

ICTP-SAFIR

School on Open Quantum Systems: theory and experiment



July 27 – August 7, 2026

**SCHOOL ON OPEN
QUANTUM SYSTEMS:
THEORY AND EXPERIMENT**

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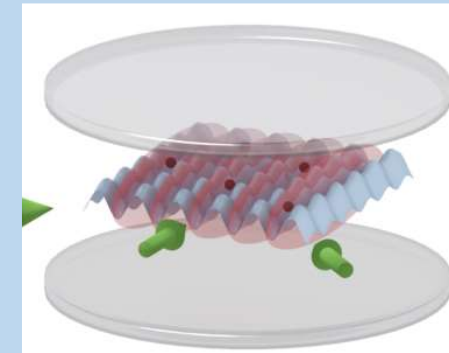
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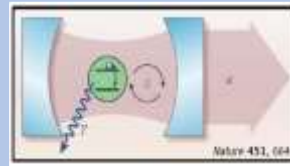


CQED with ultracold particles
Quantum gas cavity QED

Quantum Gas Cavity QED

Basic Physics

quantum optics:
*light fields confined
by fixed classical matter*



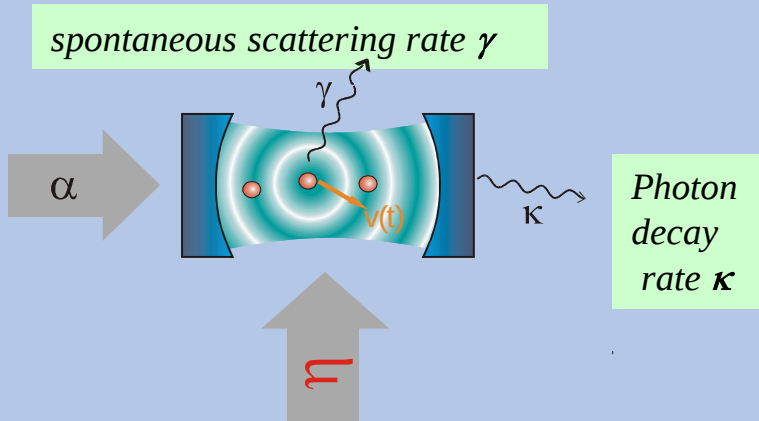
laser cooling & trapping:
particle motion controlled by
classic optical potentials

in optical setups with ultracold gases:
dynamic (quantum) description of light and particles

**light induces a
dynamic optical forces**
+
**particles induce
dynamic diffraction + refractive index**

Cavity QED: Jaynes/Tavis Cummings coupling + open system dynamics

$$H = \hbar\omega_f a^\dagger a + \frac{1}{2}\hbar\omega_a \sigma^z + \hbar g(a^\dagger \sigma^- + a \sigma^+)$$

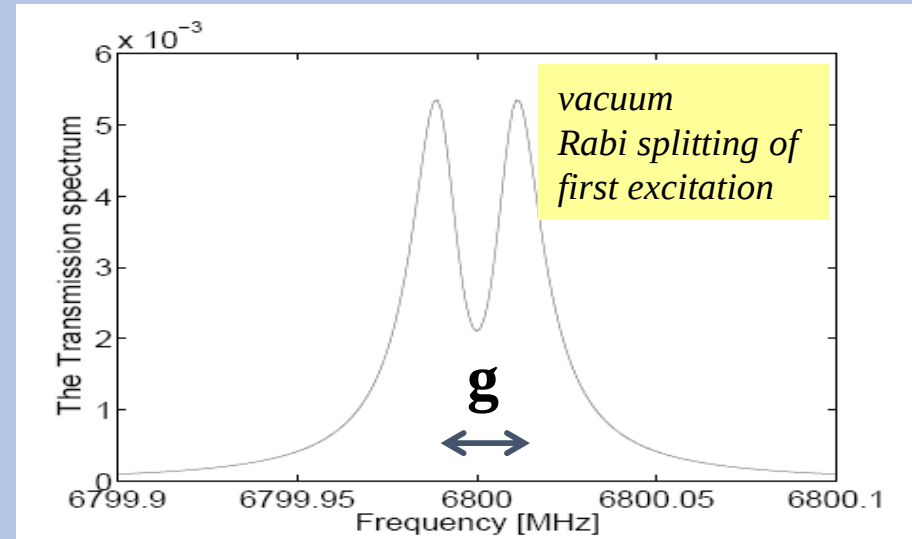


input and output channels :

- ☺ measurement + feedback control
- ☹ damping + fluctuations + decoherence

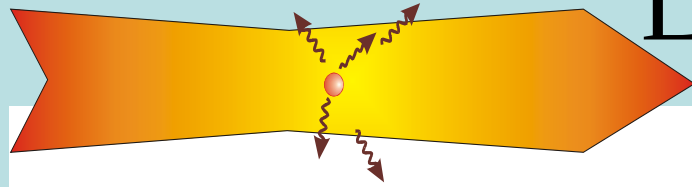
strong coupling

$$\omega_f, \omega_a \gg g \gg (\kappa, \gamma)$$



- Nonlinear atomic response at less than a single photon
- Single atom splits cavity resonance by more than a line width

Laser light forces on atoms



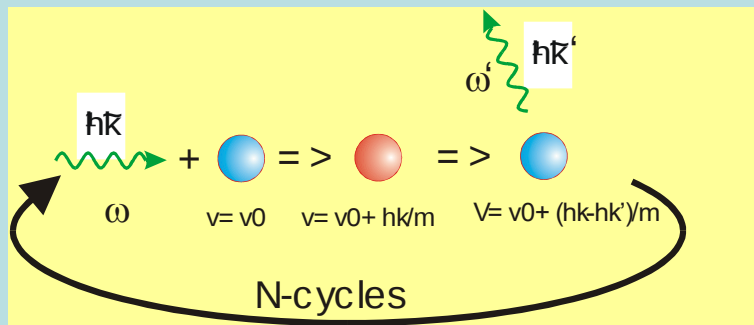
Two classes of forces:

Radiation pressure

(absorption + spontaneous re-emission)

Dipole force

(absorption + stimulated emission)

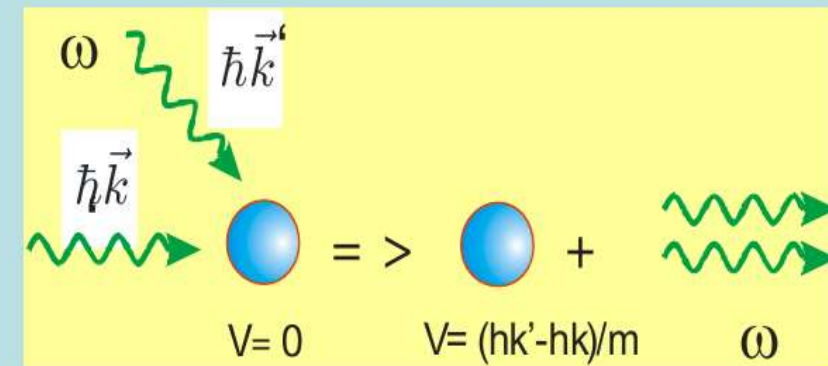


$$\vec{v} = \vec{v}_0 + N\hbar\vec{k} - \sum_{i=1}^N \hbar\vec{k}' \approx N\vec{k}$$

emissions have random directions and cancel
N **depends on frequency and intensity**

$$\vec{F}_{rad} = \hbar\vec{k} \frac{\gamma}{2} \frac{\frac{1}{2}\Omega^2}{(\omega_l - \omega_a)^2 + \Gamma^2/4 + \frac{1}{2}\Omega^2}$$

- dissipative (random) force
- maximum force at resonance $F = \hbar k \gamma/2$ ($\sim 10^5$ g)
- **momentum diffusion** due to random emission



coherent transfer of momentum,
which **depends on relative phases** of fields

$$\vec{F}_{dip} = \hbar(\omega_l - \omega_a) \frac{\frac{1}{2}(\vec{\nabla}\Omega)^2}{(\omega_l - \omega_a)^2 + \Gamma^2/4 + \frac{1}{2}\Omega^2}$$

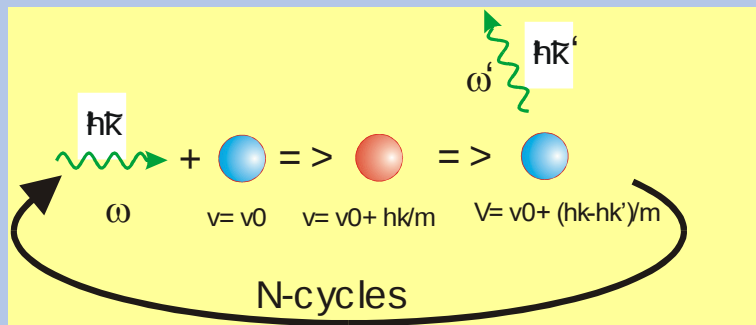
- conservative force with no upper limit =>
optical potential $U(x) \sim I(x)/(\omega_l - \omega_a)$
- high field minima for red detuning

Light force basics

Two classes of forces (CCT,JD) :

Radiation pressure

(absorption + spontaneous re-emission)



incoherent + random emissions

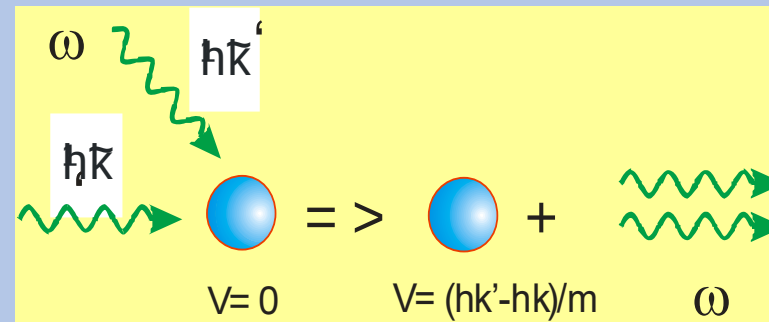
$$\vec{F}_{rad} = \frac{\hbar\omega_l}{c} \frac{\frac{1}{2}(\vec{\nabla}\Omega)^2}{\Gamma^2/4 + \frac{1}{2}\Omega^2}$$

*negligible
in dispersive
large detuning
limit !*

- dissipative interaction
- need resonance + spontaneous emission
- scales as $1/\text{detuning}^2$

Dipole force

(absorption + stimulated emission)

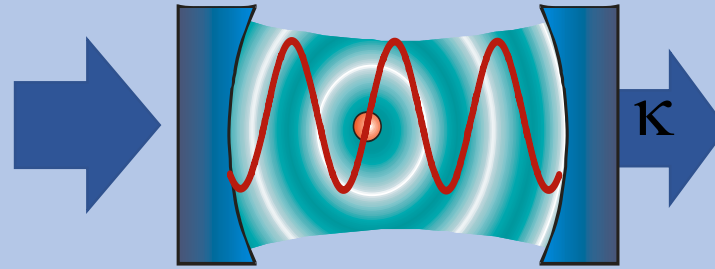


coherent transfer of momentum,
which **depends on relative phases** of fields

$$\vec{F}_{dip} = \hbar(\omega_l - \omega_a) \frac{\frac{1}{2}(\vec{\nabla}\Omega)^2}{(\omega_l - \omega_a)^2 + \Gamma^2/4 + \frac{1}{2}\Omega^2}$$

- conservative optical potential $\sim$ intensity
- scales as $1/\text{detuning}$
- optical tweezers, opto-mechanics

Lightforces on polarizable particles in optical resonators



***Light forces** of resonatorfield determine atomic motion*

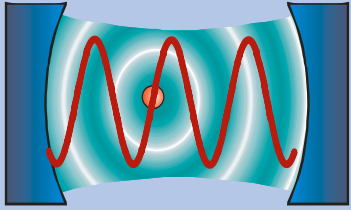
interaction

self - trapping

friction + diffusion

correlation and entanglement

***Cavity QED** : atoms change resonator field dynamics*



dispersive Cavity-QED at large detuning

$$\Delta = \omega_a - \omega_L \gg \gamma, \kappa$$

g_0 ... coupling strength
 γ ... atomic width
 κ ... cavity linewidth

eliminate upper atomic state $\Rightarrow$ effective atom-field interaction potential

$$U(x) := \frac{\Delta_a}{\Delta_a^2 + \gamma_0^2} g_0^2 \cos^2(kx)$$

$$\gamma(x) := \frac{\gamma_0}{\Delta_a^2 + \gamma_0^2} g_0^2 \cos^2(kx)$$

$U(x)$ = **optical potential per photon** =
 cavity **frequency shift** per atom
 based on dipole force

$>$

$\gamma(x)$ = **photon loss per particle** $\ll \kappa$
 radiation pressure / photon

Strong **dispersive** coupling limit :

$$U \gg \kappa \gg \gamma$$

$\Rightarrow$ single atom shifts cavity in & out of resonance
 $\Rightarrow$ single photon creates an optical trap for an atom

