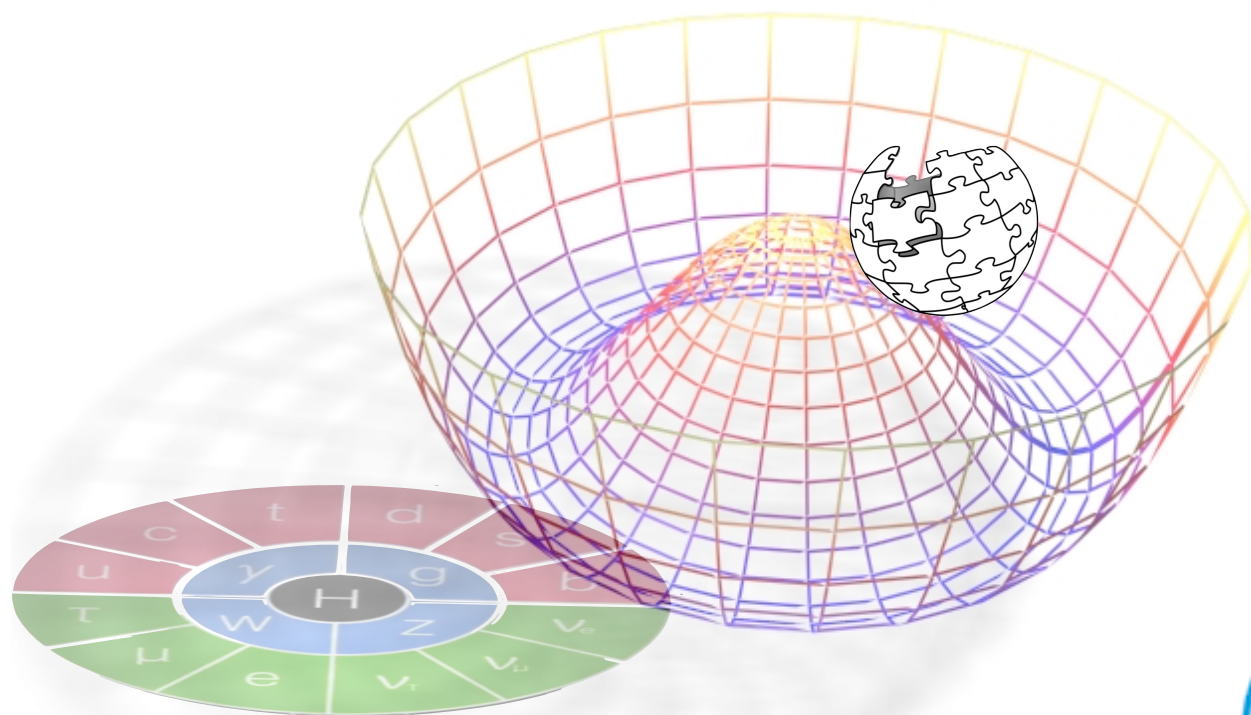


towards ~~into~~ Journeys into Effective Field Theories

Journeys into Theoretical Physics

Sao Paulo, July 2026

Lecture 1/4



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Outline

□ Wednesday: Role of symmetries

- Lagrangians
- Lorentz symmetry - scalars, fermions, gauge bosons
- Gauge/local symmetry as dynamical principle - Example: $U(1)$ electromagnetism

□ Thursday AM: Weak interactions - dimensional analysis

- Nuclear decay, Fermi theory and weak interactions: $SU(2)$
- Strong interactions: $SU(3)$
- Dimensional analysis: cross-sections and life-time computations made simple

