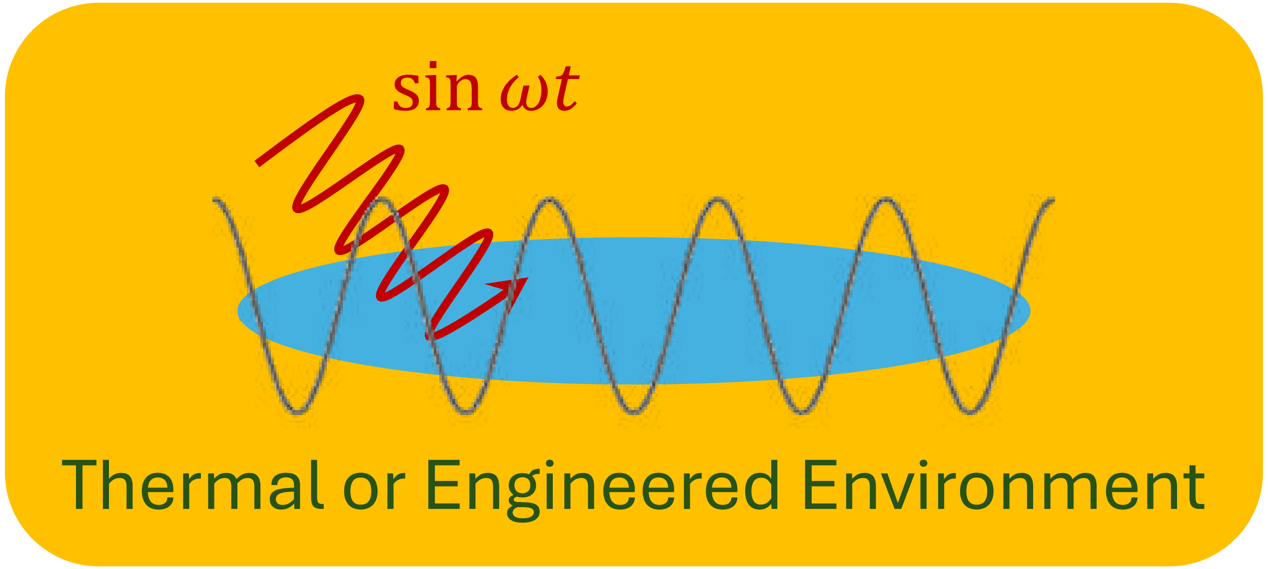
Floquet Engineering of Open Systems



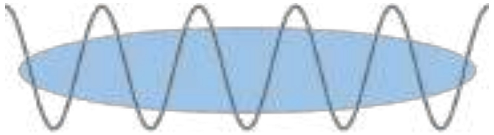
André Eckardt
Technische Universität Berlin

School on Open Quantum Systems

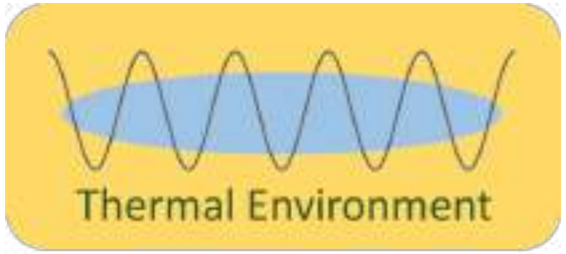
IFT-UNESP, Sao Paulo, August 4-7

Driven-dissipative systems

Thermal equilibrium

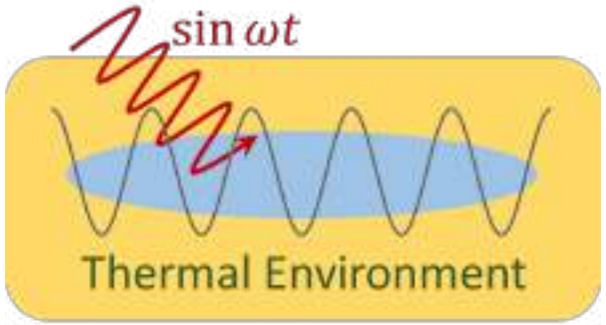


Universal concept: **Statistical Mechanics**



$$\langle \hat{O} \rangle = \mathrm{tr}(\hat{\rho}\hat{O}) \qquad \hat{\rho} = Z^{-1} e^{-\beta \hat{H}}$$

Far from equilibrium



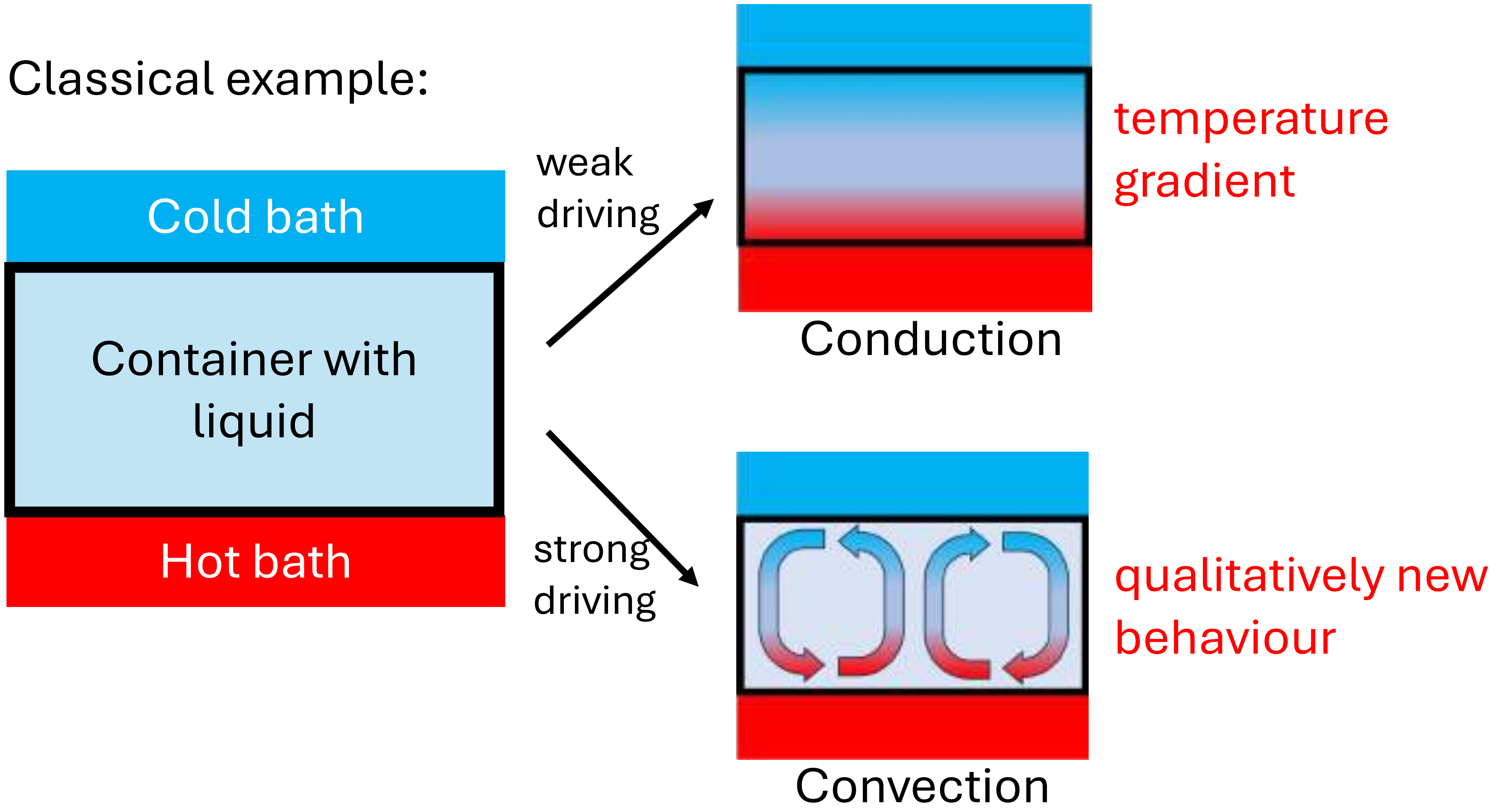
No such universal concept

Often
a challenge

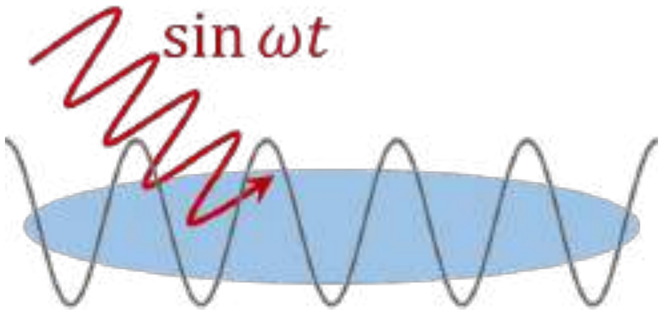
Opportunity
Control states & properties
beyond constraints of equilibrium

Driven-dissipative systems

Classical example:



Floquet systems



$$H(t) = H(t+T) = \sum_\mu H_\mu e^{i\mu\omega t}$$

Floquet states ("Bloch states in time")

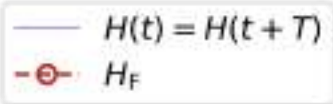
quasienergy ε_n

$$|\psi_n(t)\rangle = |u_n(t)\rangle\, e^{-i\varepsilon_n t/\hbar}$$

$= |u_n(t+T)\rangle$ (Floquet mode)

Time-independent Floquet Hamiltonian

$$H_F = \sum_n \varepsilon_n \, |u_n(0)\rangle\langle u_n(0)|$$

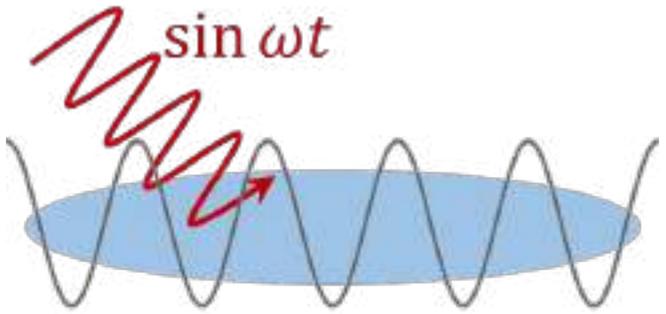


Describes stroboscopic time evolution

$$|\psi(\nu T)\rangle = \exp\left(-\frac{i}{\hbar}\,\nu T H_F\right)|\psi(0)\rangle$$

Control system by engineering H_F ! (?)

Floquet systems



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Quasienergy defined modulo $\hbar\omega$ only

$$\varepsilon_{nm} = \varepsilon_n + m\hbar\omega, \quad |u_{nm}\rangle = |u_n\rangle e^{im\omega t}$$

Time-independent Floquet Hamiltonian

$$H_F = \sum_n \varepsilon_n \, |u_n(0)\rangle\langle u_n(0)|$$

Many-body Floquet modes $|u_n(0)\rangle$
superpositions of very different energies!

H_F describes heating (unlike static Hamiltonian)

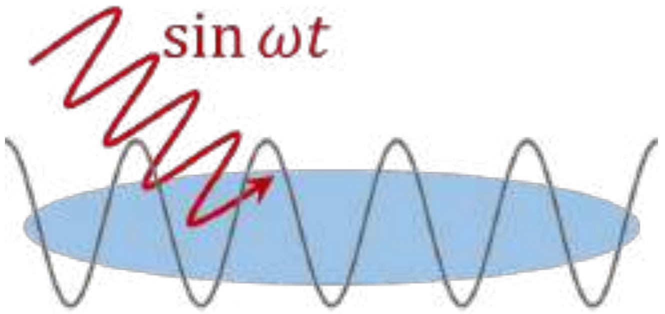
Describes stroboscopic time evolution

$$|\psi(\nu T)\rangle = \exp\left(-\frac{i}{\hbar}\,\nu T H_F\right)|\psi(0)\rangle$$

Not that simple!

Control system by engineering H_F ! (?)