3 regimes of interaction

dispersive interaction

$$U \gg \gamma$$

- ⇒ atom-field interaction via **optical potential** U
- ⇒ **dipole force** dominates radiation pressure

strong coupling re-defined

$$U \gg \kappa \gg \gamma$$

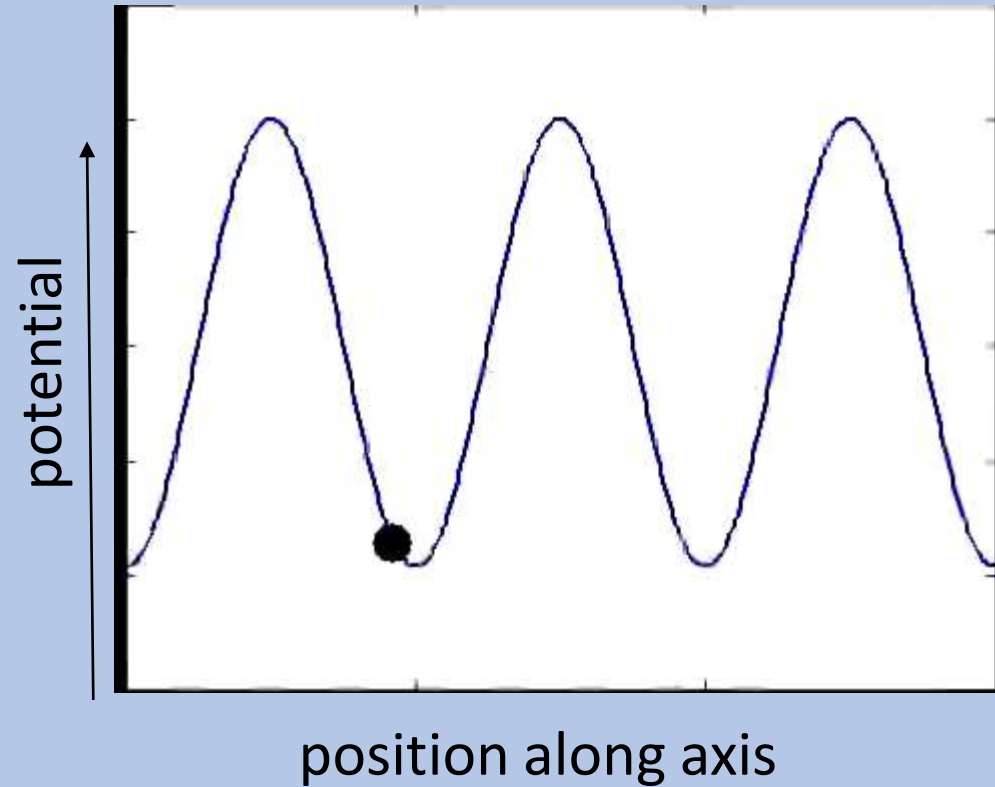
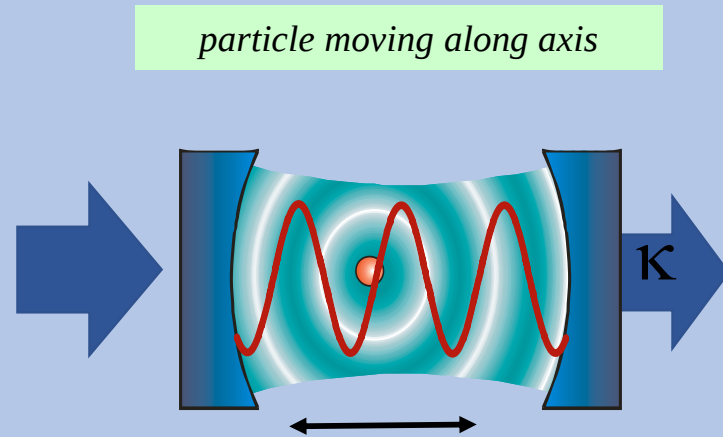
- ⇒ single atom shifts cavity in or out of resonance
- ⇒ single photon creates an optical trap for an atom

ultrastrong dispersive coupling

$$U \gg \Delta\omega_l \gg \kappa$$

- ⇒ single mode picture breaks down
- ⇒ nonlinear coupled multi mode model

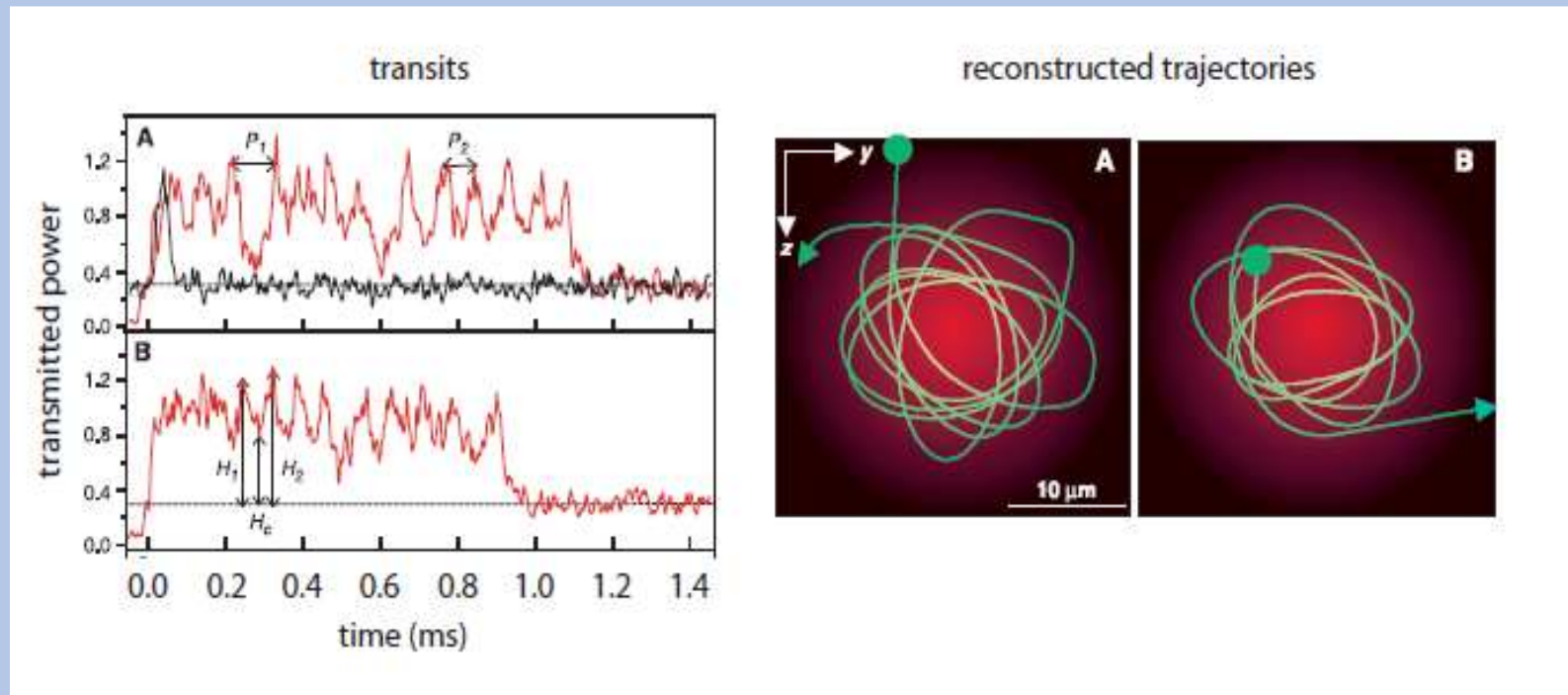
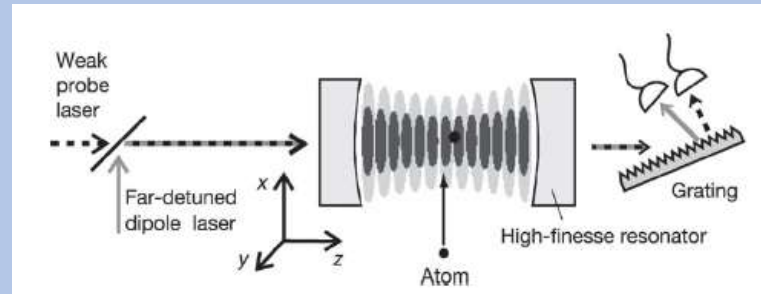
Point particle dynamics in optical resonator:



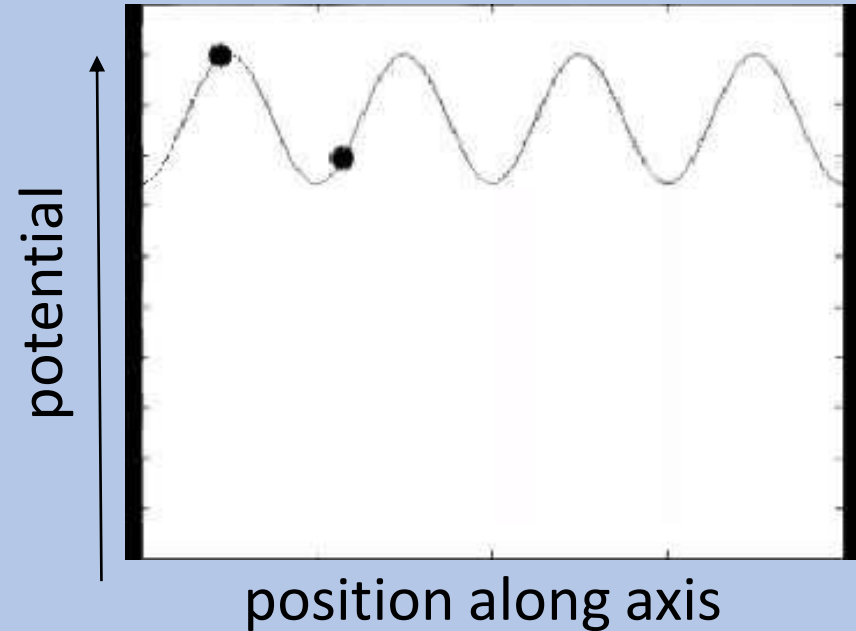
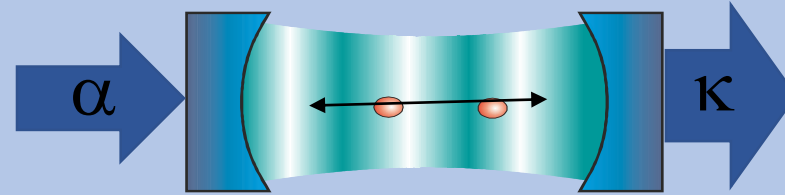
Lewenstein , PRL 95: ions
Horak, PRL 97: atoms
Vuletic, Chu, PRL 00: atoms
Vitali, PRL 02: mirrors

trapping + cavity cooling

Experiment confirms:
Single atom can be trapped by **single photon**



***„long“ range interaction:
two particles in a driven optical resonator***



friction and „long“ range interaction

cavity cooling towards zero temperature: $\kappa < \omega_r$

several cooling steps in time
for optimized cooling sequence
to ground state = 'BEC'

bosons

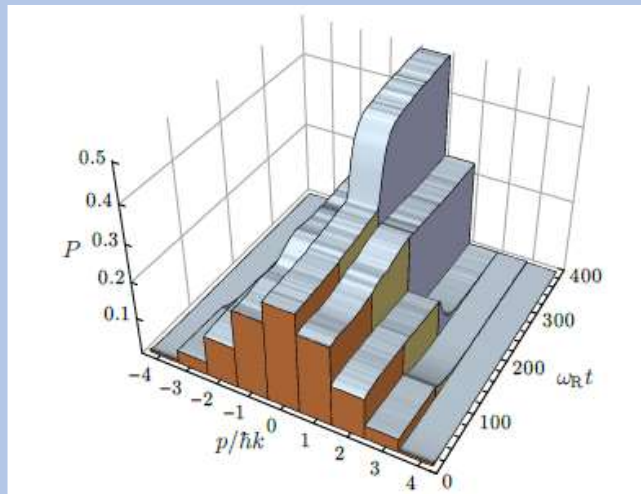
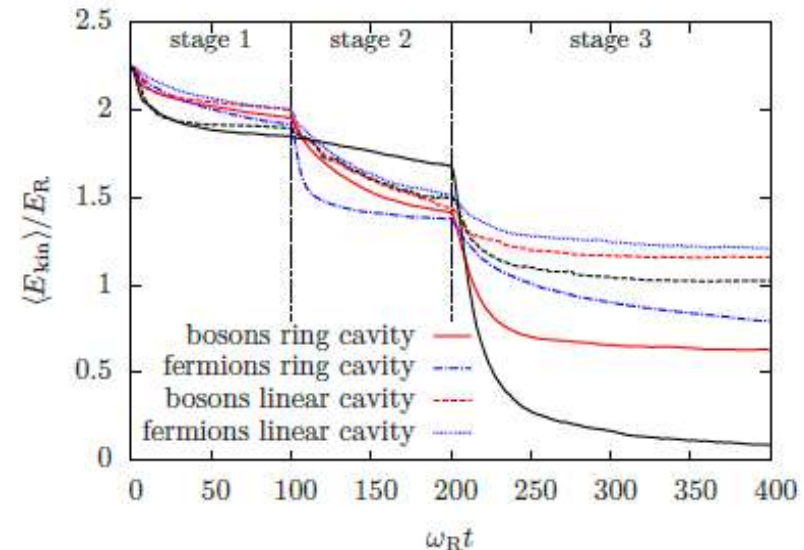


Fig. 4: (Colour on-line) Single-particle momentum distribution for $\Delta_c/\omega_R = -14.75|-12|-7$ (ring cavity bosons).

