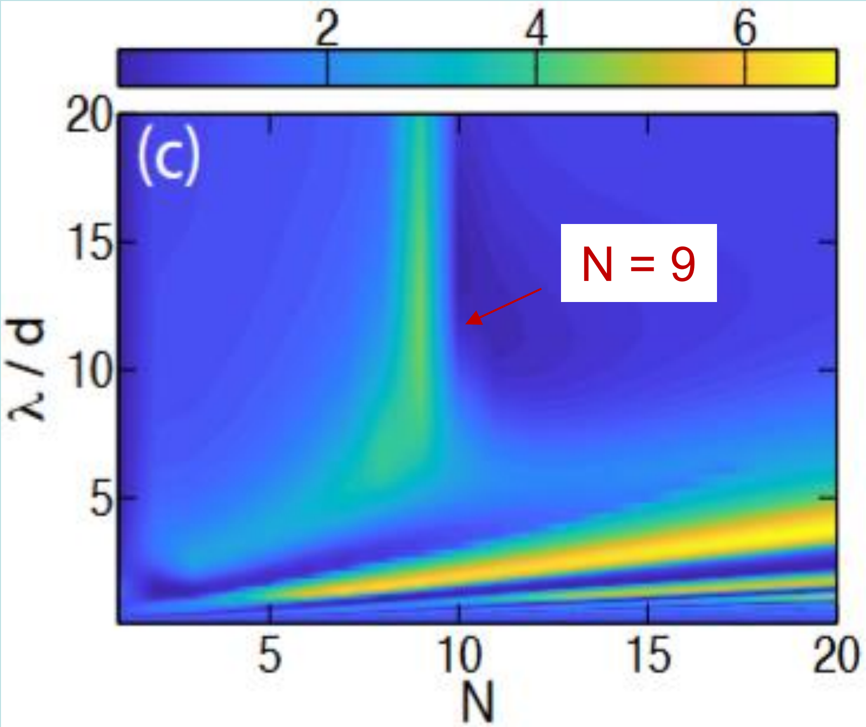
How much energy from an incoming field ends at center absorber state |g> ?

*T. Polivka, Chem. Rev.*20041044 2021-2072 , 18, 2004 https://doi.org/10.1021/cr020674n

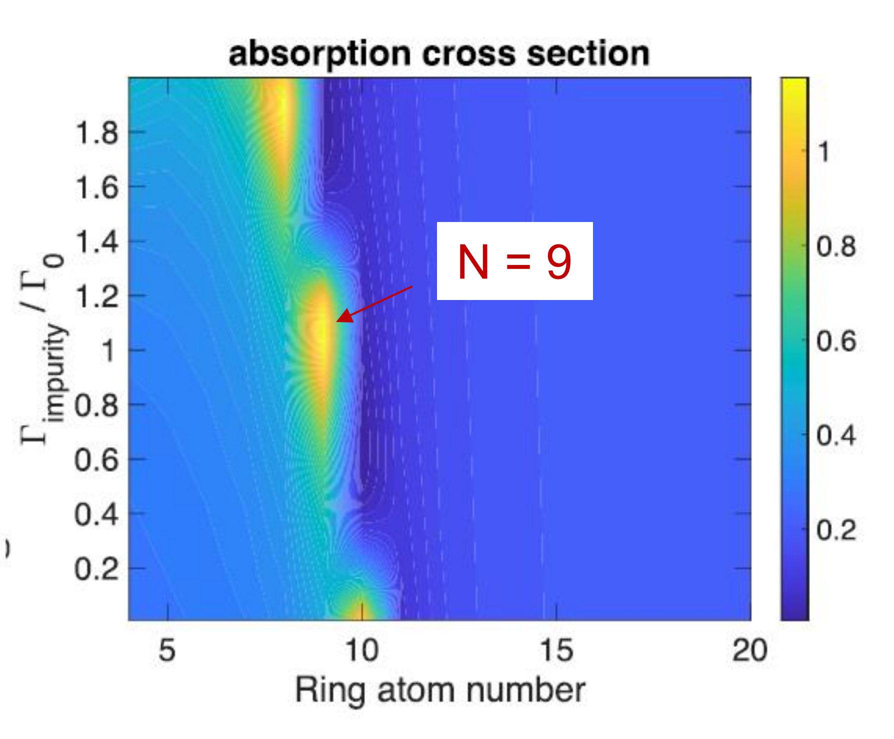
absorption efficiency of a ring antenna



ring size + dipole number



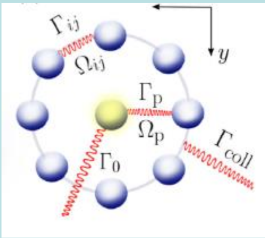
acceptor strength + dipole number



a symmetric **9 dipoles** ring offers largest antenna efficiency

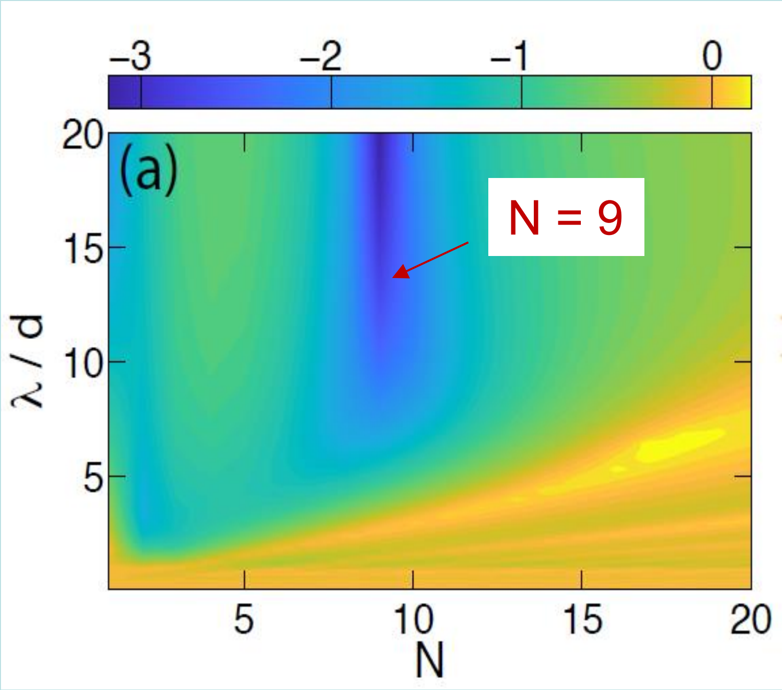
M. Cardoner,
R. Holzinger

why 9 ?

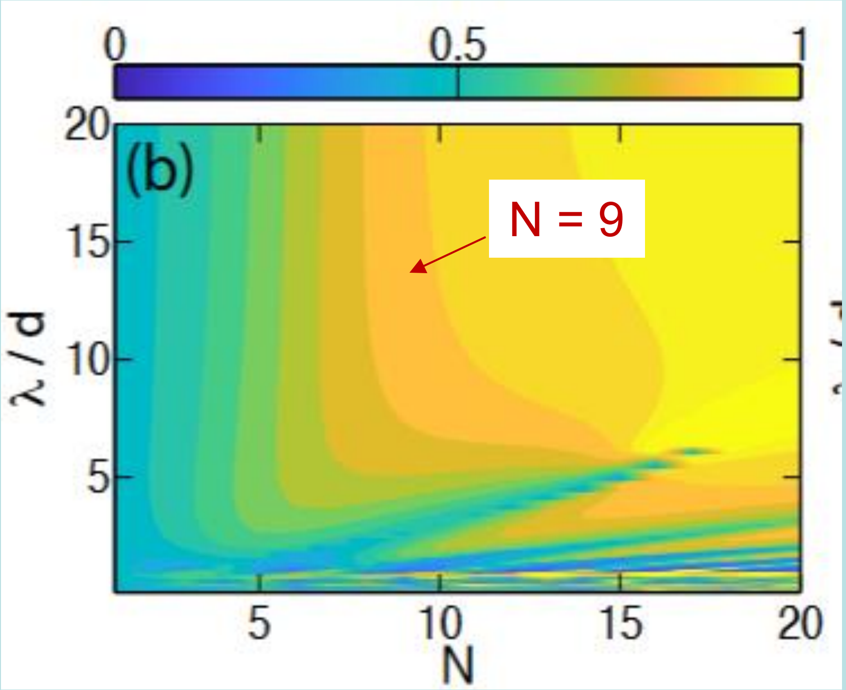


Assume all dipoles and center circularily polarized

minimal decay rate over all states
(most dark state)



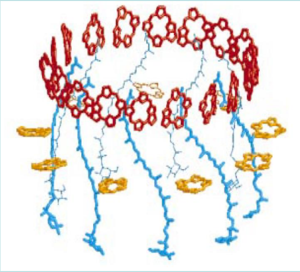
center impurity occupation of most dark state



- **most dark state for N = 9**
- **dark state exhibits dominant central occupation**

M. Cardoner,
R. Holzinger

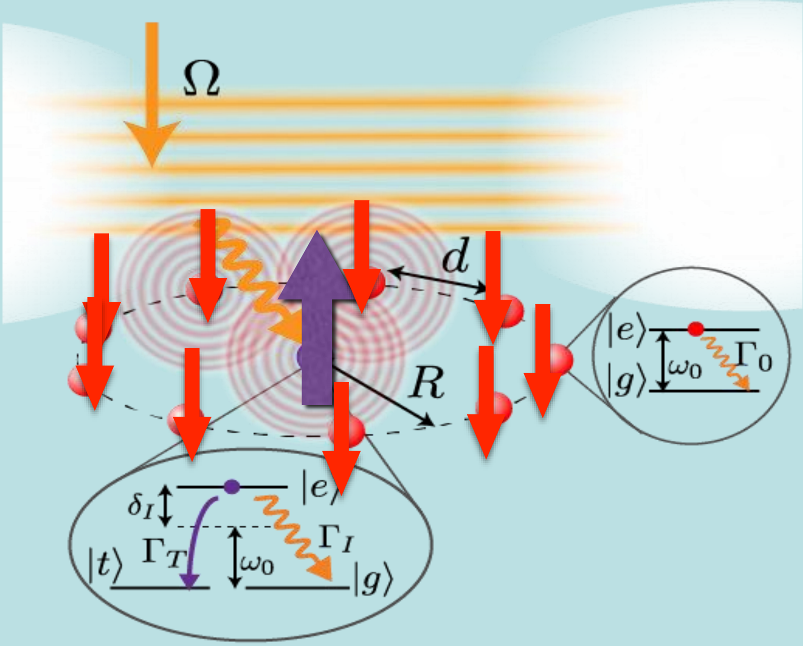
Why are 9 dipoles in a ring best ?



Enhancement is based on:

dark eigenmode
with
dominant central occupation

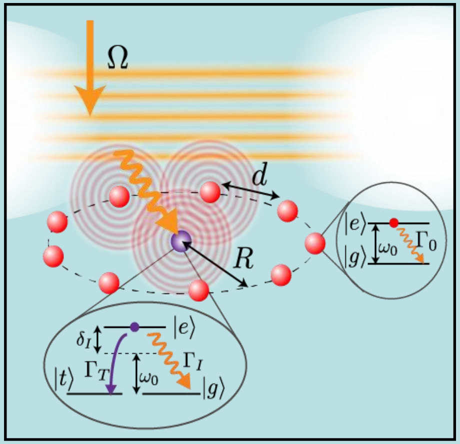
for identical emitters this is best fullfilled
for N = 9

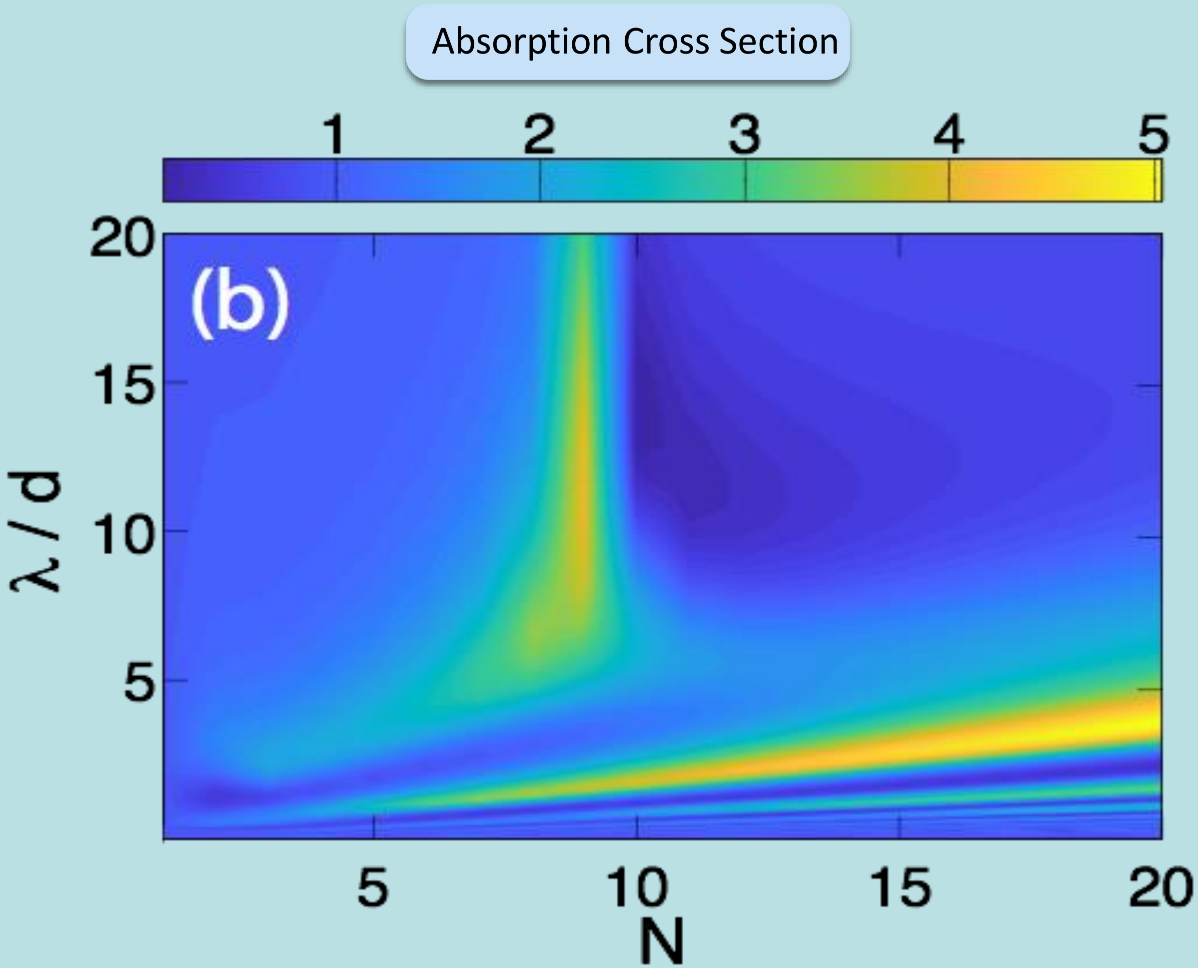


simple geometric argument:

$$\frac{9}{R^3} =\sim \sum_j \frac{1}{d_{ij}\hat{}3}$$

Absorption efficiency of broadband radiation

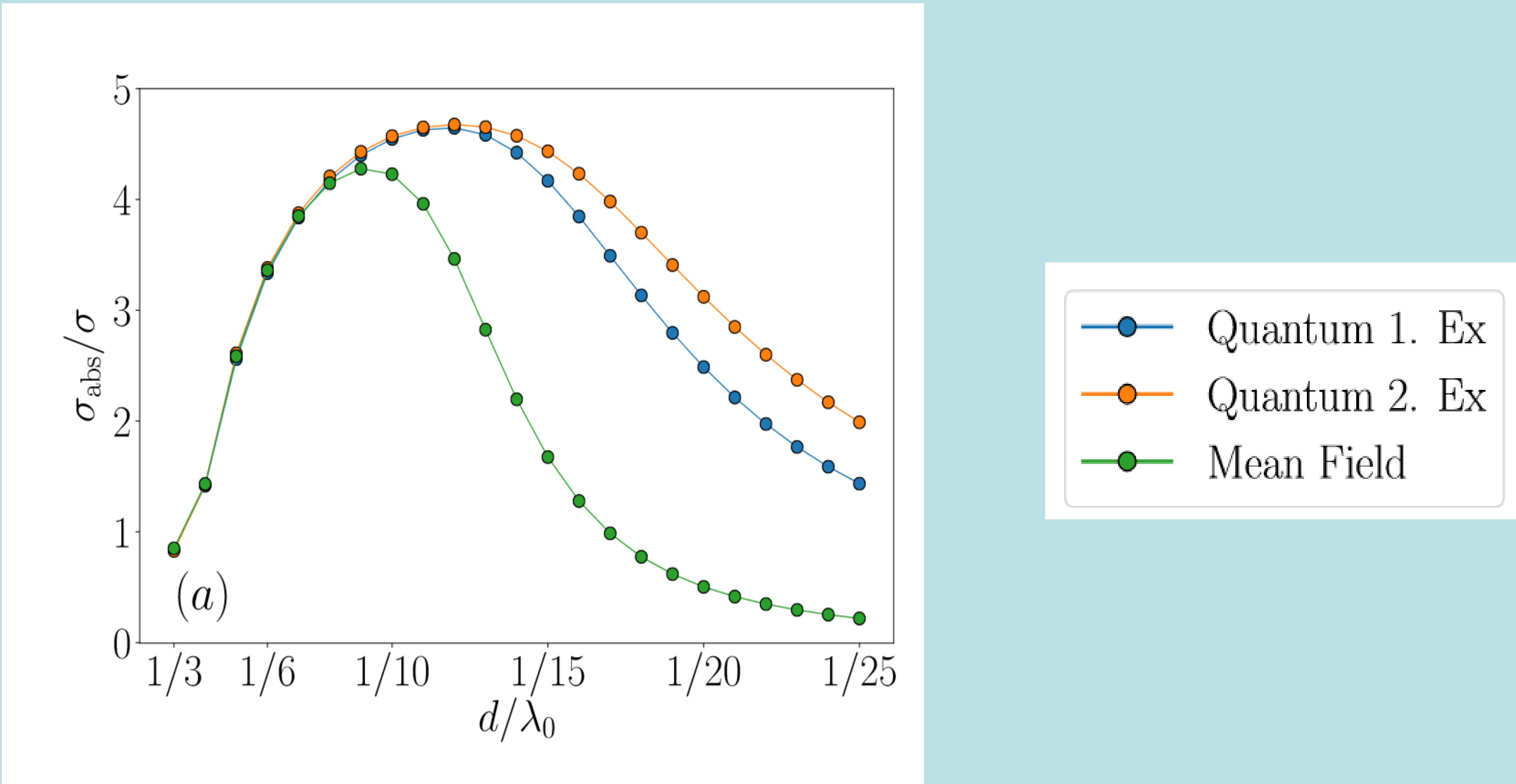




Enhancement also present for temporally incoherent light

Is it a **quantum** effect ?

Compare classical dipoles and full quantum model

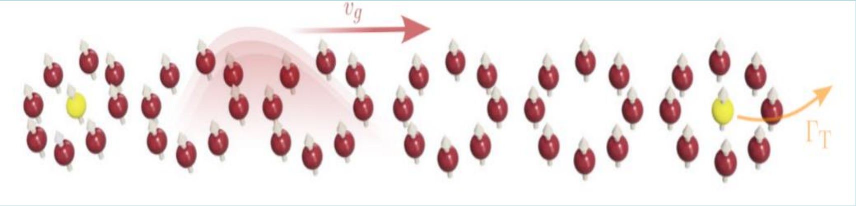


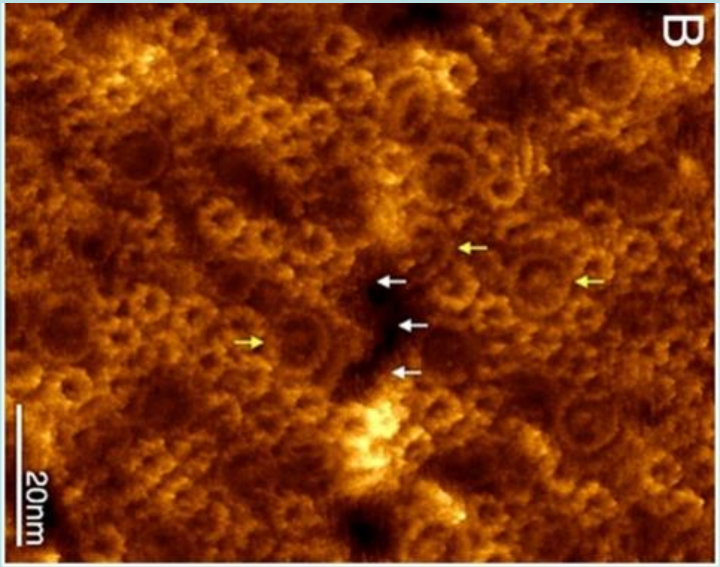
enhancement also appears for classical dipoles in mean field

but

quantum improvement appears for very small rings

excitation transport using rings

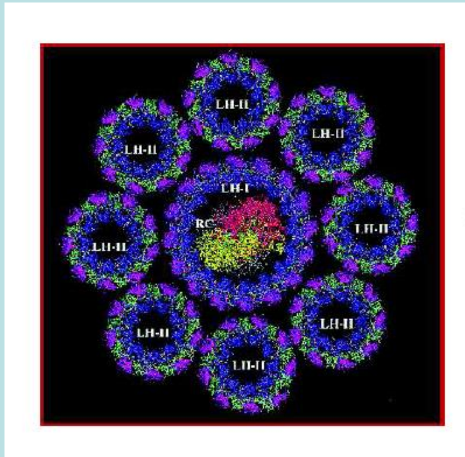


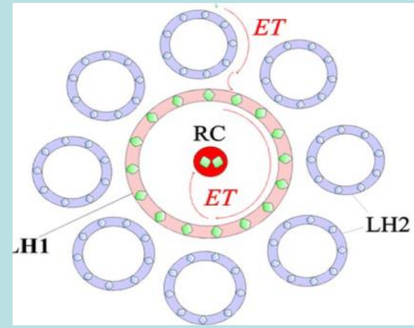


light harvesting membrane

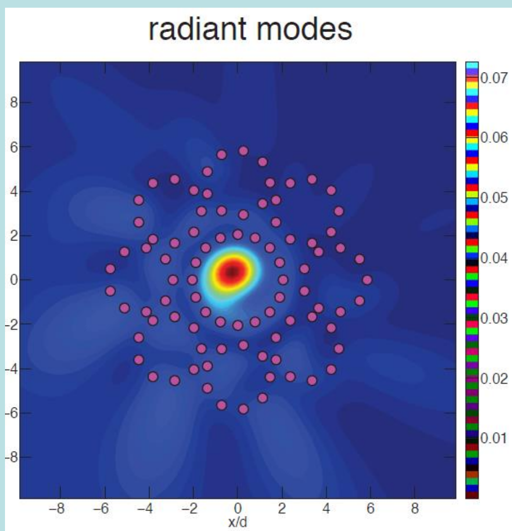
Nanorings antenna structures in natural LHC – systems

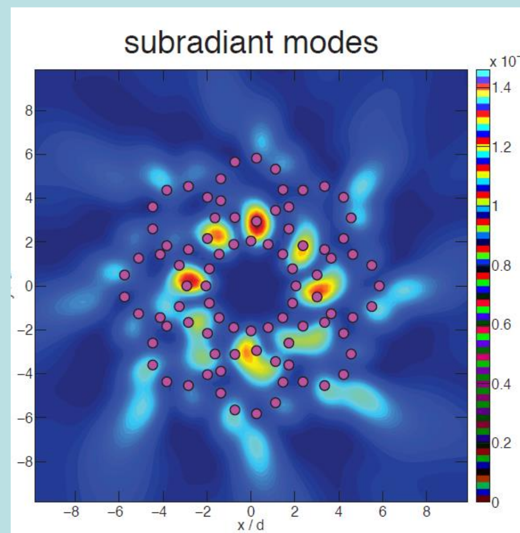
(purple bacteria)





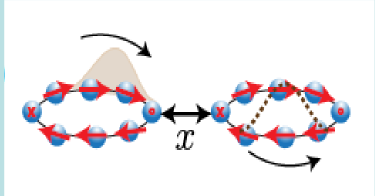
8 rings of 9 dipoles
+ central ring of 16
=> 88 - dipoles





radiant eigen-modes concentrate light at center

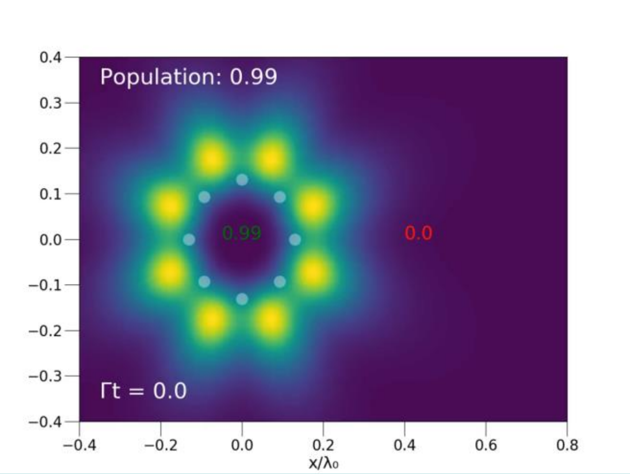
T. Polivka, Chem. Rev. 200410442021-2072, 2004



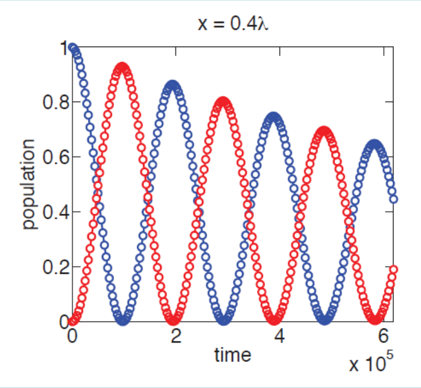
dipole coupling of two rings:

how do dark states interact ?

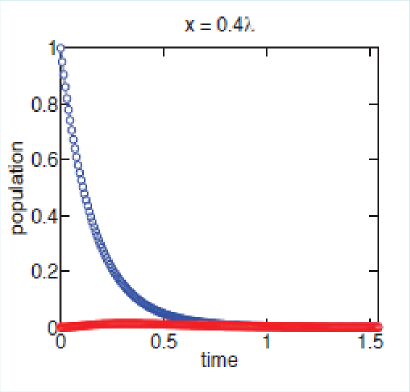
exciton – transfer between neighboring rings



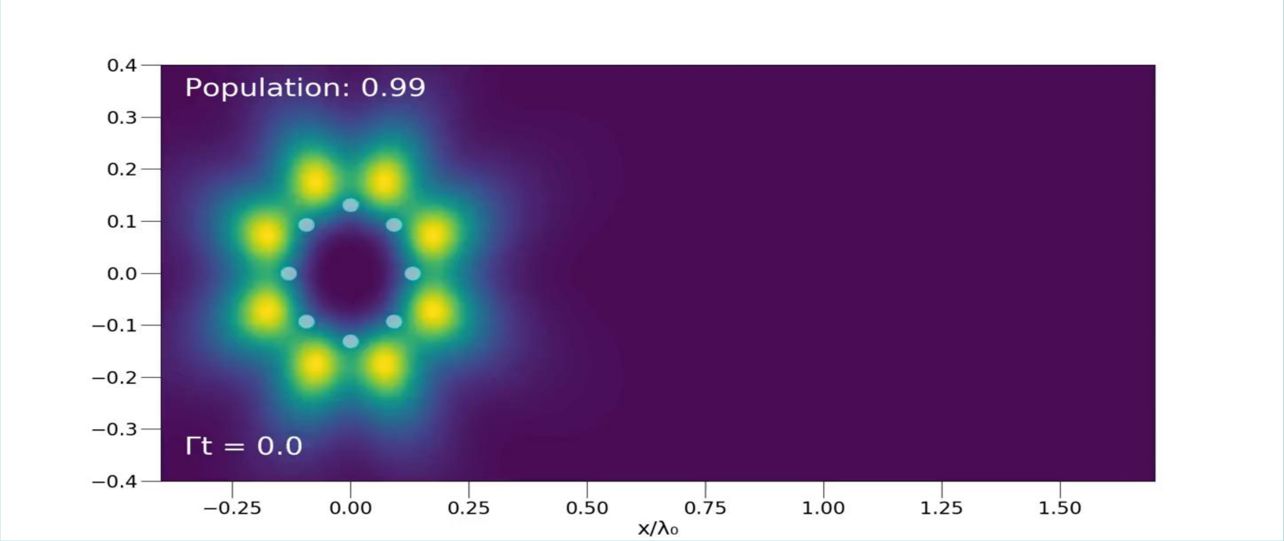
sub-radiant



super-radiant



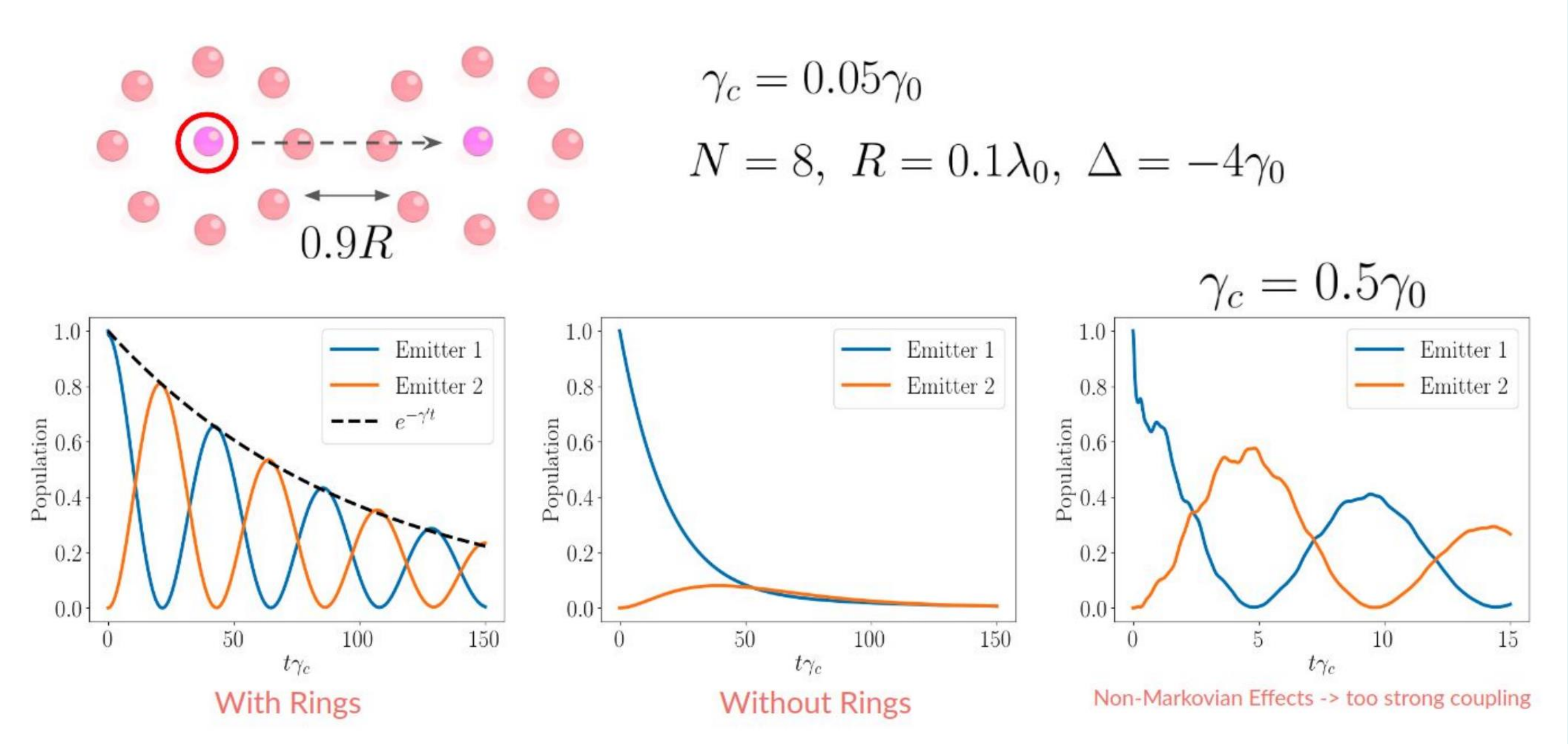
dark states transport energy with low decay !