Floquet systems



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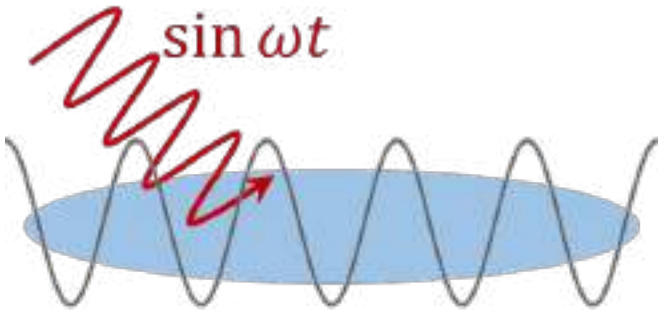
However: Heating can be suppressed on finite time scale:

$$|\psi(\nu T)\rangle \approx e^{-i\,\nu T H_{\mathrm{eff}}/\hbar}\, |\psi(0)\rangle \qquad \text{for} \qquad t < t_{\mathrm{heating}}$$

H_{eff} regular static many-body Hamiltonian approximating H_F

A.E., Weiss, Holthaus PRL 2005

Floquet systems



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$$|\psi_n(t)\rangle = |u_n(t)\rangle\, e^{-i\varepsilon_n t/\hbar} \qquad = |u_{nm}(t)\rangle\, e^{-i\varepsilon_{nm} t/\hbar}$$

H_{eff} **heating**

$\sim t_{\text{heating}}$ time

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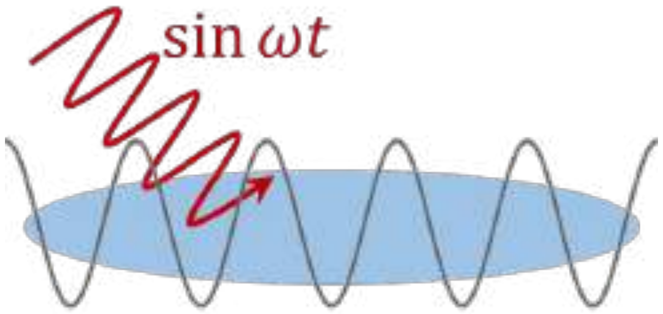

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Eigenvalue problem of **quasienergy operator** Q [Sambe 73]

$$\underbrace{[H(t) - i\hbar\partial_t]}_{Q}\, |u_{nm}\rangle\rangle = \varepsilon_{nm} |u_{nm}\rangle\rangle$$

in Floquet space (time-periodic states)

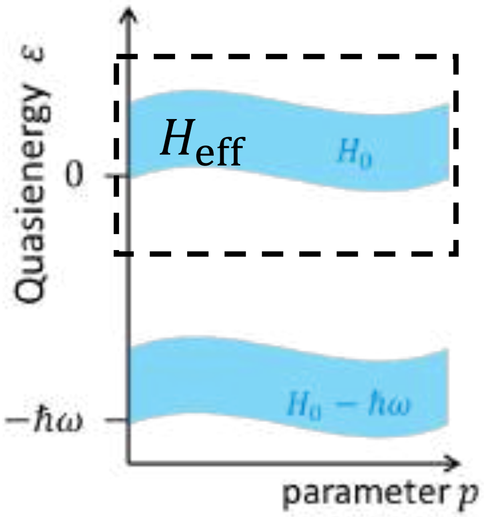
scalar product: $\langle\langle u|v\rangle\rangle = \frac{1}{T}\int_0^T dt\, \langle u(t)|v(t)\rangle$

basis $|\alpha m\rangle\rangle$: $|\alpha\rangle e^{im\omega t}$ (static basis $\{|\alpha\rangle\}$ & $m \in \mathbb{Z}$)

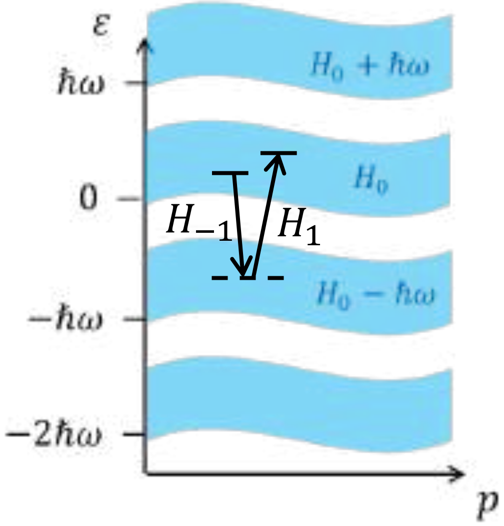
$$\langle\langle \alpha' m' | Q | \alpha m \rangle\rangle = \langle \alpha' | H_{m-m'} | \alpha \rangle + \delta_{m'm}\delta_{\alpha'\alpha} m\hbar\omega$$

$$\begin{matrix} \ddots & & & \\ & \begin{bmatrix} H_0 + \hbar\omega & H_1 & H_2 \\ H_{-1} & H_0 & H_1 \\ H_{-2} & H_{-1} & H_0 - \hbar\omega \end{bmatrix} & \\ & & \ddots \end{matrix}$$

Single particle, finite bandwidth



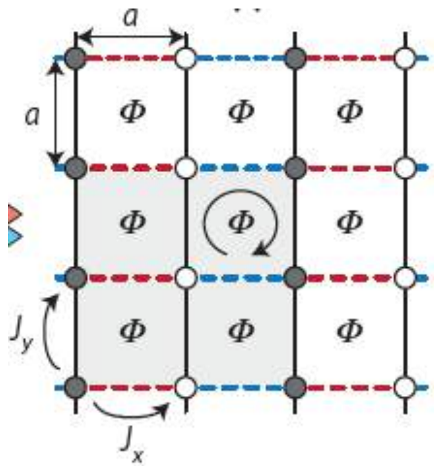
High frequencies



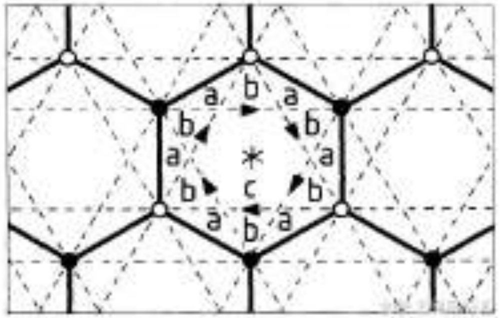
Moderately large freq.

$$H_{\text{eff}} \simeq H_0 = \frac{1}{T}\int_0^T H(t)dt$$

$$H_{\text{eff}} \simeq H_0 + \sum_{m\neq 0} \frac{H_m H_{-m}}{m\hbar\omega} + \cdots$$



Quantum-gas experiments:
Munich, MIT, Harvard



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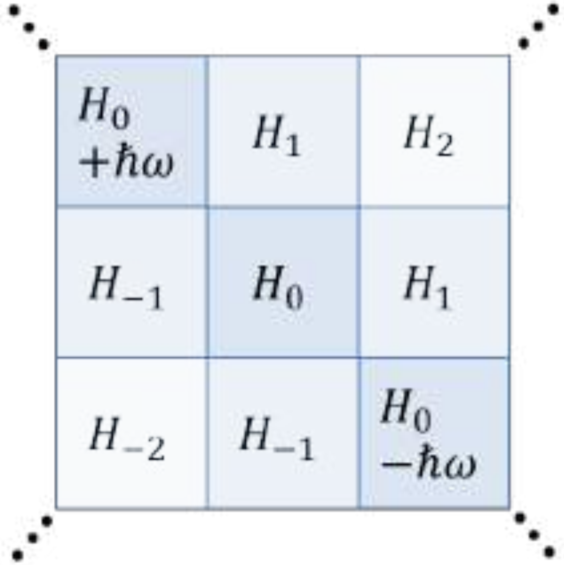

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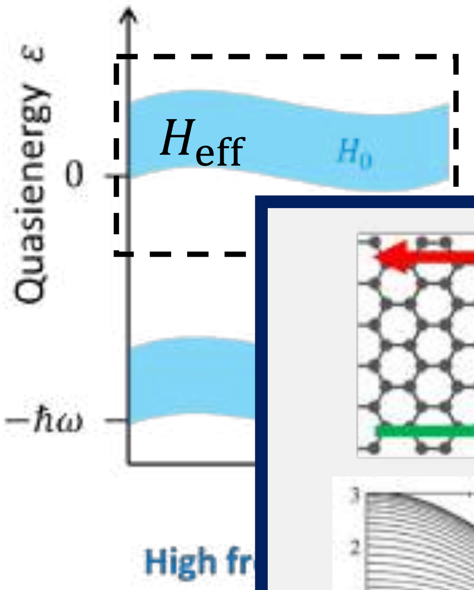
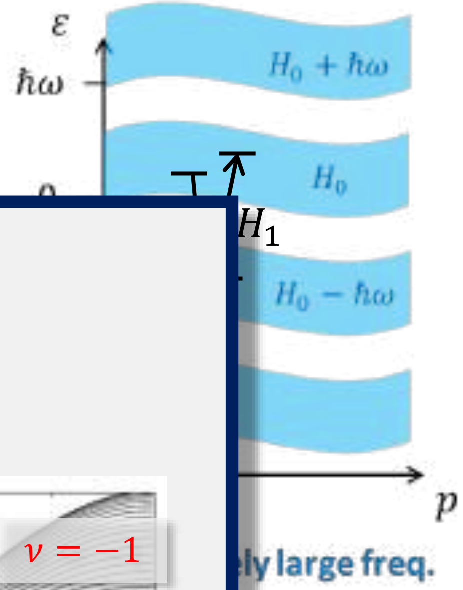
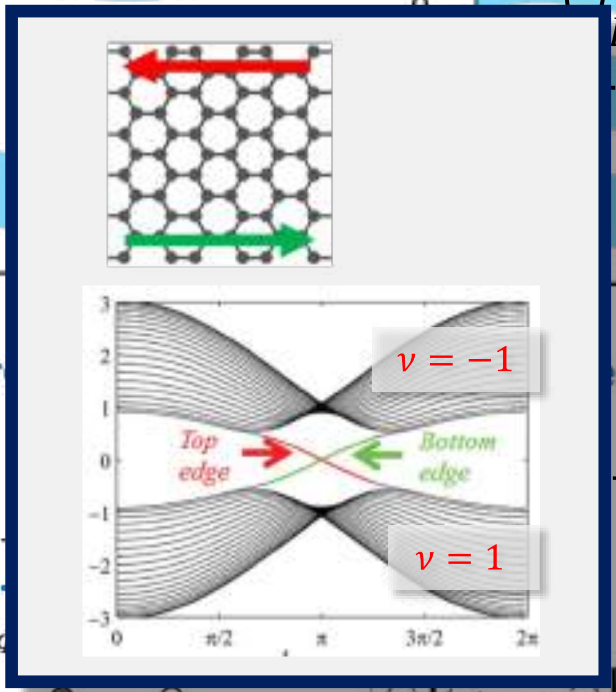
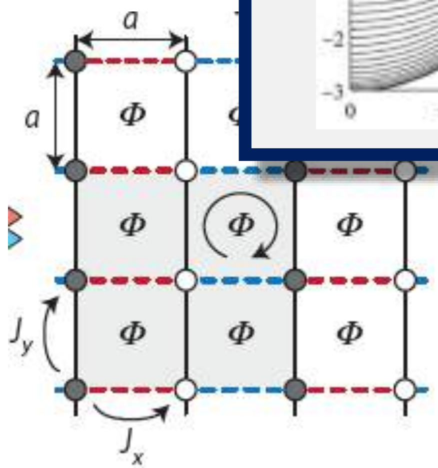






$H_{\mathrm{eff}} \simeq H_0 =$ … $+ \sum_{m\neq 0} \frac{H_m H_{-m}}{m\hbar\omega} + \cdots$



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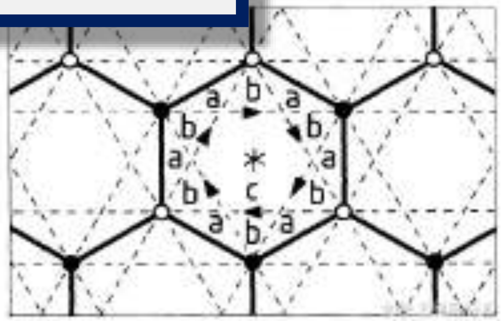

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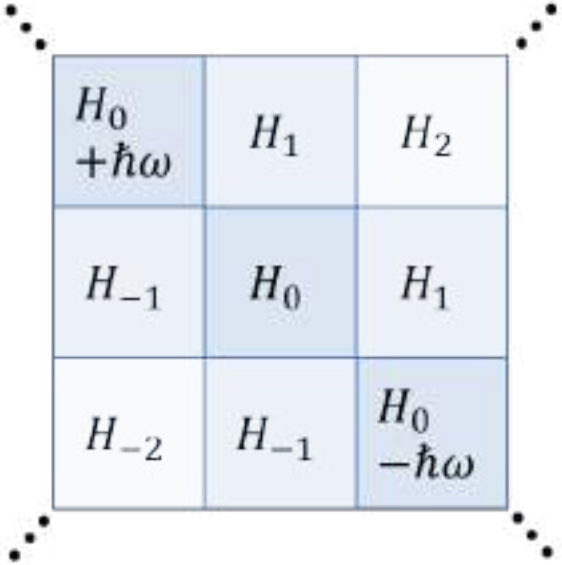
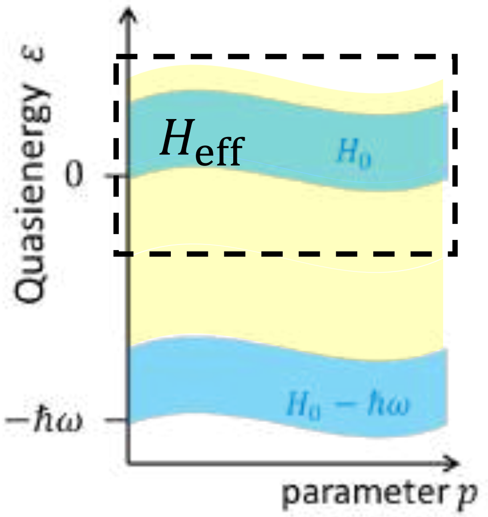


~~Single particle, finite bandwidth~~



High frequencies

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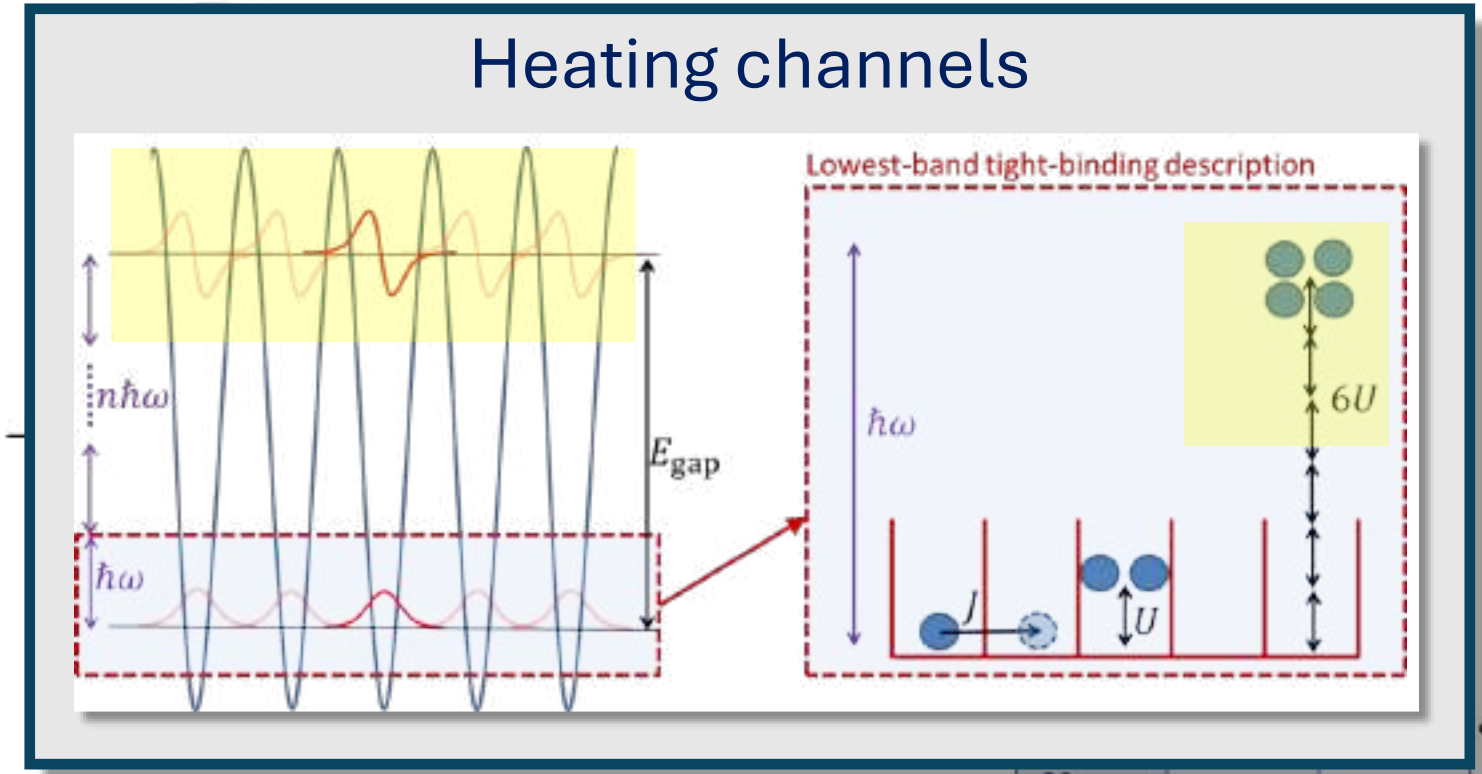