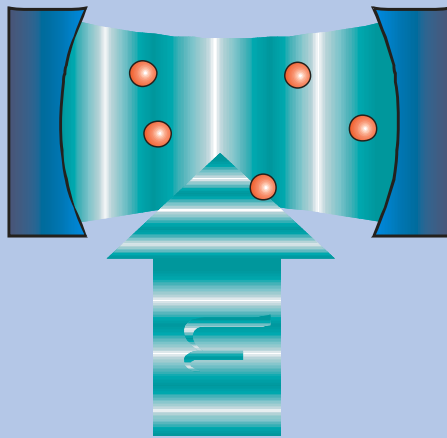
Cooling dynamics
for different particle quantum statistics



Cavity cooling has no principle temperature limit
 $\Rightarrow$ one can reach degeneracy !

Crystallisation of atoms via collective light scattering

transverse geometry



excitation of atoms from side !

phase of scattered light depends on position x, z

$$\begin{aligned}\dot{\sigma}_i &= (i\Delta_A - \gamma)\sigma_i - g(z_i)a + \eta_x \xi_A \\ \dot{a} &= (i\Delta_C - \kappa)a + \underbrace{\sum_{i=1}^N g^*(z_i)\sigma_i}_{\text{collective pump strength } R} + \xi_i\end{aligned}$$

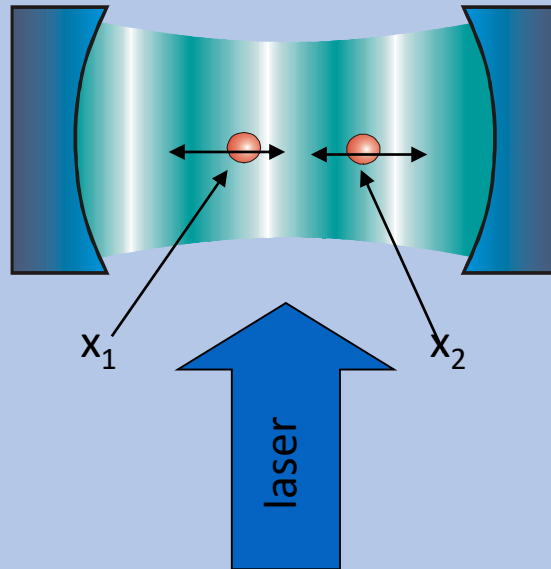
collective pump strength R

Field in cavity generated by **collective scattering** by atoms

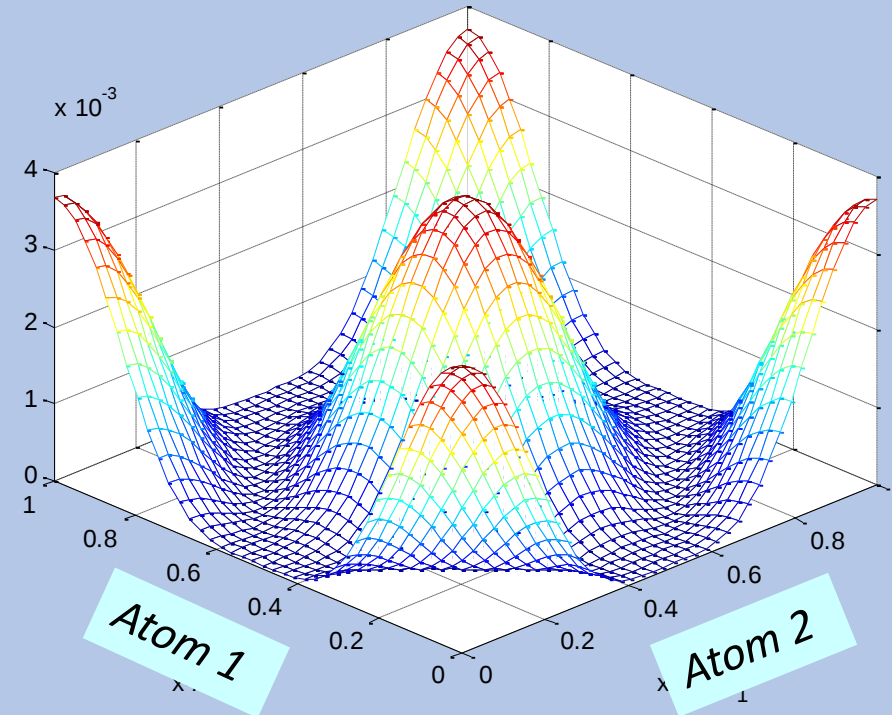
$R = 0$ for random atomic distribution

$R \sim Ng$ for regular lattice (superradiance = $n \sim N^2$)

Two scatterers placed along cavity axis



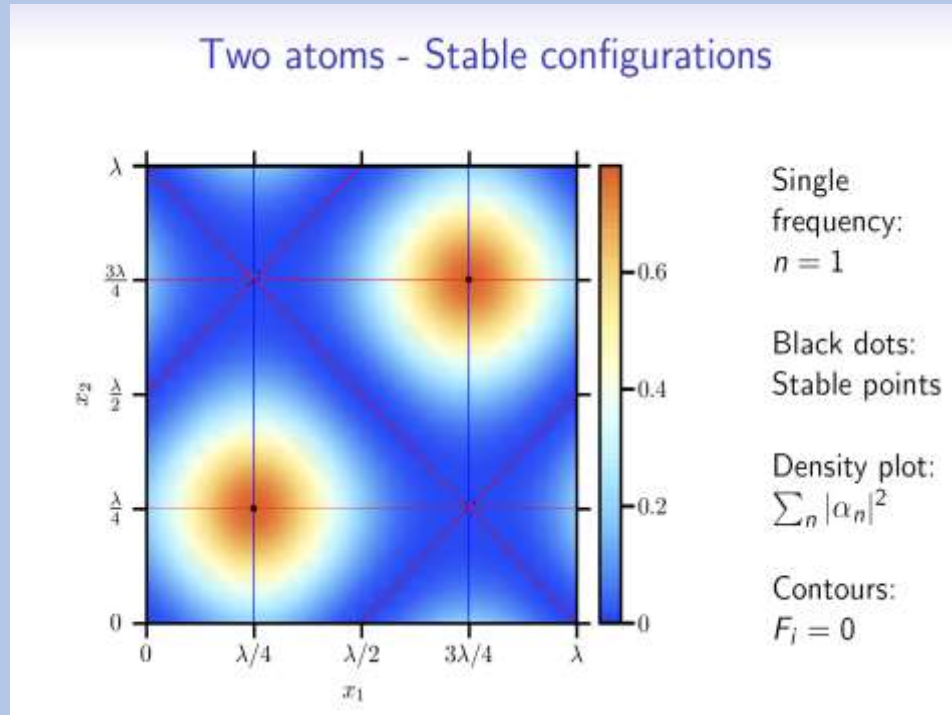
Cavity field as a function of atom position



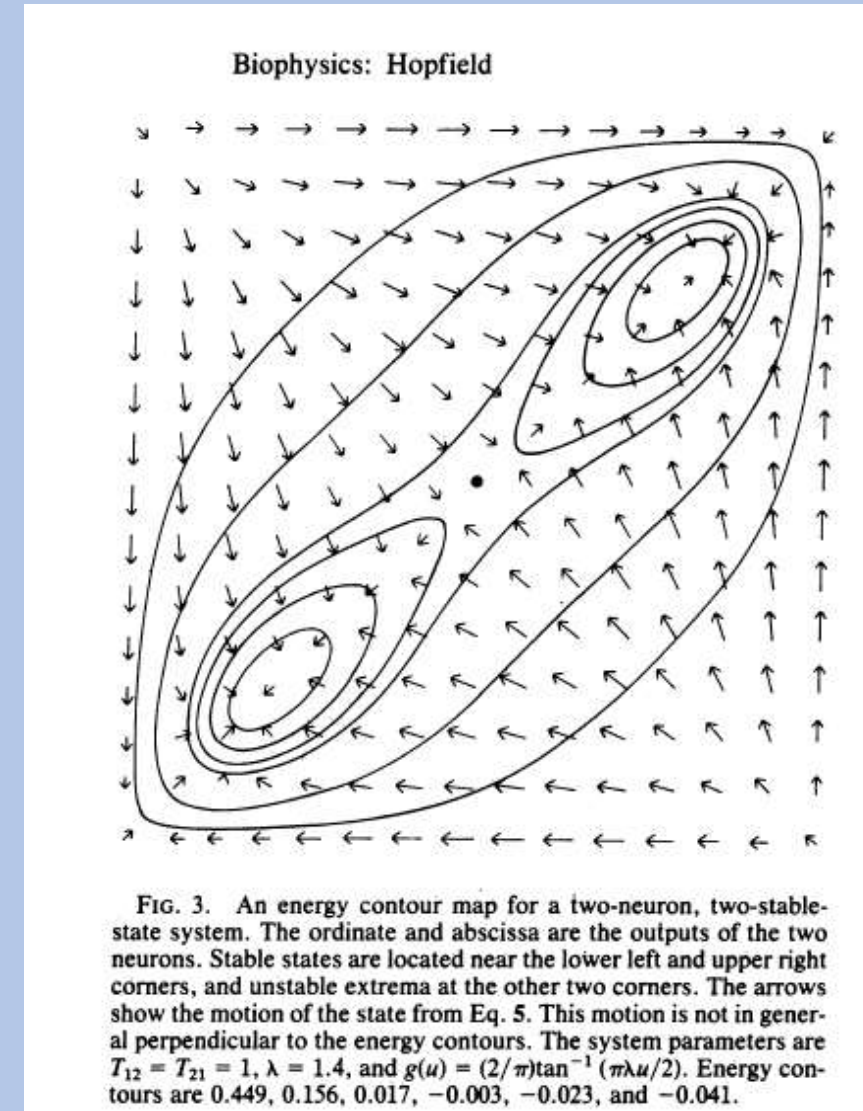
Maximum photon number for 0 and λ distance
Minimum photon number for $\lambda/2$ distance

$\Rightarrow \lambda$ distance gives maximal field
 $\Rightarrow$ for high field seekers
ordering is energetically favorable

optical potential and force field on high field seeking atoms:

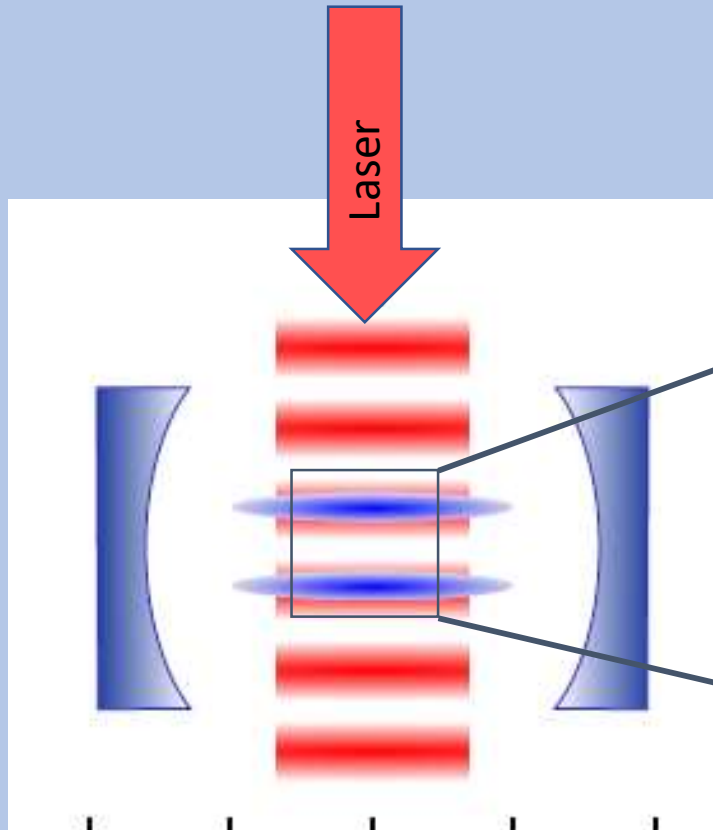


Z_2 symmetry !

