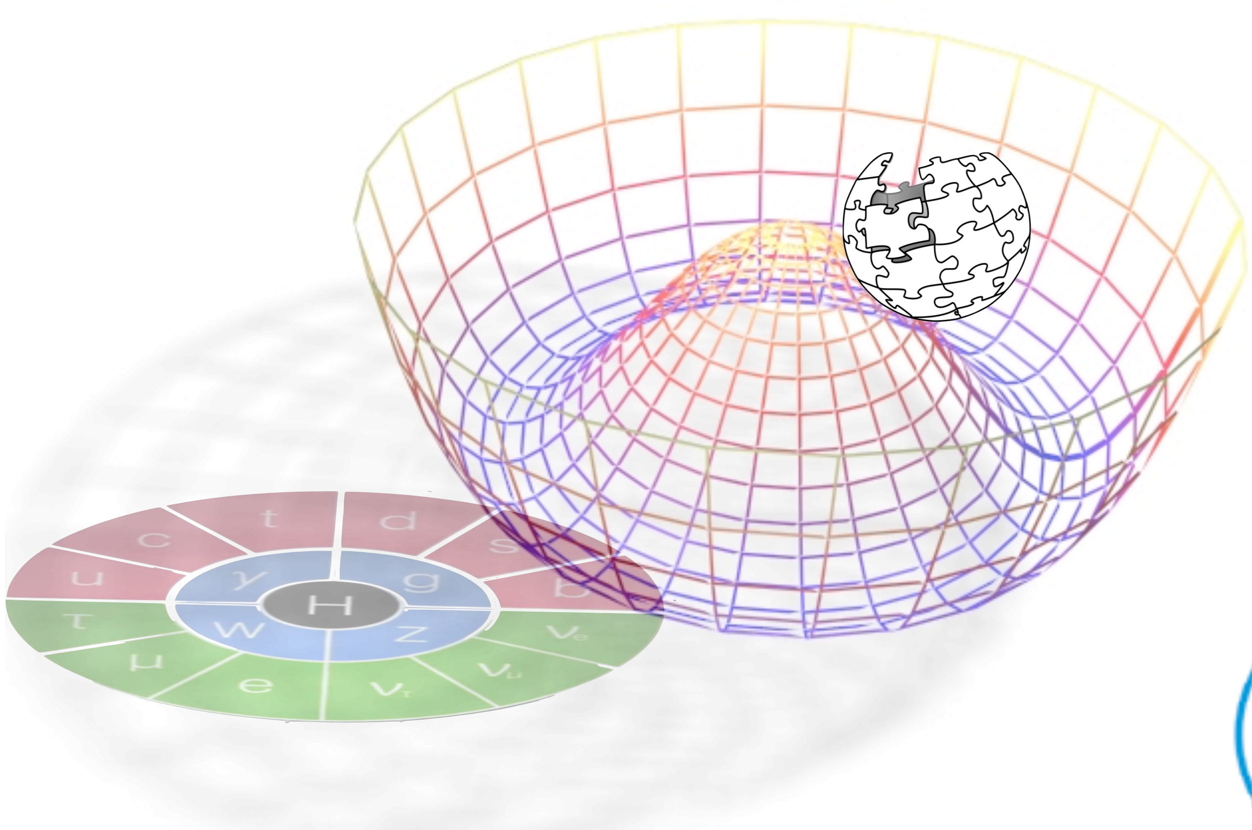
Journeys

~~into~~ towards Effective Field Theories

Journeys into Theoretical Physics

Sao Paulo, July 2026

Lecture 3/4



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Outline

Wednesday: Role of symmetries

- Lagrangians
- Lorentz symmetry - scalars, fermions, gauge bosons
- Gauge/local symmetry as dynamical principle - Example: U(1) electromagnetism

Thursday AM: Weak interactions - dimensional analysis

- Nuclear decay, Fermi theory and weak interactions: SU(2)
- Strong interactions: SU(3)
- Dimensional analysis: cross-sections and life-time computations made simple

Thursday PM: Chirality

- Chirality of weak interactions
- Pion decay

Friday: Higgs mechanism

- Spontaneous symmetry breaking and Higgs mechanism
- Lepton and quark masses, quark mixings
- Neutrino masses

Pion decay(s)

What about $\pi^{\pm}$ decay $\tau_\pi \approx 10^{-8}$s?

$$\pi^- \to \mu \bar{\nu}_\mu \qquad\qquad \pi^- \to e^- \bar{\nu}_e$$

experimentally the pions decay dominantly into muons and not electrons.

Why $\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \sim 10^{-4}$? And not $\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \sim \frac{(m_\pi - m_e)^5}{(m_\pi - m_\mu)^5} \sim 500$?

EXP TH

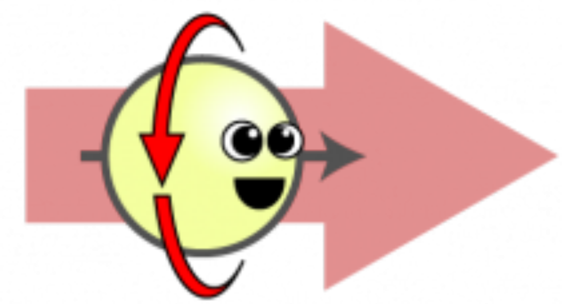
Does it mean that our way to compute decay rate is wrong?
Is pion decay mediated by another interaction?
The pion is a composite particle: does is mean that the form factors drastically change our estimates?
Is the weak interaction non universal, i.e. is the value of G_F processus dependent?

