QCD

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The CA and the anti-k_t algorithm

<u>The Cambridge/Aachen</u>: sequential algorithm like k_t, but uses only angular properties to define the distance parameters

$$d_{ij} = \frac{\Delta R_{ij}^2}{R^2} \qquad d_{iB} = 1 \qquad \Delta R_{ij}^2 = (\phi_i - \phi_j)^2 + (y_i - y_j)^2$$

Dotshitzer et. al '97; Wobisch & Wengler '99

Cambridge/Aachem algorithm ignores the soft divergence, and privileges the collinear one

<u>The anti-kt algorithm</u>: designed not to recombine soft particles together

$$d_{ij} = \min\{1/k_{ti}^2, 1/k_{tj}^2\}\Delta R_{ij}^2/R^2 \qquad d_{iB} = 1/k_{ti}^2$$

Cacciari, Salam, Soyez '08

The anti-k_t algorithm disfavours the soft divergence, and privileges the collinear one

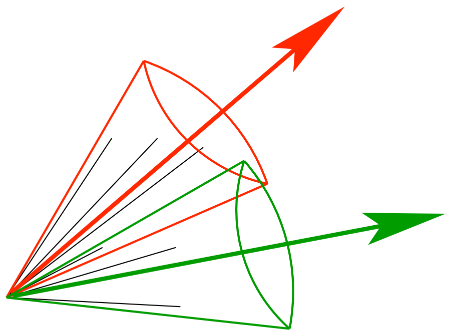
Cone algorithms

1. A particle i at rapidity and azimuthal angle (y_i, Φ_i) $\subset$ cone C iff

$$\sqrt{(y_i - y_C)^2 + (\phi_i - \phi_C)^2} \le R_{\text{cone}}$$

2. Define

$$\bar{y}_C \equiv \frac{\sum_{i\in C} y_i \cdot p_{T,i}}{\sum_{i\in C} p_{T,i}} \qquad \bar{\phi}_C \equiv \frac{\sum_{i\in C} \phi_i \cdot p_{T,i}}{\sum_{i\in C} p_{T,i}}$$



3. If weighted and geometrical averages coincide $(y_C, \phi_C) = (\bar{y}_C, \bar{\phi}_C)$ a stable cone (⇒ jet) is found, otherwise set $(y_C, \phi_C) = (\bar{y}_C, \bar{\phi}_C)$ & iterate

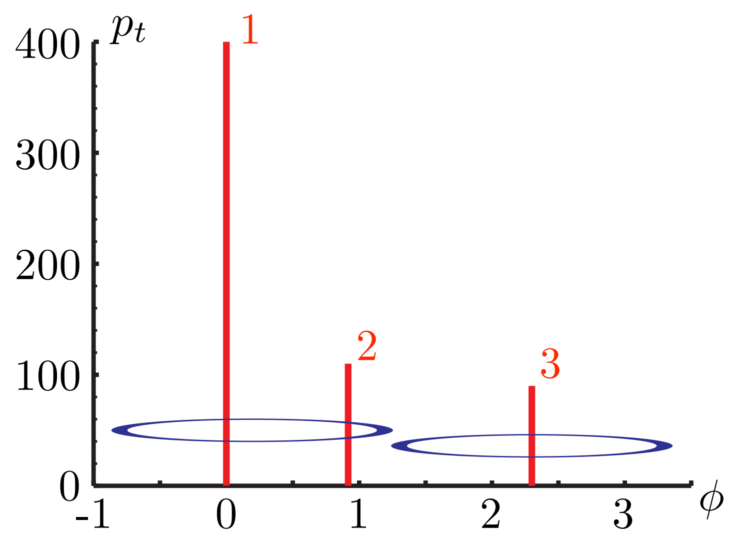
4. Stable cones can overlap. Run a split-merge on overlapping jets: merge jets if they share more than an energy fraction f, else split them and assign the shared particles to the cone whose axis they are closer to.
Remark: too small f (<0.5) creates very large jets, not recommended

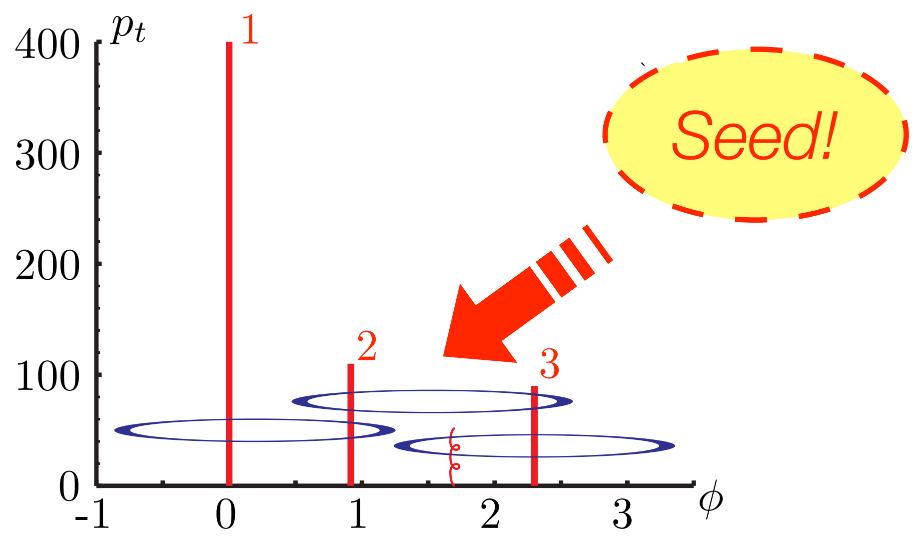
Cone algorithms

- The question is where does one start looking for stable cone ?
- The direction of these trial cones are called seeds
- Ideally, place seeds everywhere, so as not to miss any stable cone
- Practically, this is unfeasible. Speed of recombination grows fast with the number of seeds. So place only some seeds, e.g. at the (ϕ, y)-location of particles.

Seeds make cone algorithms infrared unsafe

Jets: infrared unsafety of cones





3 hard ⇒ 2 stable cones   3 hard + 1 soft ⇒ 3 stable cones

Soft emission changes the hard jets ⇒ algorithm is IR unsafe

<u>Midpoint algorithm:</u> take as seed position of emissions and midpoint between two emissions (postpones the infrared safety problem)

Seedless cones

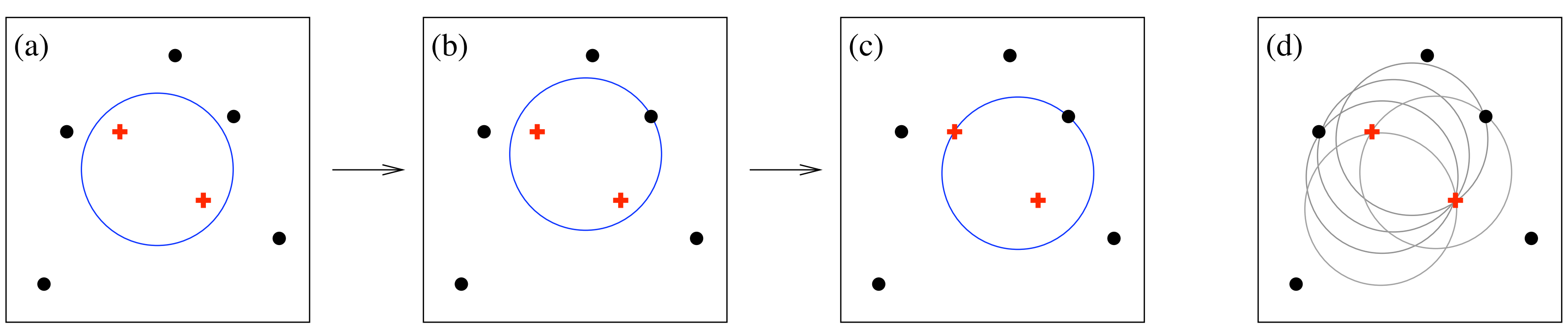
Solution:
use a seedless algorithm, i.e. consider all possible combinations of particles as candidate cones, so find all stable cones [⇒ jets]

Blazey '00

The problem:
clustering time growth as $N2^N$. So for an event with 100 particles need 10^{17} years to cluster the event ⇒ prohibitive beyond fixed order (N=4,5)

Better solution:

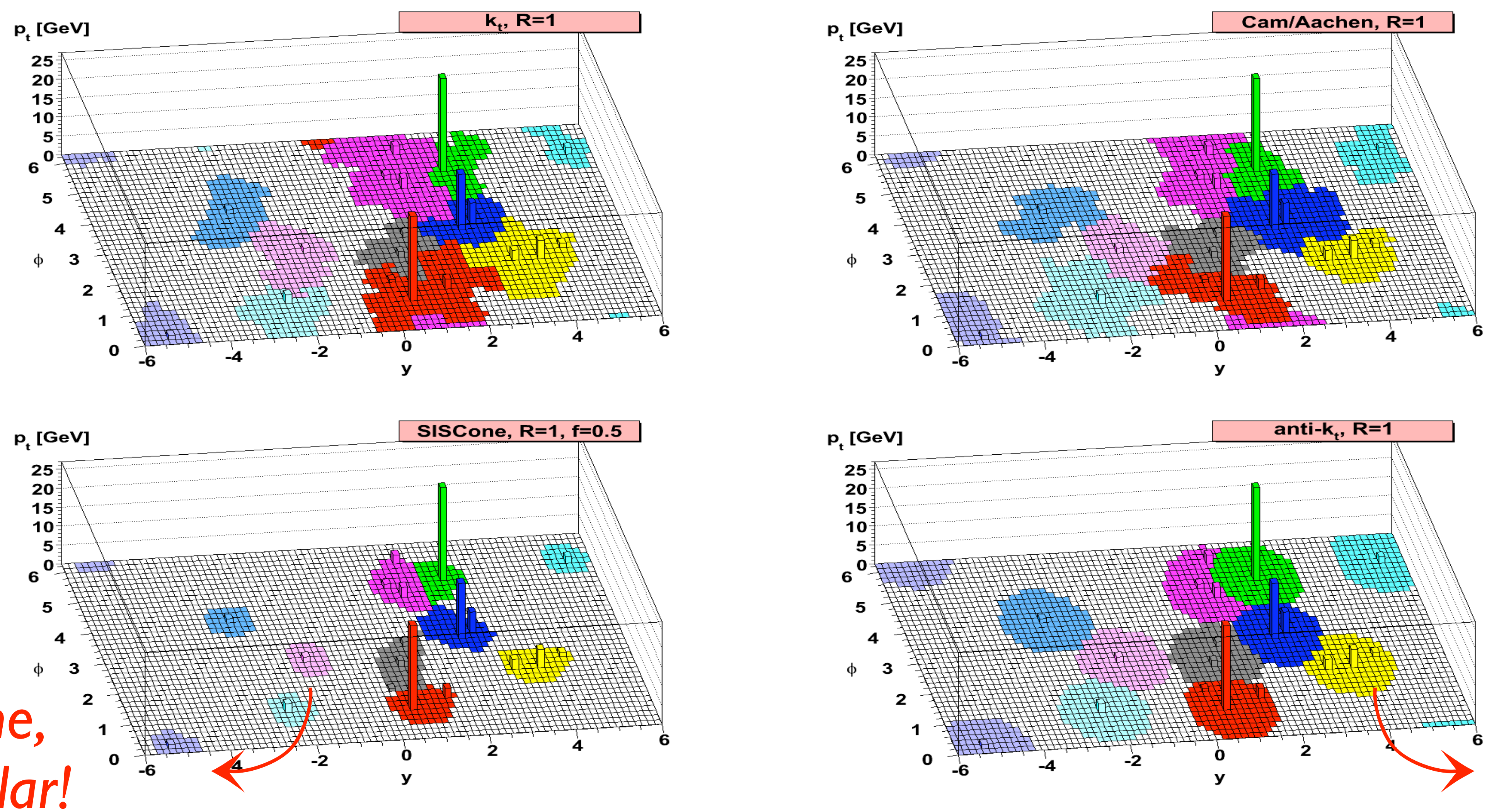
SISCone recasts the problem as a computational geometry problem, the identification of all distinct circular enclosures for points in 2D and finds a solution to that ⇒ $N^2 \ln N$ time IR safe algorithm



Salam, Soyez '07

Jet area

Jet area: Given a fast, IRC-safe jet algorithm, define the jet area by filling the event with infinitely many infinitely soft "ghost" particles uniformly distributed in (ϕ, y).The jet area is proportional to the number of ghosts clustered into the jet.



NB: cone, not circular!

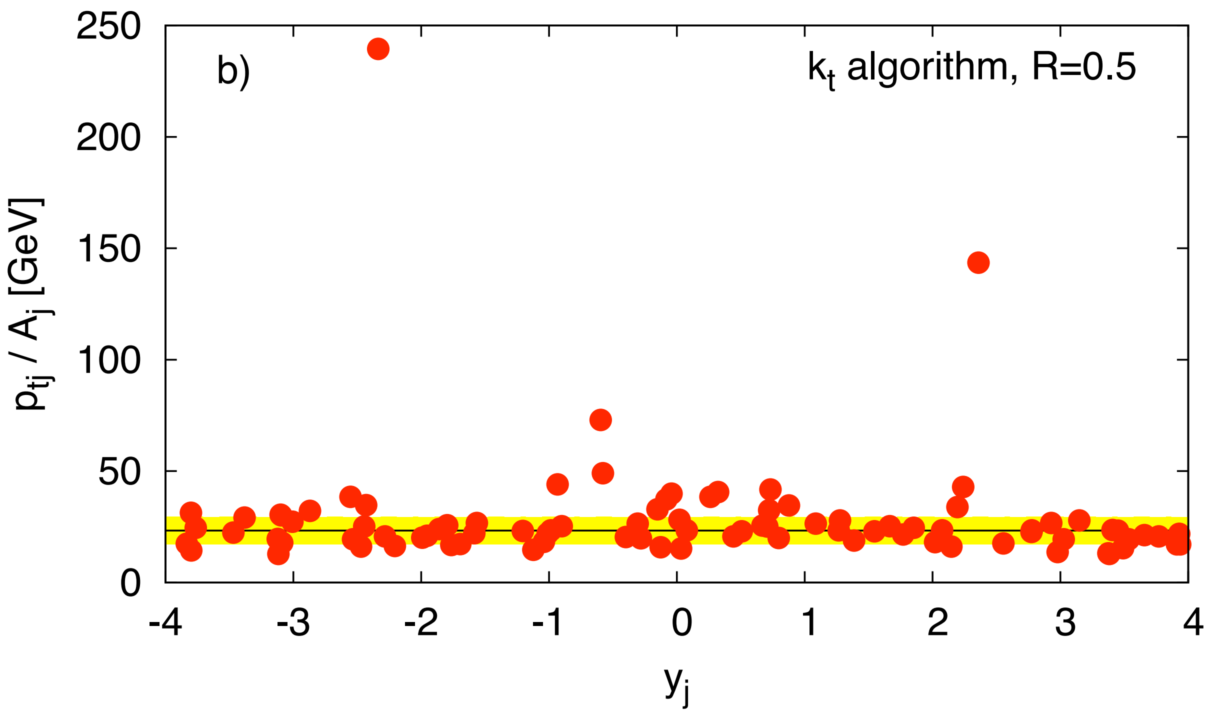
NB: anti-kt

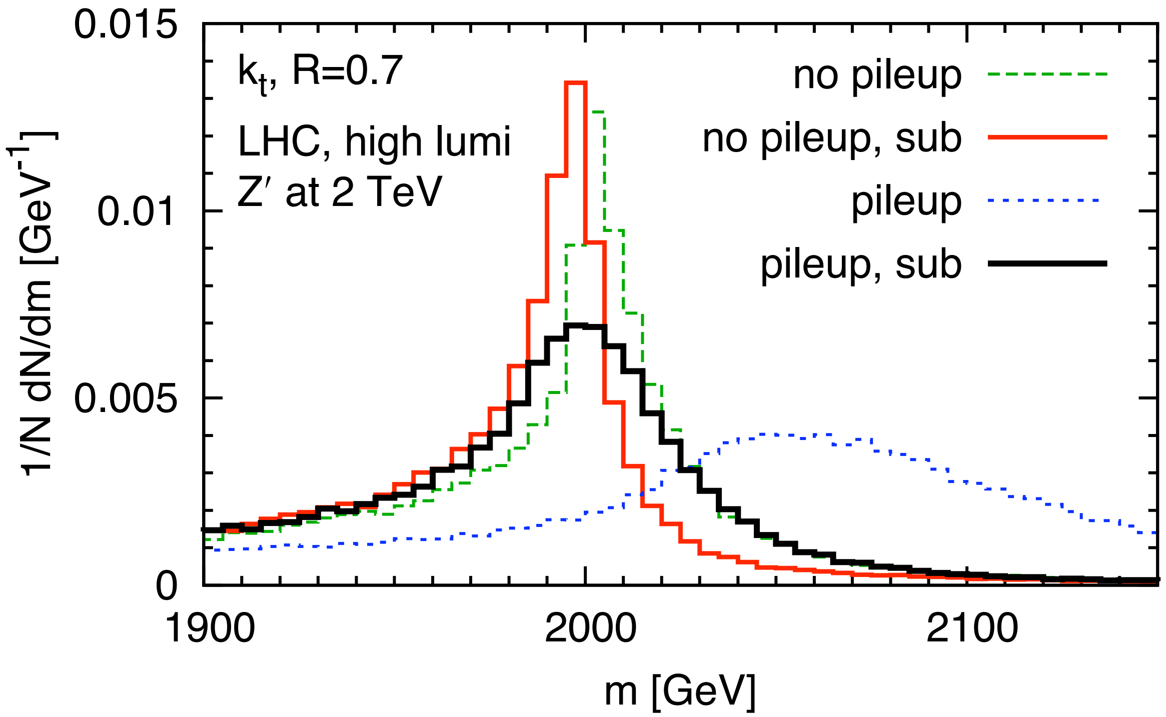
What jet areas are good for

jet-area ≡ catching area of the jet when adding soft emissions

⇒ simple area based subtraction for a variety of algorithms

Get $\rho = \frac{p_{t,j}}{A_j}$ from the majority of (pile-up) jets, define $p_j^{\rm sub} = p_j - A_j \rho$





