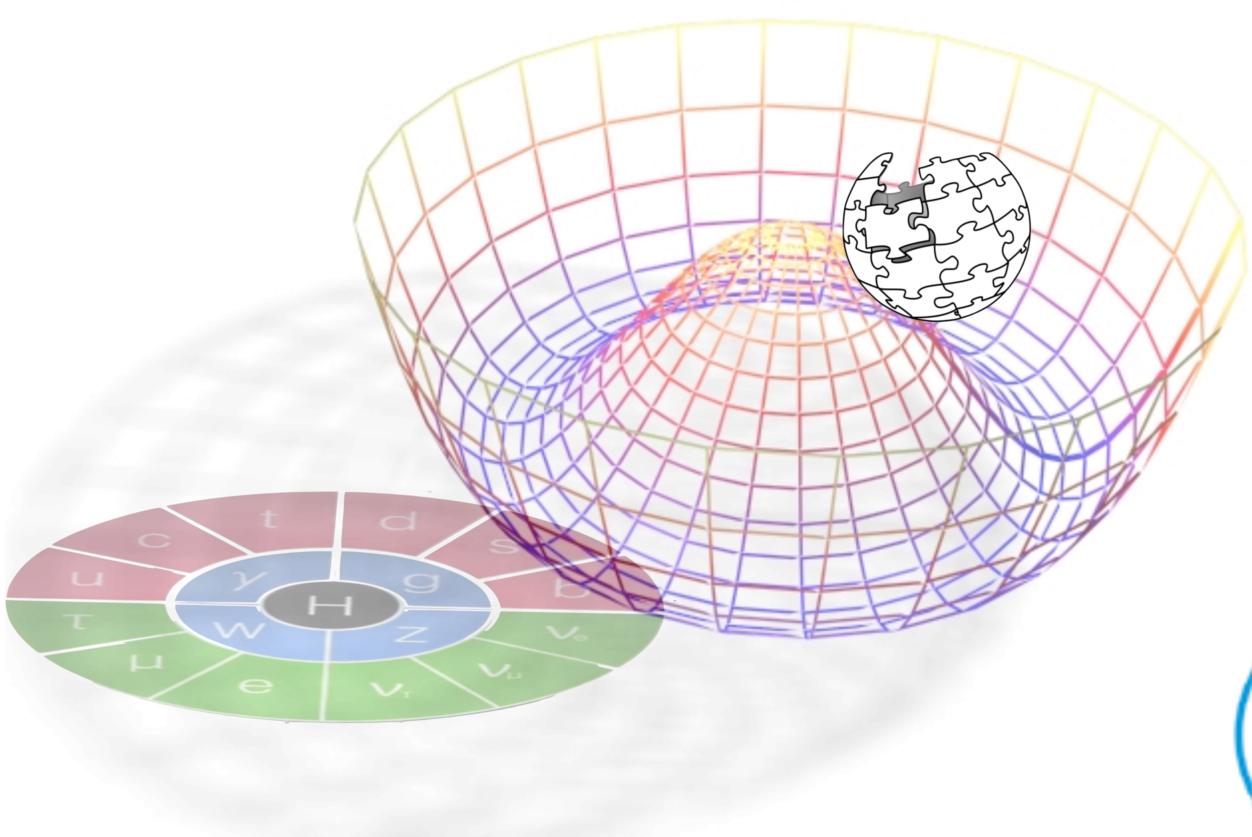
Journeys

~~into~~ towards Effective Field Theories

Journeys into Theoretical Physics

Sao Paulo, July 2026

Lecture 2/4



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Outline

Wednesday: Role of symmetries

- Lagrangians
- Lorentz symmetry - scalars, fermions, gauge bosons
- Gauge/local symmetry as dynamical principle - Example: U(1) electromagnetism

Thursday AM: Weak interactions - dimensional analysis

- Nuclear decay, Fermi theory and weak interactions: SU(2)
- Strong interactions: SU(3)
- Dimensional analysis: cross-sections and life-time computations made simple

Thursday PM: Chirality

- Chirality of weak interactions
- Pion decay

Friday: Higgs mechanism

- Spontaneous symmetry breaking and Higgs mechanism
- Lepton and quark masses, quark mixings
- Neutrino masses

Recap from Lecture #1

- **Lorentz transformation:**

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu \quad \text{with} \quad \eta_{\mu\nu} = \eta_{\mu'\nu'}\Lambda^{\mu'}{}_\mu \Lambda^{\nu'}{}_\nu$$

At <u>linear order</u>, $\Lambda^\mu{}_\nu \approx \delta^\mu{}_\nu + \omega^\mu{}_\nu$, it simply writes $\omega_{\mu\nu} + \omega_{\nu\mu} = 0$ where $\omega_{\mu\nu} \equiv \eta_{\mu\mu'}\omega^{\mu'}{}_\nu$

- **Scalar (aka spin-0) field:** $\phi(x) \to \phi'(x') = \phi(x)$

$$\boxed{\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)}$$

Eq. of motion: $\delta\mathcal{L} = 0 \Rightarrow \Box\phi = -V'(\phi)$ Klein-Gordon equation

- **Spin-1/2 field:** $\psi(x) \to \psi'(x') = \left(1_4 + \frac{1}{8}\omega_{\mu\nu}[\gamma^\mu, \gamma^\nu]\right)\psi(x)$

$$\boxed{\mathcal{L} = \psi^\dagger\gamma^0\left(i\gamma^\mu\partial_\mu - m\right)\psi}$$

Eq. of motion: $\delta\mathcal{L} = 0 \Rightarrow \left(i\gamma^\mu\partial_\mu - m\right)\psi = 0$ Dirac equation

- **U(1) gauge symmetry** $\psi \to e^{i\theta}\psi$ $\quad$ θ const. = global symm., θ(x) = local symm.