Chirality & Masslessness

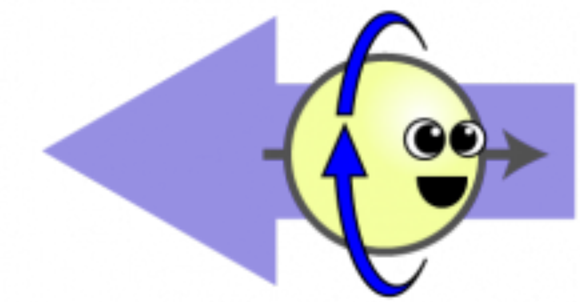
Quantum Mechanics 1.0.1

Particle of spin s has 2s+1 polarisation states

Particle spinning anticlockwise wrt its direction of motion



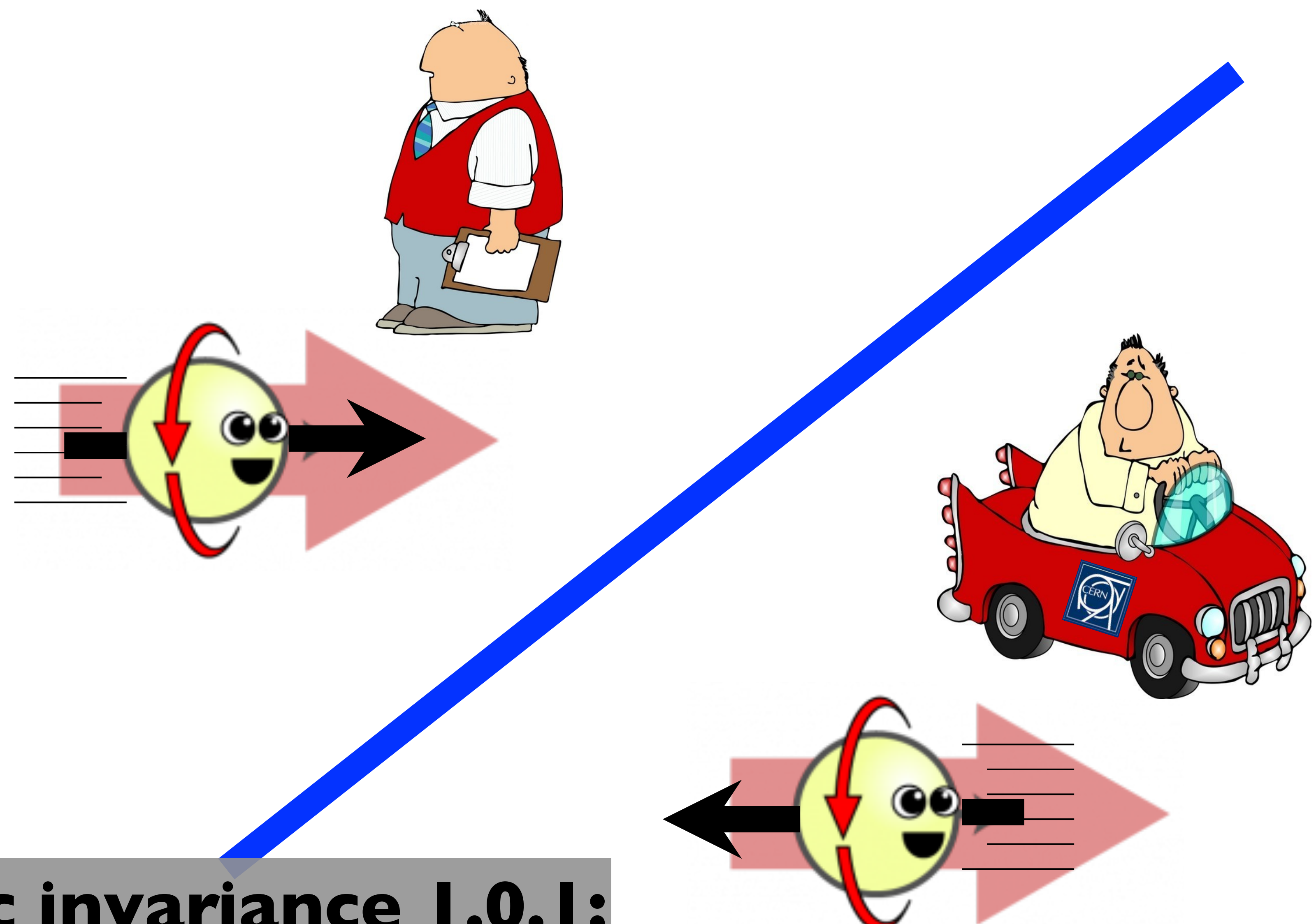
Particle spinning clockwise wrt its direction of motion



electron
has 2 polarisation

Picture courtesy to G. Giudice

Chirality & Masslessness



Picture courtesy to G. Giudice

Relativistic invariance 1.0.1:

there must be no distinction for massive particles between particles spinning clockwise or anti-clockwise

[chirality operator doesn't commute with the Hamiltonian]

Theorem

If your theory sees a difference between e_L and e_R, either your theory is wrong or m_e=0

Chirality of SM & Mass problem

Weak interaction
(force responsible for
neutron decay)
is chiral!

[e_L and e_R are fundamentally
two different particles
Only an accident of the history of
physics that they are both called
electron]