Pileup = generic p-p interaction (hard, soft, single-diffractive...) overlapping with hard scattering

Quality measures of jets

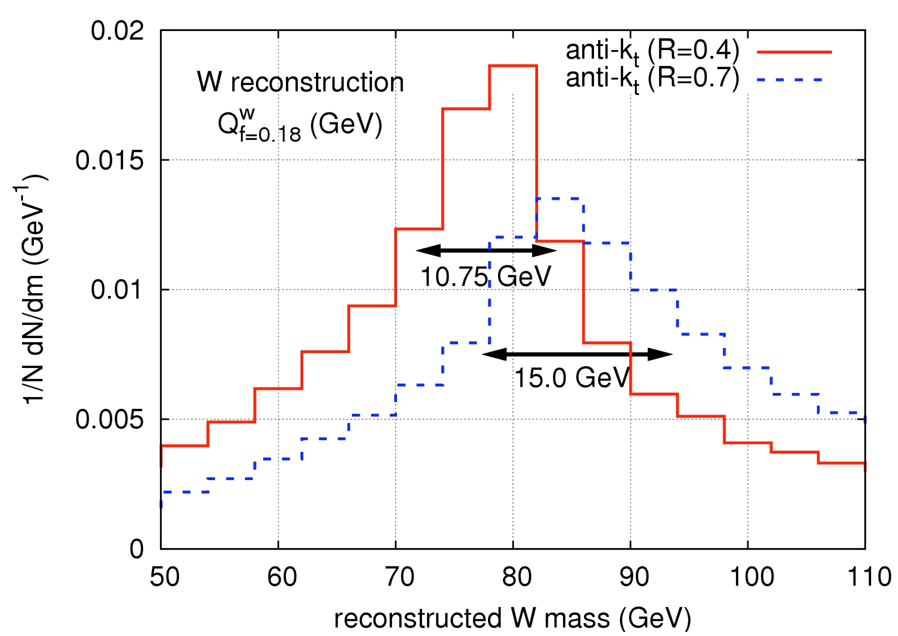
Suppose you are searching for a heavy state (H→gg, Z'→qq, ...)

The object is reconstructed through its decay products
⇒ *Which jet algorithm (JA) is best ? Does the choice of R matter?*

Define: Q^w_f(JA, R) ≡ width of the smallest mass window that contains a fraction f of the generated massive objects

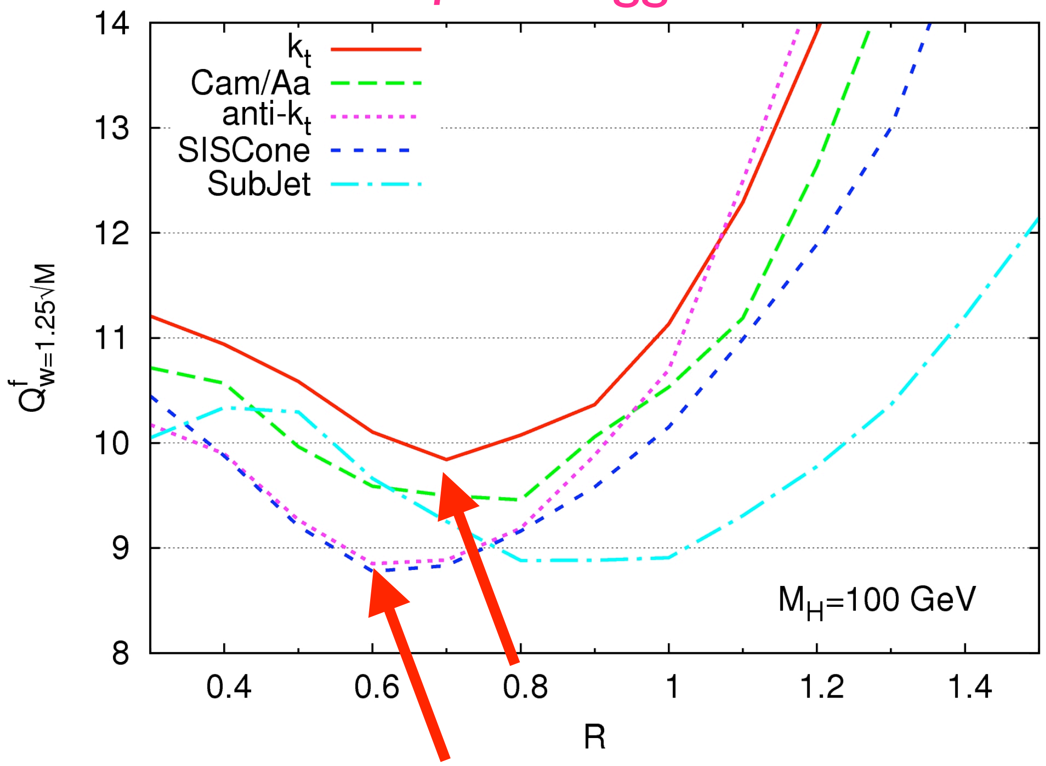
- good algo ⇔ small Q^w_f(JA, R)
- ratios of Q^w_f(JA,R): mapped to ratios of effective luminosity (with same $S/\sqrt{B}$)

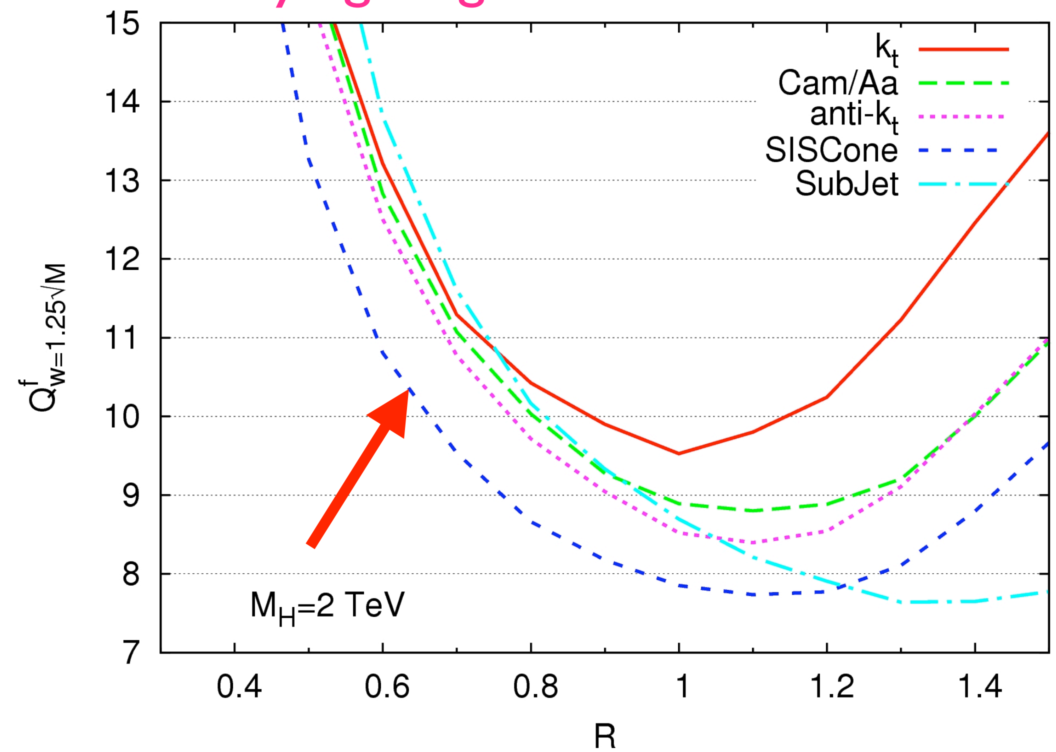
$$\mathcal{L}_2 = \rho_{\mathcal{L}} \mathcal{L}_1 \qquad \rho_{\mathcal{L}} = \frac{Q_z^f(JA_2, R_2)}{Q_z^f(JA_1, R_1)}$$



Quality measures: sample results

NB: Here "fake Higgs" = narrow resonance decaying to gluons

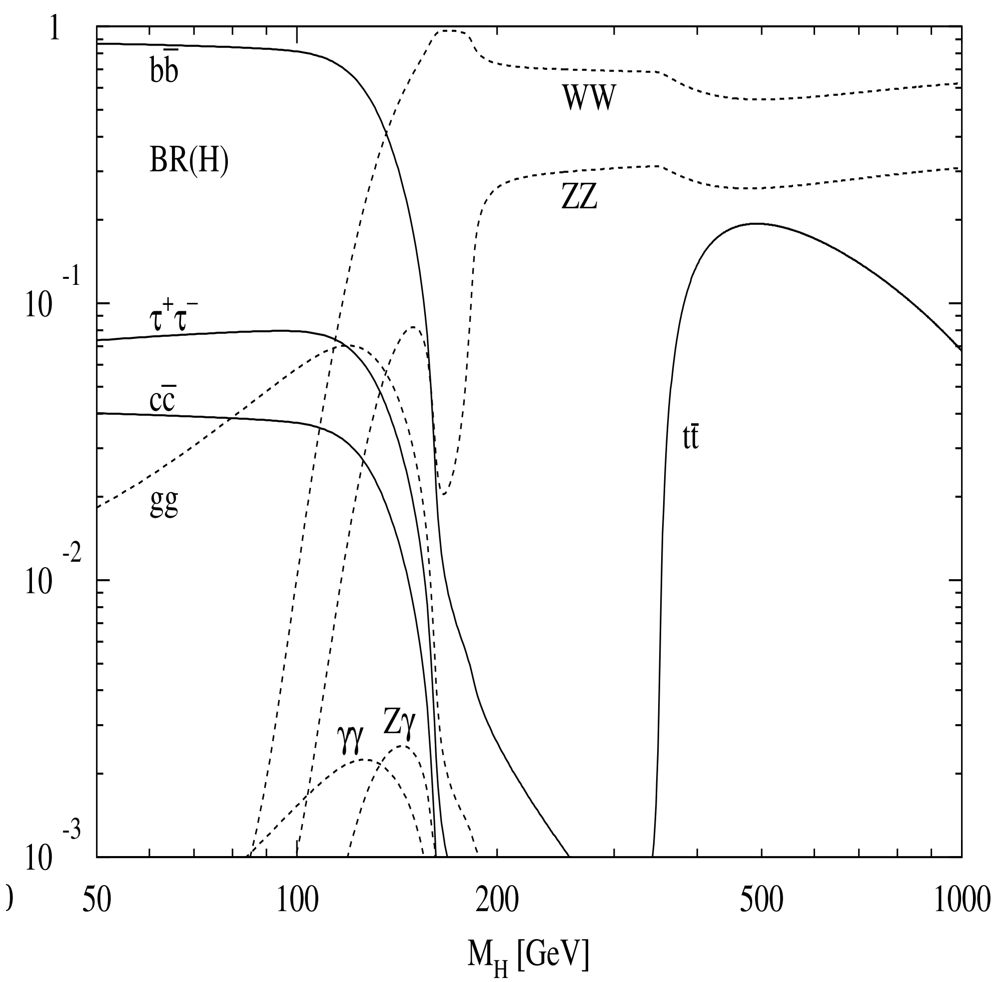


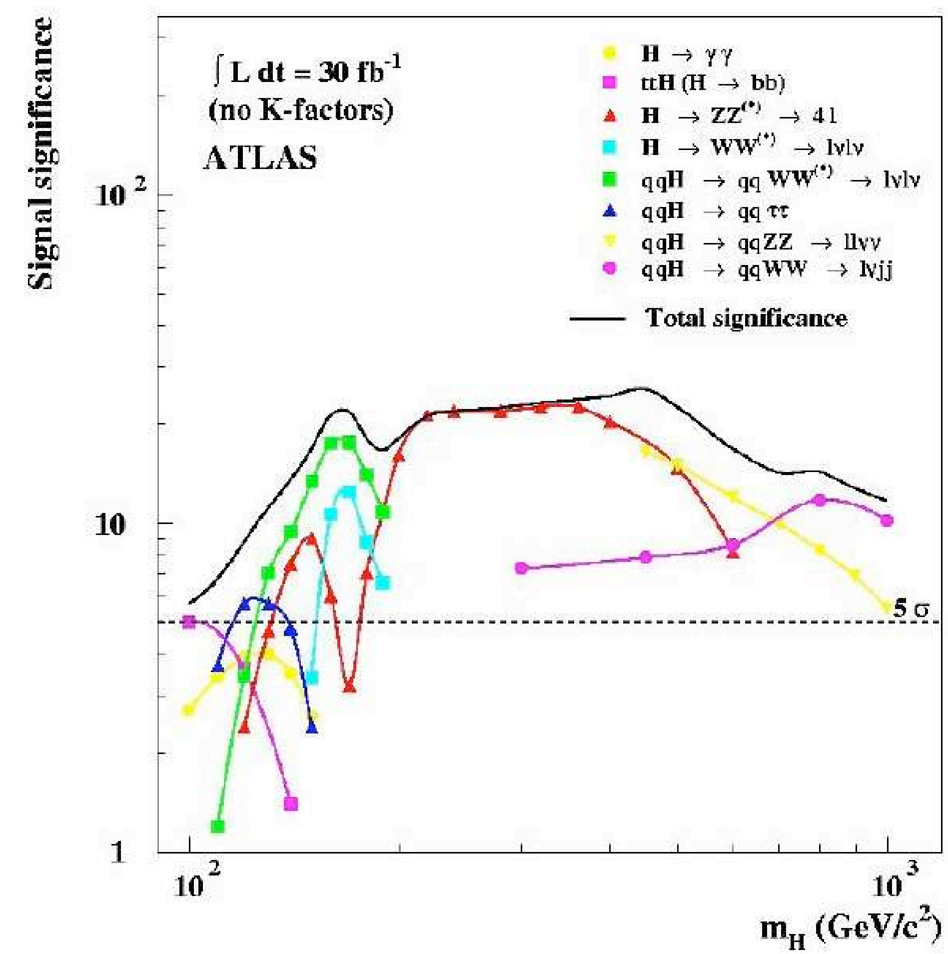


- At 100GeV: use a Tevatron standard algo (k_t, R=0.7) instead of best choice (SISCone, R=0.6 $\Rightarrow$ lose $\rho_{\mathcal{L}} = 0.8$ in effective luminosity
- At M_H=2 TeV: use M_H=100GeV best choice again loose in effective luminosity

A good choice of jet-algorithm does matter!
Bad choice of algorithm $\Leftrightarrow$ lost in discrimination power!

