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THE OFF-SHELL SIDE OF GRAVITATIONAL WAVES: PARTICLE PRODUCTION FROM INHOMOGENEITIES

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PARTICLE PRODUCTION = UNAVOIDABLE

KEYWORDS:

- Hidden dark sectors
- Gravitational Freeze-in ~ thermal
- Gravitational Particle Production ~ time dependence
- CFTs
- Generic metric backgrounds
- Applications: FOPTs…

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MODEL INDEPENDENT PRODUCTION?

GRAVITATIONAL PRODUCTION OF DARK MATTER

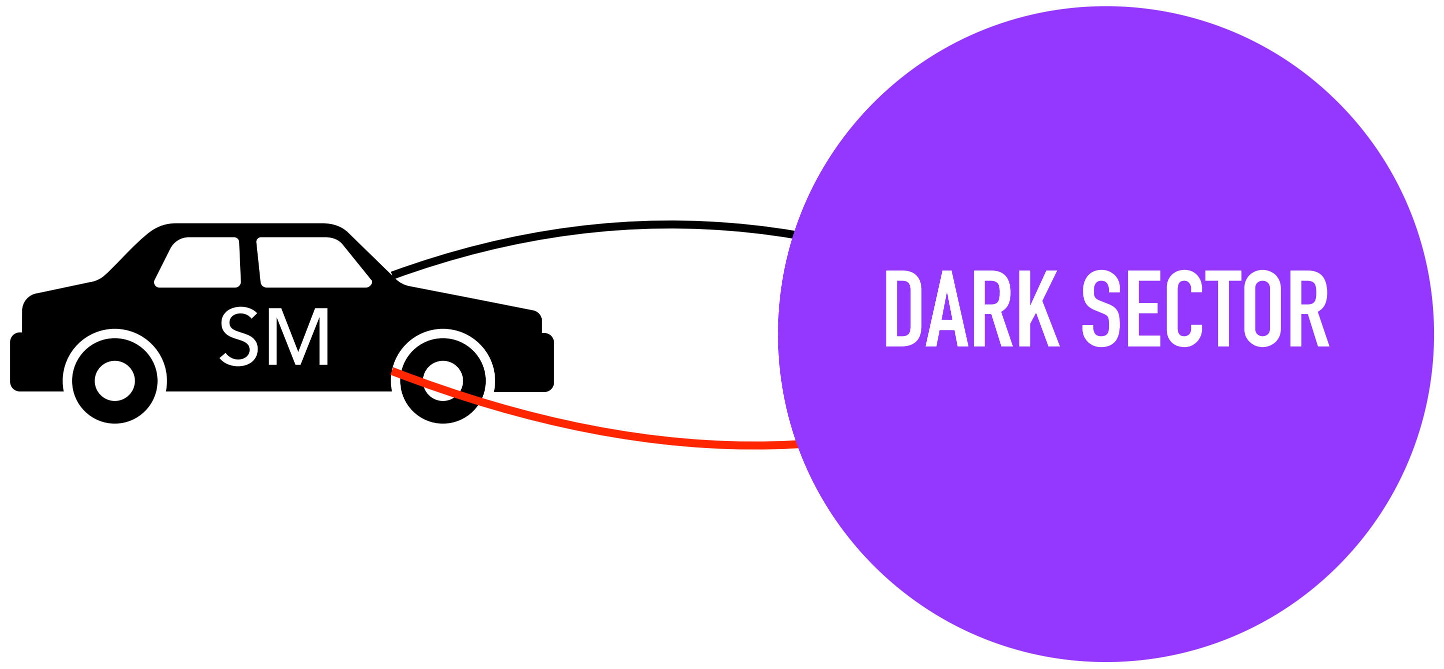
- Assume SM and standard cosmology and nothing else
- How can we ever populate the dark sector?
- Only chance is thanks to “gravitational effects”

ONLY OBSERVABLE IS DM ABUNDANCE..

A REAL CHALLENGE: INITIAL PRODUCTION

SECTOR CAN BE SELF-INTERACTING, BUT IT'S INITIALLY EMPTY

- We focus on the 'jump start'
- Only source of energy is the SM background



IN THE SM ONLY A COUPLE OF POSSIBILITIES:

GRAVITATIONAL FREEZE-IN $\rho_{\rm SM} \sim T^4$

- Pump energy into the dark sector thanks to graviton mediated 2-to-dark scatterings
- Short distance/flat space

GRAVITATIONAL PARTICLE PRODUCTION

- Pump energy into the dark sector thanks to non-adiabatic evolution of quantum fields
- Feels the expansion