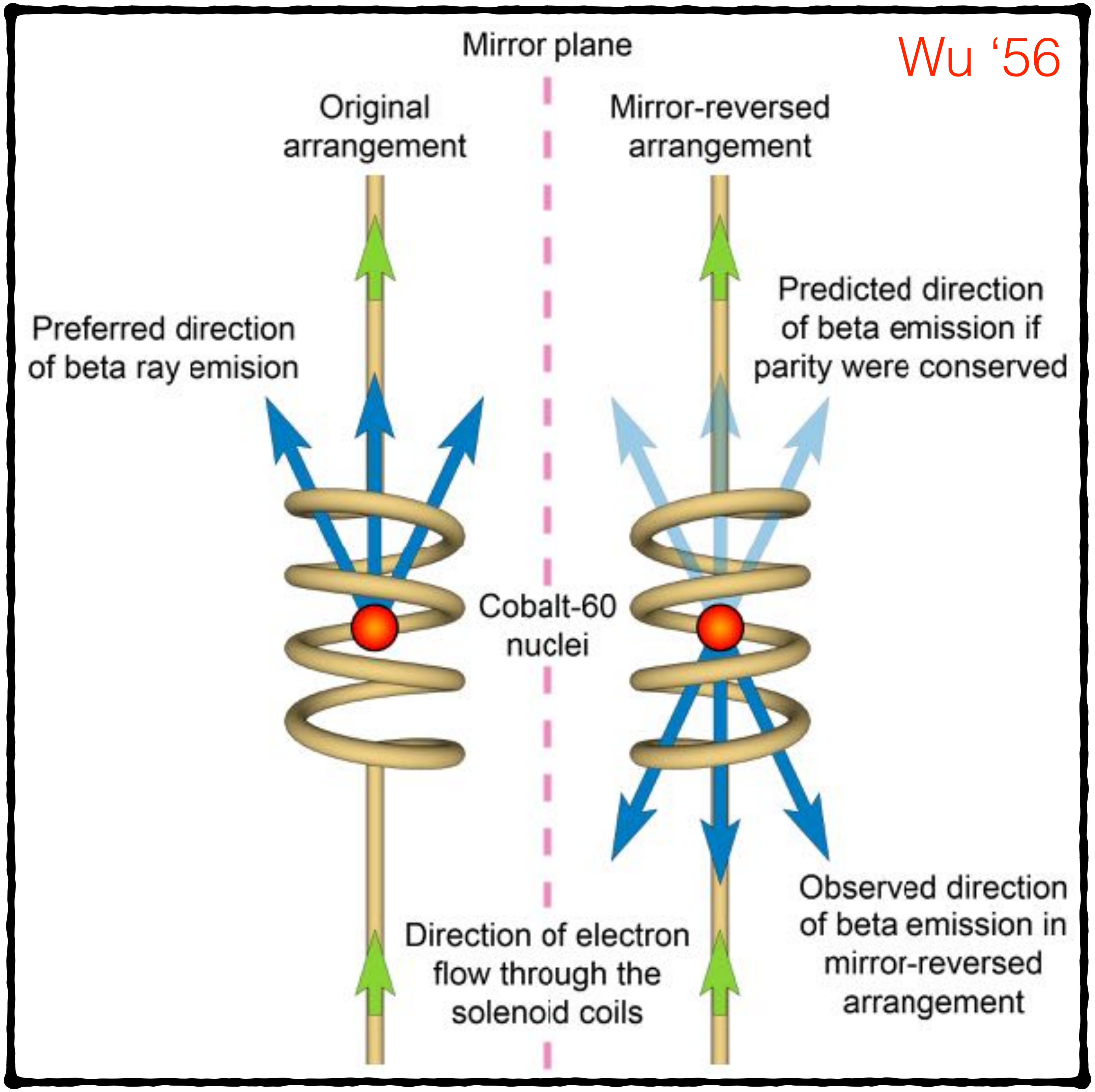
↓

m_e=0

but since we know it is not true, we

need a new phenomena to generate mass: Higgs mechanism

TH: Yang&Lee '56. EXP: Wu '57



Chirality of SM & Mass problem

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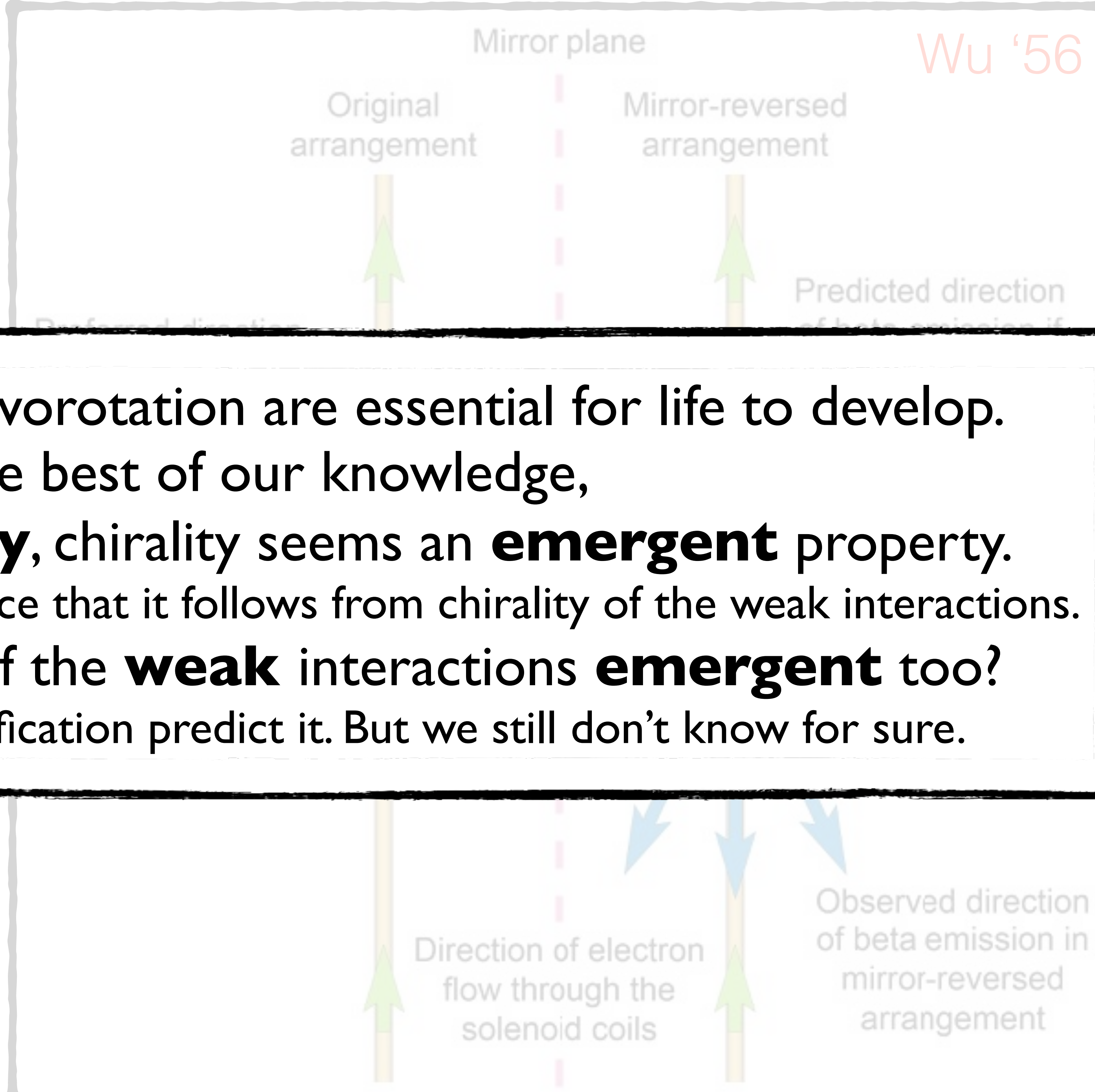
Weak interaction (force responsible for neutron decay) is chiral!

[e_L and e_R are fundamentally



Dextrorotation and Levorotation are essential for life to develop.
To the best of our knowledge,
in **molecular biology**, chirality seems an **emergent** property.
At least, there is no clear evidence that it follows from chirality of the weak interactions.
Are the chiral nature of the **weak** interactions **emergent** too?
Some models of grand unification predict it. But we still don't know for sure.

But since we know it is not true, we

need a new phenomena to generate mass: Higgs mechanism

SM is a Chiral Theory

Weak interactions maximally violates P

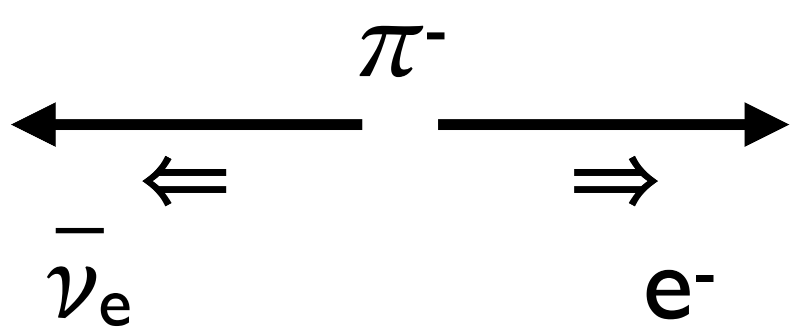
$^{60}_{27}\mathrm{Co} \to ^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e$ only left-handed (LH) e^- produced

Weak interactions act only on LH particles (and RH anti-particles)

this property has an important consequence (aka selection rule) for pion decay

$\rightarrow$ direction of momentum

$\Rightarrow$ direction of spin



Conservation of momentum and spin imposes to have a RH e^-

Weak decays proceed only w/ LH e^-
So the amplitude is prop. to m_e

$$\mathcal{L}_{\mathrm{Dirac}} = \bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R \gamma^\mu \partial_\mu \psi_R + m \left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \right)$$

Lorentz structure of fermion mass

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \propto \frac{m_e^2}{m_\mu^2} \sim 2 \times 10^{-5} \sim 10^{-4}_{\mathrm{obs}}$$