M. Moreono-Cardoner et.al, PRA 2019

Single photon communication

donor-ring as sender => acceptor ring as receiver



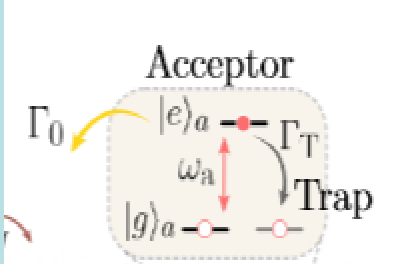
Efficiency is close to the case of dark-state ring transfer

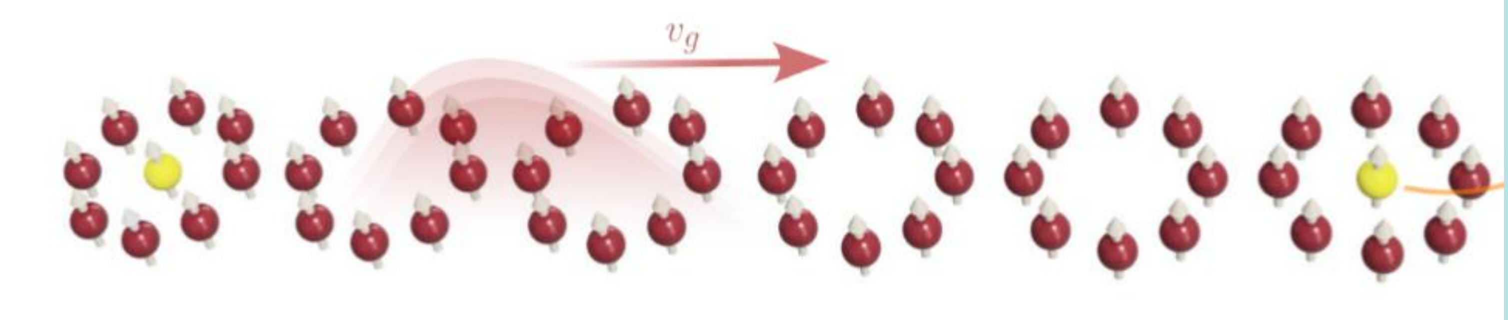
Quantum communication

Long range transport efficiency:

optimize ring size

and storage transfer rates





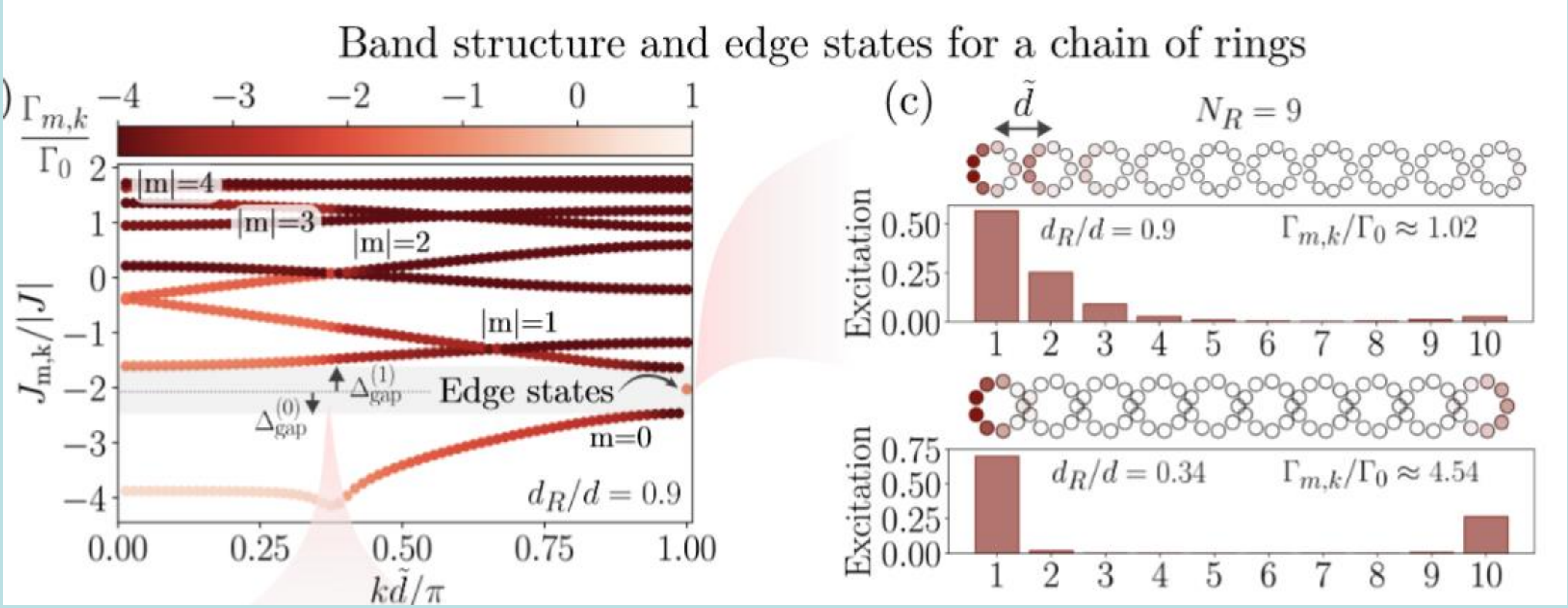
nature's preferred choice of 8 and 9 still gives close to optimal performance

R Holzinger, JS Peter, S Ostermann, H R., S Yelin, Optica Quantum 2 (2)

What is the physics behind?

$$\hat{\mathcal{H}}_{\text{lattice}} = \sum_{n,m} \left(J_{nm} - i\frac{\Gamma_{nm}}{2} \right) \hat{\sigma}_n^\dagger \hat{\sigma}_m,$$

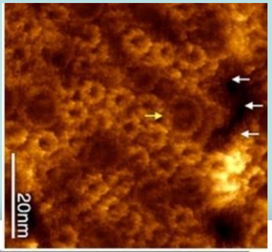
Symmetry determines band-structure: 9 energy bands for 9-rings

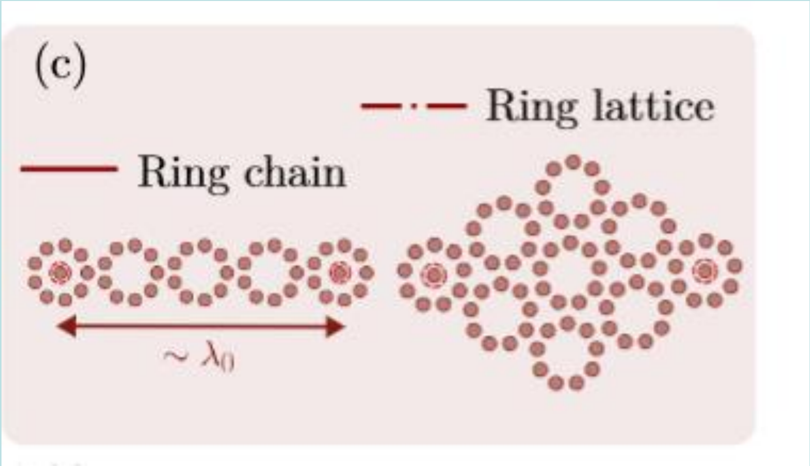


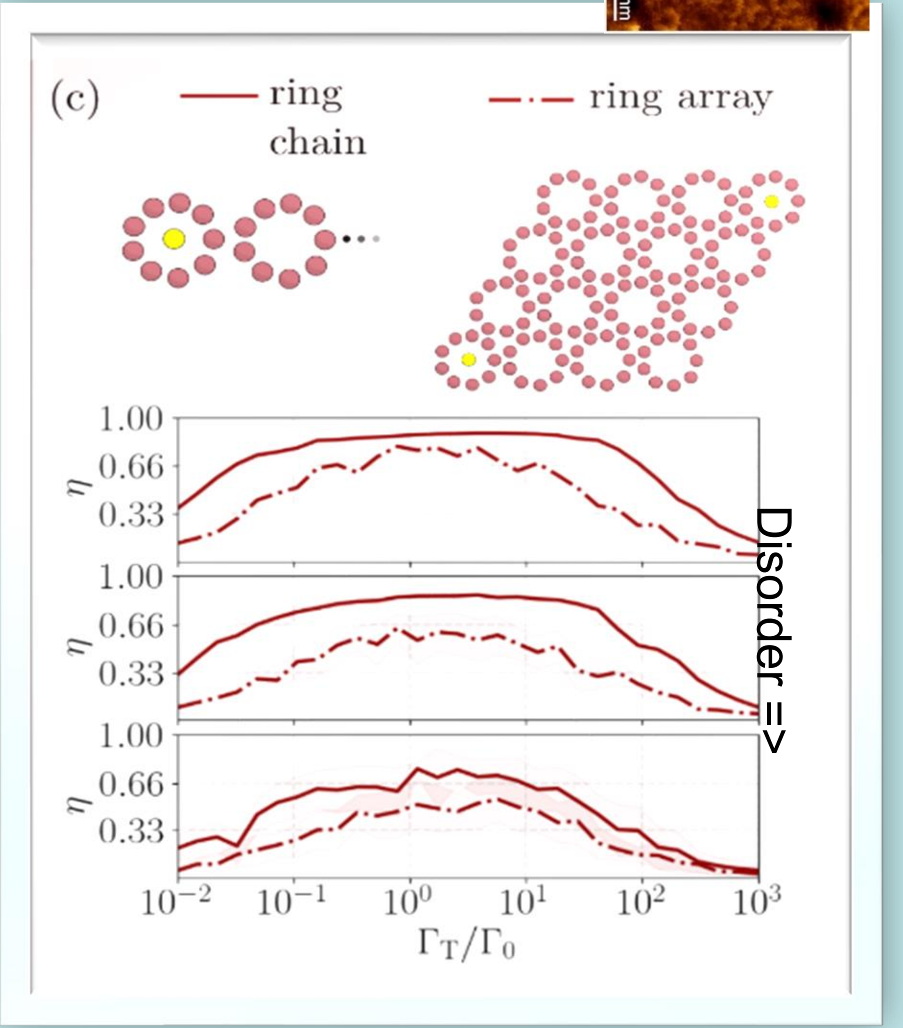
delocalized edge states, which are protected by a gap enhance lossless transfer

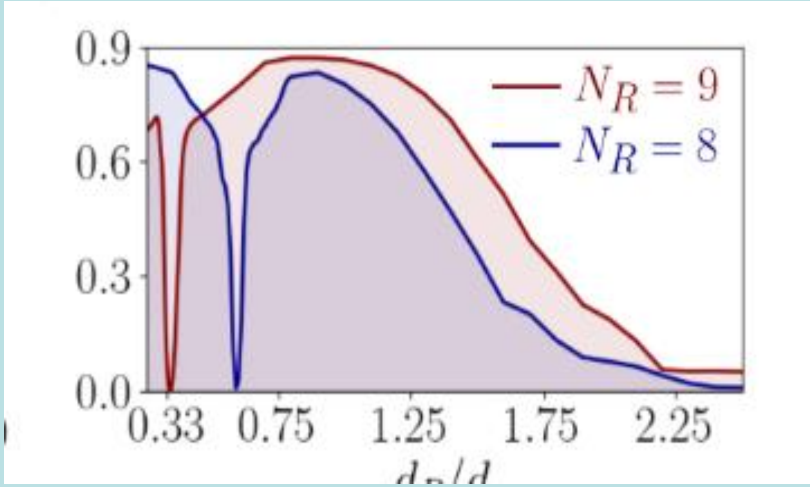
R, Holzinger, et.al. Optica Quantum 2 (2), 57-63

2d ring disorderd arrays for in plane transport







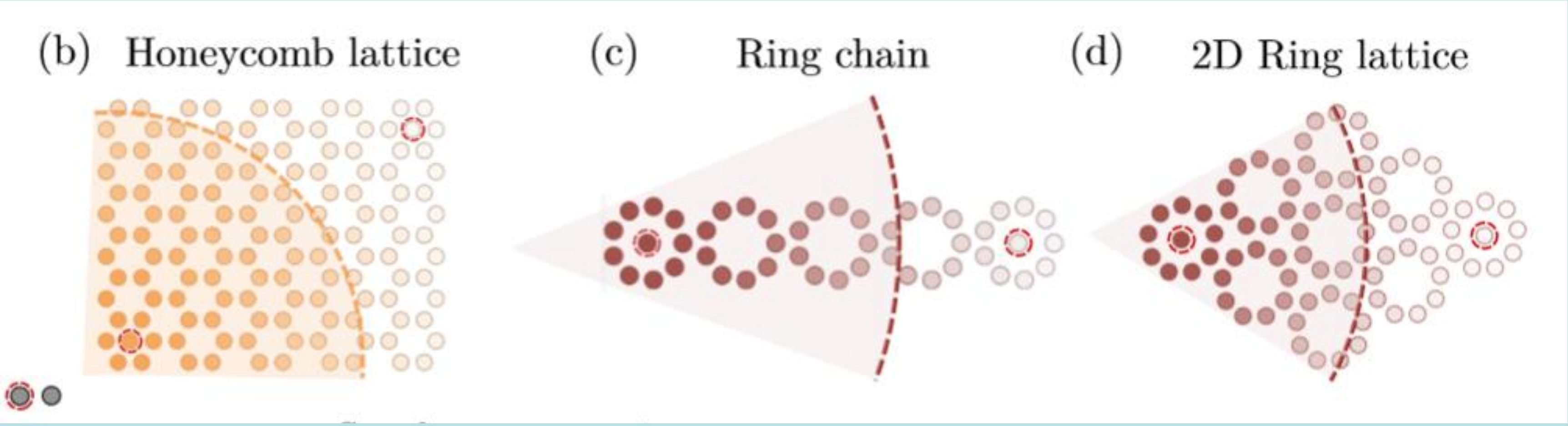


Improved transport in 9-rings with no fluorescence visible between ends

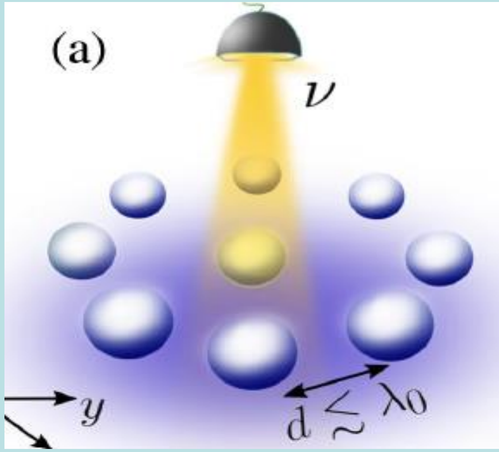
Experiment: Van Hulst, Science 21 Jun 2013,Vol 340,

absorption and transfer after local laser illumination

Gaussian beam around donor site



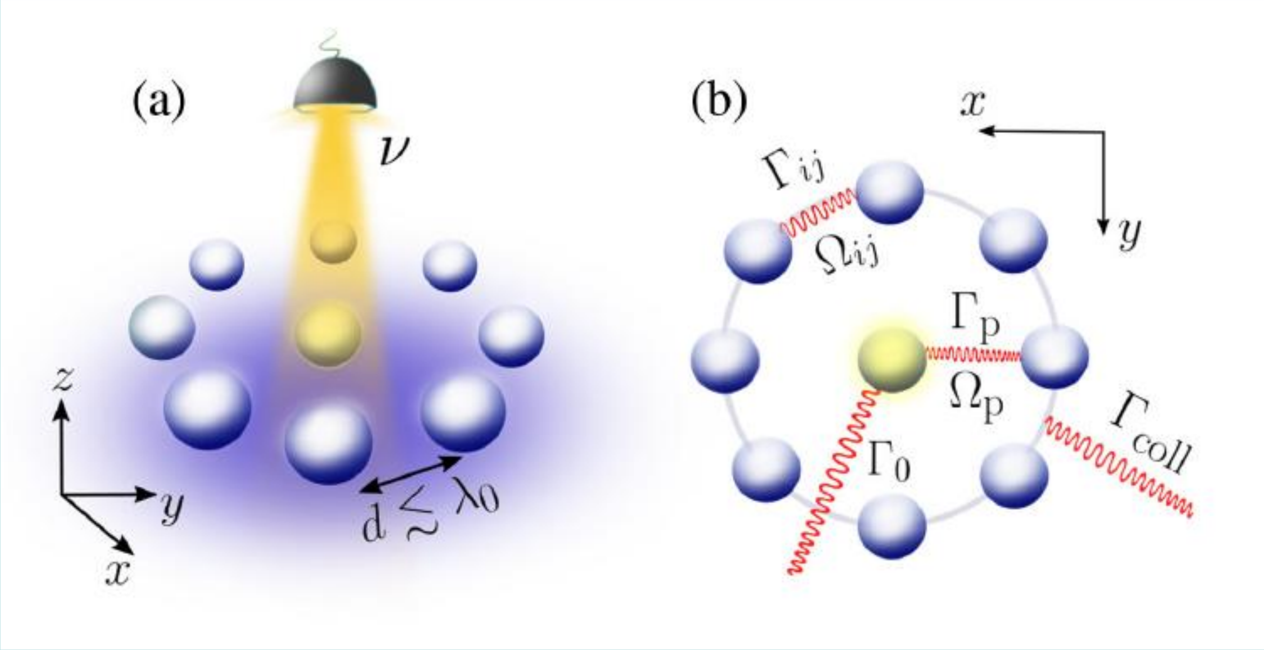
Active nano-optical devices
as
nonclassical light sources



A "bio-inspired" nanoscale laser

reverse operation !