Jet substructure: Z/W+ H (→bb)





⇒ Light Higgs hard: promising channel associated production with mainly produced in association with Z/W, with decay H→bb is dominant, but overwhelmed by QCD backgrounds

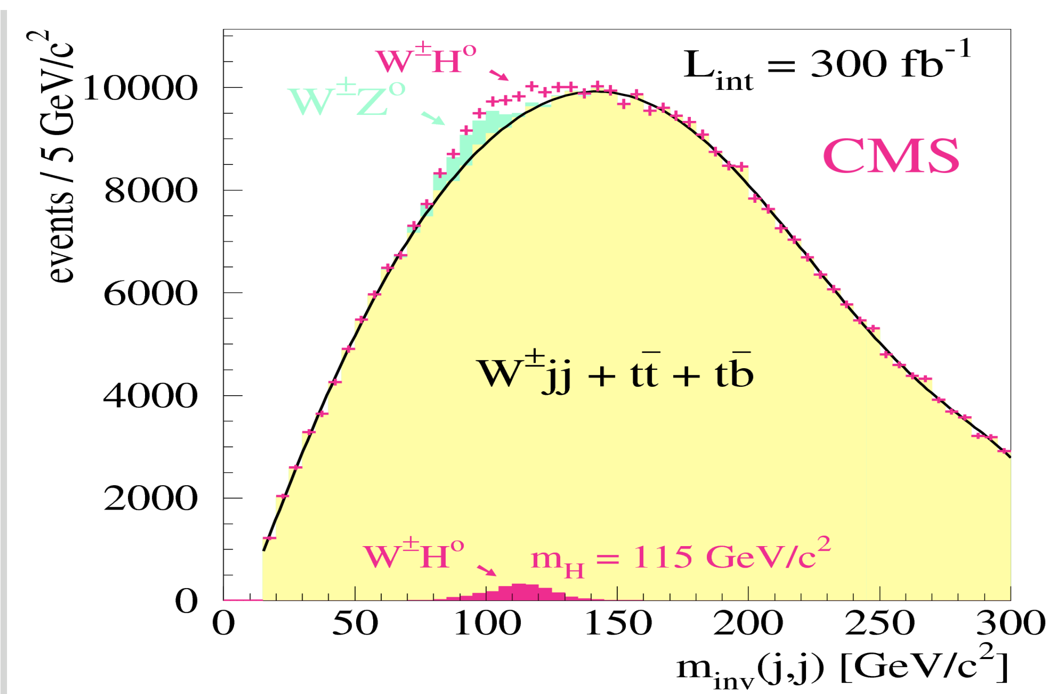
Z/W+ H (→bb)

Recall why searching for pp →WH(bb) is hard:

$\sigma(\mathrm{pp} \to WH(bb)) \sim \text{few pb}$ $\quad \sigma(\mathrm{pp} \to Wbb) \sim \text{few pb}$

$\sigma(\mathrm{pp} \to tt) \sim 800\text{pb}$ $\quad \sigma(\mathrm{pp} \to Wjj) \sim \text{few } 10^4\text{pb}$ $\quad \sigma(\mathrm{pp} \to bb) \sim 400\text{pb}$

⇒ signal extraction very difficult



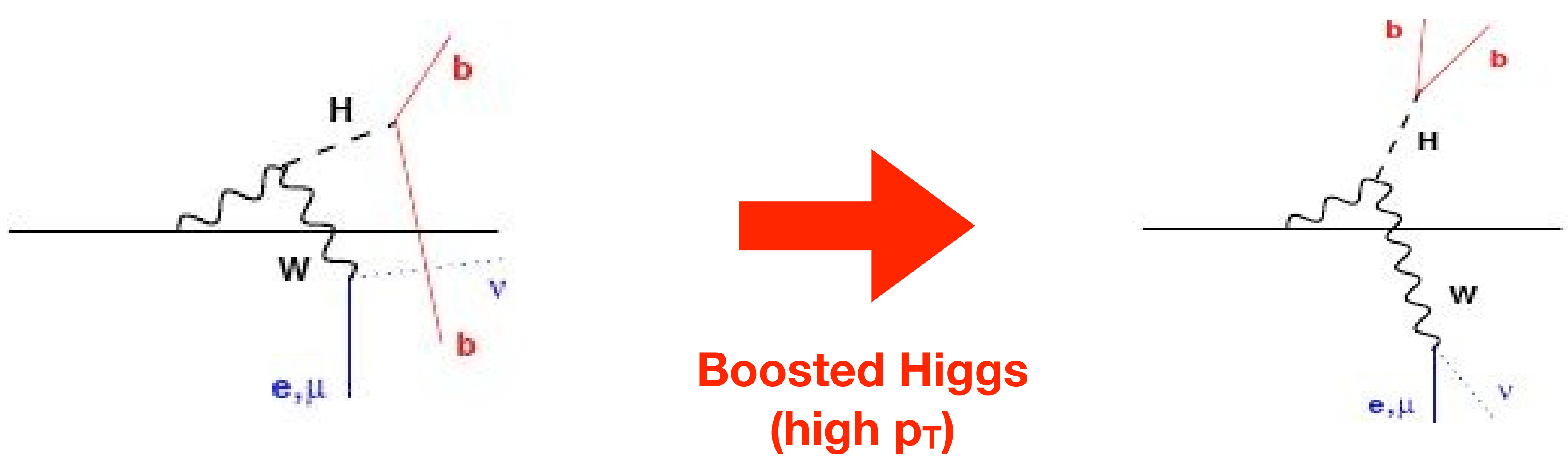
__Conclusion [ATLAS TDR]:__
The extraction of a signal from H → bb decays in the WH channel will be very difficult at the LHC even under the most optimistic assumptions [...]

Z/W+ H (→bb) rescued

But ingenious suggestions open up to window of opportunity

Central idea: require high-p_T W and Higgs boson in the event

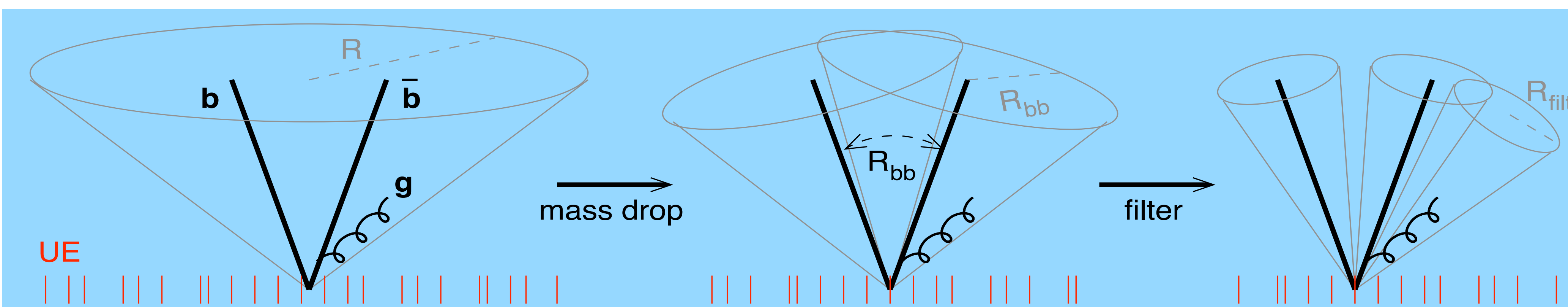
- leads to back-to-back events where two b-quarks are contained within the same jet
- high p_T reduces the signal but reduces the background much more
- improve acceptance and kinematic resolution



Z/W+ H (→bb) rescued ?

Then use a jet-algorithm geared to exploit the specific pattern of H → bb versus g → gg, q → gg

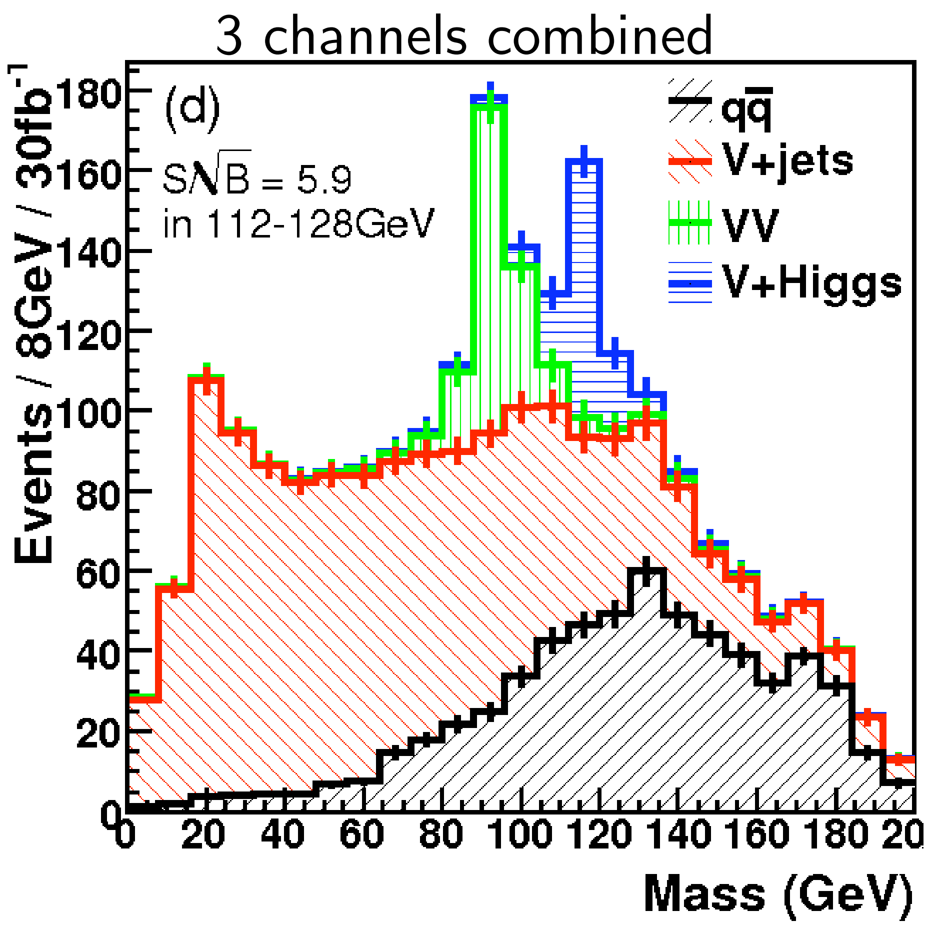
- QCD partons prefer soft emissions (hard → hard + soft)
- Higgs decay prefers symmetric splitting
- try to beat down contamination from underlying event
- try to capture most of the perturbative QCD radiation



1. cluster the event with e.g. CA algo and large-ish R
2. undo last recomb: large mass drop + symmetric + b tags
3. filter away the UE: take only the 3 hardest sub-jets

Z/W+ H (→bb) rescued ?

Mass of the three hardest sub-jets:



- with common & channel specific cuts: p_{tV}, p_{tH} > 200GeV , ...
- real/fake b-tag rate: 0.7/0.01
- NB: very neat peak for WZ (Z →bb) Important for calibration

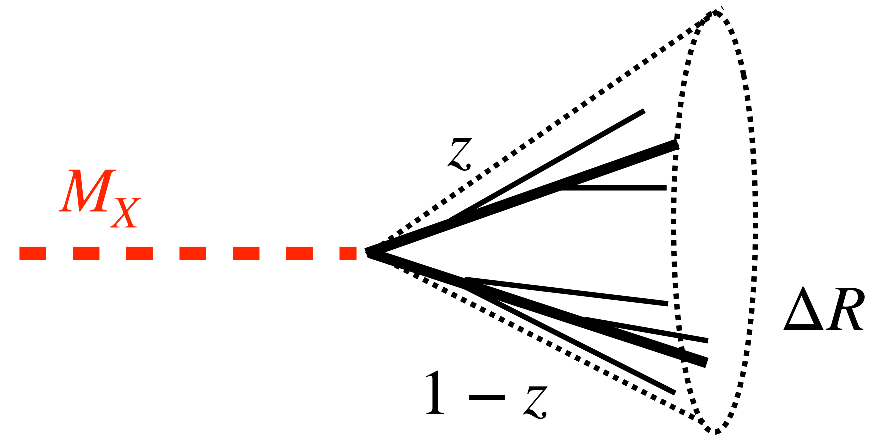
Butterworth, Davison, Rubin, Salam '08

This and other works opened a new field of jet-substructure…
(would be a whole new lecture)

Boosted mass reconstruction

Take an object decaying into a two-prong event

$$M_X^2 \sim E_X^2 \, z(1-z) \; (1-\cos\theta)$$
$$\sim p_{t,X}^2 \, z(1-z) \Delta R^2$$
$$\sim \frac{1}{4} p_{t,X}^2 \, \Delta R^2$$



The two decay products end up in a single jet if $\Delta R < R$

Which means

$$\boxed{p_{t,X} > \frac{2M_X}{R}}$$

Boosted mass reconstruction

<u>Question</u>: what jet radius should one chose so that the hadronic decay products of a boosted W boson with $p_{t,W} \sim 300\,\mathrm{GeV}$ are reconstructed in the same jet?

<u>Answer</u>:

$$R > \frac{2M_X}{p_{t,X}} \sim \frac{160\,\mathrm{GeV}}{300\,\mathrm{GeV}} \sim 0.5$$

<u>Lesson</u>:

The heavier the resonance, the larger the jet-radius required.

The more boosted the resonance, the smaller the jet-radius required.

Tagging and grooming

Widely used terms in the field of jet-substructure, but no precise or universal definition of these terms

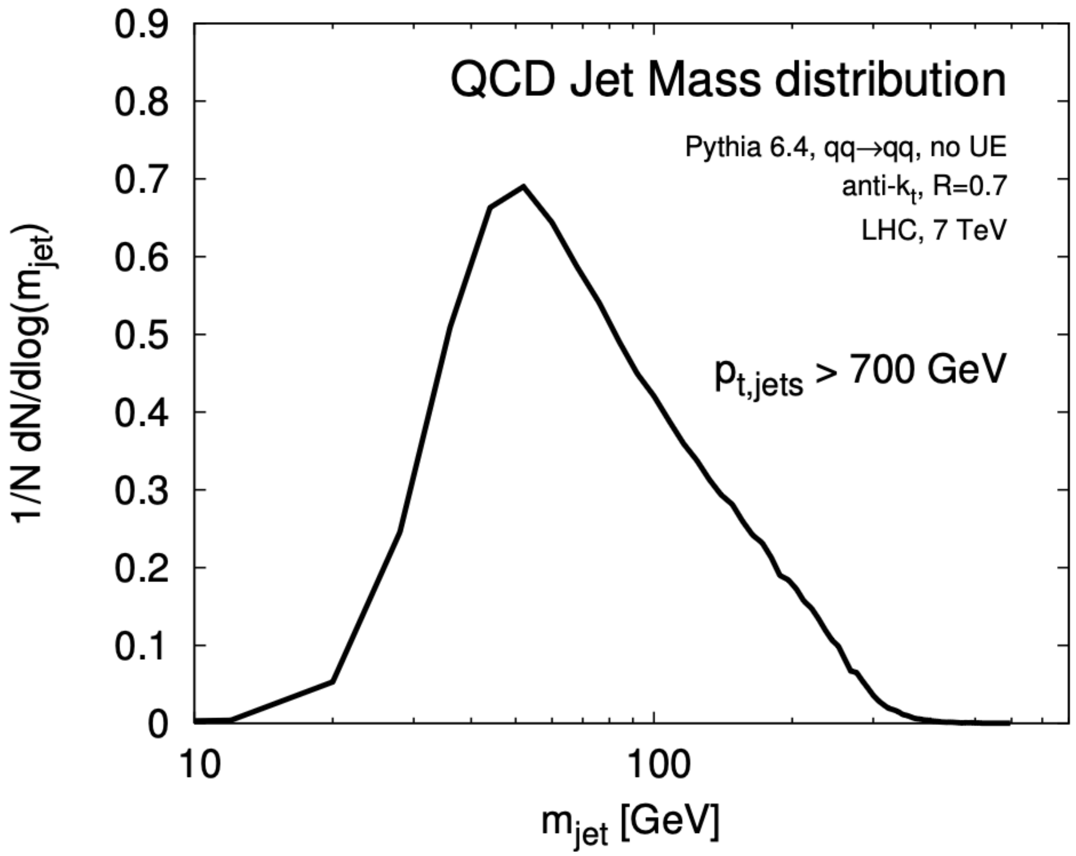
Tagging: aims at reducing the background while keeping most of the signal. Relies on the underlying radiation pattern