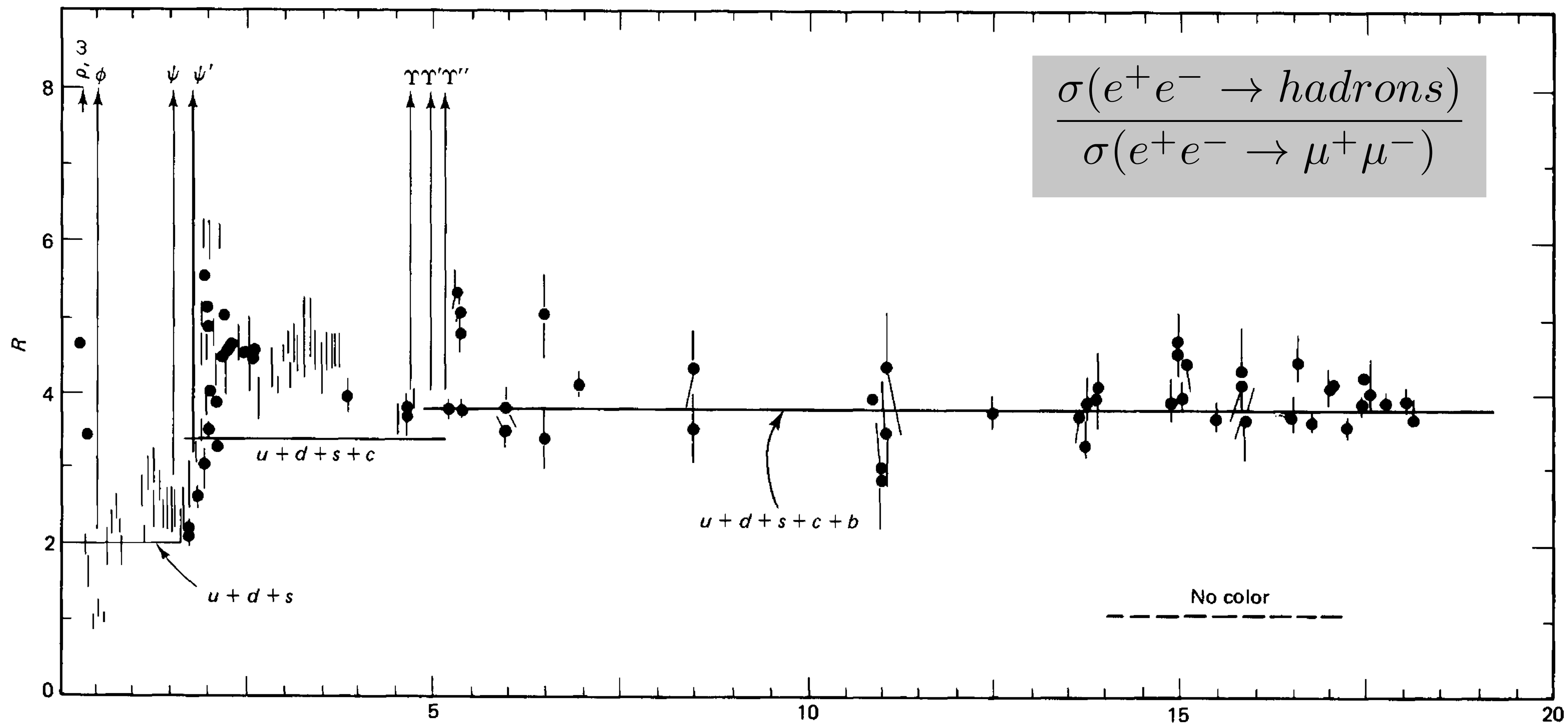
↑ Extra phase-space factor

SU(3) QCD

Experiments in the 60's revealed the internal structure of the neutrons and protons
Gell-Mann and others proposed that they are made of "**quarks**"

Up quark (up, charm, top): spin-1/2, Q=2/3
Down quark (down, strange, bottom): spin-1/2, Q=-1/3

SU(2) weak symmetry that changes neutrino into electron also changes up-quark into down-quark



counts the number of quarks and gives their electric charges
another remarkable feature: at high energy, the quarks behaves like muons,
i.e., not sensitive to strong interactions
Asymptotic freedom of QCD!

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Quarks carry yet another quantum number: "**colour**"
There are 3 possible colours and Nature is colour-blind, i.e, Lagrangian should remain the same when the colours of the quarks are changed, i.e., when we perform a rotation in the colour-space of quarks.

$$Q^a \to U^a{}_b Q^b$$

U: 3x3 matrix satisfying $U^\dagger U = 1_3$
such that the quark kinetic term is invariant

SU(3)

hadrons (spin-1/2, #hadronic=1):	$p = uud$	$n = udd$
mesons (spin-0, #hadronic=0):	$\pi^0 = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$	$\pi^+ = u\bar{d}$ $\quad \pi^- = d\bar{u}$

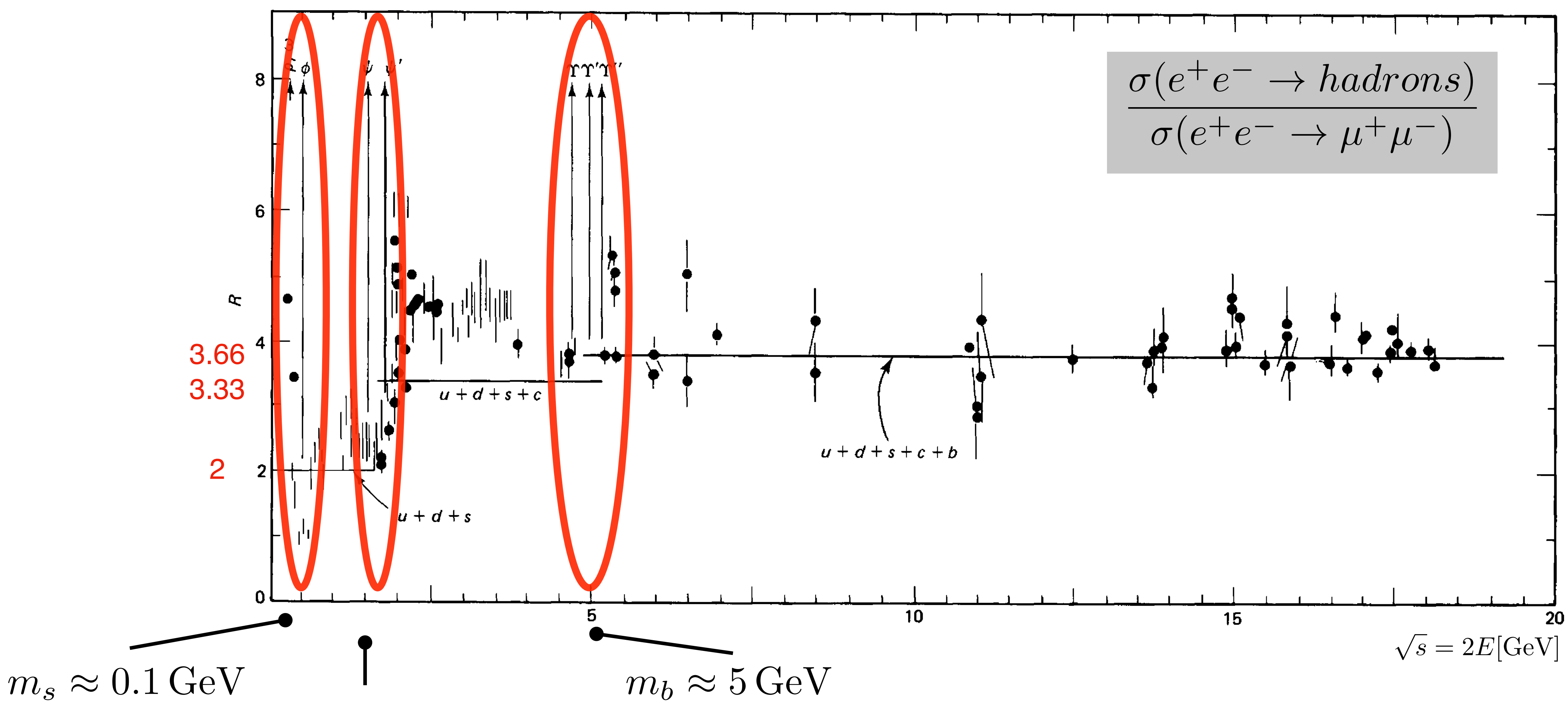
(Each quark carries a baryon number =1/3)