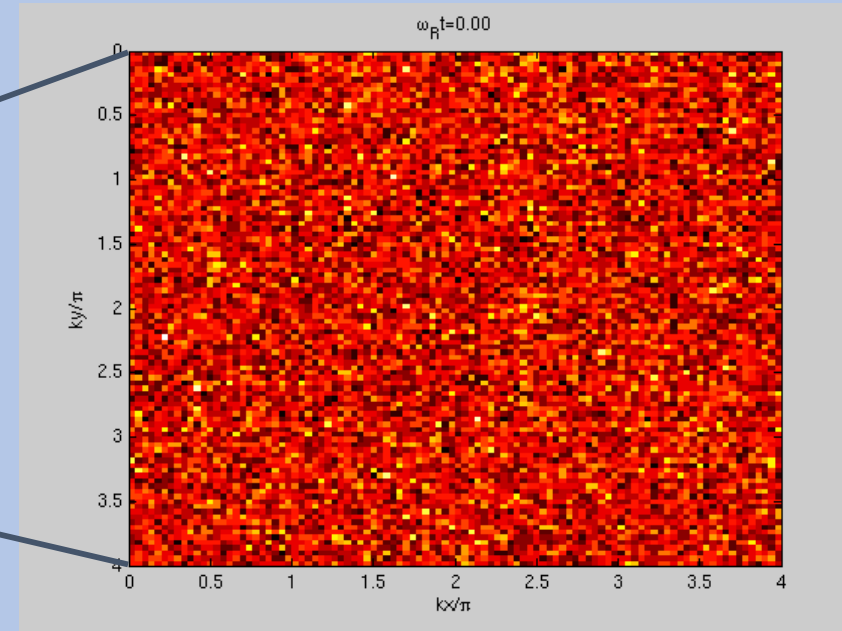


picture taken from "old" Hopfield paper

Laser-illuminated Atoms between Mirrors



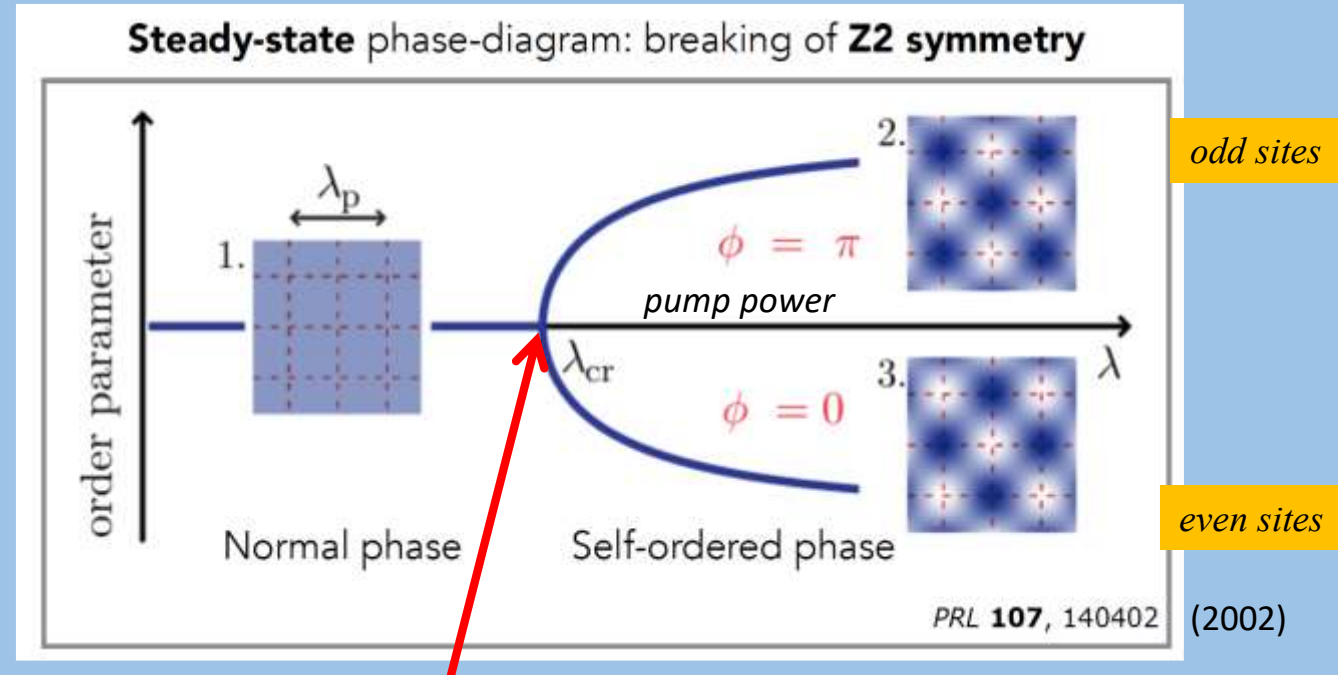
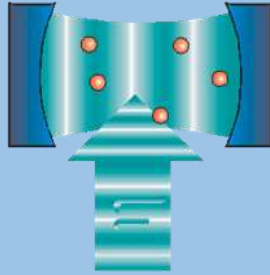
random cloud of particles or BEC



for strong laser power the particles form a crystal
which scatters many photons into the resonators

Collective enhancement through order !

Selfconsistent atom-field distribution



critical point: Z_2 – symmetry breaking

$$U_0 N V_{opt} > \kappa k_B T$$

frequency shift of cavity

pump laser opt. potential

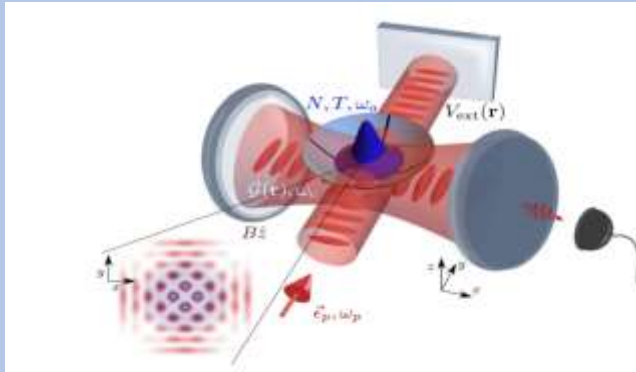
cavity damping

temperature

Selforganisation is an *open system phase transition*

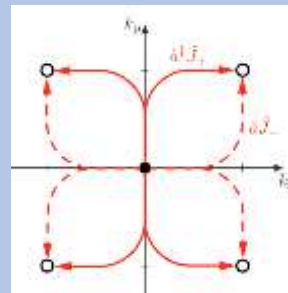
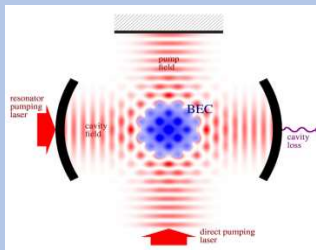
*Selforganisation is an **open system phase transition***
*Is it a **quantum phase transition** at $T = 0$?*

experiment at ETH: selforganization of a BEC at $T \sim 0$



$$H = -\Delta_C a^\dagger a + \int_0^L \Psi^\dagger(x) \left[-\frac{\hbar}{2m} \frac{d^2}{dx^2} + U_0 a^\dagger a \cos^2(kx) + i\eta_t \cos kx (a^\dagger - a) \right] \Psi(x) dx,$$

Two-mode BEC approximation
=> Tavis-Cummings model



$$\Psi(x) = \frac{1}{\sqrt{L}} c_0 + \sqrt{\frac{2}{L}} c_1 \cos kx$$

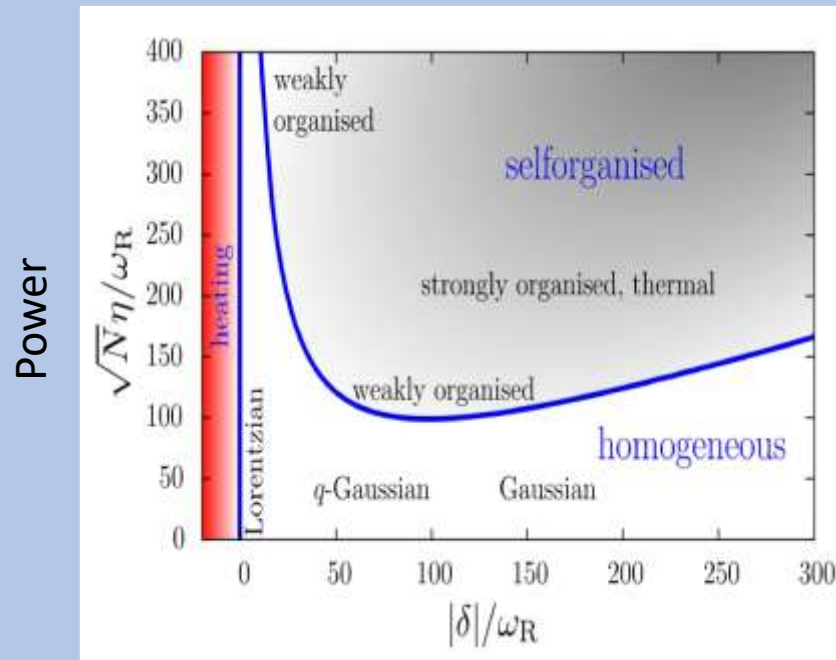
Effective Tavis-Cummings Hamiltonian

$$H = -\delta_C a^\dagger a + \omega_R \hat{S}_z + iy(a^\dagger - a) \hat{S}_x / \sqrt{N} + ua^\dagger a \left(\frac{1}{2} + \hat{S}_z / N \right)$$

large $N \Rightarrow$ Quantum Simulation
of
„Dicke Superradiant Phase Transition“
(predicted by Hepp+Lieb 1973)

compare **classical** vs **quantum** phase diagram

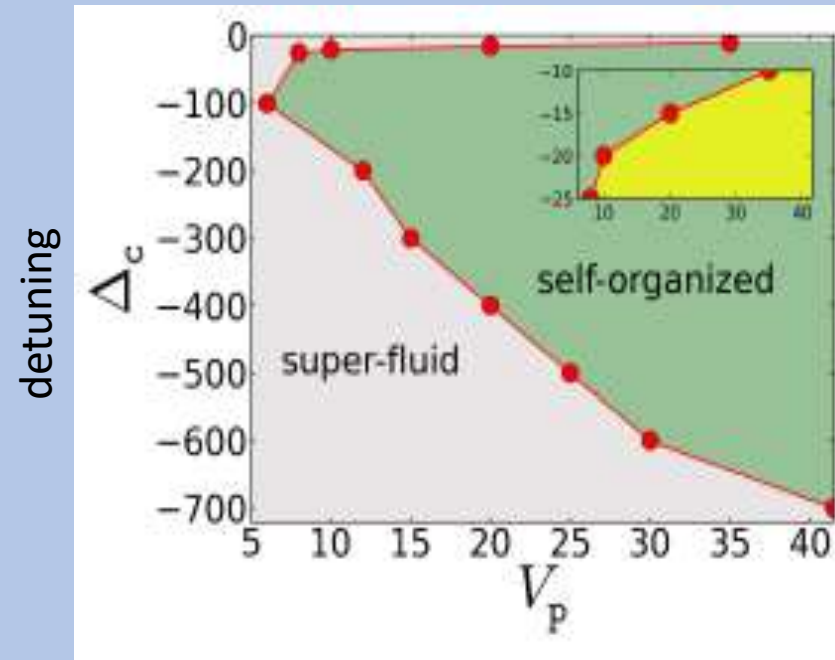
classical gas $T \gg 0$



detuning

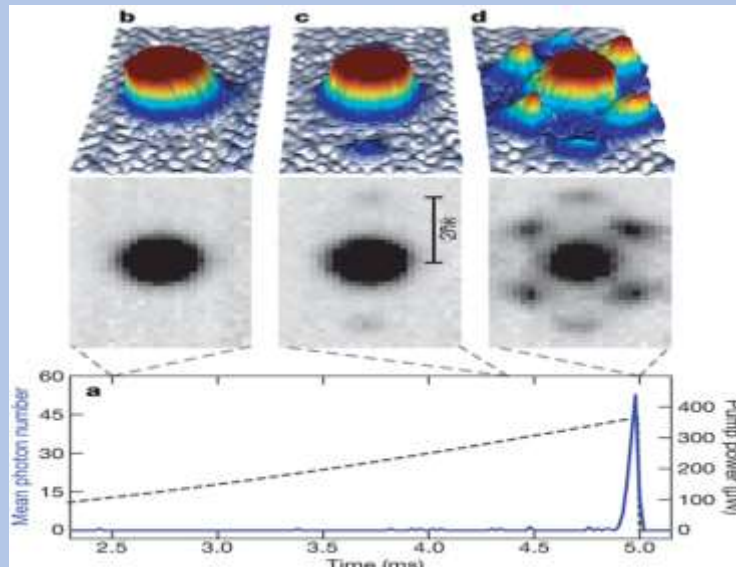
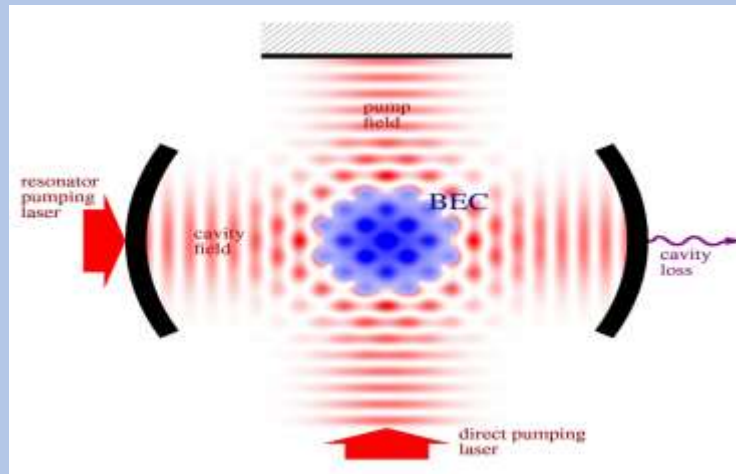
G. Morigi, H.R. 2004
W. Niedenzu, H.R., 2011