□ Thursday PM: Chirality

- Chirality of weak interactions
- Pion decay

□ Friday: Higgs mechanism

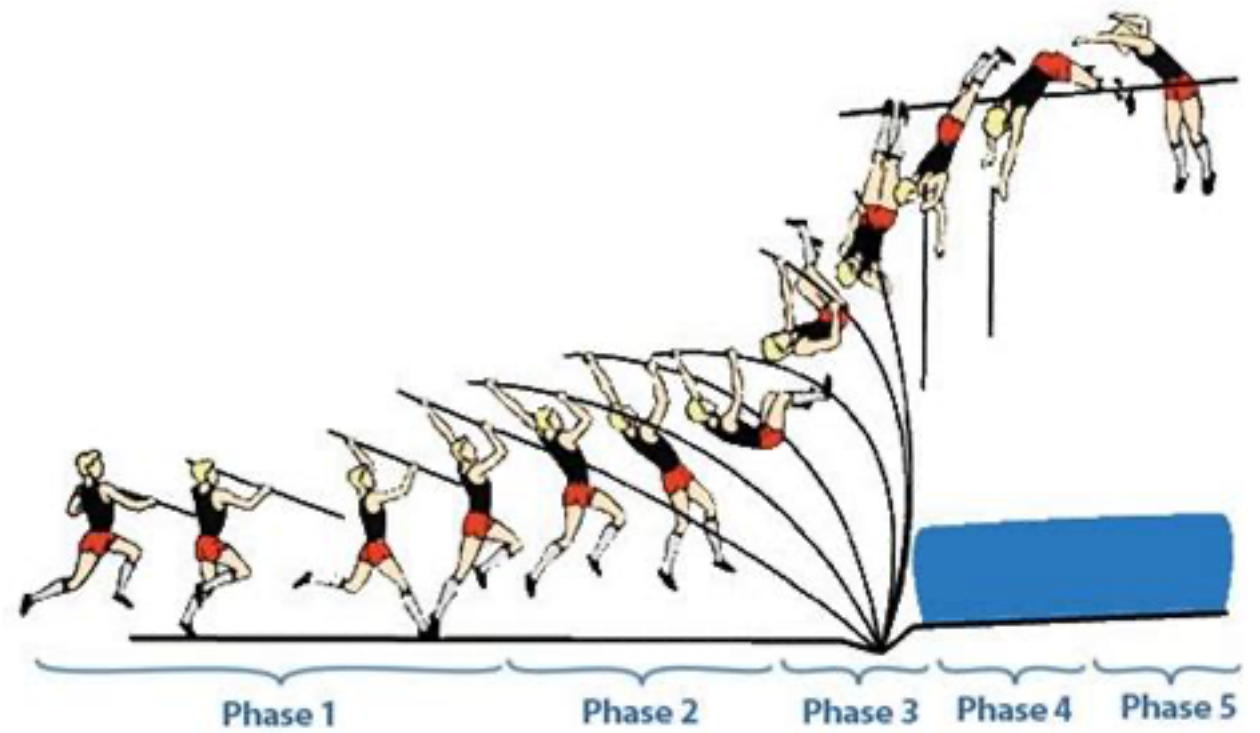
- Spontaneous symmetry breaking and Higgs mechanism
- Lepton and quark masses, quark mixings
- Neutrino masses

Citius, Altius, Fortius

How high can a human jump with a pole?



[link](#)



[link](#)

Citius, Altius, Fortius

How high can a human jump with a pole?

Physics (energy conservation) tells us that longer poles don't help!

$$\Delta h = \frac{v^2}{2g} \quad \text{feet speed: } 44.72 \text{ km/h}$$

(Usain Bolt, Berlin, August 2009, between 60m and 80m)

$$\Delta h = 7.62 \text{ m}$$

Over the years, we have learnt a few other **conservation laws** that tell us what an athlete/a particle can do or cannot do.

— Remarkable breakthrough in the understanding of Nature: —
forces among particles are associated to symmetries

- conservation of E → invariance by (time)-translation
- electro-magnetic forces → (local) invariance by phase rotation of particle wavefunctions

The Standard Model of Particle Physics
Lorentz symmetry + internal SU(3)xSU(2)xU(1) symmetry

Role(s) of Symmetry

— Selection Rules —

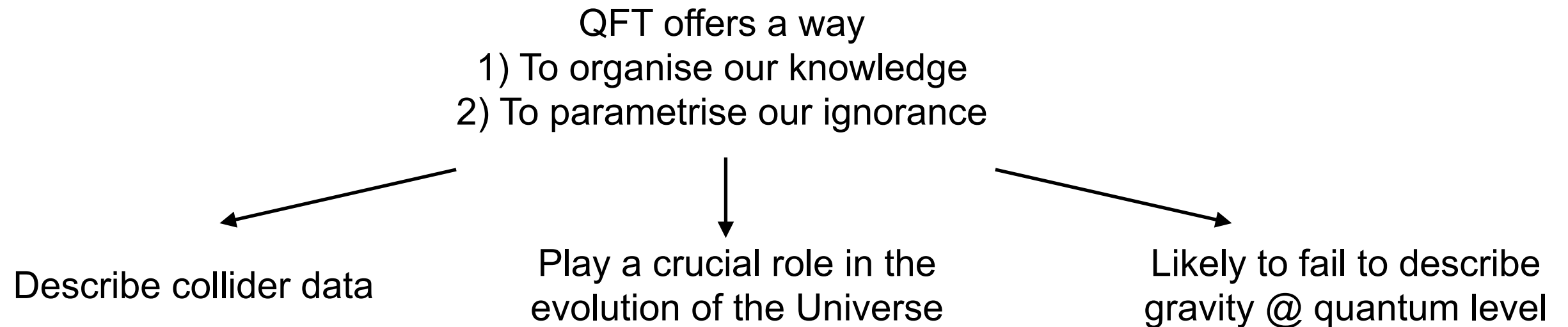
- hydrogen atom: energy levels depends on n , but not on l , nor m
(invariance under rotations as well as another symmetry that leaves the Runge-Lenz vector invariant)
- electric charge conservation: $e^+e^- \xrightarrow{\checkmark} \gamma$ but $e^+\gamma \xrightarrow{\times} e^-$

— Dynamical Principle —

Demanding that theory describing SM particles is invariant under some (local) symmetries requires the existence of interactions among these particles. And these interactions have a particular structure.