Need to promote space-time derivative to covariant derivative: $D_\mu\psi = \partial_\mu\psi + ieA_\mu\psi$

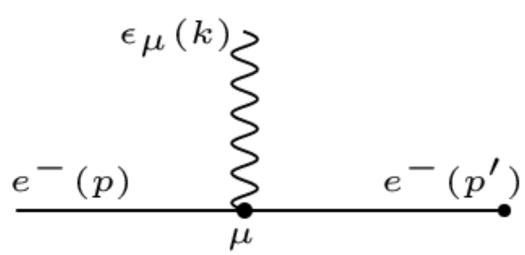
Gauge field, Aμ, transforms non-trivially under gauge transformation: $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\theta$

$$\boxed{\mathcal{L}_{\text{kin}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

$\Downarrow$

Maxwell equations

Dictate EM interactions photon-electron:



Natural & Planck Units

- $[G_N]$=mass^{-1} L^3 T^{-2}
- $[\hbar]$=mass L^2 T^{-1}
- $[c]$=L T^{-1}

- Planck mass: $M_{\rm Pl} = \sqrt{\frac{\hbar c}{G_{\rm N}}} \sim 10^{19}\,{\rm GeV/c^2} \sim 2\times 10^{-5}\,{\rm g}$
- Planck length: $l_{\rm Pl} = \sqrt{\frac{\hbar G_{\rm N}}{c^3}} \sim 10^{-33}\,{\rm cm}$
- Planck time: $\tau_{\rm Pl} = \sqrt{\frac{\hbar G_{\rm N}}{c^5}} \sim 10^{-44}\,{\rm s}$

In High Energy Physics, it is a current practise to use a system of units for which $\hbar$=1 and c=1

energy~ mass ~ distance^{-1} ~ time^{-1}

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Unit conversion: SI ↔ HEP

E	T	L
1eV	10^{-16}s	10^{-7}m
10^{-16}eV	1s	10^{9}m
10^{-7}eV	10^{-9}s	1m

- The string theorists will remember:

$$M_{\rm Pl} \sim 10^{19}\,{\rm GeV} \quad \leftrightarrow \quad \tau_{\rm Pl} \sim 10^{-44}\,{\rm s} \quad \leftrightarrow \quad l_{\rm Pl} \sim 10^{-33}\,{\rm cm}$$

- The nuclear physicists will remember:

$$\hbar c \sim 200\,{\rm MeV}\cdot{\rm fm}$$
$$10^{8}\,{\rm eV} \quad \leftrightarrow \quad 10^{-15}\,{\rm m} \quad \leftrightarrow \quad 10^{-24}\,{\rm s}$$

- The others will remember:

average mosquito m~10^{-3}g=100M_{Pl}
Compton wavelength 0.01L_{Pl}=10^{-35}cm, Schwarzschild radius 100L_{Pl}=10^{-31}cm
(much smaller than its physical size, so a mosquito is not a Black Hole)

Compton vs Schwarzschild Scales

Compton radius: for an object of mass m, one can define a length scale that will measure its quantum size

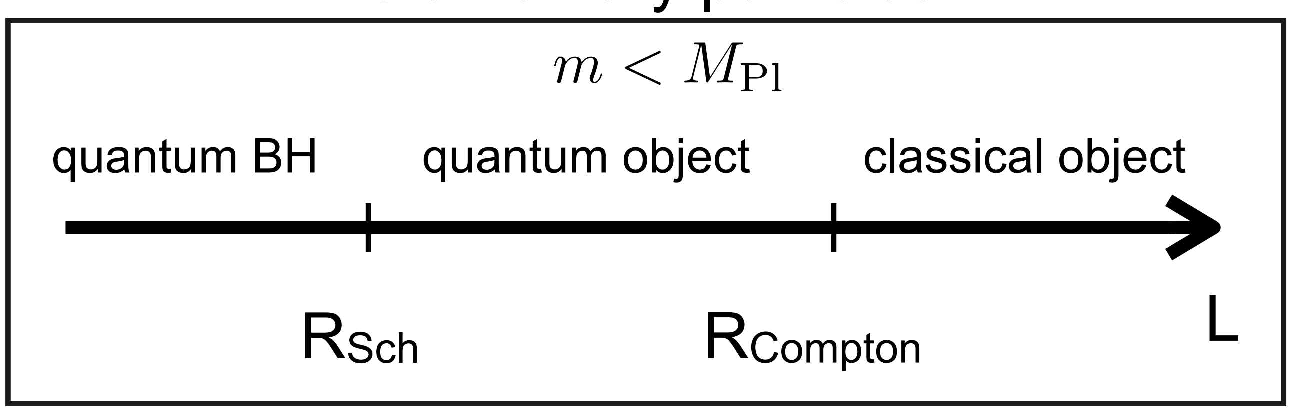
$$\boxed{R_{\text{Compton}} = \frac{\hbar}{mc}}$$

