

Causal Inference: Classical and Quantum

Lecturer: Robert Spekkens
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Journeys,
SAIFR, ICTP, Brazil



Causarum Investigatio
“Investigate the causes”

Simpson's paradox
in statistics
and its causal resolution

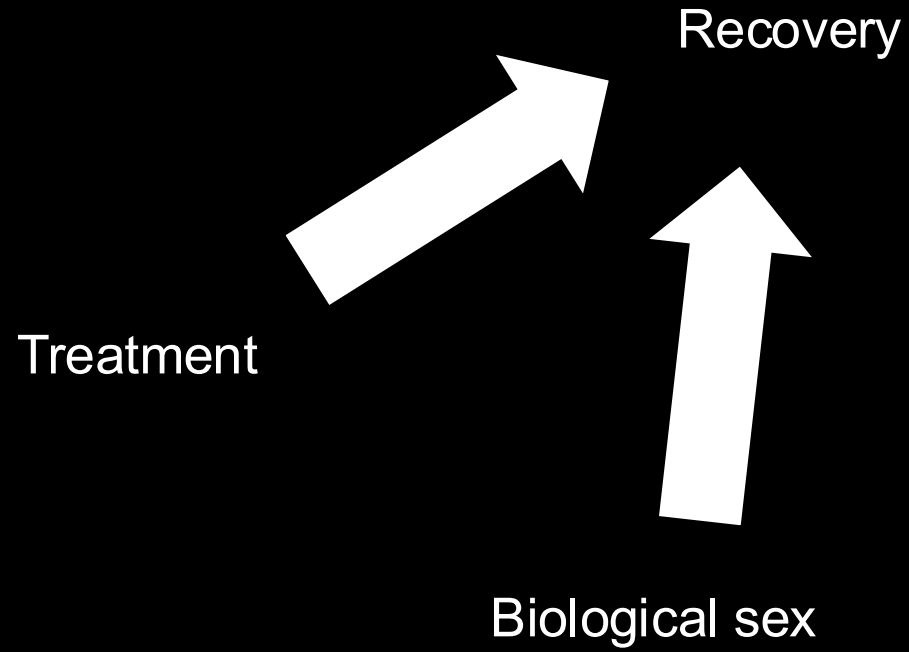
$$P(\text{recovery} \mid \text{drug}) > P(\text{recovery} \mid \text{no drug})$$

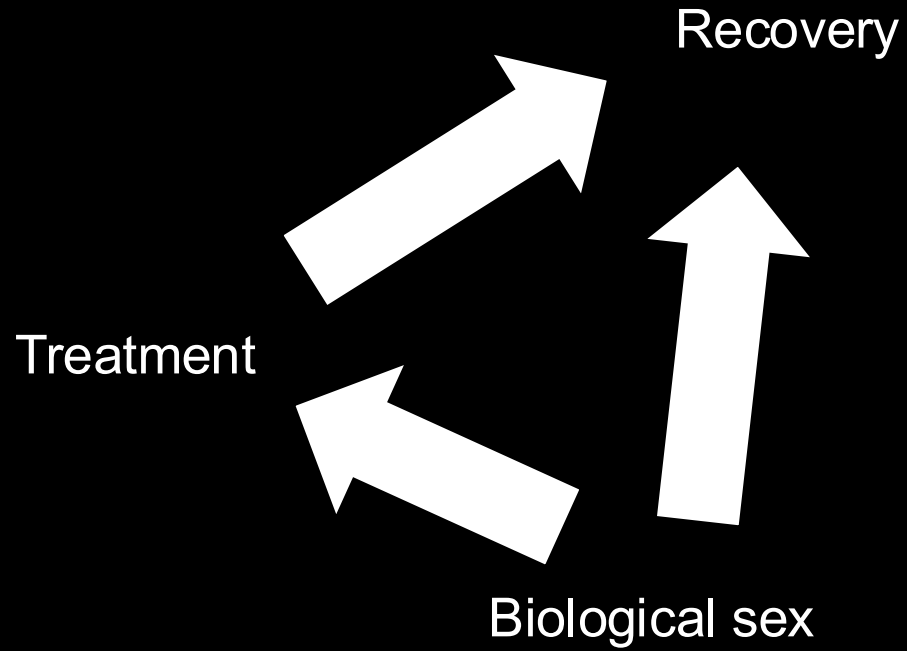
$$P(\text{recovery} \mid \text{drug, male}) < P(\text{recovery} \mid \text{no drug, male})$$

$$P(\text{recovery} \mid \text{drug, female}) < P(\text{recovery} \mid \text{no drug, female})$$

Recovery probability

	drug	no drug
male	180/300 = 60%	70/100 = 70%
female	20/100 = 20%	90/300 = 30%
combined	200/400 = 50%	160/400 = 40%



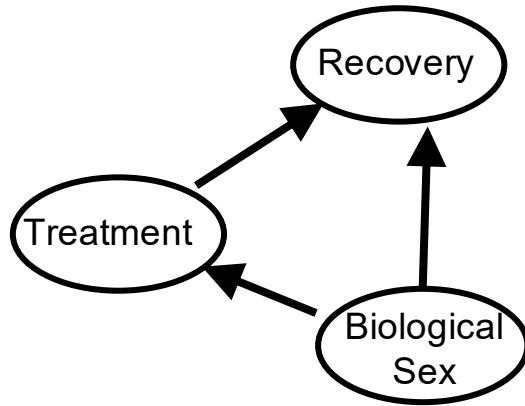


$$P(\text{recovery} \mid \text{drug}) > P(\text{recovery} \mid \text{no drug})$$

$$P(\text{recovery} \mid \text{drug, male}) < P(\text{recovery} \mid \text{no drug, male})$$



$$P(\text{recovery} \mid \text{drug, female}) < P(\text{recovery} \mid \text{no drug, female})$$



Therefore: stratify the data by the common cause

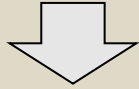
“statisticians are invariably motivated by causal questions but the peculiar nature of these questions is that they cannot be answered, or even articulated, in the traditional language of statistics.

[...]

causation is not merely an aspect of statistics; it is an addition to statistics, an enrichment that allows statistics to uncover workings of the world that traditional methods cannot”

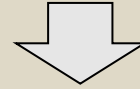
From: J. Pearl, M. Glymour and N. Jewell, ‘Causal Inference in Statistics’

Statistical
paradigm



Causal
paradigm

Operational
paradigm



Realist
paradigm

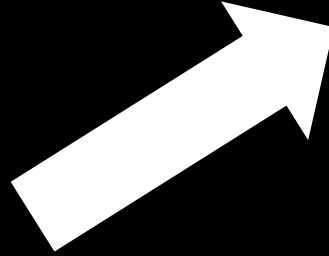
Causal structures with hidden variables

Maximum wages
in future career
above some
threshold?

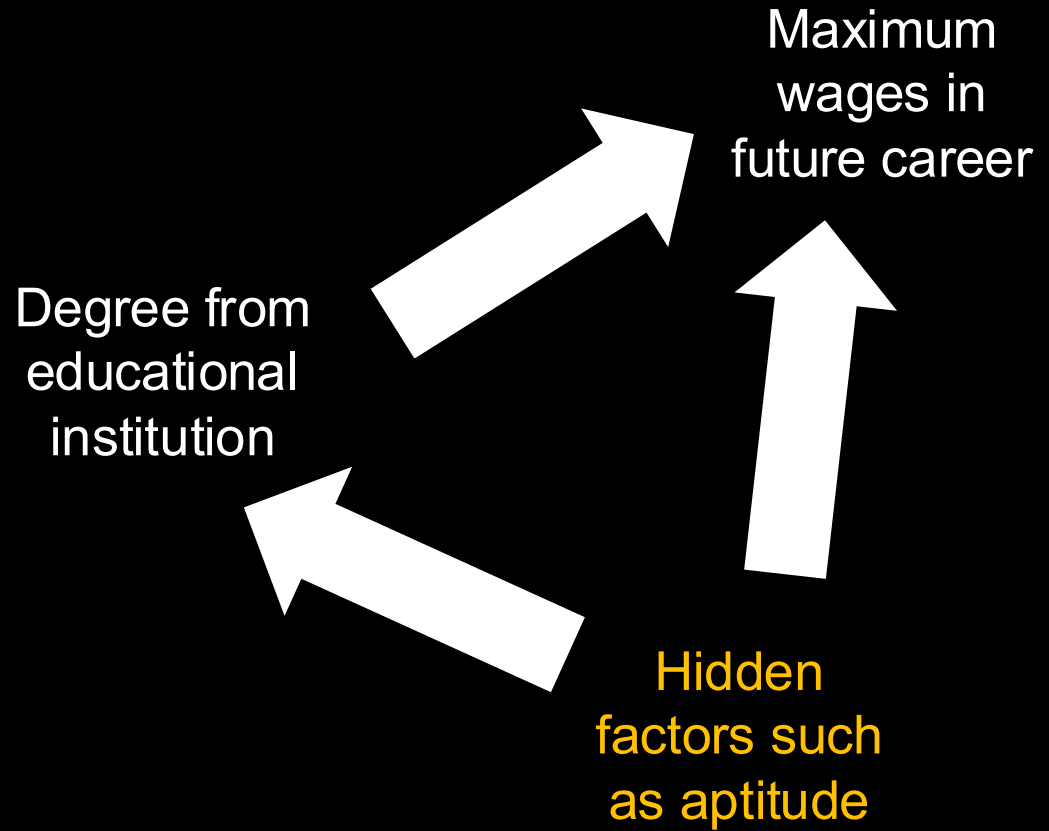
Degree from
educational
institution?

	Yes	No
Yes	79%	21%
No	43%	57%

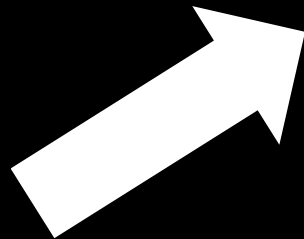
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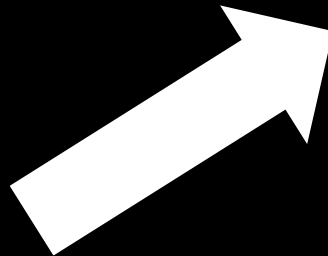
Maximum
wages in
future career



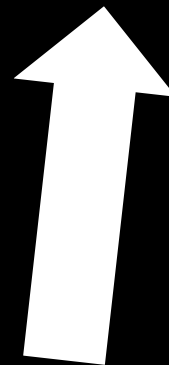
Coin
Flip



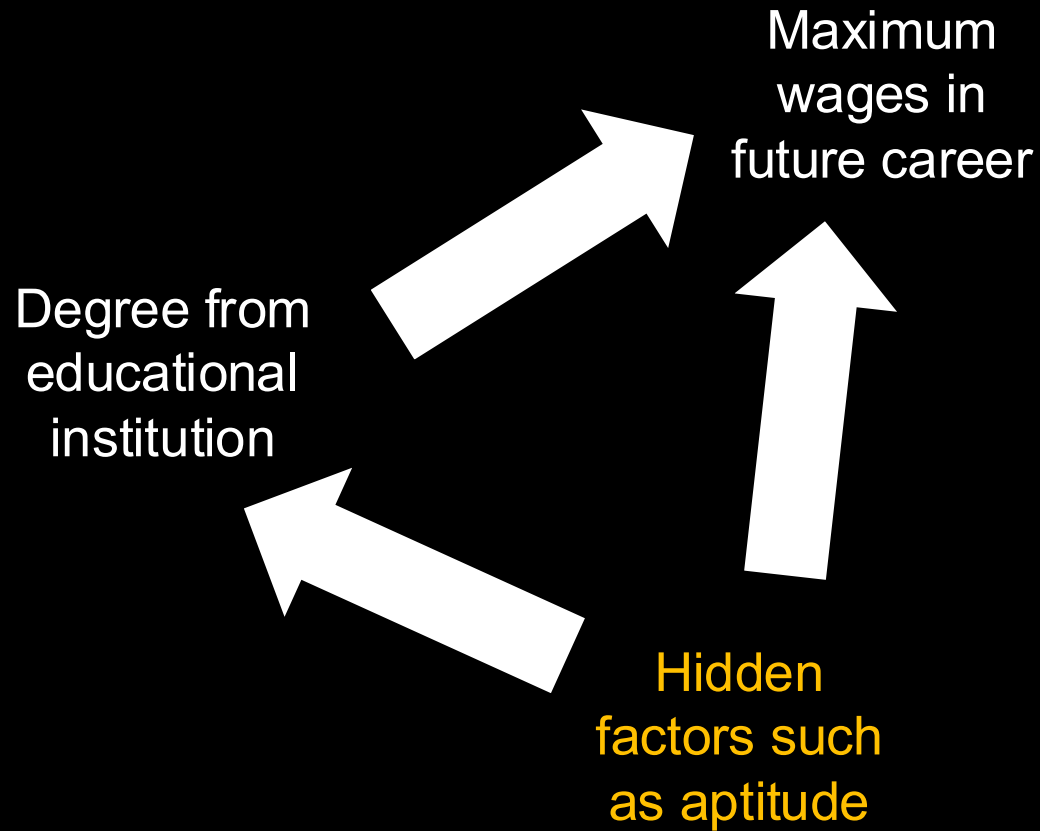
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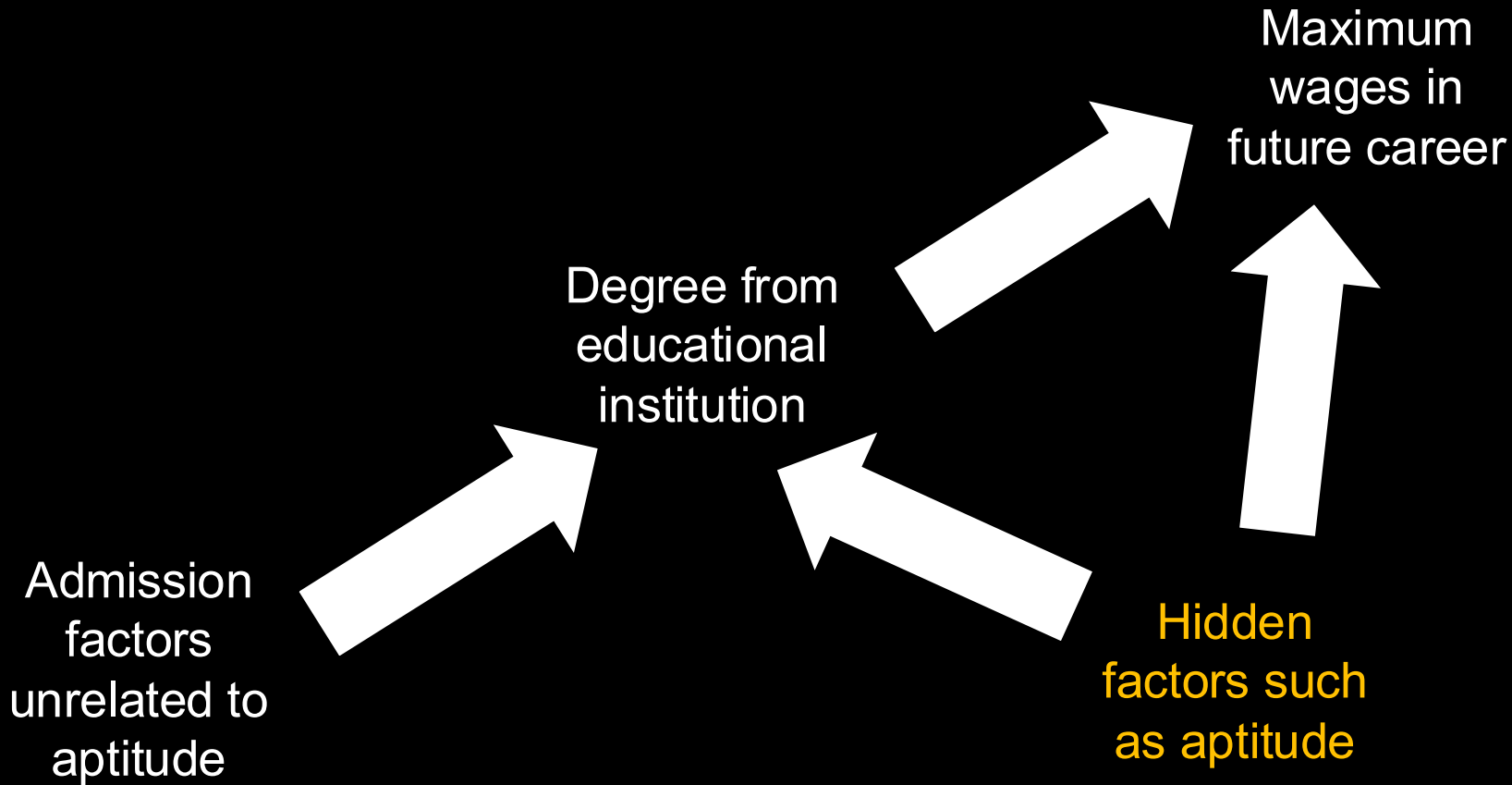