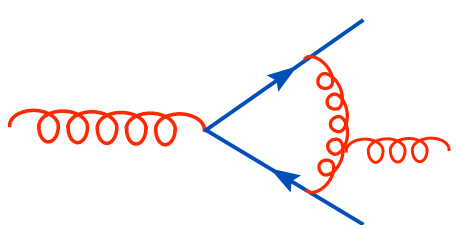
There are (heavier) quarks and hence other baryons and mesons

All the interactions of the SM preserve baryon and lepton numbers

$\mu \to e \nu_\mu \bar{\nu}_e$ $\qquad n \to p\, e\, \bar{\nu}_e$ $\qquad \pi^- \to \mu^- \bar{\nu}_\mu$ $\qquad \pi^0 \to \gamma\gamma$ $\qquad p \not\to \pi^0 \bar{e}$

SU(3) QCD





$$\boxed{R(E) = \sum_{m_i < E} Q_i^2} \times \left(1 + \frac{\alpha_s}{\pi} + \frac{\alpha_s^2}{\pi^2}(c + \pi b \log \Lambda^2/E^2)\right)$$

$$2 = 3 \times \left(\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right)$$

below 2 GeV
3 particles of charges ±2/3 and 6 particles of charge ±1/3

$$\frac{10}{3} = 3 \times \left(\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right)$$

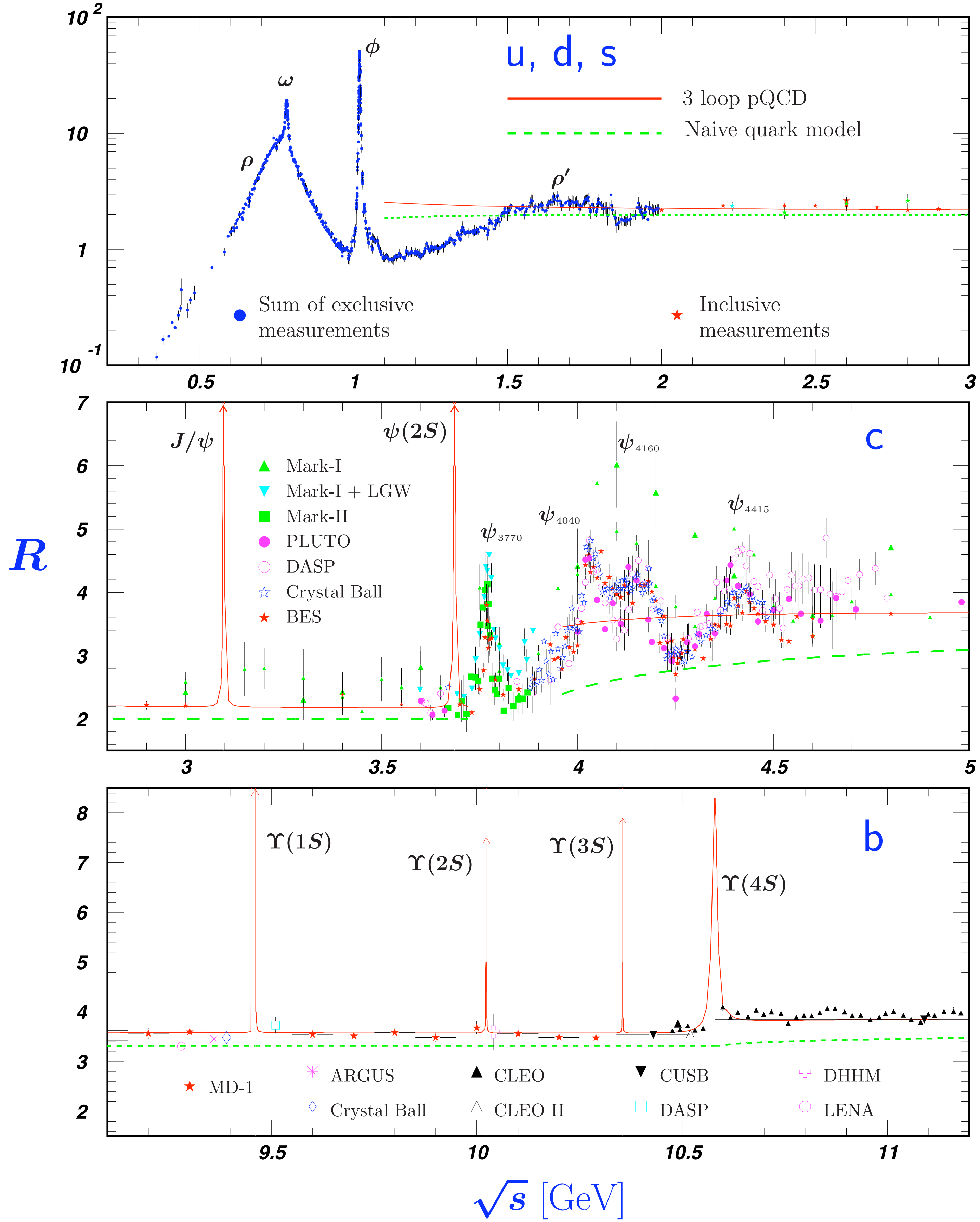
at 2 GeV, 3 more particles of charge ±2/3

$$\frac{11}{3} = 3 \times \left(\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 \left(-\frac{1}{3}\right)^2\right)$$