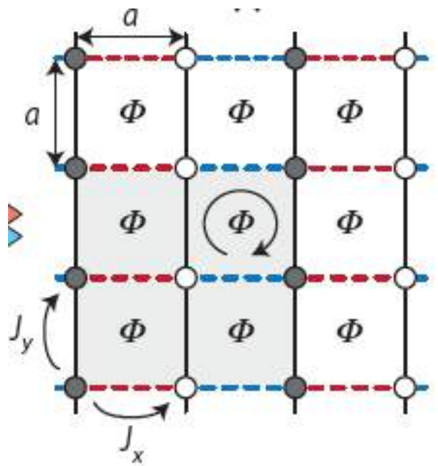

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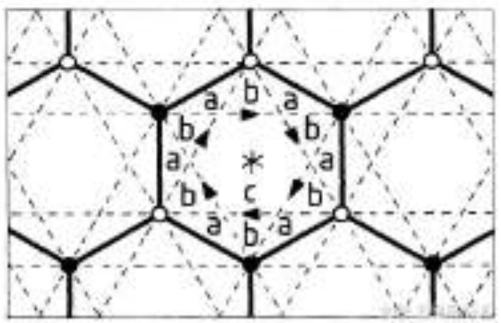

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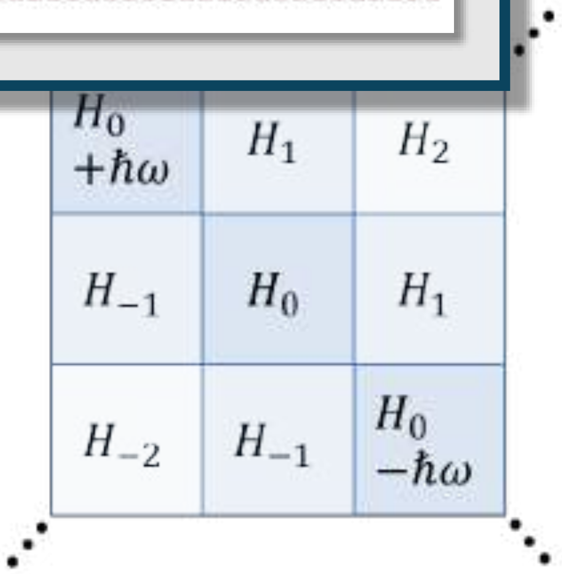
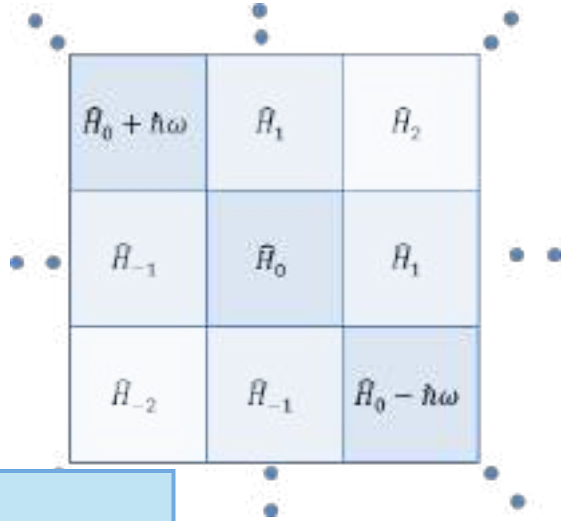
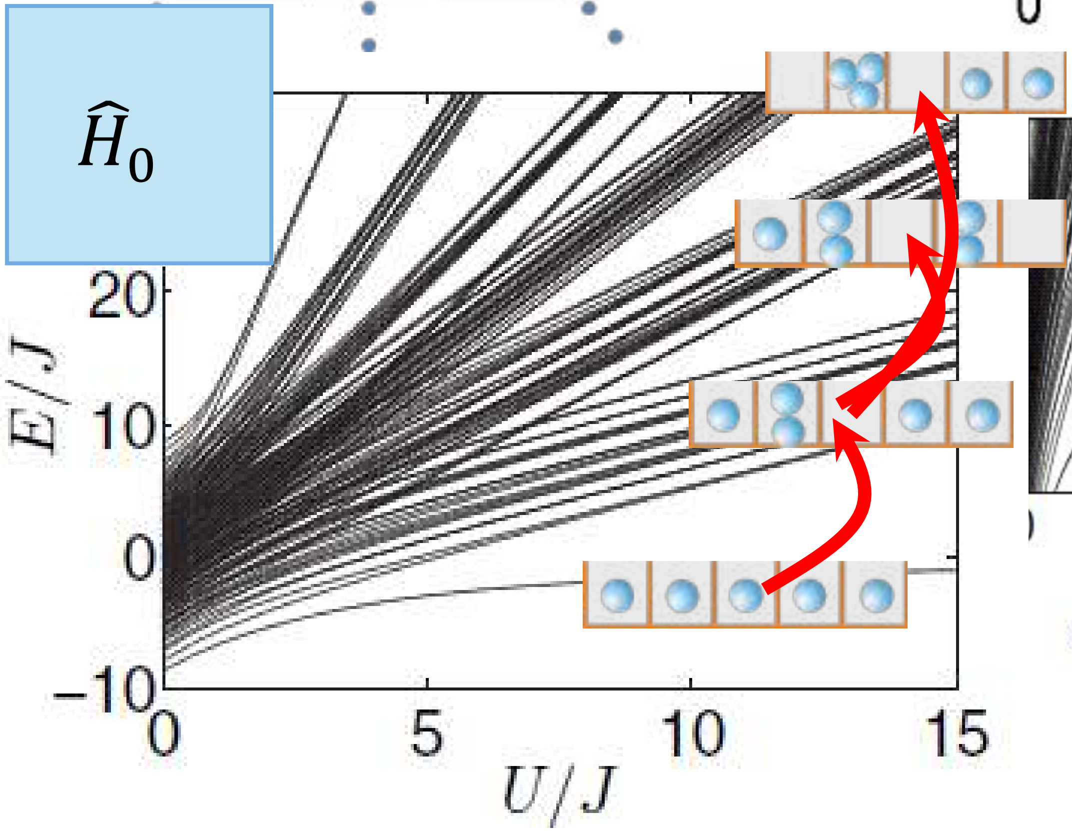
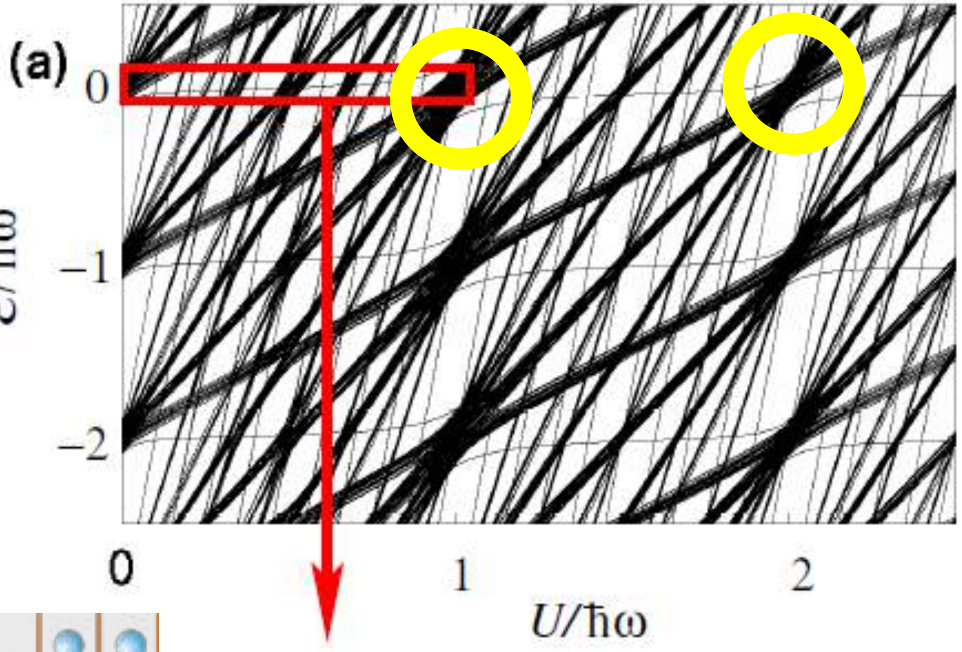
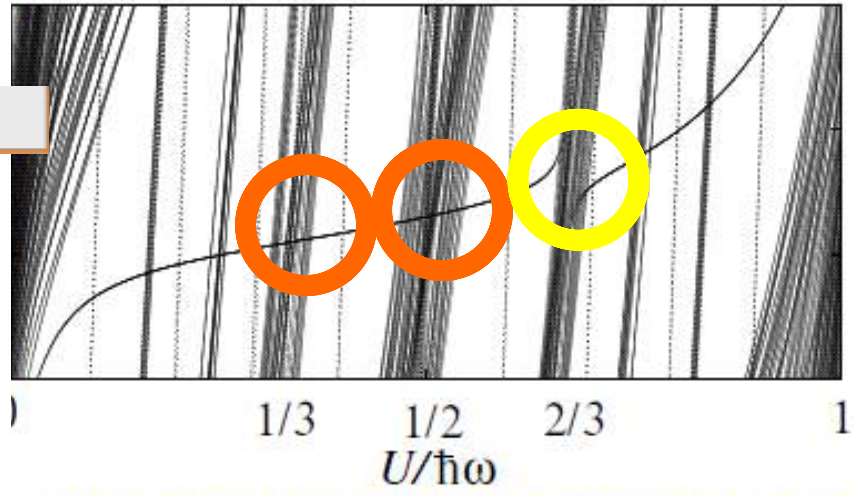


Example for Floquet heating





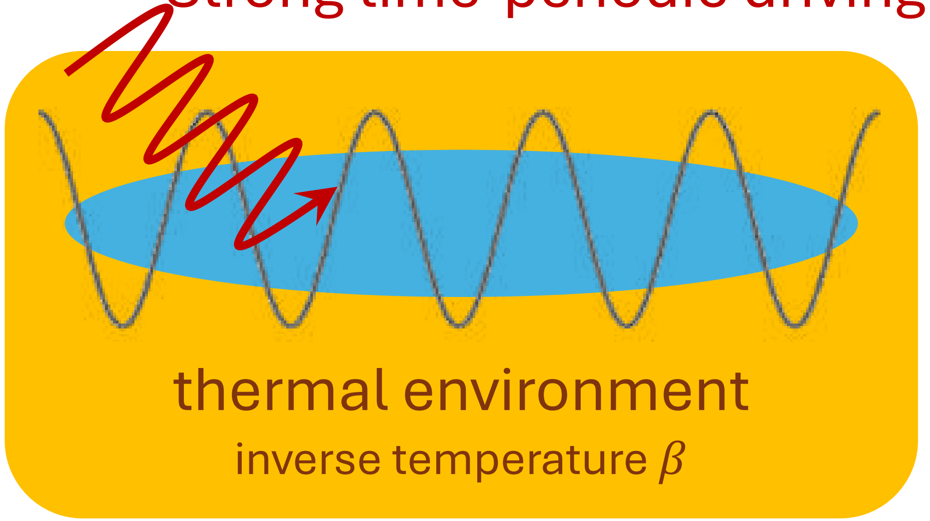




Quasienergy spectrum for 5 particles on 5 sites with $\hbar\omega/J = 20$ and $K_\omega/\hbar\omega = 2$

A.E. & M. Holthaus, PRL 2008
A.E. & E. Ansimovas, NJP 2009

Open Floquet systems



$$H_{\text{full}}(t) = H(t) + \gamma H_{\text{SB}} + H_{\text{B}}$$

$$H(t) = H(t+T)$$

Weak system-bath coupling

(Floquet-Born-Markov Theory + secular/RWA [Blümel et al. PRA 1991])

