QCD

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3rd Lecture

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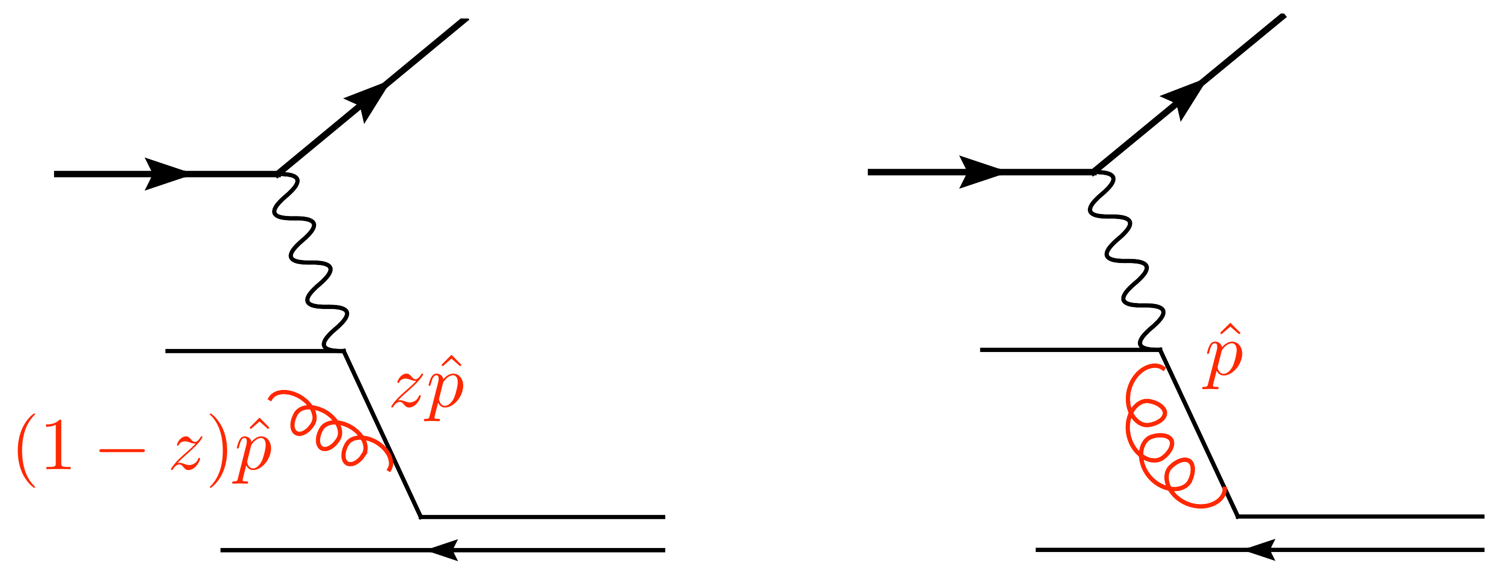
Radiative corrections from initial state

To first order in the coupling:
need to consider the emission of one real gluon and a virtual one



Adding real and virtual contributions, the partonic cross-section reads

$$\sigma^{(1)} = \frac{C_F \alpha_s}{2\pi} \int dz \frac{dk_\perp^2}{k_\perp^2} \frac{1+z^2}{1-z} \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

Partial cancellation between real (positive), virtual (negative), but real gluon changes the energy entering the scattering, the virtual does not

Radiative corrections

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int dz \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right), \qquad P(z) = C_F \frac{1+z^2}{1-z}$$

Soft limit: singularity at z=1 cancels between real and virtual terms

Collinear singularity: $k_\perp \rightarrow 0$ with finite z. Collinear singularity does not cancel because partonic scatterings occur at different energies

$\Longrightarrow$ naive parton model does not survive radiative corrections

Similarly to what is done when renormalizing UV divergences, collinear divergences from initial state emissions are absorbed into parton distribution functions

The plus prescription

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_0^1 dz\, P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

Plus prescription makes the universal cancelation of singularities explicit

$$\int_0^1 dz f_+(z) g(z) \equiv \int_0^1 f(z) \left(g(z) - g(1) \right)$$

The partonic cross section becomes

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int dz \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} P_+(z) \sigma^{(0)}(z\hat{p}) \,, \quad P(z) = C_F \left(\frac{1+z^2}{1-z} \right)$$

Collinear singularities still there, but they factorize.

Factorization scale

Schematically use $$\ln\frac{Q^2}{\lambda^2} = \ln\frac{Q^2}{\mu_F^2} + \ln\frac{\mu_F^2}{\lambda^2}$$

$$\sigma = \sigma^{(0)} + \sigma^{(1)} = \left(1 + \frac{\alpha_s}{2\pi}\ln\frac{\mu_F^2}{\lambda^2}P_+\right) \times \left(1 + \frac{\alpha_s}{2\pi}\ln\frac{Q^2}{\mu_F^2}P_+\right)\sigma^{(0)}$$

So we define

$$f_q(x,\mu_F) = f_q(x) \times \left(1 + \frac{\alpha_s}{2\pi}\ln\frac{\mu_F^2}{\lambda^2}P_{qq}^{(0)}\right) \qquad \hat{\sigma}(p,\mu_F) = \left(1 + \frac{\alpha_s}{2\pi}\ln\frac{Q^2}{\mu_F^2}P_{qq}^{(0)}\right)\sigma^{(0)}(p)$$

NB:

- universality, i.e. the PDF redefinition does not depend on the process
- choice of $\mu_F \sim Q$ avoids large logarithms in partonic cross-sections
- PDFs and hard cross-sections don't evolve independently
- the factorization scale acts as a cut-off, it allows to move the divergent contribution into non-perturbative parton distribution functions

Improved parton model

Naive parton model:

$$\cancel{\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1) f_2^{(P_2)}(x_2) \hat{\sigma}(x_1 x_2 s)} \qquad \hat{s} = x_1 x_2 s$$

After radiative corrections:

$$\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1, \mu^2) f_2^{(P_2)}(x_2, \mu^2) \hat{\sigma}(x_1 x_2 s, \mu^2)$$

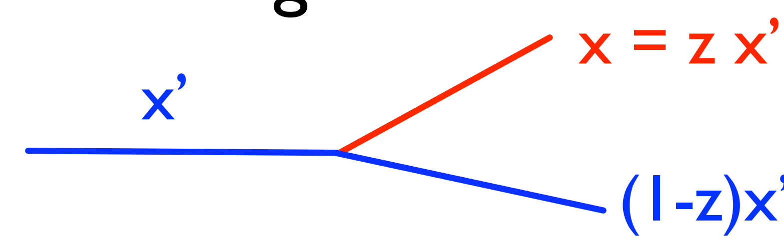
Intermediate recap

- With initial state parton collinear singularities do not cancel
- Initial state emissions with $k_\perp$ below a given scale are included in PDFs
- This procedure introduces a scale μ_F, the so-called factorization scale which factorizes the low energy (non-perturbative) dynamics from the perturbative hard cross-section
- As for the renormalization scale, the dependence of cross-sections on μ_F is due to the fact that the perturbative expansion has been truncated
- The dependence on μ_F becomes milder when including higher orders
- The redefinition of PDFs is universal and process-independent

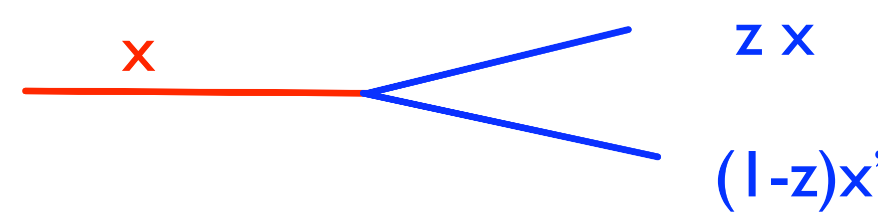
Evolution of PDFs

A parton distribution at momentum fraction x changes if

- a different parton splits and produces it



- the parton itself splits



$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_0^1 dx' \int_x^1 dz \frac{\alpha_s}{2\pi} P(z) f(x',\mu^2)\delta(zx'-x) - \int_0^1 dz \frac{\alpha_s}{2\pi} P(z) f(x,\mu^2)$$

$$= \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu^2\right) - \int_0^1 dz \frac{\alpha_s}{2\pi} P(z) f\left(x,\mu^2\right)$$

$$= \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_+(z) f\left(\frac{x}{z},\mu^2\right)$$

The plus prescription $\quad \int_0^1 dz f_+(z) g(z) \equiv \int_0^1 dz f(z)\left(g(z)-g(1)\right)$

DGLAP equation

$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z}, \mu^2\right)$$

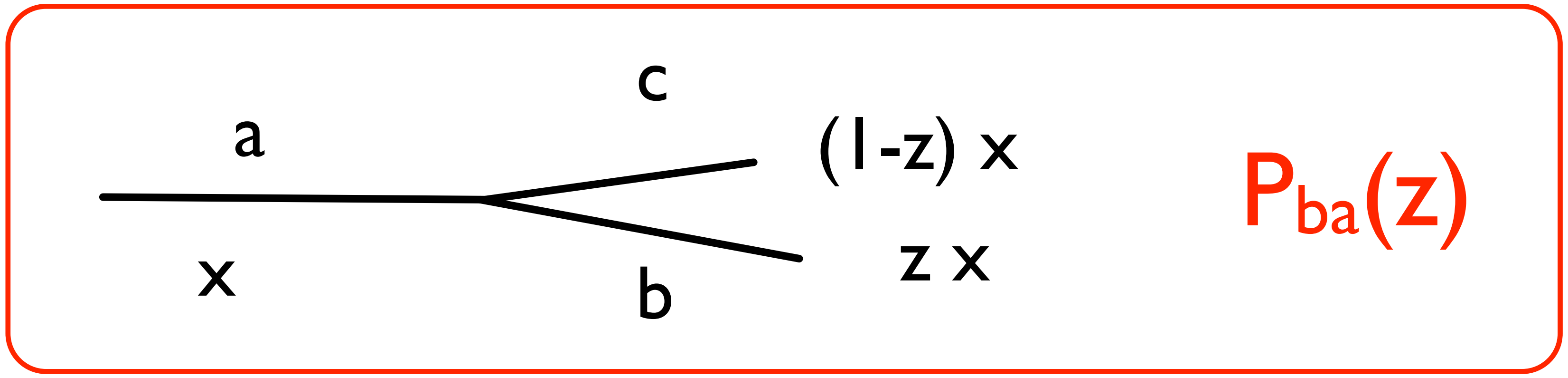
Altarelli, Parisi; Gribov-Lipatov; Dokshitzer '77

Master equation of QCD: we can not compute parton densities, but we can predict how they evolve from one scale to another

Universality of splitting functions: we can measure pdfs in one process and use them as an input for another process

Conventions for splitting functions

There are various partons types. Standard notation:



Accounting for the different species of partons the DGLAP equations become:

$$\mu^2 \frac{\partial f_i(x,\mu^2)}{\partial \mu^2} = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z) f_j\left(\frac{x}{z},\mu^2\right)$$

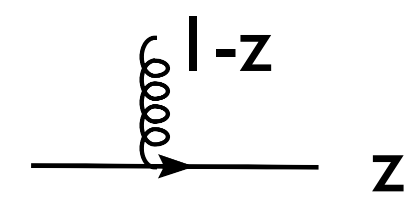
This is a system of coupled integro/differential equations