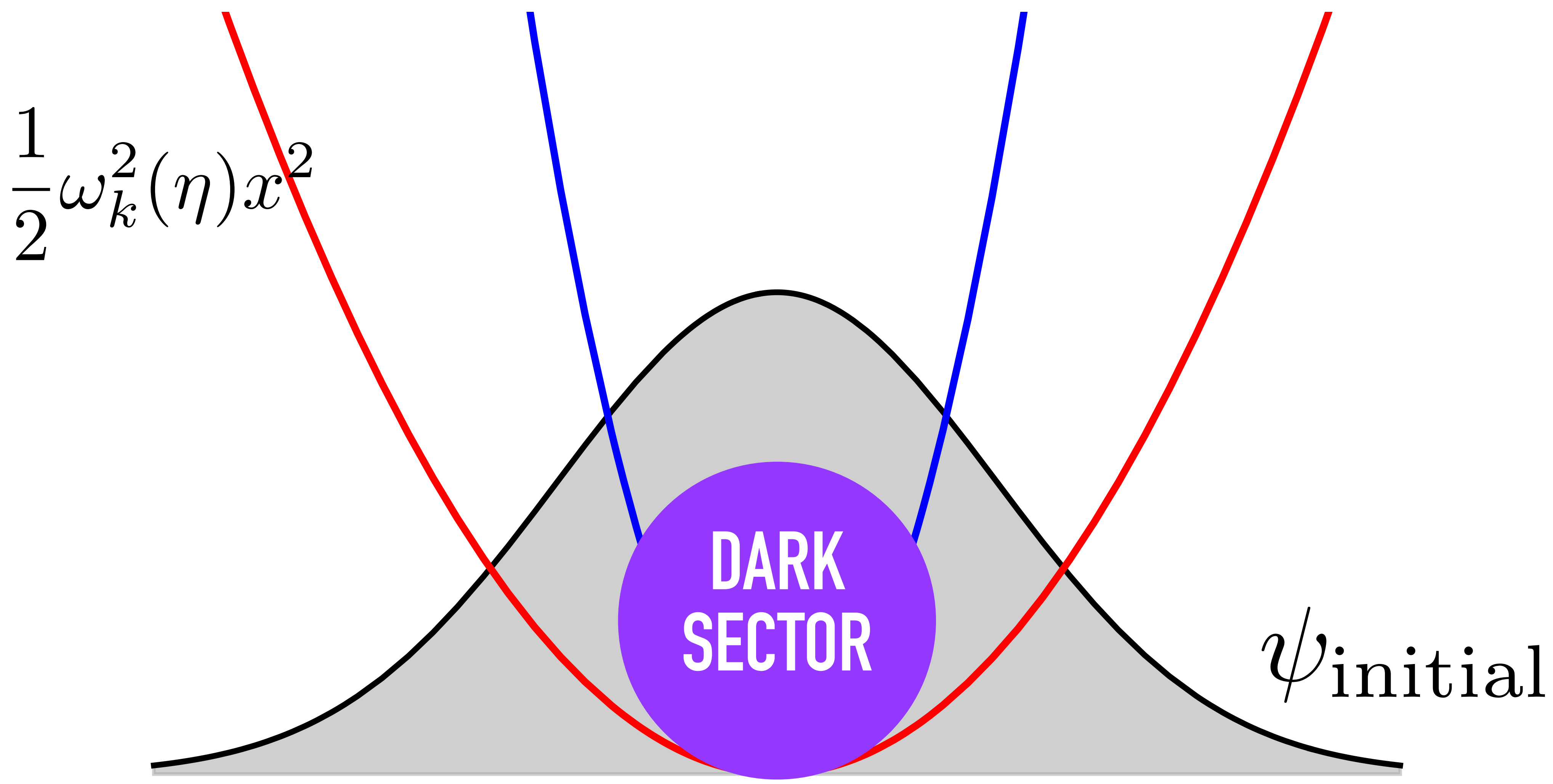
GRAVITATIONAL PARTICLE PRODUCTION

(IN THE HOMOGENEOUS FRW UNIVERSE)

PARTICLE PRODUCTION = NON ADIABATICITY

[Schroedinger '39]

OFTEN SAID THAT PP NEEDS TIME DEPENDENCE



$$a_k|0\rangle_{\text{in}} = b_k|0\rangle_{\text{out}} = 0$$

BASED ON BOGOLIUBOV-TRANSFORMATIONS

$$a_k^\dagger = \alpha_k^\dagger b_k^\dagger + \beta_k^{\star\dagger}\,\xi_k^{\star\dagger}\,b_{-k}^\dagger$$

$$\langle 0_{\text{in}}|b_k^\dagger b_k|0_{\text{in}}\rangle = |\beta_k|^2$$

- Sudden variation of frequency
- Old state has non-zero overalp with excited states of new hamiltonian
- In the new vacuum -> particle production

PARTICLE PRODUCTION NEEDS ALSO VIOLATION OF WEYL INVARIANCE [Ford '87]

PRODUCTION OF QUANTUM FIELDS

- Time dependence is not enough (necessary but not sufficient)
- Clear difference between goldstone bosons and chiral fermions/gauge fields

$$g_{\mu\nu}(x) \to \Omega(x)^2 g_{\mu\nu}(x)$$ + action on fields $$g_{\mu\nu}|_{\mathrm{FRW}} = a^2(\tau)\eta_{\mu\nu} \qquad C_{\mu\nu\rho\sigma}|_{\mathrm{FRW}} = 0$$

LARGE SCALAR FLUCTUATION FROM INFLATION

- Inflaton fluctuations tested by CMB = consequence of maximal breaking of Weyl
- Kinetic term for scalar is not Weyl invariant

TARGET: COMPUTE ENERGY/NUMBER DENSITY

$$\frac{d\rho}{d\log k} = \frac{\omega_k k^3}{2\pi^2}|\beta_{\vec{k}}|^2 \qquad \lim_{\eta\to+\infty}\langle T_X^{00}(\eta)\rangle$$

many tools:
Bogoliubov, power spectrum, in-in..

MAXIMAL WEYL VIOLATION: NAMBU-GOLDSTONE BOSONS

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

MOST EFFICIENT PARTICLE PRODUCTION FOR LIGHT SCALARS

- Computation can be done in several ways (bogoliubov/power spectrum)
- Massless during inflation, M << H_I

$$\Omega_{\rm DM} \sim \sqrt{\frac{M}{10^{-5}{\rm eV}}} \left(\frac{H_I}{10^{14} GeV}\right)^2$$

$$\frac{1}{s}\frac{d\rho}{d\log k} \approx \frac{H_I^2}{(2\pi)^2}\frac{\sqrt{M}}{M_{Pl}^{3/2}} \begin{cases} \frac{k_*}{k} & k \gg k_* \\ 1 & k \ll k_* \end{cases}$$

$$k_* = a_{\rm eq}\sqrt{M H_{\rm eq}}$$

ISOCURVATURES

- Energy density is flat in k at large distances
- Fluctuations are not synchronized with the inflaton clock
- Model is strongly excluded by CMB data / misalignement partially helps (pre-inflationary QCD axion)
- Need to consider other scenarios…

FERMIONS AND GAUGE FIELDS ARE CONFORMALLY COUPLED

CLEAR OBSTRUCTION

- Chiral fermions and gauge fields are conformally coupled
- They need to be heavy to feel the universe expansion

IN FRW AFTER WEYL RESCALING / FERMIONS / GAUGE FIELDS:

$$\Box\chi = M^2 a^2 \chi$$

- Mass x scale factor = breaks Weyl invariance
- Maximal production when k ~ a M ~ a H
- Production is actually **post-inflationary**
- Calculation is non-peturbative