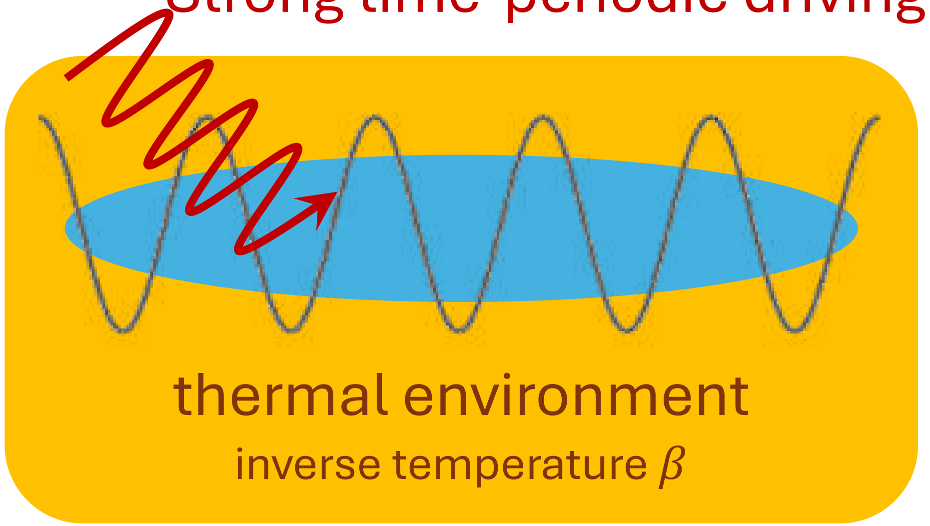
Floquet modes of $H(t)$

$$\dot{\rho} = \frac{1}{i}[H,\rho] + \gamma^2 \mathcal{D}(\rho) \xrightarrow{\gamma \ll \Delta\varepsilon} \rho_{\text{steady}} = \sum_n p_n |u_n\rangle\langle u_n|$$

p_n determined by environment

$$\dot{p}_n = \sum_m \left(R_{nn'} p_{n'} - R_{n'n} p_n\right) = 0$$

Open Floquet systems



$$H_{\mathrm{full}}(t) = H(t) + \gamma H_{\mathrm{SB}} + H_{\mathrm{B}}$$

$$H(t) = H(t+T)$$

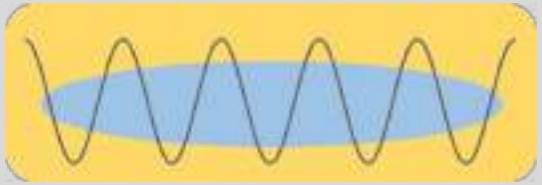
Rates from Floquet-Fermi golden rule:

$$R_{n'n} = \sum_{\mu=-\infty}^{\infty} R_{n'n}^{(\mu)} \quad \text{(processes, where } \Delta E_{\mathrm{bath}} = -(\varepsilon_{n'} - \varepsilon_n + \mu\hbar\omega)$$

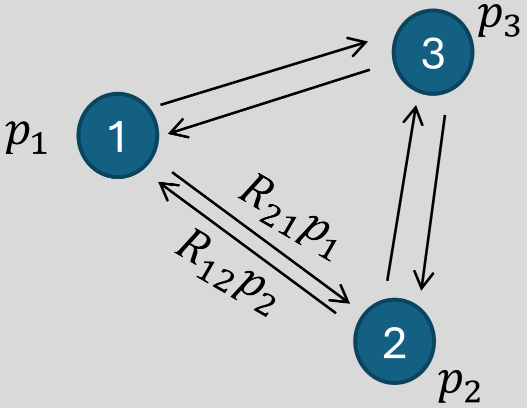
$$\frac{R_{n'n}}{R_{nn'}} \neq e^{-\beta(\varepsilon_{n'} - \varepsilon_n)}$$

$$\dot{p}_n = \sum_m (R_{nn'} p_{n'} - R_{n'n} p_n) = 0 \qquad \rho_{\mathrm{steady}} = \sum_n p_n |u_n\rangle\langle u_n|$$

Equilibrium:



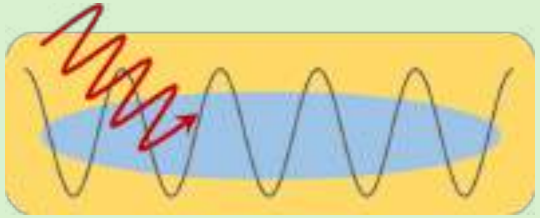
$p_n \propto e^{-\beta E_n}$ **(Gibbs state)**



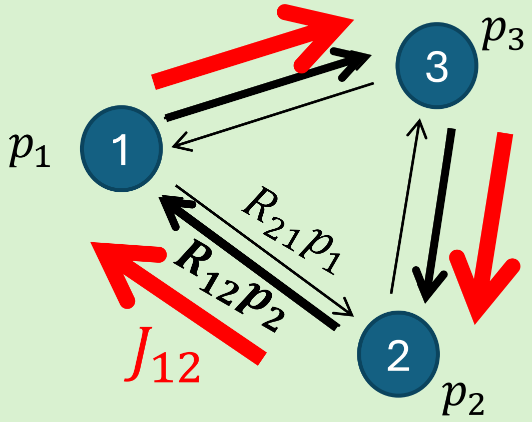
$$\frac{R_{mn}}{R_{nm}} = e^{-\beta(E_m - E_n)}$$

Detailed balance $J_{nn'} = 0$

Nonequilibrium steady state (NESS):



p_n **depend on details of environment**



$$\frac{R_{mn}}{R_{nm}} \neq e^{-\beta(\varepsilon_m - \varepsilon_n)}$$

No detailed balance $J_{nn'} \neq 0$

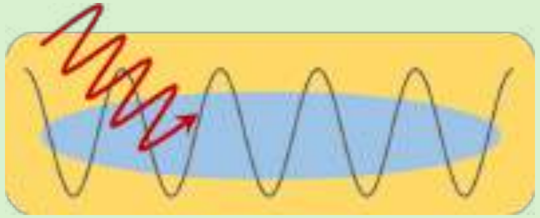
$$\dot{p}_n = \sum_m \underbrace{(R_{nn'}p_{n'} - R_{n'n}p_n)}_{J_{nn'}\text{(net probability current from } n' \text{ to } n)} = 0 \qquad \rho_{\text{steady}} = \sum_n p_n |u_n\rangle\langle u_n|$$

Dynamics & Relaxation **Non-equilibrium Steady State (NESS)**

$\sim t_{\mathrm{relax}}$ time

Robust dissipative state engineering

Nonequilibrium steady state (NESS):



p_n **depend on details of environment**

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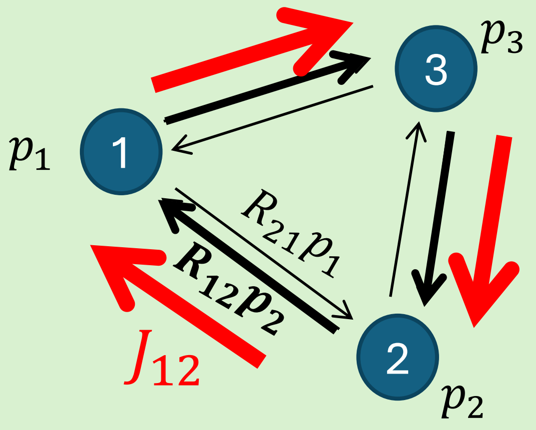
No detailed balance $J_{nm} \neq 0$



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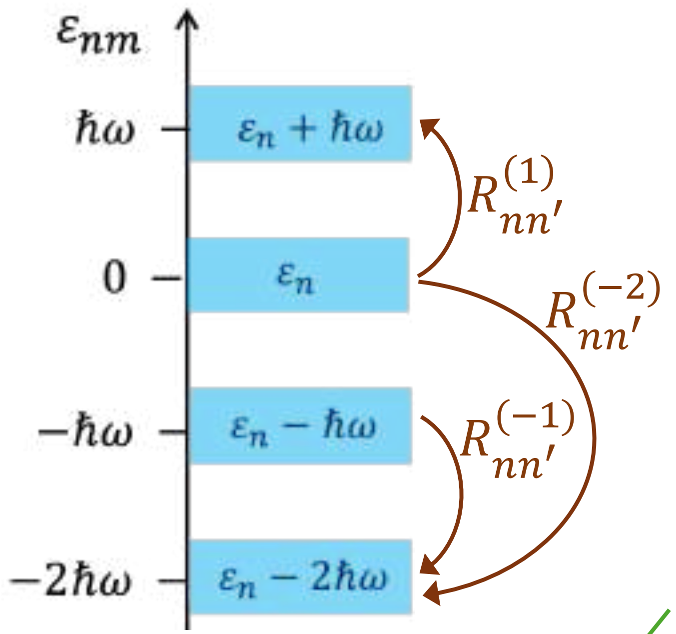
Intuitive picture in Floquet space [L. Wanckel, A.E., arXiv:2604.01291]

Distinguish Floquet copies $|u_{nm}\rangle = |u_n\rangle e^{im\omega t}$ with quasienergies $\varepsilon_{nm} = \varepsilon_n + m\hbar\omega$

Interpretation:
System state n
"Photon" number m



Rewrite: $\rho = \sum_n p_n |u_n\rangle\langle u_n| = \sum_{nm} p_{nm} |u_{nm}\rangle\langle u_{nm}|$ $\quad |u_n\rangle\langle u_n| = |u_{nm}\rangle\langle u_{nm}|$

with $p_n = \sum_m p_{nm}$, $\quad p_{nm}$: Interpreted as population of $|u_{nm}\rangle$ (not unique)

$$\dot{p}_{nm} = \sum_{n'm'} \left(R_{(nm)(n'm')} p_{n'm'} - R_{(n'm')(nm)} p_{n'm'} \right)$$

similar to Floquet two-time formalism [HP Breuer, M Holthaus, Phys. Lett. A **140**, 507 (1989)]

$$R_{(nm)(n'm')} \equiv R^{(m-m\prime)}_{nn'}$$

Gives intuition!
But no steady state $\rho \propto e^{-\beta\varepsilon_{nm}}$ as ε_{nm} not bounded from below!

equivalent to

Looks equilibrium like $\frac{R_{nm,n'm'}}{R_{n'm',nm}} = e^{-\beta(\varepsilon_{nm} - \varepsilon_{n'm'})}$

$$\dot{p}_n = \sum_m \underbrace{(R_{nn'} p_{n'} - R_{n'n} p_n)}_{J_{nn'}\text{(net probability current from } n' \text{ to } n)} = 0 \qquad \rho_{\text{steady}} = \sum_n p_n |u_n\rangle\langle u_n| \qquad R_{n'n} = \sum_{\mu=-\infty}^{\infty} R^{(\mu)}_{n'n}$$

Alex Schnell
(TU Berlin)

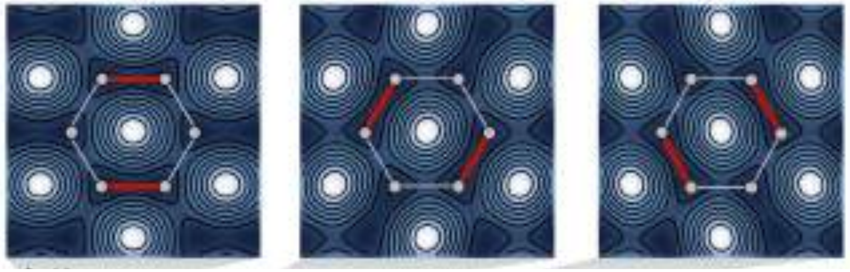
Christof Weitenberg
(U Dortmund)

Stabilizing Floquet topological insulators

SciPost Phys. 17, 052 (2024)

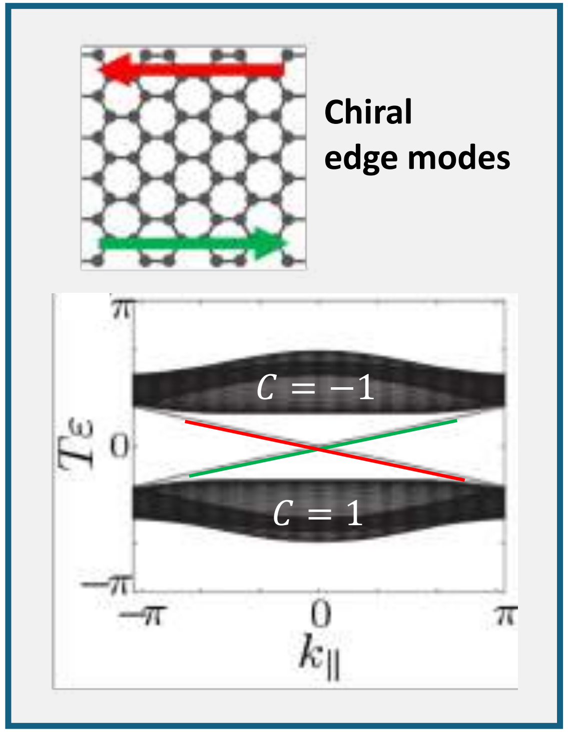
Floquet-topological system:

Theory: Oka & Aoki, PRB 2009, Kitagawa et al. PRB 2010, …
Quantum gas experiments: Aidelsburger/Bloch, Esslinger, Greiner, Weitenberg/Sengstock, …



from Wintersperger et al., Nat. Phys. 2020

H_{eff}

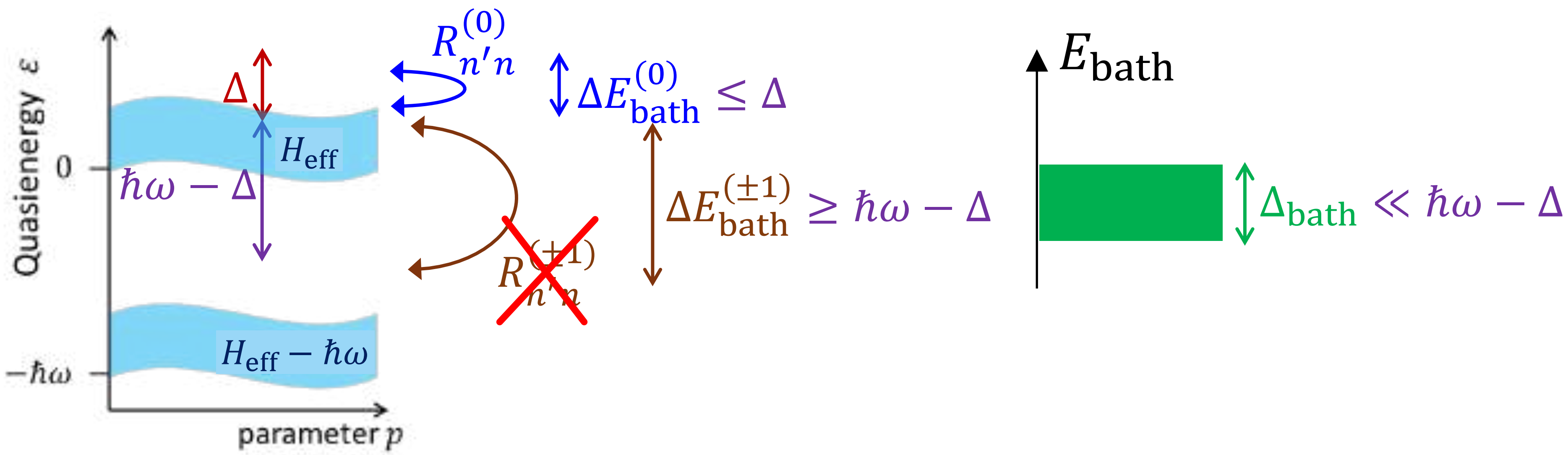


Topological Chern insulator
(integer quantum Hall sate):