The above convolution in compact notation:

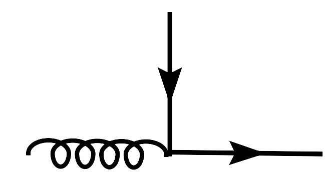
$$\mu^2 \frac{\partial f_i(x,\mu^2)}{\partial \mu^2} = \sum_j P_{ij} \otimes f_j(\mu^2)$$

Properties of splitting functions

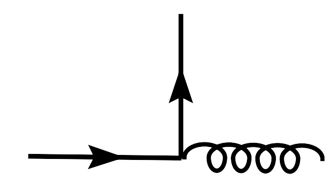
$$P_{qq}^{(0)} = P_{\bar{q}\bar{q}}^{(0)} = C_F \left[\left(\frac{1+z^2}{1-z} \right)_+ \right]$$



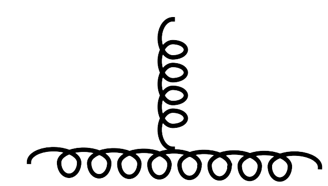
$$P_{qg}^{(0)} = P_{\bar{q}g}^{(0)} = T_R \left(z^2 + (1-z) \right)$$



$$P_{gq}^{(0)} = P_{g\bar{q}}^{(0)} = C_F \frac{1+(1-z)^2}{z}$$



$$P_{gg}^{(0)} = 2C_A \left[z \left(\frac{1}{(1-z)} \right)_+ + \frac{1-z}{z} + z(1-z) \right]$$



- P_{qg} anf P_{gg} symmetric under z (1-z)
- P_{qq} and P_{gg} divergence for z=1 (soft gluon)
- P_{gq} and P_{gg} divergenge for z=0 (soft gluon)
- P_{qg} no soft divergence for gluon splitting to quarks

➠ gluon PDF grows at small x

(Here given without virtual δ-terms)

Sum rules in pQCD

Beyond the naive parton model the probabilistic picture does not hold anymore. What about basic conservation principles (e.g. sum rules)?

Exercise: show that e.g.

$$\int_0^1 dx\,\left(f_u(x,\mu^2) - f_{\bar{u}}(x,\mu^2)\right) = \text{constant} \quad \text{if and only if} \quad \int_0^1 dz P_{qq}(z) = 0$$

Solution:

1. Start from DGLAP for u

$$\mu^2 \frac{\partial f_u(x,\mu^2)}{\partial \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \int_x^1 \frac{dz}{z} \left(P_{uu}(z) f_u\left(\frac{x}{z},\mu^2\right) + P_{ug}(z) f_g\left(\frac{x}{z},\mu^2\right)\right)$$

2. Subtract the same equation for $\bar{u}$ and integrate over x

Sum rules in pQCD

2. Subtract the same equation for ū and integrate over x

$$\mu^2 \frac{\partial}{\partial \mu^2} \int_0^1 dx \left(f_u(x,\mu^2) - f_{\bar{u}}(x,\mu^2)\right) = \frac{\alpha_s(\mu^2)}{2\pi} \int_0^1 dx \int_x^1 \frac{dz}{z} P_{qq}(z) \left(f_u\left(\frac{x}{z},\mu^2\right) - f_{\bar{u}}\left(\frac{x}{z},\mu^2\right)\right)$$

3. Swap x and z integration, replace x with y = x/z

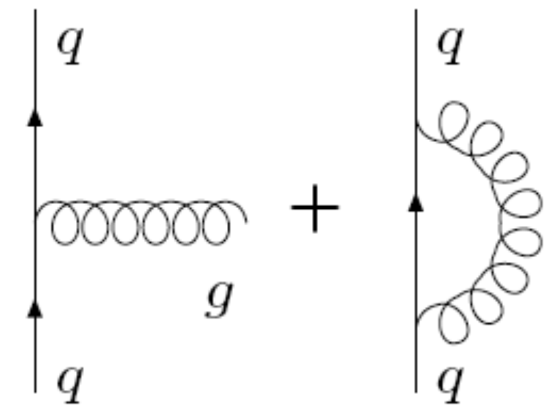
$$\mu^2 \frac{\partial}{\partial \mu^2} \int_0^1 dx \left(f_u(x,\mu^2) - f_{\bar{u}}(x,\mu^2)\right) = \frac{\alpha_s(\mu^2)}{2\pi} \int_0^1 dz P_{qq}(z) \int_0^1 dy \left(f_u\left(y,\mu^2\right) - f_{\bar{u}}\left(y,\mu^2\right)\right)$$

Conclusion: the integral $\displaystyle\int_0^1 dx \left(f_u(x,\mu^2) - f_{\bar{u}}(x,\mu^2)\right)$

does not depend on the scale if, and only if $\displaystyle\int_0^1 dz P_{qq}(z) = 0$

Properties of splitting functions

$$P_{qq}^{(0)} = P_{\bar{q}\bar{q}}^{(0)} = C_F \left[\left(\frac{1+z^2}{1-z} \right)_+ + \frac{3}{2}\delta(1-z) \right]$$



☛ the delta-term is the virtual correction (present only when the flavour does not change)

We have just seen that in order to conserve quark (baryon) number, the integral of the quark distribution can not vary with Q^2, hence, the splitting functions must integrate to zero

Exercise: use this fact to compute the coefficients of the pure delta terms in P_{qq} and P_{gg} without performing the loop integral

The singlet distribution

The flavour singlet distribution is invariant under flavour rotations

$$\Sigma(x,Q^2)=\sum_{i=1}^{n_f}\left(q_i+\bar{q}_i\right)$$

Its DGLAP equation reads

$$\frac{\partial}{\partial \ln Q^2}\begin{pmatrix}\Sigma(x,Q^2)\\ g(x,Q^2)\end{pmatrix}=\begin{pmatrix}P_{qq} & P_{qg}\\ P_{gq} & P_{gg}\end{pmatrix}\otimes\begin{pmatrix}\Sigma(x,Q^2)\\ g(x,Q^2)\end{pmatrix}$$

Which is equivalent to

$$\frac{\partial}{\partial \ln Q^2}\binom{\Sigma}{g}(x,Q^2)=\int_x^1\frac{dz}{z}\begin{pmatrix}P_{qq}(z,\alpha_s) & P_{qg}(z,\alpha_s)\\ P_{gq}(z,\alpha_s) & P_{gg}(z,\alpha_s)\end{pmatrix}\begin{pmatrix}\Sigma(x/z,Q^2)\\ g(x/z,Q^2)\end{pmatrix},$$

The singlet distribution naturally mixes with the gluons

The non-singlet distributions

The non-singlet distributions consider flavour differences, e.g.

$$q_{\rm NS} = u + \bar{u} - (d + \bar{d})$$

Since the gluon acts in the same way (QCD is flavour blind), the interaction with the gluon drops in the difference and the DGLAP equation at LO becomes

$$\frac{\partial q_{\rm NS}(x, Q^2)}{\partial \ln Q^2} = P_{\rm NS}(x, \alpha_s) \otimes q_{\rm NS}(x, Q^2)$$

A general $SU(n_f)$ group has $n_f^2 - 1$ generators, but PDF evolve only the gluon and the n_f diagonal flavours. Then, one can replace n_f flavours with one singlet and $(n_f - 1)$ non-singlet distributions.

History of splitting functions

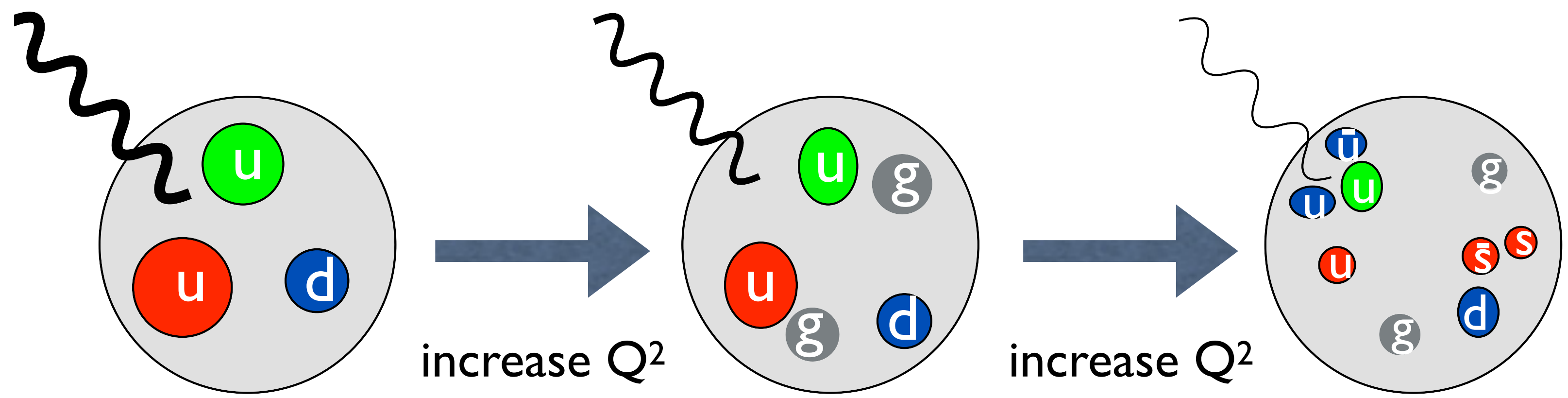
- ☑ $P_{ab}^{(0)}$: Altarelly, Parisi; Gribov-Lipatov; Dokshitzer (1977)
- ☑ $P_{ab}^{(1)}$: Curci, Furmanski, Petronzio (1980)
- ☑ $P_{ab}^{(2)}$: Moch, Vermaseren, Vogt (2004)
- ☐ $P_{ab}^{(3)}$: in progress non-singlet result in planar limit (Moch et al. 2017) and quark-quark case (Falcioni et al. 2023)

☛ Splitting functions are essential input for pdf determination

Evolution

So, in perturbative QCD we can not predict values for

- the coupling
- the masses
- the parton densities
- ...



What we can predict is the evolution with the Q^2 of those quantities.

These quantities must be extracted at some scale from data, but then the evolution is controlled by perturbative QCD

Evolution

- Lesson: not only is the coupling scale-dependent, but protons have a scale dependent sub-structure
- We started with the question of how one can access the gluon pdf:
  - Because of the DGLAP evolution, one can access the gluon pdf indirectly, through the way it changes the evolution of quark pdfs.
  - Today also direct measurements using Tevatron jet data and LHC tt and jet data

Formal definition of PDFs

PDF as matrix element of gauge invariant light-cone operators

$$f_{q/H}(x,\mu) = \frac{1}{2}\int \frac{d\xi^-}{2\pi}\, e^{-ixP^+\xi^-} \left\langle P \left| \bar{\psi}(\xi^-)\gamma^+ \, W(\xi^-,0)\, \psi(0) \right| P \right\rangle$$

Quarks

$$f_{g/H}(x,\mu) = \frac{1}{xP^+}\int \frac{d\xi^-}{2\pi}\, e^{-ixP^+\xi^-} \left\langle P \left| F_a^{+\mu}(\xi^-)\, W(\xi^-,0)\, F^+_{\mu,a}(0) \right| P \right\rangle$$

Gluons

$$W(\xi^-,0) = \mathscr{P}\exp\left[ig \int_0^{\xi^-} d\eta^-\, A^+(\eta^-) \right]$$

Wilson line

Wilson lines crucial for gauge invariance of the definitions.