SM= S(R+Q)M

The fundamental constituents of matter obey the laws of **Quantum Mechanics** and **Special Relativity**
They are described in the framework of **Quantum Field Theory (QFT)**



"Before breaking the rules, you first need to master them"

Goals of the lectures

1. Explain QFT to describe the SM particles and their interactions
2. Introduce the principles to build a model of Nature
3. Explain how to compute cross-section and decay rate
4. Unveil clues where the SM might fail

Lagrangians

The Newton law of classical mechanics

$$\vec{F} = m\vec{a} \quad \text{or} \quad V'(x) = -m\ddot{x}$$

can be obtained by requiring the least action principle

$$\delta S = 0$$

where

the action: $S = \int_{t_1}^{t_2} dt \mathcal{L}(x, \dot{x})$ with the (classical) Lagrangian: $\mathcal{L}(x, \dot{x}) = \frac{1}{2}m\dot{x}^2 - V(x)$
(Hamiltonian/energy: $\mathcal{H} = \dot{x} \frac{\delta \mathcal{L}}{\delta \dot{x}} - \mathcal{L} = \frac{1}{2}m\dot{x}^2 + V(x)$)

Euler-Lagrange
equations

$$\delta S = \int_{t_1}^{t_2} dt \left(\frac{\delta \mathcal{L}}{\delta x} - \frac{d}{dt} \frac{\delta \mathcal{L}}{\delta \dot{x}} \right) \delta x + \text{boundary terms} = 0 \quad \Rightarrow \quad \frac{\delta \mathcal{L}}{\delta x} = \frac{d}{dt} \frac{\delta \mathcal{L}}{\delta \dot{x}}$$

For the classical Lagrangian: $-V'(x) = m\ddot{x}$

Lagrangians for Particle Physics

Equations of motion, like $\vec{F} = m\vec{a}$, are **covariant** under the action of a symmetry.

Lagrangians are **invariant**.

That makes identifying the symmetries of Nature much easier.

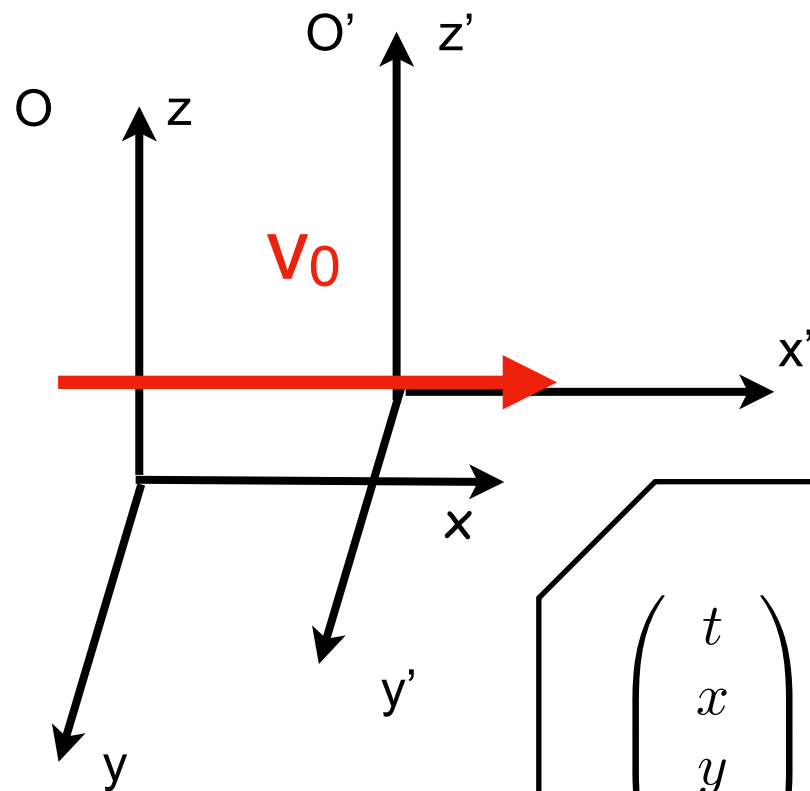
—Particle Physics—

particles $\leftrightarrow$ fields with specific transformation properties under some fundamental symmetries

build a Lagrangian (i.e. a function of these fields and their space-time derivatives) that remains invariant under the action of the symmetry transformations.

Which symmetries?

Lorentz Transformations



Consider two observers

in relative motion with a constant speed v_0 along the x-axis
they use their own systems of coordinates (t, x, y, z) and (t', x', y', z')

Galilean transformations

$$\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} t' = t \\ x' = -\beta_0 ct + x \\ y' = y \\ z' = z \end{pmatrix} \quad \text{with} \quad \beta_0 = \frac{v_0}{c}$$

in particular

$$v' = v - v_0$$

the speed can
be arbitrarily large.