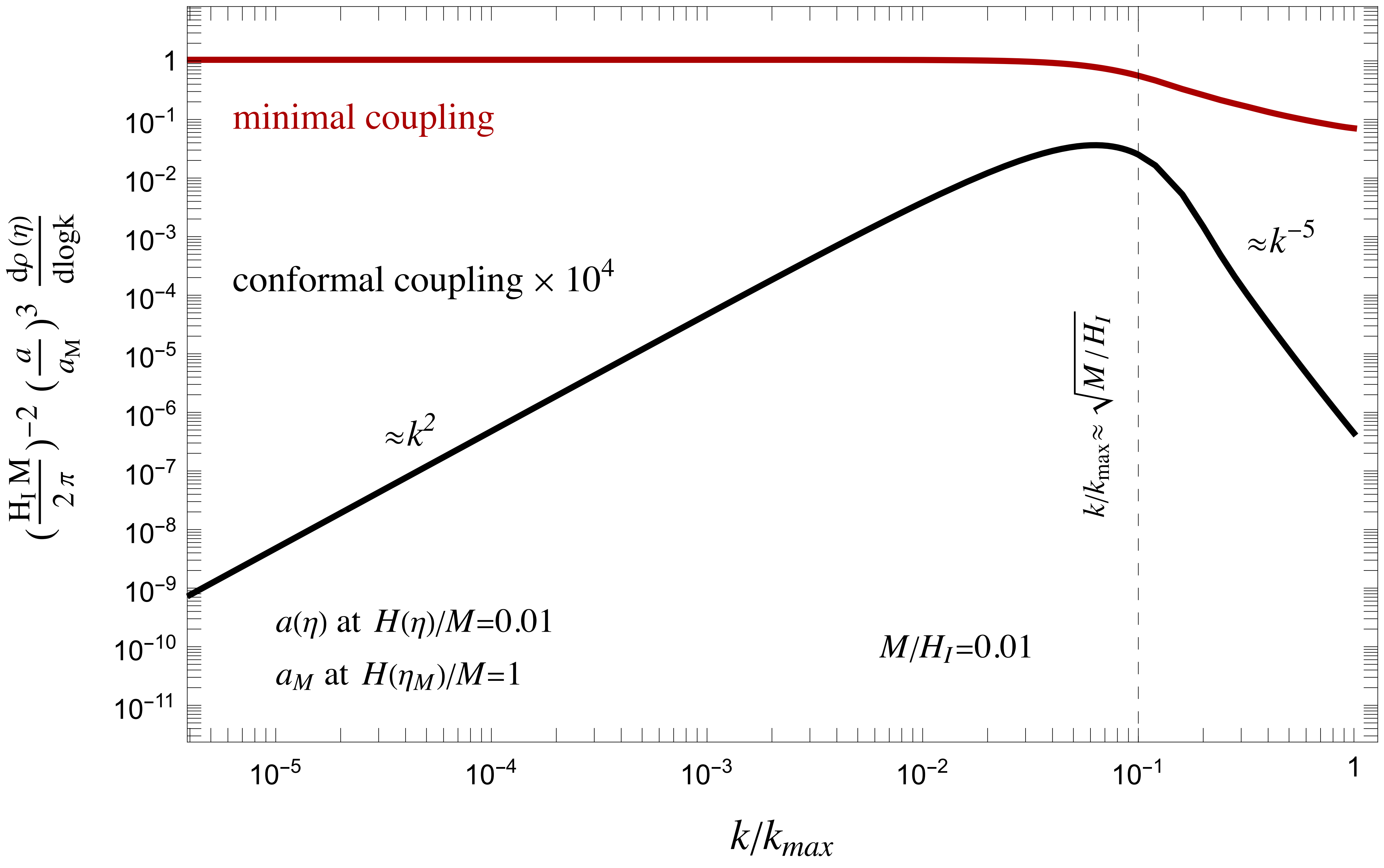
$$\Omega_{\rm DM} \sim \left(\frac{M}{10^9\,\rm GeV}\right)^{5/2}$$

CALCULATION IS DONE BY EXTRACTING BOGO-COEFF.S

[Cheung, Kolb, Riotto '90s +… + Ema, Nakayama, Tang + Long, Kolb '24]

GRAVITATIONAL PRODUCTION

HEAVY RELICS HAVE NO ISOCURVATURES



[Redi, AT]

CALCULATION IS DONE BY EXTRACTING BOGO-COEFF.S

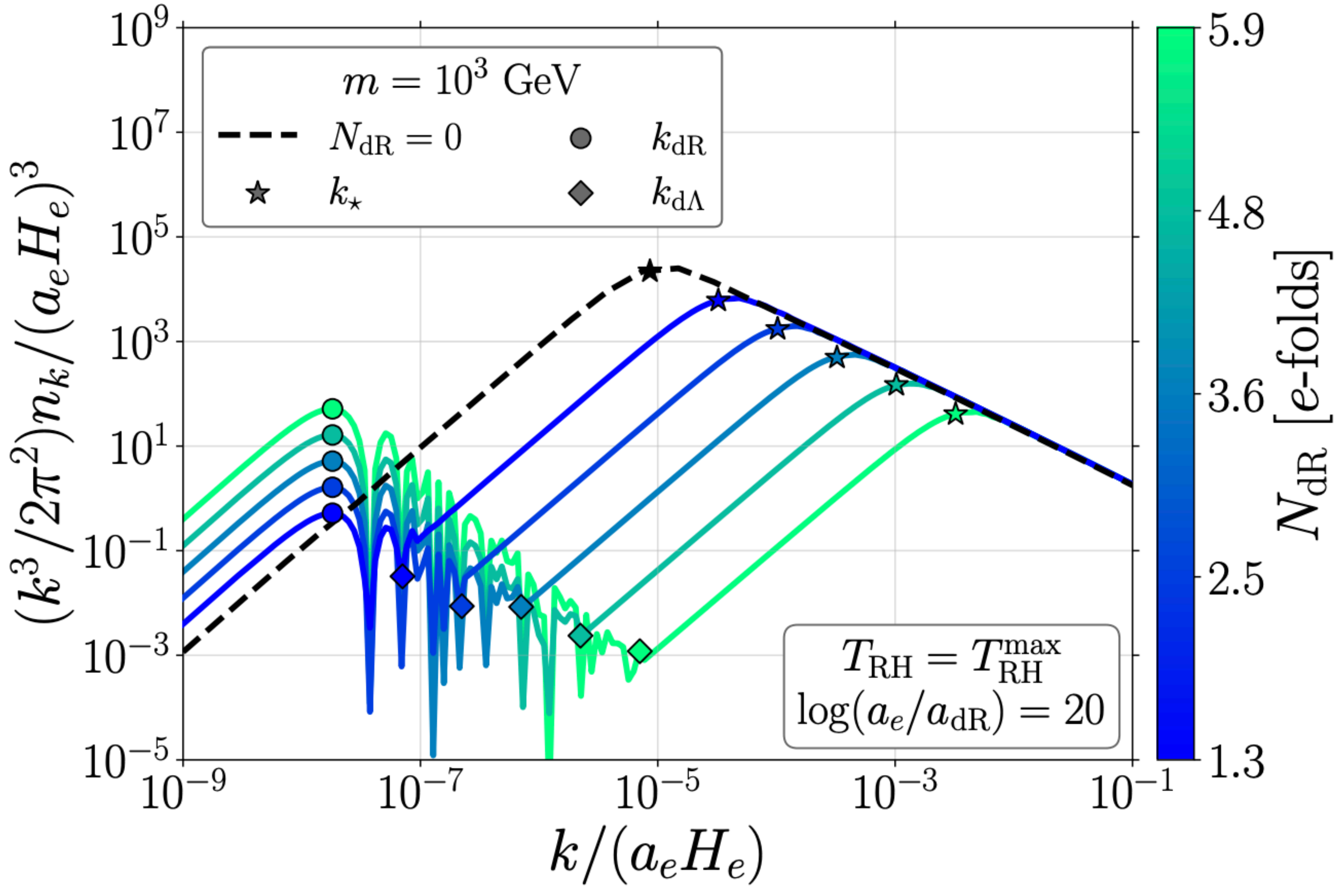
- alpha / beta
- they relate operators at early/late times

$$\frac{d\rho}{d\log k} = \frac{\omega_k k^3}{2\pi^2}|\beta_{\vec{k}}|^2, \qquad \frac{dn}{d\log k} = \frac{k^3}{2\pi^2}|\beta_{\vec{k}}|^2$$