DGLAP arises from renormalization (running)

Recap.

- Parton model: incoherent sum of all partonic cross-sections
- Sum rules (momentum, charge, flavor conservation)
- Determination of parton densities (electron & neutrino scattering)
- Radiative corrections: failure of parton model
- Factorization of initial state divergences into scale dependent parton densities
- DGLAP evolution of parton densities $\Rightarrow$ measure gluon PDF
- While PDFs loose the naive probabilistic interpretation basic conservation principle still hold (momentum sum rules, energy, flavour conservation)

DGLAP in Mellin space

How are DGLAP equations solved?

One approach is to work in Mellin space

$$f_i(N,\mu^2) = \int_0^1 dx\, x^{N-1}\, f_i(x,\mu^2)$$

The advantage of Mellin transform: convolutions ⇒ ordinary products

Exercise: show that $(f \otimes g)(N) = f(N)g(N)$

The disadvantage of Mellin transform: need to evaluate inverse Mellin transform at the end

$$f_i(x,\mu^2) = \frac{1}{2\pi i}\int_C dN\, x^{-N}\, f_i(N,\mu^2)$$

Exercise: show that the above is indeed the inverse Mellin transform

Anomalous dimensions

Evolution equation for the non-singlet in Mellin space (for simplicity)

$$\mu^2 \frac{\partial q^{\mathrm{NS}}(N,\mu^2)}{\partial \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \gamma_{qq}(N, \alpha_s(\mu^2))\ q^{\mathrm{NS}}(N,\mu^2)$$

Where the anomalous dimension is given by

$$\gamma_{qq}(N, \alpha_s(\mu^2)) = \int_0^1 dx\, x^{N-1}\, P_{qq}(x, \alpha_s)$$

And similarly for the gluon and singlet component. At leading order:

$$\gamma_{qq}^{(0)} = C_F \left\{ -\frac{1}{2} + \frac{1}{N(N+1)} - 2\sum_{k=2}^{N} \frac{1}{k} \right\}$$

$$\gamma_{qg}^{(0)} = T_R \left\{ \frac{2+N+N^2}{N(N+1)(N+2)} \right\} \qquad \gamma_{qg}^{(0)} = C_F \left\{ \frac{2+N+N^2}{N(N^2-1)} \right\}$$

$$\gamma_{gg}^{(0)} = 2C_A \left\{ -\frac{1}{12} + \frac{1}{N(N-1)} + \frac{1}{(N+1)(N+2)} - \sum_{k=2}^{N} \frac{1}{k} \right\} - \frac{2}{3} n_f T_R$$

Solution in Mellin space

Given the anomalous dimension, the equation for non-singlet is

$$\mu^2 \frac{\partial q^{\mathrm{NS}}(N,\mu^2)}{\partial \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \gamma_{qq}(N, \alpha_s(\mu^2))\; q^{\mathrm{NS}}(N,\mu^2)$$

To lowest order one has

$$\alpha_s(\mu^2) = \frac{1}{b_0 \ln \frac{\mu^2}{\Lambda^2}} \qquad\qquad \gamma_{qq}(N, \alpha_s(\mu^2)) = \gamma_{qq}^{(0)}(N)$$

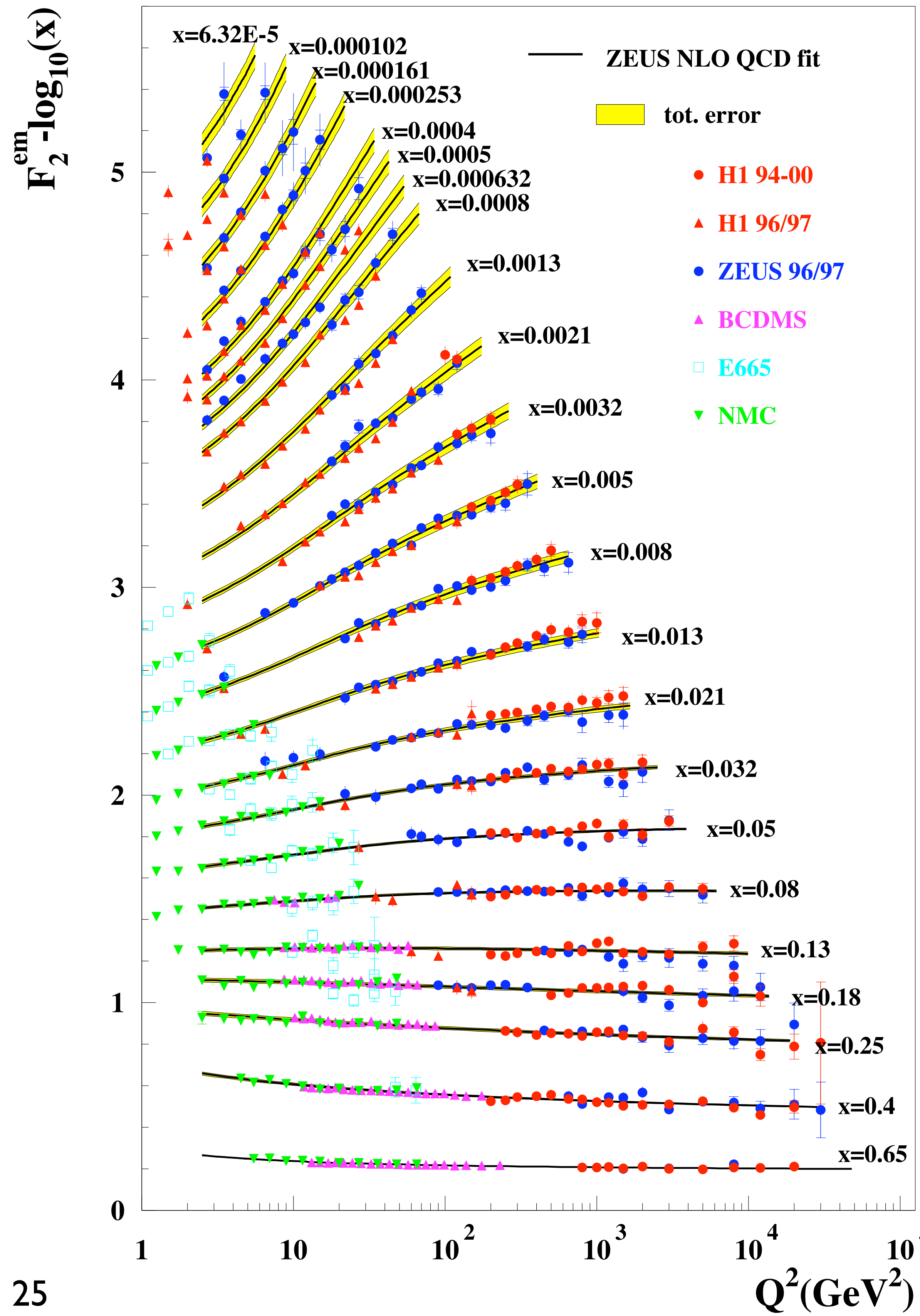
Integrate the equation

$$q^{\mathrm{NS}}(N,Q^2) = q^{\mathrm{NS}}(N,Q_0^2) \left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{d^{(0)}(N)} \qquad\qquad d^{(0)}(N) = \frac{\gamma^{(0)}(N)}{2\pi b_0}$$

Finally one needs to take in inverse Mellin transform to go back to x-space (usually this can be done only numerically)

Data: F_2

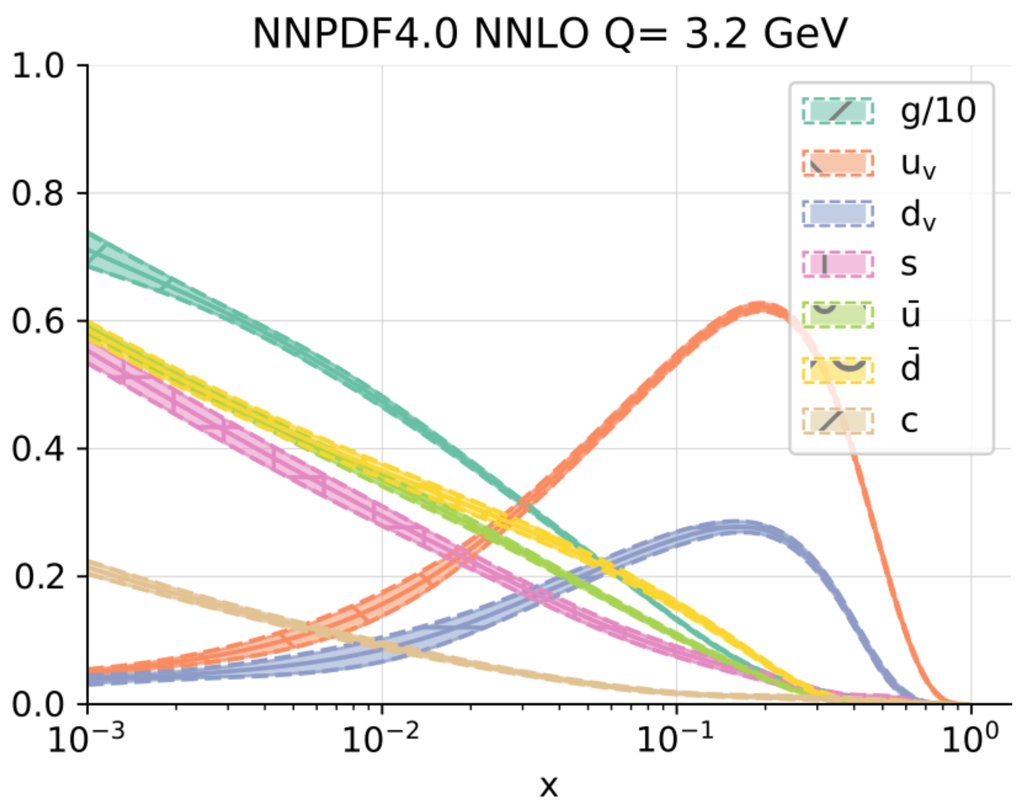
- DGLAP evolution equations allow to predict the Q^2 dependence of DIS data
- gluons crucial in driving the evolution