at 5 GeV, 3 more particles of charge ±1/3

SU(3) QCD

modern version of the plot…



SU(3) QCD

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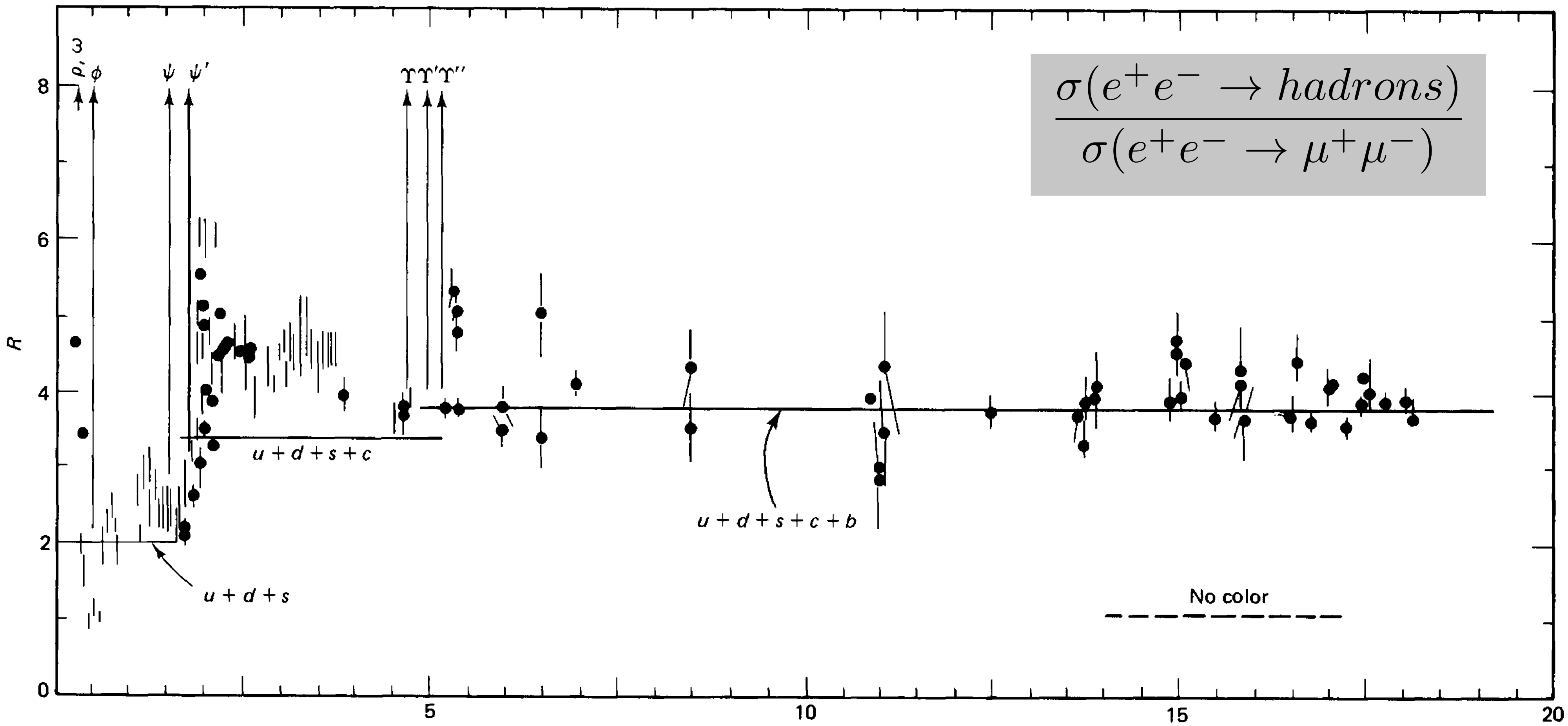

This experiment counts the number of quarks and gives their electric charges.
Another remarkable feature: at high energy, the quarks behave like muons, i.e., not sensitive to strong interactions.

Asymptotic freedom of QCD!

(consequence of non-abelian nature of strong interaction - see tomorrow lecture)

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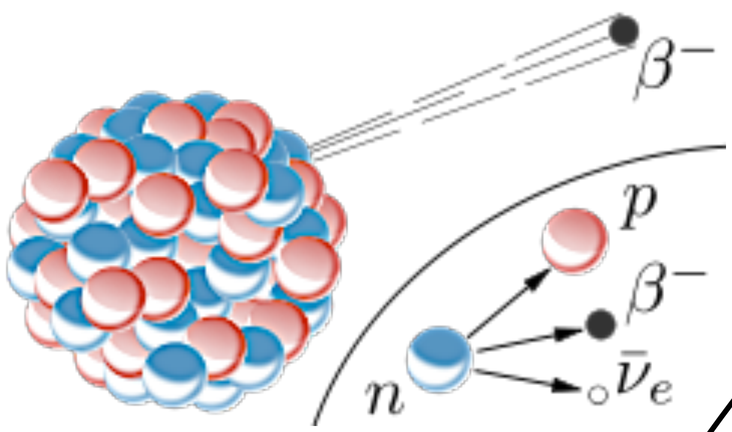