Lorentz transformations

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} ct' = \gamma_0 (ct - \beta_0 x) \\ x' = \gamma_0 (-\beta_0 ct + x) \\ y' = y \\ z' = z \end{pmatrix} \quad \text{with} \quad \beta_0 = \frac{v_0}{c} \quad \gamma_0 = \frac{1}{\sqrt{1 - \beta_0^2}}$$

in particular

$$v' = \frac{v - v_0}{1 - v \cdot v_0 / c^2}$$

The speed of light is
the same for all observers:

if $v=c$ then $v'=c$ too

Note: $\Delta^2 \equiv (ct)^2 - x^2 - y^2 - z^2 = (ct')^2 - x'^2 - y'^2 - z'^2 \equiv \Delta'^2$

Einstein Algebra

$$x^\mu \equiv \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \quad \mu = 0, 1, 2, 3$$

$$\Delta^2 \equiv (ct)^2 - x^2 - y^2 - z^2 = \eta_{\mu\nu} x^\mu x^\nu \quad \text{with} \quad \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1) \quad \text{Minkowski metric}$$

$$x^\mu \rightarrow x'^\mu = \Lambda^\mu_{\nu} x^\nu \quad \text{leaves } \Delta^2 \text{ invariant iff} \quad \eta_{\mu\nu} = \eta_{\mu'\nu'} \Lambda^{\mu'}_{\mu} \Lambda^{\nu'}_{\nu}$$

At linear order, $\Lambda^\mu_{\nu} \approx \delta^\mu_{\nu} + \omega^\mu_{\nu}$, the invariance of Δ^2 simply writes $\omega_{\mu\nu} + \omega_{\nu\mu} = 0$
 where we have defined $\omega_{\mu\nu} \equiv \eta_{\mu\mu'} \omega^{\mu'}_{\nu}$

boost along x-direction

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} ct' = \gamma_0 (ct - \beta_0 x) \\ x' = \gamma_0 (-\beta_0 ct + x) \\ y' = y \\ z' = z \end{pmatrix} \quad \text{corresponds to} \quad \Lambda^\mu_{\nu} = \begin{pmatrix} \gamma & -\gamma\beta & & \\ -\gamma\beta & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

indeed satisfies $\eta_{\mu\nu} = \eta_{\mu'\nu'} \Lambda^{\mu'}_{\mu} \Lambda^{\nu'}_{\nu}$ since $\gamma^2(1 - \beta^2) = 1$

Exercise: find the expression of Λ^μ_{ν} for a boost along a general space direction

Scalar Lagrangian

A (real) **scalar** field ϕ

is a real function of space-time coordinates that doesn't change under Lorentz transformations

$$x^\mu \rightarrow x'^\mu = \Lambda^\mu{}_\nu x^\nu$$

$$\phi(x) \rightarrow \phi'(x') = \phi(x)$$

Lorentz invariant Lagrangian for scalar field?

- any potential $V(\phi)$ is automatically invariant
- kinetic term?

$$\begin{aligned} x^\mu \rightarrow x'^\mu = \Lambda^\mu{}_\nu x^\nu \\ \phi(x) \rightarrow \phi'(x') = \phi(x) \end{aligned} \Rightarrow \partial_\mu \phi = \Lambda^\nu{}_\mu \partial'_\nu \phi' \Rightarrow \partial_\mu \phi \partial^\mu \phi = \underbrace{\eta^{\mu\nu} \Lambda^{\mu'}{}_\mu \Lambda^{\nu'}{}_\nu}_{\eta^{\mu'\nu'} \text{ (Lorentz transformation)}} \partial'_{\mu'} \phi' \partial'_{\nu'} \phi' = \eta^{\mu'\nu'} \partial'_{\mu'} \phi' \partial'_{\nu'} \phi'$$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)$$

Eq. of motion: $0 = \delta \mathcal{L} = \left(-\partial_\mu \partial^\mu \phi - \frac{\partial V}{\partial \phi} \right) \delta \phi$ i.e. $\square \phi = -V'(\phi)$ Klein-Gordon equation

Equations of Motion of Elementary Particles

Schrödinger Equation (1926):
$$\left(i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \Delta - V \right) \Phi = 0$$

$$E = \frac{p^2}{2m} + V \quad \text{classical} \leftrightarrow \text{quantum} \\ \text{correspondance}$$

$$E \rightarrow i\hbar \frac{\partial}{\partial t} \quad \& \quad p \rightarrow i\hbar \frac{\partial}{\partial x}$$