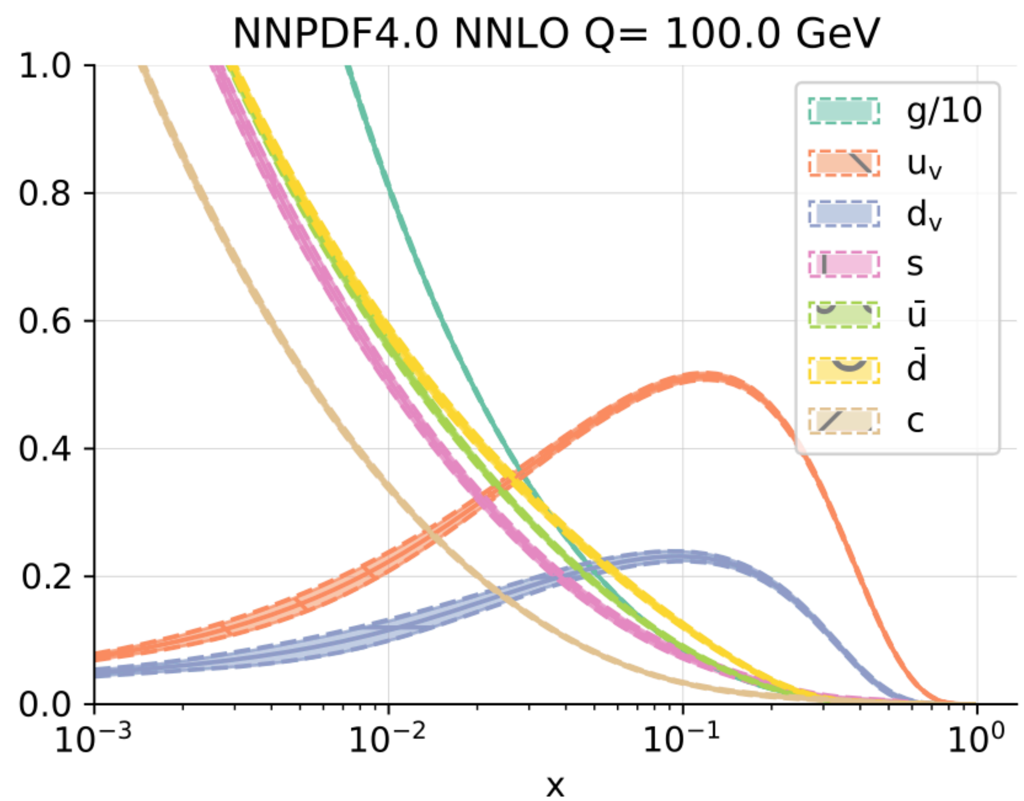
DGLAP Evolution

The DGLAP evolution is a key to precision LHC phenomenology: it allows to measure PDFs at some scale (say in DIS) and evolve upwards to make LHC (7, 8, 13, 14, 33, 100.... TeV) predictions

Measure PDFs at low Q



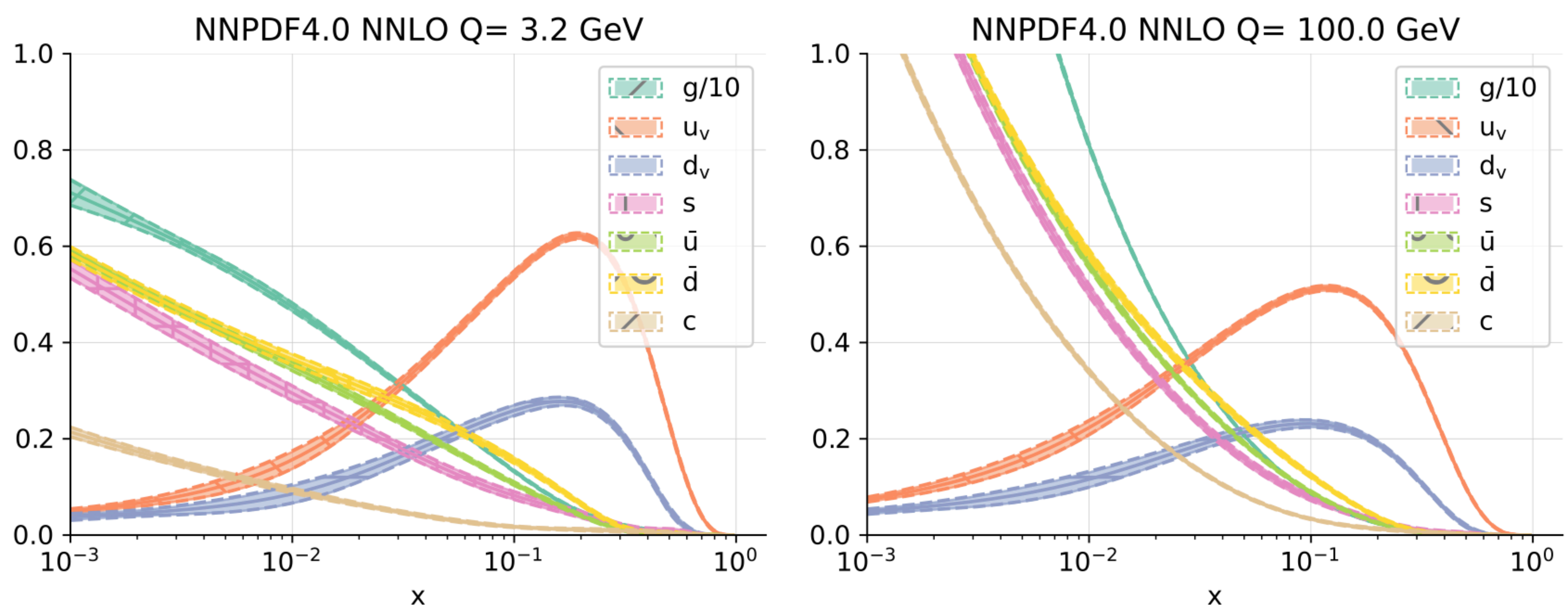
Evolve in Q² and make LHC predictions



Different PDFs evolve in different ways
(different equations + unitarity constraint)

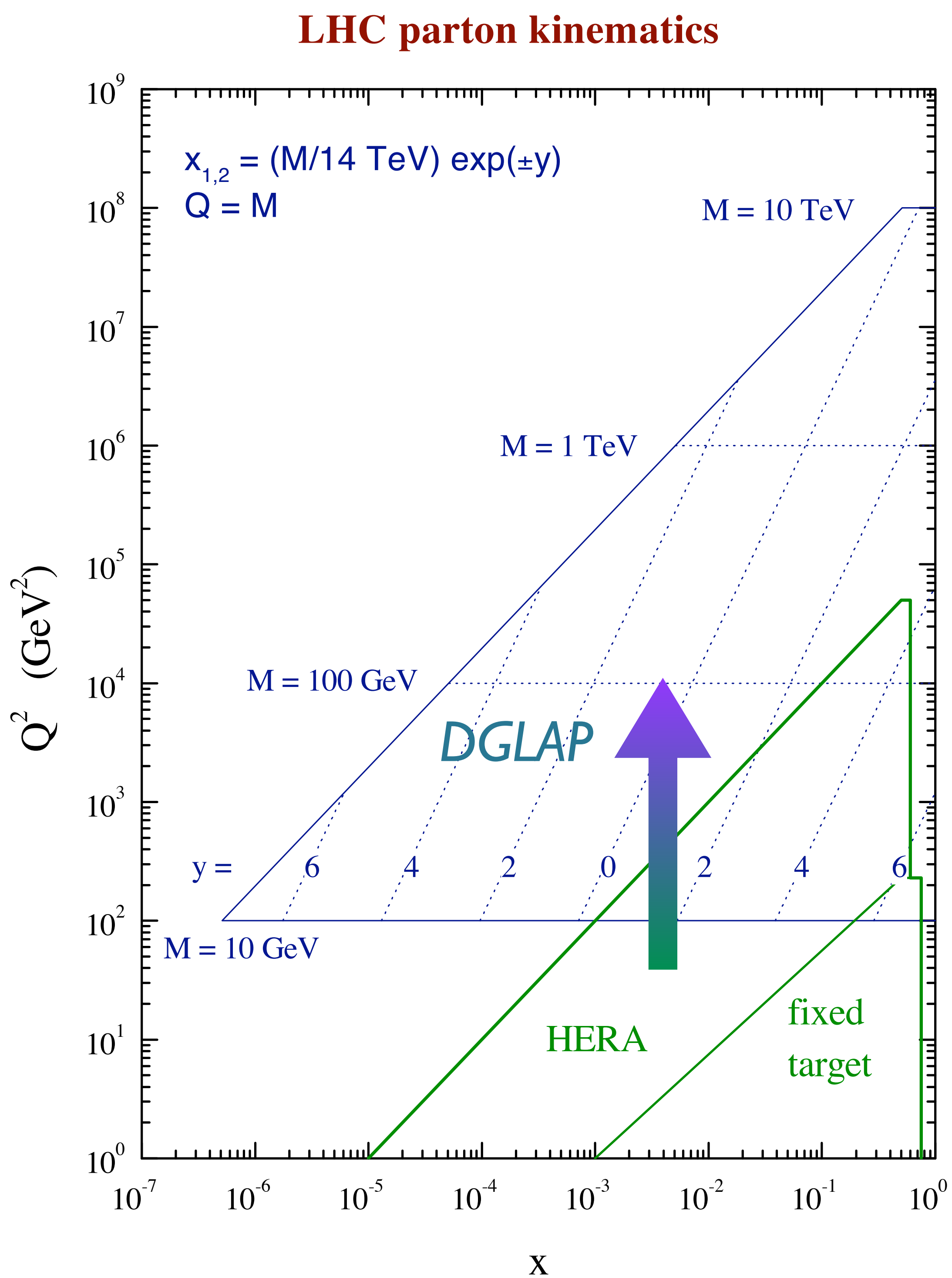
Typical features of PDFs

- vanish at $x \to 1$
- valence quarks peak at $x \simeq 1/3$
- gluon and sea distribution rise for $x \to 0$ (region dominated by gluons)