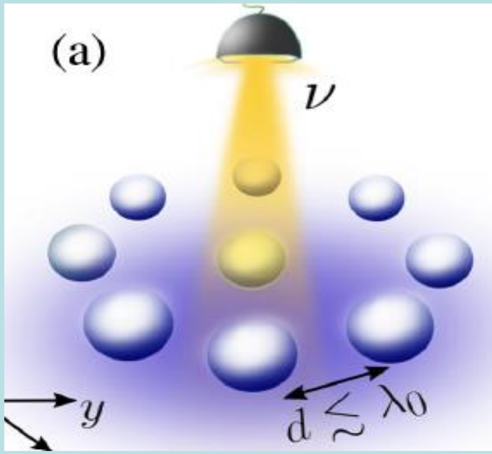
reverse operation of a light harvesting structure by pumping the center to inversion = Gain



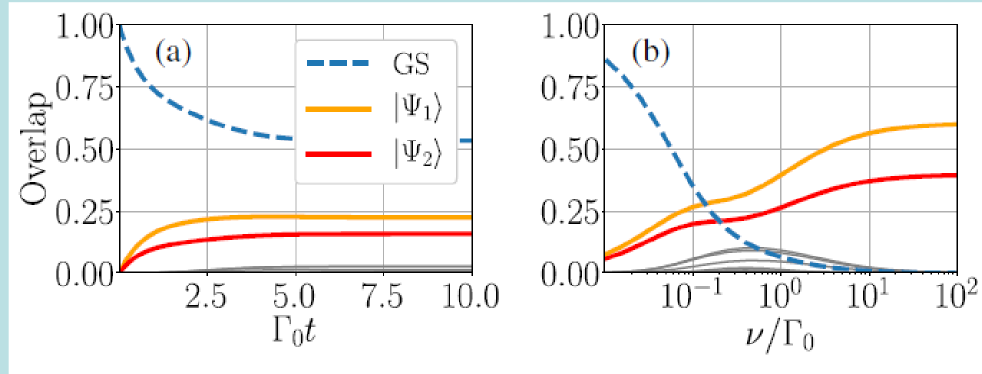
Lamp or Laser ?

R. Holzinger, et. al. PRL 124 (2020)

- incoherent pump of center
- outer ring a nano-resonator

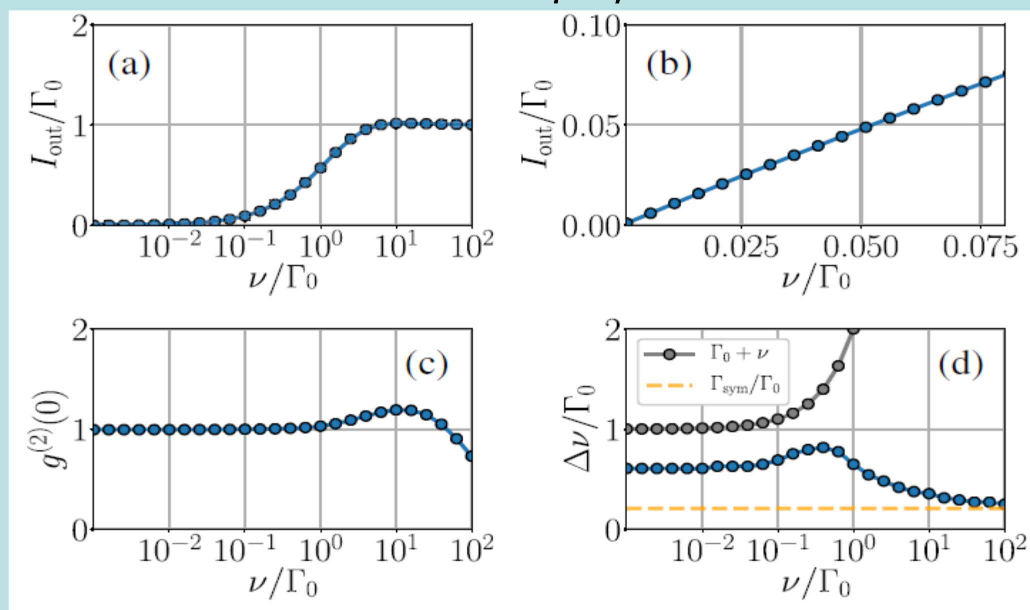


pop. time evolution *stationary state*

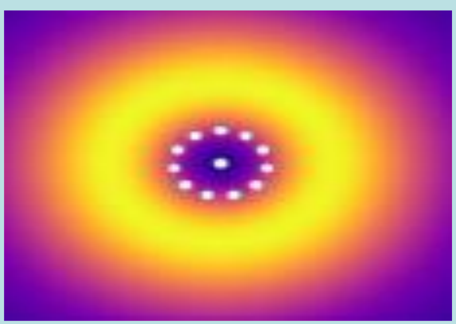


What about the emitted light statistics ?

radiated field properties



stationary field distribution



is spatially coherent

mirrorless laser : radiates temporal and spatial coherent light

„Molecules“: adding vibrational degrees of freedom

optical transitions with phonons

$$h_0^{(j)} = (\omega_0 + \lambda^2 \nu)\sigma_j^\dagger \sigma_j + \nu b_j^\dagger b_j, \qquad h_{\mathrm{Hol}}^{(j)} = -\lambda \nu \sigma_j^\dagger \sigma_j (b_j^\dagger + b_j).$$

collective basis

$$\mathcal{H} = \omega_{\mathcal{S}}^{\lambda} \mathcal{S}^\dagger \mathcal{S} + \sum_{k=1}^{\mathcal{N}-1} \omega_k^{\lambda} \mathcal{A}_k^\dagger \mathcal{A}_k.$$

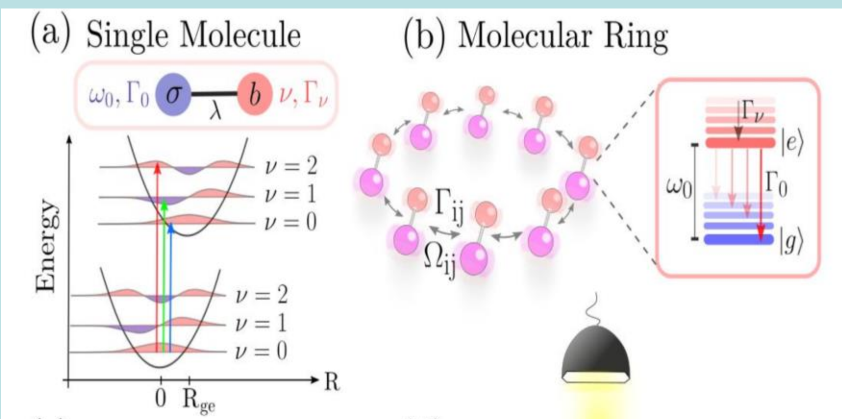
$$\mathcal{A}_k = \frac{1}{\sqrt{\mathcal{N}}} \sum_{j=1}^{\mathcal{N}} \sigma_j e^{2\pi i jk/\mathcal{N}} \qquad \mathcal{Q}_k = \frac{1}{\sqrt{\mathcal{N}}} \sum_{j=1}^{\mathcal{N}} e^{2\pi i jk/\mathcal{N}} (b_j + b_j^\dagger) \qquad \mathcal{S} = \sum_j \sigma_j / \sqrt{\mathcal{N}}$$

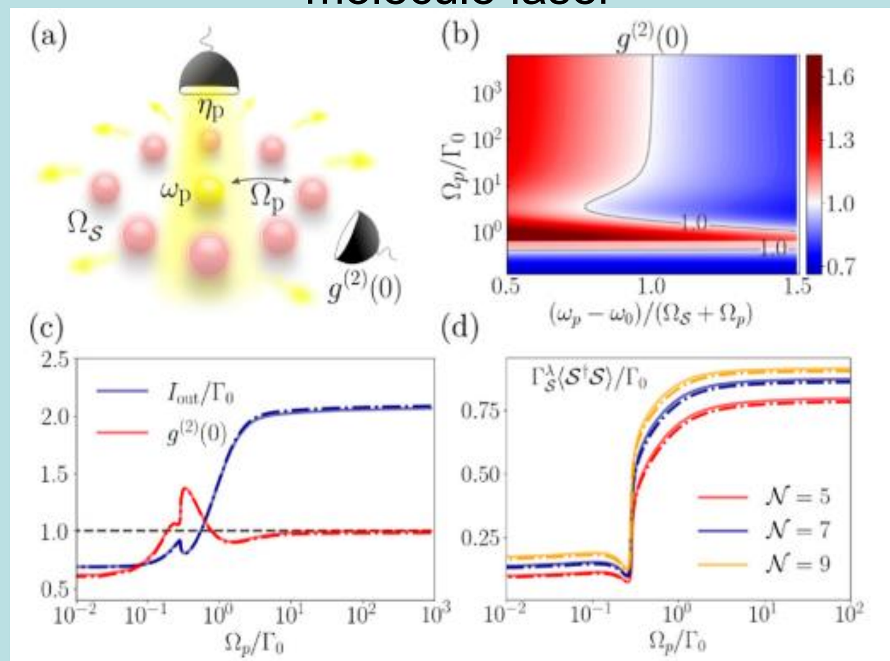
$$\mathcal{H}_{\mathrm{int}}^{\mathcal{SA}} = -\frac{\lambda \nu}{\sqrt{\mathcal{N}}} \sum_{k=1}^{\mathcal{N}-1} (\mathcal{Q}_k \mathcal{S}^\dagger \mathcal{A}_k + \mathrm{H.c.}),$$

molecule laser

collective damping

$$\tilde{\mathcal{L}}_e[\rho] = \sum_{j,j'} e^{-\lambda^2 (1+2\bar{n})(1-\delta_{jj'})} \frac{\Gamma_{jj'}}{2} [2\sigma_j \rho \sigma_{j'}^\dagger - \{\sigma_j^\dagger \sigma_{j'}, \rho\}].$$



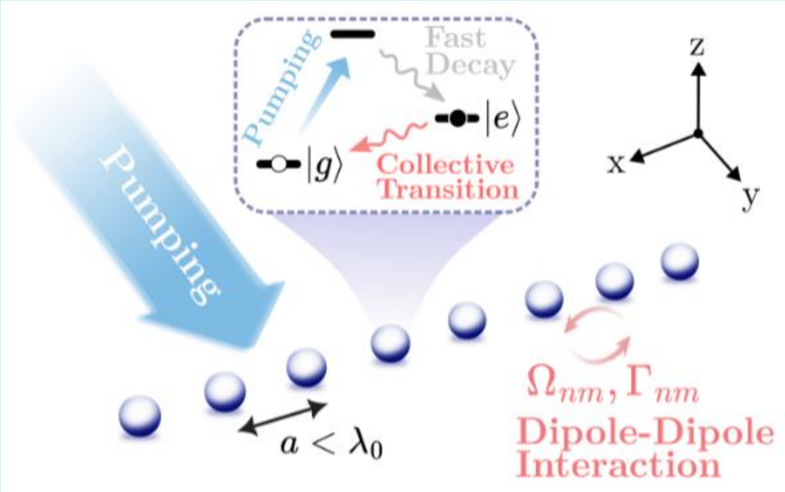


C. Genes and R. Holzmann, PRR4 (2022)

E Sánchez-Llorente, H Ritsch, M Moreno-Cardoner arXiv preprint arXiv:2605.28085

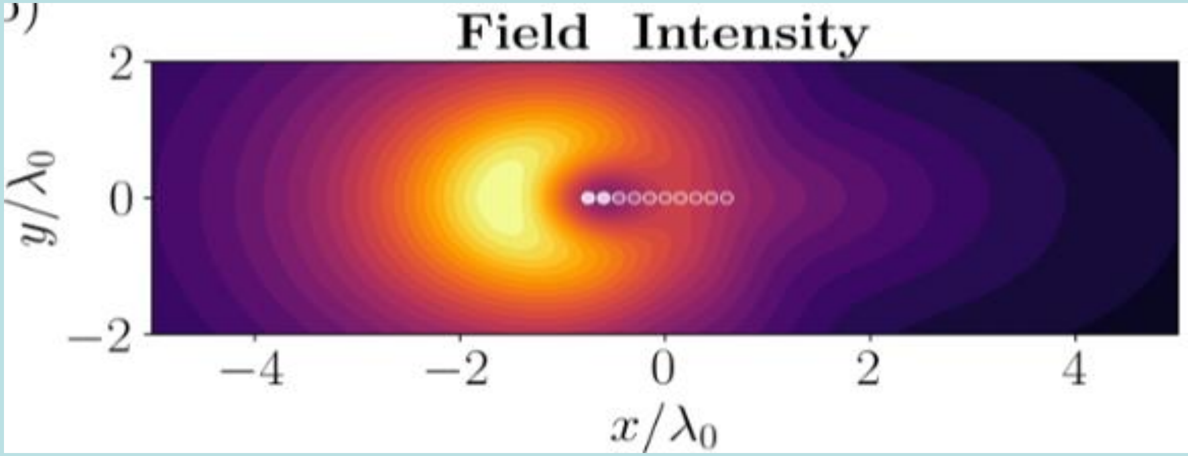
A generic model nanoscale mirrorless laser

(dense 1D array of driven dipoles)



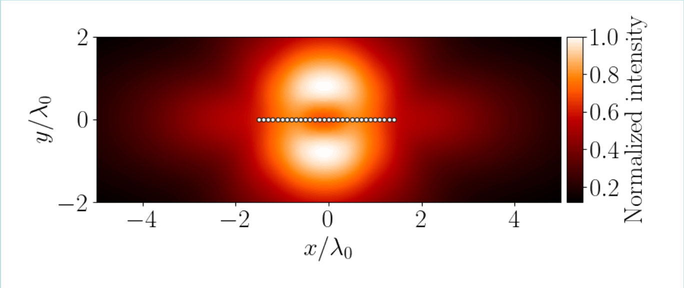
- some atoms create gain
- other atoms build resonator and outcoupler

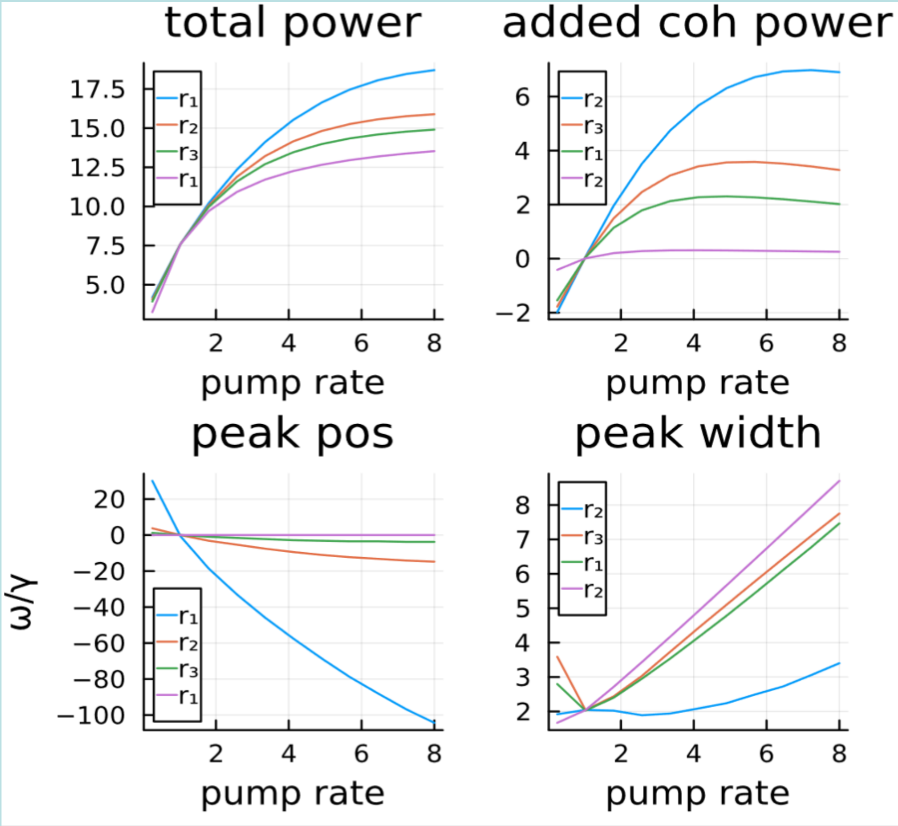
Basic example: 2 out of 10 atoms pumped at d=λ/5