Fermionic band insulator:
One band filled completely,
other band empty

Goal: Engineer bath, so that

$$\rho_{\mathrm{NESS}} \approx Z^{-1} e^{-\beta H_{\mathrm{eff}}}\,, \quad \beta E_{\mathrm{gap}}^{\mathrm{eff}} \gg 1$$



Rates from Floquet-Fermi golden rule:

$$R_{mn} = \sum_{\mu} R^{(\mu)}_{mn} \qquad \approx R^{(0)}_{mn} \qquad \text{with } \Delta E^{(\mu)}_{\text{bath}} = -(\varepsilon_m - \varepsilon_n + \mu\, \hbar\omega)$$

$$\frac{R_{mn}}{R_{nm}} \neq e^{-\beta(\varepsilon_m - \varepsilon_n)}$$

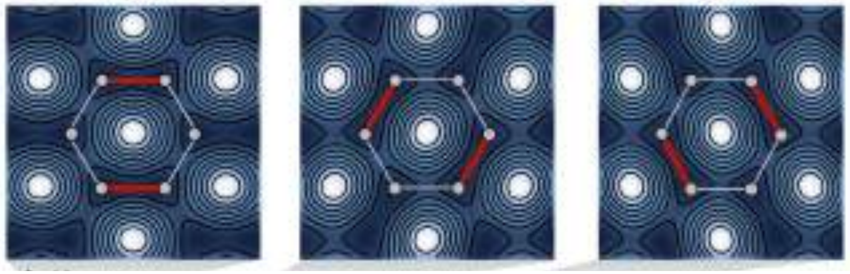
$$\frac{R^{(0)}_{mn}}{R^{(0)}_{nm}} = e^{\beta \Delta E^{(0)}_{\text{bath}}} = e^{-\beta(\varepsilon_m - \varepsilon_n)}$$

$$\Rightarrow p_n \approx Z^{-1} e^{-\beta \varepsilon_n}$$

[Iadecola, Neupert, Chamon (2015)], [Shirai, Mori, Miyashita (2015)], [Dhegani, Oka, Mitra (2015)], …

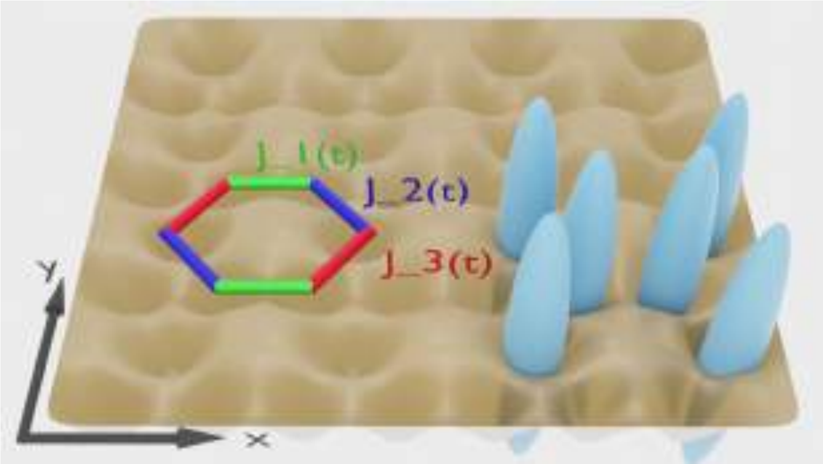
Floquet-topological system:

Theory: Oka & Aoki, PRB 2009, Kitagawa et al. PRB 2010, …
Quantum gas experiments: Aidelsburger/Bloch, Esslinger, Greiner, Weitenberg/Sengstock, …

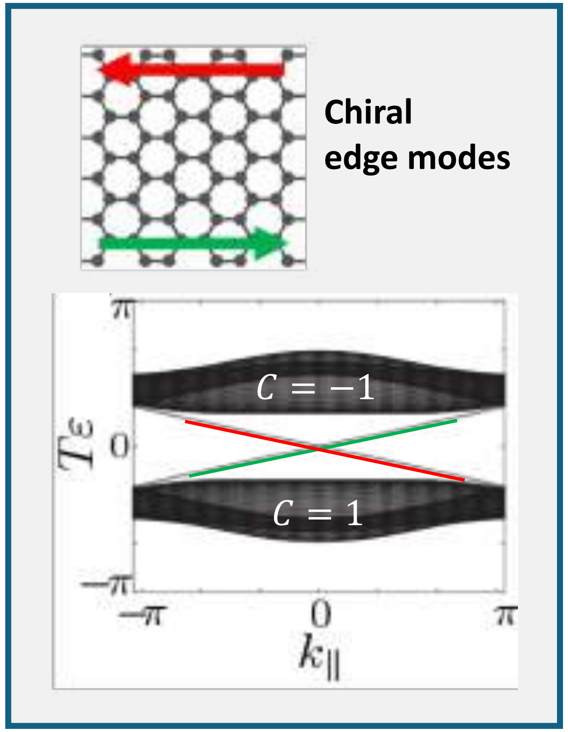


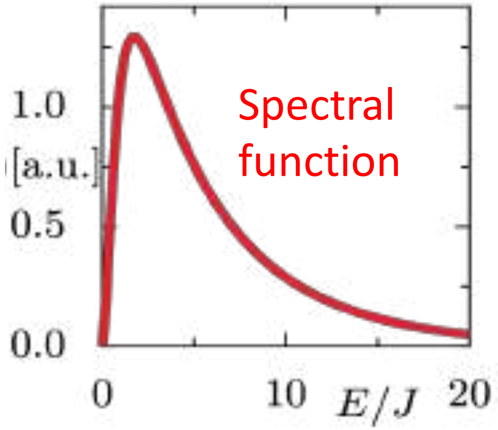
from Wintersperger et al., Nat. Phys. 2020

Microscopic model (^{6}Li - ^{87}Rb)
BEC baths in transverse 1D tubes
(Bogoliubov + Floquet-Born Markov)



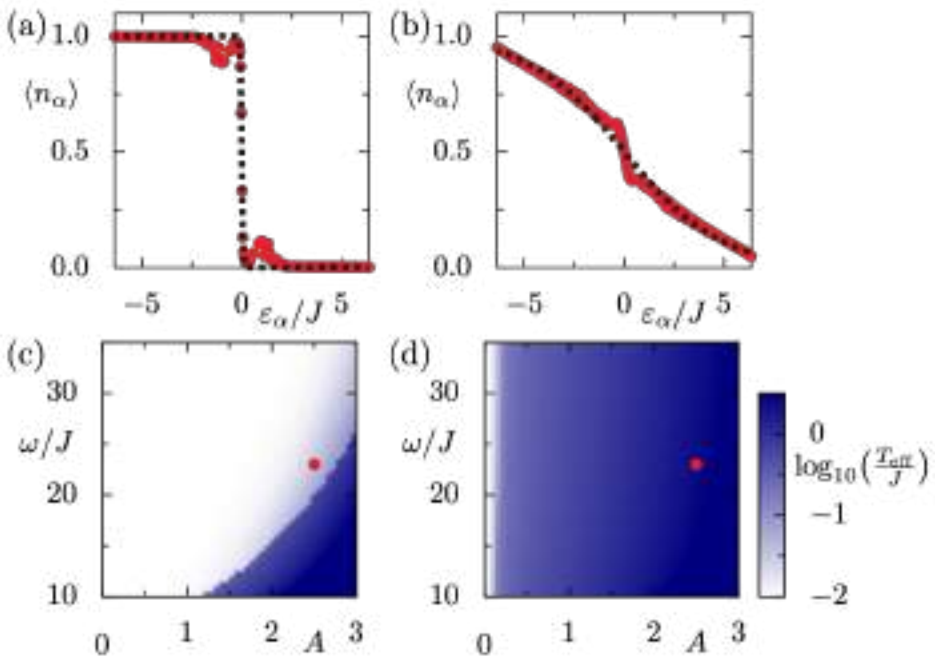
H_{eff}





Peaked
spectral
function

Vertical BEC tubes **Ohmic bath**



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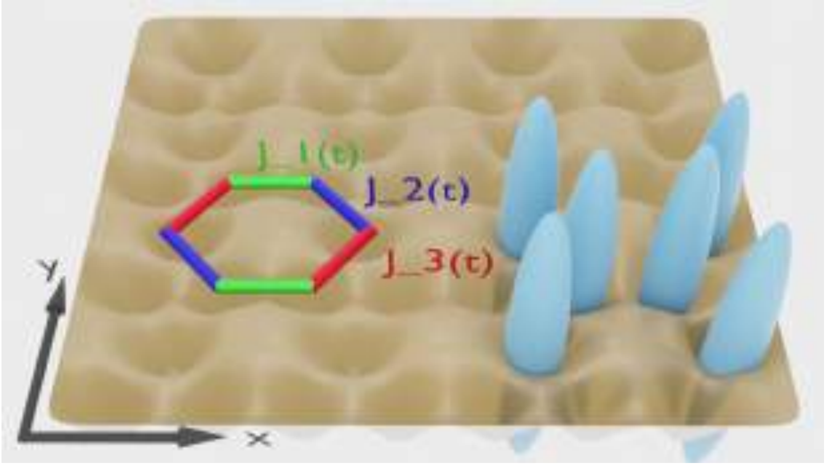
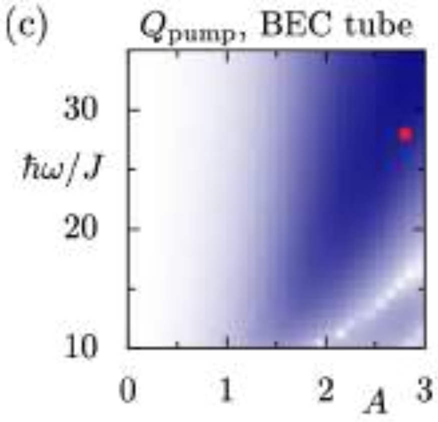
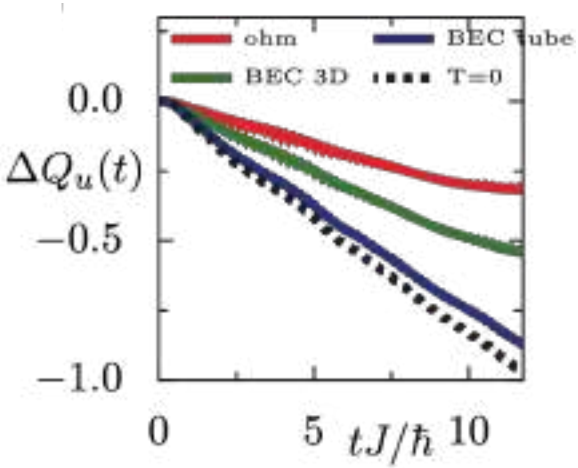
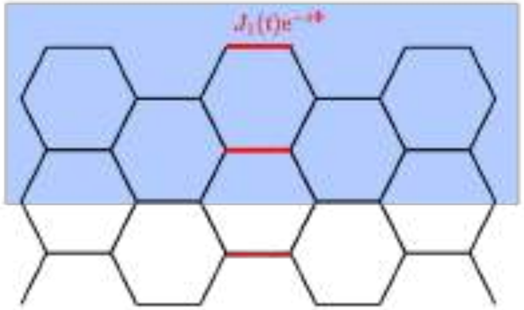


Hall response from Charge pumping (16x16 unit cells)







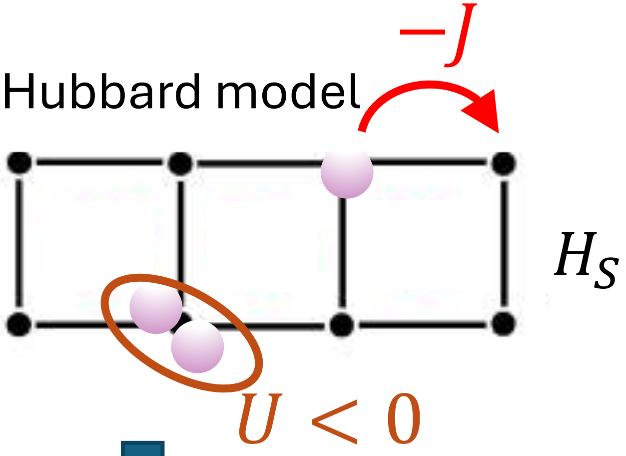
Francesco Petiziol (TU Berlin)

Cooling Floquet systems using cavity-based reservoir engineering

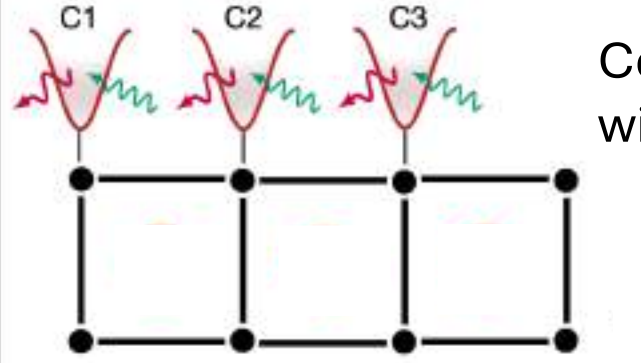
Petiziol, A.E., PRL **129**, 233601 (2022)