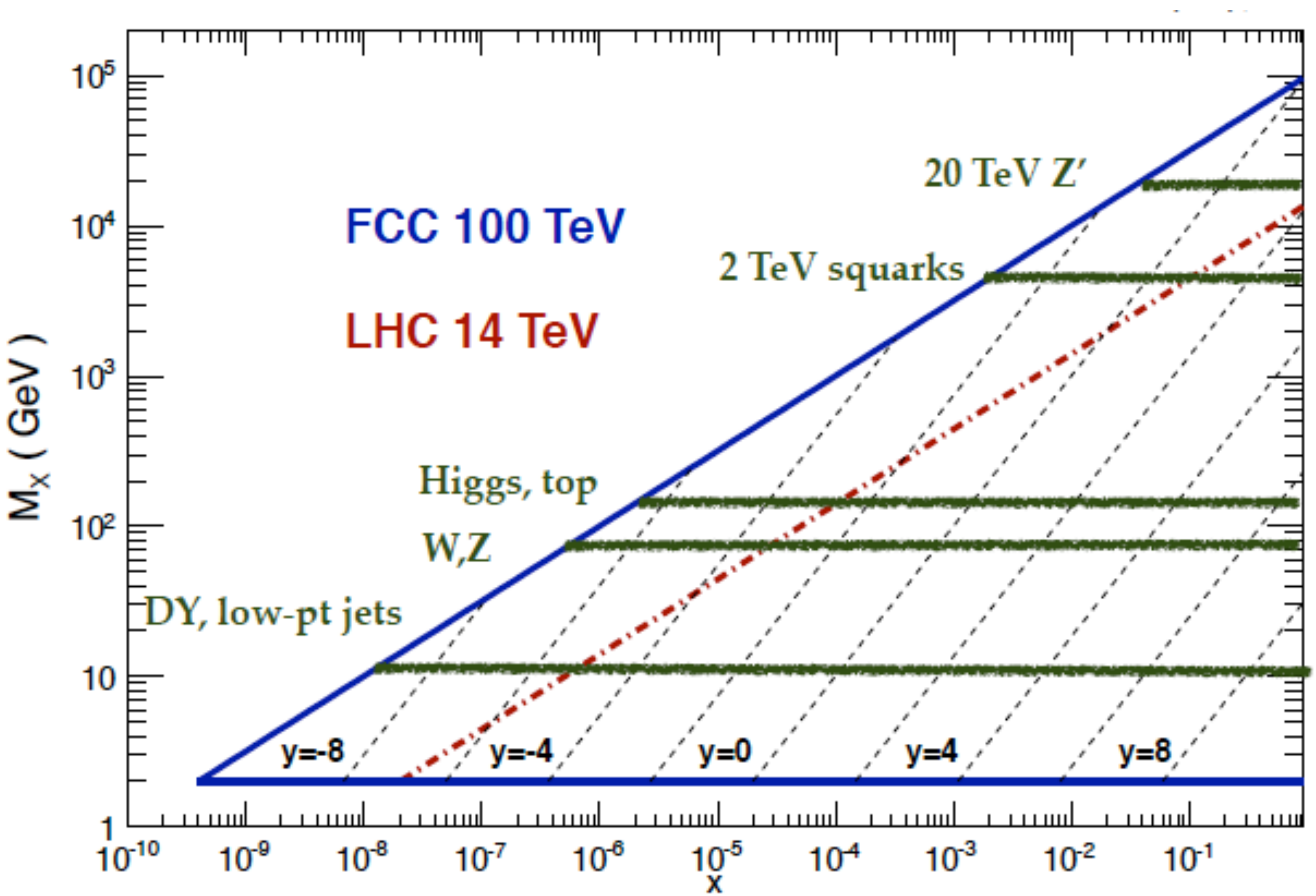
Parton density coverage

- most of the LHC x-range covered by Hera
- need 2-3 orders of magnitude Q^2-evolution
- rapidity distributions probe extreme x-values
- 100 GeV physics at LHC: small-x, sea partons
- TeV physics: large x



Parton density coverage

Coverage of 14 TeV LHC with respect to 100 TeV FCC



Today's PDF fits

PDFs are obtained from "global" fits of diverse data sets from different experiments, processes, observables

Three major collaborations involved in LHC fits

- NNPDF: relies on neural network as generic fit functions
- CT: relies on sophisticated parameterisation
- MHST: relies on sophisticated parameterisation

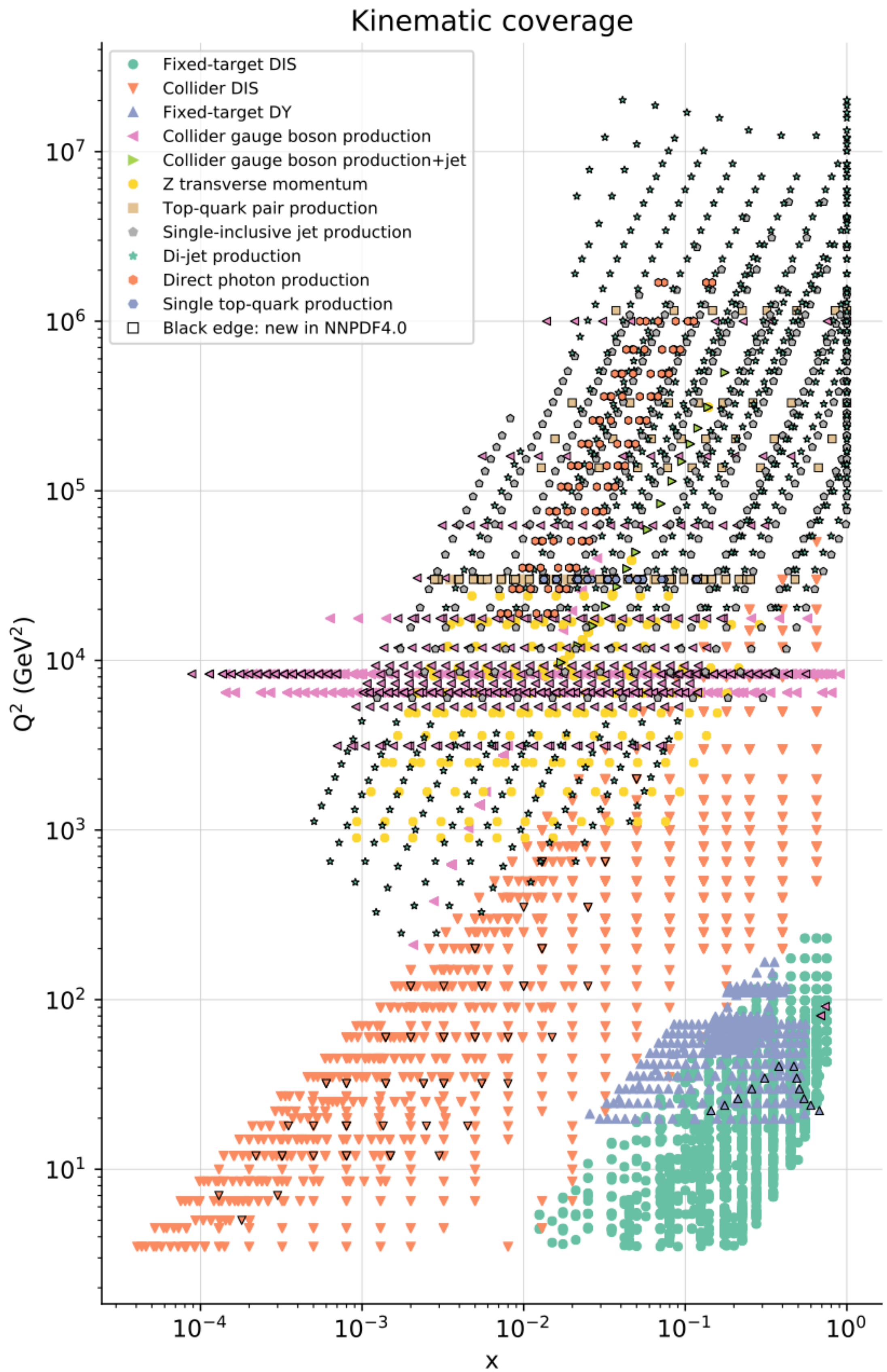
New sets released on https://www.lhapdf.org/pdfsets.html

Easy to download, install and use

Dataset

<u>Questions</u>:

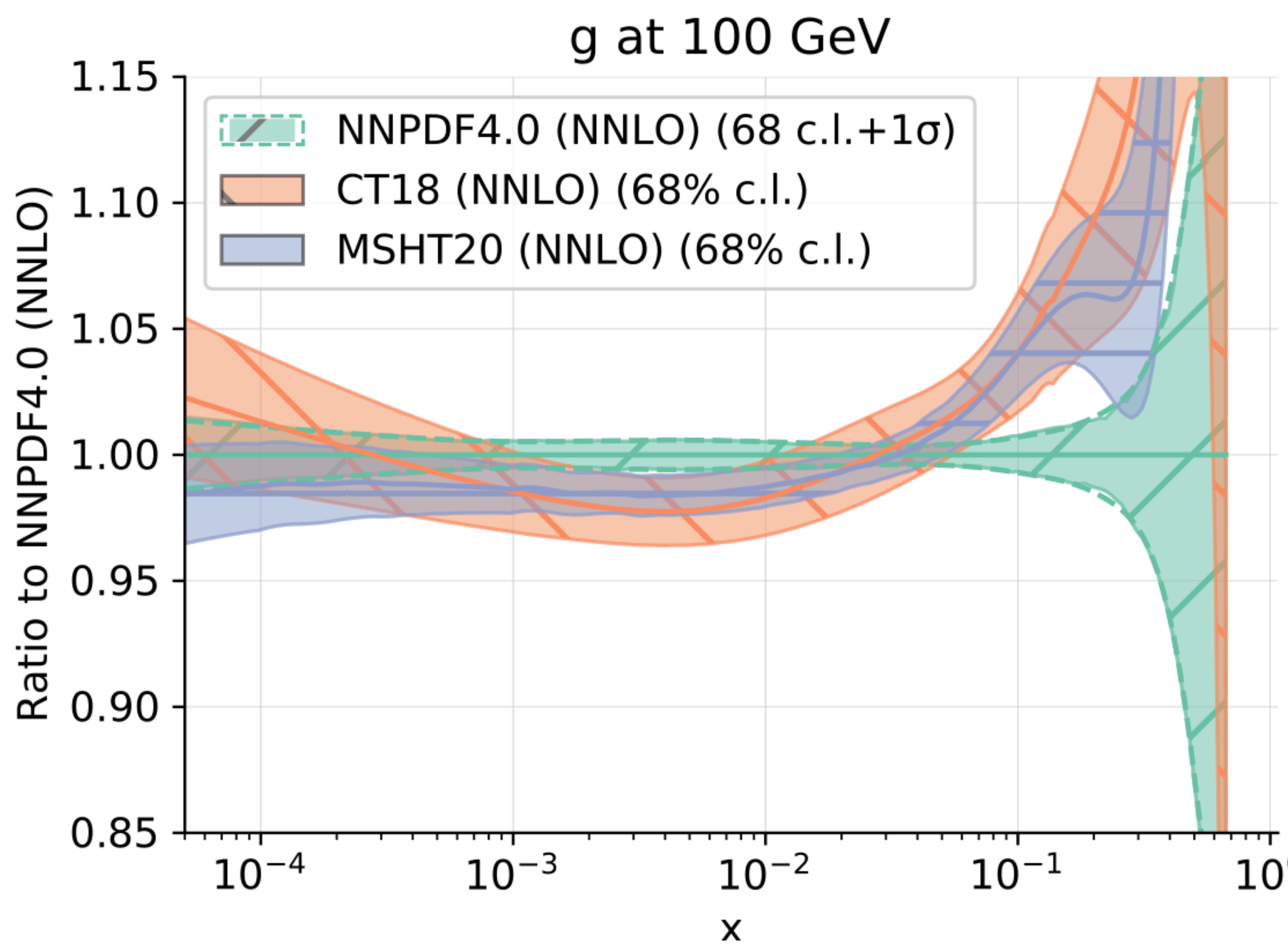
- which data to include in the fits (and how to deal with incompatible data)
- enhance relevance of some data (reduce effect of inconsistent data sets)?
- treatment of correlations and systematics



Comparison of different PDF sets

- In most of the LHC relevant regions uncertainties are about 1-2%
- Relative good agreement between independent determinations