Schwarzschild radius: for an object of mass m, one can define a length scale that will measure its gravitational strength

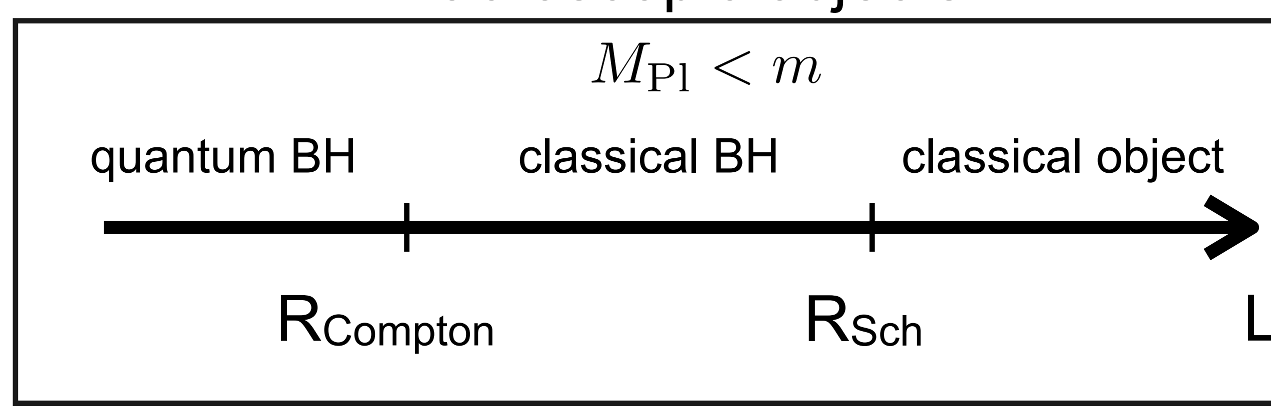
$$\boxed{R_{\text{Sch}} = \frac{G_{\text{N}} m}{c^2} = \frac{m}{M_{\text{Pl}}} l_{\text{Pl}}}$$

$$R_{\text{Compton}} < R_{\text{Sch}} \ \text{ iff } \ M_{\text{Pl}} < m$$

— elementary particles —



— macroscopic objects —

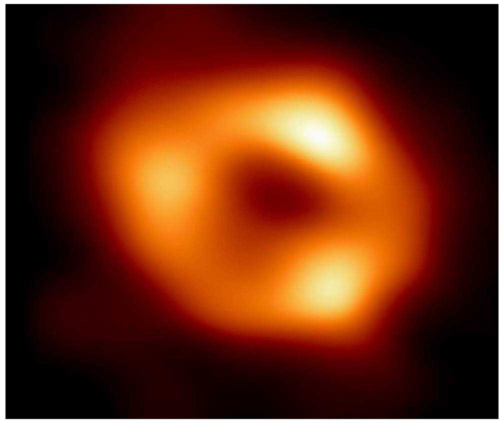


Black Holes

Neutron stars: $m \sim 10^{30}$kg, $R \sim 10^{4}$m (density of human population concentrated in a sugar cube): $R_{Sch} \sim 10^{3}$m: ~~BH~~

Stellar BHs: $m \sim 10^{31}$kg, $R \sim 10^{4}$m: $R_{Sch} \sim 10^{4}$m: BH

Supermassive BHs: $m \sim 10^{37}$kg, $R \sim 10^{10}$m: $R_{Sch} \sim 10^{10}$m: BH



Event Horizon Telescope
Sagittarius A*
$m = 4.3 \times 10^{6}\ M_{sun}$
$R = 23.5 \times 10^{6}$ km

LHC Black Holes: $m \sim 1$TeV, $R \sim 10^{-19}$m: $R_{Compton} \sim 10^{-19}$m, $R_{Sch} \sim 10^{-51}$m (ordinary gravity) but

$R_{Sch} \sim 10^{-19}$m if M_{Pl} is lowered to 1TeV as in models with large extra dimensions

Dimensional Analysis

$$[S]_m = 0 \quad \Rightarrow \quad [\mathcal{L}]_m = 4$$

$$S = \int d^4x\, \mathcal{L}$$

Scalar field $\mathcal{L} = \partial_\mu \phi \partial^\mu \phi + \ldots$ $\Rightarrow$ $[\phi]_m = 1$

Spin-1/2 field $\mathcal{L} = \psi^\dagger \gamma^0 \gamma^\mu \partial_\mu \psi$ $\Rightarrow$ $[\psi]_m = 3/2$

Spin-1 field $\mathcal{L} = F_{\mu\nu} F^{\mu\nu} + \ldots \text{ with } F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \ldots$ $\Rightarrow$ $[A_\mu]_m = 1$