Array of artificial atoms (superconducting circuit)

Bose-Hubbard model



Engineered dissipation

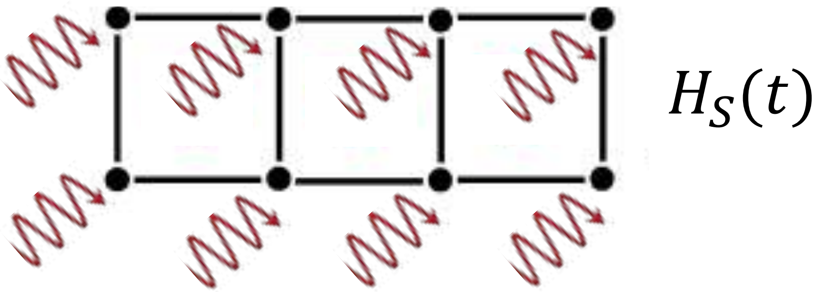


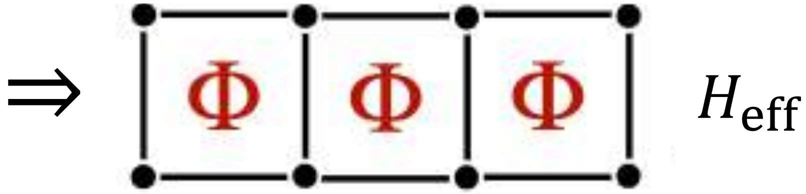
Ground-state cooling of H_S

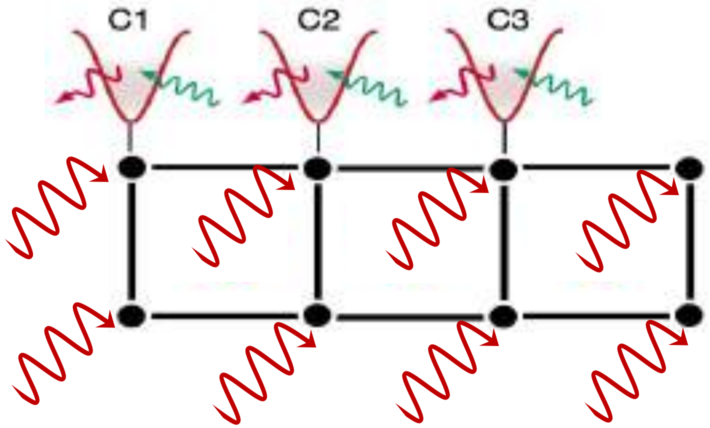
Couple to leaky cavities with red-detuned pumping

Hacohen-Gourgy,... Siddiqi, Girvin, PRL 2015

Engineered driving

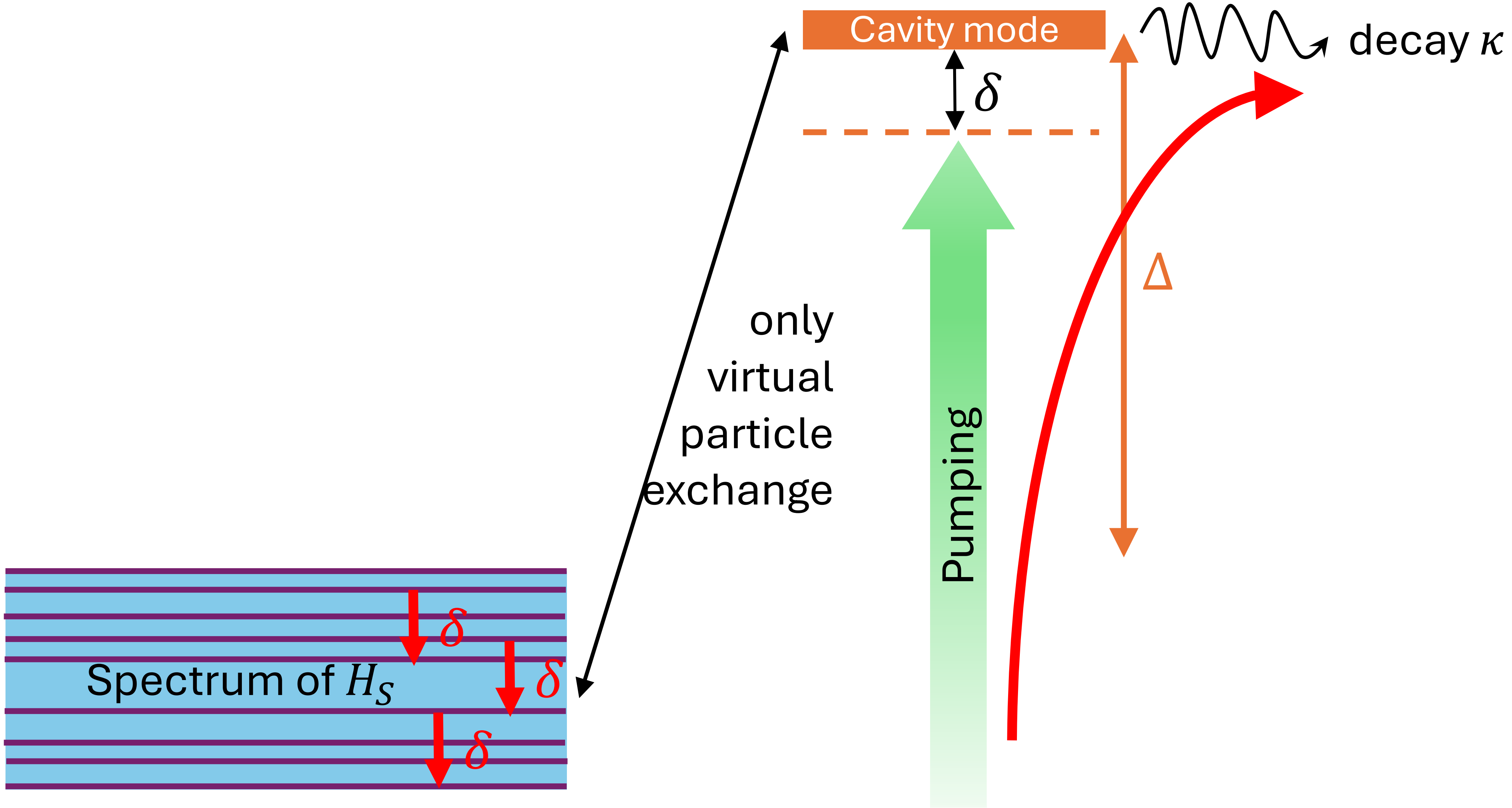




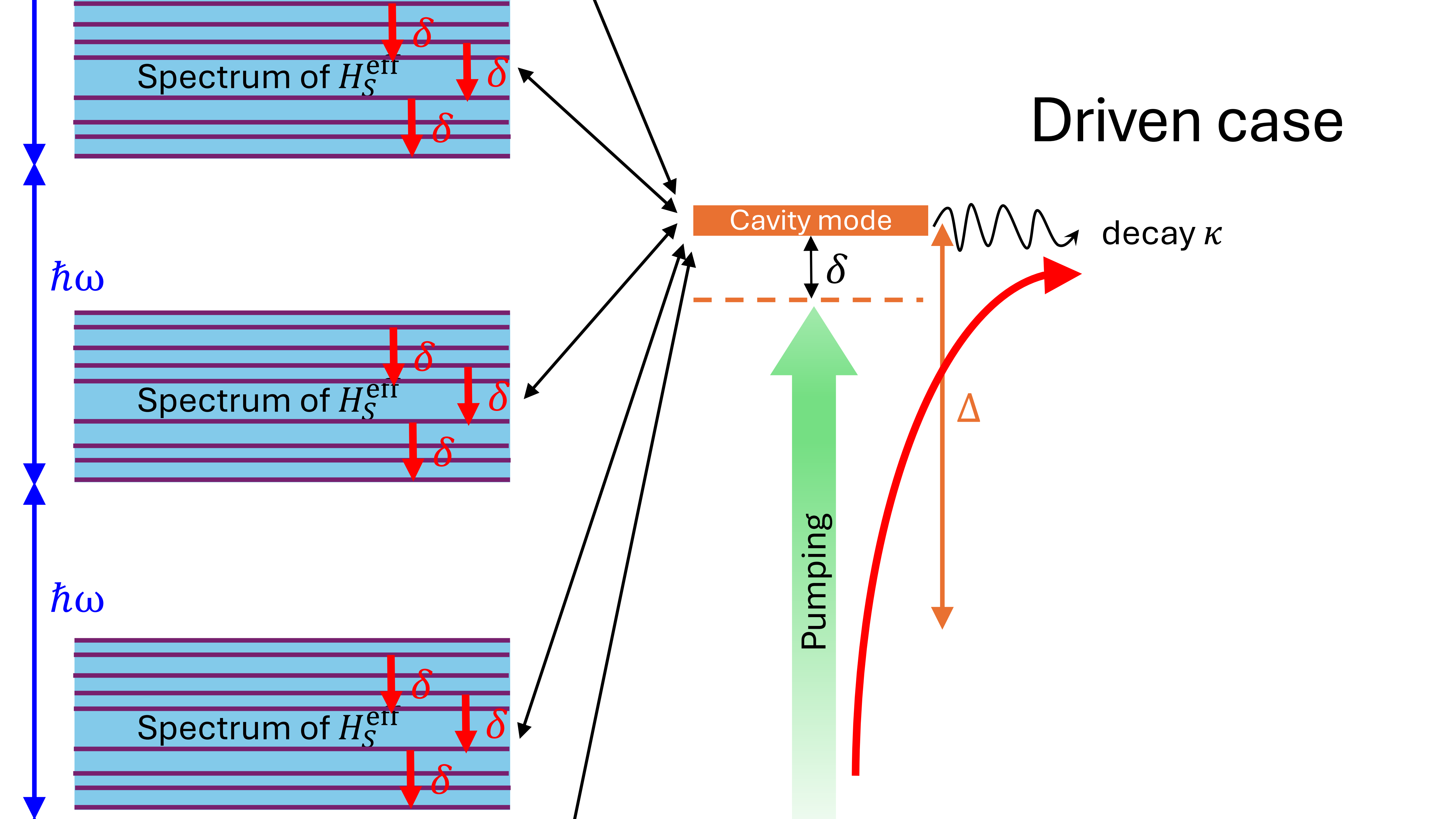


Ground-state cooling of H_{eff} ?

Undriven case

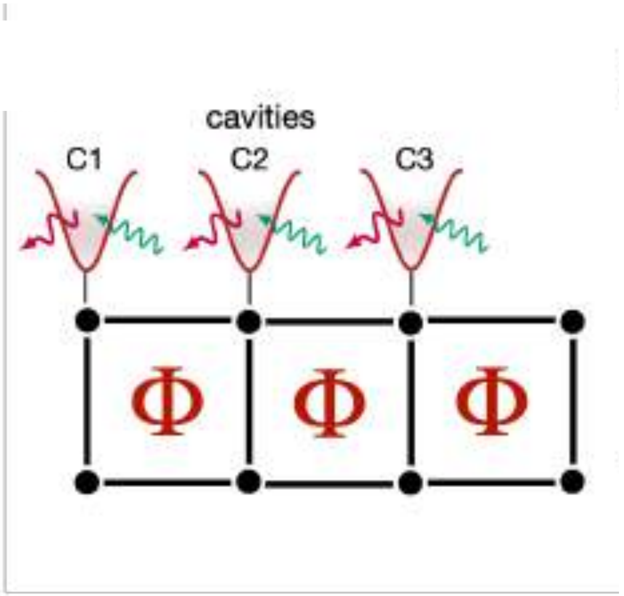


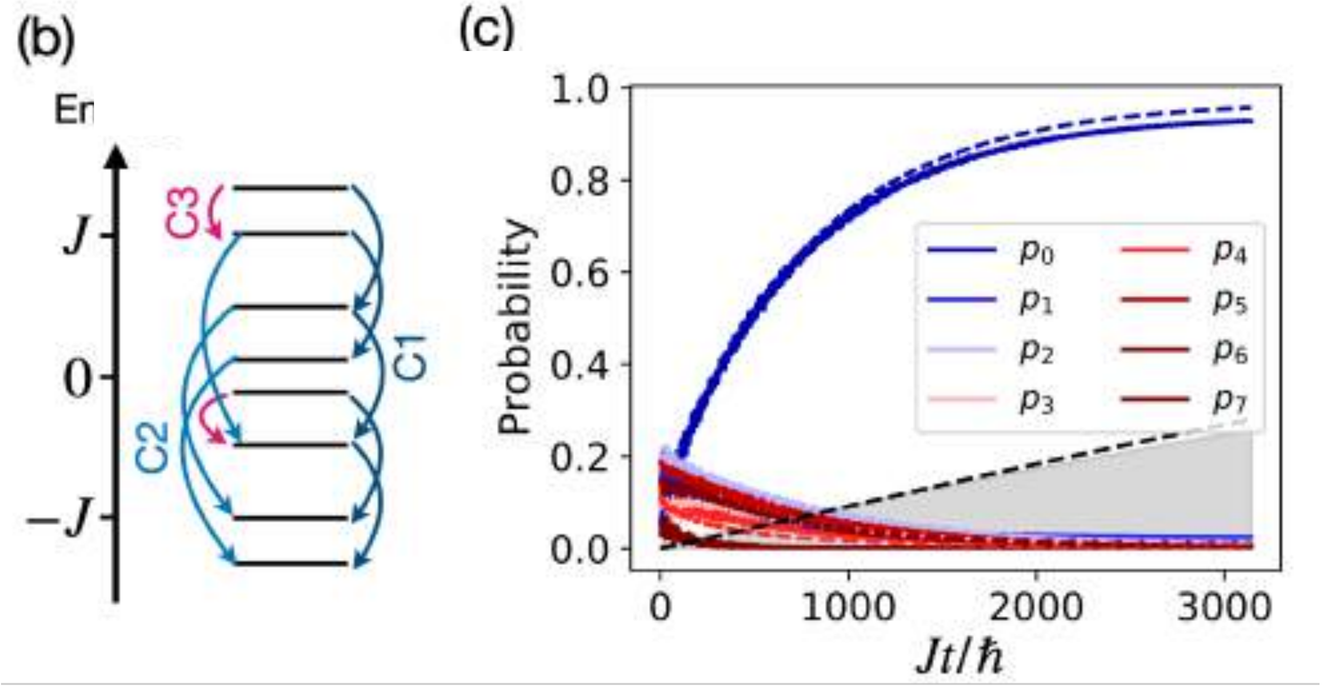
Hacohen-Gourgy, ...Siddiqi, Girvin, PRL 2015