- Bunch Davies initial state: no particles

GRAVITATIONAL PRODUCTION

SOME DEPENDENCE ON THE DETAILS OF INFLATION



MINI INFLATION AT THE RIGHT TIME: CAN INDUCE AN EFFECTIVE EXCITED STATE

- Just the case for **massive vectors**
- They are special = no isocurvature: spectrum looks the one of conformal matter
- Peak still mostly controlled by M
- No effect for conformally coupled fields

[E. Bertuzzo, G. Salla, AT, https://arxiv.org/abs/2603.03430]

GRAVITATIONAL PARTICLE PRODUCTION

(IN THE REAL UNIVERSE)

BREAKING OF WEYL INVARIANCE

MORE THAN JUST BACKGROUND

- Conformally coupled matter (fermions, gauge fields, conformal scalars) do not see scale factor
- However they do feel metric perturbations

EXPLOITING THIS ARGUMENT:

- Every departure from Weyl invariance + time dependence = particle production
- Mass is only a possible source of breaking…

$$ds^2 = a^2(\tau)[\eta_{\mu\nu} + h_{\mu\nu}(\tau, \vec{x})]dx^\mu dx^\nu$$

A TIME DEPENDENT PERTURBATION WILL GENERATE MASSLESS CONFORMALLY COUPLED PARTICLES

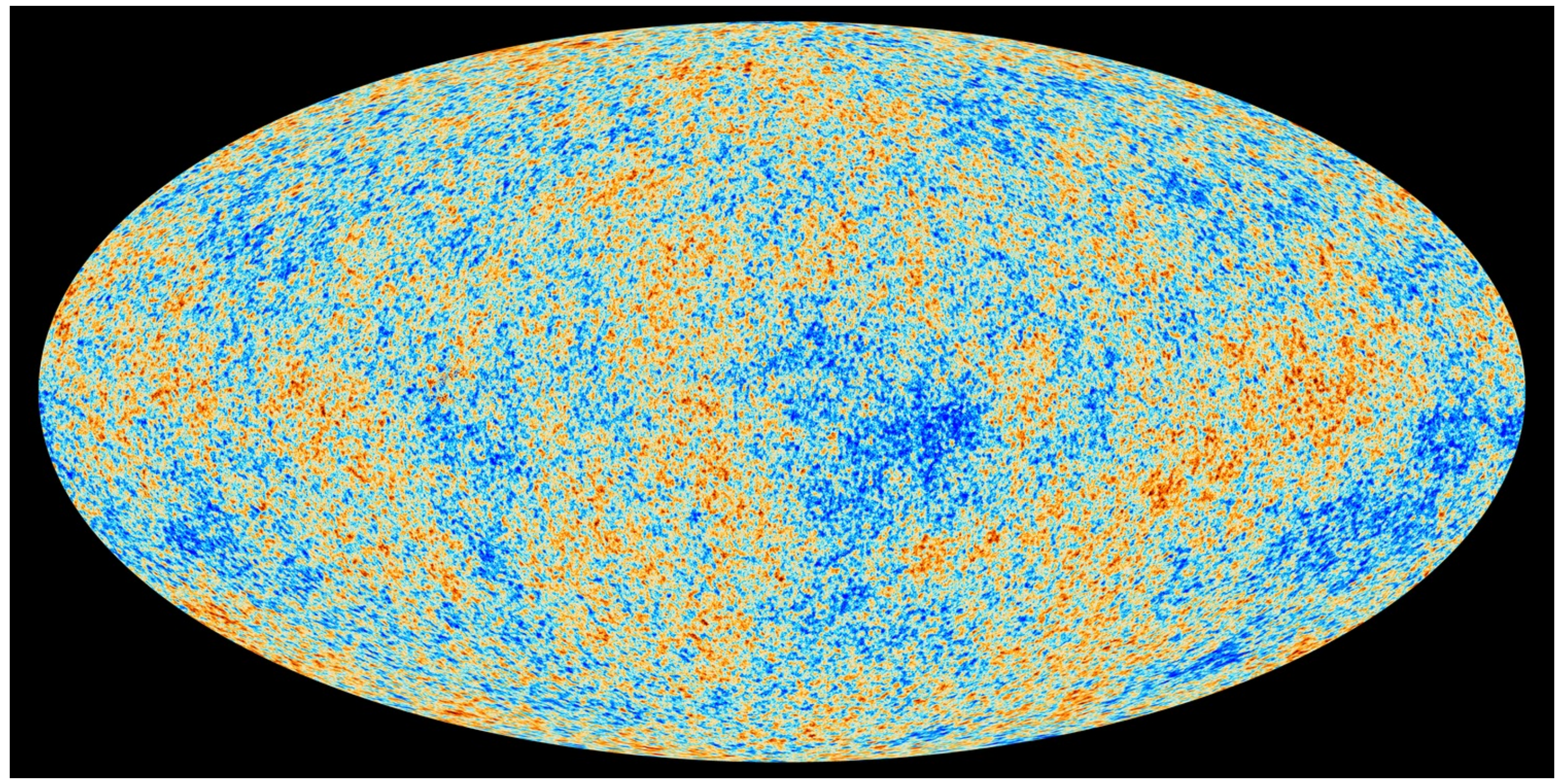
in the literature this was often not touched / gravitational pp = massive relics [see review by Long&Kolb]

[we were prompted by a couple of papers of Kopp&Maleknejad]

BREAKING OF WEYL INVARIANCE

POSSIBLE REALISTIC SOURCES

- Time dependence of **scalar gravitational potentials** (as predicted by standard cosmology)
- Gravitational Wave backgrounds (build-up and redshifts)
- Metric inhomogeneitis sources by phase transitions



$$\Delta_\zeta|_{\mathrm{CMB}} \sim 10^{-9}$$

ζ curvature perturbation / matched on super-horizon scales

FOR THE OBSERVED AMPLITUDES OF COSMOLOGICAL FLUCTUATIONS => SMALL PRODUCTION

A BIT OF TECHNICAL DETAILS

GENERALIZED BOGOLIUBOV TRANSFORMATIONS

$$\Box\chi = J(h_{\mu\nu}(\tau,\vec{x}))\cdot\chi$$

- Free quantum fields in curved backgrounds
- Upon Weyl rescaling (remove scale factor) –> departure from flat space
- Perturbations induces an effective mass-term (if you want an analogy)
- Now the computation is **perturbative**

AT FIRST ORDER IN THE PERTURBATION:

- Fourier modes solved with retarded Green's function

$$G^R_{\vec{k}}(\tau) = -\theta(\tau)\frac{e^{ik\tau} - e^{-ik\tau}}{2ik}$$

$$\chi^{\rm out}_{\vec{k}}(\tau) = \chi^{\rm in}_{\vec{k}}(\tau) + \int d\tau' G^R_{\vec{k}}(\tau-\tau')\int\frac{d^3q}{(2\pi)^3}J(\vec{q},\tau')\chi^{\rm in}_{\vec{\omega}}(\tau'), \quad \omega = k - q$$