Particle lifetime of a (decaying) particle: $[\tau]_m = -1$ Width: $[\Gamma = 1/\tau]_m = 1$

Cross-section ("area" of the target): $[\sigma]_m = -2$

Lifetime "Computations"

muon and neutron are unstable particles

$$\mu \to e\nu_\mu\bar{\nu}_e$$

$$n \to p\, e\, \bar{\nu}_e$$

We'll see that the interactions responsible for the decay of muon and neutron are of the form

$$\underset{[\text{mass}]^4}{\mathcal{L}} = \underset{[\text{mass}]^{-2}}{G_F}\, \underset{[\text{mass}]^{3/2\times 4}}{\psi^4} \quad \Rightarrow \quad \Gamma \propto G_F^2 m^5$$

$$[\text{mass}]$$

G_F = Fermi constant: $G_F \sim \dfrac{10^{-5}}{m_{\text{proton}}} \sim 10^{-5}\,\text{GeV}^{-2}$

For the **muon**, the relevant mass scale is the muon mass m_μ=105MeV:

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \sim 10^{-19}\,\text{GeV} \quad \text{i.e.} \quad \tau_\mu \sim 10^{-6}\,\text{s}$$

For the **neutron**, the relevant mass scale is (m_n-m_p)≈1.29MeV:

$$\Gamma_n = \mathcal{O}(1)\frac{G_F^2 \Delta m^5}{\pi^3} \sim 10^{-28}\,\text{GeV} \quad \text{i.e.} \quad \tau_{\text{n}} \sim 10^3\,\text{s}$$

$1 = \hbar c \sim 200\,\text{MeV}\cdot\text{fm}$

E	**T**	**L**
1eV	10^{-16}s	10^{-7}m

What if particles were spin-0?

$$\mathcal{L} = G_F \phi^4$$

$[\text{mass}]^4$ $[\text{mass}]^0$ $[\text{mass}]^{1\times 4}$

$$\Gamma \propto G_F^2 m$$

$[\text{mass}]$

$$\Gamma_\mu = \left(10^{-6\prime\prime}\right)^{-1} = \frac{G_F^2 m_\mu}{192\pi^2} \qquad \Gamma_n = (15')^{-1} = \frac{G_F^2(m_n - m_p)}{192\pi^2}$$

$$10^{11} = \frac{\Gamma_\mu}{\Gamma_n} \overset{?}{=} \frac{m_\mu}{m_n - m_p} = 10^2$$

TH prediction

It could still have been true but we would need to give up universality of the Fermi interactions.
Remember theorists like to connect phenomena are are seemingly different.
Even more true when they follow from simple assumptions.

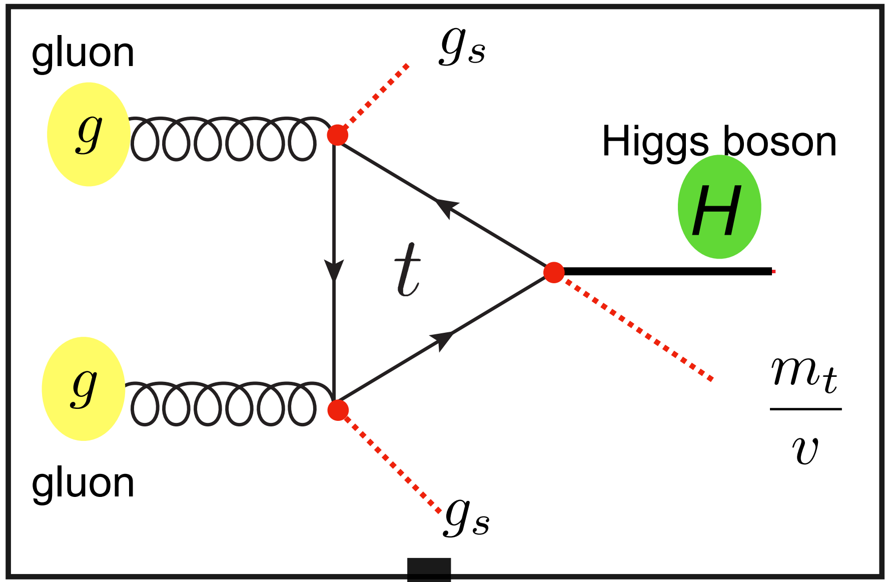
Higgs production "Computation"

At the LHC, the dominant Higgs production mode is gluon fusion

strong coupling constant $g_s \sim 1$

$$\mathcal{L} = g_s\, g^a_\mu\, \bar{t}\gamma^\mu T^a t$$

[mass]^0 [mass]^1 [mass]$^{3/2x2}$



Higgs coupling

$$g_{ttH}\, \bar{t}t H$$

[mass]^0 [mass]$^{3/2x2}$ [mass]^1

$$g_{ttH} = \frac{m_t}{v}$$

v=246 GeV

dimensional analysis allows us to compute the Higgs production cross-section

$$\sigma = \frac{1}{8\pi}\,\frac{1}{16\pi^2}\,g_s^4\,\frac{m_t^2}{v^2}\,\frac{1}{m_t^2}$$

flux, loop, couplings, dimensionally $[\sigma]_m$=-2

i.e. $\sigma \sim 10^{-25}\,\mathrm{eV}^{-2} \sim 10^{-39}\,\mathrm{m}^2 = 10\,\mathrm{pb}$