Driven case
Spectrum of H_S^{eff}
Spectrum of H_S^{eff}
Spectrum of H_S^{eff}
δ
$\hbar\omega$
$\hbar\omega$
Cavity mode
decay κ
Δ
Pumping

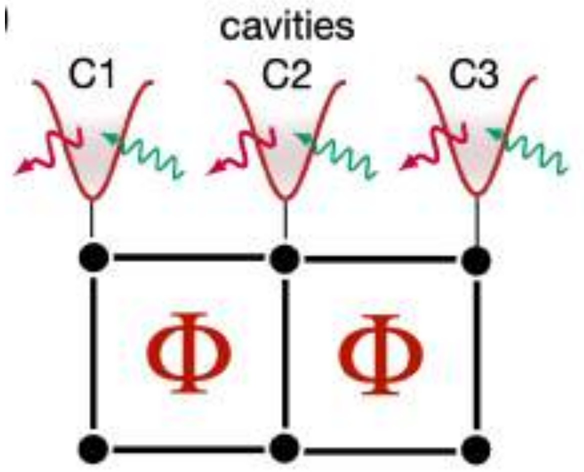
Cooling single particle on 2x4 ladder

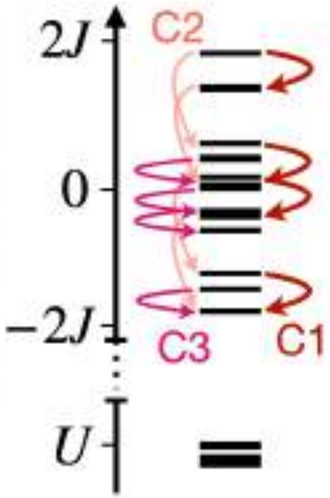


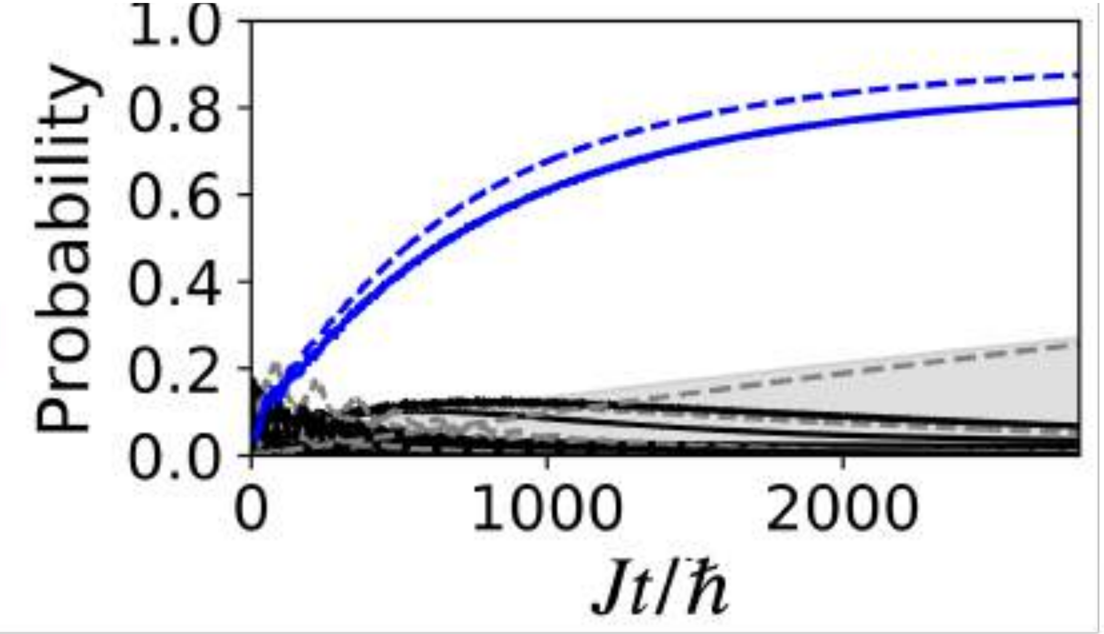


Petiziol, A.E., PRL **129**, 233601 (2022)

Cooling 2 hard-core bosons on 2x6 ladder

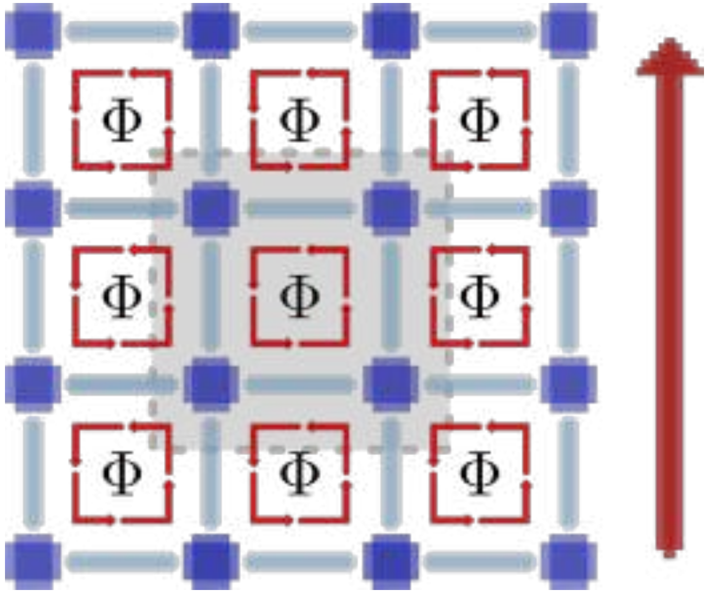


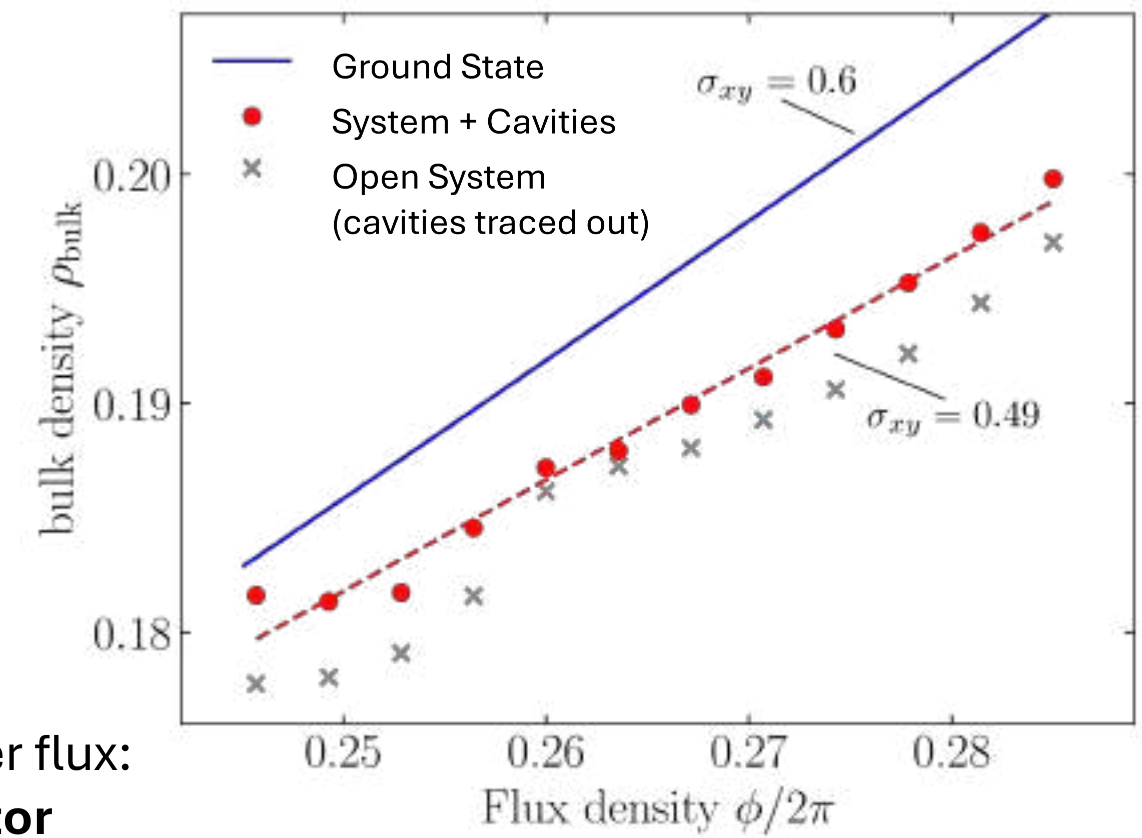




Petiziol, A.E., PRL **129**, 233601 (2022)

Cooling 2 hard-core bosons 4x4 lattice





Luis Steinfadt

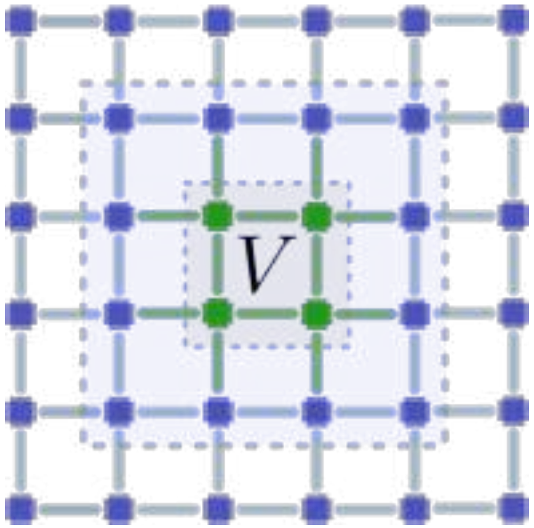
Expected ground state at quarter flux:
Baby Fractional-Chern-Insulator
(Laughlin $\nu = 1/2$-type)

Signature: Streda formula (bulk density = $\nu\times$Flux density)

Luis Steinfadt, Francesco Petiziol, A.E

Cooling 3 hard-core bosons 6x6 lattice

Luis Steinfadt



Expected ground state at quarter flux:
Baby Fractional-Chern-Insulator
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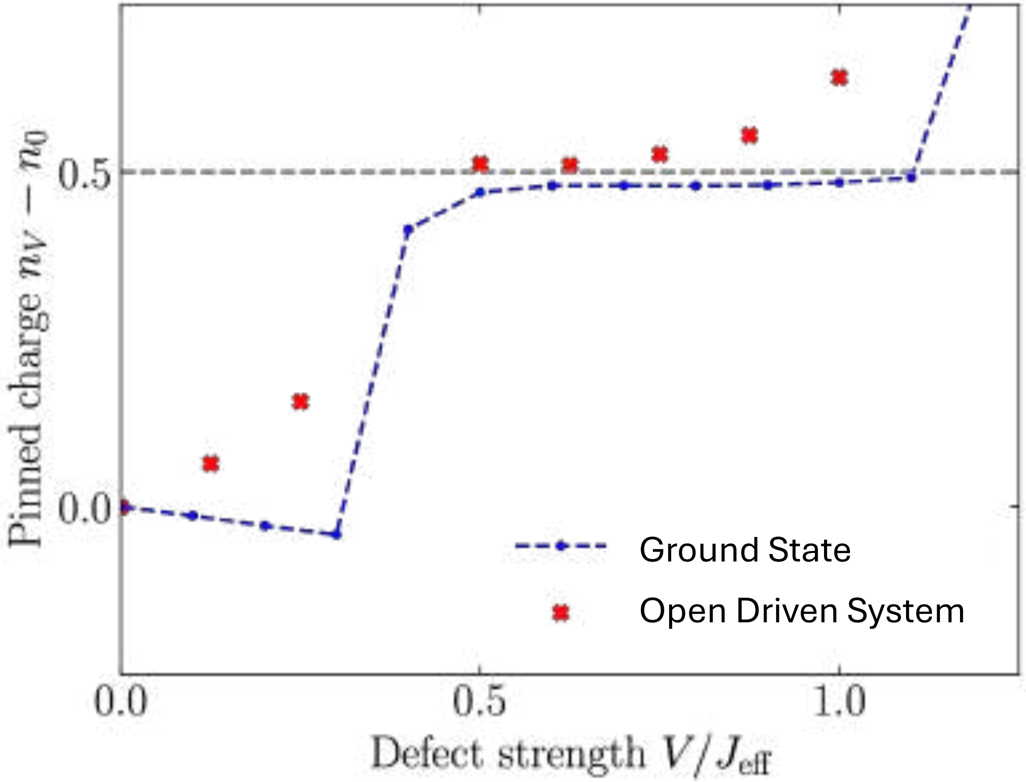
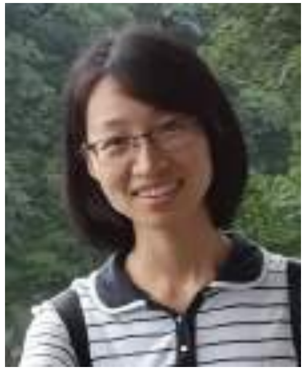


Signature: Charge fractionalization

Luis Steinfadt, Francesco Petiziol, A.E

Alexander Schnell
(TU Berlin)



