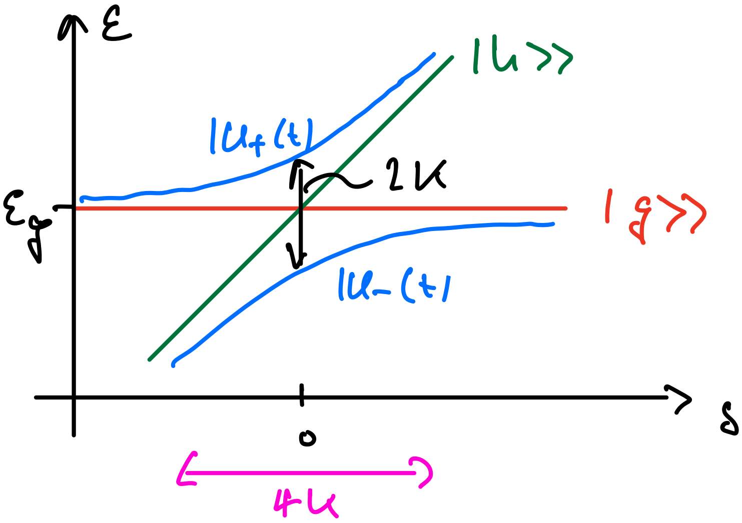
F) Supression of coherent resonant excitation

Lets assume that we have a weak avoided crossing in the quasienergy spectrum between two unperturbed Floquet states [Hone, Ketzmerick, Kohn, PRE 2009]



$$Q \approx \delta\, |h\rangle\rangle\langle\langle h| + \underbrace{K}_{>0 \text{ (for simplicity)}} \left(|h\rangle\rangle\langle\langle g| + h.c.\right)$$

with

$$\varepsilon_\pm = \varepsilon_g + \frac{\delta}{2} \pm \underbrace{\sqrt{K^2 + \delta^2/4}}_{\equiv v \;\to\; K \text{ f. } \delta \to 0}$$

$$|u_\pm\rangle = \sqrt{\frac{v \pm \delta/2}{2v}}\, |g\rangle \pm \sqrt{\frac{v \mp \delta/2}{2v}}\, |h\rangle$$

where

$$\sqrt{\frac{v \pm \delta/2}{2v}} \quad \begin{cases} \xrightarrow{\delta \to -\infty} & \begin{smallmatrix} 0 \\ 1 \end{smallmatrix} \\ \xrightarrow{\delta \to 0} & 1/\sqrt{2} \\ \xrightarrow{\delta \to \infty} & \begin{smallmatrix} 1 \\ 0 \end{smallmatrix} \end{cases} .$$

Here $|g\rangle\rangle$ and $|h\rangle\rangle$ shall correspond to the ground state $|\tilde{g}\rangle$ and a highly excited eigenstate $|\tilde{h}\rangle$ of $H_F^{(HF)}$ with energies $\varepsilon_h = \varepsilon_g + m\omega + \delta$, i.e.

$$|g(t)\rangle = \mathcal{U}_F^{HF}(t)\,|\tilde{g}\rangle$$
$$|h(t)\rangle = e^{-im\omega t}\,\mathcal{U}_F^{HF}(t)\,|\tilde{h}\rangle .$$

In the high-frequency regime $|g\rangle$ and $|h\rangle$ are very different, since $m\omega$ is large. Therefore, the coherent resonant coupling K results from some high-order perturbation theory and is the smallest energy scale in the problem. For such a scenario, we can make a number of assumptions:

The width of the avoided crossing in δ direction is of the order of K and therefore also tiny. The states $|g\rangle$ and $|h\rangle$ therefore hardly vary over this distance and can be assumed δ-independent:

$$|g_\delta(t)\rangle \simeq |g_0(t)\rangle \equiv |g(t)\rangle$$
$$|h_\delta(t)\rangle \simeq |h_0(t)\rangle \equiv |h(t)\rangle$$

Moreover, since the states $|g\rangle$ and $|h\rangle$ are structurally very different (as they have very different energies, we also have

$$A_{ge}^{(\mu)} \simeq 0 \quad \text{or} \quad \langle g|H_{SE}|e\rangle \simeq 0,$$

while $A_{gg}^{(\mu)}$ and $A_{ee}^{(\mu)}$ remain relevant.

Similarly, any other state $j \neq g, h$, is either very different from g or from h, so that

$$A_{gj}(m)\,A_{jh}(m) \approx 0 \qquad \forall\, j.$$

Near the avoided crossing, the quasienergies vary very little on the small scale κ, so that we can assume

$$g(\mu + \varepsilon_{ij}) \approx g(\mu + \varepsilon_g) = \text{const.},$$

which should be fine as long as

$$\kappa \tau_E \ll 1.$$

The crossing shall be well isolated, so that

$$\Gamma_{SE} \ll \Delta E_{other},$$

where ΔE_{other} is the quasienergy distance from other states. Via the arguments of the secular approximation (see previous chapter) for $\Gamma_{SE} \ll \Delta E_{other}$ the steady state has no coherent superposition between $|g\rangle$ or $|e\rangle$ with other states.