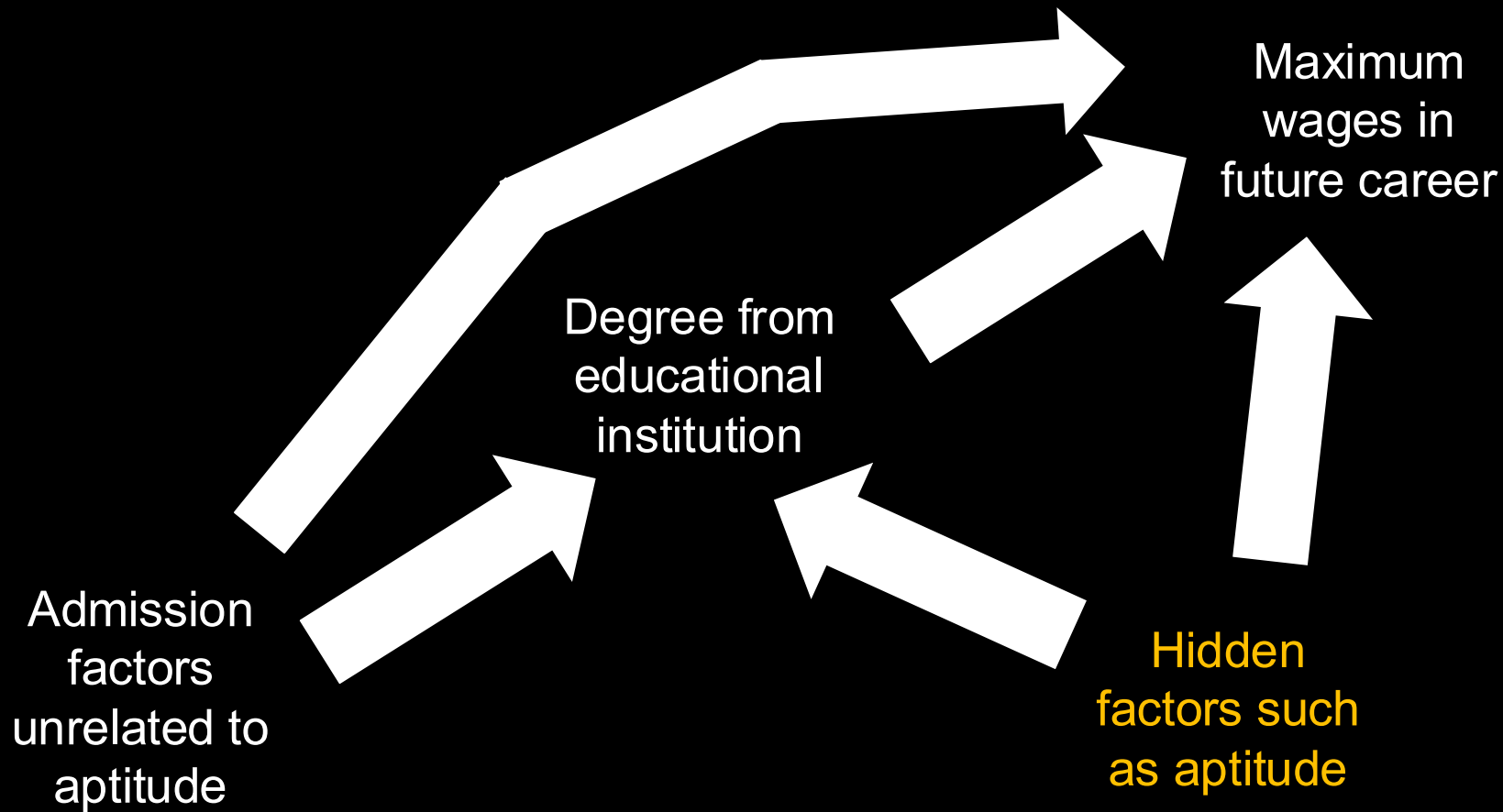
Maximum
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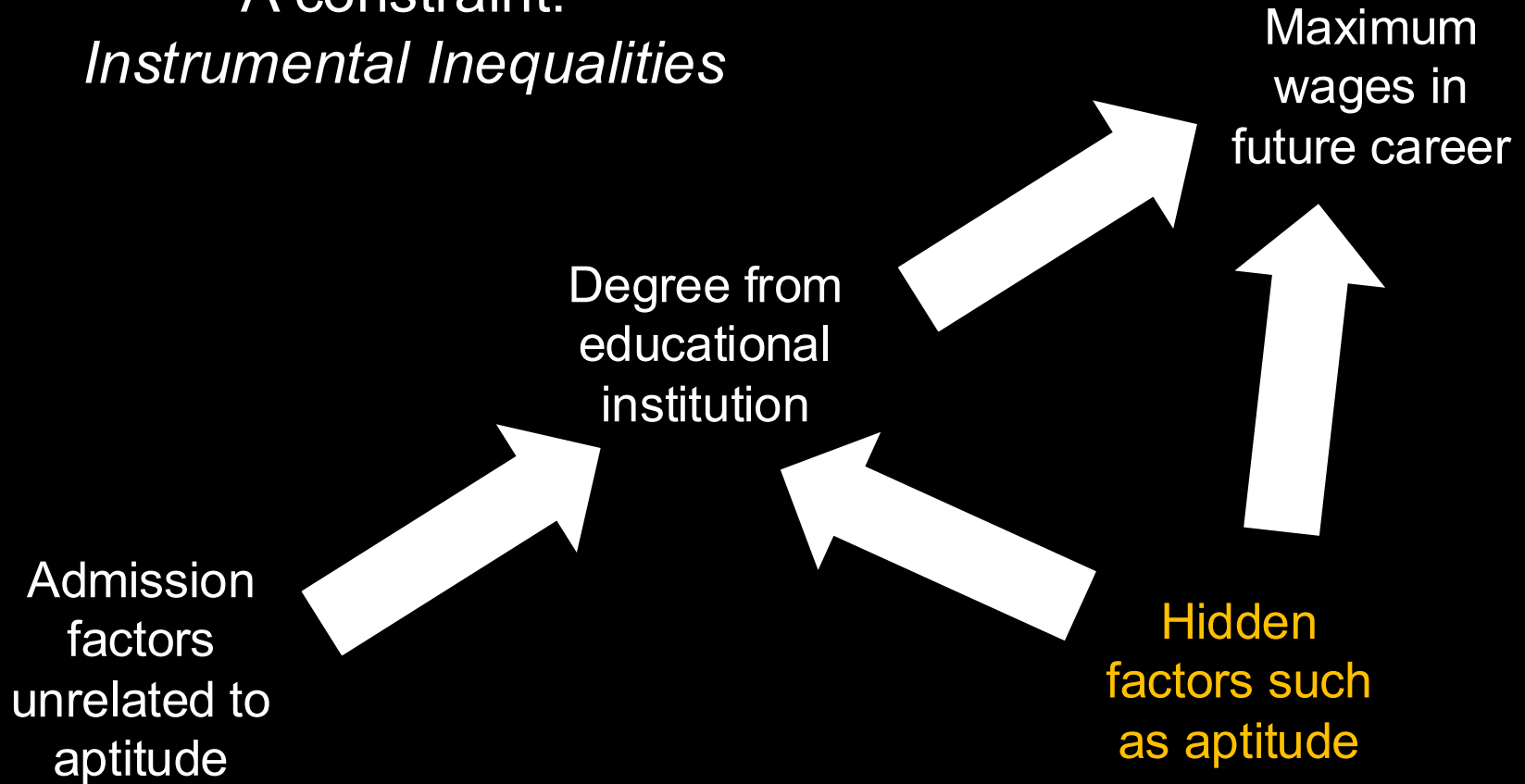
Hidden
factors such
as aptitude