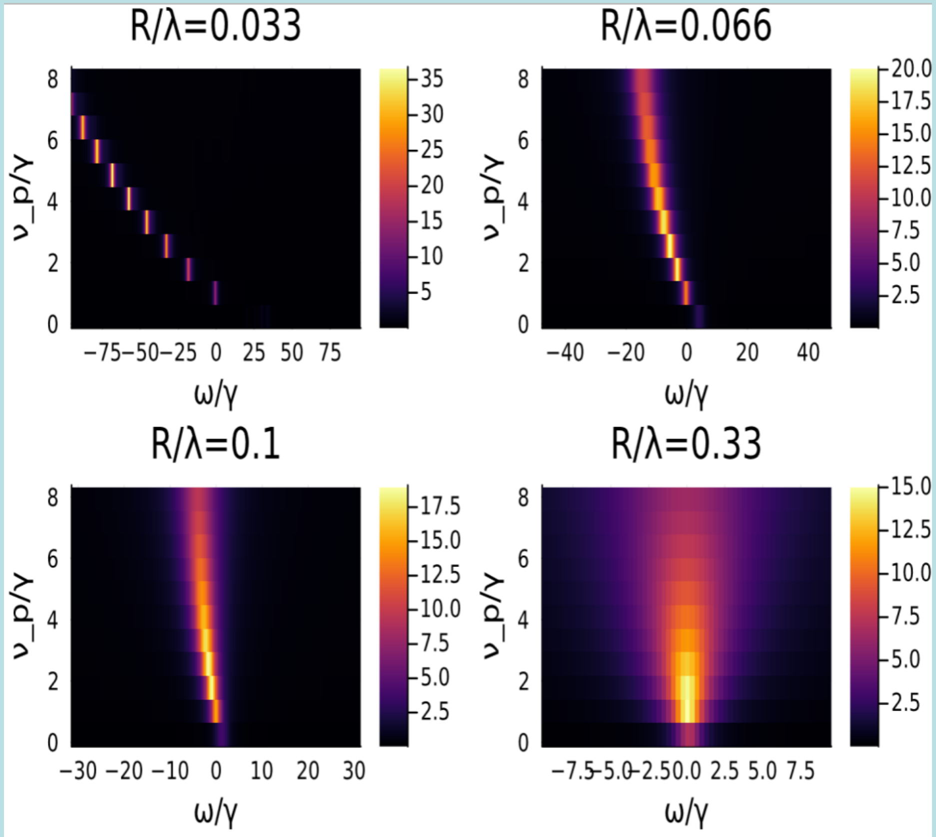


R. Holzinger & A. Bychek

emission of incoherently pumped
of whole array of N = 15 emitters



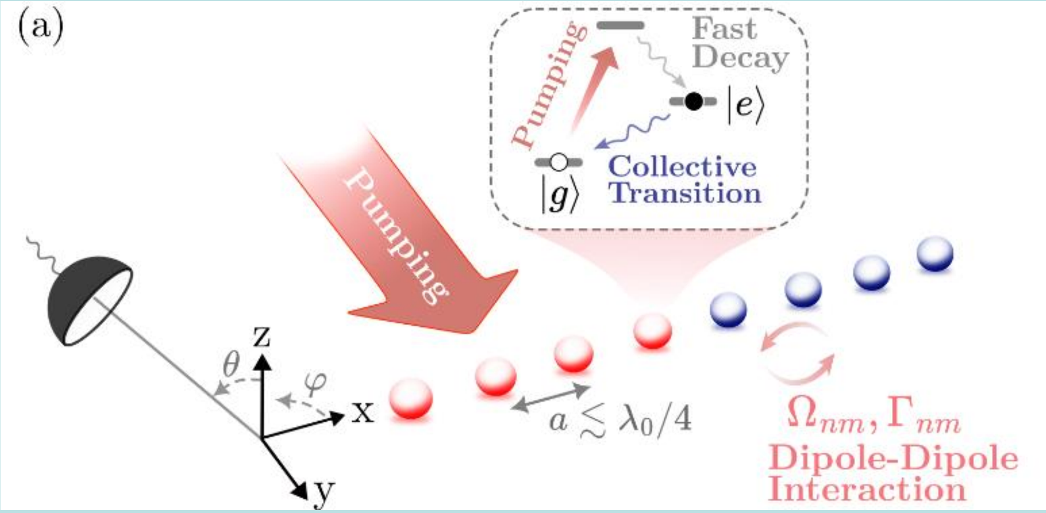




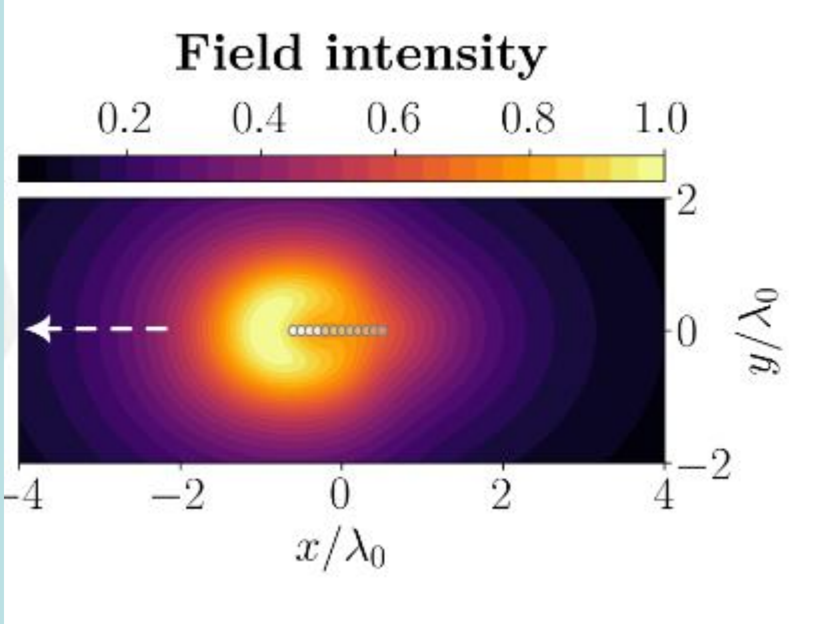
r = λ/30 r = λ/15 r = λ/10 r = λ/3

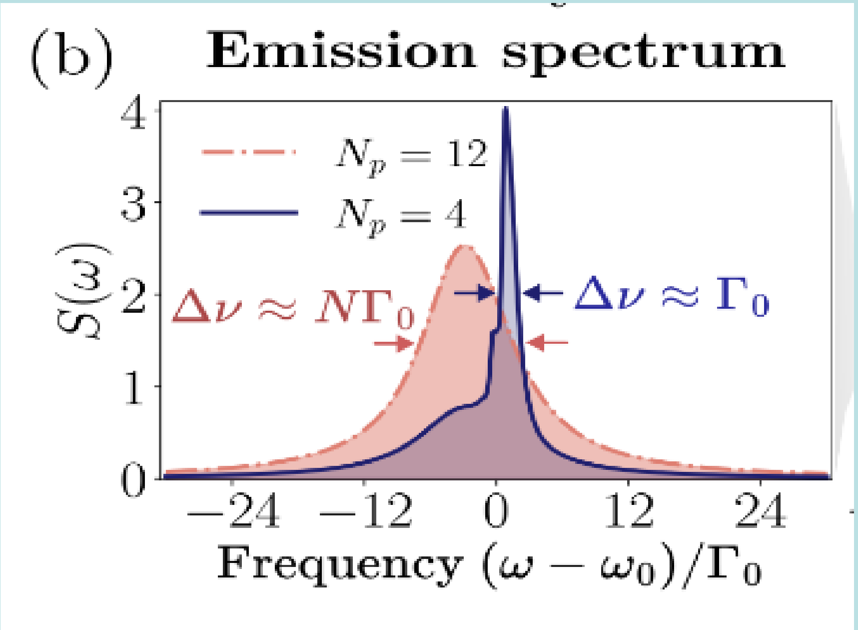
a tighter chain leads to more CW-super-radiant emission
at larger central shift

Mirrorless lasing of a linear array with partial drive

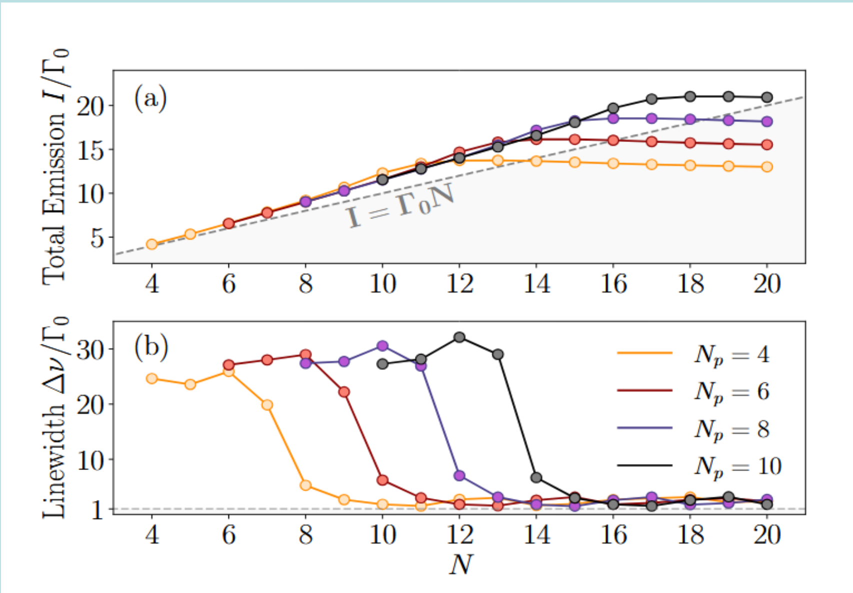


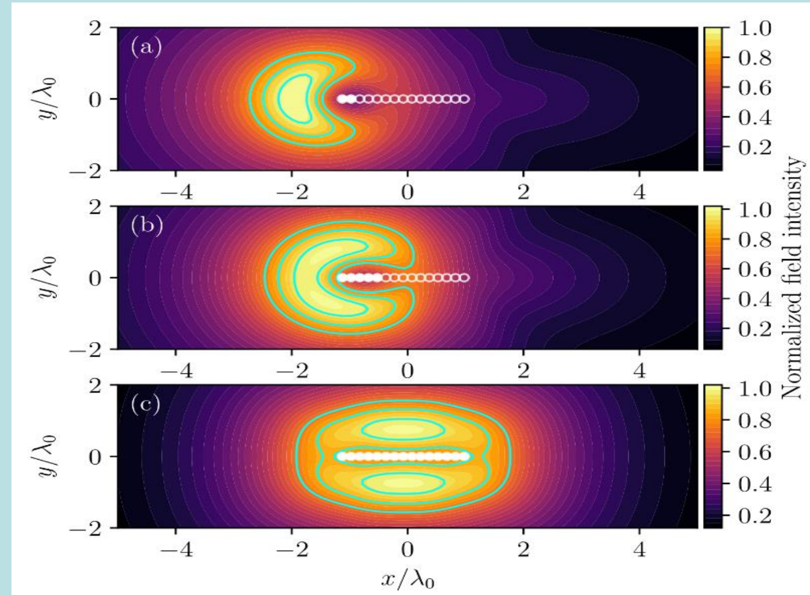
Atoms are split in
gain atoms
+
antenna atoms