- Bogoliuobov transformation mixes different k's, rotation more complex

GENERALIZED BOGOLIUBOV TRANSFORMATION

RELATION BETWEEN ALL MODES

$$a^{\text{out}}_{\vec{k}} = a^{\text{in}}_{\vec{k}} + \sum_{\omega} \beta^{*}_{\vec{k}\vec{\omega}} (a^{\text{in}}_{-\vec{\omega}})^{\dagger} \qquad N_{\vec{k}} = \sum_{\vec{\omega}} |\beta_{\vec{k}\vec{\omega}}|^2$$

- We derived formulas for all spin 0,1/2,1
- Without some assumptions, the computation gives us the total number N_k

AFTER SOME MANIPULATIONS, WE SEE:

[Verdaguer '90s]

- Bogoliubov coefficients are rather simple (perturbative) for each elicity J

$$\beta^{J}_{\vec{k}\vec{\omega}} = \frac{h_{\mu\nu}(\tilde{q})}{2\sqrt{k\omega}} \times A^{\mu\nu}_{J}(\tilde{k}\,,\tilde{\omega})$$

ON-SHELL AMPLITUDES: DECAY OF PERTURBATIONS

$$\tilde{k}^2 = \tilde{\omega}^2 = 0$$

GENERALIZED BOGOLIUBOV TRANSFORMATION

FULL RESULT (DIFFERENTIAL)

$$\frac{dN}{d^3k} = \frac{1}{(2\pi)^3}\int \frac{d^3q}{(2\pi)^3} h_{\mu\nu}(k+\omega,\vec{q})h^*_{\rho\sigma}(k+\omega,\vec{q}) \times \frac{A_J^{\mu\nu}A_J^{\rho\sigma\,*}}{4k\omega}$$

on-shell + momentum conservation
INCLUSIVE

FOR REALISTIC APPLICATIONS

- Scalar and tensor perturbations $\Theta(\tau,\vec{x}) \equiv \Phi + \Psi\,, \qquad \Sigma(\tau,\vec{x}) \equiv \Phi - \Psi$

$$ds^2 = a^2(\tau)[1+2\Phi(\tau,\vec{x})]d\tau^2 - a^2(\tau)[\delta_{ij}(1-2\Psi(\tau,\vec{x})) - h_{ij}(\tau,\vec{x})]dx^i dx^j$$

- Decomposition into helicities

$$\beta^\lambda_{\vec{k}\vec{\omega}} = \frac{\Theta(k+\omega,\vec{q})}{2\sqrt{k\omega}}(B_J)_{\vec{k}\vec{\omega}} + \frac{h^+(k+\omega,\vec{q})}{2\sqrt{k\omega}}(C_J^+)_{\vec{k}\vec{\omega}} + \frac{h^-(k+\omega,\vec{q})}{2\sqrt{k\omega}}(C_J^-)_{\vec{k}\vec{\omega}}$$

$J=0$		
B_0	C_0^+	C_0^-
$\frac{3(k-\omega)^2-q^2}{6}$	$\frac{4k^2q^2-\left(k^2+q^2-\omega^2\right)^2}{4\sqrt{2}q^2}$	$\frac{4k^2q^2-\left(k^2+q^2-\omega^2\right)^2}{4\sqrt{2}q^2}$
$J=1/2$		
$B_{1/2}$	$C_{1/2}^+$	$C_{1/2}^-$
$\frac{1}{2}(\omega-k)\sqrt{q^2-(k-\omega)^2}$	$\frac{((k+\omega)^2-q^2)(k-q-\omega)\sqrt{q^2-(k-\omega)^2}}{4\sqrt{2}q^2}$	$\frac{((k+\omega)^2-q^2)(k+q-\omega)\sqrt{q^2-(k-\omega)^2}}{4\sqrt{2}q^2}$
$J=1$		
B_1	C_1^+	C_1^-
$\frac{1}{2}(q^2-(k-\omega)^2)$	$\frac{(k+q-\omega)^2((k+\omega)^2-q^2)}{4\sqrt{2}q^2}$	$\frac{(-k+q+\omega)^2((k+\omega)^2-q^2)}{4\sqrt{2}q^2}$

GRAVITATIONAL PARTICLE PRODUCTION
(IN THE REAL UNIVERSE,
WITH THE EFFECTIVE ACTION)

SCHWINGER'S APPROACH

VACUUM PERSISTENT AMPLITUDE

$$\langle \mathrm{out}|\mathrm{in}\rangle = e^{i\Gamma_{1\mathrm{PI}}}$$

- overlap betwen in and out vacuum state
- imaginary part of the effective action = decay probability

$$N \approx 2\mathrm{Im}[\Gamma_{1\mathrm{PI}}]$$

number of decays

EFFECTIVE ACTION AT SECOND ORDER IN THE INTERACTIONS

$$S_{\mathrm{int}} = \frac{1}{2}\int d^4x h_{\mu\nu}(x) T^{\mu\nu}(x)$$

- Computing the effective action in the background of hmunu
- Applications also for more generic interactions $\lambda\phi\mathcal{O}$

EFFECTIVE ACTION IN THE BACKGROUND

EFFECTIVE ACTION CAN DEVELOP AN IMAGINARY PART

$$\Gamma = \frac{1}{4}\int \frac{d^4q}{(2\pi)^4} h^{\mu\nu}(-q)\langle T_{\mu\nu}(q)T_{\rho\sigma}(-q)\rangle h^{\rho\sigma}(q)$$

- imaginary part due to 2-point function of stress-energy tensor
- for CFTs again the results just depends on central charges

CFTS FROM METRIC PERTURBATIONS

$$\langle T_{\mu\nu}(q)T_{\rho\sigma}(q')\rangle = (2\pi)^4\delta^4(q+q')\frac{c_J}{7680\pi^2}\Pi_{\mu\nu\rho\sigma}(q)\log(-q^2)$$

$$\Pi_{\mu\nu\rho\sigma}(q) \equiv (2\pi_{\mu\nu}\pi_{\rho\sigma} - 3\pi_{\mu\rho}\pi_{\nu\sigma} - 3\pi_{\mu\sigma}\pi_{\nu\rho})\,, \qquad \pi_{\mu\nu} \equiv \eta_{\mu\nu}q^2 - q_\mu q_\nu$$

- Imaginary part for positive q^2

EFFECTIVE ACTION = PARTICLE PRODUCTION

FULL EXPRESSION

$$N = \frac{c_J}{15360\pi}\int \frac{d^4q}{(2\pi)^4}\theta(q^2)h^{\mu\nu}(q)\Pi_{\mu\nu\rho\sigma}(q)h^{\rho\sigma}(-q)$$