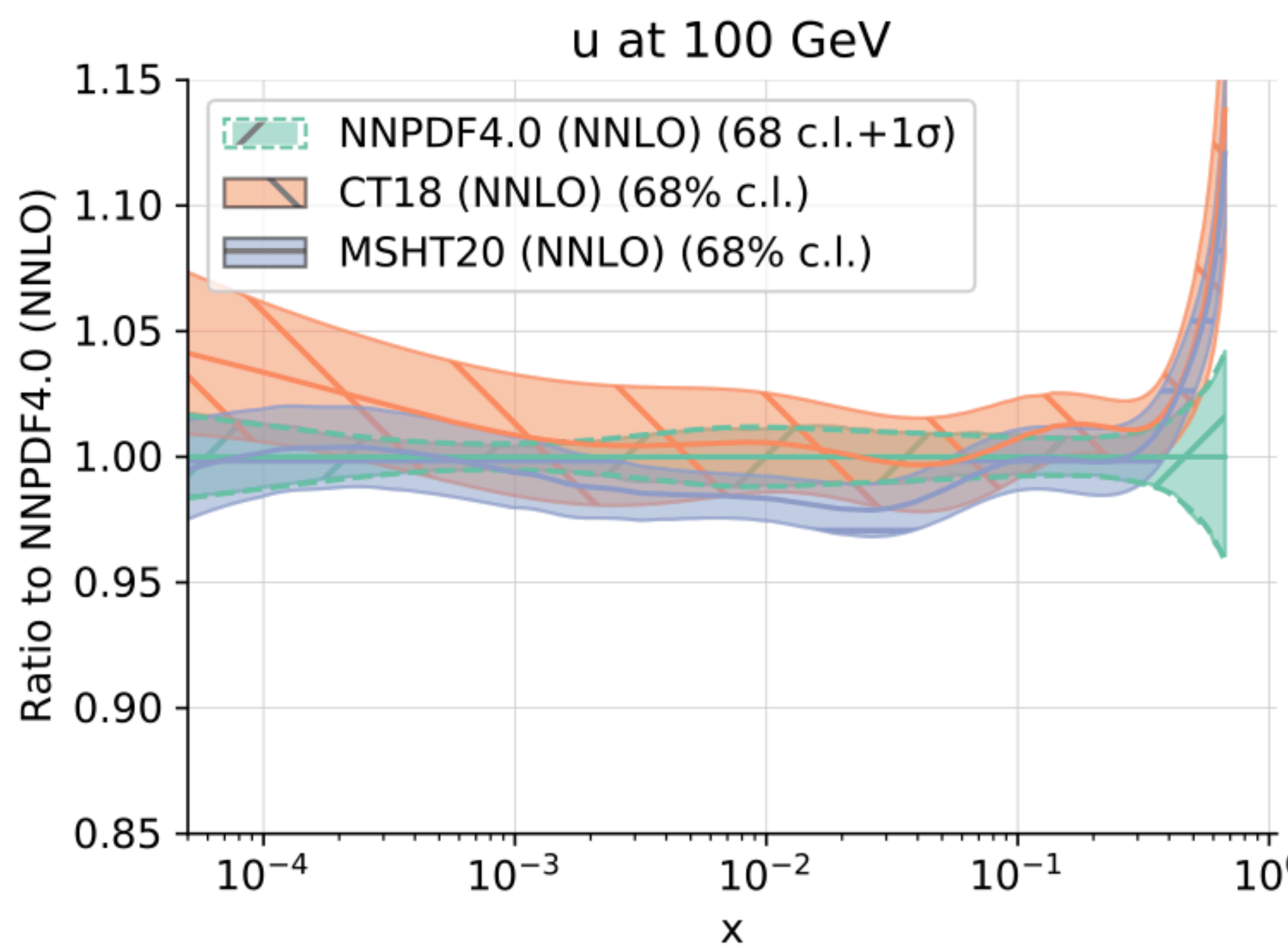
The gluon PDF



Comparison of different PDF sets

Valence PDF (e.g. up-quark)

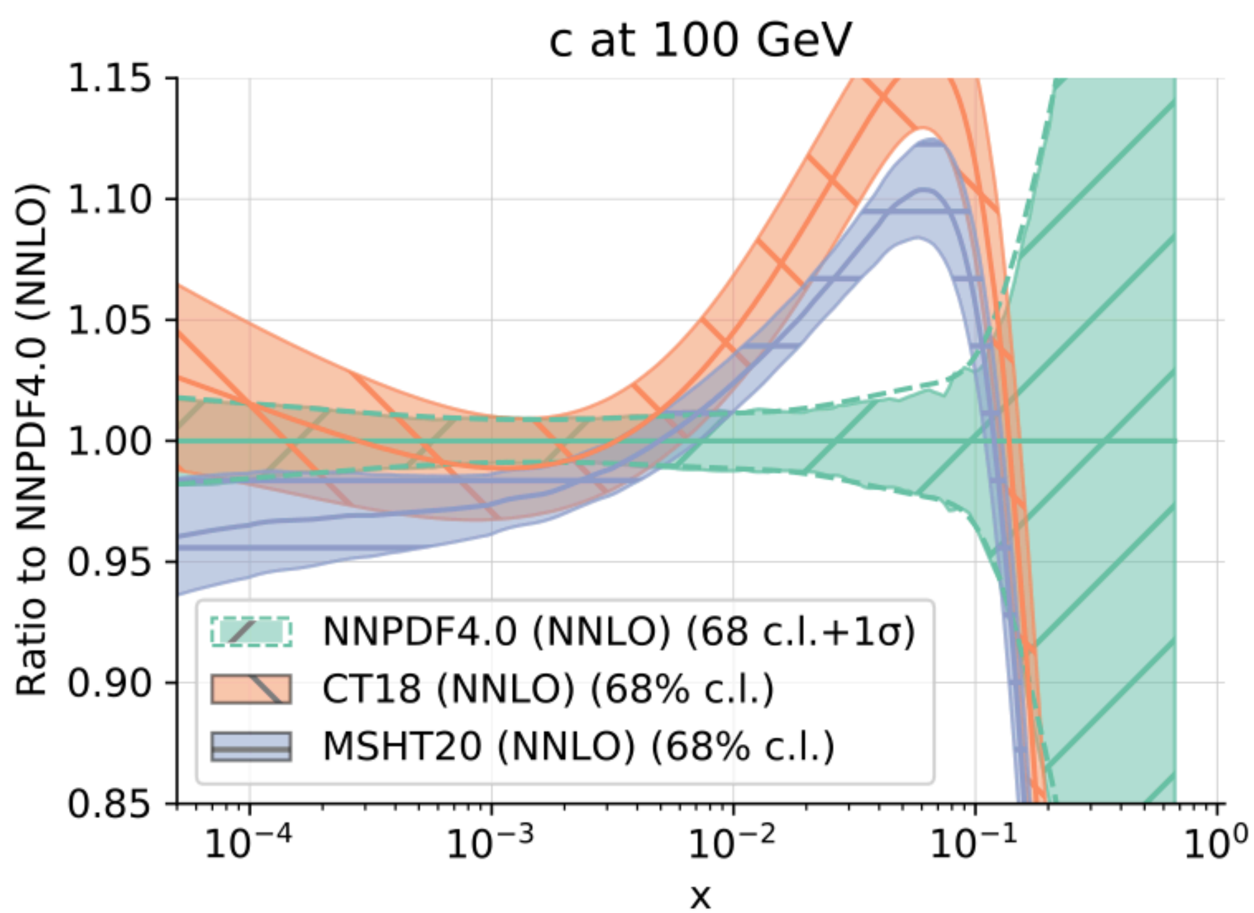
- In most of the LHC relevant regions uncertainties are about 1-2%
- Relative good agreement between independent determinations



Comparison of different PDF sets

Charm PDF

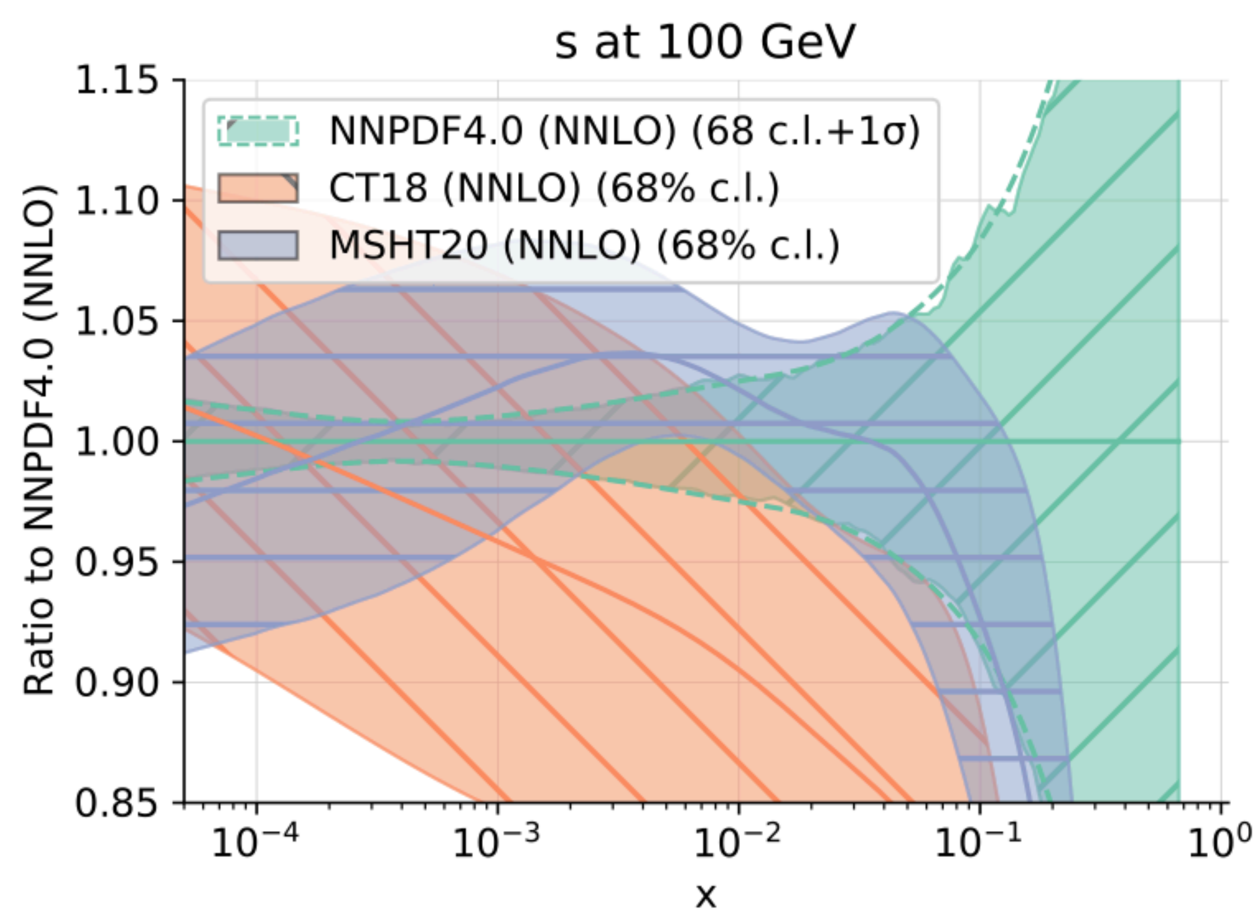
- Less good agreement for heavy-flavour
- NNPDF assumes intrinsic charm, CT18/ MSHT20 treat charm only perturbatively (evolve from charm mass)



Comparison of different PDF sets

- Less well-known PDF
- Few good experimental handles

Strange PDF



Progress in PDFs

PDFs are an essential ingredient for the LHC program.