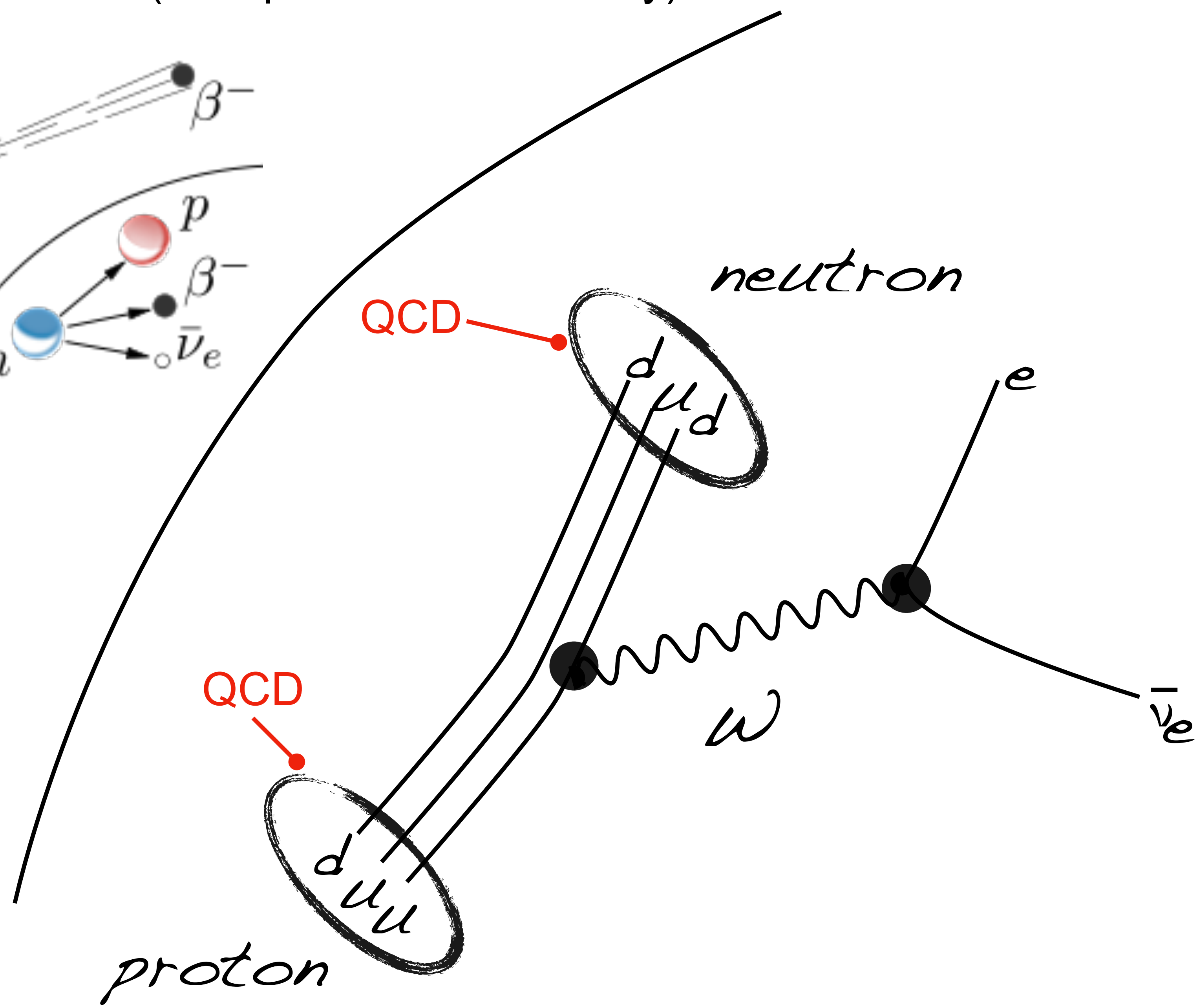

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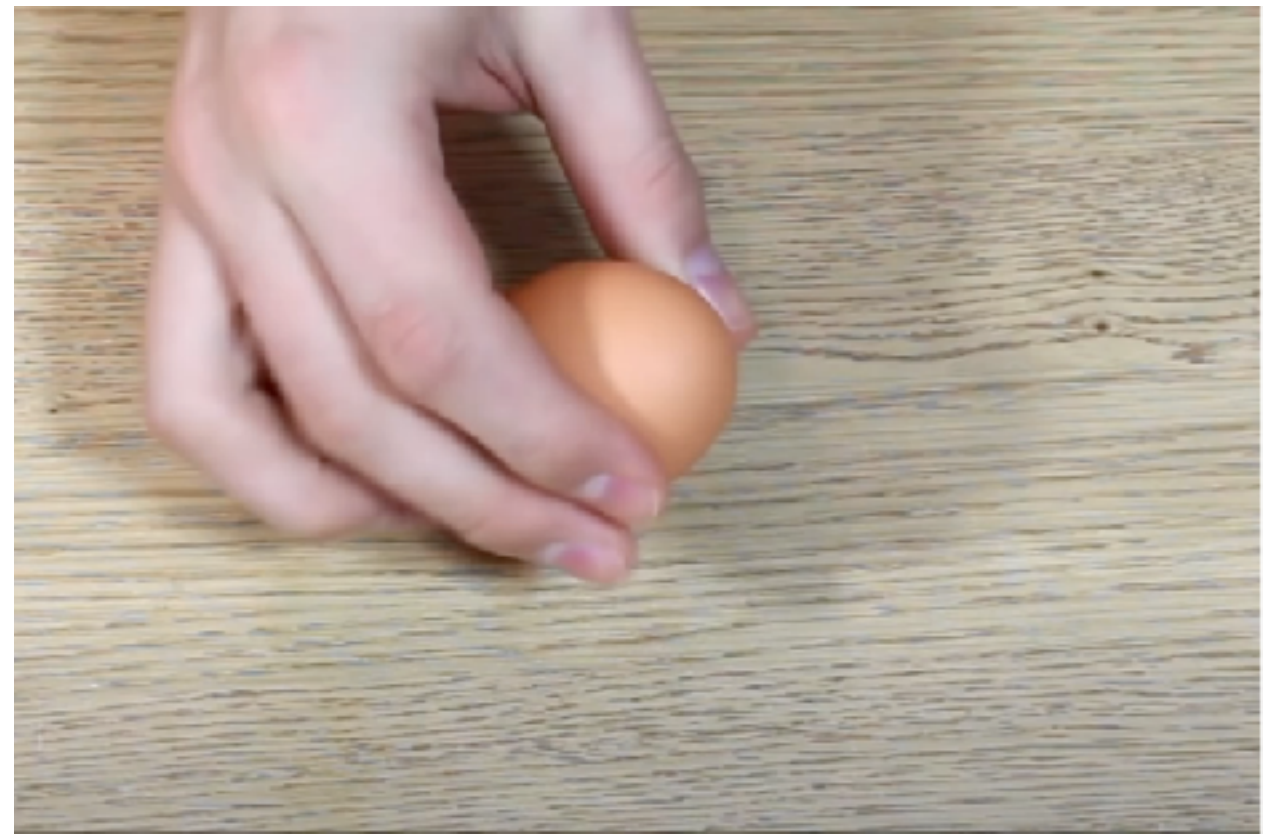
Inside Hadrons

One can break matter into pieces to learn what it is made of.
But this is not always possible (not sharp enough knife, not enough energy…).
Fortunately, remember the boiled egg experiment:

https://youtu.be/r1ygKQbcqh4



The way the egg is spinning can tell if it is boiled (one piece) or raw (internal structure with different components moving independently from each others)

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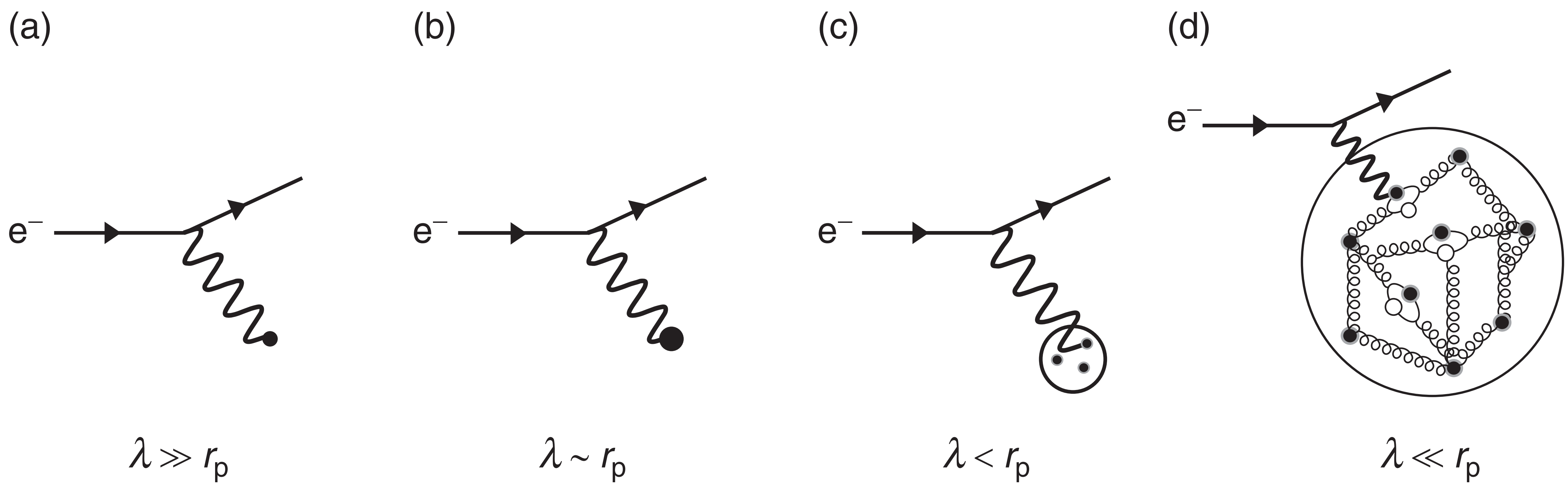

electron-proton scattering (1960's) reveals the proton intimate structure
(3 elementary spin-1/2 quarks that exist in 3 colours bounded by strong interactions that become feeble at large energies — asymptotic freedom).