Klein-Gordon Equation (1927):
$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta + \frac{m^2 c^2}{\hbar^2} \right) \Phi = 0$$

$$\frac{E^2}{c^2} = p^2 + m^2 c^2$$

Dirac Equation (1928):

$$\left(i\gamma^\mu \partial_\mu - \frac{mc}{\hbar} \right) \Psi = 0$$

$$E = \begin{cases} +\sqrt{p^2 c^2 + m^2 c^4} & \text{matter} \\ -\sqrt{p^2 c^2 + m^2 c^4} & \text{antimatter} \end{cases}$$

$$E = \vec{\alpha} \vec{p} c + \beta m c^2$$

$$\gamma^0 = \beta, \quad \gamma^i = \beta \alpha^i, \quad \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

positron (e^+) discovered by C. Anderson in 1932

Fermion Lagrangian

$$\mathcal{L} = \psi^\dagger \gamma^0 (i\gamma^\mu \partial_\mu - m) \psi$$

ψ 4-component Dirac spinor
describes a spin-1/2 particle
when quantised

γ^μ ($\mu = 0, 1, 2, 3$) are four 4x4 matrices

- **Equation of motion:**

$$0 = \delta \mathcal{L} = \psi^\dagger \gamma^0 (i\gamma^\mu \partial_\mu - m) \delta \psi$$

Dirac equation

$$(i\gamma^\mu \partial_\mu - m) \psi = 0$$

- **Lorentz invariance:** (see technical slides at the end of the lecture)

$$x^\mu \rightarrow x'^\mu = (\delta^\mu_\nu + \omega^\mu_\nu) x^\nu \quad \text{with} \quad \omega_{\mu\nu} + \omega_{\nu\mu} = 0$$

$$\psi(x) \rightarrow \psi'(x') = \left(1_4 + \frac{1}{8} \omega_{\mu\nu} [\gamma^\mu, \gamma^\nu] \right) \psi(x)$$

- **Dirac algebra:**

For this equation to be consistent with Einstein equation ($m^2 = E^2 - p^2$) or Klein-Gordon eq., the γ^μ matrices have to obey the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

- **Dirac matrices:** One particular realisation of the Dirac algebra (not unique)

$$\gamma^0 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & -1 & & \\ -1 & & & \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} & & & -i \\ & & i & \\ & i & & \\ -i & & & \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} & & & 1 \\ & & & -1 \\ -1 & & & \\ & 1 & & \end{pmatrix}$$

U(1) Gauge Symmetry — QED

Quantum ElectroDynamics : the phase of an electron is not physical and can be rotated away
(internal symmetry, same transformation in all Dirac components)

$$\psi \rightarrow e^{i\theta} \psi$$

If the phase transformation is **local**, i.e., depends on space-time coordinate, then

$$\partial_\mu \psi \rightarrow e^{i\theta} (\partial_\mu \psi + i(\partial_\mu \theta) \psi)$$

and the kinetic term is no-longer invariant due to the presence of the non-homogenous piece

To make the theory invariant under **local** transformation, one needs to introduce a **gauge field** that keeps track/memory of how the phase of the electron changes from one point to another.

For that, we build a **covariant derivative** that has nice homogeneous transformations

$$D_\mu \psi = \partial_\mu \psi + ieA_\mu \psi \rightarrow e^{i\theta} D_\mu \psi \quad \text{iff} \quad A_\mu \rightarrow A_\mu - \frac{1}{e} \partial_\mu \theta$$

$$\mathcal{L} = \psi^\dagger \gamma^0 (i\gamma^\mu D_\mu - m) \psi$$

invariant under

- Lorentz transformation
- local phase rotation

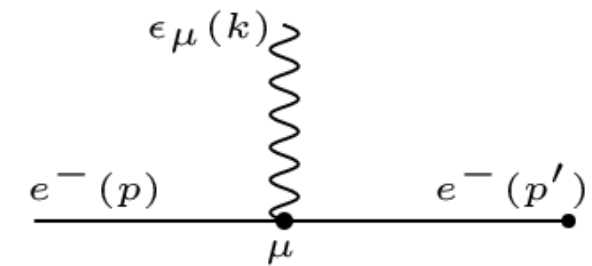
Dynamical Principle

$$\mathcal{L} = \psi^\dagger \gamma^0 (i\gamma^\mu D_\mu - m) \psi$$



interaction between
gauge field (aka photon) and electron

$$eA_\mu \psi^\dagger \gamma^0 \gamma^\mu \psi$$

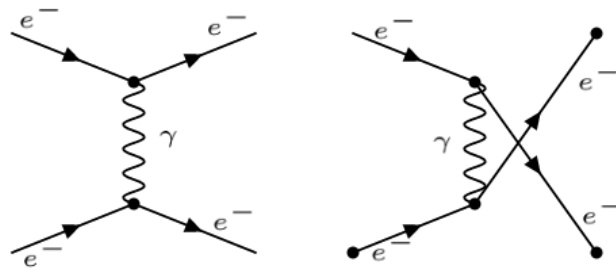


Gauge invariance is a dynamical principle: it predicts some interactions among particles.