- inclusive / equivalent to Bogoliubov computation

EVEN BETTER, PARTICLE PRODUCTION ENTIRELY CONTROLLED BY WEYL TENSOR

- Thanks to Weyl invariance of the CFT contribution proportional to C^2
- Projection onto TT 4d components

$$N = \frac{c_J}{2560\pi}\int \frac{d^4q}{(2\pi)^4}\theta(q^2)\theta(q_0)C^{\mu\nu\rho\sigma}(q)C_{\mu\nu\rho\sigma}(-q)$$

PARTICLE PRODUCTION FROM WEYL TENSOR ~ SCHWINGER PAIR PRODUCTION

INTERESTING ANALOGY (AT LEADING ORDER)

$$N = \frac{c_J}{2560\pi} \int \frac{d^4q}{(2\pi)^4} \theta(q^2)\theta(q_0) C^{\mu\nu\rho\sigma}(q) C_{\mu\nu\rho\sigma}(-q)$$

- At leading order, Schwinger pair production has the same structure c–>e^2 and C–> F
- Time-like support requires 'electric configurations'
- Electric components of the Weyl tensor crucial $C^2 = 8(E^2 - B^2)$

$$E_{ij} = \frac{1}{2}(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2)\Theta \qquad B_{ij} = 0 \qquad \text{scalar}$$

$$E_{ij} = \frac{1}{4}\left(h''_{ij} + \nabla^2 h_{ij}\right) \quad B_{ij} = -\frac{1}{4}\epsilon_{i\ell m}\partial_\ell h'_{mj} - \frac{1}{4}\epsilon_{j\ell m}\partial_\ell h'_{mi}. \qquad \text{tensor}$$

- Need for spatial gradients as well
- Result is finite for physical inputs!

NEED UNEQUAL TIME CORRELATORS

$$N \propto \int d^4q h^{\mu\nu}(q) h^{\rho\sigma}(-q) \cdots$$

IN COSMOLOGICAL APPLICATIONS:

- great interest in stochastic backgrounds (with translational invariance)

$$\langle h(\vec{q},\tau) h^*(\vec{q}\,',\tau')\rangle = (2\pi)^3 \delta^3(\vec{q}-\vec{q}\,') \frac{2\pi^2}{q^3} \Delta_h(q,\tau,\tau')$$

SCALAR FLUCTUATIONS

$$\frac{d(na^3)}{dq_0} = \frac{c_J}{240\pi^2} \int d(\log q)\, \theta(q_0^2 - q^2) q^4 \Delta_\Psi(q, q_0, -q_0)$$

TENSOR FLUCTUATIONS

$$\frac{d(na^3)}{dq_0} = \frac{c_J}{640\pi^2} \int d(\log q)\, \theta(q_0^2 - q^2)(q_0^2 - q^2)^2 \Delta_h(q, q_0, -q_0)$$

FLUCTUATIONS MATCHED ON SUPER-HORIZON SCALES:

$$\Delta(q,\tau,\tau') = T_q(\tau) T_q^*(\tau') \Delta(q;\tau_*)$$

for example: late evolution of inflaton perturbations

DARK MATTER FROM INFLATON PERTURBATIONS

EXPLOIT TIME DEPENDENCE OF GRAVITATIONAL POTENTIAL

- inflaton generates power on short scales (larger than CMB)
- later cosmological evolution provides time dependence to

$$\Delta(q,\tau,\tau') = T_q(\tau)T_q^*(\tau')\Delta(q;\tau_*)$$

MECHANISM:

- spectator conformally coupled DM gets generated when perturbations evolves
- exploit scalar perturbations (they do not evolve in matter dominance)

CASE STUDY OF MAJORANA FERMION

- mass neglected at production
- mode functions evolve due to graviational potentials

DARK MATTER FROM INFLATON PERTURBATIONS

MAJORANA FERMION

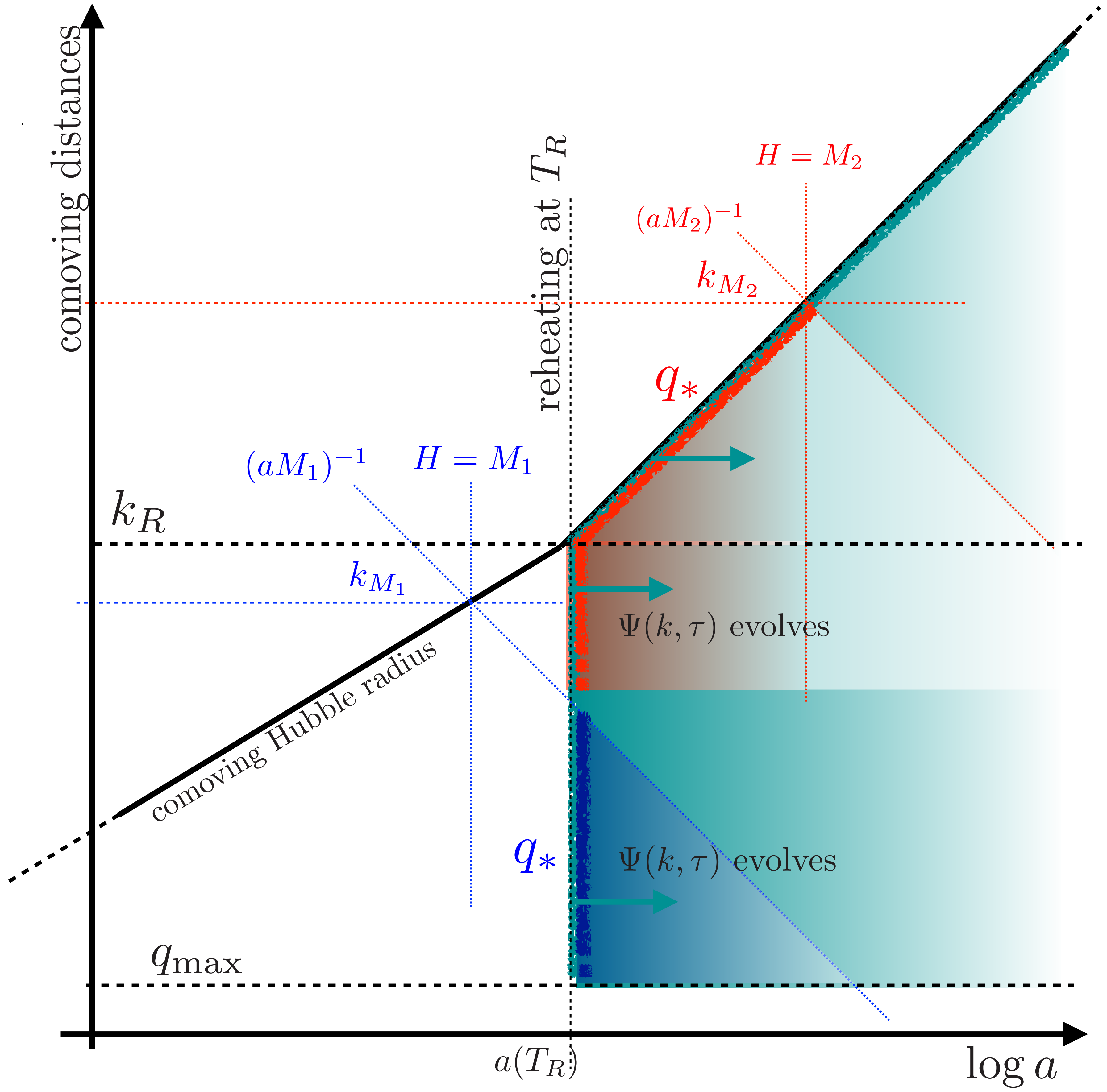
- source of the perturbations is inflation
- performing the q0 integral