The Standard Model: Interactions

- $U(1)_Y$ ***electromagnetic interactions***
  - → Photon γ
  - { light, atoms, molecules } — 10^{-5}
- $SU(2)_L$ ***weak interactions***
  - → bosons $W^{\pm}, Z^0$
  - { β decay
  - $n \xrightarrow{W^{\pm}} p + e^- + \bar{\nu}_e$
  - $e^+ + e^- \xrightarrow{Z^0} D^+_{(c\bar{s})} + D^-_{(\bar{c}s)}$ } — 10^{-2}
- $SU(3)_c$ ***strong interactions***
  - → gluons g^a
  - { atomic nuclei
  - α decay
  - ${}^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$ } — 1

strength

SM Summary

	SPIN	PARTICLES	$SU(3)_C$ (color)	× $SU(2)_L$ (chirality)	× $U(1)_Y$ (hypercharge)	T_{3_L} (weak isospin)	$Q = T_{3_L} + Y$ (electric charge)	g_{eff} (effective coupling to Z boson)	MEANING
LEPTONS	1/2	$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$	1	2	$\begin{pmatrix} -1/2 \\ -1/2 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 + \sin^2\theta_W \end{pmatrix}$	doublet under SU(2), singlet under SU(3)
		e_R	1	1	-1	0	-1	$\sin^2\theta_W$	singlet under SU(2) and SU(3)
QUARKS		$Q = \begin{pmatrix} u \\ d \end{pmatrix}_L$	3	2	$\begin{pmatrix} 1/6 \\ 1/6 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$	$\begin{pmatrix} 1/2 - \frac{2}{3}\sin^2\theta_W \\ -1/2 + \frac{1}{3}\sin^2\theta_W \end{pmatrix}$	doublet under SU(2), triplet under SU(3)
		u_R	3	1	2/3	0	2/3	$-\frac{1}{3}\sin^2\theta_W$	singlet under SU(2), triplet under SU(3)
		d_R	3	1	-1/3	0	-1/3	$\frac{1}{3}\sin^2\theta_W$	singlet under SU(2), triplet under SU(3)
HIGGS	0	$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$	1	2	$\begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	×	doublet under SU(2), singlet under SU(3)

Technical Details for Advanced Students

Chirality

Chirality matrix

$$\boxed{\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3}$$

A few remarkable properties

$$\left(\gamma^5\right)^2 = 1_4$$

$$\{\gamma^5, \gamma^\mu\} = 0$$

$$\gamma^{5\dagger} = \gamma^5 = -\gamma^0\gamma^5\gamma^0$$

Chiral/Weyl spinor

A **chiral/Weyl** spinor is an eigenvector of the chirality matrix $\psi_{L,R} = \pm\gamma^5\psi_{L,R}$

From the Lorentz-transformation law of a spinor, it is obvious that the chirality condition is frame-independent

A Dirac spinor can also be written as a sum of two chiral spinors

$$\psi = \frac{1}{2}\left(1_4 + \gamma^5\right)\psi + \frac{1}{2}\left(1_4 - \gamma^5\right)\psi \equiv \psi_L + \psi_R$$

Charge conjugation

In general, ψ and ψ^* do not transform in the same way under Lorentz transformations and the naive reality condition $\psi = \psi^*$ is frame dependent

But it is possible to find a matrix C, called charge conjugation matrix, such that

$$\psi \quad \text{and} \quad \psi_C = C\,\psi^*$$

transform in the same way under Lorentz transformations

The matrix C needs to satisfy $C\gamma^* = -\gamma^\mu C$

In the Dirac and Weyl representations, $C = i\gamma^2$

In the Majorana representation, $C = 1_4$

Basic properties of the charge conjugation matrix: $C^2 = 1_4,\ C^\dagger = C,\ C^* = C$

The charge conjugated spinor, ψ_C , satisfies the same Dirac equation as ψ, with the same mass but opposite electric charge (when the spinor is minimally coupled to a U(1) gauge field)

A **Majorana** spinor satisfies the (Lorentz invariant!) condition $\psi = \psi_C$

Note that in 4D, a spinor cannot be simultaneously chiral and Majorana

Dirac and Majorana Masses

By construction, the following two mass terms in the Lagrangian are Lorentz-invariant

Dirac mass: $\mathcal{L}_{\text{Dirac}} = m\bar{\psi}\psi$ (conserves fermion number)

Majorana mass: $\mathcal{L}_{\text{Majorana}} = m\bar{\psi}_C\,\psi$ (changes fermion number by 2)

These two mass terms have different a chirality structure

$$\mathcal{L}_{\text{Dirac}} = m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$$

$$\mathcal{L}_{\text{Majorana}} = m\left(\bar{\psi}_{L_C}\psi_L + \bar{\psi}_{R_C}\psi_R\right)$$

A chiral fermion can have a Majorana mass
A Dirac mass requires spinors of opposite chirality

Whether or not a Dirac or a Majorana mass can be included in the Lagrangian depends on transformation laws of the spinors under the gauge transformations

Within the SM (with the Higgs field), a Dirac mass can written for the charged leptons and the quarks while a Majorana mass can be written for the neutrinos.

Higgs Lifetime “Computation”

Using dimensional analysis arguments,
compute the Higgs boson lifetime (or its inverse aka as the Higgs decay width)

— Hints —

Higgs couplings proportional are proportional to the mass of the particles it couples to.
It will therefore decay predominantly decay into the heaviest particle that is lighter than $m_H/2$

$$\Gamma \sim \frac{1}{8\pi}\left(\frac{m_b}{v}\right)^2 m_h \sim \frac{1}{10}\left(\frac{4\,\mathrm{GeV}}{246\,\mathrm{GeV}}\right)^2 125\,\mathrm{GeV} \sim 1\,\mathrm{MeV}$$

phase space

couplings to b-quark

dimensionally $[\Gamma]_m=1$

$\tau \sim 10^{-22}\,\mathrm{s}$

E	T	L
1eV	10^{-16}s	10^{-7}m

Putting all factors and considering the other decay modes, Higgs width = 4MeV in the SM

(for Z gauge boson: $\Gamma_Z = \frac{7}{48\pi} g^2 m_Z \sim 2\,\mathrm{GeV}$ i.e. $\tau_Z \sim 10^{-25}\,\mathrm{s}$)