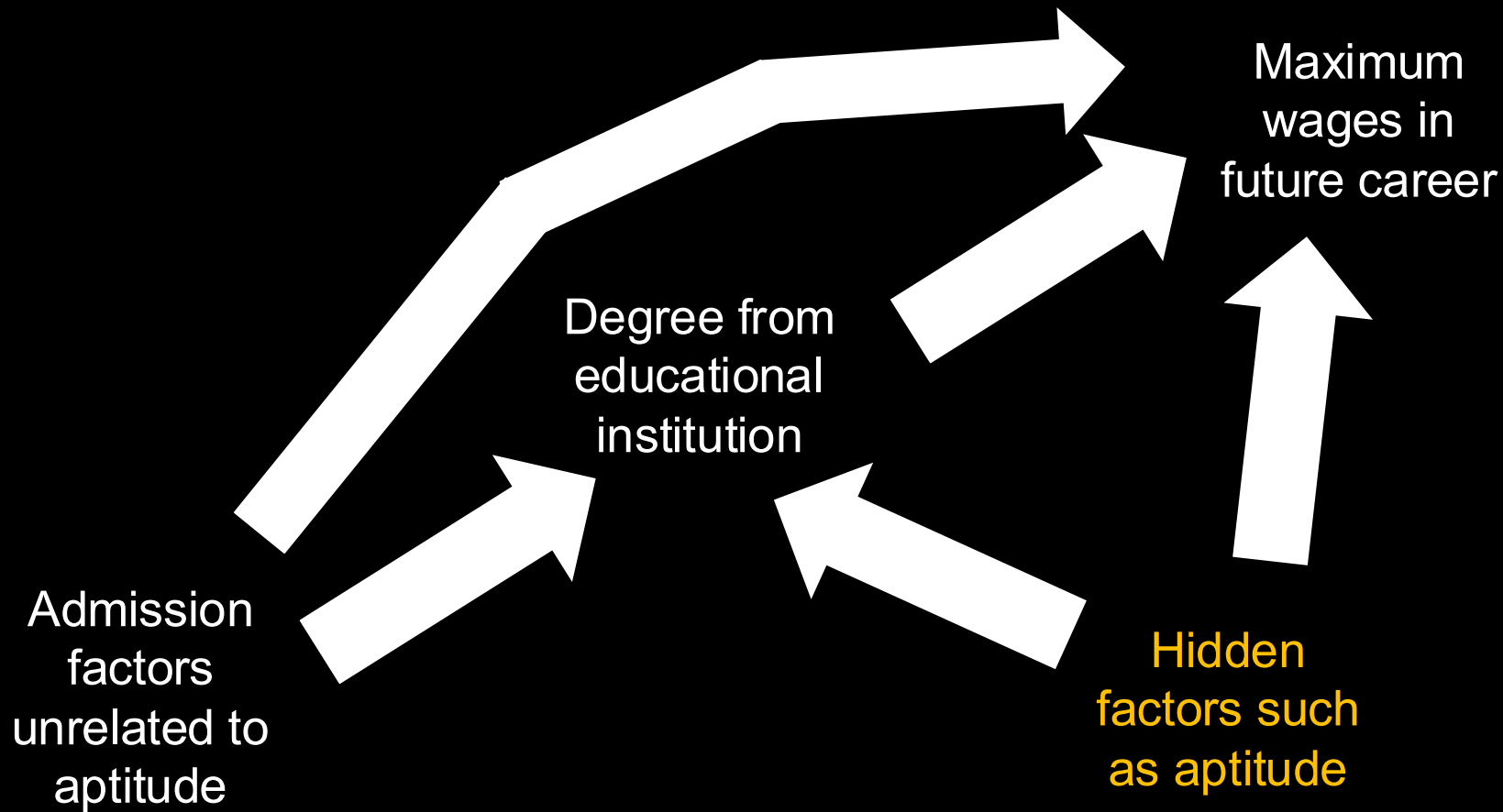




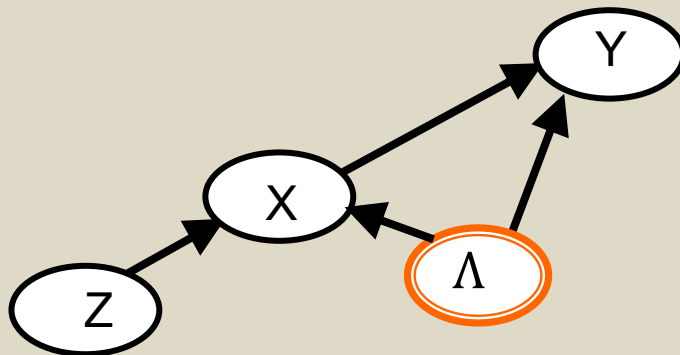


A constraint:
Instrumental Inequalities





Causal structure



Parameters

$$P_{X|\Lambda Z}$$

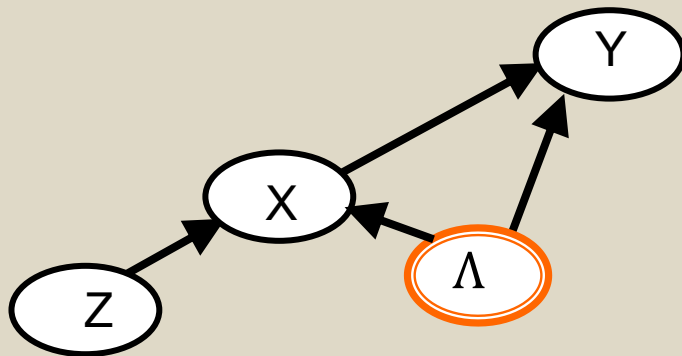
$$P_{Y|\Lambda X}$$

$$P_{\Lambda}$$

$$P_Z$$

Causal structure

Parameters



$$P_{X|\Lambda Z}$$

$$P_{Y|\Lambda X}$$

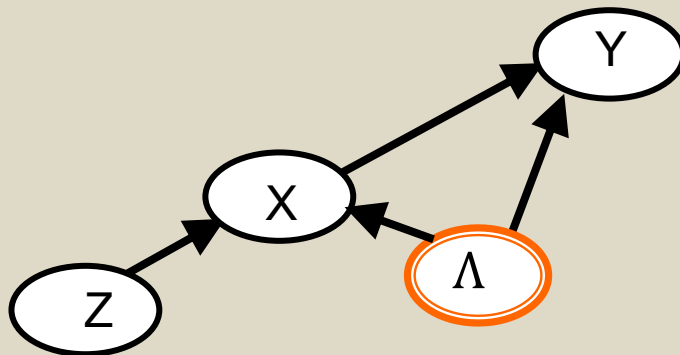
$$P_{\Lambda}$$

$$P_Z$$

$$P_{XYZ} = \sum_{\Lambda} P_{Y|X\Lambda} P_{X|Z\Lambda} P_{\Lambda} P_Z$$

Causal structure

Parameters



$$P_{X|\Lambda Z}$$

$$P_{Y|\Lambda X}$$

$$P_{\Lambda}$$

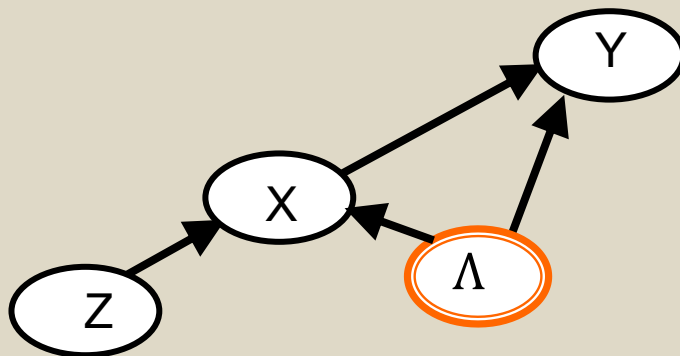
$$P_Z$$

$$P_{XYZ} = \sum_{\Lambda} P_{Y|X\Lambda} P_{X|Z\Lambda} P_{\Lambda} P_Z$$

A distribution is said to be **compatible** with a given causal structure if there are parameters that yield it

Causal structure

Parameters



$$P_{X|\Lambda Z}$$

$$P_{Y|\Lambda X}$$

$$P_{\Lambda}$$

$$P_Z$$

$$P_{XYZ} = \sum_{\Lambda} P_{Y|X\Lambda} P_{X|Z\Lambda} P_{\Lambda} P_Z$$

Example of causal compatibility constraint:

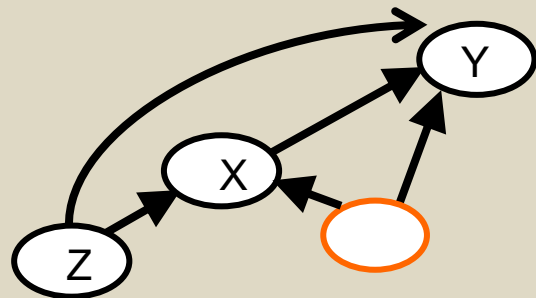
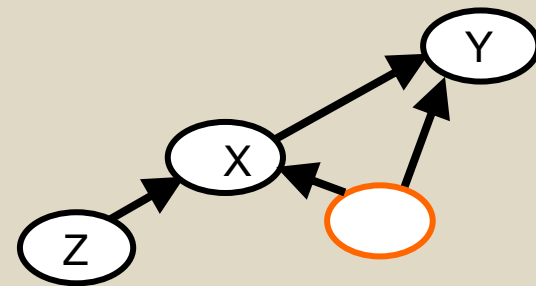
$$P_{XY|Z}(00|0) + P_{XY|Z}(01|1) \leq 1$$

Pearl, 1993

The evidence

Z=0		Y=0	Y=1
	X=0	0.79	0.21
	X=1	0.43	0.57
Z=1		Y=0	Y=1
	X=0	0.59	0.41
	X=1	0.39	0.61

The hypotheses

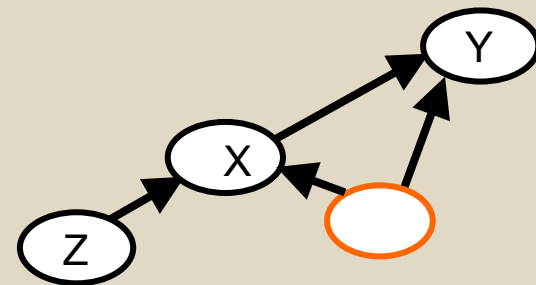


The evidence

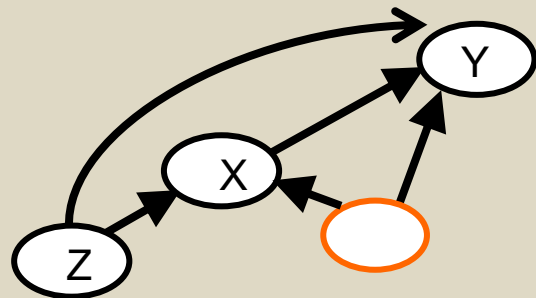
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Violates the instrumental inequality!

The hypotheses



Implies a constraint:
the instrumental inequality

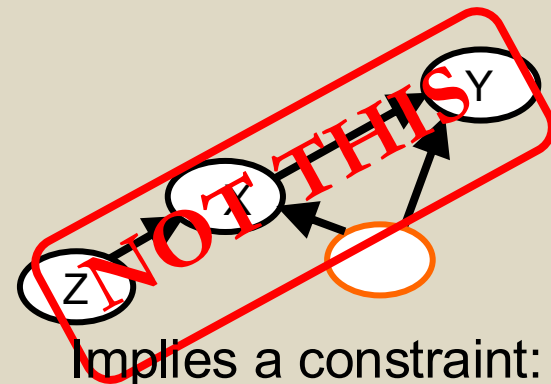


The evidence

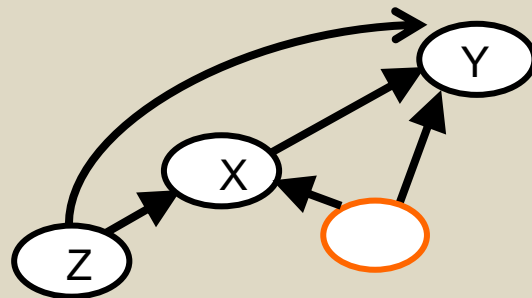
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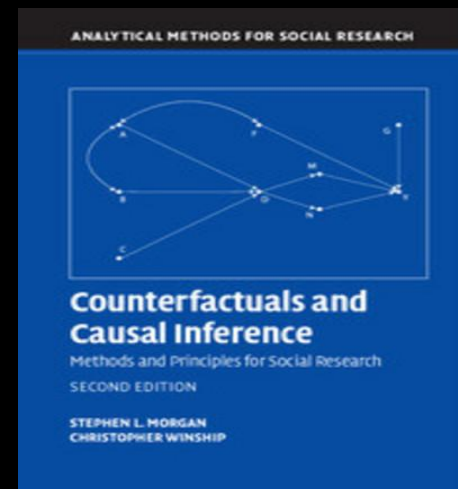
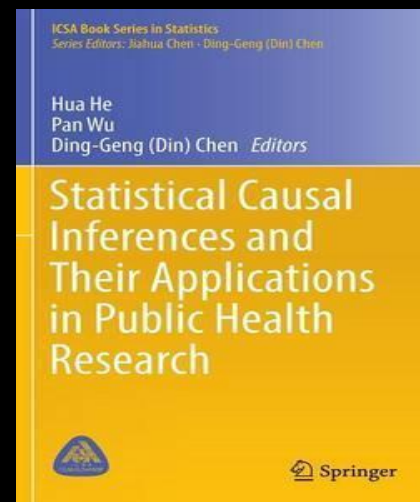
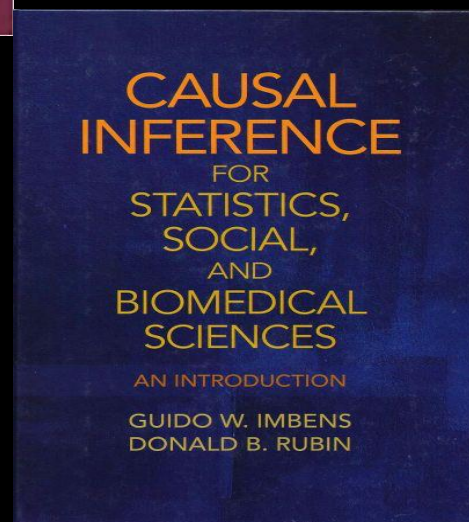
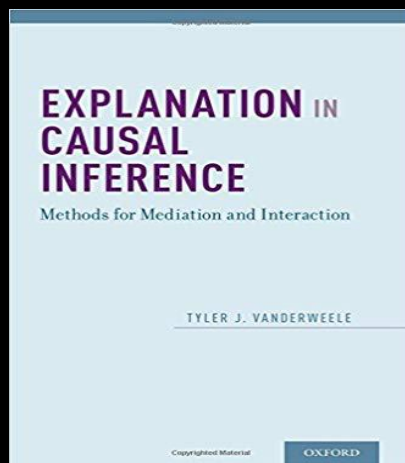
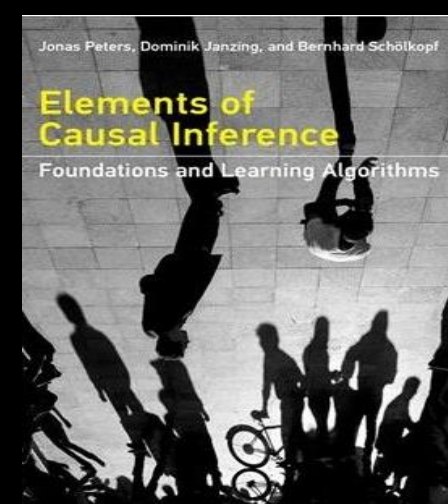
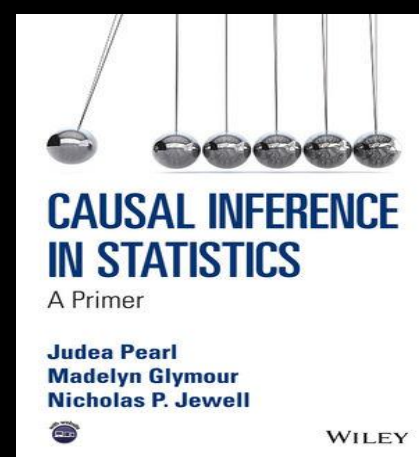
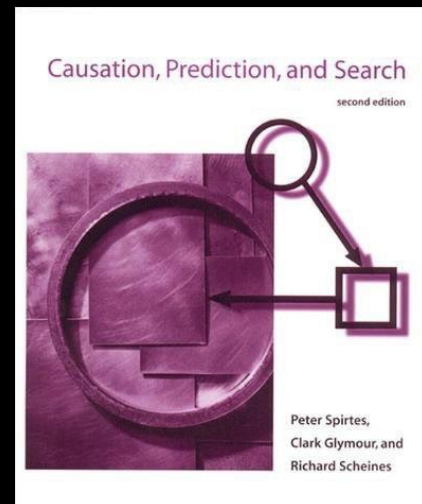
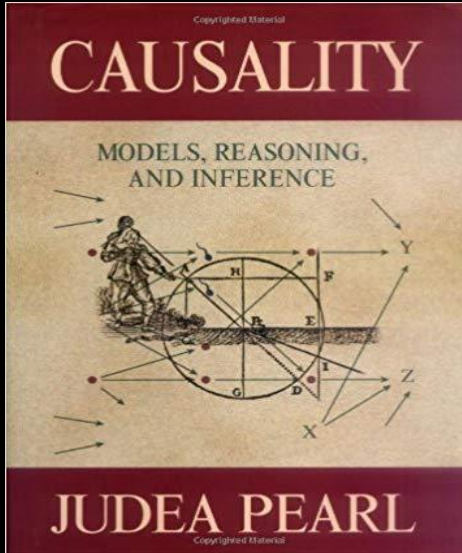
Violates the instrumental inequality!

The hypotheses



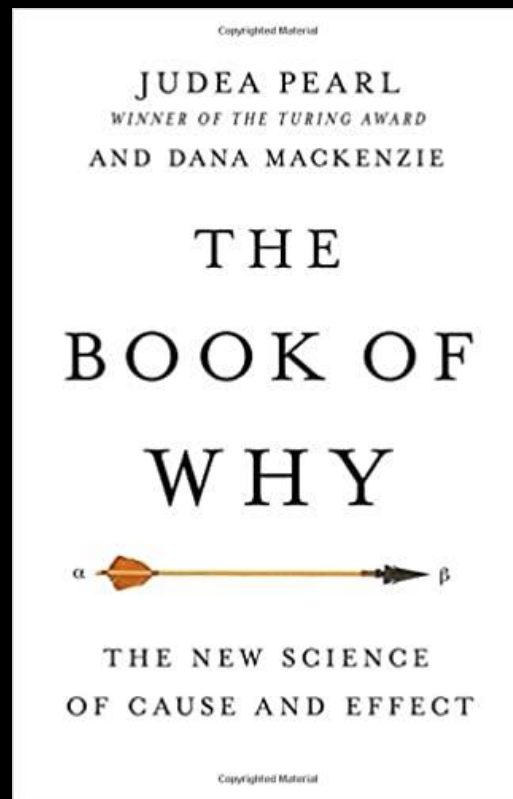
Implies a constraint:
the instrumental inequality







Judea Pearl

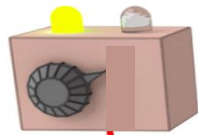


QUANTUM THEORY



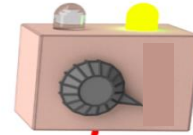
John Stuart
Bell

Left outcome

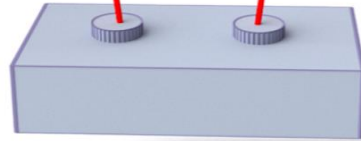


Left
setting

Right outcome



Right
setting

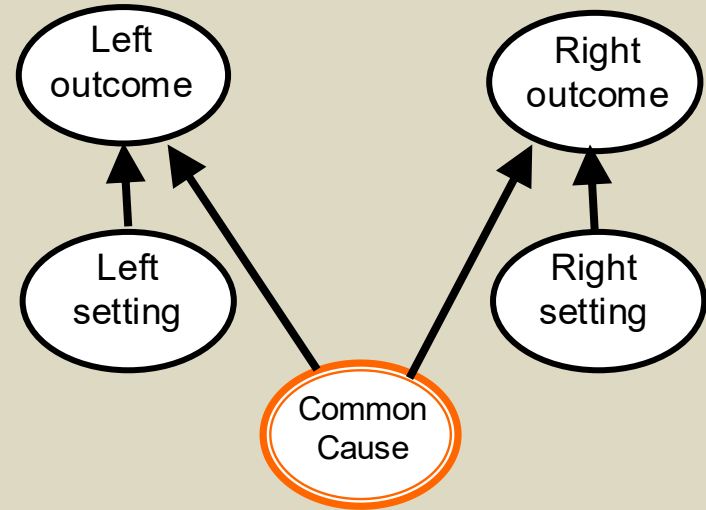


		Left outcome and Right outcome			
Left setting and Right setting		0 and 0	0 and 1	1 and 0	1 and 1
	0 and 0	43%	7%	7%	43%
	0 and 1	43%	7%	7%	43%
	1 and 0	43%	7%	7%	43%
	1 and 1	7%	43%	43%	7%