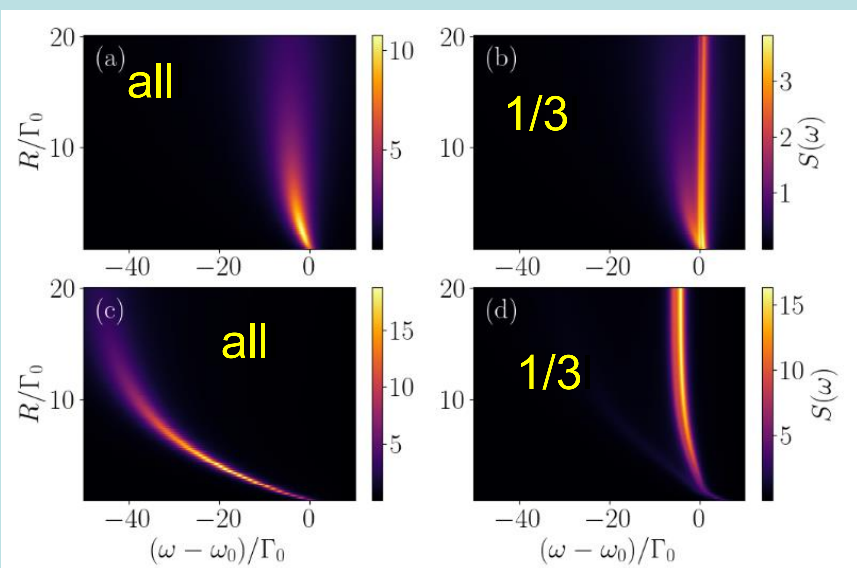


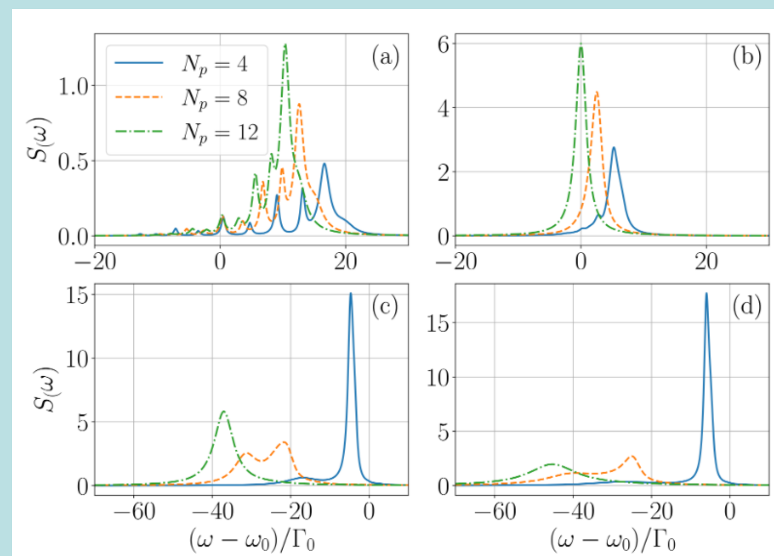


Power and spectra for varying pump fraction



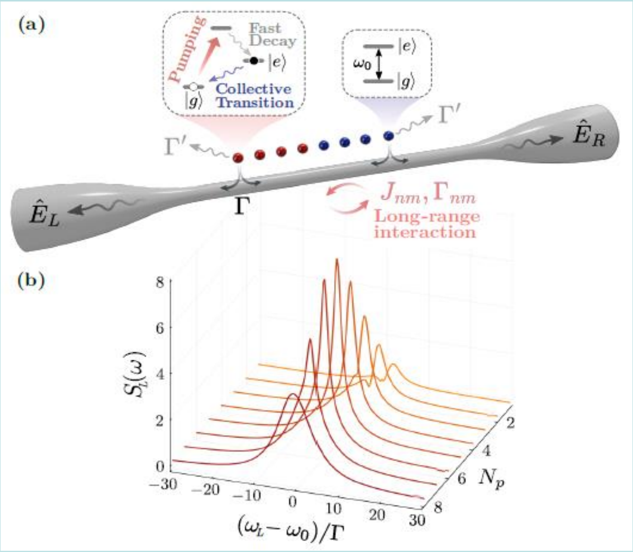




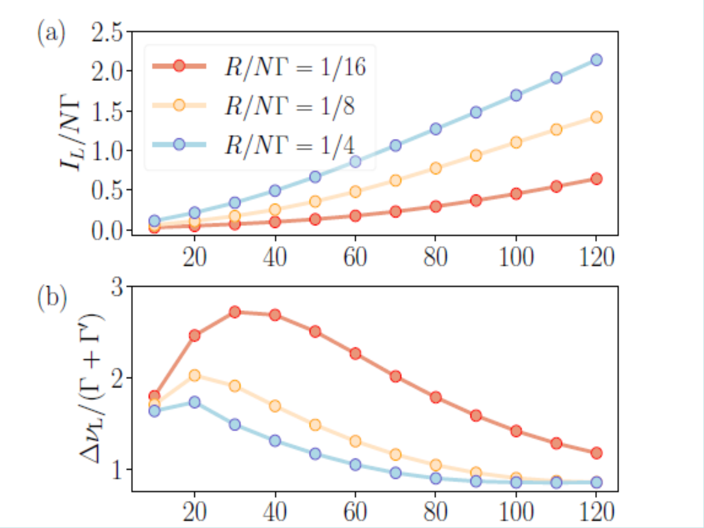


Narrow-Line Waveguide Superradiance from Partially Pumped Emitter Ensembles

Anna Bychek
Raphael Holzinger

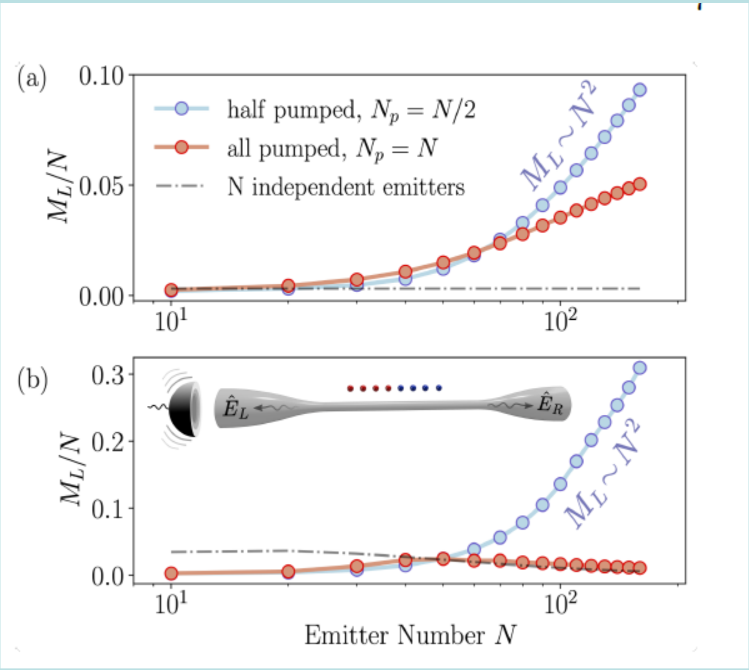


Optimal output near half ensemble pumping



Heuristic performance number:

$$M_{L/R} \equiv \frac{I_{L/R}\Gamma}{\Delta\nu_{L/R}^2 + 4(\omega_{L/R} - \omega_0)^2}$$



(Thorium frequency reference ?)

Summary and Conclusions:

- **Dipole interaction dominates collective dynamics in sub-λ structures**

  => exponential increase of excitation lifetimes in rings
  => almost all eigenstates of nano-arrays are dark

- ***Dark states enable low loss energy transfer***

- ***Rings make good nanoscopic antennas***

  a 9-ring exhibits superior absorption properties

- ***Ring structures are well suited for robust long range transport***

- ***Rings with gain can implement nanoscopic coherent light sources***

- ***Nano-arrays allow tailored nonlinear scattering as nonclassical light sources***

- ***protected sub-space Ramsey spectroscopy using collective dark states***

Open source quantum dynamics software

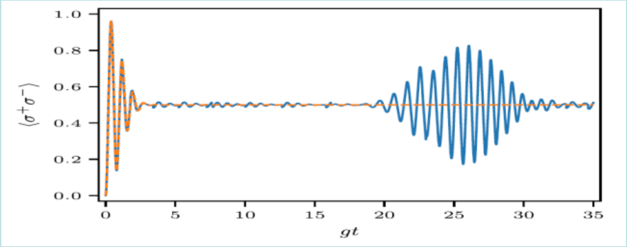
QuantumOptics.jl

Quantum Optics toolbox in *Julia*

www.qojulia.org

```
a = destroy(bc) ⊗ one(ba)
σ⁻ = one(bc) ⊗ sigmam(ba)

# Construct Hamiltonian
H = Δ*dagger(a)*a + g*(dagger(a)*σ⁻ + a*dagger(σ⁻))

# Define initial state
ψ₀ = coherentstate(bc, α) ⊗ spindown(ba)

# Define list of time steps
T = [0:0.01:35;]

# Evolve in time according to Schrödinger's equation
tout, ψₜ = timeevolution.schroedinger(T, ψ₀, H)

# Calculate atomic excitation
excitation = expect(dagger(σ⁻)*σ⁻, ψₜ)
```

Code sample 1: *Jaynes-Cummings model.*



CollectiveSpins.jl

Simulate Dipole-Dipole Coupled Spin Systems

https://qojulia.github.io/CollectiveSpins.jl/

dipole-dipole package

QuantumCumulants.jl

analytic package for
automated cumulant expansions

https://github.com/qojulia/QuantumCumulants.jl


Thanks for the invitation !

Use tailored cavity sub-radiance for improved Ramsey spectroscopy

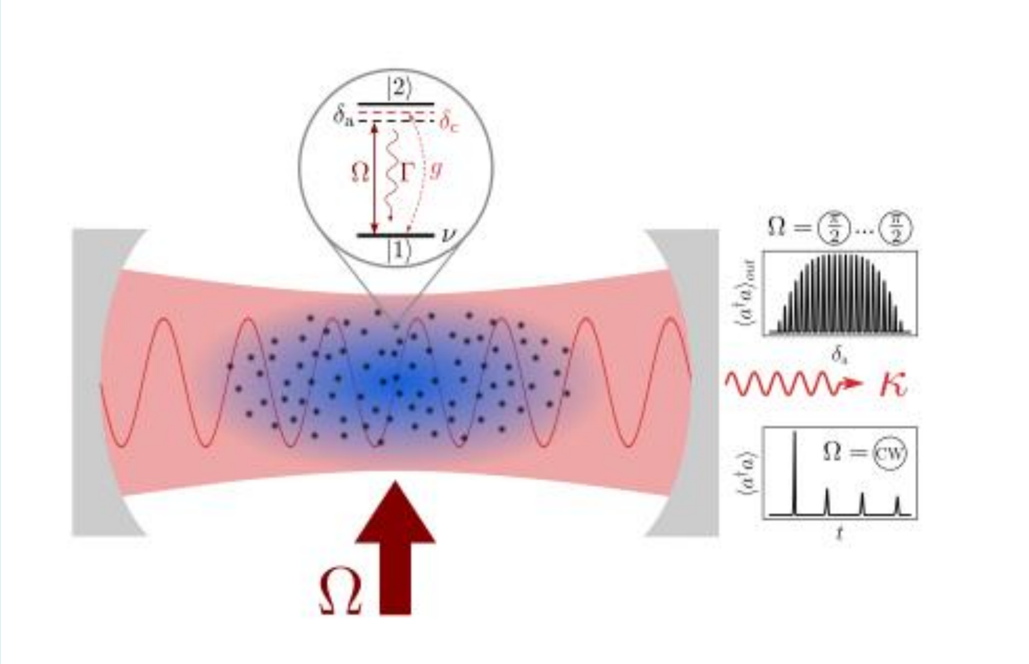


Figure 1. *Cavity Enhanced Spectroscopy Setup.* We consider a

transverse pump excitation of dark states in trapped ensemble at *$\lambda/2$ distance*

Bohr, et.al. Nature Communications | (2024)15:1084

Tailored cavity/waveguide sub-radiance for improved Ramsey spectroscopy

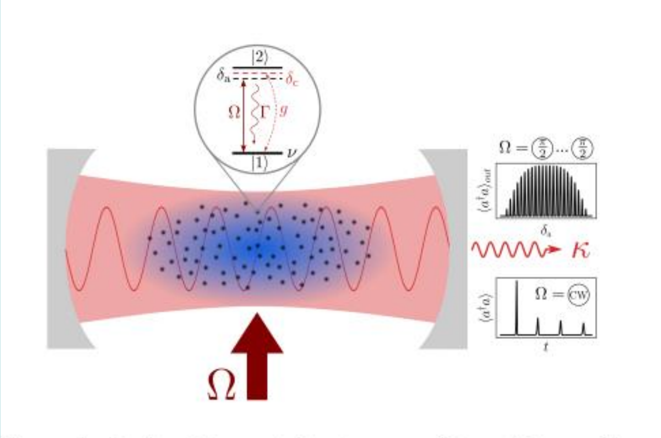
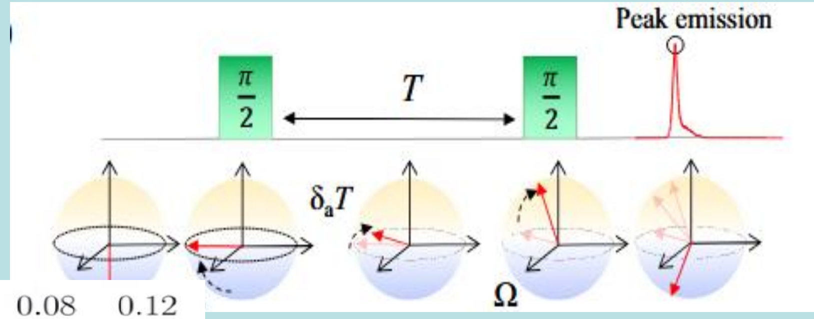
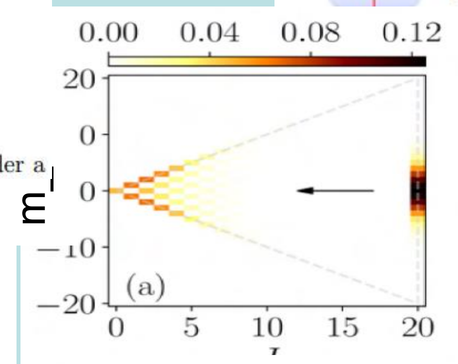


Figure 1. *Cavity Enhanced Spectroscopy Setup.* We consider a

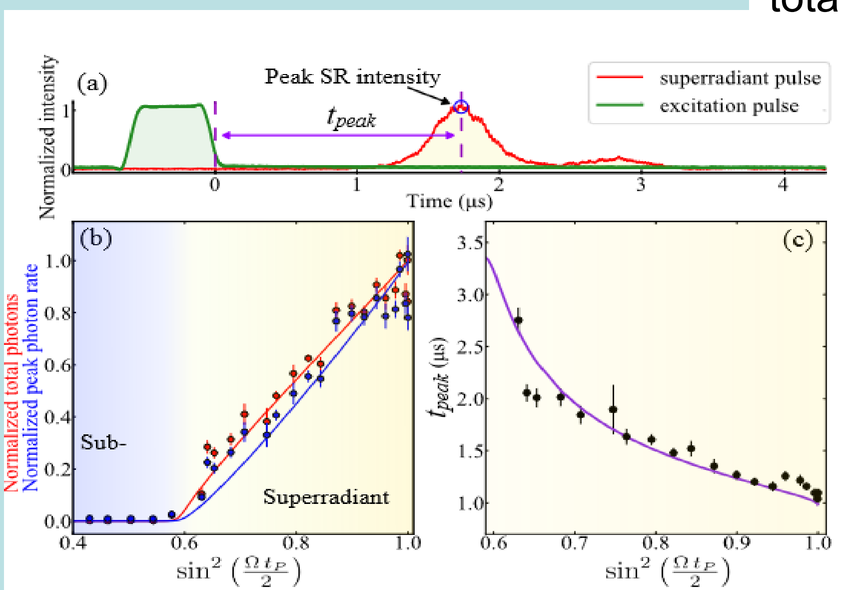


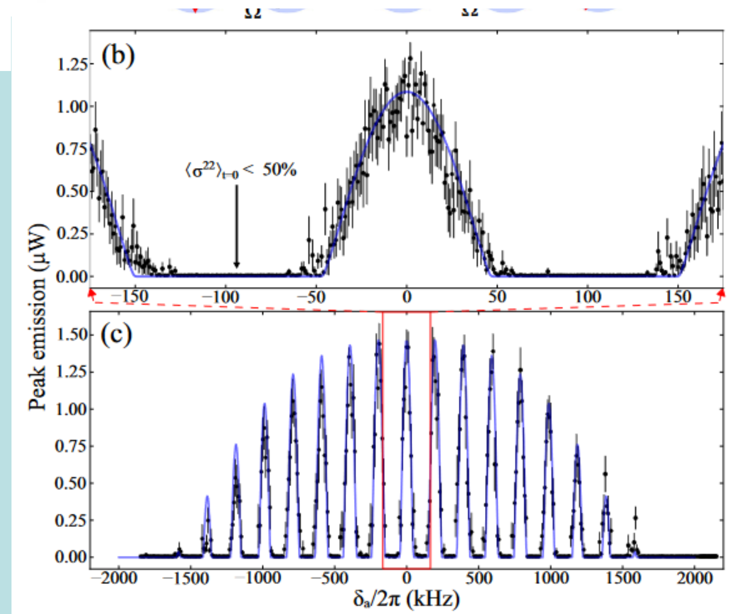
first pulse < π/2 creates collective dark state protected from collective decay



total spin J

second pulse < π/2 creates collective state with enhanced cavity decay for >50% excitation

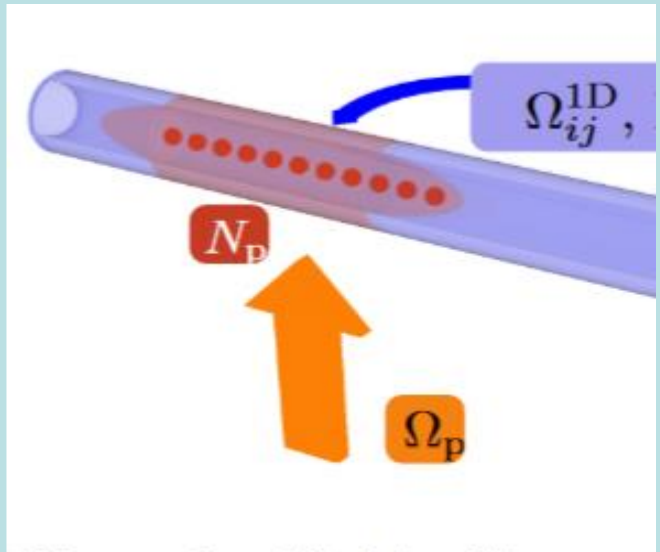




Bohr, et.al. Nature Communications | (2024)15:1084

combines precision and fast readout

Use tailored cavity/waveguide sub-radiance for collective transfer / pumping



transverse pump excitation of dark states in trapped ensemble at $\lambda/2$ *distance*

“classic” experiments of waveguide superradiance (Benabid, Katori …)

Sub-radiance and super-absorption for fast long range excitation transport

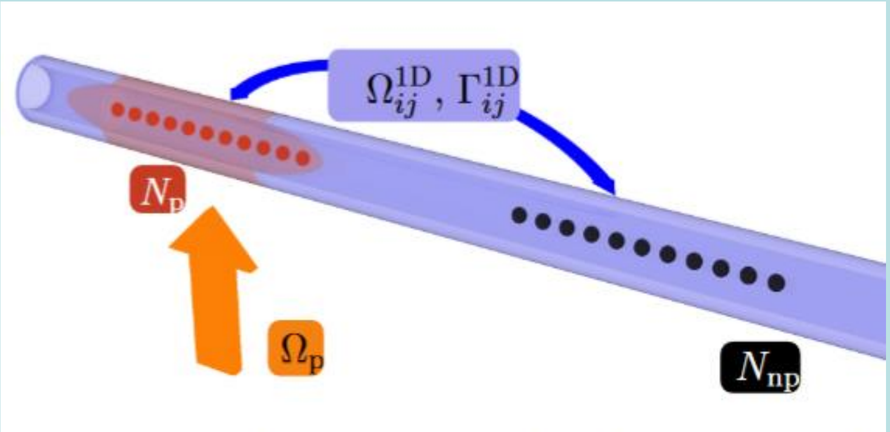
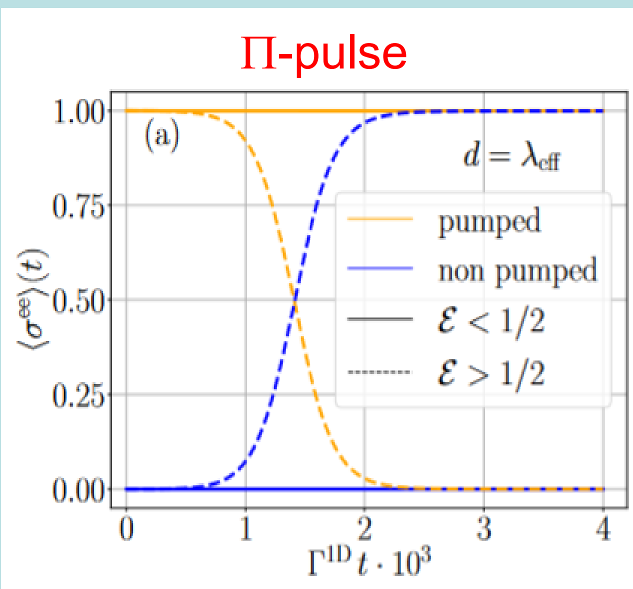