Recent progress includes

- better assessment of uncertainties (e.g. different groups now agree at the 1σ level where data is available)
- exploit wealth of new information from LHC Run II and Run III measurements
- progress in tools and methods to include these data in the fits
- inclusion of PDFs for photons, leptons, treatment of heavy flavour

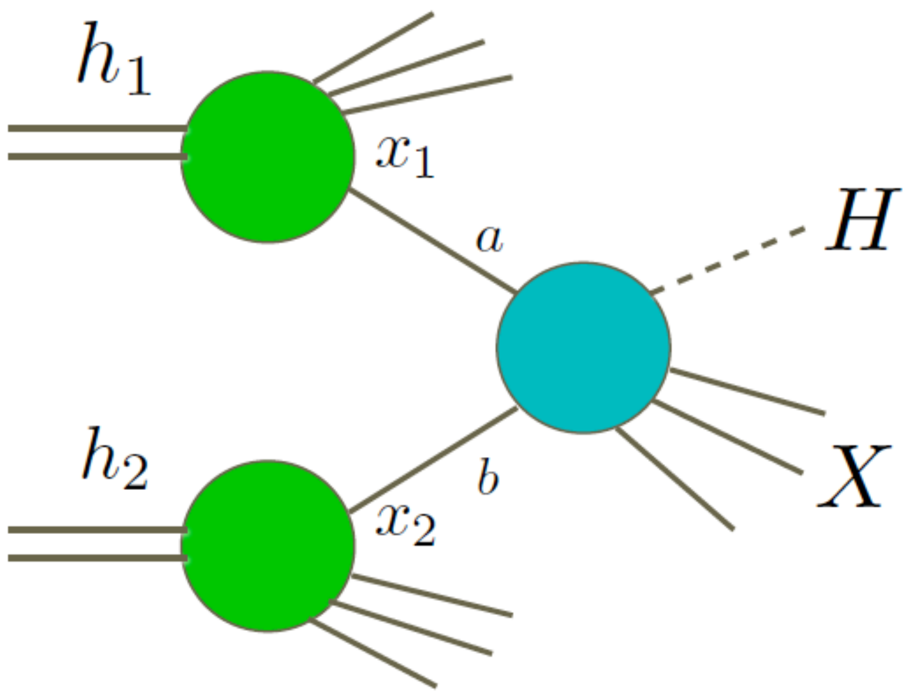
Progress in PDFs

Some other issues under discussion

- heavy-quark treatment and masses
- parameterization for PDFs (theoretical bias, reduced in Neural Network PDFs)
- include theoretical improvement (e.g. resummation) for some observables
- unphysical behaviour close to x=0 and x=1
- meaning of uncertainties. How reliable are they?
- α_s as external input or fitted with PDFs
- how not to "fit away" New Physics effects in PDFs

Parton luminosities

Parton luminosity are very informative as they enter directly the cross-section calculations, they are defined as

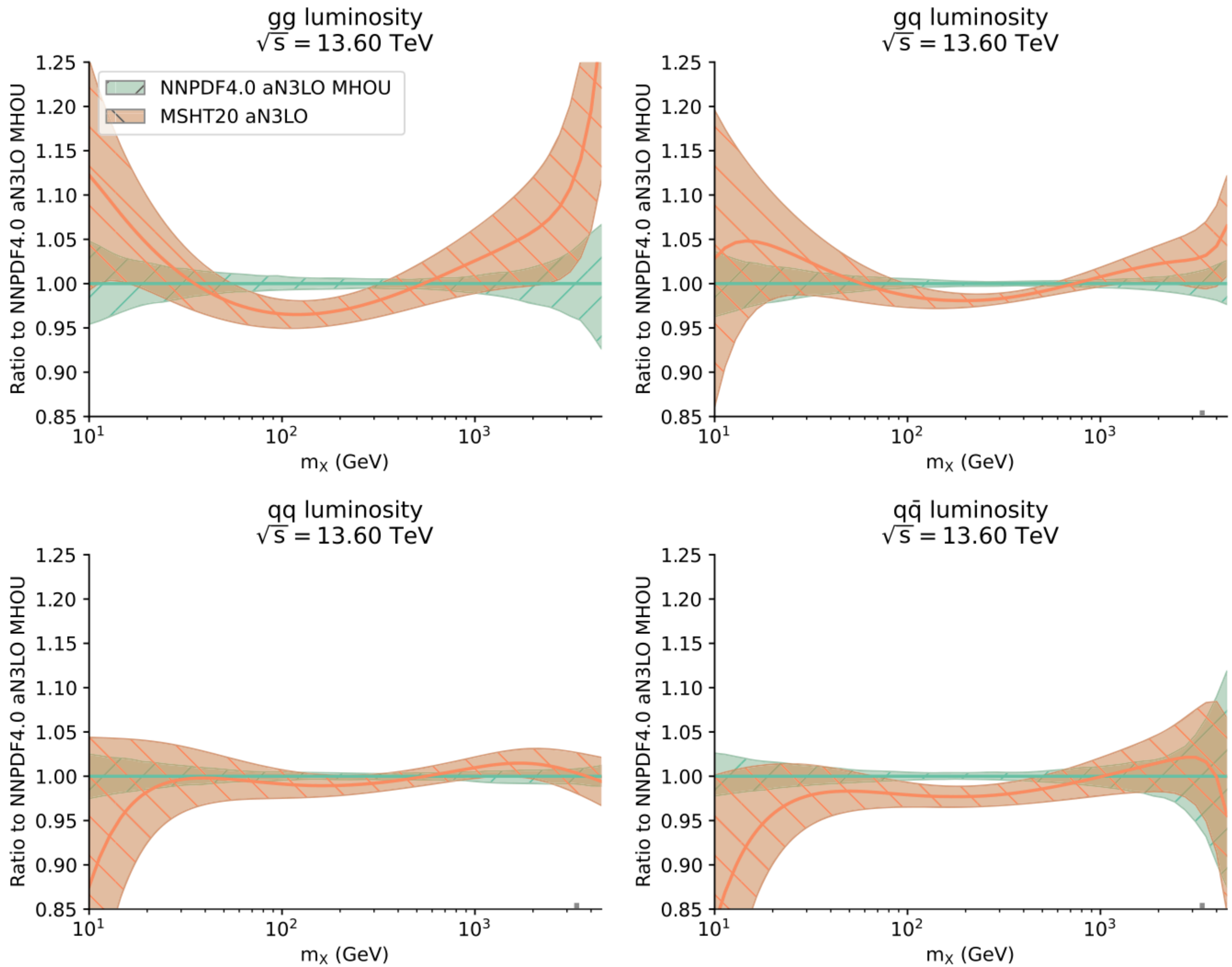


$$\sigma(S) = \sum_{i,j} \int d\tau \left[\frac{1}{S} \frac{dL_{ij}}{d\tau} \right] [\hat{s}\hat{\sigma_{ij}}]$$

$$\tau \frac{dL_{ij}}{d\tau} = \int_0^1 dx_1 dx_2 x_1 f_i(x_1, \mu_F^2) \times x_2 f_j(x_2, \mu_F^2) \delta(\tau - x_1 x_2)$$

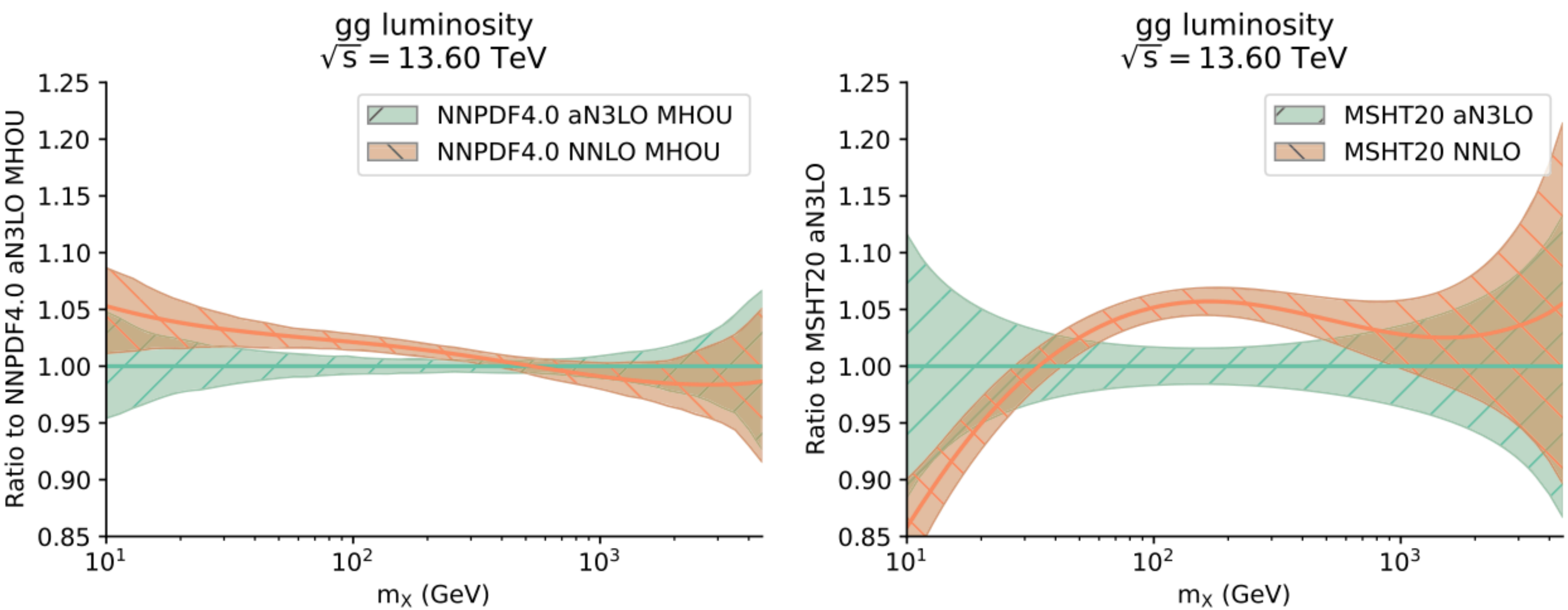
Comparison of luminosity

Example: comparison between two approximate N^{3}LO accurate luminosities



Comparison of luminosity

Example: for gg luminosity, compare to previous order



Intermediate summary

We have seen that

- In the QCD parton model, hadrons are treated as bound states of quasi-fee point-like quarks is very successful to explain DIS measurements
- In this model, the probability to find a parton with a given momentum fraction is given by the (scale independent) parton distribution function
- The model breaks down once one includes initial state radiation since collinear divergences do not cancel
- This leads to scale dependent parton distribution functions
- The dependence is governed by the DGLAP evolution equations
- QCD factorisation means that PDFs are universal and process-independent quantities: they can be measured in some process, at some scale, and use in a different process at a different scale
- PDFs are today determined by global fits to data. Many open questions in their determination, including the full assessment of theoretical uncertainties

Next: jets

Where do jets enter? Essentially everywhere at colliders

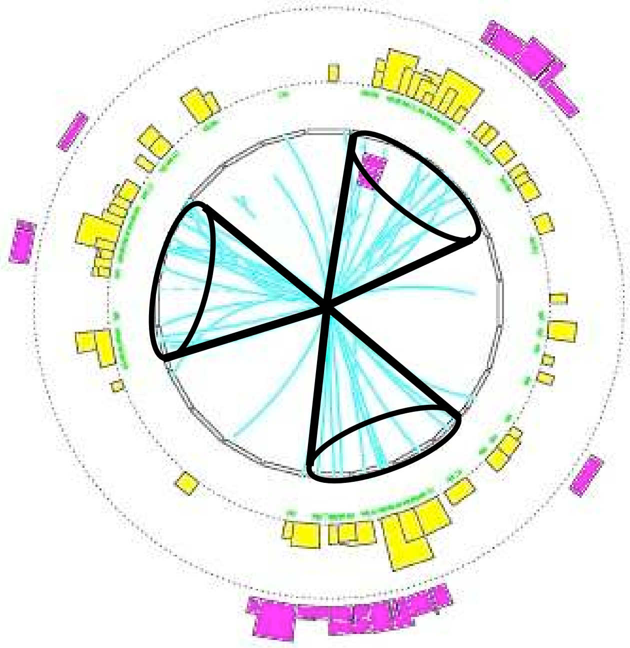
Jets are an essential tool for a variety of studies, e.g.