E	L
1eV	10^{-7}m

$1\,\mathrm{barn} = 10^{-28}\,\mathrm{m}^2$

$1\,\mathrm{pb} = 10^{-12}\,\mathrm{barn}$

One could think that all the quarks should give a similar contribution to the Higgs production since m_t factors cancel.
But it can be shown that this cancelation holds only for quarks heavier than the Higgs
(value of Higgs production xs excludes a heavy fourth generation of quarks).
LHC collides protons which contain gluons. How many gluons are inside the quarks depends on the energy of the protons. The production cross-section depends on the energy of the collider (40 pb at 14TeV, at 100TeV)

How many Higgs bosons produced at LHC? $\sigma \times \int dt\, L = 10\,\mathrm{pb} \times 100\,\mathrm{fb}^{-1} \sim 10^6$

(L = luminosity= nbr of collisions)

Higgs Lifetime “Computation”

exercice:

Using similar dimensional analysis arguments,
compute the Higgs boson lifetime (or its inverse aka as the Higgs decay width)

— Hints —

Higgs couplings proportional are proportional to the mass of the particles it couples to.
It will therefore decay predominantly decay into the heaviest particle that is lighter than $m_H/2$

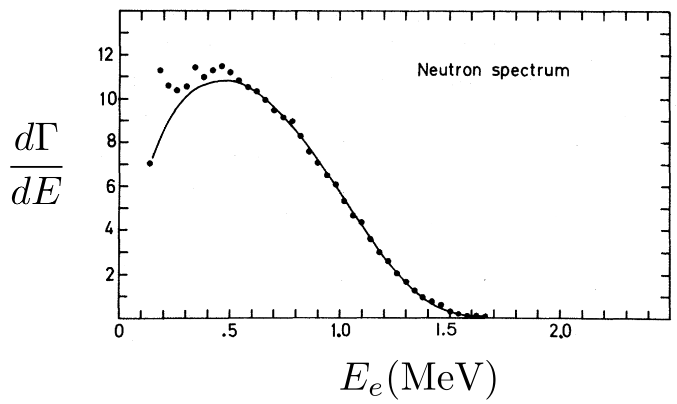
Beta decay

$${}^{40}_{19}\mathrm{K} \to {}^{40}_{20}\mathrm{Ca}^+ + e^- \qquad {}^{64}_{29}\mathrm{Cu} \to {}^{64}_{30}\mathrm{Zn}^+ + e^- \qquad {}^{3}_{1}\mathrm{H} \to {}^{3}_{2}\mathrm{He}^+ + e^-$$

- Two body decays: A→B+C

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A} c^2 \qquad p = \frac{\sqrt{\lambda(m_A, m_B, m_C)}}{2m_A} c$$

$$\lambda(m_A, m_B, m_C) = (m_A + m_B + m_C)(m_A + m_B - m_C)(m_A - m_B + m_C)(m_A - m_B - m_C)$$



EXP measurements

fixed energy of daughter particles

(pure SR kinematics, independent of the dynamics)

TH prediction

⇛ non-conservation of energy?

Pauli '30: ∃ **neutrino**, very light since end-point of spectrum is close to 2-body decay limit

ν first observed in '53 by Cowan and Reines

- N-body decays: A→B$_1$+B$_2$+...+B$_N$

$$E_{B_1}^{\min} = m_{B_1} c^2 \qquad E_{B_1}^{\max} = \frac{m_A^2 + m_{B_1}^2 - (m_{B_2} + \ldots + m_{B_N})^2}{2m_A} c^2$$

— How are neutrinos produced? —

$\pi \to \mu\bar{\nu}$ (more about pion decay later later)

$\mu \to e\bar{\nu}_e\nu_\mu$ need 2 neutrino flavours and flavour conservation since $\mu \not\to e\gamma$

Lederman, Schwartz, Steinberger '62: $p\,\bar{\nu}_\mu \to n\,\mu^+$ but $p\,\bar{\nu}_\mu \not\to n\,e^+$

Fermi theory '33

(paper rejected by Nature: declared too speculative !)

$$\mathcal{L} = G_{\mathcal{F}}(\bar{n}p)(\bar{\nu}_e e)$$

exp: $G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$

We'll see later that the structure is a bit more complicated

Universality of Weak Interactions

$$\mu \to e \nu_\mu \bar{\nu}_e \qquad\qquad n \to p\, e\, \bar{\nu}_e$$

$\tau_\mu \approx 10^{-6}$s $\qquad$ $\tau_n \approx 900$s

$$\mathcal{L} = G_F\, \psi^4$$

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \sim 1/10^{-6\prime\prime} \qquad\qquad \Gamma_n = \frac{G_F^2 \Delta m^5}{192\pi^3} \sim 1/15^\prime$$

[factor 192 not exactly correct
because n and p are not elementary particles:
form factors are involved]

$$\mathcal{L} \stackrel{?}{=} G_F\ \left(\bar{n} p \bar{e} \nu_e + \bar{\mu} \nu_\mu \bar{e} \nu_e\right)$$