Figure 1. *Model.* Two ensembles of atoms coupled t

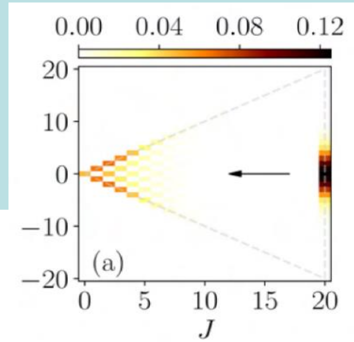
- two λ/2 ensembles at large distance prepared in ground state
- Coherent transverse excitation of first ensemble shows critical transition from sub-radiance to superradiant transfer to second ensemble



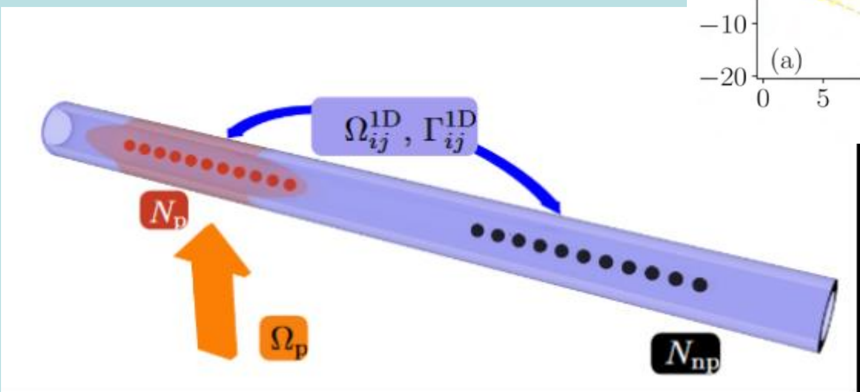
perfect inversion of distant ensemble

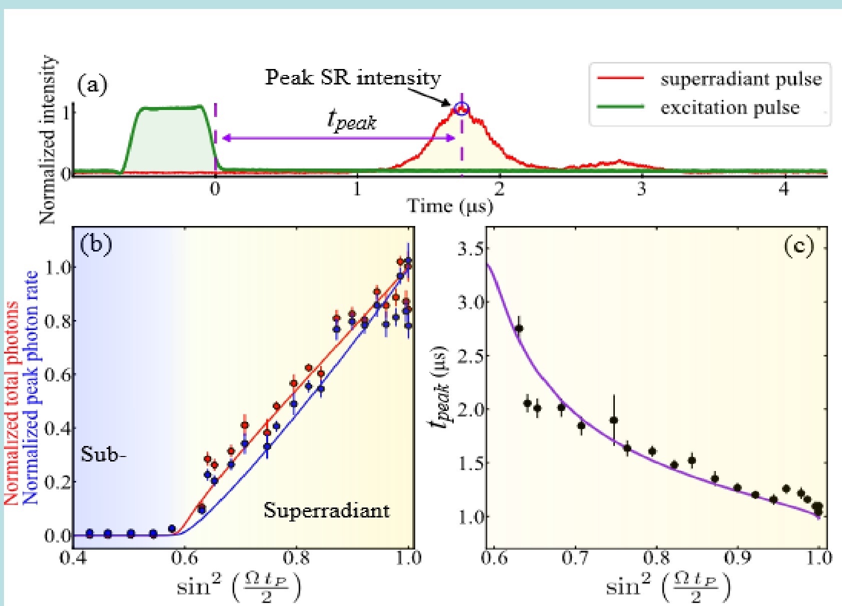
Tailored cavity/waveguide sub-radiance for improved Ramsey spectroscopy

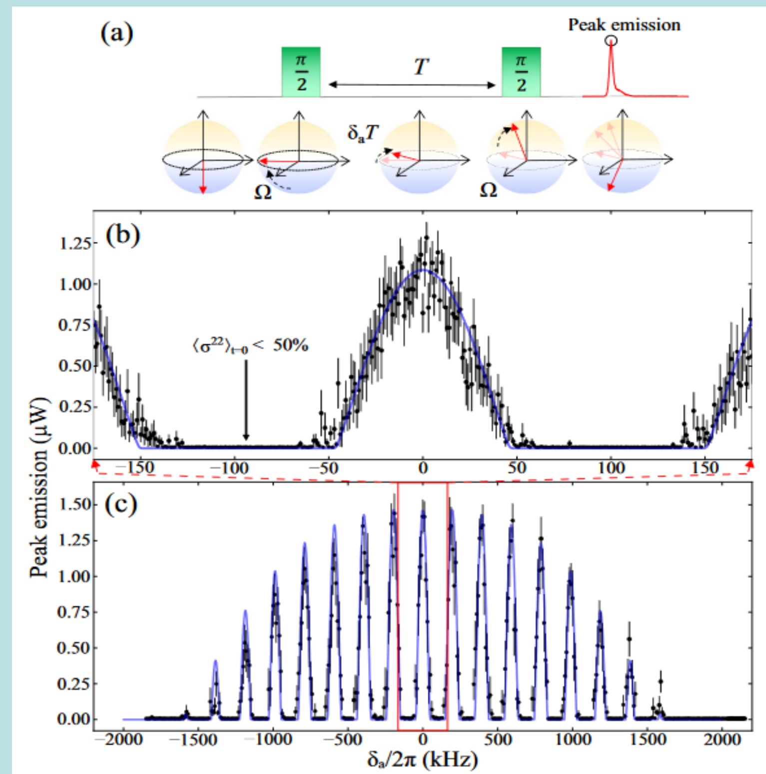
transverse pulse < π/2 creates collective dark state protected from waveguide decay



second pulse < π/2 creates collective dark state protected from cavity decay



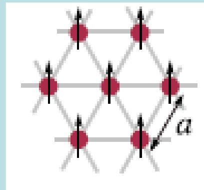




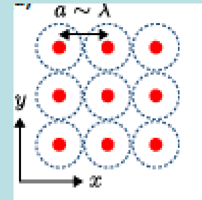
combines precision and fast readout

Bohr, et.al. Nature Communications | (2024)15:1084

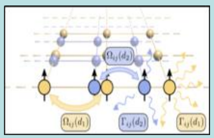
extensive work for 2D/3D - geometries (some examples):



R.J., Bettles, et.al.
PRL 116.10 (2016)

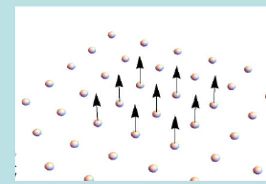


E. Shahmoon, et.al.
PRL 118.11 (2017)

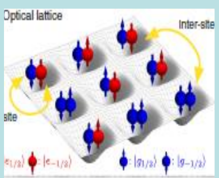


S. Krämer, et.al.,
EPJD 69, (2015)

+ many more
more recent

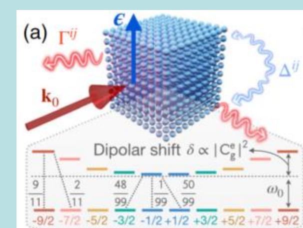


E. Ballantine, et.al.
PRR 2, (2020)

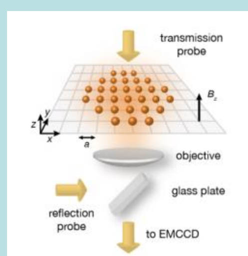


A.Orioli, et.al.
PRAb101.4 (2020)

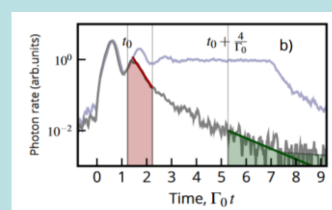
3D



Cidrim , et.al.
PRL, (2021)



Experiment:
J.Rui, et.al. , Nature (2020)



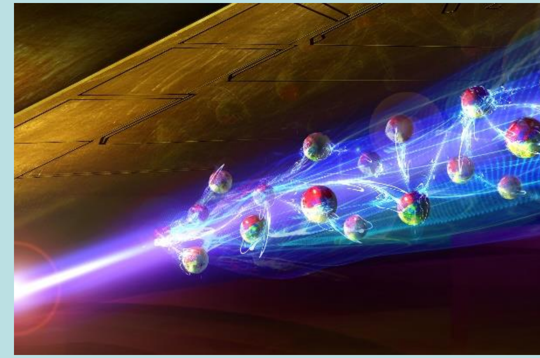
A. Glicenstein,
Opt.Lett. 2022

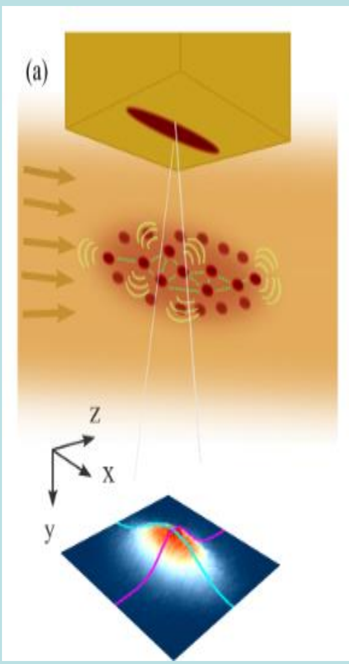
Conclusion

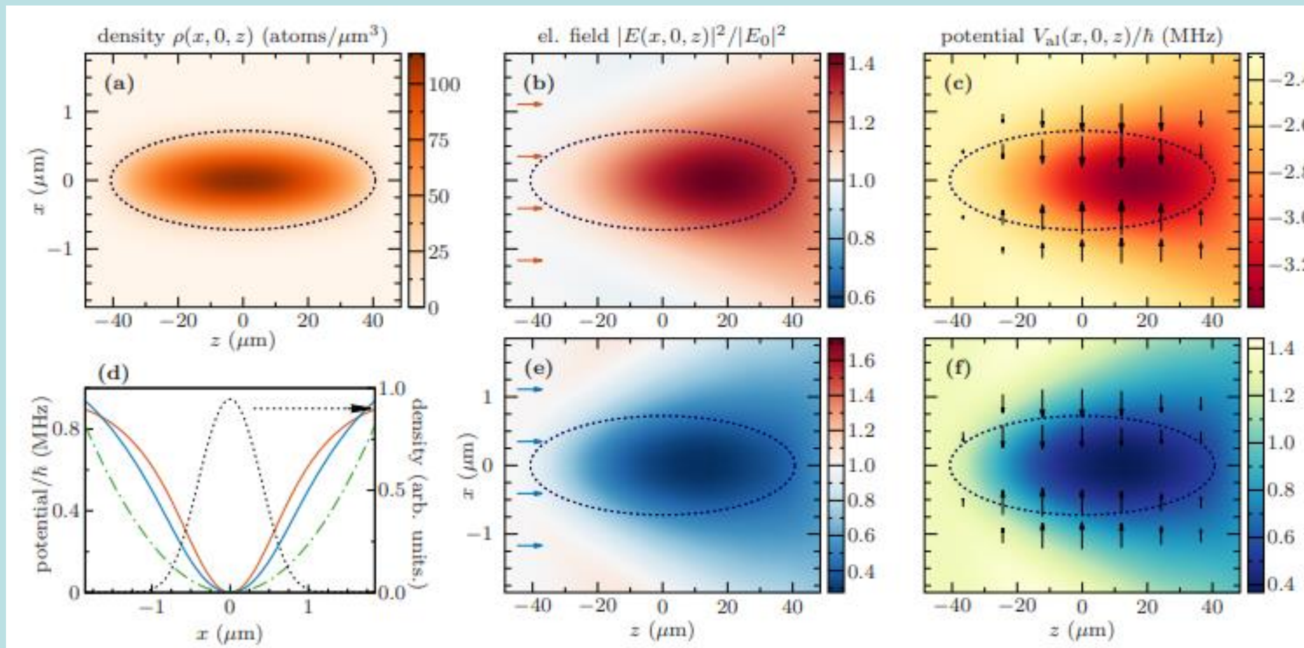
Nanophotonic structure or resonator induced
coherent light scattering
is a versatile tool to study
fundamental physics problems
from nonlinear dynamics and self-ordering
to a full quantum simulator

Transverse optical binding of a BEC and light

very long trap in 3D
=>
light guiding by the atoms trapped by the light







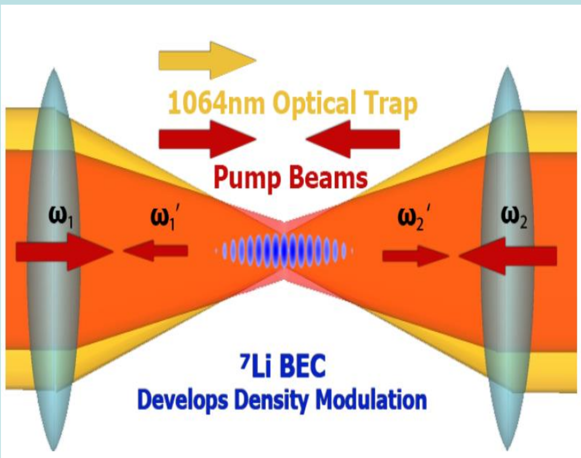
atomic compression occurs for red abd blue detuned light !