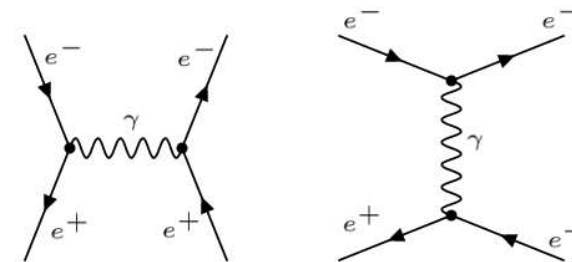
It also explains why the QED interactions are universal
(an electron interacts with a photon in the same way on Earth, on the Moon and at the outskirts of the Universe)

— Some examples of QED processes —

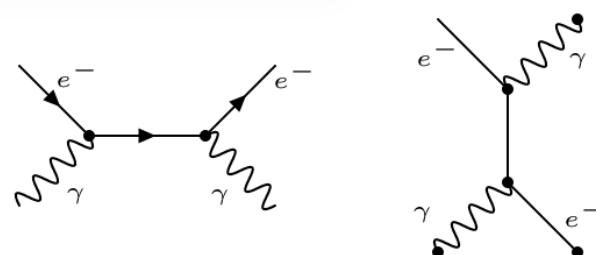
- Moeller scattering : $e^- + e^- \rightarrow e^- + e^-$



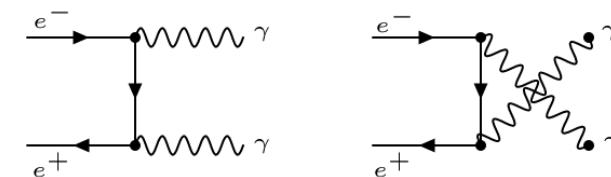
- Bhabha scattering : $e^- + e^+ \rightarrow e^- + e^+$



- Compton scattering : $e^- + \gamma \rightarrow e^- + \gamma$



- Pair annihilation : $e^- + e^+ \rightarrow \gamma + \gamma$



$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

Gauge Field Kinetic Term

To build the QED Lagrangian, we had to introduce a new field A_μ
it is propagating degree of freedom we need to add a kinetic term in the Lagrangian.

Tensor field strength: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

- **Lorentz transformations:**
$$\begin{aligned} x^\mu &\rightarrow x'^\mu = \Lambda^\mu{}_\nu x^\nu \\ A^\mu &\rightarrow A'^\mu = \Lambda^\mu{}_\nu A^\nu \end{aligned} \quad \Rightarrow \quad F^{\mu\nu} \rightarrow F'^{\mu\nu} = \Lambda^\mu{}_\rho \Lambda^\nu{}_\sigma F^{\rho\sigma}$$
- **U(1) gauge transformations:**
$$A_\mu \rightarrow A_\mu - \frac{1}{e} \partial_\mu \theta \quad \Rightarrow \quad F_{\mu\nu} \rightarrow F_{\mu\nu}$$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

invariant under

- Lorentz transformation
- local phase rotation

equations of motion $\leftrightarrow$ **Maxwell equations** of electromagnetism

A^0 =EM scalar potential, $A^{i=1,2,3}$ = EM vector potential

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla} \wedge \vec{A} \quad F^{\mu\nu} = \begin{pmatrix} 0 & -\vec{E}_x & -\vec{E}_y & -\vec{E}_z \\ \vec{E}_x & 0 & -\vec{B}_z & \vec{B}_y \\ \vec{E}_y & \vec{B}_z & 0 & -\vec{B}_x \\ \vec{E}_z & -\vec{B}_y & \vec{B}_x & 0 \end{pmatrix} \quad \Rightarrow \quad \partial_\mu F^{\mu\nu} = J^\nu$$

Remark: no interaction among photons (photons only interact with electrically charged fields)

SU(N) non-Abelian Gauge Symmetry

We generalise the QED construction by considering general transformation of a N-vector

$$\phi \rightarrow U\phi$$

We build a **covariant derivative** that again has nice homogeneous transformations

$$D_\mu \phi = \partial_\mu \phi + igA_\mu \phi \rightarrow UD_\mu \phi \quad \text{iff} \quad A_\mu \rightarrow UA_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$$