quantum gas $T=0$

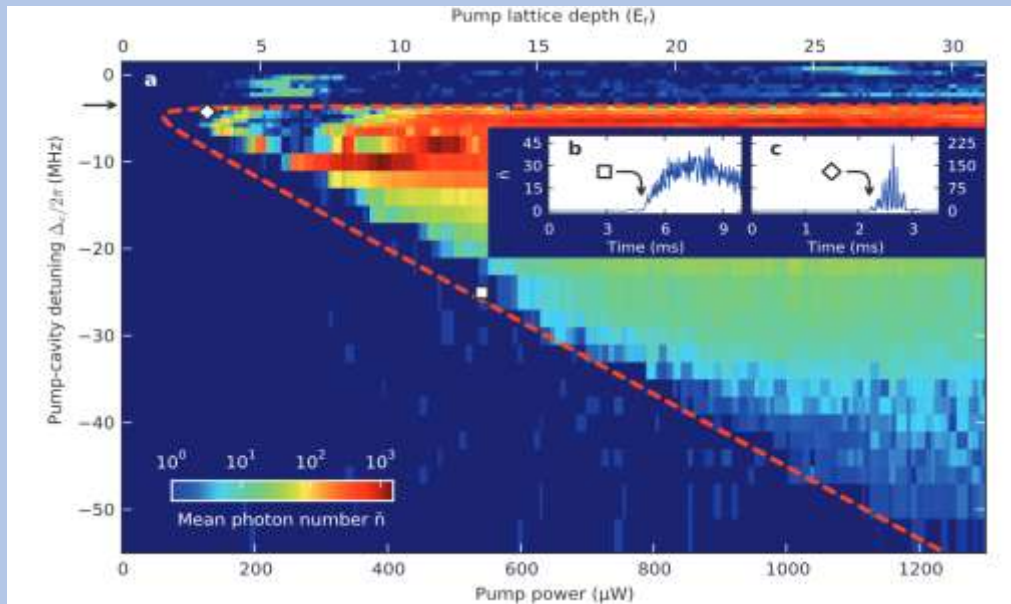


Power

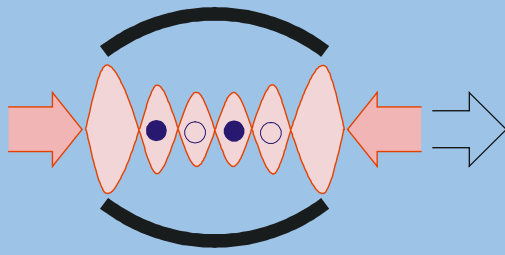
Domokos, Esslinger, Donner 2010
Hofstetter 2013, Thorwart 2014
Piazza, Zwerger, 2013 (fermions)



Experiment follows prediction
of
Dicke Superradiant Phase Transition



„quantum simulation“ of $T=0$ phase transition
with Z_2 symmetry breaking
predicted by Hepp+Lieb 1973



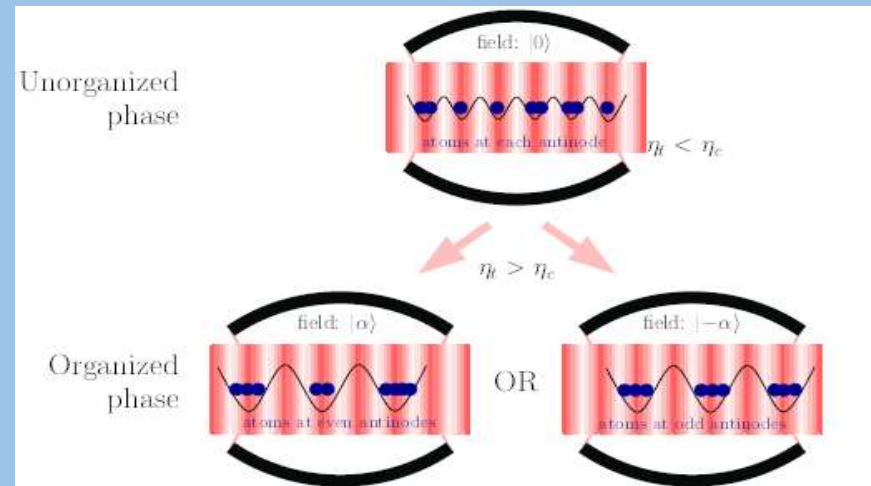
- atoms in an optical lattice in cavity
- cavity field from scattered lattice light => interactions

Effective Hubbard type Hamiltonian:

$$H = \sum_{k,l} E_{k,l} b_k^\dagger b_l + \hbar U_0 \eta' g \sum_{k,l} J_{k,l} b_k^\dagger b_l + \hbar \eta' (a + a^\dagger) \sum_{k,l} \tilde{J}_{k,l} b_k^\dagger b_l - \hbar (\Delta_c - U_0) a^\dagger a$$



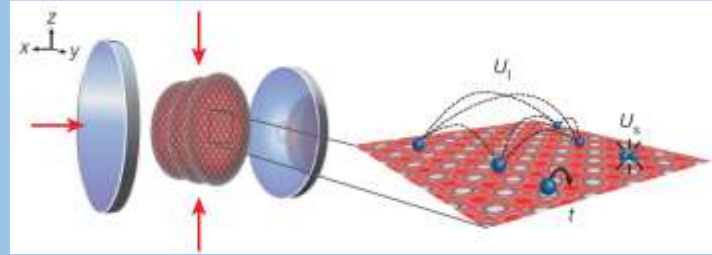
cavity field determined by atomic distribution



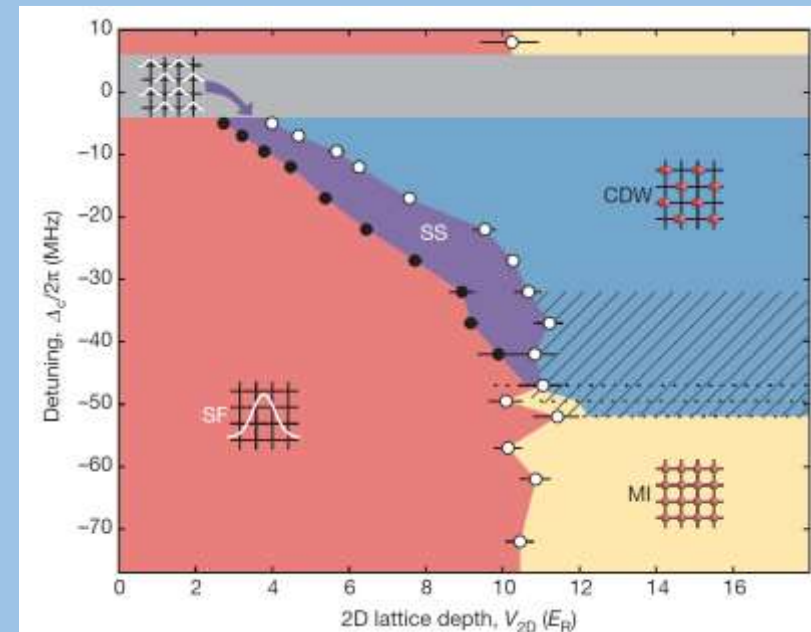
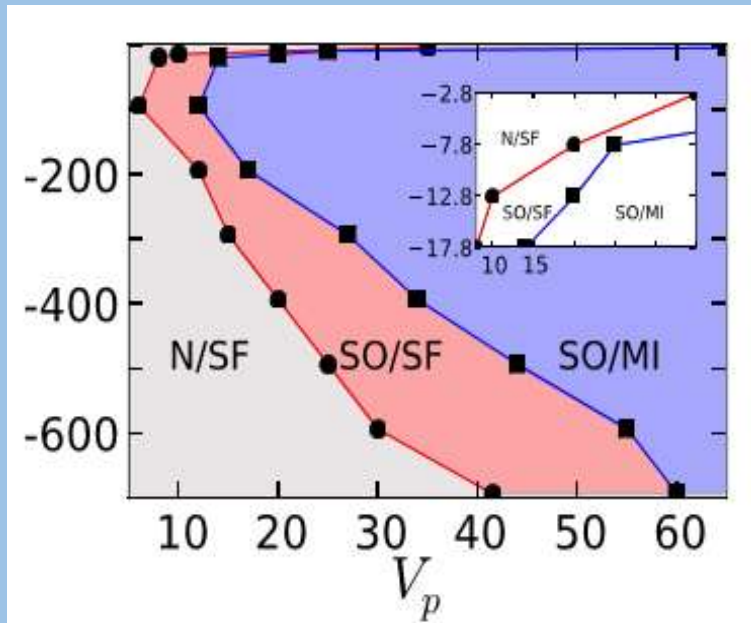
Z₂ – Symmetry Breaking

quantum regime (BH): self ordering phase transition in optical lattice

Theory



Experiments: ETH (Hamburg, Stanford)



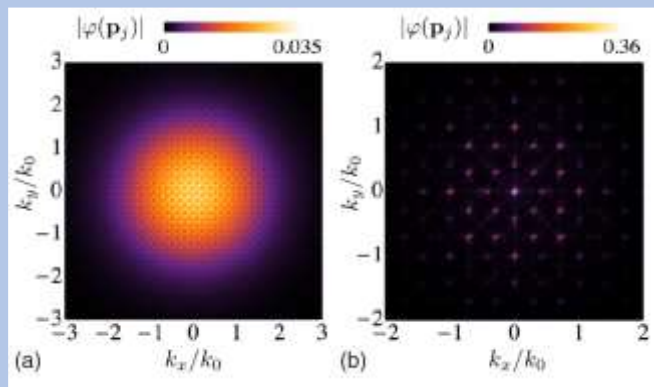
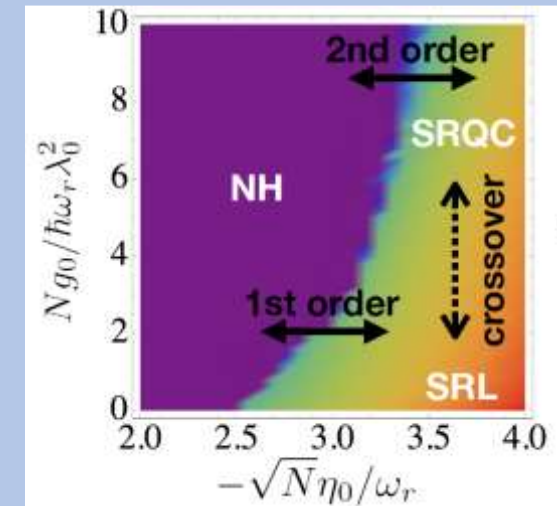
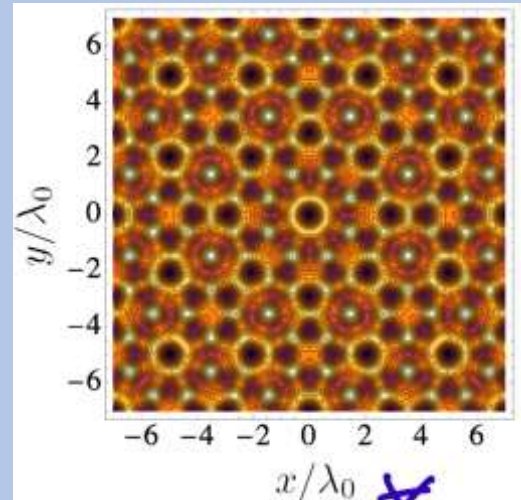
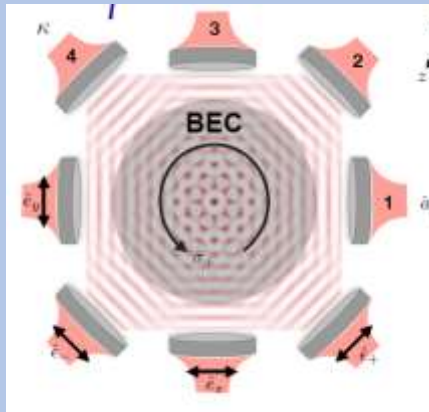
intermediate phase with coherence + diagonal order => „supersolid“