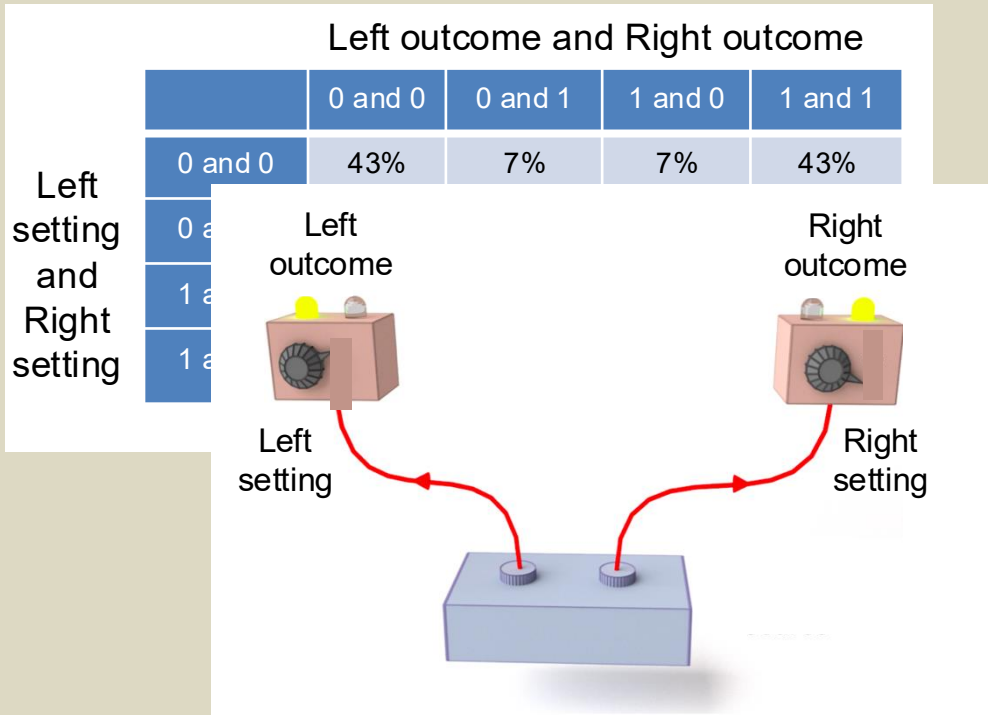
The evidence

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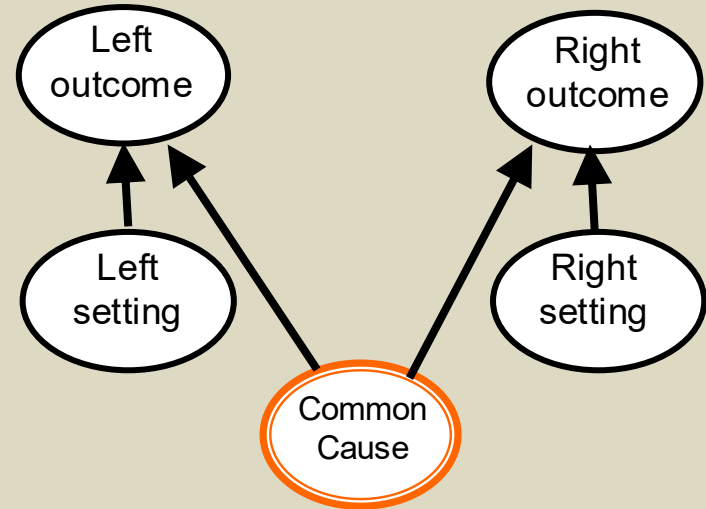
The natural hypothesis



The evidence



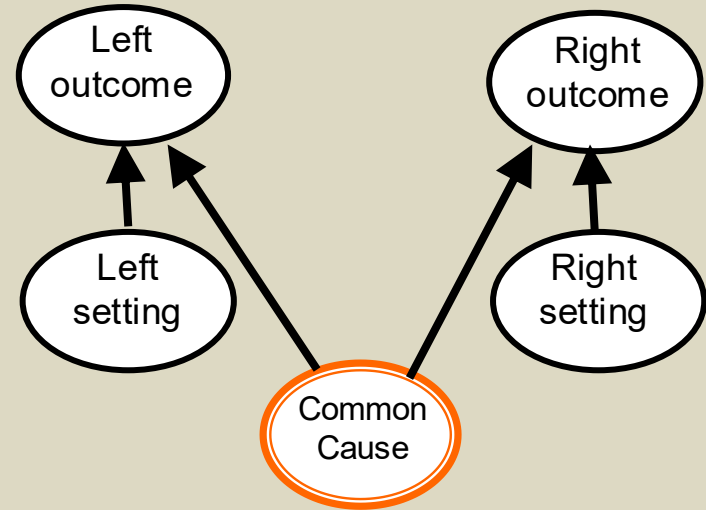
The natural hypothesis



The evidence

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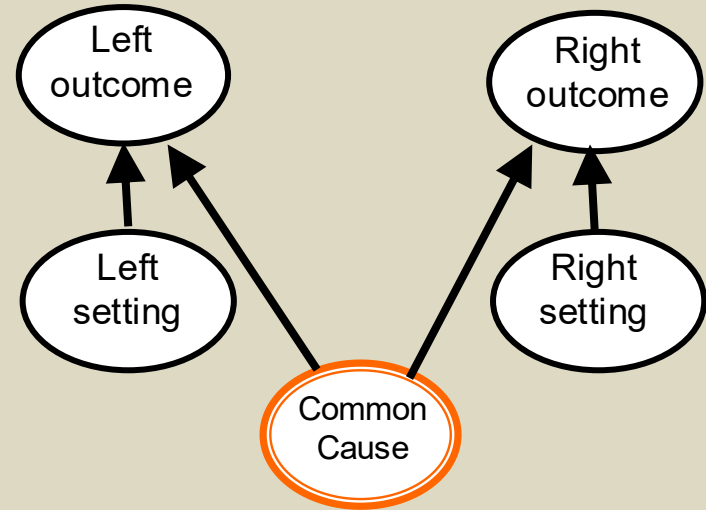
The natural hypothesis



The evidence

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The natural hypothesis



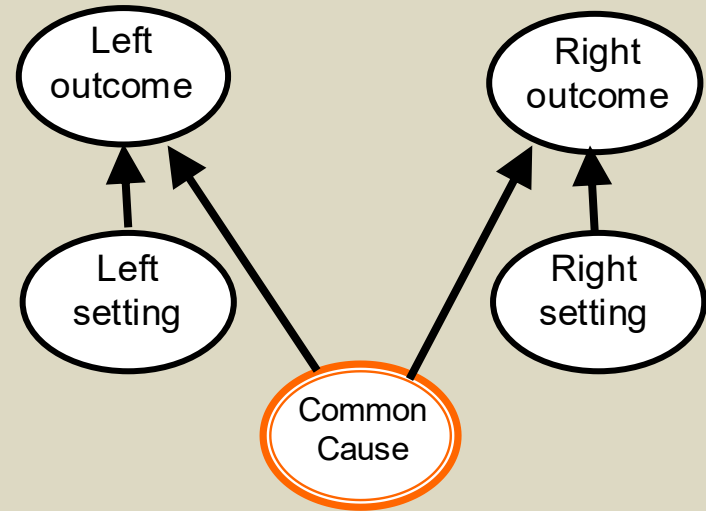
Implies a constraint:
Bell Inequalities

The evidence

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	1 and 1	7%	43%	43%	7%

Violates the
Bell Inequalities!

The natural hypothesis



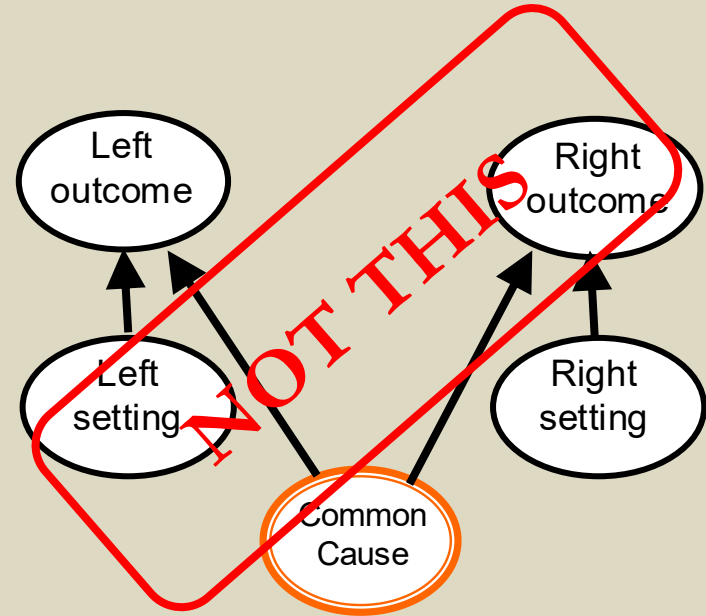
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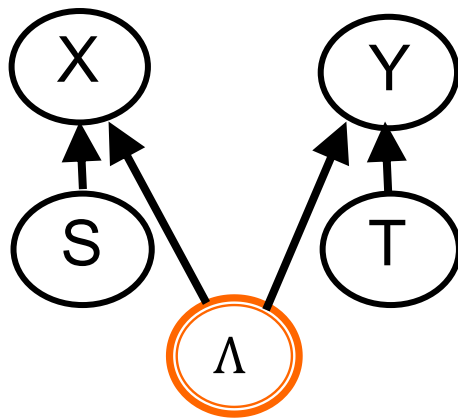
The natural hypothesis



Implies a constraint:
Bell Inequalities

Incompatible

Causal structure



Parameters

$$P_{X|S\Lambda}$$

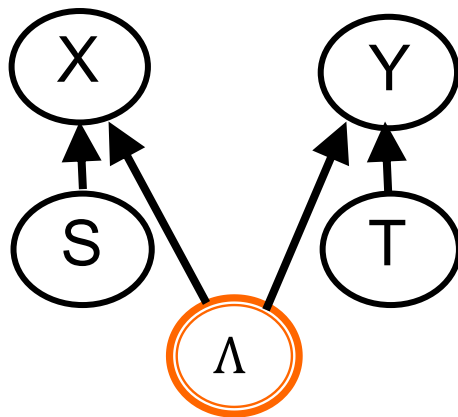
$$P_{Y|T\Lambda}$$

$$P_{\Lambda}$$

$$P_S$$

$$P_T$$

Causal structure



Parameters

$$P_{X|S\Lambda}$$

$$P_{Y|T\Lambda}$$

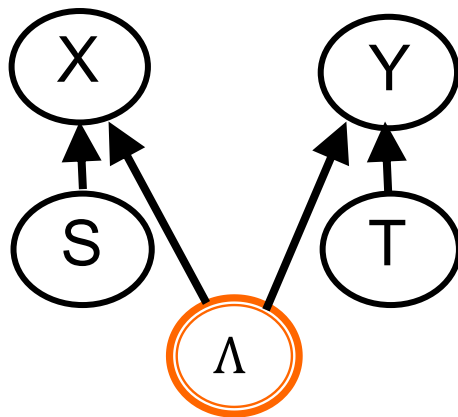
$$P_{\Lambda}$$

$$P_S$$

$$P_T$$

$$P_{XYST} = \sum_{\Lambda} P_{Y|T\Lambda} P_{X|S\Lambda} P_{\Lambda} P_S P_T$$

Causal structure

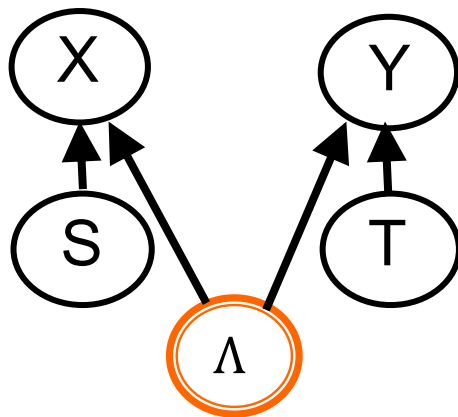


Parameters

$$P_{X|S\Lambda}$$
$$P_{Y|T\Lambda}$$
$$P_{\Lambda}$$

$$P_{XY|ST} = \sum_{\Lambda} P_{Y|T\Lambda} P_{X|S\Lambda} P_{\Lambda}$$

Causal structure



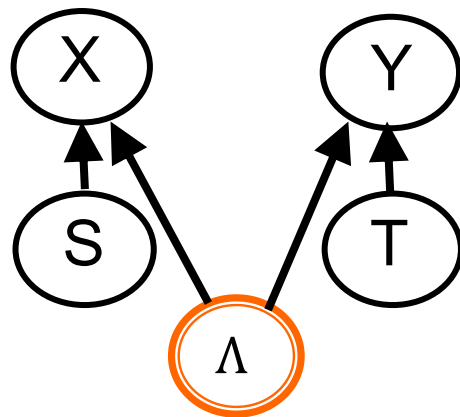
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$$P_{XY|ST} = \sum_{\Lambda} P_{Y|T\Lambda} P_{X|S\Lambda} P_{\Lambda}$$

Examples of causal compatibility constraints:

Causal structure



Parameters

$$P_{X|S\Lambda}$$
$$P_{Y|T\Lambda}$$
$$P_{\Lambda}$$

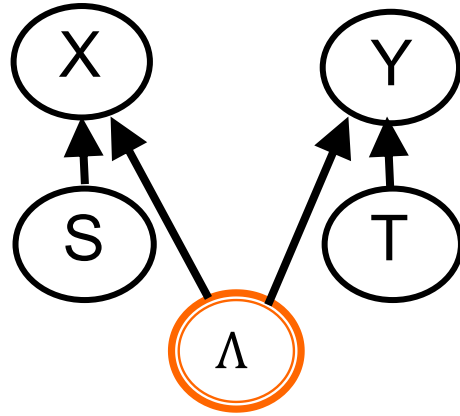
$$P_{XY|ST} = \sum_{\Lambda} P_{Y|T\Lambda} P_{X|S\Lambda} P_{\Lambda}$$

Examples of causal compatibility constraints:

$$P_{X|ST} = P_{X|S}$$
$$P_{Y|ST} = P_{Y|T}$$

Causal structure

Parameters



$$P_{X|S\Lambda}$$

$$P_{Y|T\Lambda}$$

$$P_{\Lambda}$$

$$P_{XY|ST} = \sum_{\Lambda} P_{Y|T\Lambda} P_{X|S\Lambda} P_{\Lambda}$$

Examples of causal compatibility constraints:

$$P_{X|ST} = P_{X|S}$$

$$P_{Y|ST} = P_{Y|T}$$

$$\frac{1}{4} \sum_{x=y} P_{XY|ST}(xy|00) + \frac{1}{4} \sum_{x=y} P_{XY|ST}(xy|01) + \frac{1}{4} \sum_{x=y} P_{XY|ST}(xy|10) + \frac{1}{4} \sum_{x \neq y} P_{XY|ST}(xy|11) \leq \frac{3}{4}$$

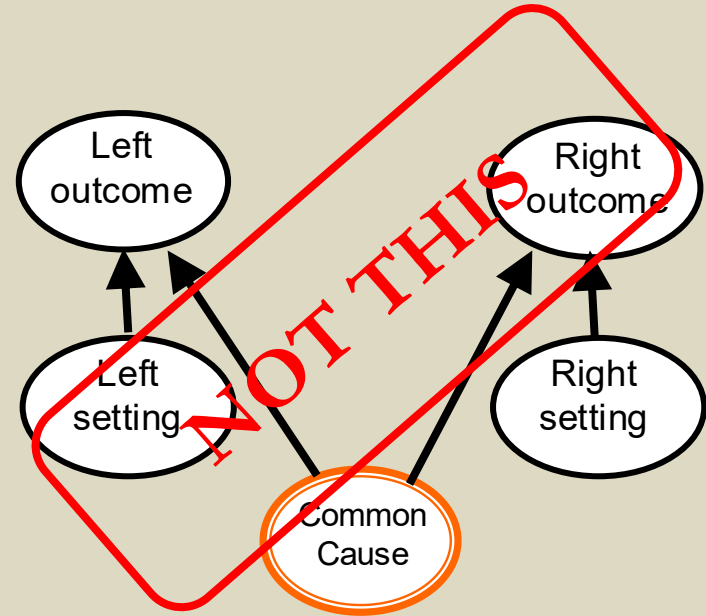
Clauser, Horne, Shimony and Holte, Phys. Rev. Lett. 23, 880 (1967)

The evidence

		Left outcome and Right outcome			
		0 and 0	0 and 1	1 and 0	1 and 1
Left setting and Right setting	0 and 0	43%	7%	7%	43%
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Violates the
Bell Inequalities!