We can now introduce the basis of unperturbed Floquet modes $|\alpha\rangle$

$$\{|\alpha\rangle\} = \{|g\rangle, |u\rangle, \text{ other modes}\}$$

and expand the Redfield equation in terms of them.

For the dissipator, we find the same form as for the perturbed basis

$$\langle\alpha|\mathcal{D}[\varrho]|\beta\rangle = -\sum_{\eta\nu}\left[\bar{R}^{(0)}_{\alpha\eta,\nu\eta}\varrho_{\nu\beta} - \bar{R}^{(0)}_{\nu\beta,\eta\alpha}\varrho_{\eta\nu} + c.c.\right].$$

where

$$\bar{R}^{(0)}_{\kappa\beta,\eta\nu} = \sum_{\mu} A^{(-\mu)}_{\kappa\beta} A^{(\mu)}_{\eta\nu}\, g(\mu+\varepsilon_{\eta\nu}).$$

In turn, for the left-hand-side of the equation, we find (where i,j is still the perturbed basis)

$$\langle\alpha|\dot{\varrho}|\beta\rangle = \sum_{ij}\langle\alpha|\Big(|u_i\rangle\dot{\varrho}_{ij}\langle u_j|$$

$$+ \underbrace{|\dot{u}_i\rangle}_{(i\varepsilon_i+\frac{1}{i}H)|u_i\rangle}\varrho_{ij}\langle u_j| + |u_i\rangle\varrho_{ij}\underbrace{\langle\dot{u}_j|}_{\langle u_j|(-i\varepsilon_j-\frac{1}{i}H)}\Big)|\beta\rangle$$

(as derived earlier)

$$= \sum_{ij}\langle\alpha|u_i\rangle\left(\dot{\varrho}_{ij} + i\varepsilon_{ij}\varrho_{ij}\right)\langle u_j|\beta\rangle + \langle\alpha|\tfrac{1}{i}[H,\varrho]|\beta\rangle,$$

The last term cancels again with the identical term on the right-hand side, describing the coherent evolution.

Moreover, in the quasi steady state $\dot{\varrho}_{ij} = 0$, so that we need to consider only

$$\sum_{i,j} \langle \alpha | u_i \rangle \, i \varepsilon_{ij} \, \varrho_{ij} \, \langle u_j | \beta \rangle$$

$$= i \, 2v \left(\langle \alpha | u_+ \rangle \, \varrho_{+-} \langle u_- | \beta \rangle - \langle \alpha | u_- \rangle \, \varrho_{-+} \langle u_+ | \beta \rangle \right),$$

where we have used

$$\varepsilon_{ij} = \begin{cases} \varepsilon_+ - \varepsilon_- = 2v \,, & i = + \,, \; j = - \\ 0 \,, & i = j \\ \varepsilon_- - \varepsilon_+ = -2v \,, & i = - \,, \; j = + \,. \end{cases}$$

Furthermore

$$\begin{aligned} \varrho_{+-} = \varrho^*_{-+} &= \langle u_+ | \varrho | u_- \rangle \\ &= \underbrace{\langle u_+ | g \rangle}_{\alpha} \underbrace{\langle g | \varrho | g \rangle}_{\varrho_{gg}} \underbrace{\langle g | u_- \rangle}_{\beta} \\ &\quad + \underbrace{\langle u_+ | g \rangle}_{\alpha} \underbrace{\langle g | \varrho | h \rangle}_{\varrho_{gh}} \underbrace{\langle h | u_- \rangle}_{-\alpha} \\ &\quad + \underbrace{\langle u_+ | h \rangle}_{\beta} \underbrace{\langle h | \varrho | g \rangle}_{\varrho_{hg}} \underbrace{\langle g | u_- \rangle}_{\beta} \\ &\quad + \underbrace{\langle u_+ | h \rangle}_{\beta} \underbrace{\langle h | \varrho | h \rangle}_{\varrho_{hh}} \underbrace{\langle h | u_- \rangle}_{-\alpha} \\ &= \alpha\beta \, (\varrho_{gg} - \varrho_{hh}) + \alpha^2 \underbrace{\varrho_{gh}}_{\varrho^*_{hg}} + \beta^2 \underbrace{\varrho_{hg}}_{\varrho^*_{gh}} \,, \end{aligned}$$