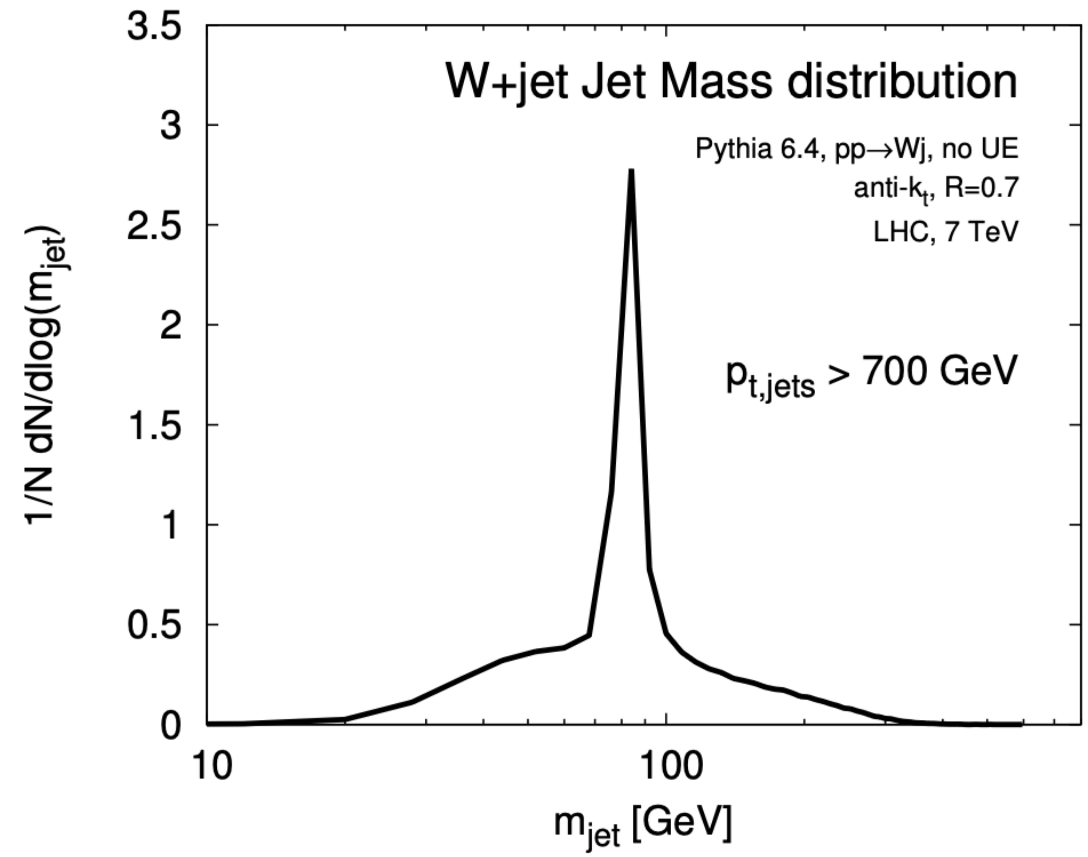
Grooming: aims at improving mass resolution (removing underlying event) without affecting considerably signal or backgrounds

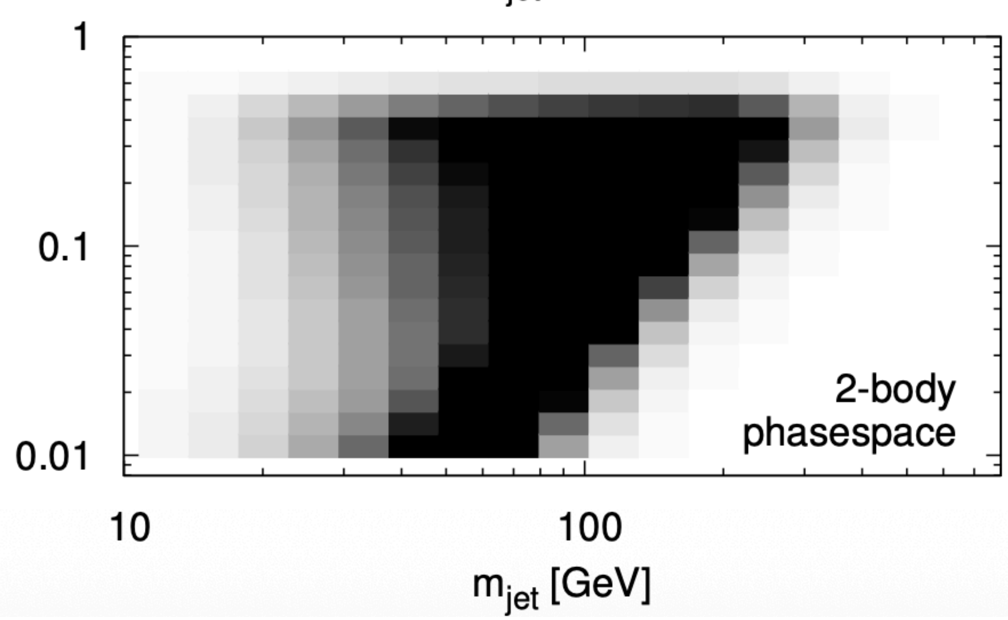
One core idea of tagging: relies on the fact that $W \to q\bar{q}$ splitting is symmetry, while $g \to gg$ or $q \to qg$ splitting are very asymmetric in energy fractions (gluons wants to have small z)

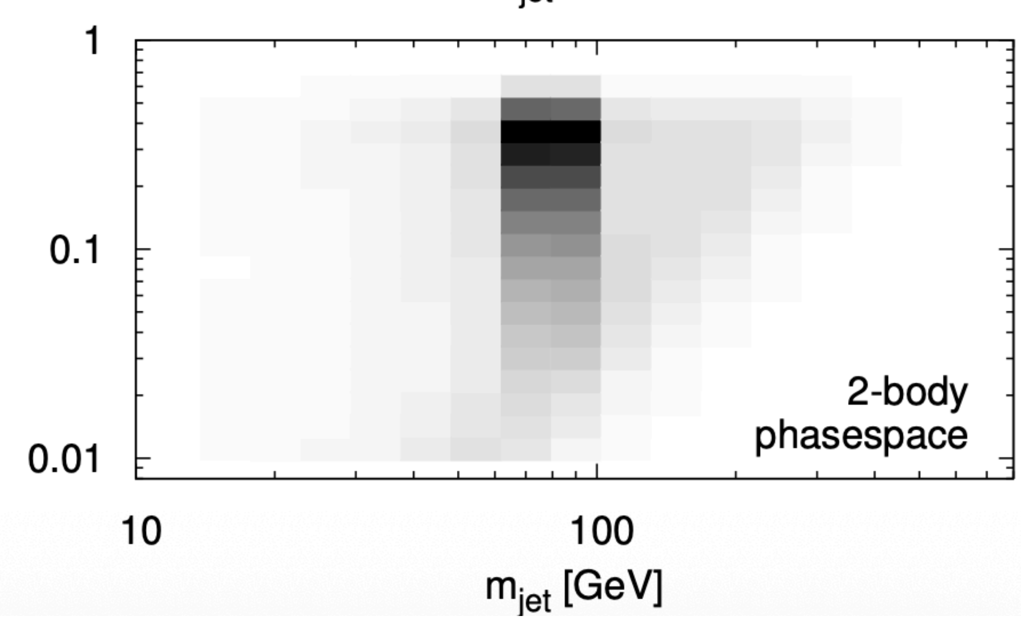
Example: aim is to reconstruct a hadronically decaying W boson, while reducing the QCD background