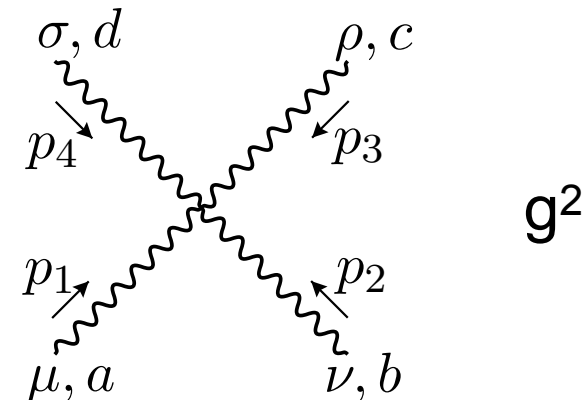
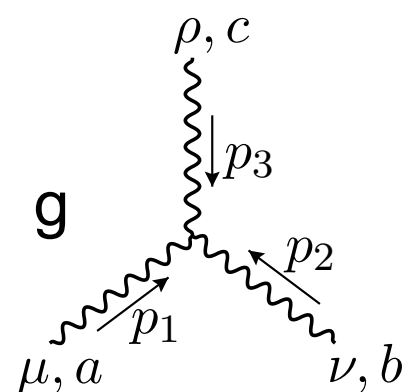
g is the gauge coupling and defines the strength of the interactions

For the field strength to transform homogeneously, one needs to add a non-Abelian piece

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \rightarrow UF_{\mu\nu}U^{-1}$$

Contrary to the Abelian case, the gauge fields are now charged and interact with themselves

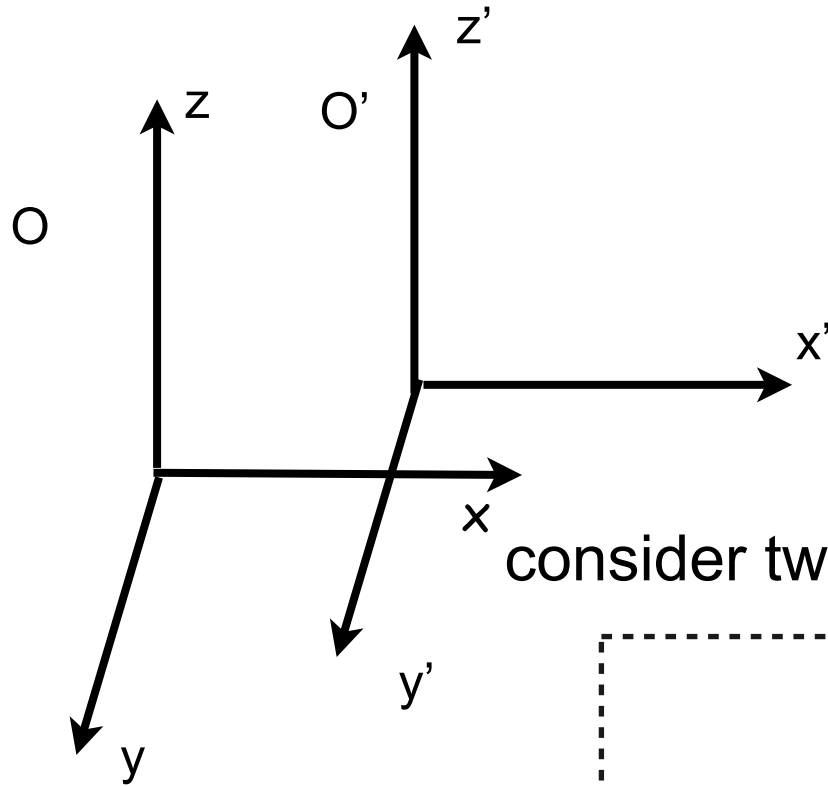
$$\mathcal{L}_{\text{kin}} = \text{Tr} F_{\mu\nu} F^{\mu\nu} \supset g \partial A A A + g^2 A A A A$$



$\exists$ gauge boson self-interactions

Technical Details for Advanced Students

Time-ordering $\neq$ Causality



$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} ct' = \gamma_0 (ct - \beta_0 x) \\ x' = \gamma_0 (-\beta_0 ct + x) \\ y' = y \\ z' = z \end{pmatrix}$$

“time dilation + space contraction”

consider two events E_1 and E_2 characterised by their space-time coordinates

$$\begin{array}{cc} E_1 \\ t_1 = 0 & t'_1 = 0 \\ x_1 = 0 & x'_1 = 0 \end{array}$$

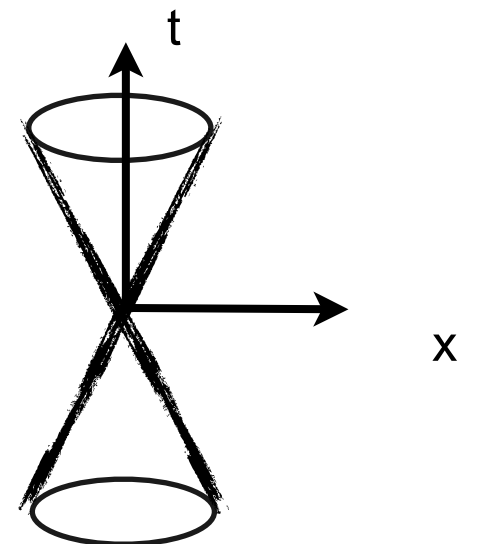
$$\begin{array}{cc} E_2 \\ t_2 > 0 & ct'_2 = \gamma (ct_2 - \beta x_2) \\ x_2 > 0 & x'_2 = \gamma (-\beta ct_2 + x_2) \end{array}$$

t'_2 can be positive or negative
causality $\neq$ time ordering

Proper space-time distance Δ is independent of the observer:

$$\Delta'^2 = (ct'_2)^2 - (x'_2)^2 = (ct_2)^2 - x_2^2 = \Delta^2$$

Only events inside the past/future light cones are causally connected
The light cones are invariant under Lorentz transformations