By analogy with electromagnetism, one can see the Fermi force as a current-current interaction (vector-vector interaction instead of scalar-scalar interaction)

$$\mathcal{L} = G_F\, J^*_\mu J^\mu \qquad \text{with} \qquad J^\mu \stackrel{?}{=} (\bar{n}\gamma^\mu p) + (\bar{e}\gamma^\mu \nu_e) + (\bar{\mu}\gamma^\mu \nu_\mu) + \ldots$$

it can be shown (thanks to the transformation law of spin-1/2 field given before) that this Lagrangian is invariant under Lorentz transformation

The cross-terms generate both neutron decay and muon decay.
The life-times of the neutron and muon tell us that the relative factor between the e and the μ in the current is of order one: the weak force has the **same strength for e and μ**.

Pathology at High Energy

What about weak scattering process, e.g . $e\nu_e \to e\nu_e$?

$$\mathcal{L} = G_F\, J^*_\mu J^\mu \qquad \text{with} \qquad J^\mu = (\bar{n}\gamma^\mu p) + (\bar{e}\gamma^\mu \nu_e) + (\bar{\mu}\gamma^\mu \nu_\mu) + \dots$$

The same Fermi Lagrangian will thus also contain a term

$$G_F\, (\bar{e}\gamma^\mu \nu_e)(\bar{\nu}_e\gamma^\mu e)$$

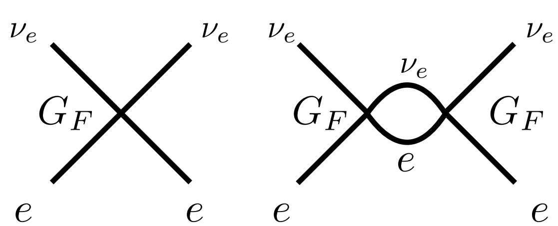
that will generate e-ν_e scattering whose cross-section can be guessed by dimensional arguments

$$\sigma \propto G_F^2 E^2$$

$[\text{mass}]^{-2}$ $\quad$ $[\text{mass}]^{-2\times 2}$ $\quad$ $[\text{mass}]^2$

non conservation of probability
(non-unitary theory)
inconsistent at high energy

It means that, at high-energy, the quantum corrections to the classical contribution can be sizeable:



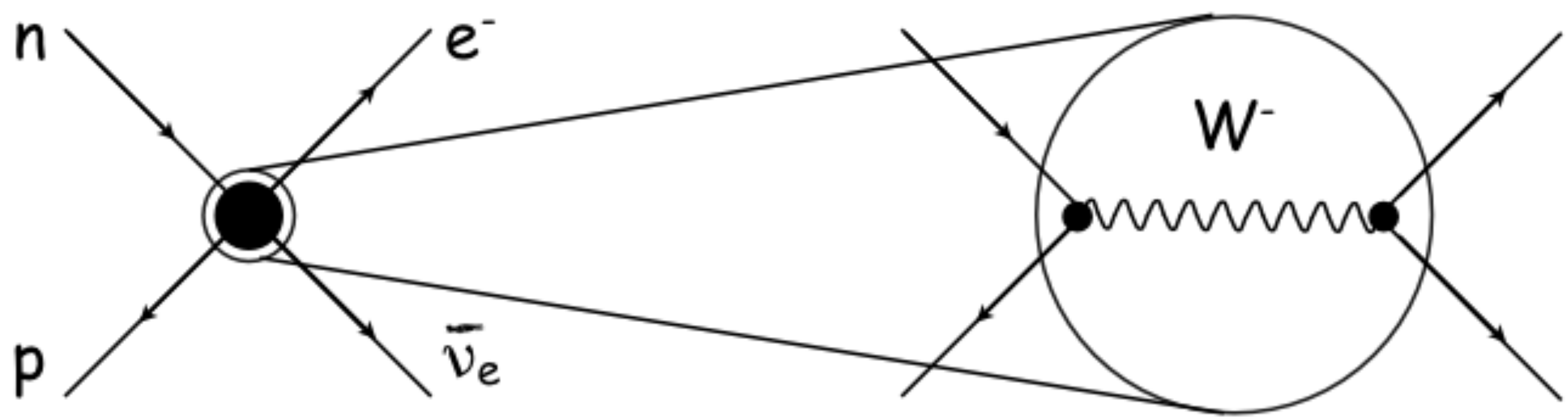
$$\sigma \propto G_F^2 E^2 + \frac{1}{16\pi^2} G_F^4 E^6 + \dots$$

The theory becomes non-perturbative at an energy $E_{\max} = \dfrac{2\sqrt{\pi}}{\sqrt{G_F}} \sim 100\,\text{GeV}\text{–}1\,\text{TeV}$

unless new degrees of freedom appear before to change the behaviour of the scattering