Tagging and grooming

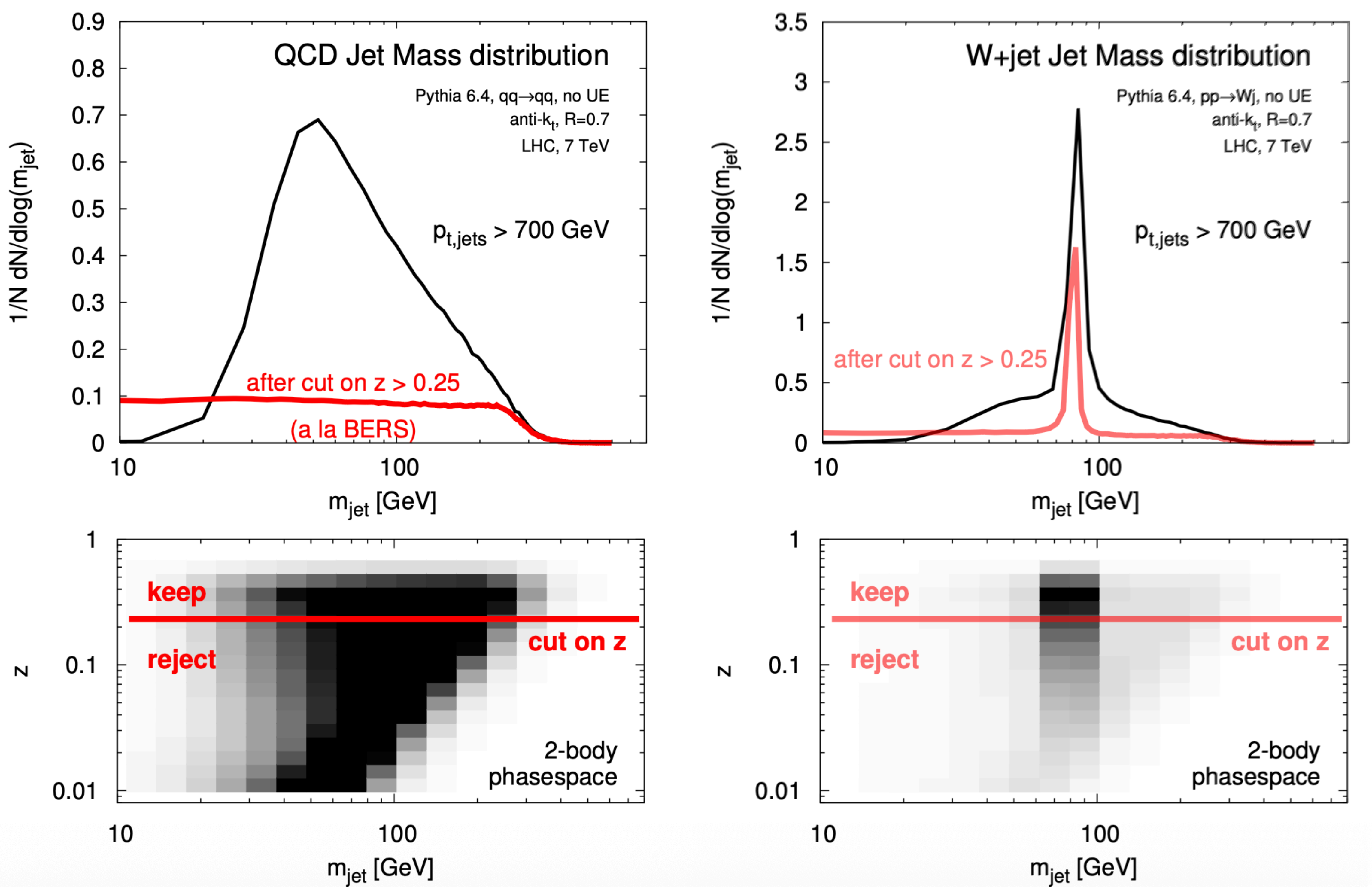




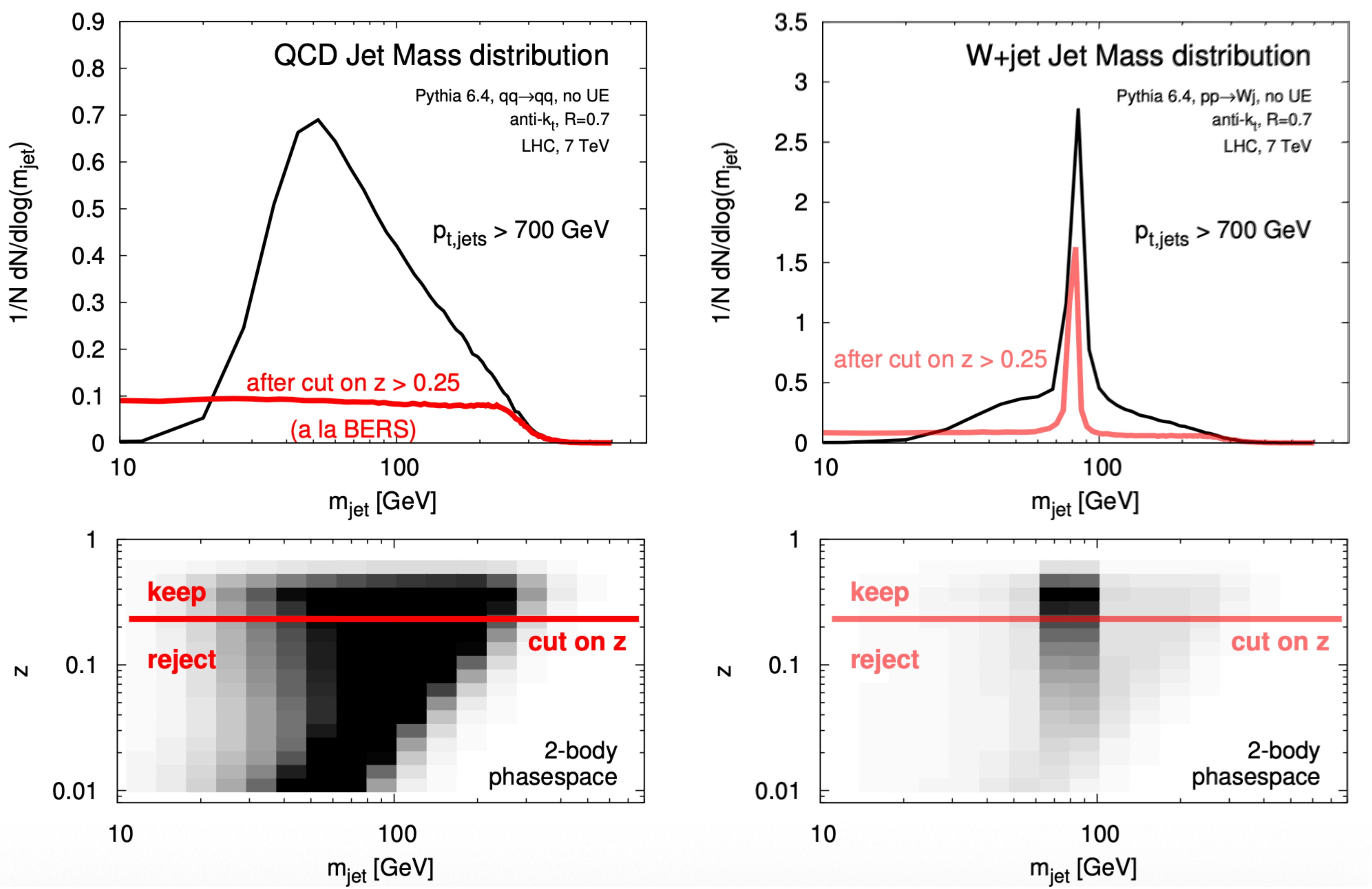




Tagging and grooming

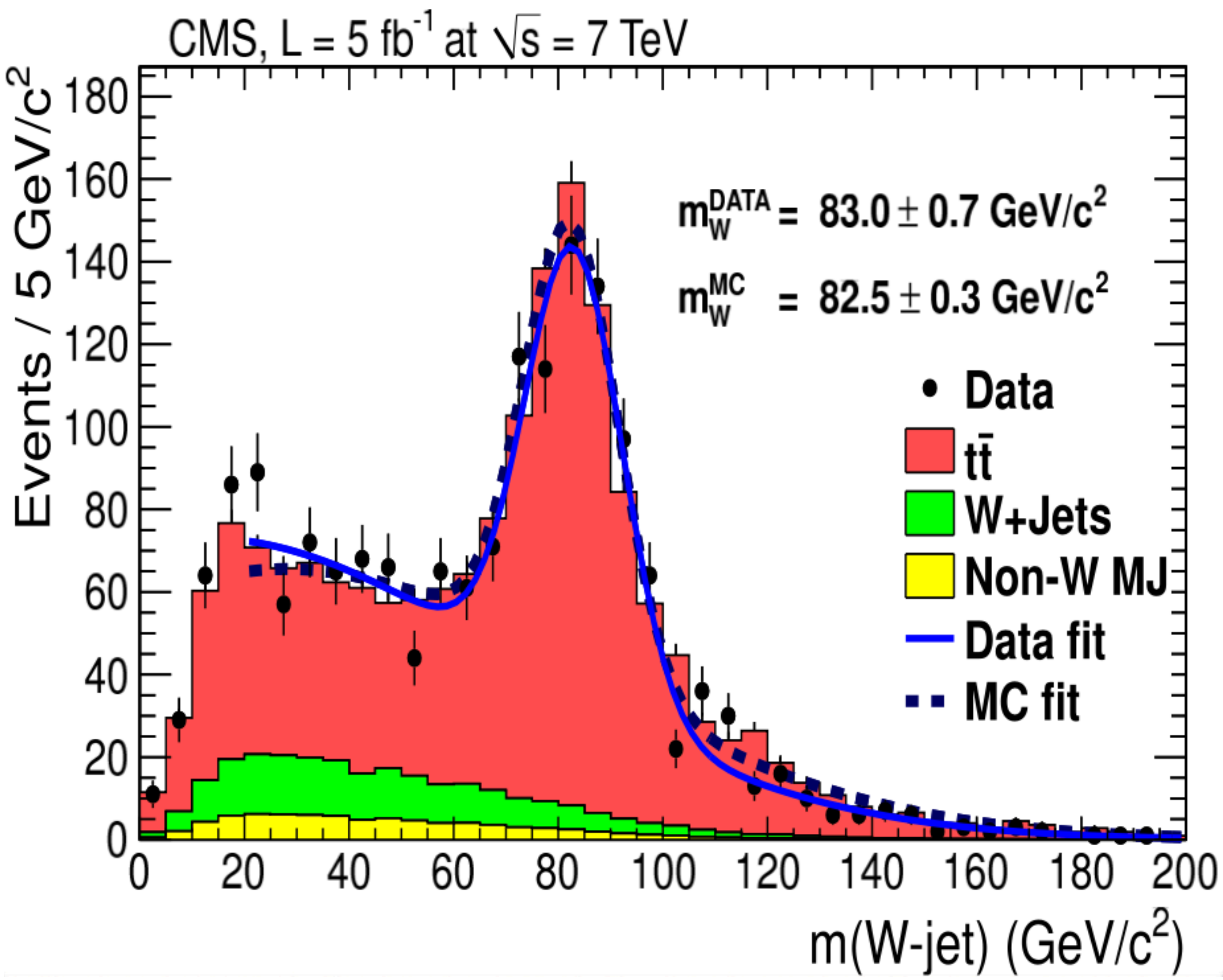


Tagging and grooming

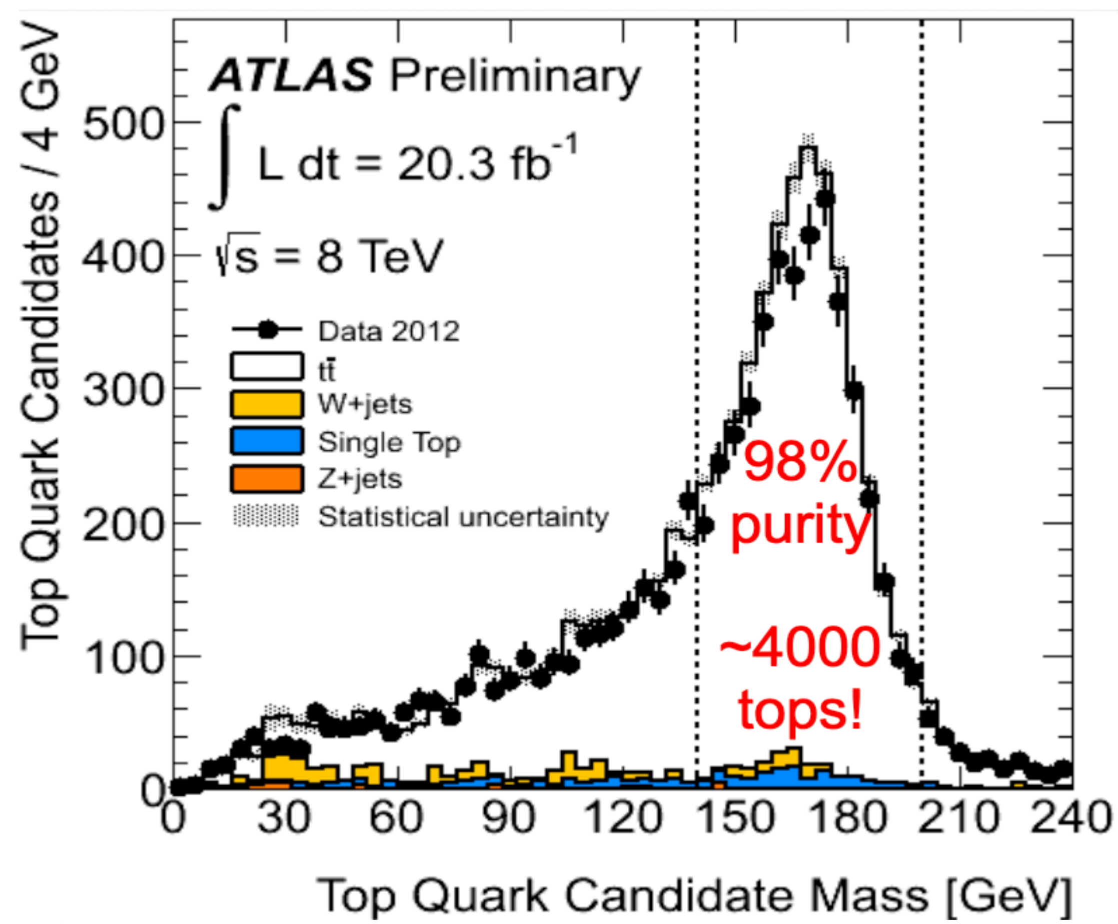


W and top in a single jet

W-boson reconstructed in a single jet by CMS



Top-quark decay reconstructed in a single jet by ATLAS



Jet flavour

Where does jet flavour enter?

- Top reconstruction (top mass)
- Instrumental for QCD studies, e.g. inclusive b-jet (⇒ b-PDF)
- Z + charm-jet (⇒ charm PDF)
- W +charm-jet (⇒ strange PDF)
- Higgs to bottom (⇒ di-Higgs studies)
- Jet-substructure (mass reconstructions)
- …

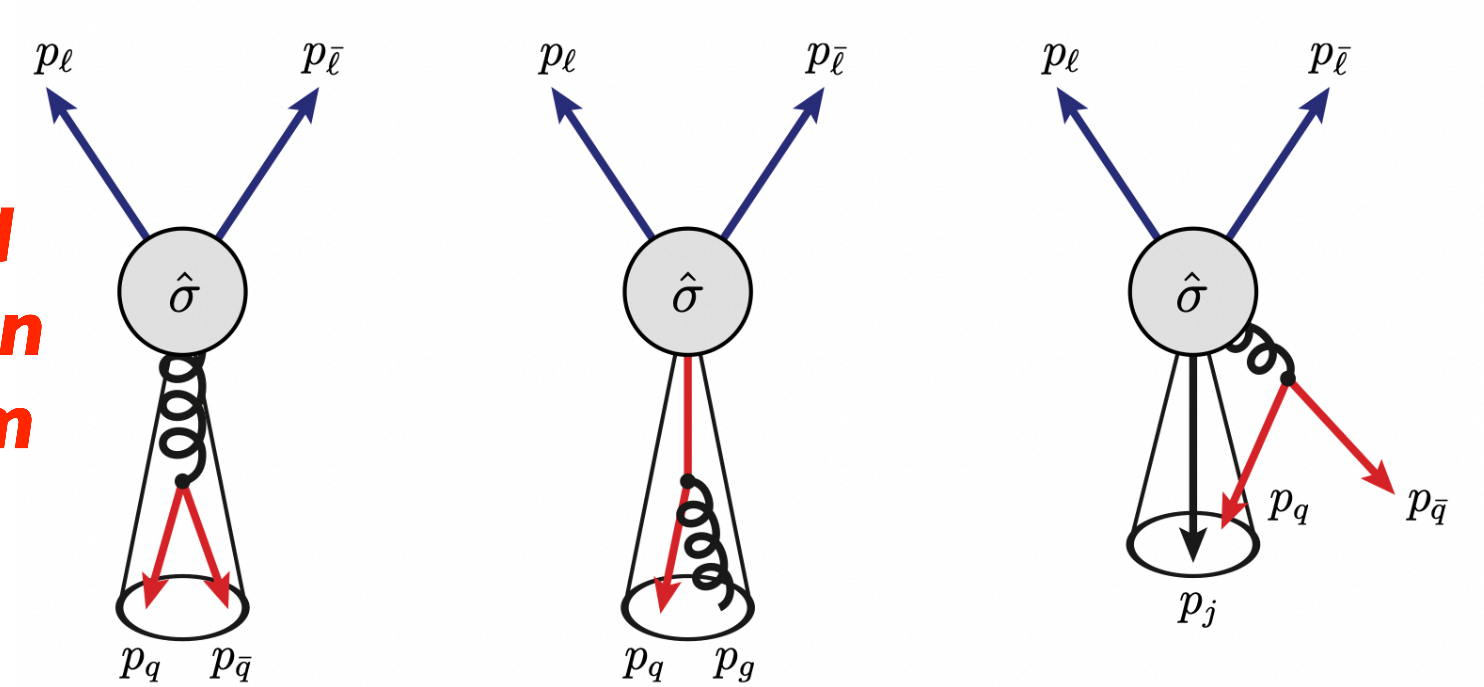
Jet flavour

LHCb 2109.08084

Example: LHCb charm-jet definition

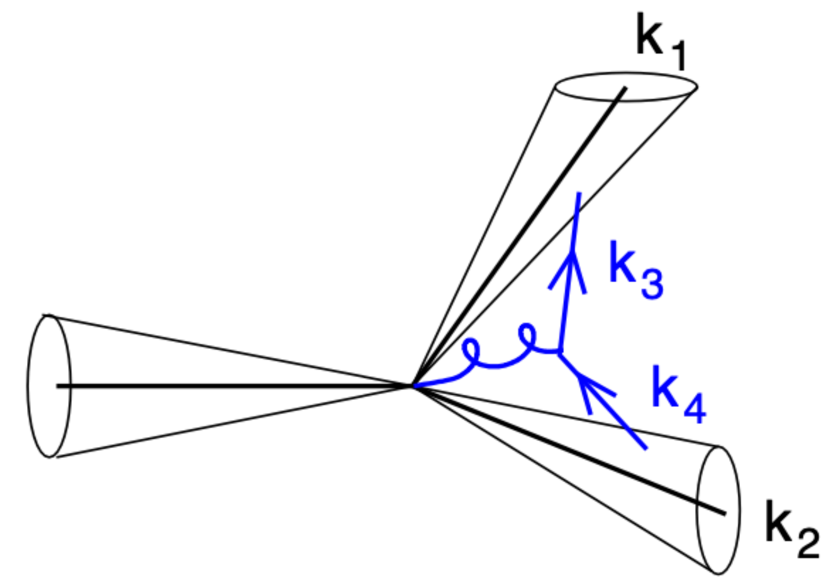
- reconstruct jets with anti-k_t algorithm
- require that the leading jet passes fiducial cuts
- the leading jet is considered a charm jet if there is at least one c-hadron satisfying $p_{t,\text{c-hadron}}$ > 5 GeV and ΔR(jet,c-hadron) < 0.5

This definition is infrared and collinear unsafe when applied to massless charm



Jet flavour

The problem was addressed in 2006 (before the anti-k_t) and the proposed definition relies on a modification of the k_t-algorithm



hep-ph/0601139

Two key elements:

1) modification of the distance for flavoured particles

$$d_{ij}^{(F)} = (\Delta\eta_{ij}^2 + \Delta\phi_{ij}^2) \times \begin{cases} \max(k_{ti}^2, k_{tj}^2) & \text{if softer of } i, j \text{ is flavoured} \\ \min(k_{ti}^2, k_{tj}^2) & \text{if softer of } i, j \text{ is flavourless} \end{cases}$$

2) classify a jet containing flavour and anti-flavour as gluon jet

Because of the k_t-like distance and the fact that it requires tagging two nearby flavoured particles, the algorithm was not adopted in practice at the LHC

Jet flavour

Recent proposals:

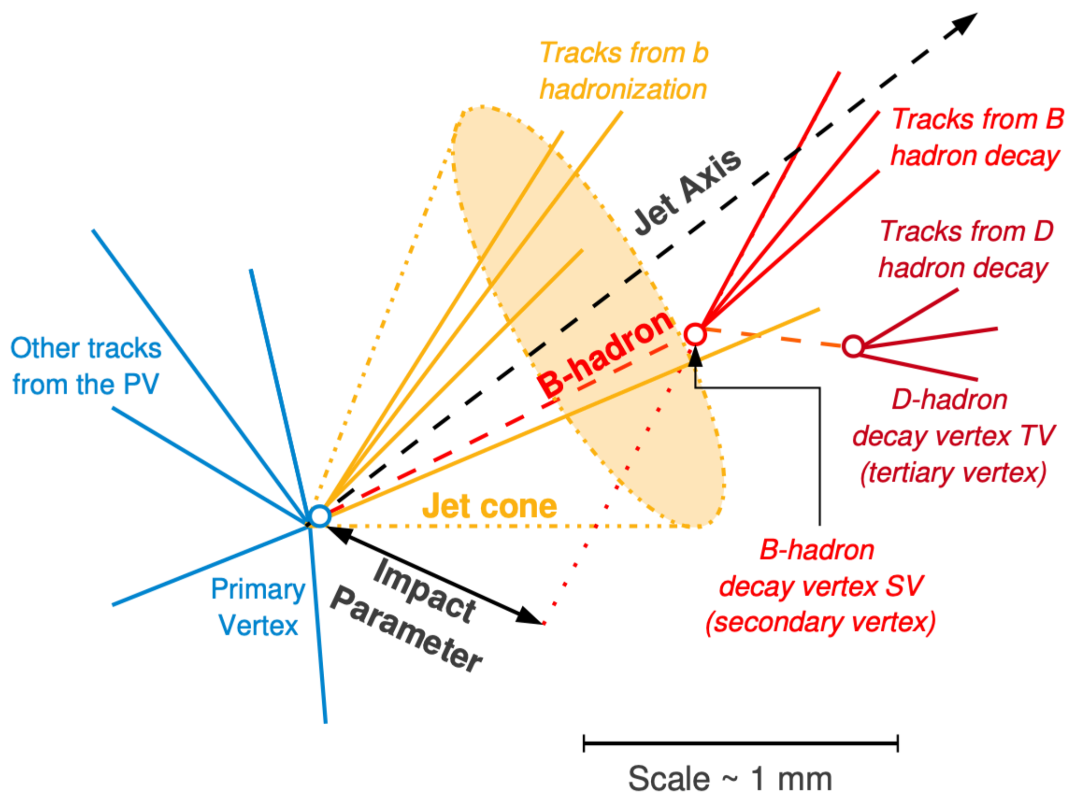
- Practical jet flavour through NNLO — Caletti et al. '22
- Infrared-safe flavoured anti-k_t jets — Czakon et al. '22
- A dress of flavour to suit any jets — Gauld et al. '22
- Flavoured jets with exact anti-k_t kinematics — Caola et al '23

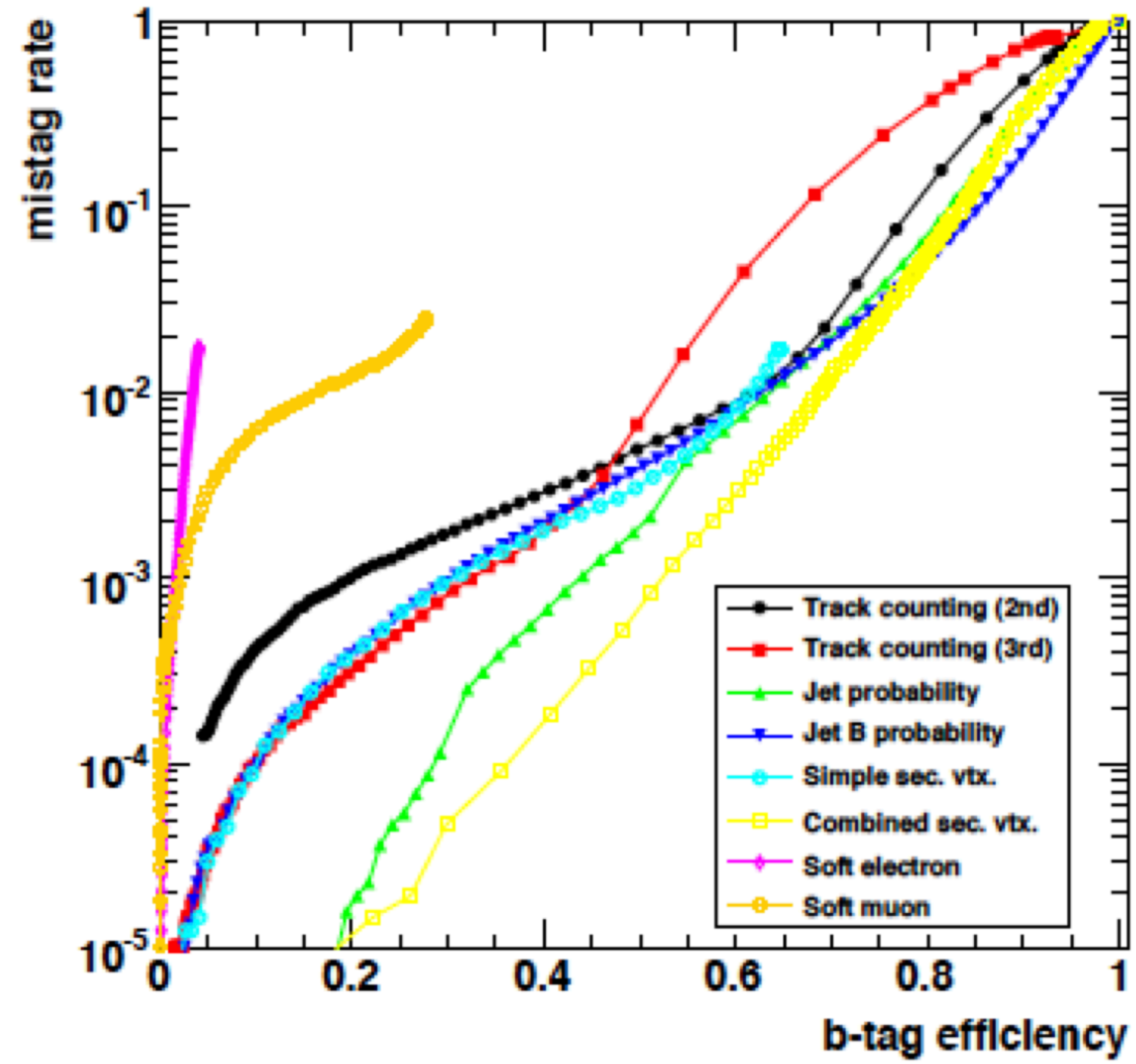
Goals (in some cases not fully met yet…)

- anti-k_t like kinematics
- infrared-safe to all orders
- flavour information, e.g. for jet-substructure
- experimentally feasible

Whether or not these novel jet definitions will be used in realistic experimental analyses remains to be seen…

Heavy flavour tagging





Very many methods to tag heavy flavour, mostly relying on the fact that B-and D-meson live long enough to travel a measurable distance before decaying. Handles: impact parameter, presence of secondary vertex, decay topology, soft leptons (from B-decay), jet kinematics. Most taggers now do not rely on simple cuts but on ML techniques.