DM ABUNDANCE RELATED TO INFLATON POWER SPECTRUM ζ

- Time evolution gives the overal coefficient

$$na^3 = c_J \frac{A^\zeta}{4\pi^2} \int d(\log q) q^3 \Delta_\zeta(q)$$

- No dilution from entropy injection for q*>kR



DARK MATTER FROM INFLATON PERTURBATIONS

PRODUCTION IS DOMINATED TOWARDS THE END OF INFLATION

$$\Omega_{\rm DM} \approx 10^{-4} c_J \frac{M q_{\rm max}^3}{3M_{\rm Pl}^2 H_0^2} \Delta_\zeta(q_{\rm max}) \approx c_J \frac{\Delta_\zeta(q_{\rm max})}{10^{-2}} \frac{M}{10^7\,{\rm GeV}}$$

- recall that in our universe qmax~ 10^-7 eV
- need sizable power spectrum towards the end of inflation

COMPARISON WITH OTHER MECHANISMS:

- gravitational freeze-in

$$\Omega_{\rm DM}|_{\rm GFI} \approx 10^{-5} \times c_J \frac{M k_R^3}{3M_{\rm Pl}^2 H_0^2}$$

- gravitational particle-production

$$\Omega_{\rm DM}|_{\rm GPP} \approx 10^{-2} \frac{M\, k_*^3}{3M_{\rm Pl}^2 H_0^2} \times \eta_{\rm eff}$$

COMMENTS:

GIVEN A METRIC PERTURBATION ONE CAN COMPUTE DM ABUNDANCE EASILY

- Not important how they are generated
- So far particle production happens thanks to cosmological evolution:
    - given a power spectrum from inflation –> transfer functions give time dependencies
    - transfer functions often as in standard cosmological perturbation theory

WHAT HAPPENS IF WE FOCUS ON THE SOURCES:

- Connection with physics generating the perturbation
- Time dependence during the build-up of the perturbations
- Can give stronger time dependencies (larger than Hubble inverse time) –> new scale
- Basically we go sub-horizon…

GRAVITATIONAL PARTICLE PRODUCTION

(IN THE REAL UNIVERSE,

WITH THE EFFECTIVE ACTION,

INSIDE THE HORIZON..)

A SOURCE FOR PERTURBATIONS

BUILD-UP VS DECAY

- In realistic context, perturbations are created and then redshifted
- Particle production happens at all times
- Possible impact of energy scales larger than Hubble

POSSIBLE APPLICATIONS:

- Phase transitions / supercooled
- Pre-heating / re-heating
- Connection with gravitational waves

$$T(q,\tau) = \frac{1}{2}[1 + \tanh(\beta(\tau))]$$

typical transfer function

FROM SOURCE TO PERTURBATIONS TO PARTICLE PRODUCTION

MICROPHYSICS SOURCES PERTURBATIONS VIA EINSTEIN'S EQ.S IN FLAT SPACE

$$\Gamma = \frac{1}{4} \int \frac{d^4q}{(2\pi)^4} h^{\mu\nu}(-q) \langle T_{\mu\nu}(q) T_{\rho\sigma}(-q) \rangle h^{\rho\sigma}(q)$$

- New formula in terms of the stress-energy tensor of the source
- Imaginary part from the dark sector –> cJ
- Integral over the 2-point function of the source –> <Ts Ts>

$$h_{\mu\nu}(q_0\,, \vec{q}) = \frac{2}{M_{\rm Pl}^2((q_0 + i\epsilon)^2 - \vec{q}^{\,2})} P_{\mu\nu\rho\sigma} T_S^{\rho\sigma}(q_0, \vec{q})$$

FROM SOURCE TO PERTURBATIONS TO PARTICLE PRODUCTION

PARTICLE PRODUCTION INSIDE THE HORIZON

$$N_{\rm dec} = \frac{c_J}{7680\pi}\frac{1}{M_{\rm Pl}^4}\int\frac{d^4q}{(2\pi)^4}\theta(q^2)\frac{1}{(q^2)^2}T_S^{\mu\nu}(q)\Pi_{\mu\nu\rho\sigma}(q)T_S^{\rho\sigma}(-q)$$

- Possible to decompose in scalar, vector, tensor perturbation
- Each helicity sourced by a given 2-point function constructed with Ts
- Given any <TT> possible to compute particle production

EFFECT OF SUPER-HORIZON EVOLUTION: IT SLIGHTLY MODIFIES THE INTEGRAND (BETTER IN REAL TIME)

WHAT ABOUT GRAVITATIONAL FREEZE-IN?

FROM OUR DERIVATION: JUST A SPECIAL CASE OF THERMAL <T T>

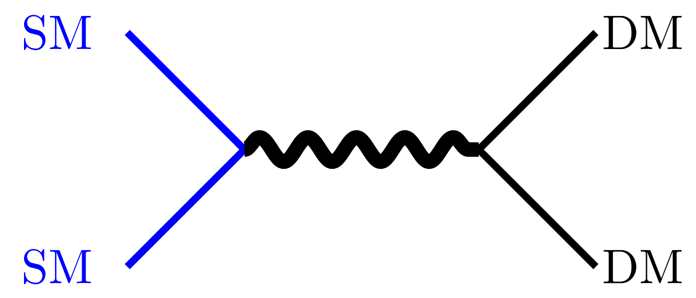
- By computing the thermal averaged <TT>

$$\langle T_{\mu\nu}(q) T_{\rho\sigma}(-q)\rangle_T \equiv \int d^4x d^4y e^{+iq(x-y)} \mathrm{tr}[\rho_T T_{\mu\nu}(x) T_{\rho\sigma}(y)] = V_4 \int d^4x e^{iqx} \mathrm{tr}[\rho_T T_{\mu\nu}(x) T_{\rho\sigma}(0)]$$