The natural hypothesis



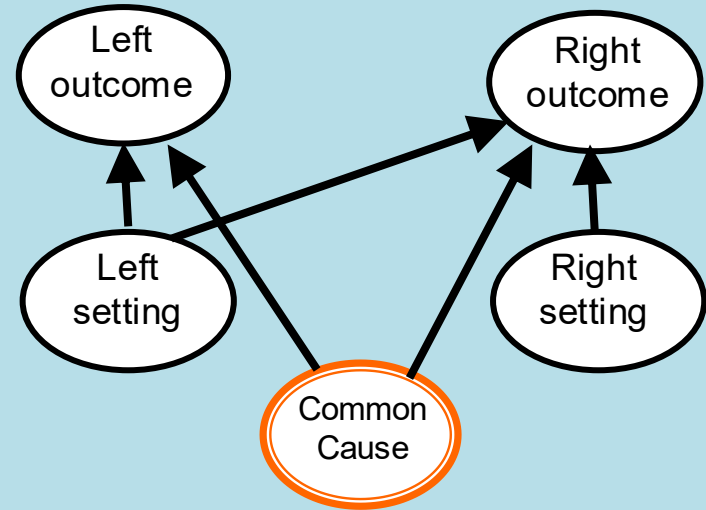
Implies a constraint:
Bell Inequalities

Incompatible

The evidence

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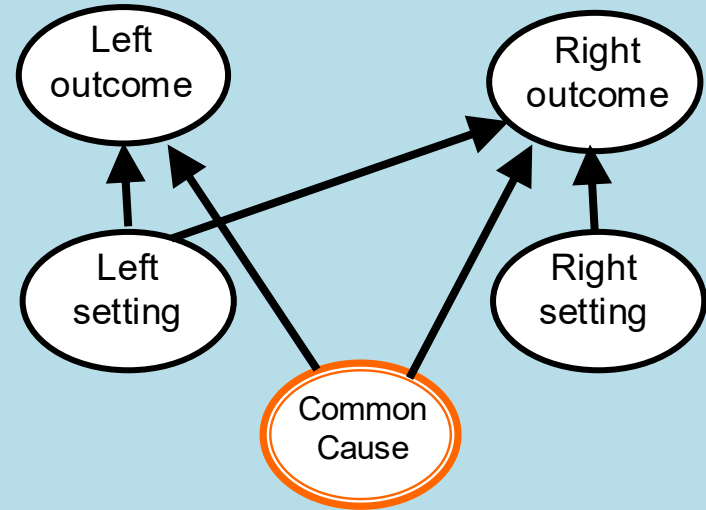
The 2nd possibility



The evidence

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The 2nd possibility



Compatible

QUANTUM THEORY



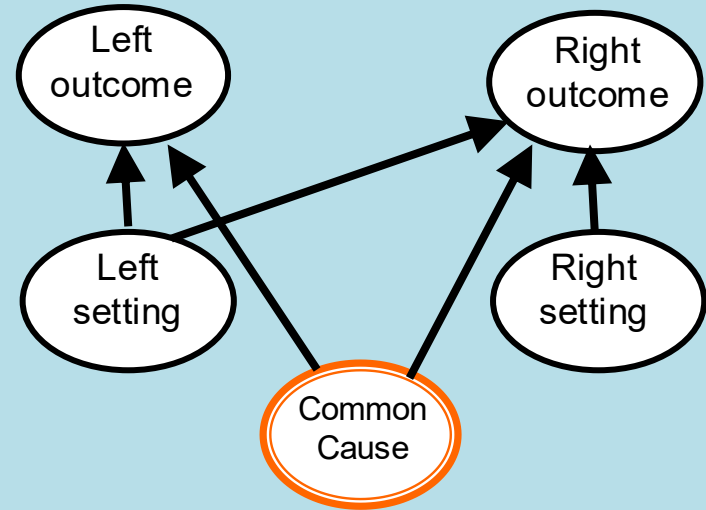
Diego Delso / CC BY-SA

RELATIVITY

The evidence

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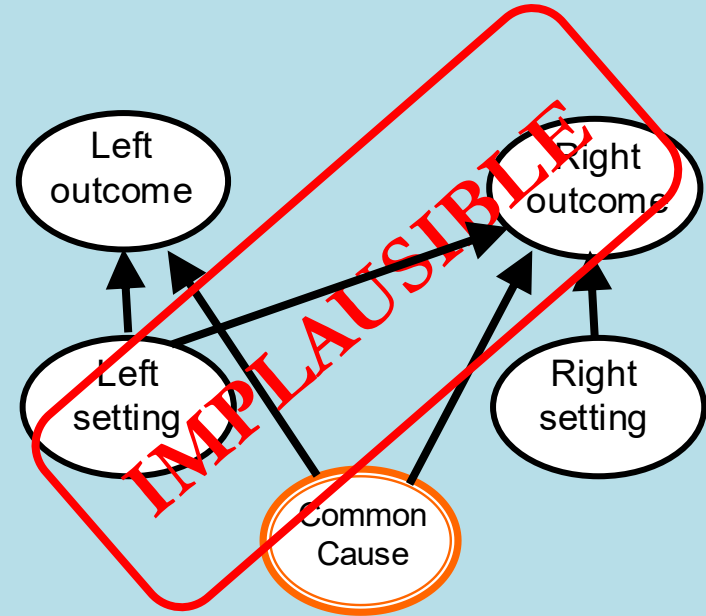
The 2nd possibility



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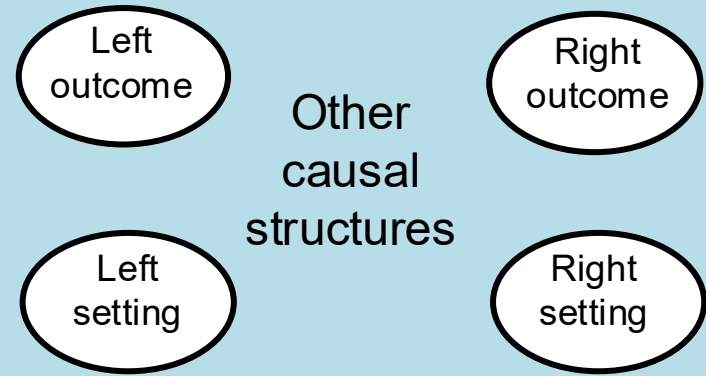
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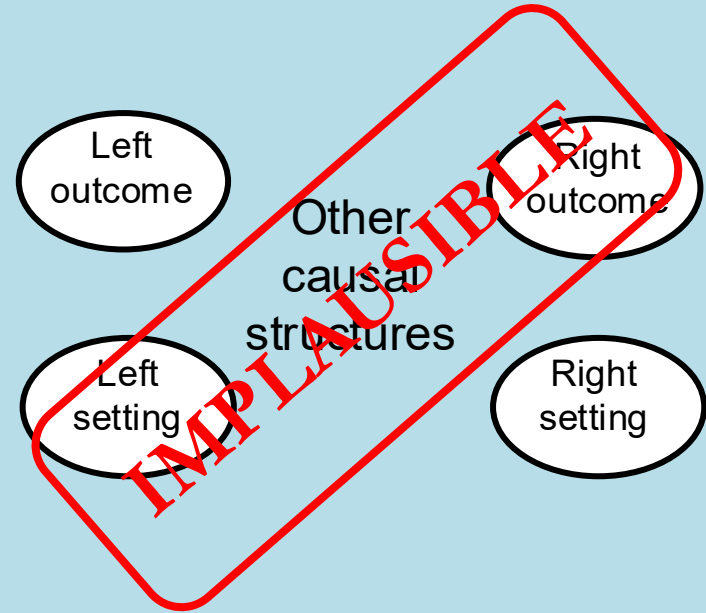
The usual suspects



The evidence

		Left outcome and Right outcome			
		0 and 0	0 and 1	1 and 0	1 and 1
Left setting and Right setting	0 and 0	43%	7%	7%	43%
	0 and 1	43%	7%	7%	43%
	1 and 0	43%	7%	7%	43%
	1 and 1	7%	43%	43%	7%

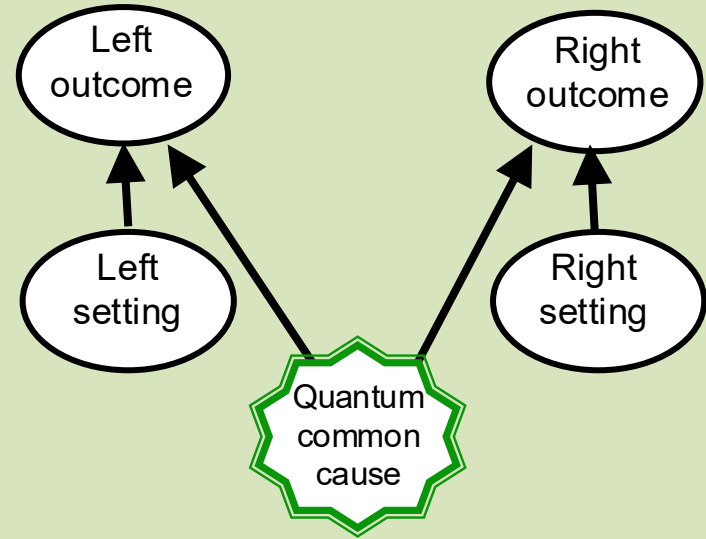
The usual suspects



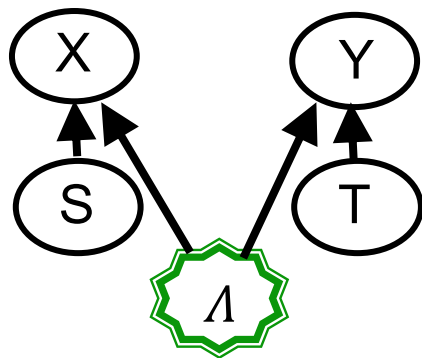
The evidence

		Left outcome and Right outcome			
Left setting and Right setting		0 and 0	0 and 1	1 and 0	1 and 1
	0 and 0	43%	7%	7%	43%
	0 and 1	43%	7%	7%	43%
	1 and 0	43%	7%	7%	43%
	1 and 1	7%	43%	43%	7%

A new possibility



Quantum causal model



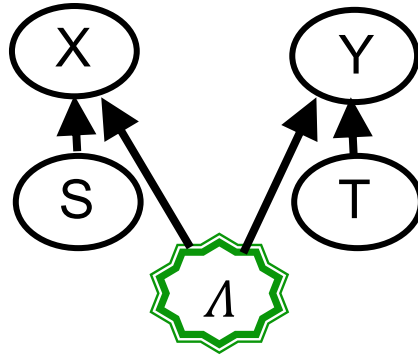
$$\rho_{X|S\Lambda}$$

$$\rho_{Y|T\Lambda}$$

$$\rho_{\Lambda}$$

$$[\rho_{X|S\Lambda}, \rho_{Y|T\Lambda}] = 0$$

Quantum causal model



$$\rho_{X|S\Lambda}$$

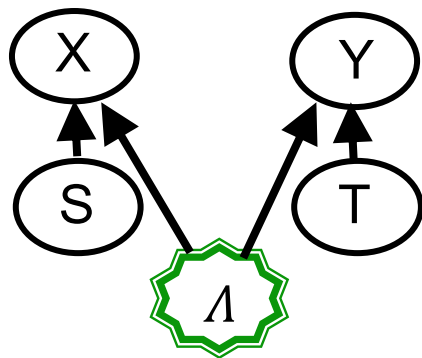
$$\rho_{Y|T\Lambda}$$

$$\rho_{\Lambda}$$

$$[\rho_{X|S\Lambda}, \rho_{Y|T\Lambda}] = 0$$

$$P_{XY|ST} = \text{Tr}_{\Lambda}(\rho_{X|S\Lambda}\rho_{Y|T\Lambda}\rho_{\Lambda})$$

Quantum causal model



$$\rho_{X|S\Lambda}$$

$$\rho_{Y|T\Lambda}$$

$$\rho_{\Lambda}$$

$$[\rho_{X|S\Lambda}, \rho_{Y|T\Lambda}] = 0$$

$$P_{XY|ST} = \text{Tr}_{\Lambda}(\rho_{X|S\Lambda} \rho_{Y|T\Lambda} \rho_{\Lambda})$$

Leifer and RWS, PRA 88, 052130 (2013)

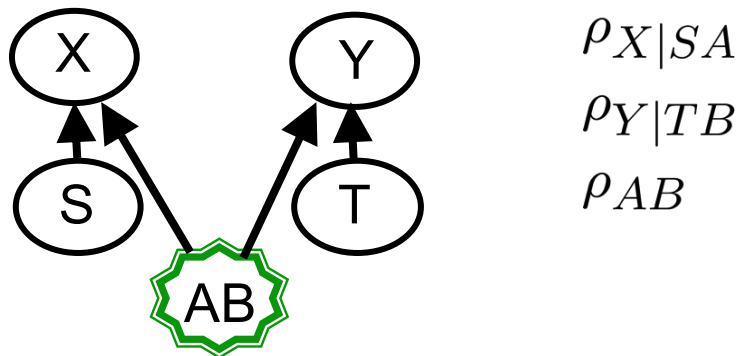
Henson, Lal & Pusey NJP 16, 113043 (2014)