Summary on jets

- Two major jet algorithm classes sequential recombination (k_t, CA, anti-k_t…) and cone algorithms
- Many approaches to defining jets
- Traditional seed-based cone algorithms are infrared unsafe
- SISCone: seedless infrared safe cone algorithm
- Anti-kt: default jet algorithm at the LHC
- IRC safe algorithm admit a well-defined jet area (pileup-subtraction)
- Jet substructure: key tool for identifying boosted heavy particles sonance (often based on C/A algorithm)
- Practical IRC-safe jet-flavour definition remains challenging

Perturbative calculations

Perturbative calculations rely on the idea of an order-by-order expansion in the small coupling

$$\sigma \sim A + B\alpha_s + C\alpha_s^2 + D\alpha_s^3 + \ldots$$

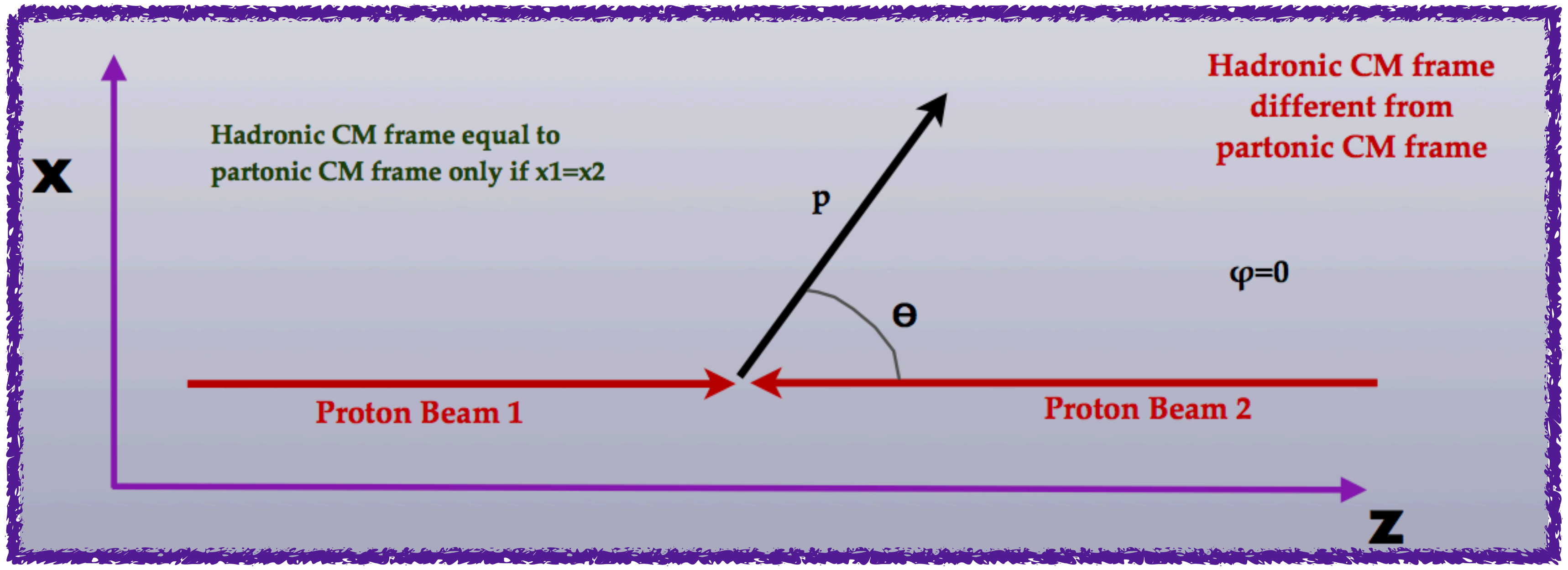
LO NLO NNLO NNNLO

- How are higher orders computed?
- In QCD (or in a generic QFT) the coupling depends on the energy (renormalization scale). So changing scale the result changes. By how much? What does this dependence mean?
- In the following will discuss these points

LHC kinematics

We want to apply perturbative QCD to high-energy LHC collisions

Before discussing calculations, it is important to understand the kinematics in these collisions



$$(E, p_x, p_y, p_z) = (\sqrt{\vec{p}^2 + m^2}, |\vec{p}| \sin\theta \cos\phi, |\vec{p}| \sin\theta \sin\phi, |\vec{p}| \cos\theta)$$

The total longitudinal momentum of the colliding system is unknown (one can measure missing transverse momentum, but not missing longitudinal one)

LHC kinematics

A more common parametrisation relies on rapidity and transverse mass

$$(E, p_x, p_y, p_z) = (m_T \cosh y, |p_T| \cos\phi, |p_T| \sin\phi, m_Y \sinh y)$$

With

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \qquad p_T = \sqrt{p_x^2 + p_y^2} \qquad m_T = \sqrt{p_T^2 + m^2}$$

Exercise: check that the two parameterisations are equivalent

Exercise: check that the rapidity transform linearly under a longitudinal boost

Exercise: given two particles, can you easily construct a boost-invariant quantity?

LHC kinematics

For particles with negligible mass the rapidity coincides with the pseudo-rapidity

$$y = \eta \equiv \frac{1}{2}\log\frac{1+\cos\theta}{1-\cos\theta} = -\log\tan\frac{\theta}{2}$$

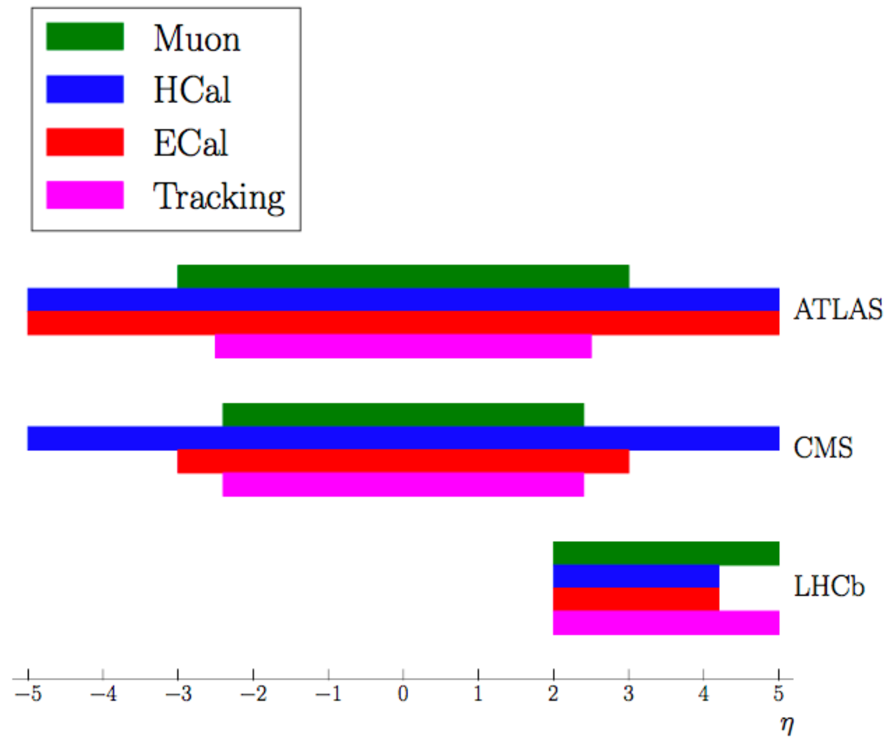
The pseudo-rapidity can then be easily translated to the detector geometric acceptance as used in experimental measurements

Θ	10^{-4}	10^{-3}	10^{-2}	0.1	0.5	1	$\pi/2$
Y	9.9	7.6	5.3	3	1.36	0.6	0

(The other hemisphere has same but negative numbers)

Rapidity coverage of LHC detectors

The achieved maximum rapidity coverage is important in LHC detectors



- For ATLAS and CMS: muons can be detected only in the central regions, while for jets and hadrons, hadronic calorimetry extends up to 4.5-5 (essential for processes like vector boson fusion Higgs production)
- LHCb covers better the forward region, but only forward one
- Studies are ongoing to determine the required/possible rapidity coverage of future detectors

LHC kinematics

Rapidity is also interesting from a theoretical point of view, as the single particle phase space is uniform in rapidity

$$\frac{d^3p}{2E(2\pi)^3} = \frac{1}{2(2\pi)^3} d^2p_T dy$$

Exercise: derive the above expression

The above relation has already deep implications: for instance incoherent radiation (e.g. soft underlying event) is to a large extend uniform in rapidity

Leading order

Born level cross section straightforward in principle

$$\sigma_{LO} = \int_m d\Phi_m |\mathcal{M}^{(0)}(\{p_i\})|^2 S(\{p_i\})$$

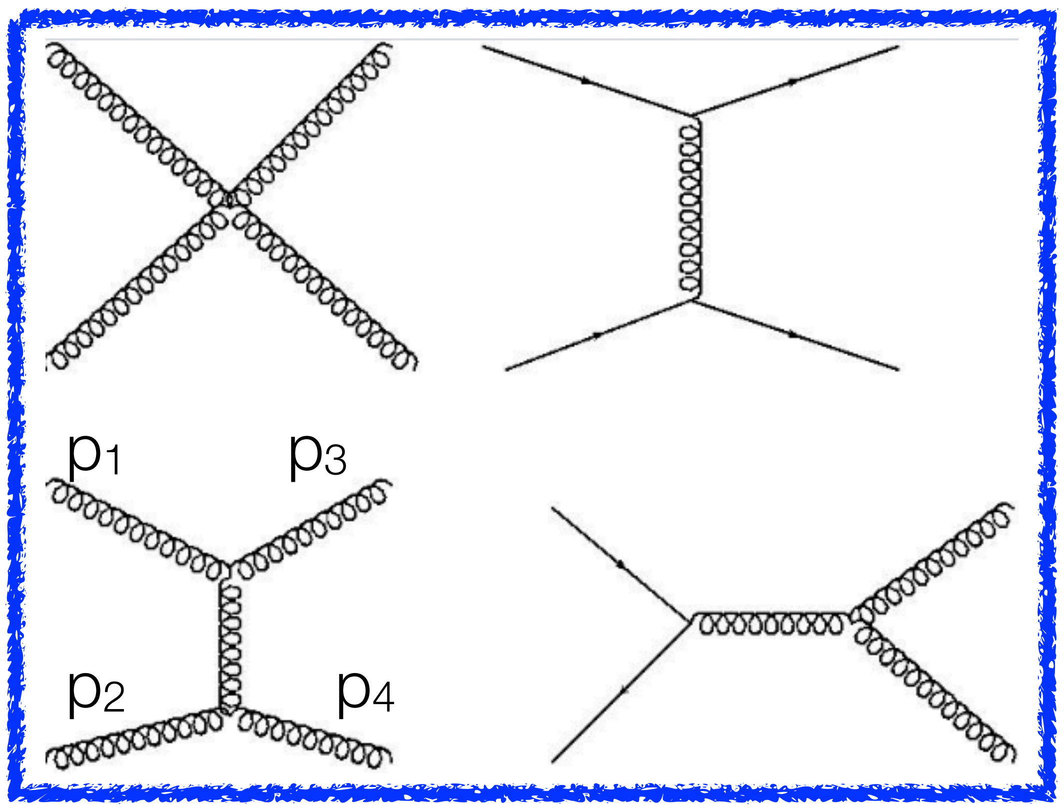
m-particle phase space
(e.g. Vegas)

Matrix element

measurement function
(constraint on phase space)

Example: Dijet production

Sample diagrams (all must be included)



Many partonic subprocesses contribute

Process	$\frac{d\hat{\sigma}}{d\Phi_2}$
$qq' \to qq'$	$\frac{1}{2\hat{s}}\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$
$qq \to qq$	$\frac{1}{2}\frac{1}{2\hat{s}}\left[\frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right)-\frac{8}{27}\frac{\hat{s}^2}{\hat{u}\hat{t}}\right]$
$q\bar{q} \to q'\bar{q}'$	$\frac{1}{2\hat{s}}\frac{4}{9}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$
$q\bar{q} \to q\bar{q}$	$\frac{1}{2\hat{s}}\left[\frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right)-\frac{8}{27}\frac{\hat{u}^2}{\hat{s}\hat{t}}\right]$
$q\bar{q} \to gg$	$\frac{1}{2}\frac{1}{2\hat{s}}\left[\frac{32}{27}\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}-\frac{8}{3}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]$
$gg \to q\bar{q}$	$\frac{1}{2\hat{s}}\left[\frac{1}{6}\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}-\frac{3}{8}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]$
$gq \to gq$	$\frac{1}{2\hat{s}}\left[-\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{s}\hat{u}}+\frac{\hat{u}^2+\hat{s}^2}{\hat{t}^2}\right]$
$gg \to gg$	$\frac{1}{2}\frac{1}{2\hat{s}}\frac{9}{2}\left(3-\frac{\hat{t}\hat{u}}{\hat{s}^2}-\frac{\hat{s}\hat{u}}{\hat{t}^2}-\frac{\hat{s}\hat{t}}{\hat{u}^2}\right)$

Mandelstam variables: $\hat{s}=(p_1+p_2)^2 \quad \hat{t}=(p_1+p_3)^2 \quad \hat{u}=(p_1+p_4)^2$

Dijet production

The hadronic cross-section **PDFs for initial state partons** **Phase space**

$$d\sigma^{\text{dijet}} = \sum_{ijkl} \frac{1}{1+\delta_{kl}} \int dx_1 dx_2 f_i(x_1, \mu^2) f_j(x_2, \mu^2) \frac{d\hat{\sigma}_{ij\to kl}}{d\Phi_2} d\Phi_2 \Theta_{\text{cuts}}$$

Symmetry factor **Matrix elements** **Measurement function**

We have seen that in the LAB frame

$$p_3 = (p_T \cosh y_3, p_T \cos\phi, p_T \sin\phi, p_T \sinh y_3)$$

$$p_4 = (p_T \cosh y_4, -p_T \cos\phi, -p_T \sin\phi, p_T \sinh y_4)$$

Exercise: show that the rapidities are related to the Bjorken-x variables by

$$x_1 = \frac{p_T}{\sqrt{s}} \left(e^{y_3} + e^{y_4}\right) \qquad x_2 = \frac{p_T}{\sqrt{s}} \left(e^{-y_3} + e^{-y_4}\right)$$