- Translational invariance + thermal state $\rho_T = e^{-H/T}$
- Full production rate need insertion of complete set of states. By focusing on vacuum insertion:

$$\frac{c_J}{7680\pi} \frac{1}{M_{\mathrm{Pl}}^4} \int \frac{d^4q}{(2\pi)^4} \theta(q^2) \frac{1}{(q^2)^2} e^{-q_0/T} 2\mathrm{Im}[i\langle T T_{\mu\nu}(q) T_{\rho\sigma}(-q)\rangle] \Pi^{\mu\nu\rho\sigma}(q)$$

same expression



GRAVITATIONAL FI = THERMAL PARTICLE PRODUCTION INSIDE THE HORIZON

GRAVITY WAVES + PARTICLES

CLEAR MESSAGE: ANY TIME GWS ARE PRODUCED, ALSO PARTICLES ARE PRODUCED

- Given a source: compare GWs + particle produced
- It can be done systematically for all helicities

$$\begin{aligned} T^{\mathrm{TT}}_{ij} &= (P^{\mathrm{T}}_{ik}P^{\mathrm{T}}_{jl} - \frac{1}{2}P^{\mathrm{T}}_{ij}P^{\mathrm{T}}_{kl})\Sigma_{kl} \\ T_{\Theta} &= -T^0_0 + \frac{3}{2}P^{\mathrm{T}}_{ij}\Sigma_{ij} = \delta\rho + \frac{3}{2}(p+\rho)\sigma \\ T^{0,\mathrm{T}}_i &= P^{\mathrm{T}}_{ij}T^0_j\,. \end{aligned}$$

KEY INGREDIENT IS UNEQUAL TIME TT CORRELATOR

$$\langle \Pi_s(\vec{q}, q_0)\Pi_{s'}(\vec{q}\,', q_0')\rangle = (2\pi)^3\delta^3(\vec{q}+\vec{q}\,')\delta_{ss'}\Pi^{\mathrm{GW}}_s(q, q_0, q_0')$$

GW SPECTRUM:

$$\frac{d(\rho_{\mathrm{GW}}a^3)}{d\log q} = \frac{1}{16\pi^2}\frac{q^3}{M^2_{\mathrm{Pl}}}\Pi_{\mathrm{GW}}(q\,, q\,, -q)$$

PARTICLE/DM SPECTRUM:

$$\frac{d(n_J a^3)}{dq_0} = \frac{c_J}{640\pi^2}\frac{4}{M^4_{\mathrm{Pl}}}\int d(\log q)\theta(q_0^2 - q^2)\frac{q^3}{2\pi^2}\Pi_{\mathrm{GW}}(q, q_0, -q_0)\,.$$

same quantity in different kinematic limits

similar formulas for scalar perturbations [less discussed in the literature]

ESTIMATES FOR FIRST ORDER PHASE TRANSITIONS

FIRST ORDER PHASE TRANSITIONS

$$Y_{\rm rad} \sim 10^{-2} c_J \left(\frac{\beta}{H_*}\right)^3 \left(\frac{T_*}{M_{\rm Pl}}\right)^3 A(q_*)$$

- Assuming a build up of the source on a time scale 1/beta
- Power spectro of gravitational waves A ~ 1/beta^2
- Important to explore connection between DM production and GWs

OTHER APPLICATIONS:

PRE-HEATING ~ AXION-GAUGE INTERACTIONS

- We are completing an explicit example with source = gauge fields sourced by axion
- Connection with the full 2-point function in the background of an oscillating axion

MORE GENERIC SOURCES

$$\phi_b \mathcal{O}$$

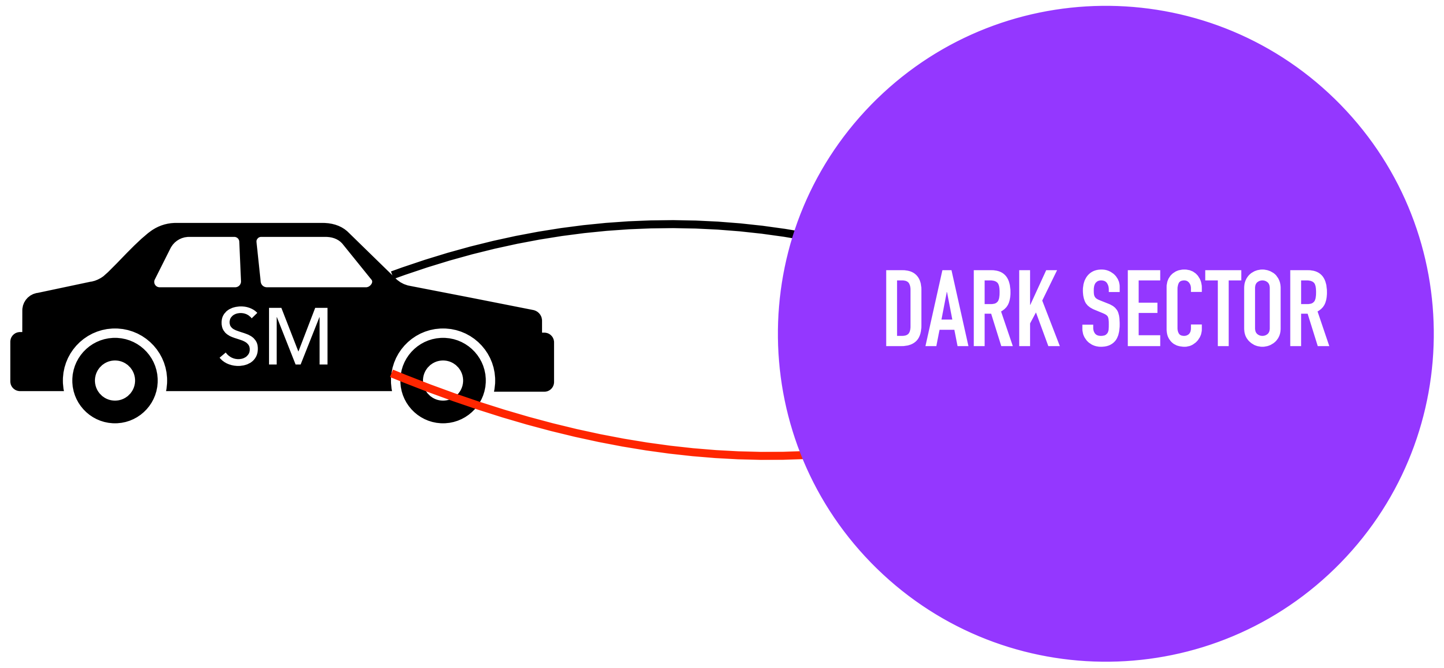
production rate of O:

$$\text{rate} \propto \int d^4p\,\phi_b(p)\phi_b(-p)\,\text{Im}[\langle \mathcal{O}(p)\mathcal{O}(-p)\rangle]$$

- potentially relevant for particle production from bubble collision
- in general maybe second order is not sufficient…

CONCLUSIONS

PRODUCTION OF DARK MATTER



- Often linked to inflationary dynamics / reheating
- Inhomogeneitis allow for production of massless relics
- Clear target for associated gravitational wave signals
- Less explored impact of scalar perturbations
- …

THANK YOU!