Spinor Transformation

Transformation law: $\psi(x) \rightarrow \psi'(x') = S(\Lambda)\psi(x)$

We want the Dirac equation to take the same form in the two systems of coordinates x and x'

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \qquad (i\gamma^\mu \partial'_\mu - m)\psi' = 0$$

This implies the condition: $S\gamma^\mu \Lambda^\nu{}_\mu S^{-1} = \gamma^\nu$

We consider small Lorentz transformations: $\Lambda_\mu{}^\nu = \delta_\mu^\nu + \omega^\mu{}_\nu$ $S = 1 - \frac{i}{4}\sigma^{\mu\nu}\omega_{\mu\nu}$

The covariance of the Dirac equation then implies that the matrices $\sigma_{\mu\nu}$ have to satisfy the relation

$$[\gamma^\nu, \sigma^{\rho\sigma}] = 2i(\eta^{\nu\rho}\gamma^\sigma - \eta^{\nu\sigma}\gamma^\rho)$$

It is easy to check that the following matrices fit the bill: $\sigma^{\rho\sigma} = \frac{i}{2}[\gamma^\rho, \gamma^\sigma]$

$$x^\mu \rightarrow x'^\mu = (\delta^\mu{}_\nu + \omega^\mu{}_\nu)x^\nu \quad \text{with} \quad \omega_{\mu\nu} + \omega_{\nu\mu} = 0$$

$$\psi(x) \rightarrow \psi'(x') = \left(1_4 + \frac{1}{8}\omega_{\mu\nu}[\gamma^\mu, \gamma^\nu]\right)\psi(x)$$

Lorentz-invariant Lagrangian

$\mathcal{L} = \psi^\dagger M (i\gamma^\mu \partial_\mu - m) \psi$ is Lorentz-invariant iff $\gamma^0[\gamma^\nu, \gamma^\mu]\gamma^0 M + M[\gamma^\mu, \gamma^\nu] = 0$

$M = \gamma^0$ is a solution and it defines the Dirac Lagrangian. $\bar{\psi} \equiv \psi^\dagger \gamma^0$

Symmetries and invariants

SU(N)

the transformations among the components of a complex N-vector that leaves its norm invariant

$$|\phi|^2 = \phi_1^* \phi_1 + \dots \phi_N^* \phi_N \rightarrow |\phi'|^2 = |\phi|^2$$

SU(N,M)

the transformations among the components of a complex (N+M)-vector that leaves its (N,M) norm invariant

$$|\phi|^2 = \phi_1^* \phi_1 + \dots \phi_N^* \phi_N + \phi_{N+1}^* \phi_{N+1} - \dots - \phi_{N+M}^* \phi_{N+M} \rightarrow |\phi'|^2 = |\phi|^2$$

SO(N)

the transformations among the components of a real N-vector that leaves its norm invariant

$$|\phi|^2 = \phi_1^2 + \dots \phi_N^2 \rightarrow |\phi'|^2 = |\phi|^2$$

SO(N,M)

the transformations among the components of a real (N+M)-vector that leaves its (N,M) norm invariant

$$|\phi|^2 = \phi_1^2 + \dots \phi_N^2 + \phi_{N+1}^2 - \dots - \phi_{N+M}^2 \rightarrow |\phi'|^2 = |\phi|^2$$

The Lorentz group is thus SO(1,3)

Lorentz transformation

$\text{SO}(1,3)$

The elements of $\text{SO}(1,3)$ satisfy $U^t \eta U = \eta$ where $\eta = \text{diag}(1, -1, -1, -1)$

The infinitesimal transformations are $U = e^{\theta^a T^a} \approx 1 + \theta^a T^a + \dots$

The generators satisfy the constraints: $T^{at} \eta + \eta T^a = 0$

One particular generator is $T = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

We obtain $e^{\theta T} = \begin{pmatrix} \cosh \theta & \sinh \theta & 0 & 0 \\ \sinh \theta & \cosh \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

We indeed recover the usual Lorentz transformation with the identification

$$\gamma = \cosh \theta \quad \text{and} \quad \beta\gamma = \sinh \theta$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \quad \Leftrightarrow \quad \cosh^2 \theta - \sinh^2 \theta = 1$$