Dijet production

Exercise: show that the rapidities in the partonic centre-of-mass frame are given by

$$\hat{y}_3 = \frac{1}{2}\left(y_3 - y_4\right) = -\hat{y}_4$$

Exercise: show that the scattering angle in the partonic frame is given by

$$\cos\hat{\theta} = \tanh\hat{y}_3 = \tanh\left(\frac{y_3 - y_4}{2}\right)$$

this relation shows that the difference in rapidities between the jets gives direct access to the dynamics in the partonic frame

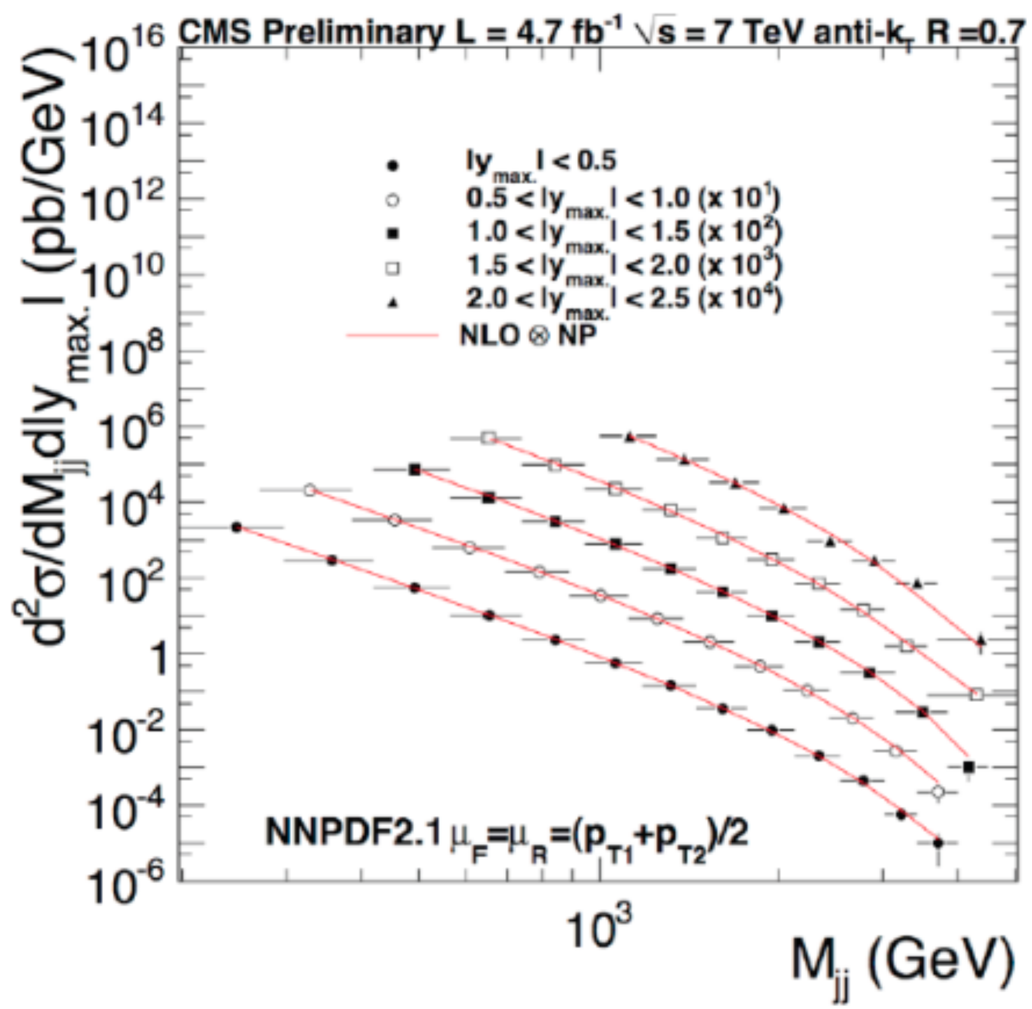
Exercise: show that in terms of rapidities the cross-section becomes

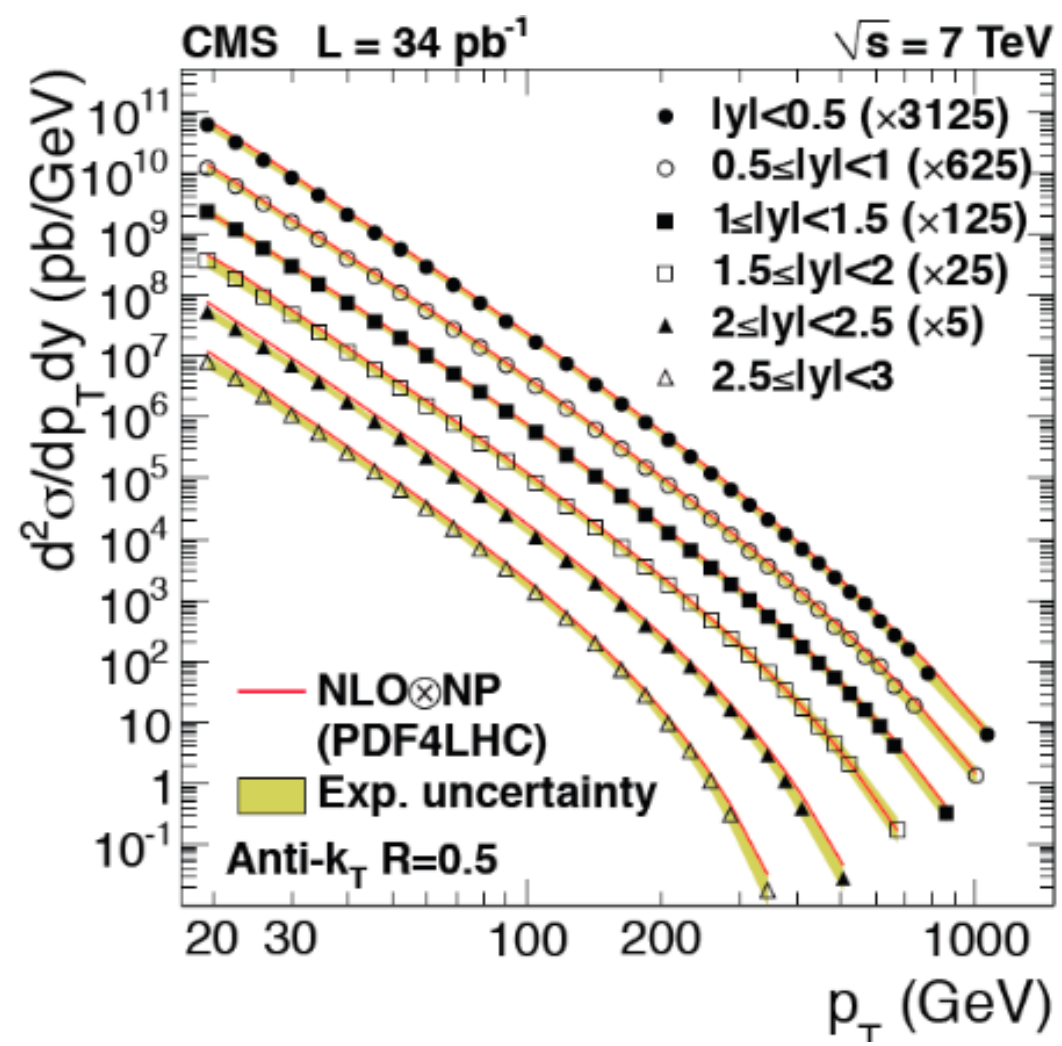
$$\frac{d^3\sigma^{\rm dijet}}{dy_1 dy_2 dp_T^2} = \frac{1}{16\pi s}\sum_{ijkl}\frac{1}{1+\delta_{kl}}\int\frac{dx_1}{x_1}\frac{dx_2}{x_2} f_i(x_1,\mu^2) f_j(x_2,\mu^2)\frac{d\hat{\sigma}_{ij\to kl}}{d\Phi_2}$$

The above expression can be integrated numerically and provides a leading order estimate of the cross section

Dijet production: theory vs data

Inclusive and dijet production are extensively studies at the LHC, both for SM measurements and in searches for New physics



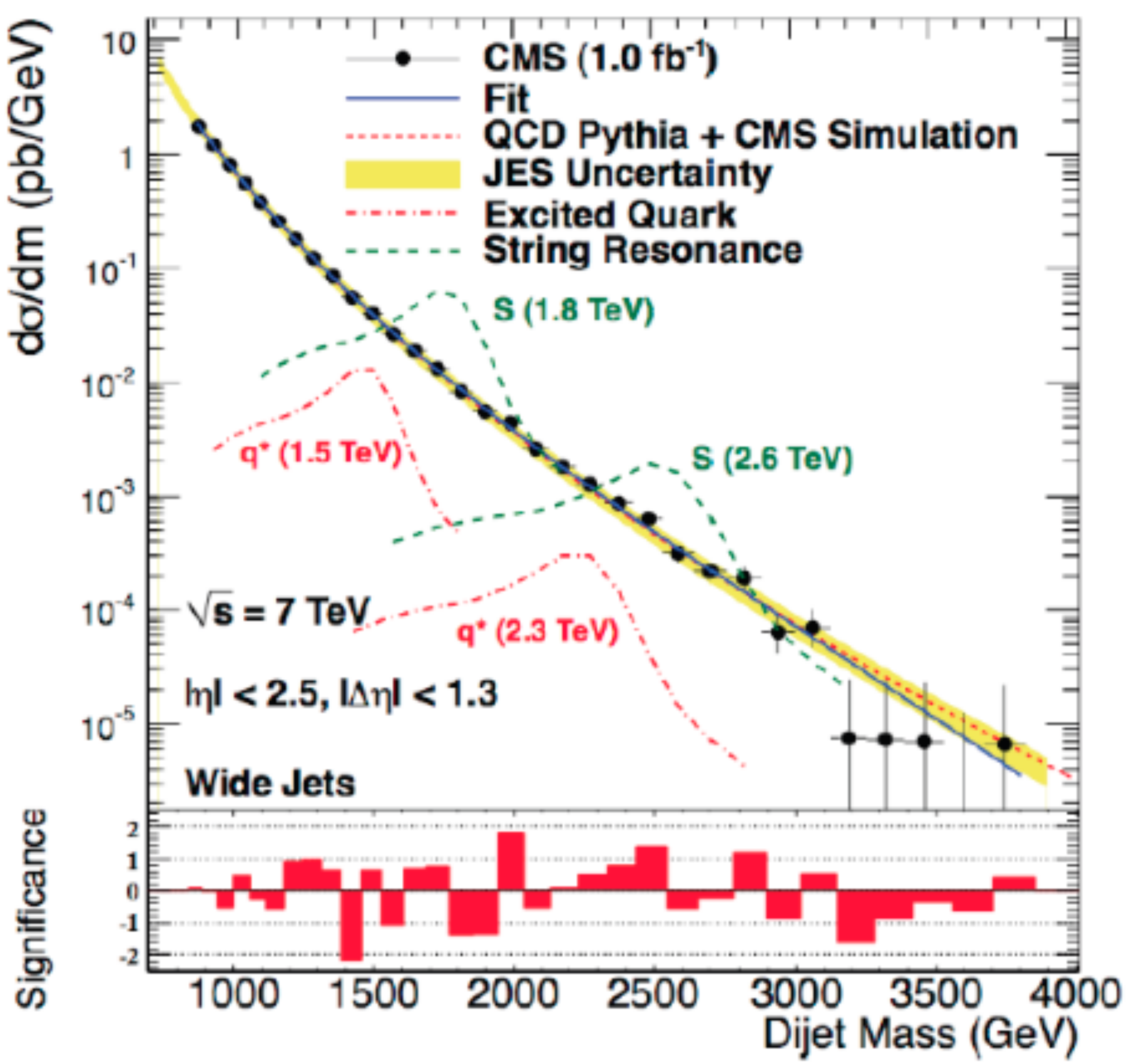


Direct determination of gluon PDF, constraints of other PDFS, measurement of α_s, probe of QCD running at TeV scales …

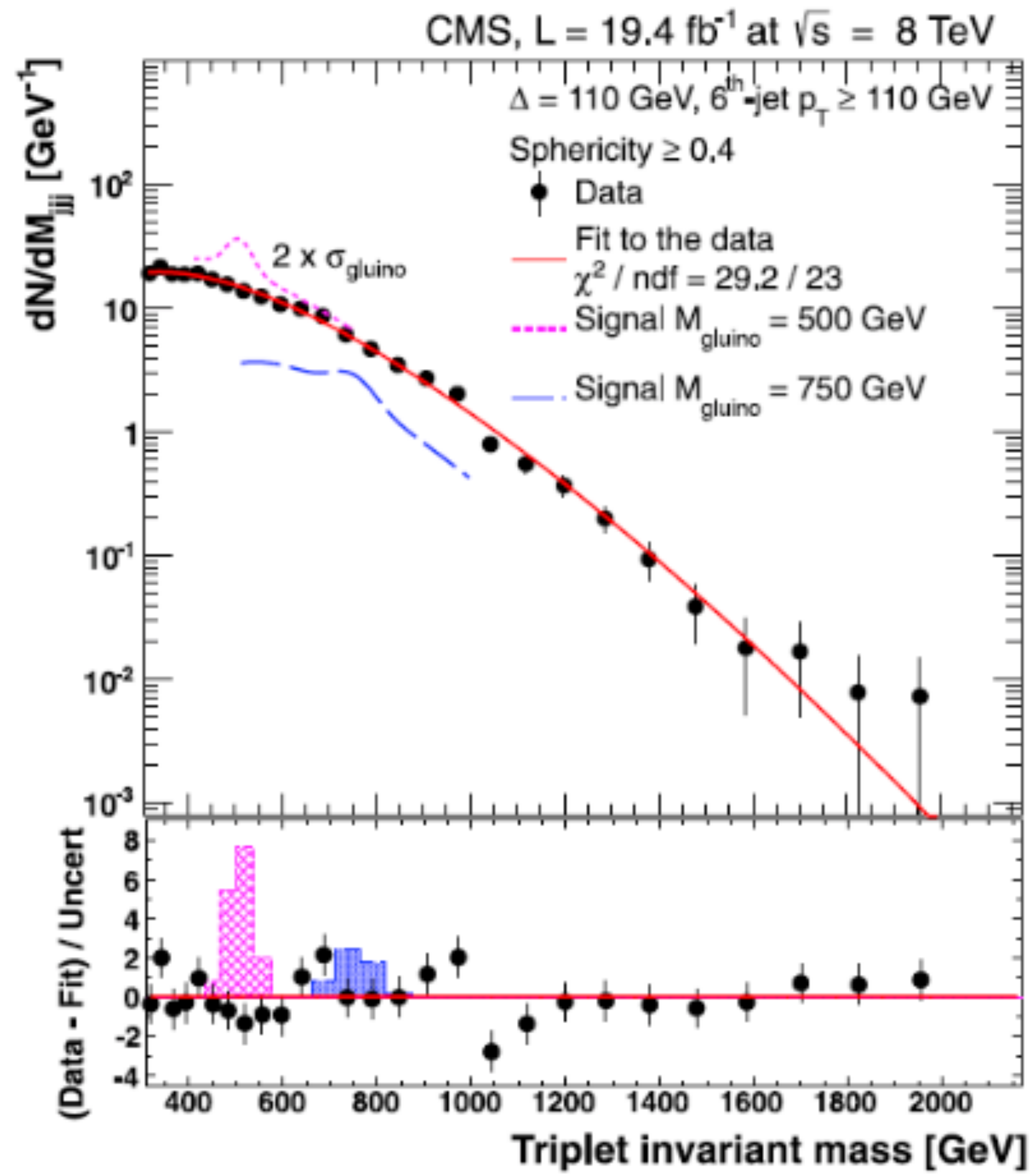
Dijet production

Inclusive and dijet production are extensively studies at the LHC, both for SM measurements and in searches for New physics