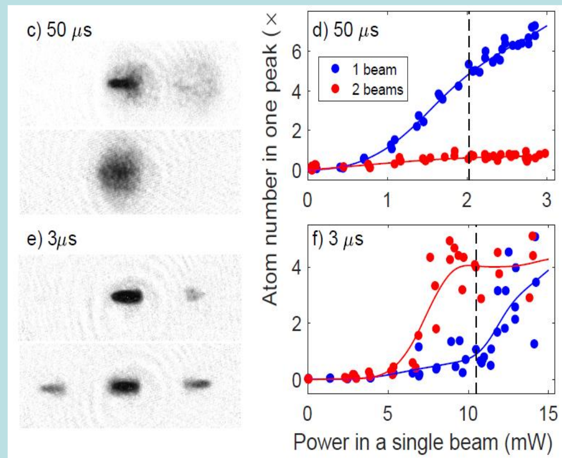
Maiwöger et.al., PRX 12, 031018

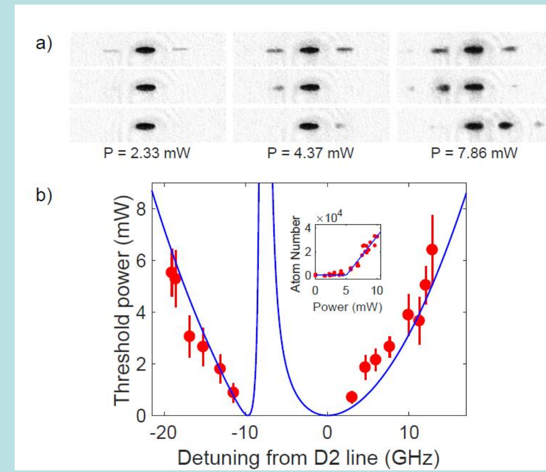
Experiment: Ketterle group (2018)

Observation of New Regimes of Superradiance in Bose-Einstein Condensates

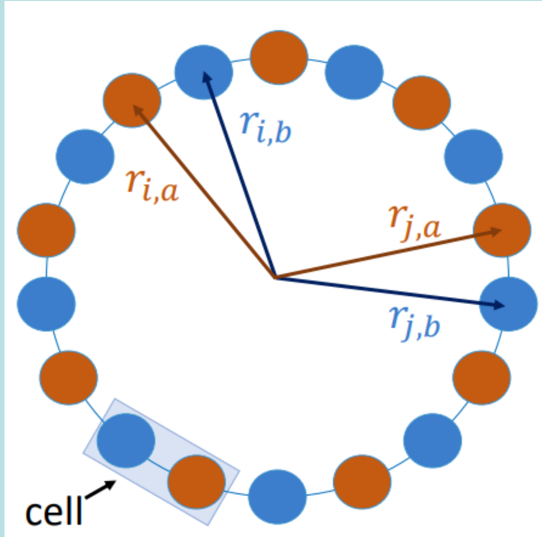
Ivana Dimitrova,[1] William Lunden,[1] Jesse Amato-Grill,[1,2] Niklas Jepsen,[1] Yichao Yu,[1,*] Michael Messer,[1,†] Thomas Rigaldo,[1,‡] Graciana Puentes,[1,§] David Weld,[1,¶] and Wolfgang Ketterle[1]

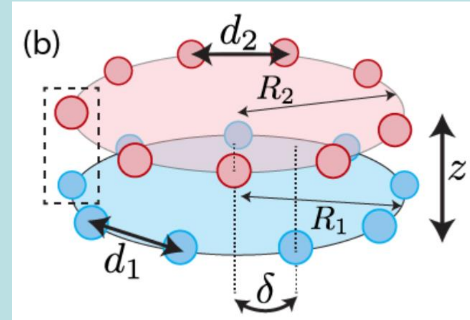


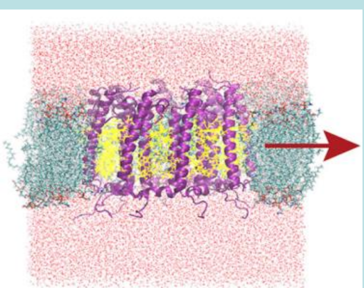


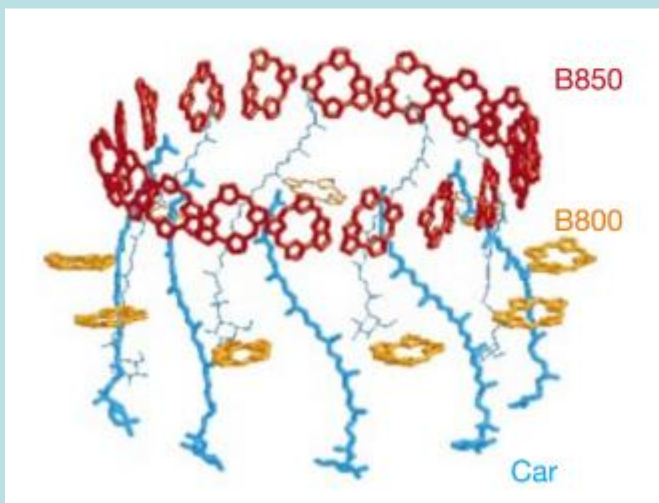


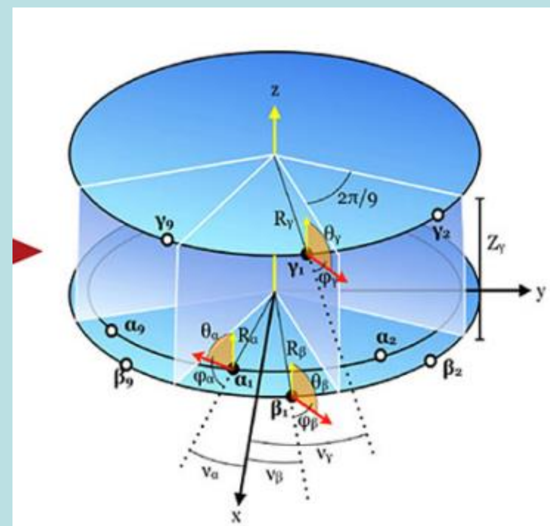
- evidence of crystallisation instability observed, but no stable crystal due to short lifetime
- elongated trap in 3D: light guiding through atoms adds tranverse confinement









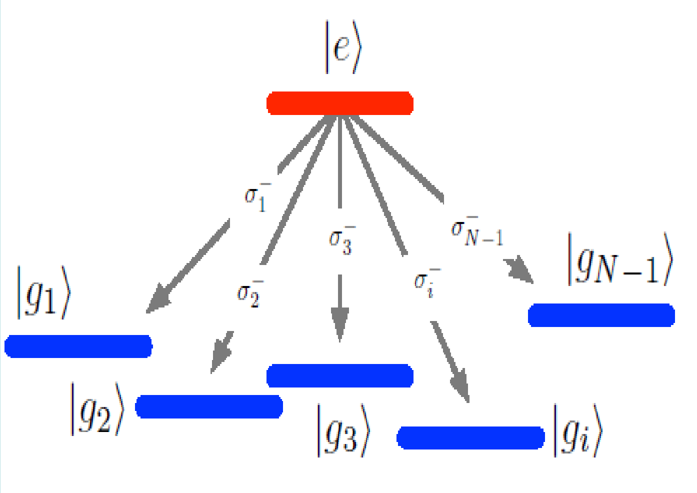


D. Montemayor, E. Rivera, and S. Jang, "Computational modeling of exciton-bath hamiltonians for lh2 and lh3 complexes of purple photosynthetic bacteria at room temperature," *The Journal of Physical Chemistry B*, vol. 122, 03 2018.

R_α / Å	R_β / Å	θ_α / °	θ_β / °	ν_α / °
26.0 ± 0.1	27.5 ± 0.1	96.5 ± 0.7	97.3 ± 0.9	-10.2 ± 0.07

ν_β / °	ϕ_α / °	ϕ_β / °	μ_α / D	μ_β / D
10.2 ± 0.07	-106.6 ± 0.6	60.0 ± 0.9	6.41 ± 0.02	6.30 ± 0.02

Generalized dark state



$$|\psi_{\mathrm{d}}^N\rangle = \frac{1}{\sqrt{N!}} \sum_{\pi \in S_N} \mathrm{sgn}(\pi) \bigotimes_i |s_{\pi(i)}\rangle$$

generalized dark state for **N decay** channels requires **N+1 atoms** !

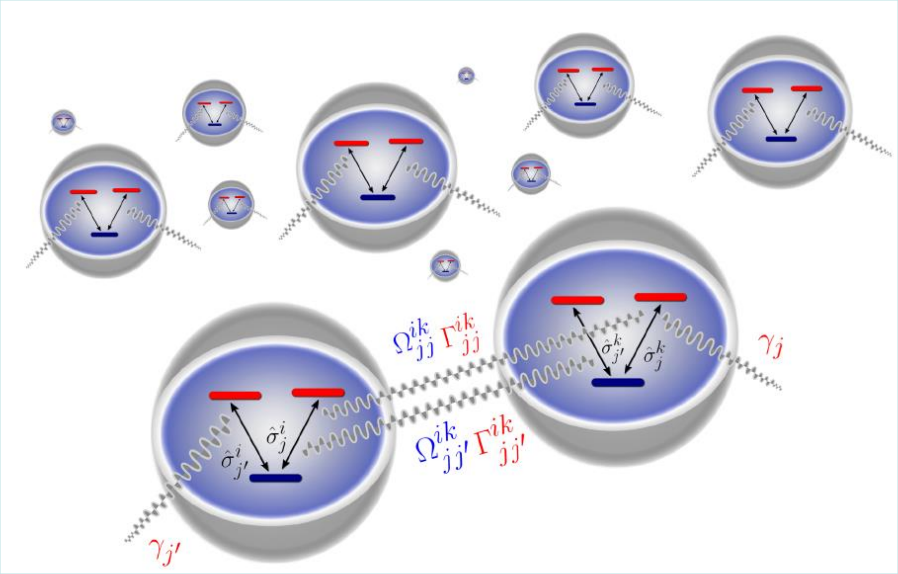
➔

at least one atom per final state

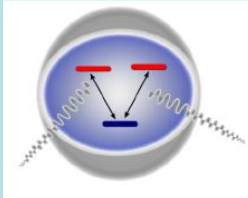
- all dipole moments vanish for totally antisymmetric state
- bipartite entanglement is maximal for any pairs
- no individual information in one atom
- Sub-radiance (darkness) is non-classical and non-local !!!

M. Hebenstreit, B. Kraus, L. Ostermann, H. R. - PRL 2017)

Subradiance and entanglement for states with several excitations

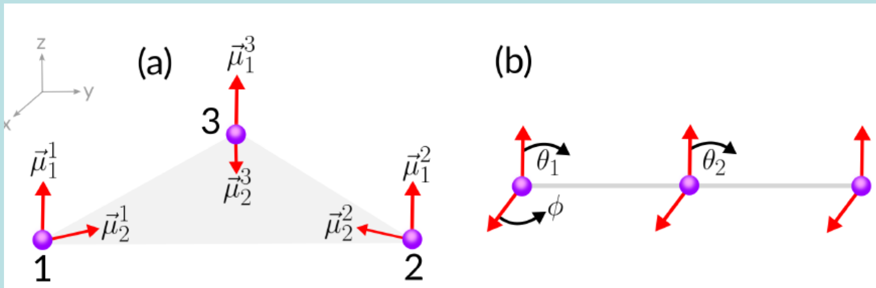


Are there long lived states with two or more excitations ?



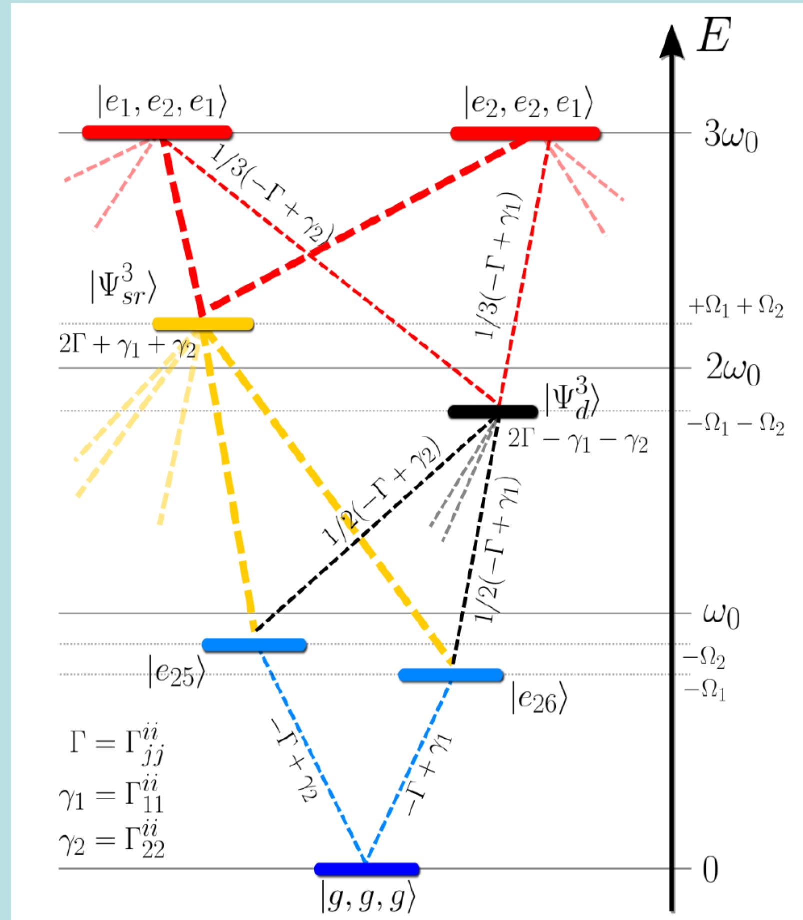
3 V-type atoms with two transitions to same state

level diagram complicated

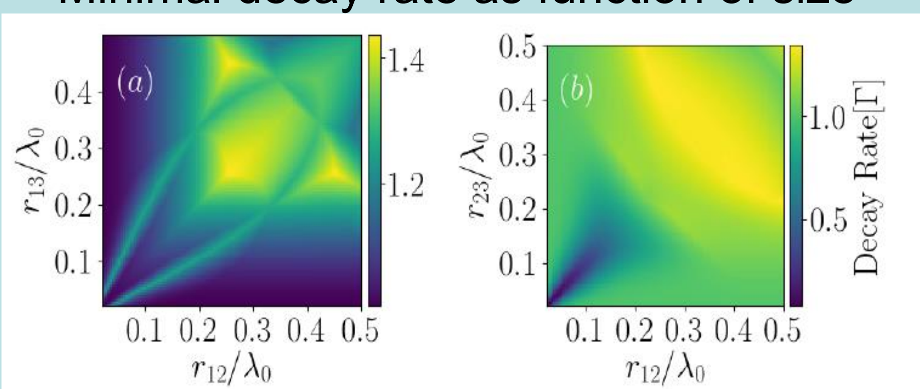


2 -excitation antisymmetric state

$$|\Psi_d^3\rangle = \frac{1}{\sqrt{6}}\{|e_1e_2g\rangle + |ge_1e_2\rangle + |e_2ge_1\rangle - |e_1ge_2\rangle - |e_2e_1g\rangle - |ge_2e_1\rangle\}$$



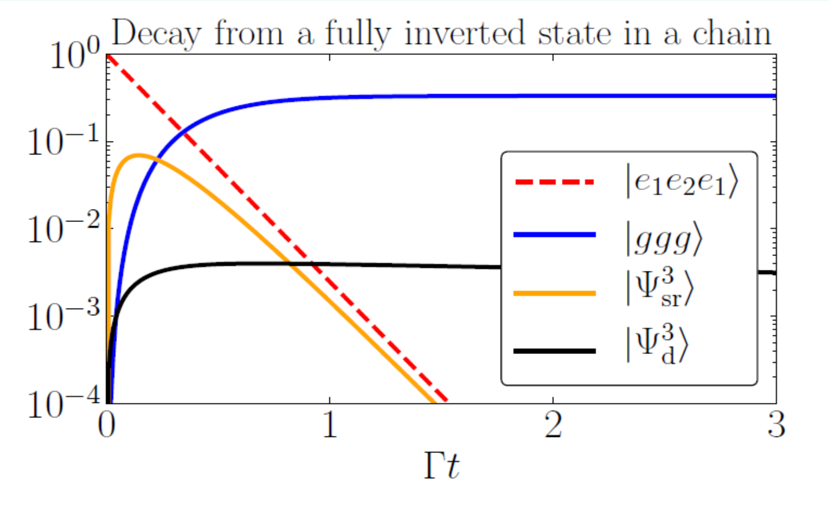
Minimal decay rate as function of size



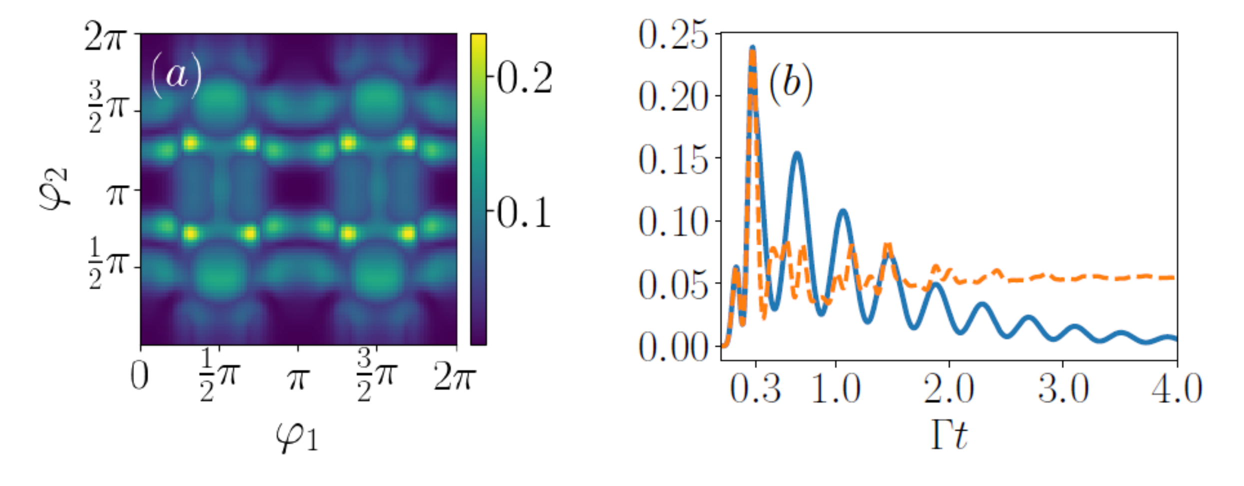
triangle   chain

- two excitation dark states exist !
- single ground state atom can stabilize N-excitations

Preparation via decay from totally inverted states:



Preparation via continuous laser excitation with phase control:



Two excitation dark states can be probablistically prepared !