W. Hofstetter (2010), F. Piazza ,
R. Bakhtiari, M. Thorwart, HR (2014)
G. Morigi 2016, A. Lode 2017,
C. Kollath 2019, 2020

ETH: T. Donner, Nature 533, 2016
Hamburg: A. Hemmerich, PRL 2016
Stanford: B. Lev, PRL 2016
EPFL: J.P. Brantut, Nature 2021
Tübingen: Zimmermann, PRL 2021

collective scattering in more complex symmetries

4-fold rotation symmetry: self ordering in 4 cavities



Systems above threshold attains C-8 symmetry

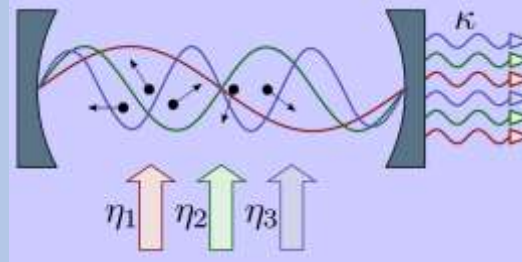
$\Rightarrow$

quasi-crystal formation

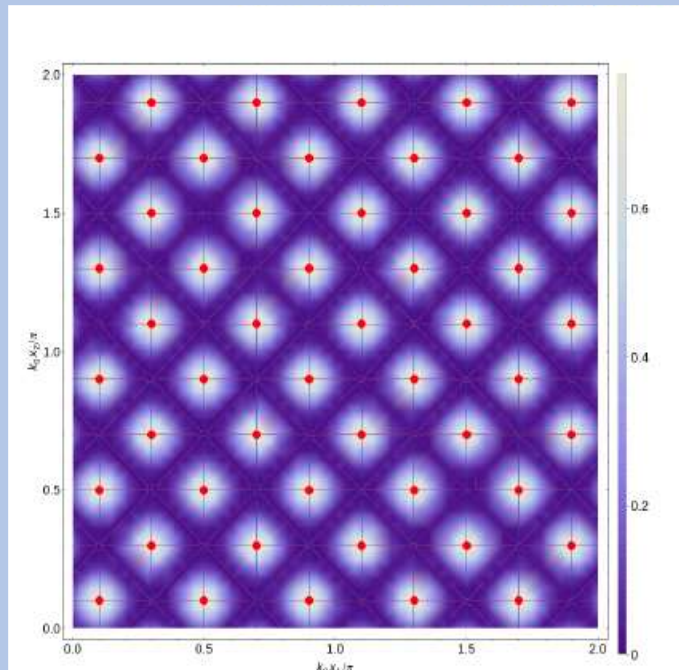
$\Rightarrow$

emerging new symmetry

Self-ordering with multicolor light (RGB)

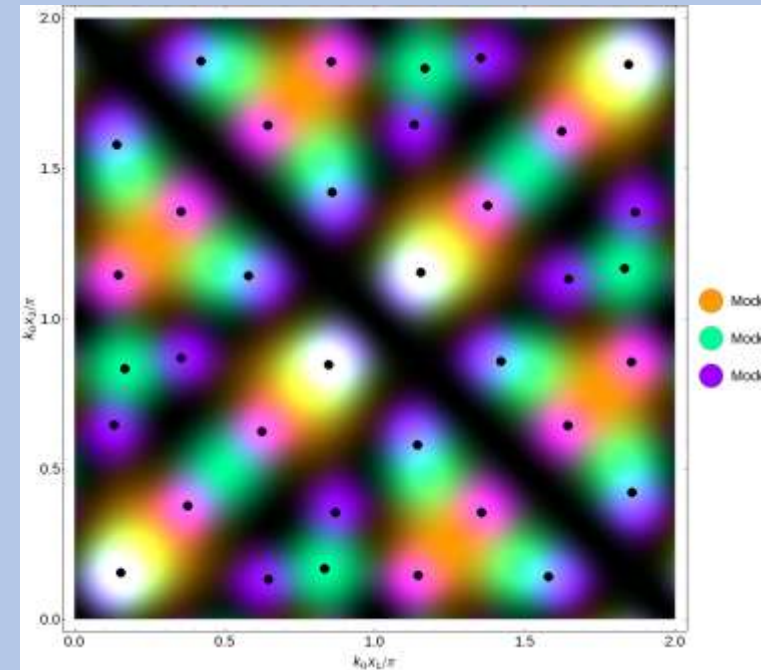


monochrome



*regular crystal maximizes
light collection*

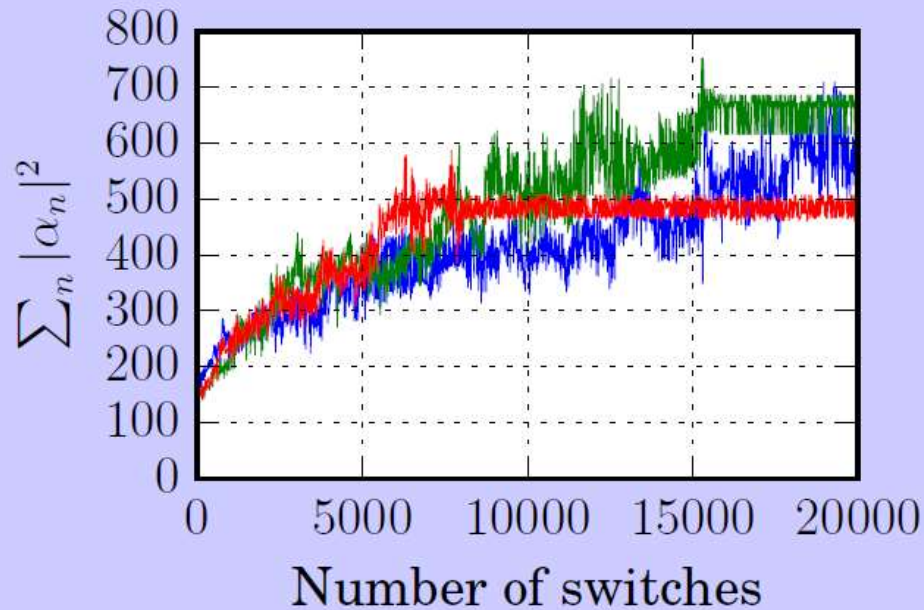
RGB - Laser



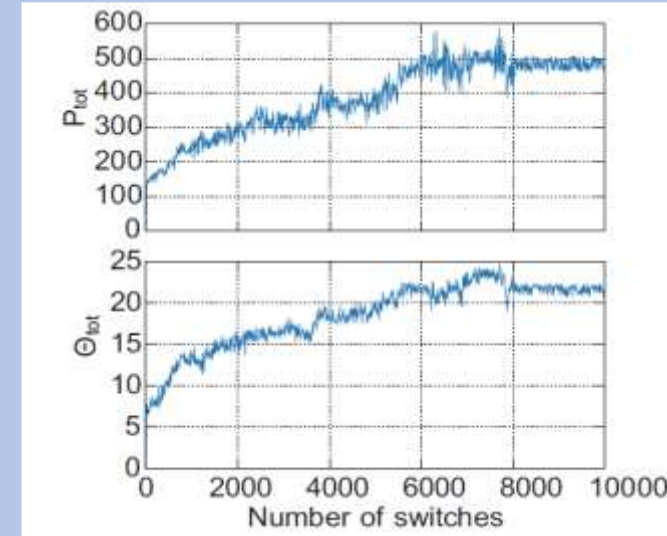
*irregular order optimizes
light collection for all colors
=> „new solar cells ? “*

Adaptive dynamics in time-varying illumination

Time evolution of 100 particles in time-varying illumination. We choose 5 illuminations, each consisting of about 50 pump lasers close to high order modes ($n > 1000$), which are applied in a **random sequence**.



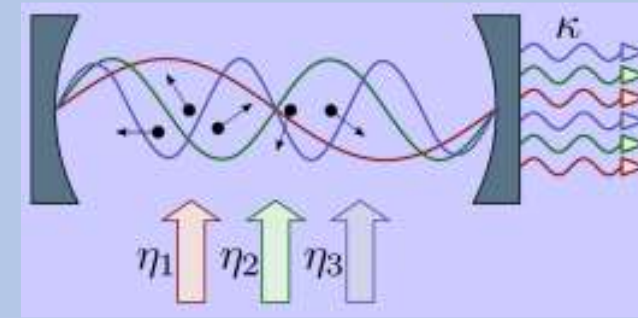
Sum of order parameters:



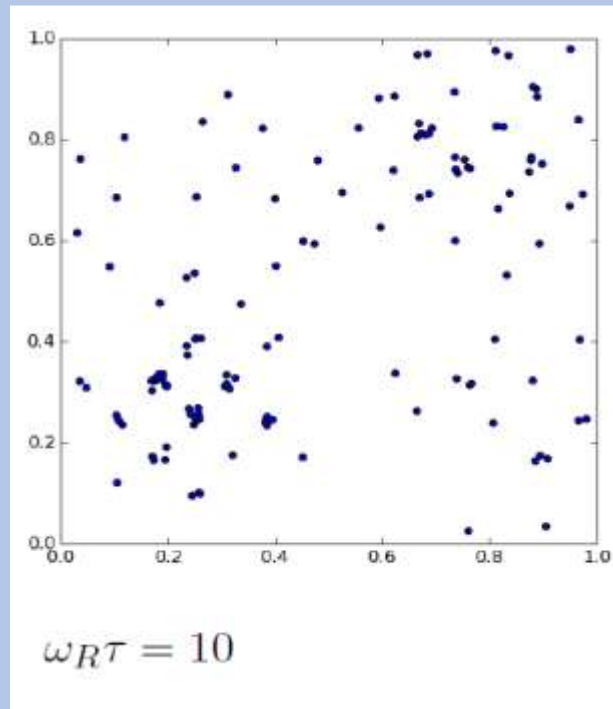
*system optimizes
scattering and „learns“
from the past*

adaptive + learning light collection system

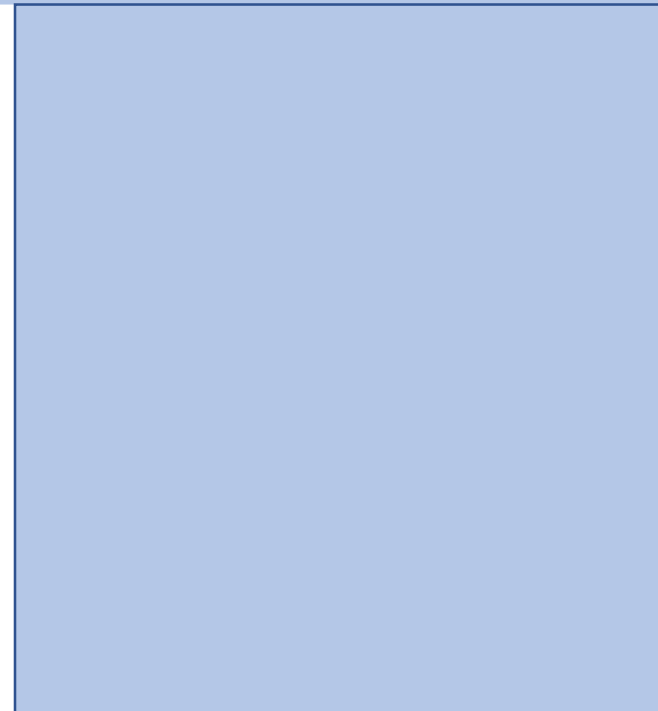
„dissipative“ annealing
=>
slow turn on 2.+ 9. mode illumination



fast switch on



slow switch on

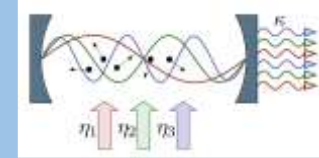


system converges mostly to states optimal for all modes

Interacting trapped **quantum** particles within a **multimode** cavity

particle-field
Hamiltonian

$$H = -J \sum_i (b_{i+1}^\dagger b_i + b_i^\dagger b_{i+1}) + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i + 1) - \hbar \sum_m \delta_{c,m} a_m^\dagger a_m + \hbar \sum_m \eta_m (a_m + a_m^\dagger) \sum_i v_m^i \hat{n}_i$$