Search for excited quarks



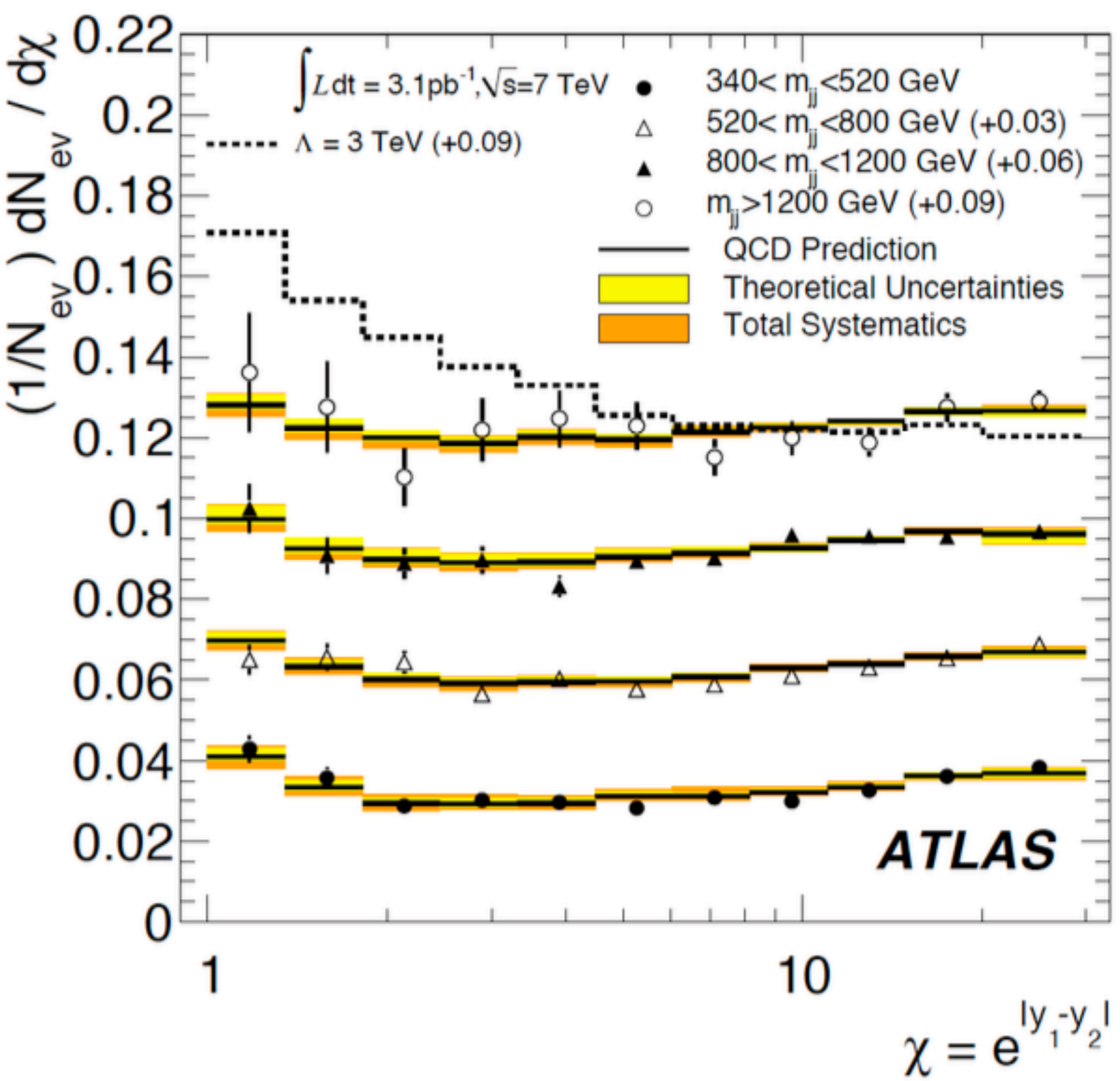
Search for gluions



Dijet production

Inclusive and dijet production are extensively studies at the LHC, both for SM measurements and in searches for New physics

Explore substructure of quarks



It is clear that the smaller the uncertainties, the more one can exclude exotic scenarios.
Above we sketched a leading order calculation, in the following we'll discuss higher-order corrections in a more generic case

Leading order with Feynman diagrams

Get *any* LO cross-section from the Lagrangian

1. draw all Feynman diagrams
2. put in the explicit Feynman rules and get the amplitude
3. do some algebra, simplifications
4. square the amplitude
5. integrate over phase space + flux factor + sum/average over outgoing/incoming states

Automated tools for (1-3): FeynArts/Qgraf, Mathematica/Form etc.

Bottlenecks

a) number of Feynman diagrams diverges factorially

b) algebra becomes more cumbersome with more particles

But given enough computer power everything can be computed at LO

Diagrams for gluon amplitudes

Number of diagrams for gg $\rightarrow$ n gluons

n	2	3	4	5	6	7	8
Nr. of Diagrams	4	25	220	2485	34300	559405	10525900

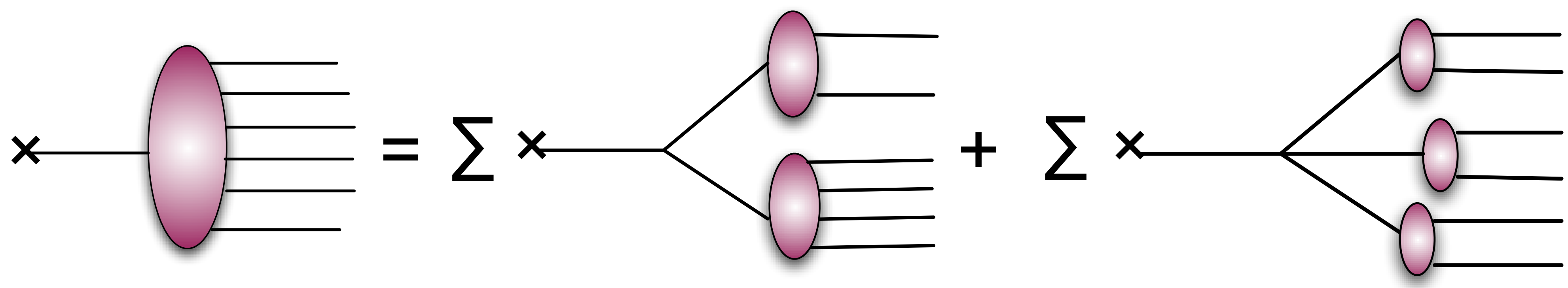
- number of diagrams grows very fast
- complexity of each diagrams grows with n

Alternative methods?

Techniques beyond Feynman diagrams

✓ <u>Berends-Giele relations:</u> compute helicity amplitudes recursively using off-shell currents

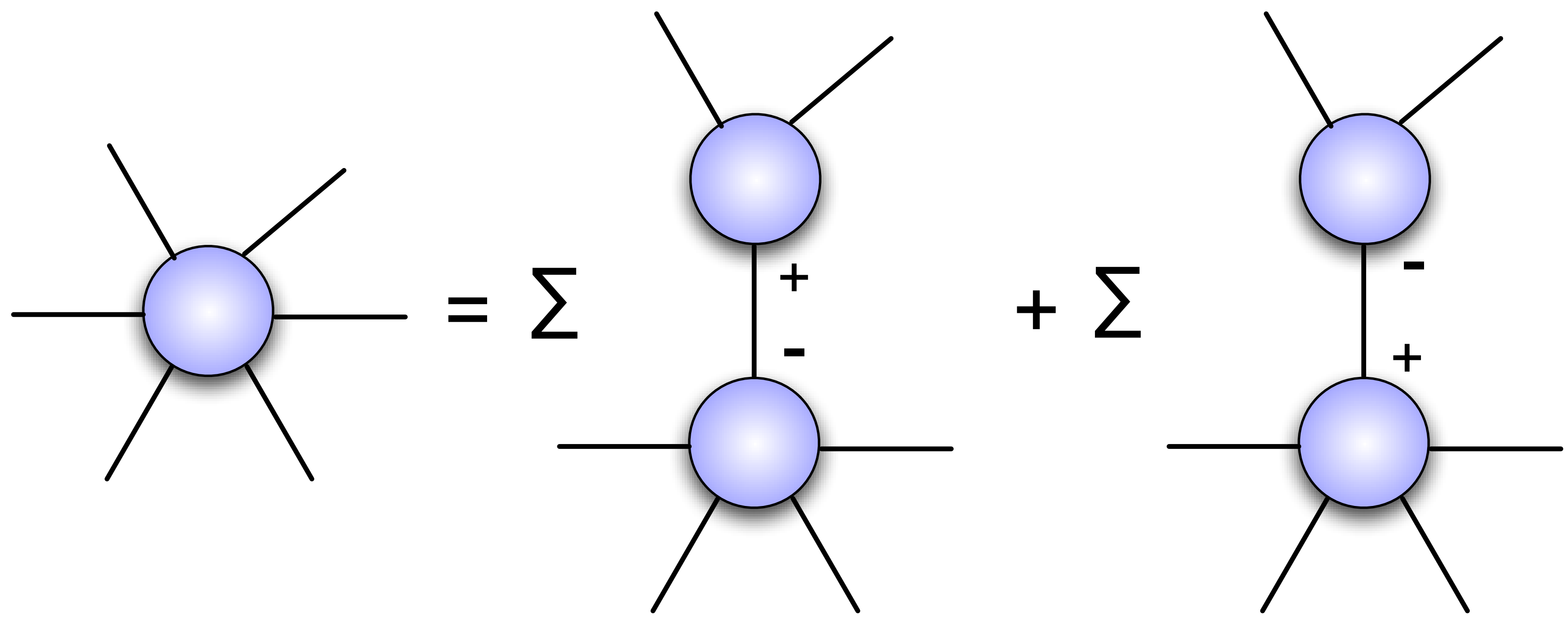
Berends, Giele '88



Techniques beyond Feynman diagrams

✓ BCF relations: compute helicity amplitudes via on-shell recursions (use complex momentum shifts)

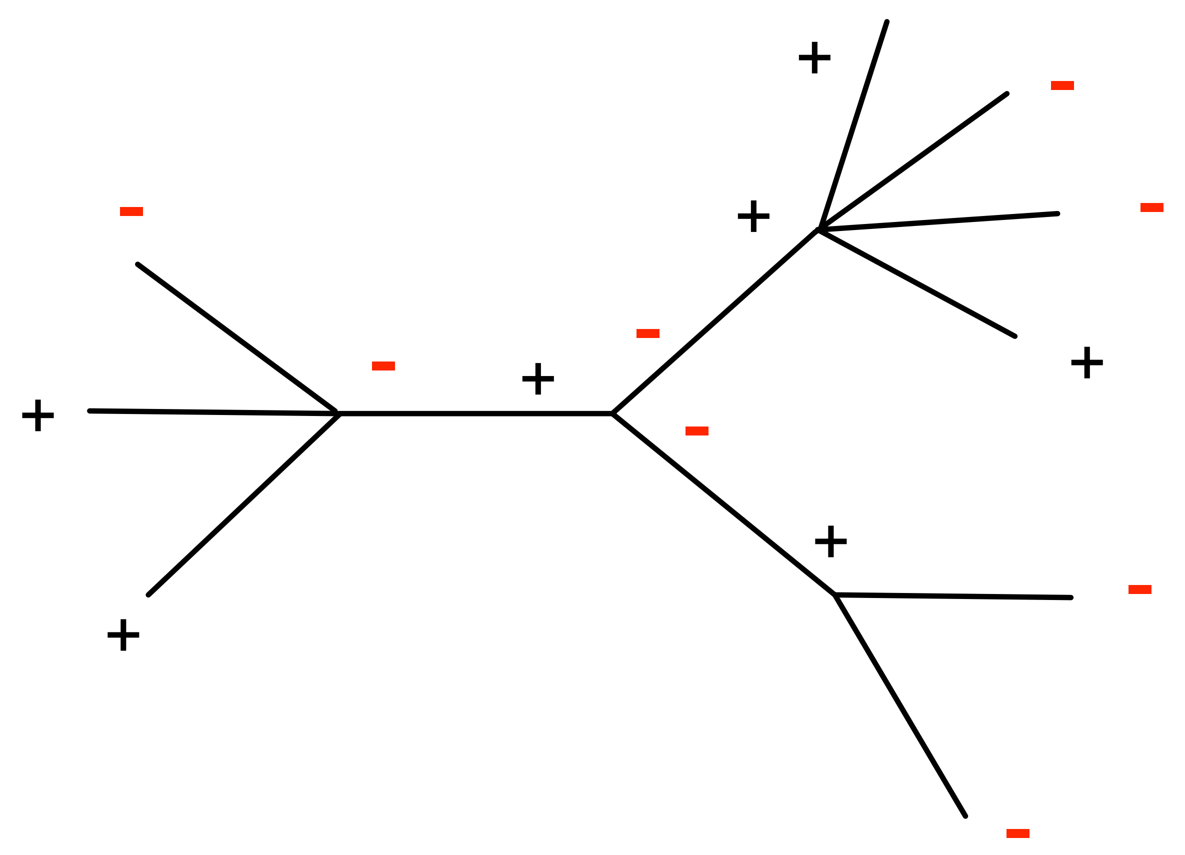
Britto, Cachazo, Feng '04



Techniques beyond Feynman diagrams

✓ CSW relations: compute helicity amplitudes by sewing together MHV amplitudes [- - + + ... +]

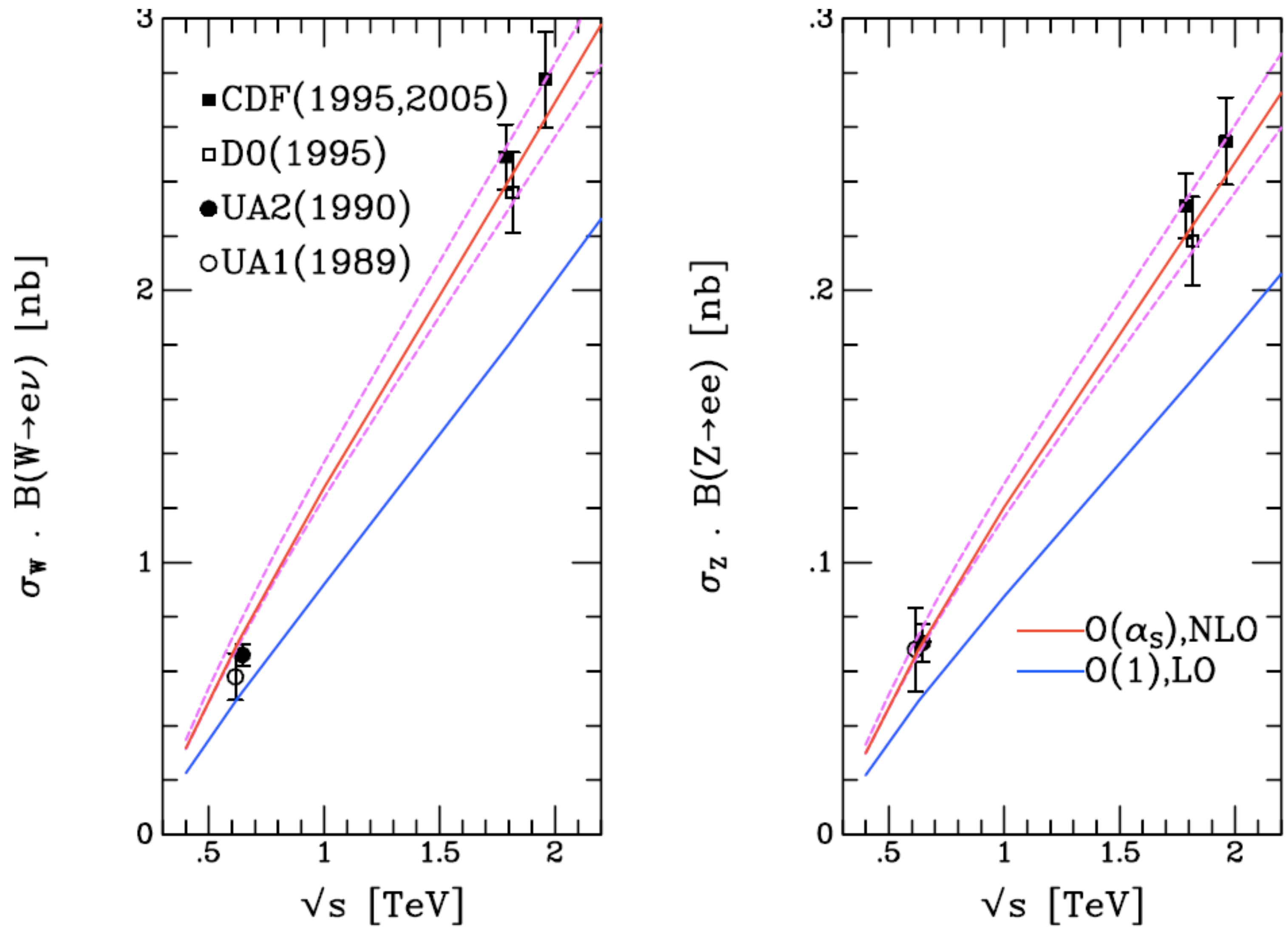
Cachazo, Svrcek, Witten '04



Is it necessary to go beyond LO?

Very early observation:

at least NLO corrections are needed to describe data



Drell Yan production is one of the first processes for which NLO corrections have been computed