Electroweak Interactions

Low energy | High energy



$\sigma \propto G_F^2 E^2$

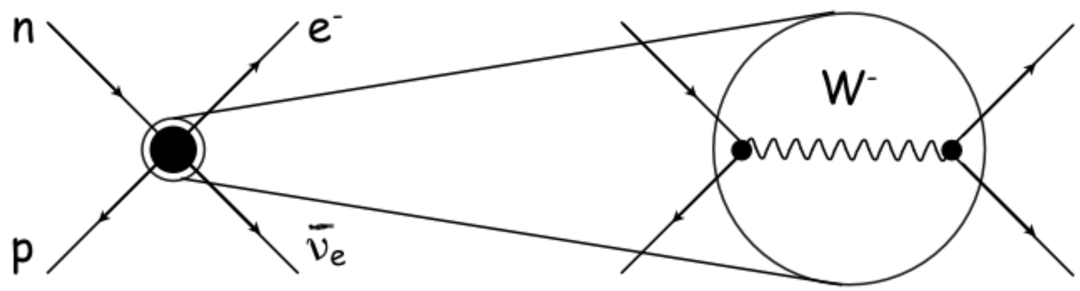
$$\sigma \propto g^4 \frac{E^2}{m_W^2 \left(E^2 + m_W^2\right)}$$

— matching —

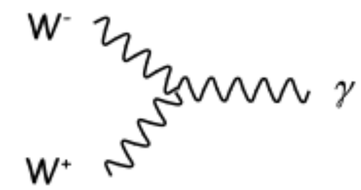
$$G_F \propto \frac{g^2}{m_W^2}$$

The Fermi interaction is not a fundamental interaction of Nature.
It is a low energy effective interaction.

Electroweak Interactions



charged W ⇛ must couple to photon:



⇛ non-abelian gauge symmetry $[Q,T^{\pm}]=\pm T^{\pm}$

1. No additional "force" (Georgi, Glashow '72) mathematical consistency ⇛ **extra matter**

$SU(2)$	$SU(1,1)$	E_2
$[T^a,T^b]=i\epsilon^{abc}T^c$ $[T^+,T^-]=Q \qquad [Q,T^{\pm}]=+\pm T^{\pm}$ $T^{\pm}=\frac{1}{\sqrt{2}}(T^1\pm iT^2)$ $\mathrm{Tr}_{\mathrm{irrep}}T^3=0$ ⇛ extra matter $\begin{pmatrix} X_L \\ \nu_L \\ e_L \end{pmatrix} \begin{pmatrix} X_R \\ \nu_R \\ e_R \end{pmatrix}$	$[T^+,T^-]=-Q$ $[Q,T^{\pm}]=+\pm T^{\pm}$ non-compact unitary rep. has dim ∞	2D Euclidean group $[T^+,T^-]=0$ $[Q,T^{\pm}]=+\pm T^{\pm}$ only one unitary rep. of finite dim. = trivial rep.

2. No additional "matter" (Glashow '61, Weinberg '67, Salam '68): SU(2)xU(1)

⇛ **extra force**

$$Q=T^3?$$

as Georgi-Glashow

⇛ extra matter

$$Q=Y?$$

$$Q(e_L)=Q(\nu_L)$$

$$Q=T^3+Y!$$

Gell-Mann '56, Nishijima-Nakano '53

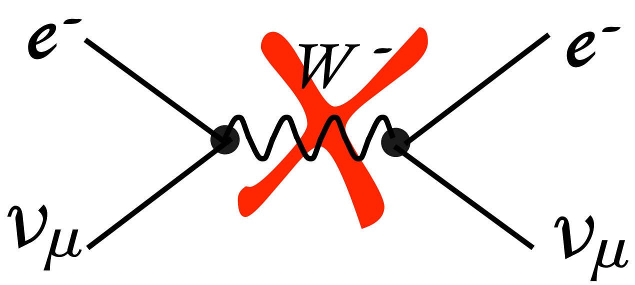
Electroweak Interactions

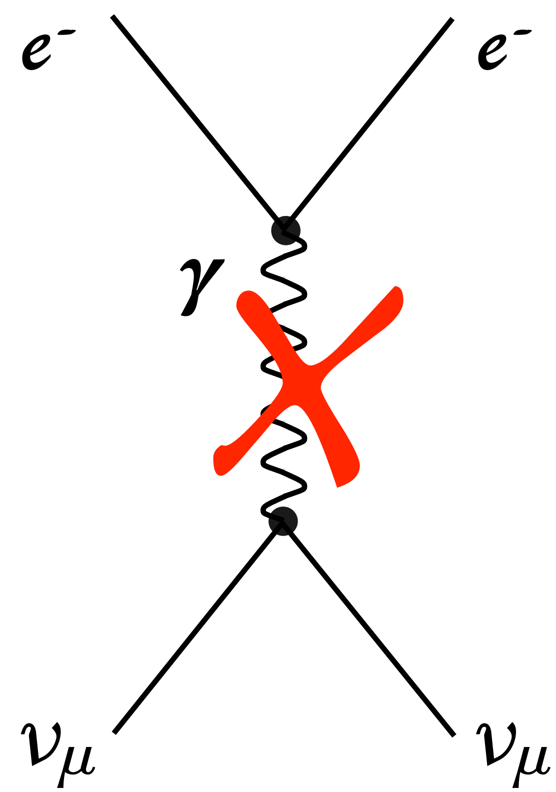
Gargamelle experiment '73 first established the SU(2)xU(1) structure