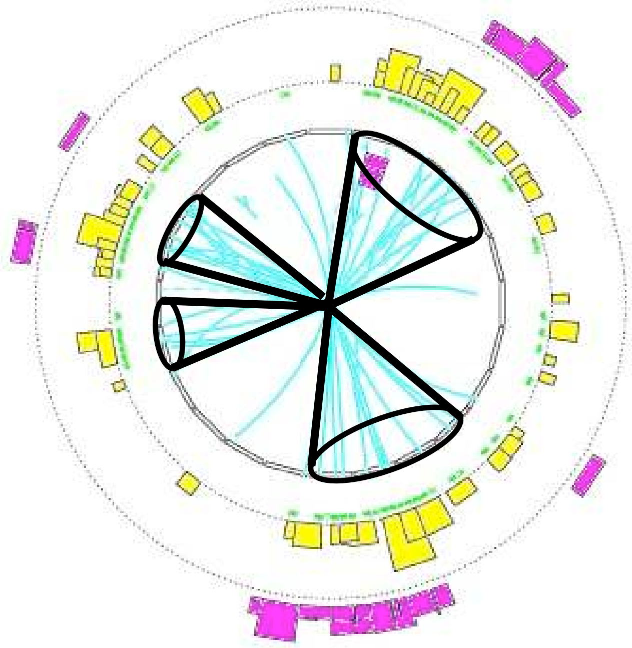
- top reconstruction
- mass measurements
- most Higgs and new physics searches
- are a general tool to attribute structure to an event
- instrumental for QCD studies, e.g. inclusive-jet measurements
  $\Rightarrow$ important input for PDF determinations

Jets

Jets provide a way of projecting away the multiparticle dynamics of an event ⇒ leave a simple quasi-partonic picture of the hard scattering

The projection is fundamentally ambiguous ⇒ jet physics is a rich subject



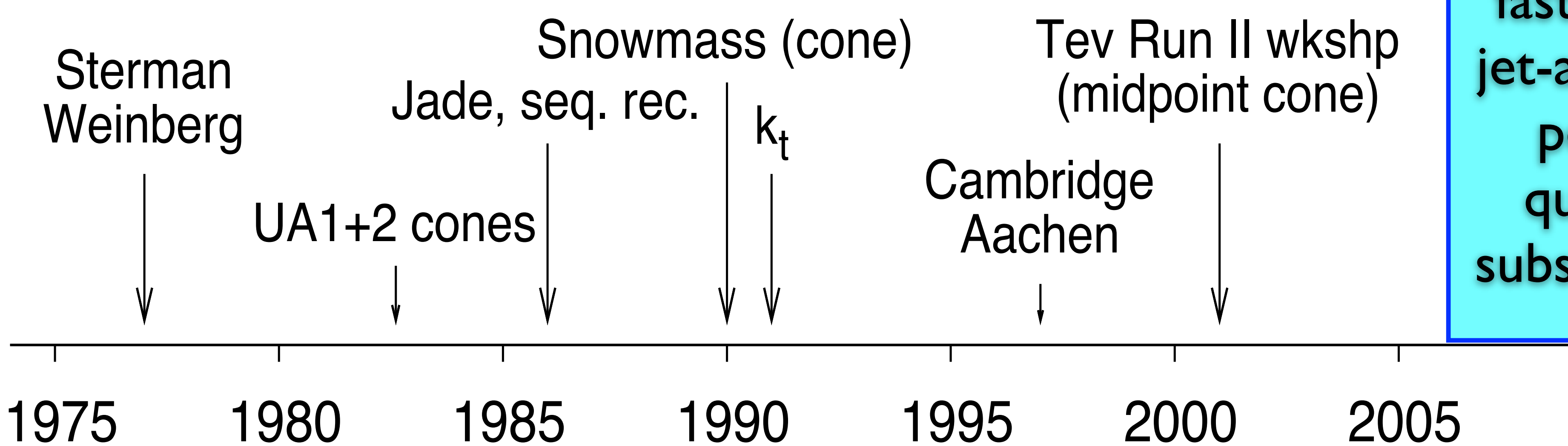


Ambiguities:

1) Which particles should belong to a same jet ?

2) How does recombine the particle momenta to give the jet-momentum?

Jet developments



fast-kt, SISCone, anti-kt, jet-areas, jet-flavour, non-perturbative effects, quality measures, jet-substructure, grooming ...

Two broad classes of jet algorithms

Today many extensions of the original Sterman-Weinberg jets.
Modern jet-algorithms divided into two broad classes

Jet algorithms

Cone type (UA1,JetCLU, Midpoint, SISCone..)	Sequential (kt-type, Jade, Cambridge/ Aachen...)
top down approach: cluster particles according to distance in coordinate-space	bottom up approach: cluster particles according to distance in momentum-space
Idea: put cones along dominant direction of energy flow	Idea: undo branchings occurred in the PT evolution

Inclusive k_t/Durham-algorithm

Catani et. al ’92-’93; Ellis&Soper ’93

<u>Inclusive algorithm:</u>

1. For any pair of final state particles i,j define the distance

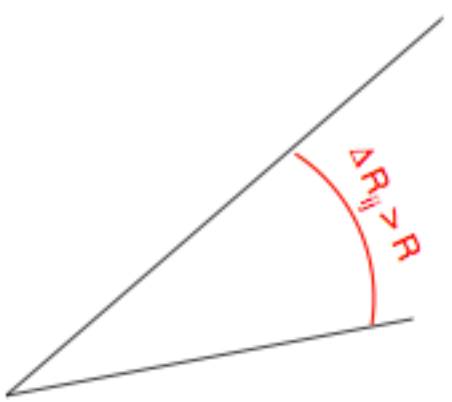
$$d_{ij} = \frac{\Delta y_{ij}^2 + \Delta\phi_{ij}^2}{R^2} \min\{k_{ti}^2, k_{tj}^2\}$$

2. For each particle i define a distance with respect to the beam

$$d_{iB} = k_{ti}^2$$

3. Find the smallest distance. If it is a d_{ij} recombine i and j into a new particle ($\Longrightarrow$ recombination scheme); if it is d_{iB} declare i to be a jet and remove it from the list of particles

   NB: if $\Delta R_{ij} \equiv \Delta y_{ij}^2 + \Delta\phi_{ij}^2 < R$ then partons (ij) are always recombined, so R sets the minimal interjet angle



4. repeat the procedure until no particles are left

Exclusive k_t/Durham-algorithm

Inclusive algorithm gives a variable number of jets per event, according to the specific event topology

Exclusive version: run the inclusive algorithm but stop when either

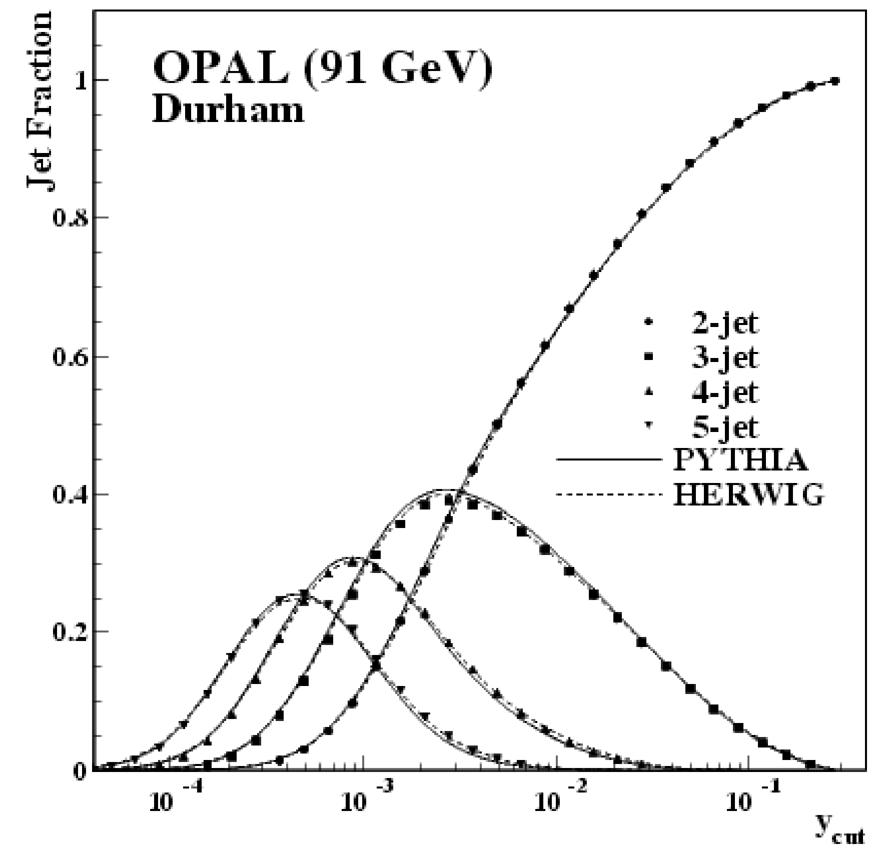
- all d_{ij}, $d_{iB} > d_{cut}$ or
- when reaching the desired number of jets n

k_t/Durham-algorithm in e^+e^-

k_t originally designed in e^+e^-, most widely used algorithm in e^+e^- (LEP)

$$y_{ij} = 2\min\{E_i^2, E_j^2\}\,(1-\cos\theta_{ij}^2)$$

- can classify events using y_{23}, y_{34}, y_{45}, y_{56} ...
- resolution parameter related to minimum transverse momentum between jets



Satisfies fundamental requirements:

1. Collinear safe: collinear particles recombine early on
2. Infrared safe: soft particles do not influence the clustering sequence

$\Rightarrow$ collinear + infrared safety important: it means that cross-sections can be computed at higher order in pQCD (no divergences)!

The CA and the anti-k_t algorithm

The Cambridge/Aachen: sequential algorithm like k_t, but uses only angular properties to define the distance parameters

$$d_{ij} = \frac{\Delta R_{ij}^2}{R^2} \qquad d_{iB} = 1 \qquad \Delta R_{ij}^2 = (\phi_i - \phi_j)^2 + (y_i - y_j)^2$$

Dotshitzer et. al '97; Wobisch & Wengler '99

The anti-kt algorithm: designed not to recombine soft particles together

$$d_{ij} = \min\{1/k_{ti}^2, 1/k_{tj}^2\} \Delta R_{ij}^2 / R^2 \qquad d_{iB} = 1/k_{ti}^2$$

Cacciari, Salam, Soyez '08