Costa, Shrapnel NJP 18(6) (2016)

Allen, Barrett, Horsman, Lee & RWS, PRX 7, 031021 (2017)

Barrett, Lorenz, Oreshkov, arXiv:1906.10726

Quantum causal model



$$P_{XY|ST} = \text{Tr}_{AB}(\rho_{X|SA}\rho_{Y|TB}\rho_{AB})$$

Leifer and RWS, PRA 88, 052130 (2013)

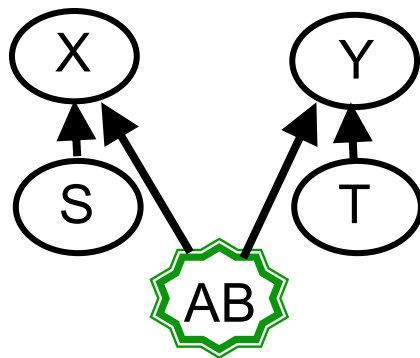
Henson, Lal & Pusey NJP 16, 113043 (2014)

Costa, Shrapnel NJP 18(6) (2016)

Allen, Barrett, Horsman, Lee & RWS, PRX 7, 031021 (2017)

Barrett, Lorenz, Oreshkov, arXiv:1906.10726

Quantum causal model



$\{E_{x|s}^A\}_x$ for each s

$\{E_{y|t}^B\}_y$ for each t

ρ_{AB}

$$P_{XY|ST}(xy|st) = \text{Tr}_{AB}((E_{x|s}^A \otimes E_{y|t}^B)\rho_{AB})$$

Leifer and RWS, PRA 88, 052130 (2013)

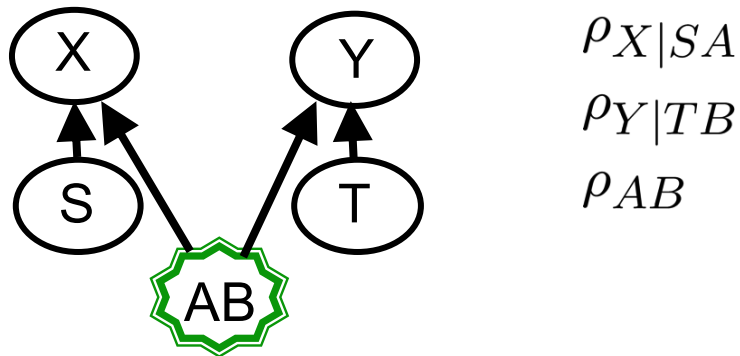
Henson, Lal & Pusey NJP 16, 113043 (2014)

Costa, Shrapnel NJP 18(6) (2016)

Allen, Barrett, Horsman, Lee & RWS, PRX 7, 031021 (2017)

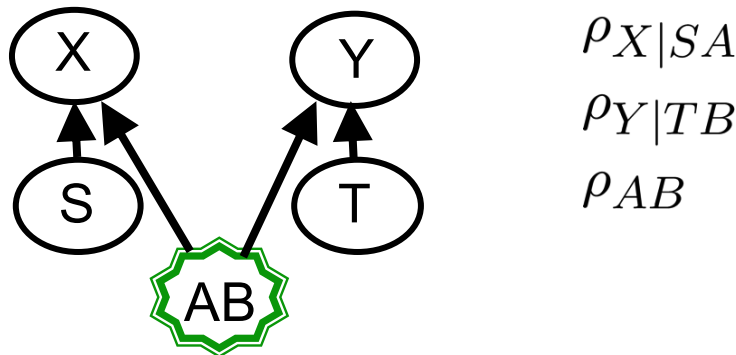
Barrett, Lorenz, Oreshkov, arXiv:1906.10726

Quantum causal model



$$P_{XY|ST} = \text{Tr}_{AB}(\rho_{X|SA}\rho_{Y|TB}\rho_{AB})$$

Quantum causal model



$$P_{XY|ST} = \text{Tr}_{AB}(\rho_{X|SA}\rho_{Y|TB}\rho_{AB})$$

Causal compatibility constraints:

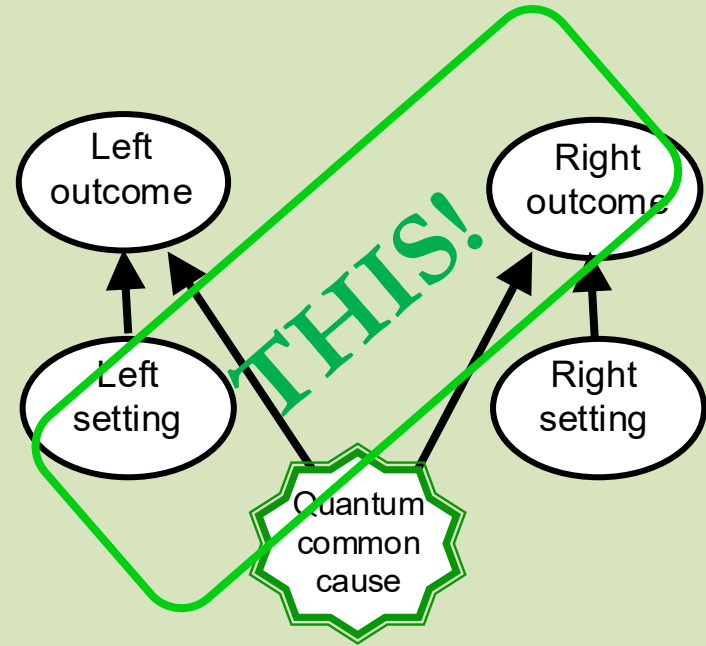
$$\begin{aligned} P_{X|ST} &= P_{X|S} \\ P_{Y|ST} &= P_{Y|T} \end{aligned}$$

$$\begin{aligned} &\frac{1}{4} \sum_{x=y} P_{XY|ST}(xy|00) + \frac{1}{4} \sum_{x=y} P_{XY|ST}(xy|01) \\ &\frac{1}{4} \sum_{x=y} P_{XY|ST}(xy|10) + \frac{1}{4} \sum_{x \neq y} P_{XY|ST}(xy|11) \leq \frac{1}{2} + \frac{1}{2\sqrt{2}} \end{aligned}$$

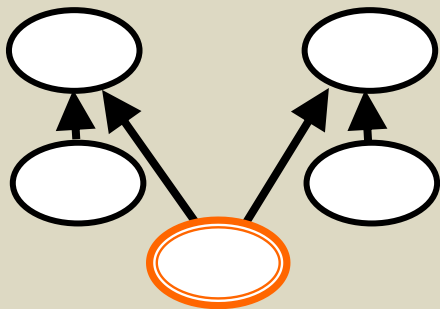
The evidence

		Left outcome and Right outcome			
		0 and 0	0 and 1	1 and 0	1 and 1
Left setting and Right setting	0 and 0	43%	7%	7%	43%
	0 and 1	43%	7%	7%	43%
	1 and 0	43%	7%	7%	43%
	1 and 1	7%	43%	43%	7%

A new possibility



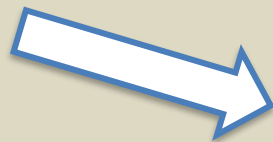
Compatible



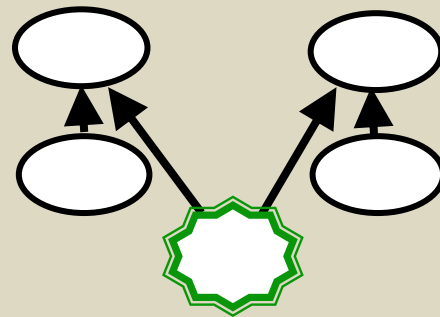
Violation of Bell
inequalities



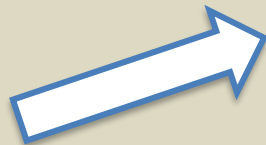
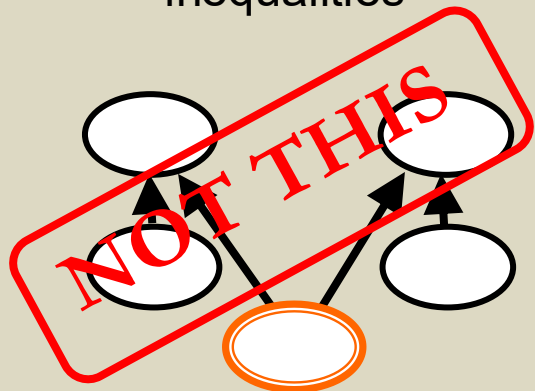
Violation of Bell
inequalities



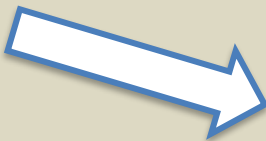
Witnessing
quantumness



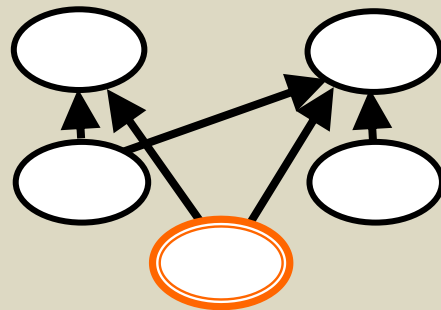
Violation of Bell inequalities



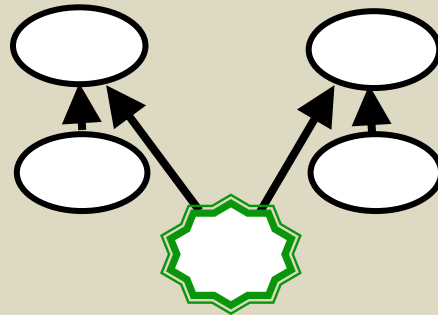
or

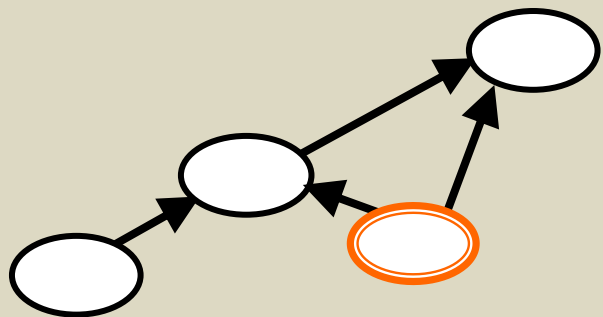


Witnessing need for
different structure

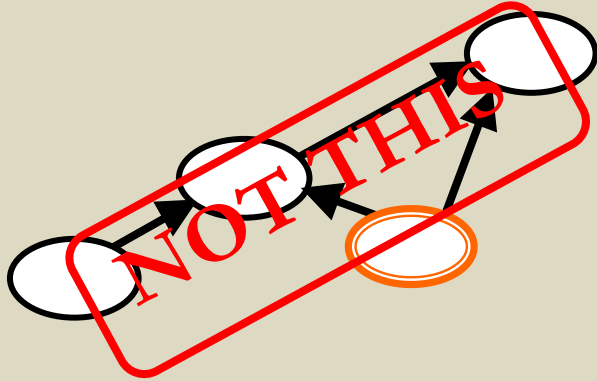


Witnessing
quantumness

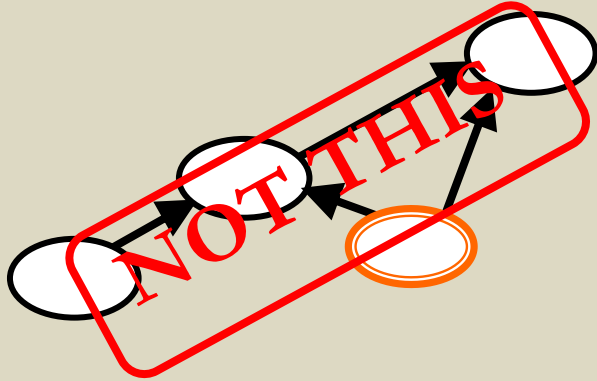




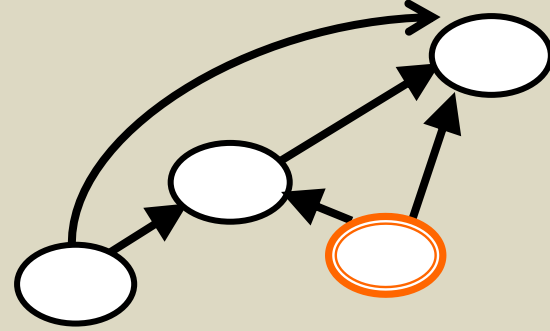
Violation of
Instrumental
Inequalities



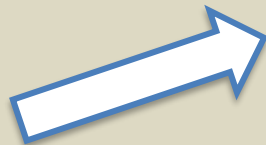
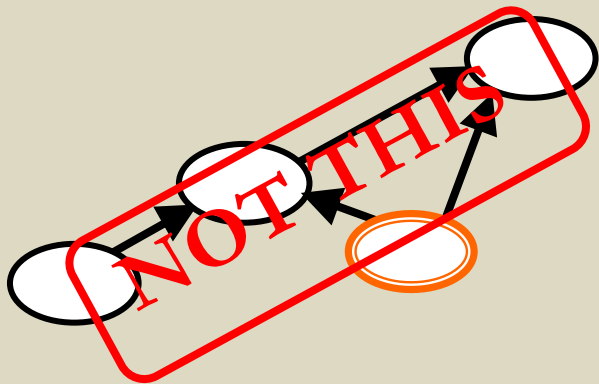
Violation of
Instrumental
Inequalities



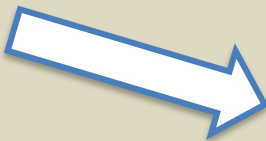
Witnessing need for
different structure



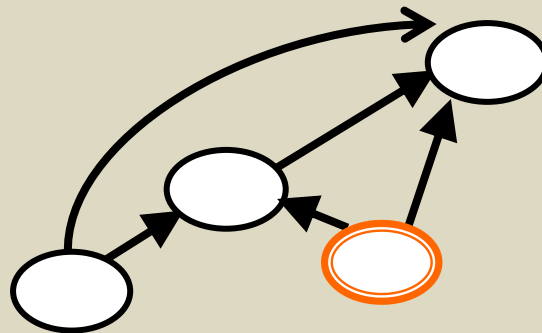
Violation of
Instrumental
Inequalities



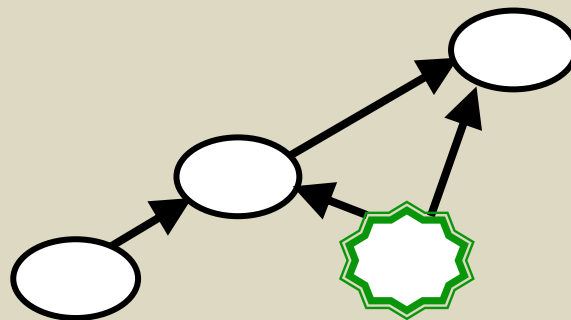
or



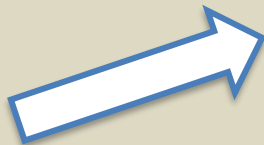
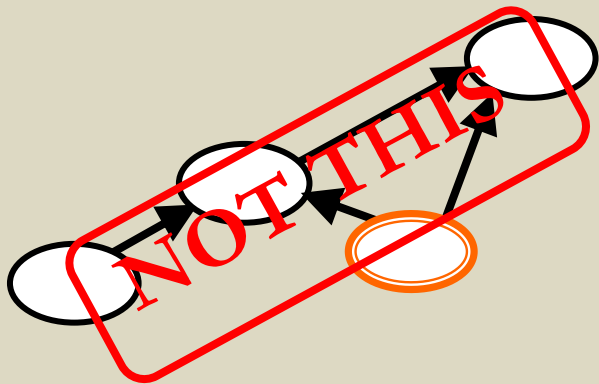
Witnessing need for
different structure