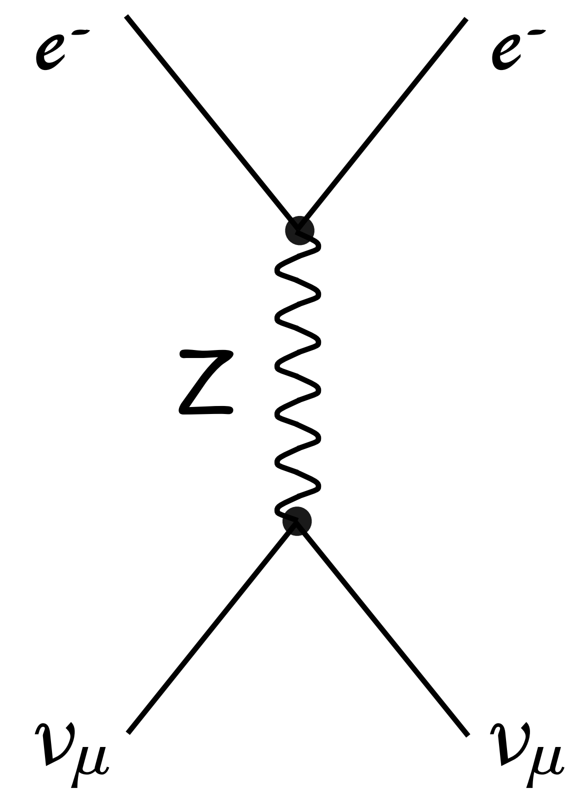
How?

rely on a particle that doesn't interact with photon to prove the existence a new neutral current process!

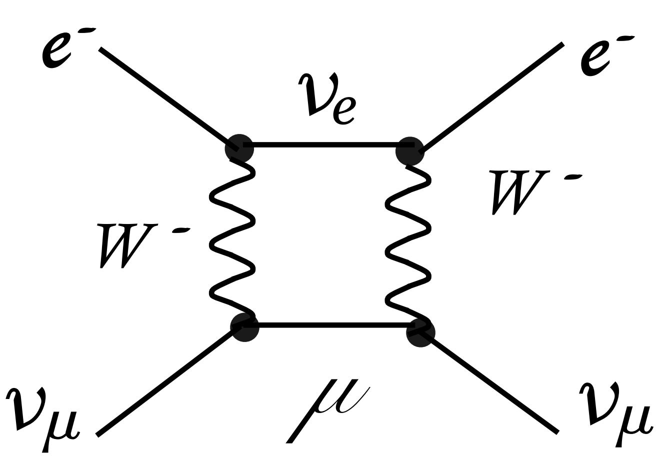
$\nu_\mu e^- \to \nu_\mu e^-$







loop-suppressed contribution from W:



From Gauge Theory to Fermi Theory

We can derive the Fermi current-current contact interactions by "integrating out" the gauge bosons, i.e., by replacing in the Lagrangian the W's by their equation of motion. Here is a simple derivation:
(a better one should take taking into account the gauge kinetic term and the proper form of the fermionic current that we'll figure out tomorrow, for the moment, take it as a heuristic derivation)

$$\mathcal{L} = -m_W^2 W_\mu^+ W_\nu^- \eta^{\mu\nu} + g W_\mu^+ J_\nu^- \eta^{\mu\nu} + g W_\nu^- J_\nu^+ \eta^{\mu\nu}$$

$$J^{+\mu} = \bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \ldots \qquad \text{and} \qquad J^{-\mu} = \left(J^{+\mu}\right)^*$$

The equation of motion for the gauge fields: $\dfrac{\partial \mathcal{L}}{\partial W_\mu^+} = 0 \qquad \Rightarrow \qquad W_\mu^- = \dfrac{g}{m_W^2} J_\mu^-$

Plugging back in the original Lagrangian, we obtain an *effective Lagrangian* (valid below the mass of the gauge bosons):

$$\boxed{\mathcal{L} = \frac{g^2}{m_W^2} J_\mu^+ J_\nu^- \eta^{\mu\nu}}$$

which is the Fermi current-current interaction. The Fermi constant is given by
(the correct expression involves a different normalisation factor)

$$\boxed{G_F = \frac{g^2}{m_W^2}}$$

But what is the origin of the W mass?
By the way, it is not invariant under SU(2) gauge transformation…
That's what the Higgs mechanism will take care of!

Weak Interactions and Higgs Physics

The decays of the neutrons and muons are controlled by the value of the Fermi constant

G_F = Fermi constant: $G_F \sim \dfrac{10^{-5}}{m_{\rm proton}} \sim 10^{-5}\,{\rm GeV}^{-2}$

Higgs physics is controlled by the value of the Vacuum Expectation Value

v = 246 GeV

We can notice that

$$G_F = \frac{1}{\sqrt{2}v^2}$$

This relation is the consequence of the description of the weak interactions in terms of a local internal symmetry and its spontaneous breaking in the vacuum.
That's what we'll figure out in the next lectures.