$\mathbb{1} = |g\rangle\langle g| + |h\rangle\langle h|$
within the subspace

where we introduced

$$\alpha = \sqrt{\frac{r+\delta/2}{2r}} = \langle g|u_+\rangle = -\langle h|u_-\rangle$$

$$\beta = \sqrt{\frac{r-\delta/2}{2r}} = \langle h|u_+\rangle = \langle g|u_-\rangle,$$

with $r = \sqrt{\kappa^2+\delta^2/4}$. With that and equating the expression to the dissipator, we finally obtain for the quasi-steady state

$$\langle\alpha|\mathring{\rho}|\beta\rangle =$$

$$i2r\Big[\langle\alpha|u_+\rangle\langle u_-|\beta\rangle\big(\alpha\beta(\rho_{gg}-\rho_{hh}) + \alpha^2\rho_{gh} + \beta^2\rho_{hg}\big)$$

$$-\langle\alpha|u_-\rangle\langle u_+|\beta\rangle\big(\alpha\beta(\rho_{gg}-\rho_{hh}) + \alpha^2\underbrace{\rho^*_{gh}}_{\rho_{hg}} + \beta^2\underbrace{\rho^*_{hg}}_{\rho_{gh}}\big)\Big]$$

$$= \langle\alpha|D[\rho]|\beta\rangle$$

$$= -\sum_{\eta\nu}\Big[\bar{R}^{(0)}_{\alpha\eta,\nu\eta}\rho_{\nu\beta} - \bar{R}^{(0)}_{\nu\beta,\eta\alpha}\rho_{\eta\nu} + \text{c.c.}\Big]$$

Let us now, for the sake of simplicity neglect all states, but $|g\rangle$ and $|h\rangle$ and evaluate the equation for $\alpha = g$ and $\beta = h$.

The left-hand-side then simplifies to

$\langle g|\dot{\rho}|h\rangle$

$$= i2v\Big[-\alpha^2\big(\alpha\beta(\rho_{gg}-\rho_{hh})+\alpha^2\rho_{gh}+\beta^2\rho_{hg}\big)$$
$$-\beta^2\big(\alpha\beta(\rho_{gg}-\rho_{hh})+\alpha^2\underbrace{\rho^*_{gh}}_{\rho_{hg}}+\beta^2\underbrace{\rho^*_{hg}}_{\rho_{gh}}\big)\Big]$$

$$= i2v\Big[-(\alpha\beta(\rho_{gg}-\rho_{hh})-\underbrace{(\alpha^4+\beta^4+2\alpha^2\beta^2)}_{(\alpha^2+\beta^2)^2=1}\rho_{gh}\Big]$$

$$= -i\Big[\underbrace{\sqrt{v^2-\delta^2/2}}_{=K\equiv\frac{1}{2}E_{gap}}(\rho_{gg}-\rho_{hh})+\underbrace{2v}_{=2\sqrt{K^2+\delta^2/4}\equiv\varepsilon_{+-}}\rho_{gh}\Big],$$

whereas for the dissipator we obtain

$\langle g|D[\rho]|h\rangle$

$$= -\sum_{\eta\nu}\Big[\underbrace{\bar{R}^{(0)}_{g\eta,\nu\eta}}_{\substack{\neq 0\\ \text{f. } \eta=g=\eta}}\rho_{\nu h}-\underbrace{R^{(0)}_{\nu h,\eta g}}_{\substack{\neq 0\\ \text{f. } \nu=h,\ \eta=g}}\rho_{\eta\nu}+c.c.\Big]$$

$$= -\underbrace{\big(\bar{R}^{(0)}_{gg,gg}-R_{hh,gg}\big)}_{\equiv\Gamma=\mathcal{O}(\Gamma_{SE})}\rho_{gh}.$$

Here we have introduce the quasienergy gap of the avoided crossing E_{gap}, the δ-dependent level splitting $\varepsilon_{+-}=\varepsilon_+-\varepsilon_-=2\sqrt{K^2+\delta^2/4}$, as well as the dissipation parameter Γ, which is of the order of Γ_{SE}.

Equating both expressions, we get

$$\varrho_{gh} = \frac{i\,\mathcal{E}_{gap}}{\Gamma - i\,\mathcal{E}_{+-}}\,(\varrho_{gg} - \varrho_{hh})$$

and with that and

$$|\varrho_{gh}| \leq \frac{\mathcal{E}_{gap}}{|\Gamma - i\,\mathcal{E}_{+-}|}\,|\varrho_{gg} + \varrho_{hh}|$$

Thus in the limit

$$\Gamma_{SE} \gg \mathcal{E}_{gap}$$

the density matrix becomes diagonal in the unperturbed basis

$$\begin{pmatrix} \varrho_{gg} & \varrho_{gh} \\ \varrho_{hg} & \varrho_{hh} \end{pmatrix} \longrightarrow \begin{pmatrix} P_g & 0 \\ 0 & P_h \end{pmatrix}.$$

Thus, the dissipation destroys the coherent resonant coupling of the unperturbed states. It can, thus, suppress unwanted Floquet heating, associated with these coherent processes.

Remark:

- The density matrix becomes diagonal also in the limit $\mathcal{E}_{\pm} \gg \mathcal{E}_{gap}$, which corresponds to the limit $|\delta| \gg |\kappa|$, where the unperturbed states $|g\rangle, |h\rangle$ approach the exact Floquet states $|u_+\rangle, |u_-\rangle$.