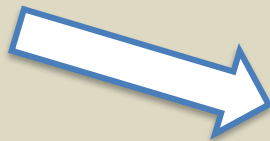
Witnessing
quantumness



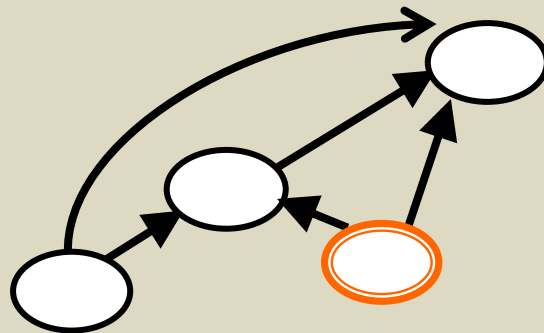
Violation of
Instrumental
Inequalities



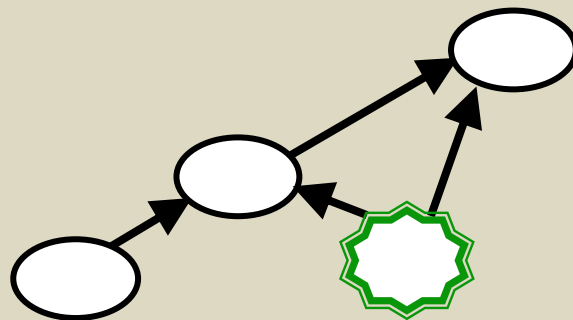
or



Witnessing need for
different structure



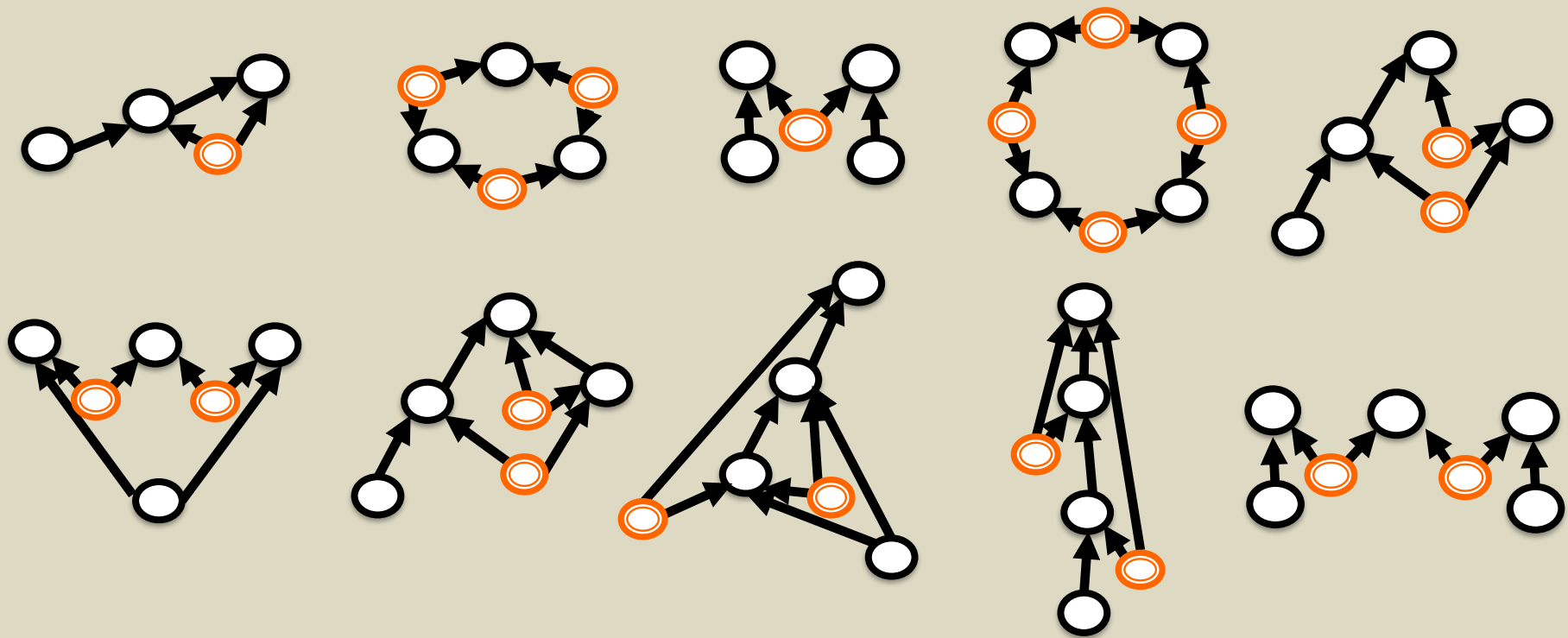
Witnessing
quantumness



van Himbeeck, Bohr Brask, Pironio, Ramanathan, Sainz & Wolfe,
Quantum 3, 186 (2019)

Chaves, Carvacho, Agresti, Di Giulio, Aolita, Giacomini & Sciarrino,
Nat. Phys. 47, 291296 (2018)

Some causal structures that admit of inequality constraints



JOURNAL OF CAUSAL INFERENCE

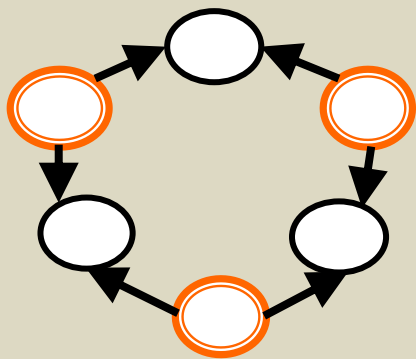
Elie Wolfe*, Robert W. Spekkens, and Tobias Fritz

The Inflation Technique for Causal Inference with Latent Variables

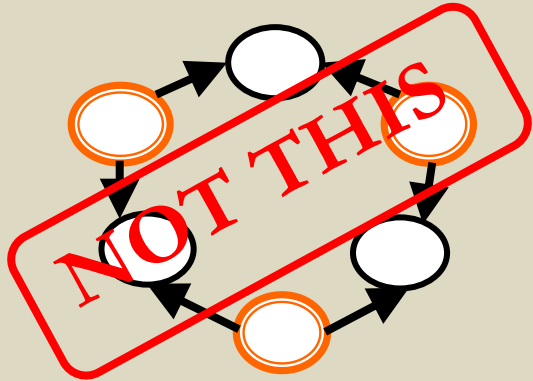
<https://doi.org/10.1515/jci-2017-0020>

Received August 16, 2017; revised August 31, 2018; accepted June 10, 2019

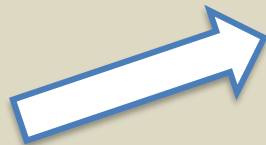
Abstract: The problem of causal inference is to determine if a given probability distribution on observed variables is compatible with some causal structure. The difficult case is when the causal structure includes latent variables. We here introduce the *inflation technique* for tackling this problem. An inflation of a causal structure is a new causal structure that can contain multiple copies of each of the original variables, but where the ancestry of each copy mirrors that of the original. To every distribution of the observed variables that is compatible with the original causal structure, we assign a family of marginal distributions on certain subsets of the copies that are compatible with the inflated causal structure. It follows that compatibility constraints for the inflation can be translated into compatibility constraints for the original causal structure. Even if the constraints at the level of inflation are weak, such as observable statistical independences implied by disjoint causal ancestry, the translated constraints can be strong. We apply this method to derive new inequalities whose violation by a distribution witnesses that distribution's incompatibility with the causal structure (of which Bell inequalities and Pearl's instrumental inequality are prominent examples). We describe an algorithm for deriving all such inequalities for the original causal structure that follow from ancestral independences in the inflation. For three observed binary variables with pairwise common causes, it yields inequalities that are stronger in at least some aspects than those obtainable by existing methods. We also describe an algorithm that derives a weaker set of inequalities but is more efficient. Finally, we discuss which inflations are such that the inequalities one obtains from them remain valid even for quantum (and post-quantum)



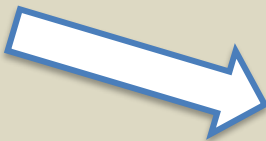
Violation of certain
causal compatibility
inequalities



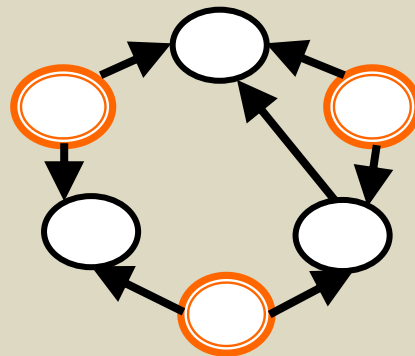
Violation of certain
causal compatibility
inequalities



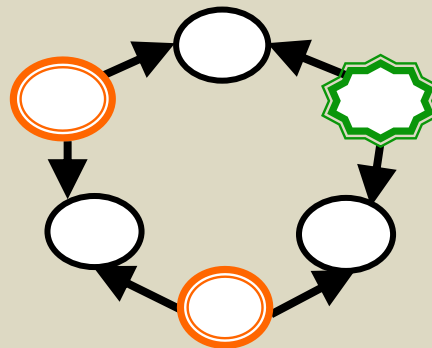
or



Witnessing need for
different structure



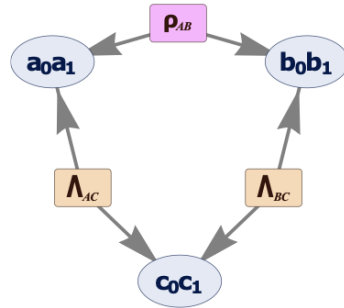
Witnessing
quantumness



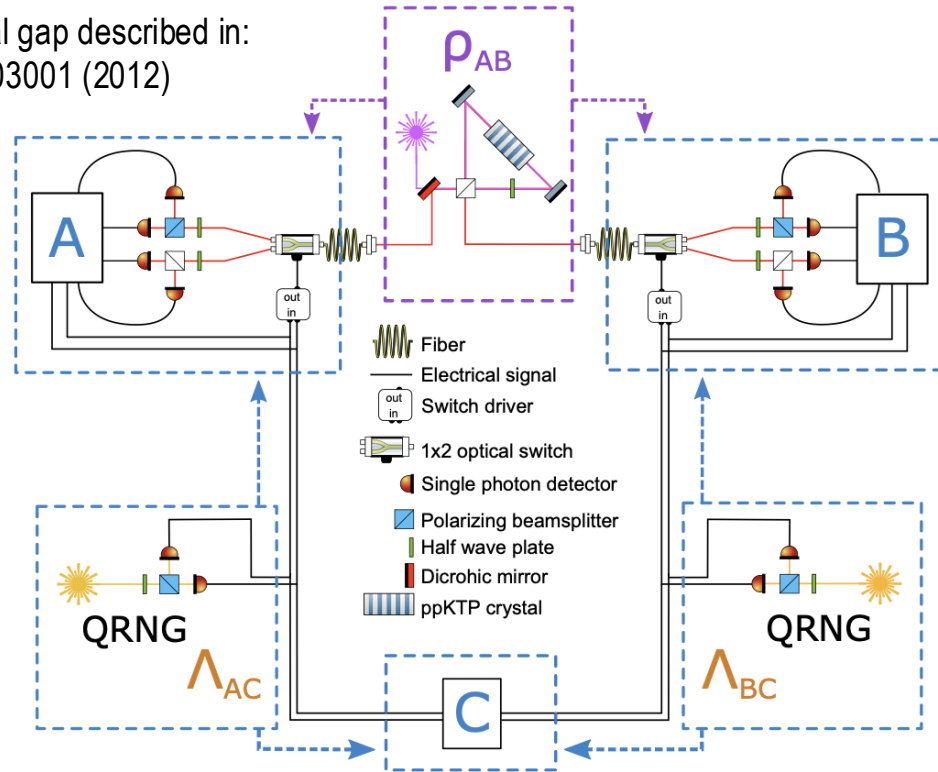
Collaboration with experimental group of Fabio Sciarrino

Polino et al., Nature Communications 14, 909 (2023)

Targeting the quantum-classical gap described in:
Fritz, New J. Phys. 14, 103001 (2012)



Experimental DAG

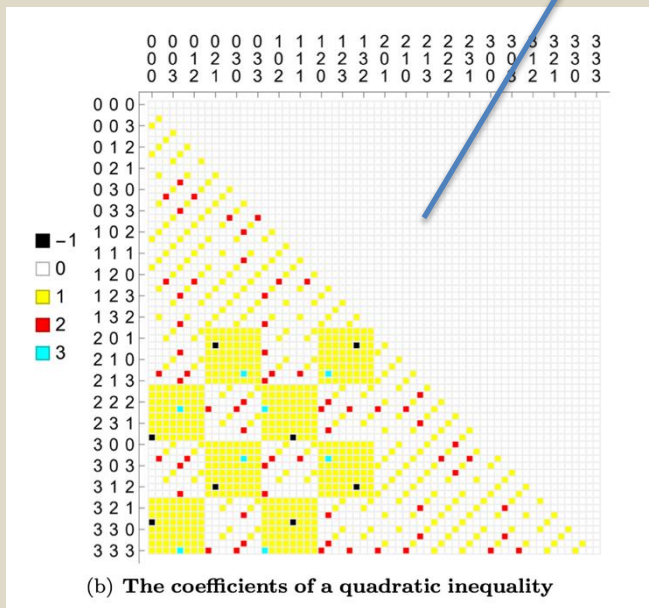


Data-seeded inflation technique

$$V := \sum_{a,b,c,a',b',c'} y_{abca'b'c'} P_{ABC}(abc) P_{ABC}(a'b'c') \geq 0$$

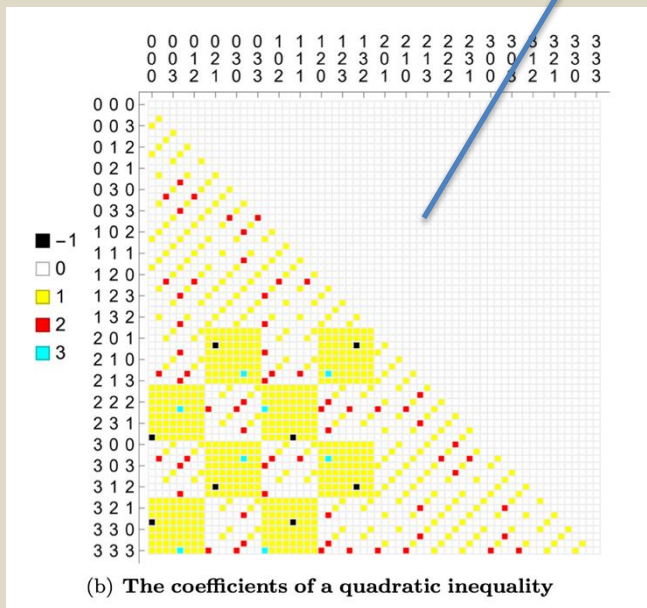
Data-seeded inflation technique

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Data-seeded inflation technique