Effective Hamiltonian after field elimination

$$H = -J \sum_i (b_{i+1}^\dagger b_i + b_i^\dagger b_{i+1}) + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i + 1) - \mu \sum_{i,j} A_{ij} \hat{n}_i \hat{n}_j$$

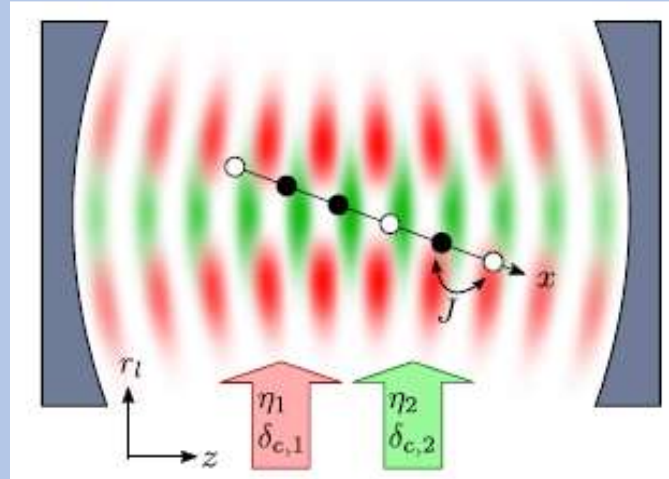
with the real and symmetric interaction matrix

$$A = \sum_m f_m \text{Re}(\mathbf{v}_m \otimes \mathbf{v}_m^*) / \mu$$

yes, we can engineer general coupling matrices A_{ij}
but: and needs $\sim N^2$ pump lasers for N sites

Quantum annealing in a cavity lattice

$T = 0$



*general coupling matrix A
can be constructed
via pump mode design*

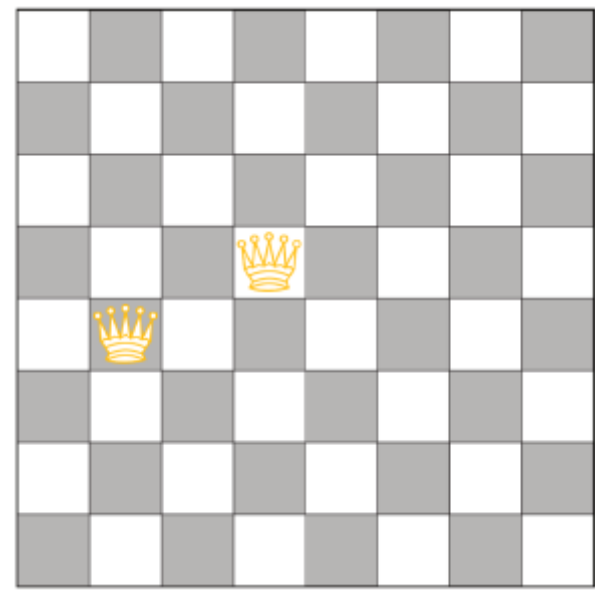
$$H = -J \sum_i (b_{i+1}^\dagger b_i + b_i^\dagger b_{i+1}) + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i + 1) - \mu \sum_{i,j} A_{ij} \hat{n}_i \hat{n}_j$$

with the real and symmetric interaction matrix

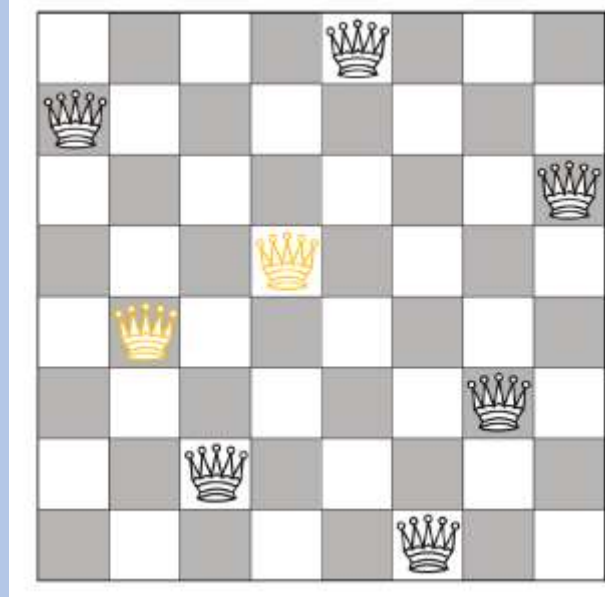
$$A = \sum_m f_m \operatorname{Re}(\mathbf{v}_m \otimes \mathbf{v}_m^*) / \mu$$

A quantum N-Queens simulator

N Queens problem:



a solution



Is it difficult ?

Total # of queen configurations:

$$\binom{8^2}{8} \approx 4 \times 10^9$$

Placed queens + 1 queen / row:

$$8^6 = 261,144$$

N-Queens completion scales exponentially with hard instances ($N > 21$)

=> NP - complete

I. Gent, C. Jefferson, and P. Nightingale,
J. Artif. Intell. Res. **59**, 815 – 848 (2017)

Optimization problem → minimize a cost function.

Minimize $f(s)$ subject to $s \in S$

$f(s) : D \rightarrow \mathbb{R}$

D ... configuration space

S ... set of feasible solutions (constraints)

Mathematics:

Cost function $f(s)$



Physics:

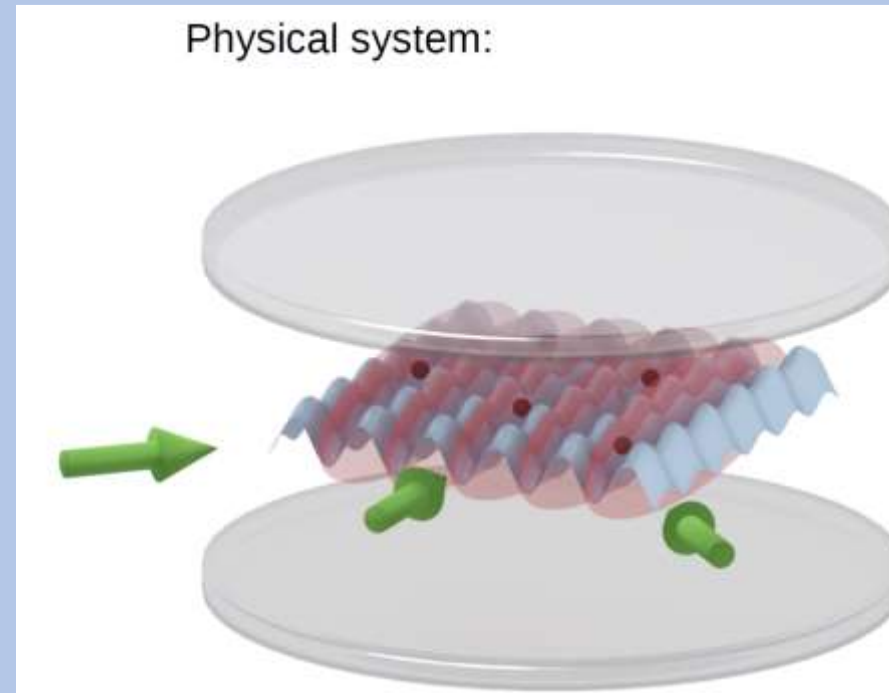
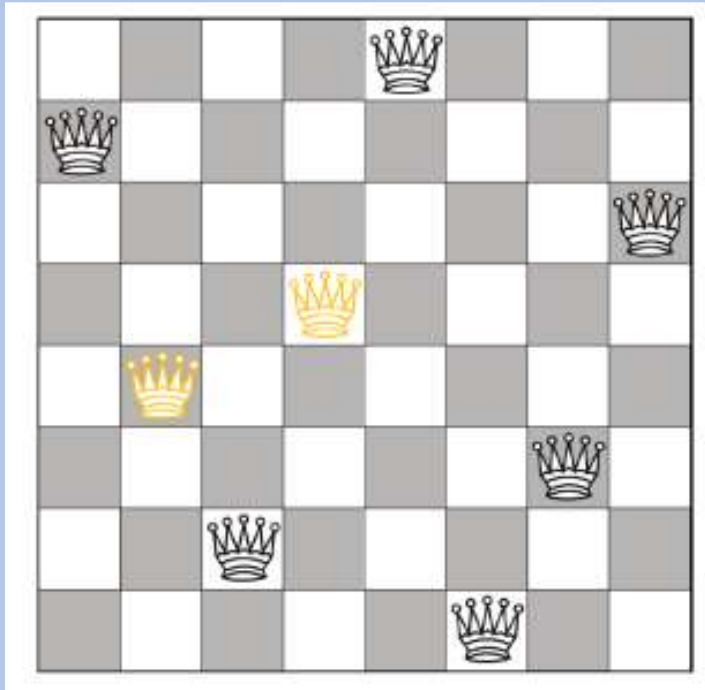
Energy $E(s)$

Solve the problem.



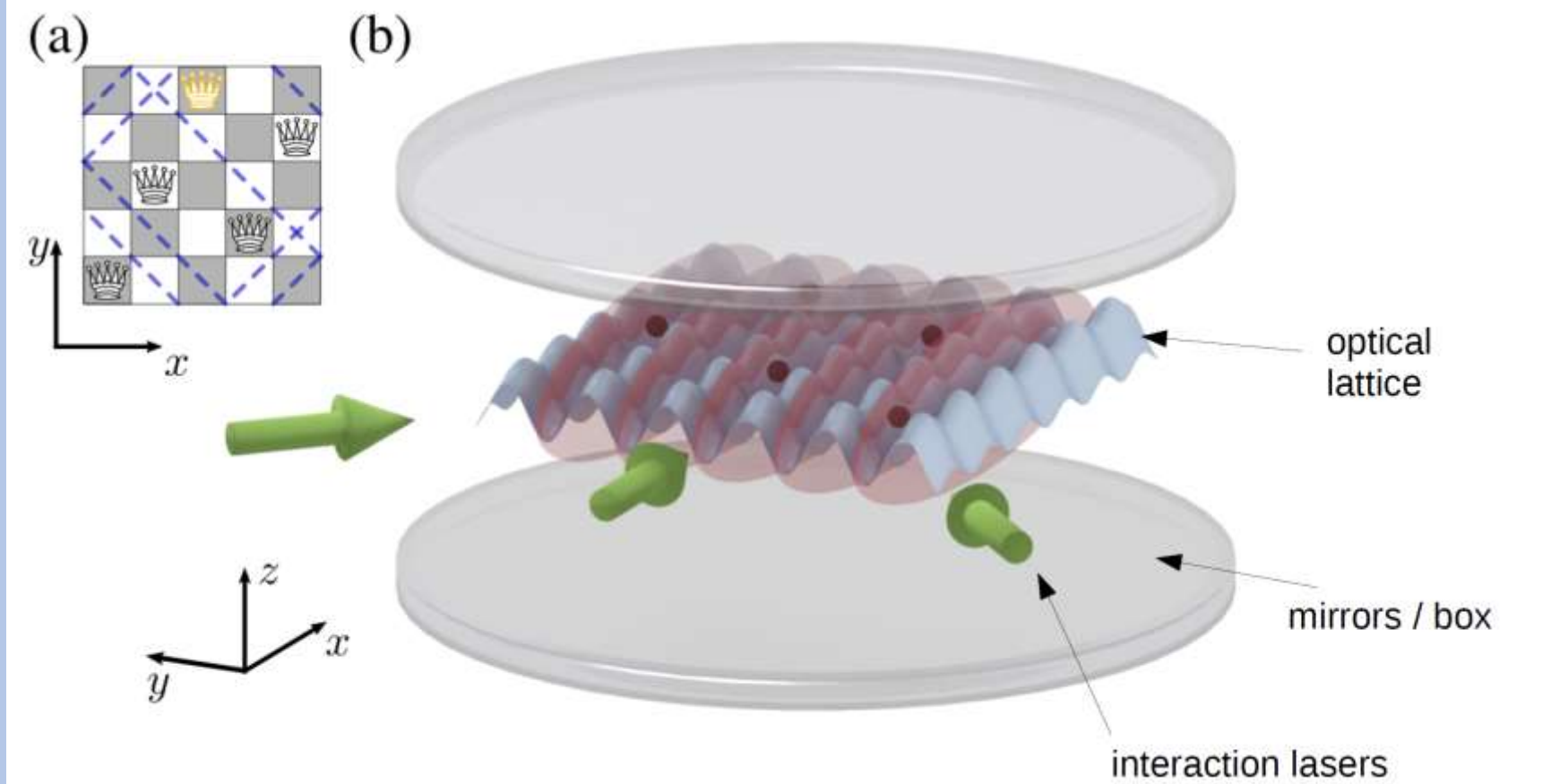
Find the ground state.

Quantum Optimization for Classical Problems : The N-Queens Problem



Queens	=> Atoms
Chess Board	=> 2D optical lattice between mirrors
Chess rules	=> Laser beams to create interactions

N-Queens completion is NP-complete = exponentially difficult !