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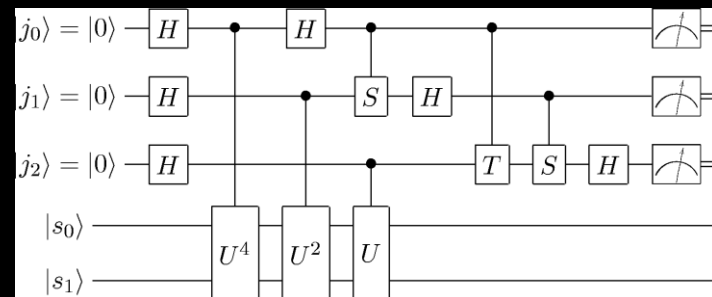
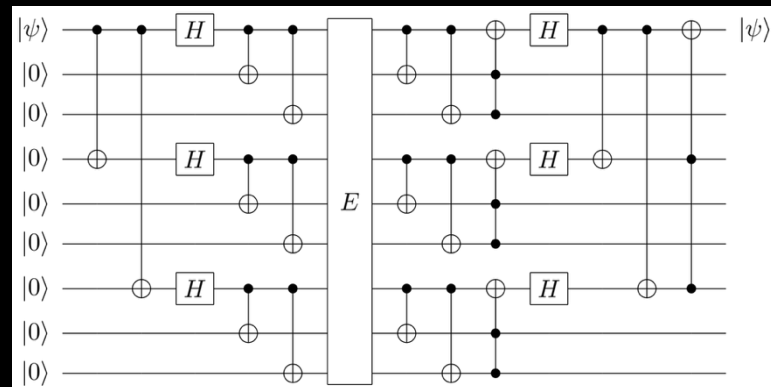
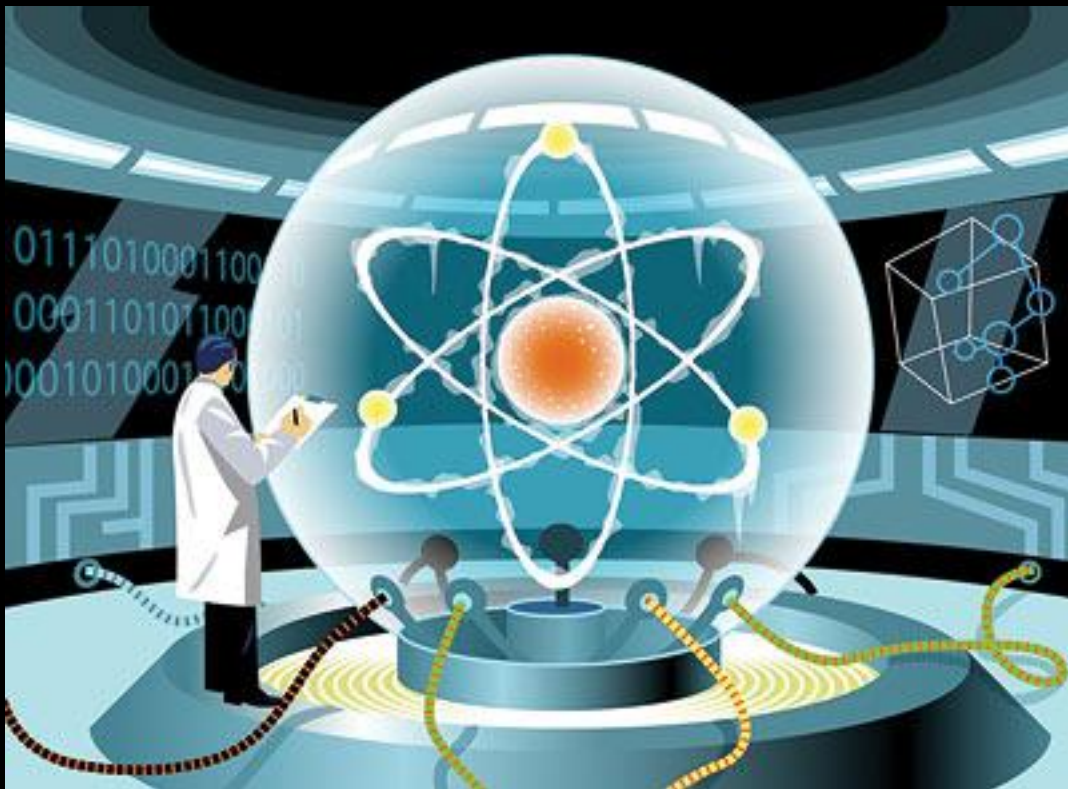


This causal compatibility inequality is violated by the data

$$V_{\text{exp}} = -0.02436 \pm 0.00016$$

Applications for Quantum Technology





Applications to Quantum Physics

The formalism and conceptual scheme of causal inference resolved various puzzles of statistics (e.g., Simpson's paradox)

The lesson:
We must unscramble the omelette of inference and causation both conceptually and in the formalism

But what hope do we have of succeeding
in the quantum context if we do not
understand how to do so in the classical
context?

Quantum
Causation and Inference

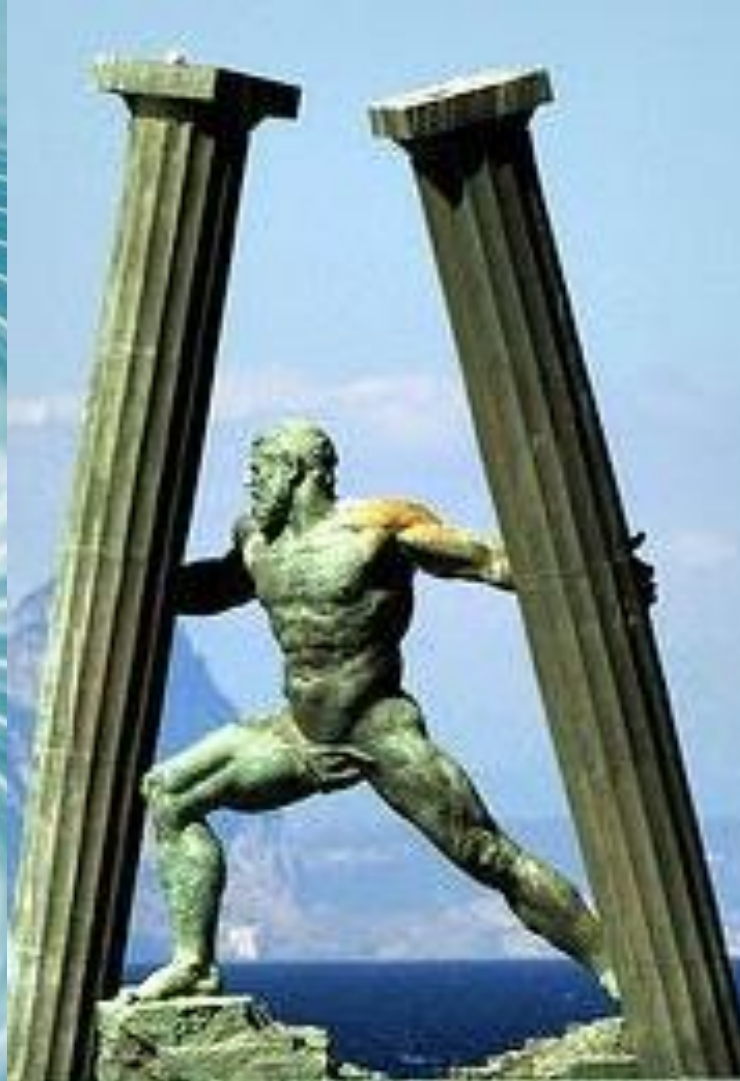
Classical
Causation and Inference



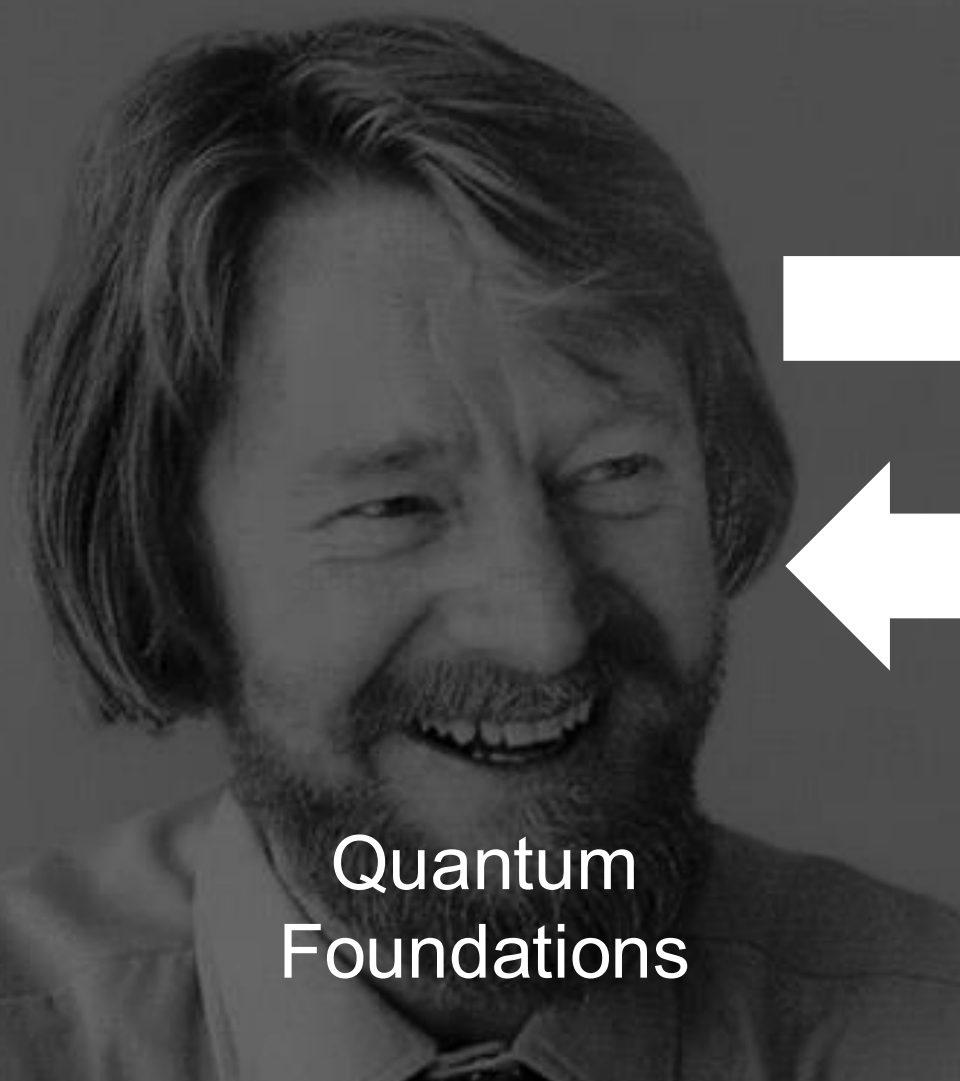
Relativistic
Notions of Space
and Time

PreRelativistic
Notions of Space
and Time

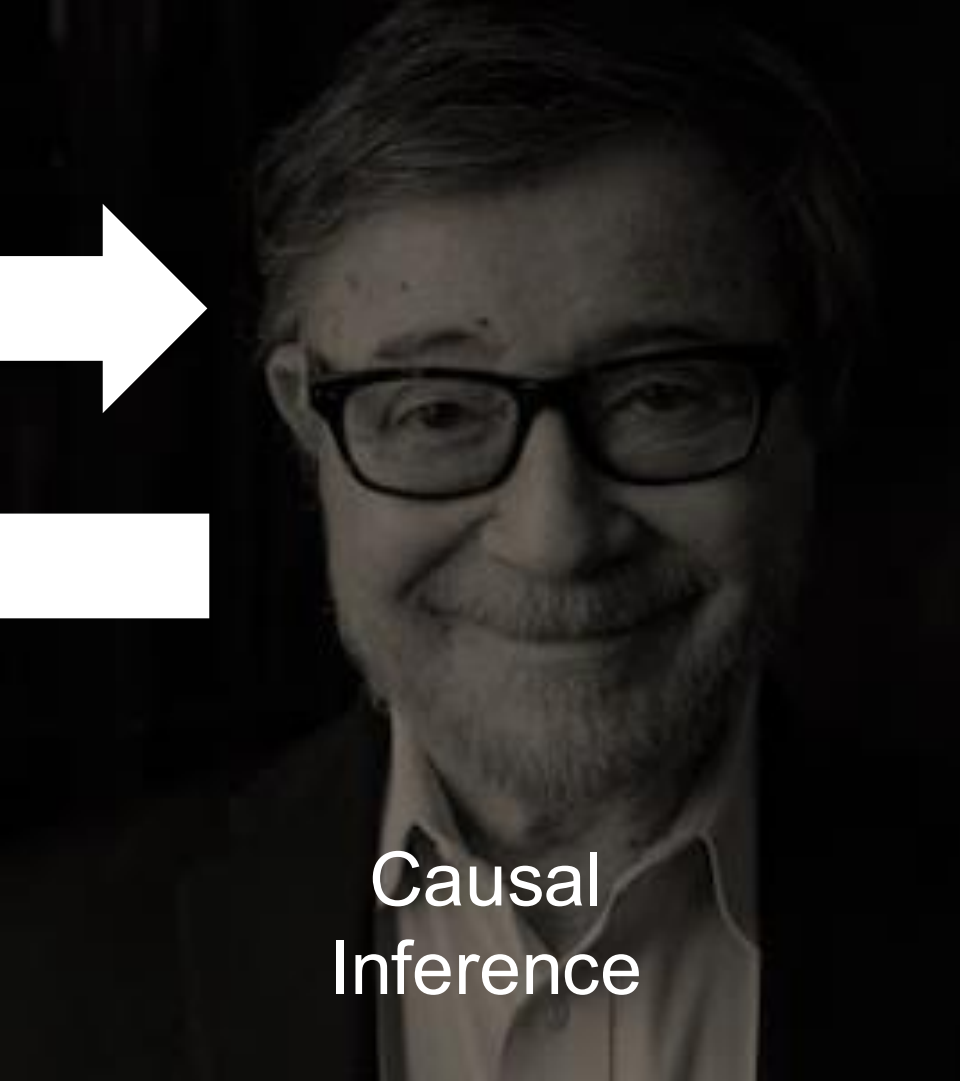
QUANTUM
THEORY



RELATIVITY



Quantum
Foundations



Causal
Inference