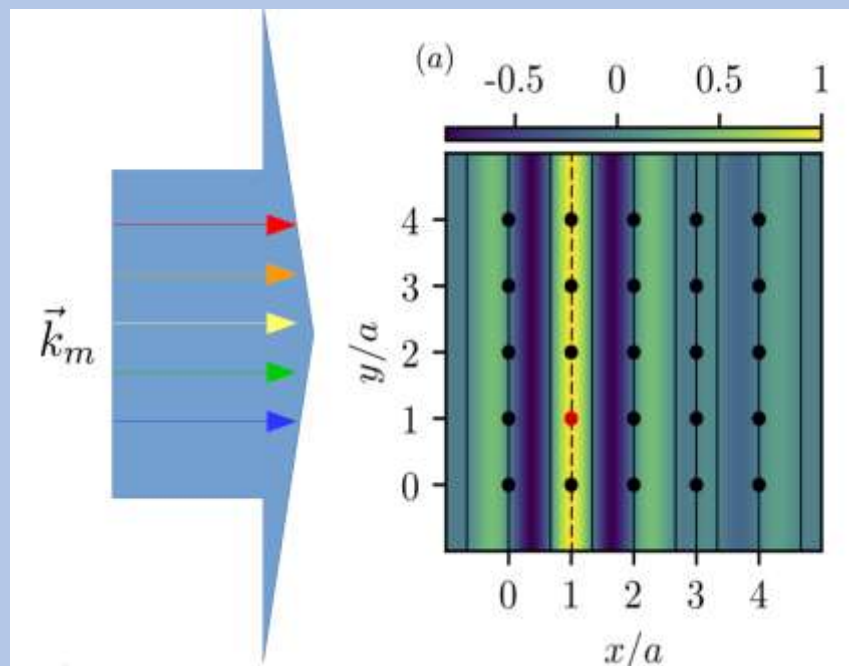


$$V_{\text{interaction}}(x_1, x_2) \propto \Delta_c \sum_{i=1}^2 \sum_{j=1}^2 \cos(k(x_i - x_j))$$

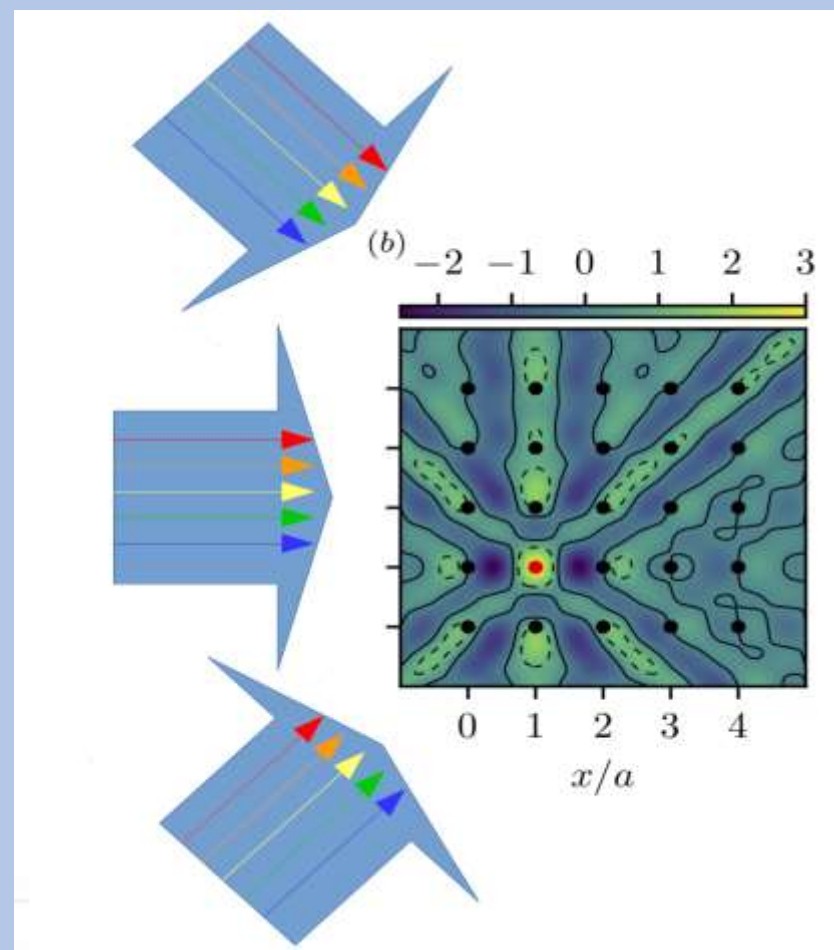
Interaction between lattice site (i,j) and (k,l):

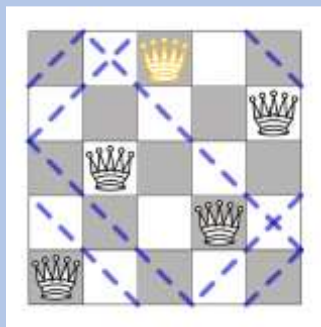
$$A_{ijkl} = \sum_m f_m \cos(\vec{k}_m(\vec{x}_{ij} - \vec{x}_{kl}))$$

one laser direction



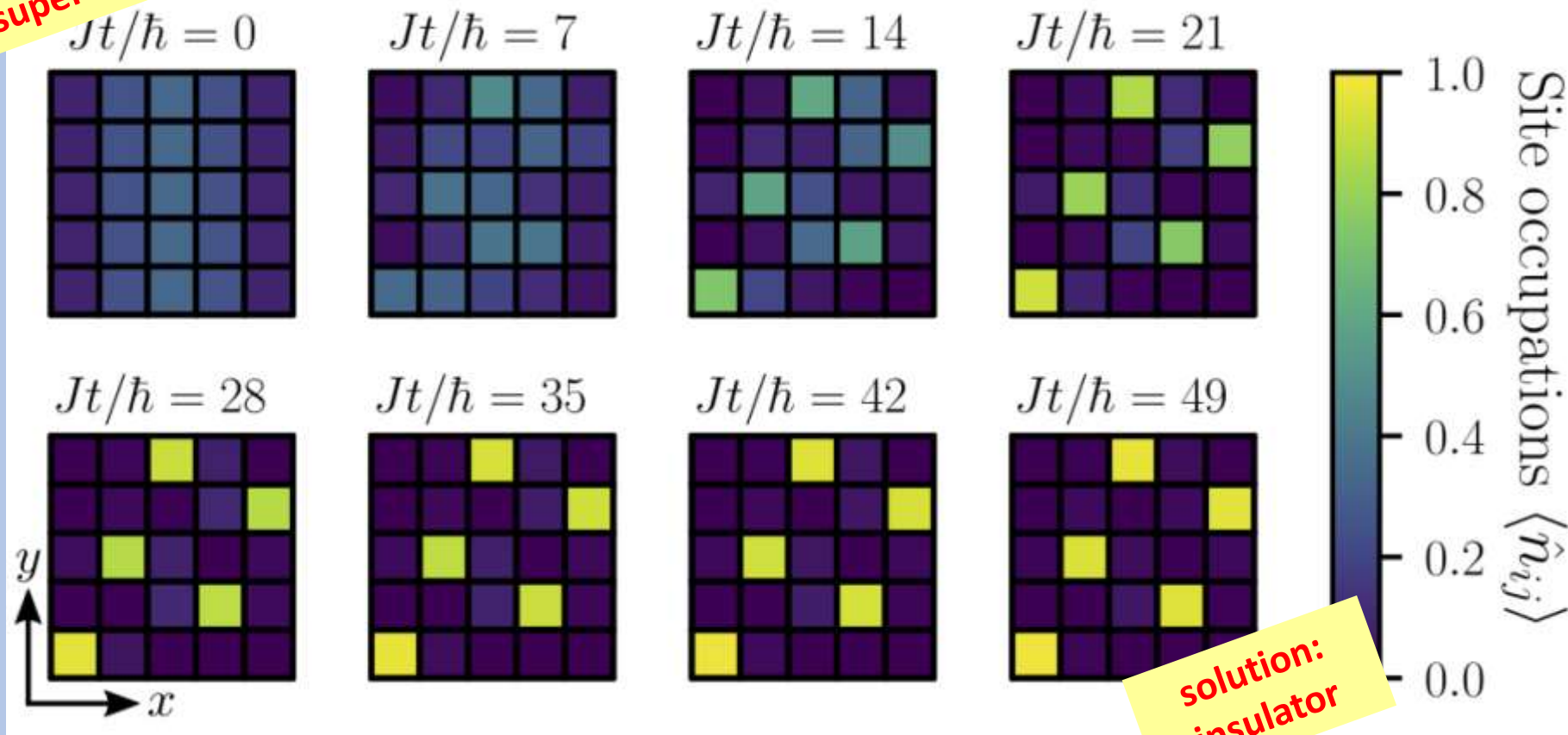
three laser directions





Lasers slowly turned on
=> atoms converge to solution

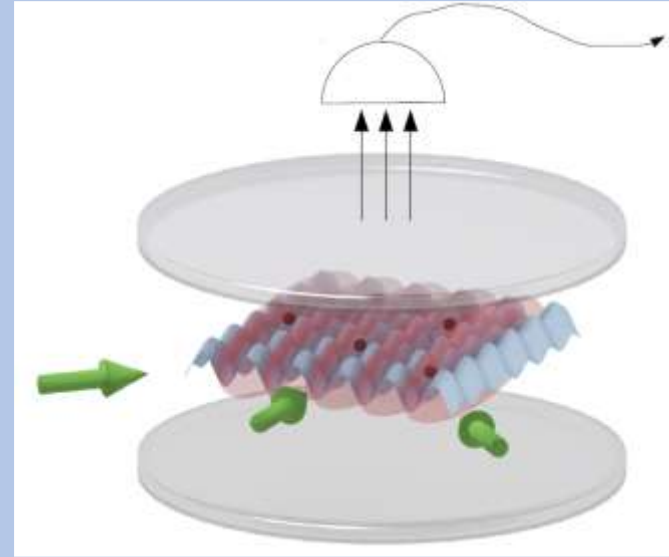
superfluid



solution:
insulator

VT, P. Aumani, H. Ritsch, and W. Lechner
"A quantum N-queens solver",
arXiv:1803.00735 [quant-ph] (2018)

readout:
cavity output field
shows appearance
of a solution



Suitable for testing quantum advantage.

N-queens problem:

→ there exist hard instances for $N > 21$

Implementation fits problem naturally:

→ No qubit overhead – as many atoms as queens

→ Laser resources scale linearly with N

Disclaimer: polynomial scaling of gap has to be shown !



Open source quantum dynamics software

www.qojulia.org



Quantum Optics toolbox in *Julia*

```
a = destroy(bc) * one(ba)
σ- = one(bc) * sigmam(ba)

# Construct Hamiltonian
H = Δ*dagger(a)*a + g*(dagger(a)*σ- + a*dagger(σ-))

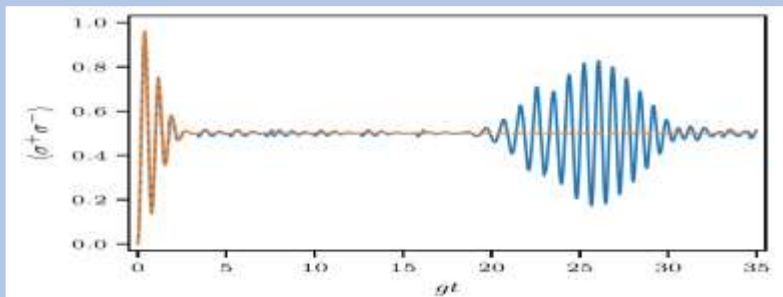
# Define initial state
ψ0 = coherentstate(bc, α) * spindown(ba)

# Define list of time steps
T = [0:0.01:35;]

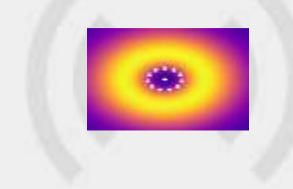
# Evolve in time according to Schrödinger's equation
tout, ψi = timeevolution.schroedinger(T, ψ0, H)

# Calculate atomic excitation
excitation = expect(dagger(σ-)*σ-, ψi)
```

Code sample 1: Jaynes-Cummings model.



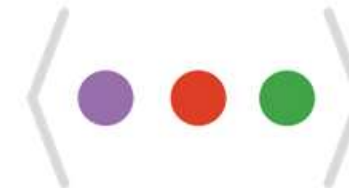
dipole-dipole package



CollectiveSpins.jl

Simulate Dipole-Dipole Coupled Spin Systems

<https://qojulia.github.io/CollectiveSpins.jl/>



QuantumCumulants.jl

symbolic & numerical package for
automated cumulant expansions

<https://github.com/qojulia/QuantumCumulants.jl>