Lingna Wu
(TU Berlin)
now U Hainan

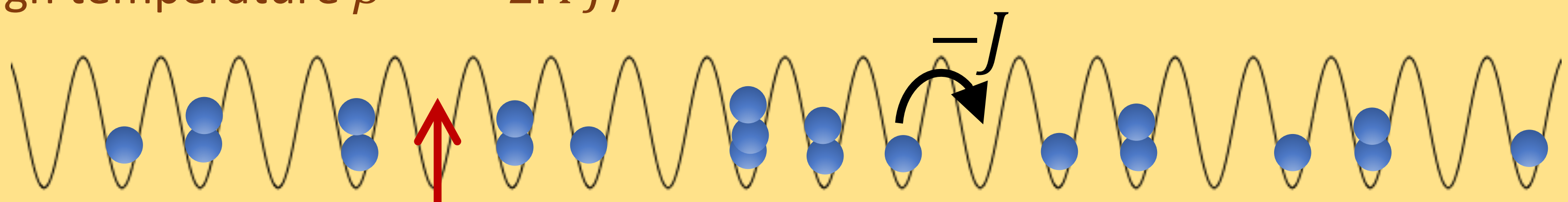
Artur Widera
(Kaiserslautern)

Floquet-heating induced Bose condensation

Schnell, Wu, Widera, A.E. Phys. Rev. A **107**, L021301 (2023)

Floquet-heating-induced Bose condensation:

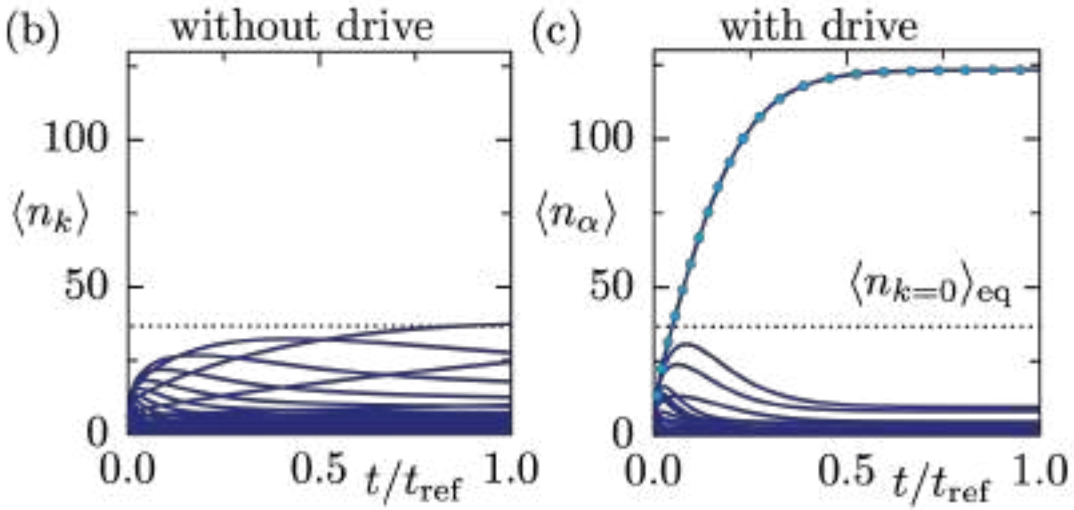
Bath: 3D BEC of interacting ^{87}Rb atoms
(@ high temperature $\beta^{-1} = 2.4\,J$)



$A\cos(\omega t)$

1D system: non-interacting bosonic ^{39}K atoms in optical lattice (200 particles on 49 sites)

Resonant local driving at site ℓ
($\ell = 20$, $A = J$, $\hbar\omega = 3/8$ band width)



Mean occupations of single-particle eigenstates

Mean Occupations of single-particle Floquet states

Intuitive explanation?

Schnell, Wu, Widera, A.E. Phys. Rev. A **107**, L021301 (2023)

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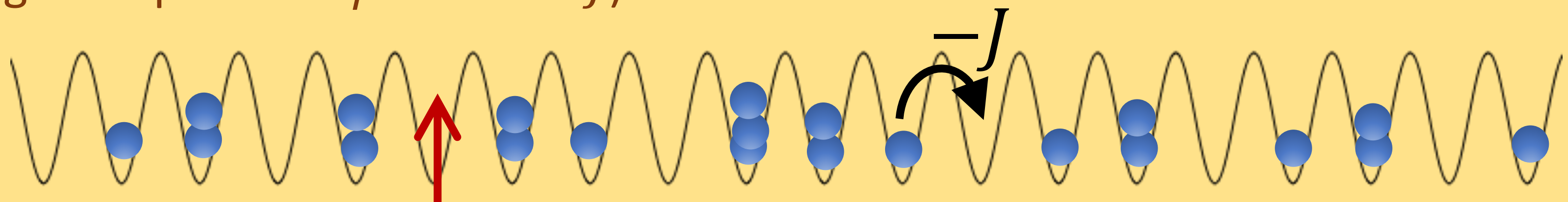

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Undriven eigenmodes:

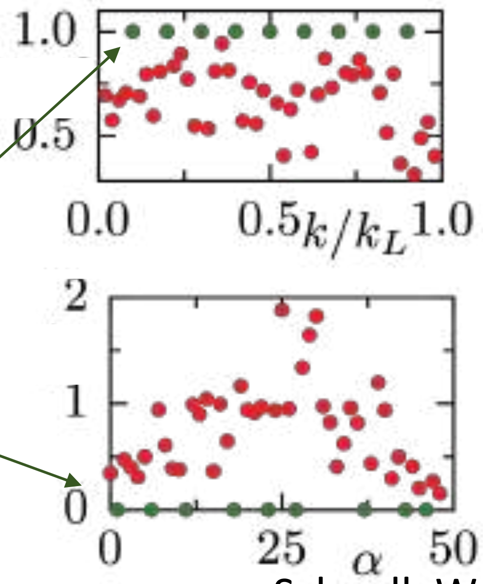
$$\psi_k(i) \sim \sin(ki)$$

$$k = \frac{\pi n}{M+1}; \quad n = 1,2,\dots,M$$

Modes k_r with $\sin(k_r \ell) = 0$,

$$k_r \approx \frac{2\pi n}{\ell} \quad \text{where } r = 1,\dots,\ell-1$$

decouple from drive.



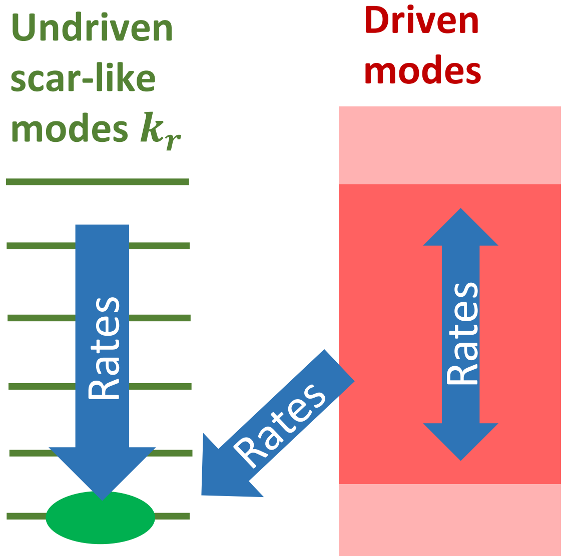
$\max_\alpha \left|\langle \psi_k | \psi_\alpha^{Floquet} \rangle\right|^2$

Quantum-scar-like Floquet modes k_r!

Energy fluctuations (units J) of Floquet modes $|\psi_\alpha^{Floquet}\rangle$

Schnell, Wu, Widera, A.E. Phys. Rev. A **107**, L021301 (2023)

Floquet-heating-induced Bose condensation:



- Minority
- Sharp energies
- Large level spacing

Non-equilibrium condensation in excited mode $k_1 = 2\pi/\ell$

- Majority
- Broad in energy
- Low-energy modes hybridize with modes of higher energy (Floquet heating)

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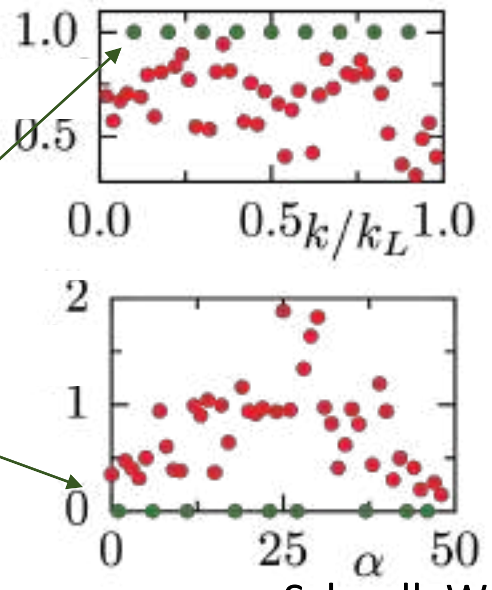

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Schnell, Wu, Widera, A.E. Phys. Rev. A **107**, L021301 (2023)

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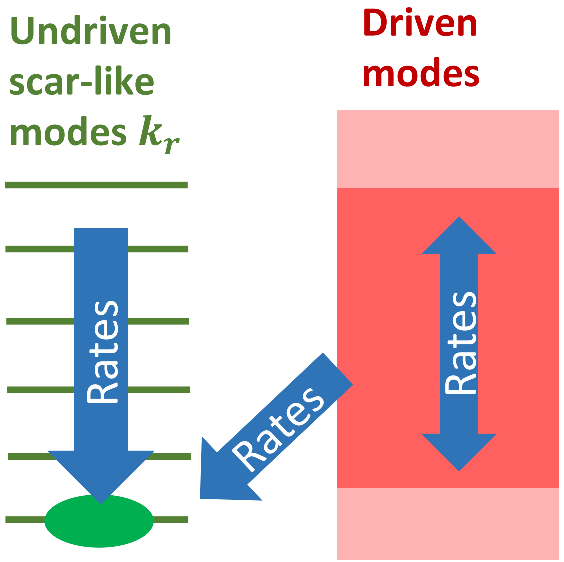
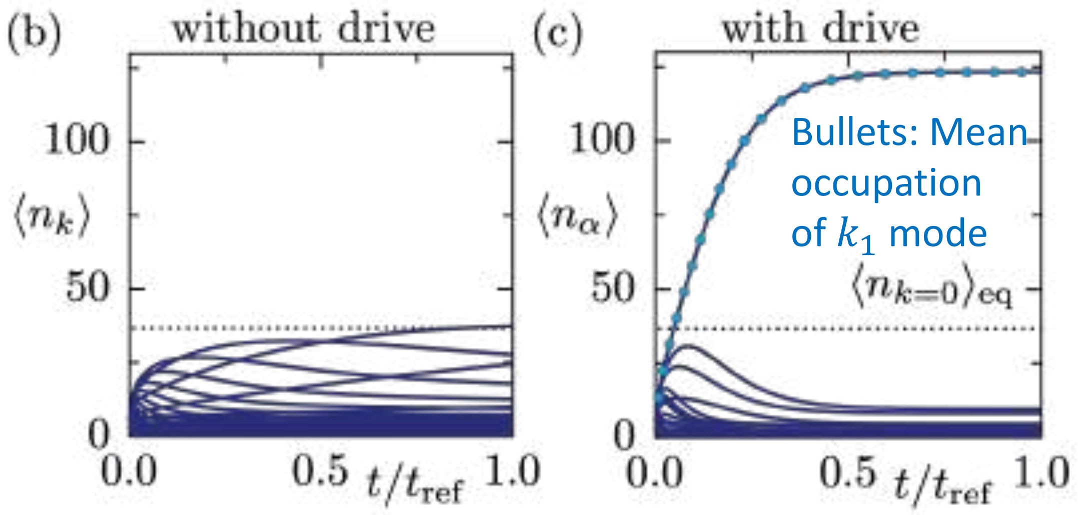


Non-equilbrium condensation in excited mode $k_1 = 2\pi/\ell$

- Majority
- Broad in energy
- Low-energy modes hybridize with modes of higher energy (Floquet heating)

A thermoelectric effect



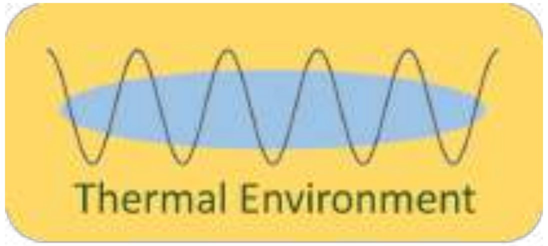
Mean occupations of single-particle eigenstates

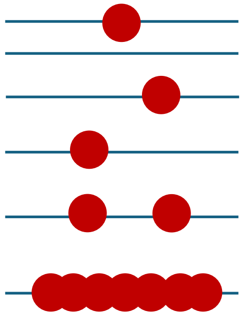
Mean Occupations of single-particle Floquet states

Schnell, Wu, Widera, A.E. Phys. Rev. A **107**, L021301 (2023)

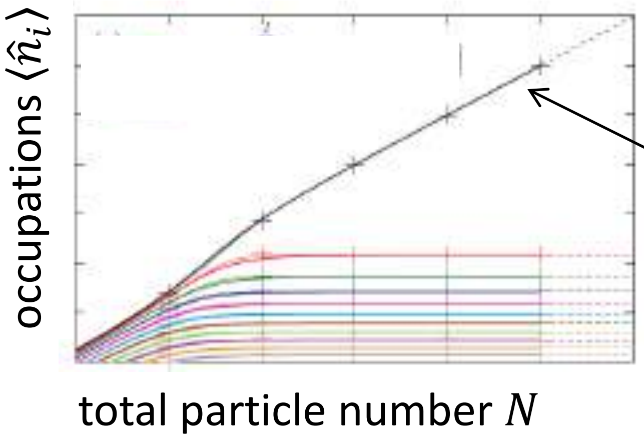
Non-equilibrium Bose-Einstein condensation?

Equilibrium





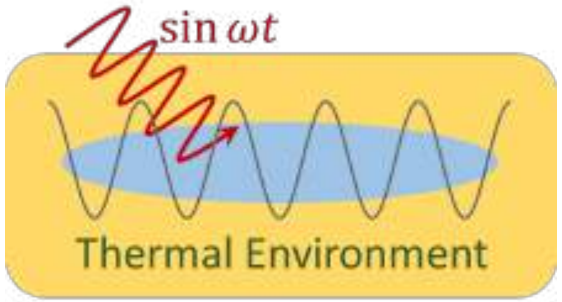
$N_0 \sim N$

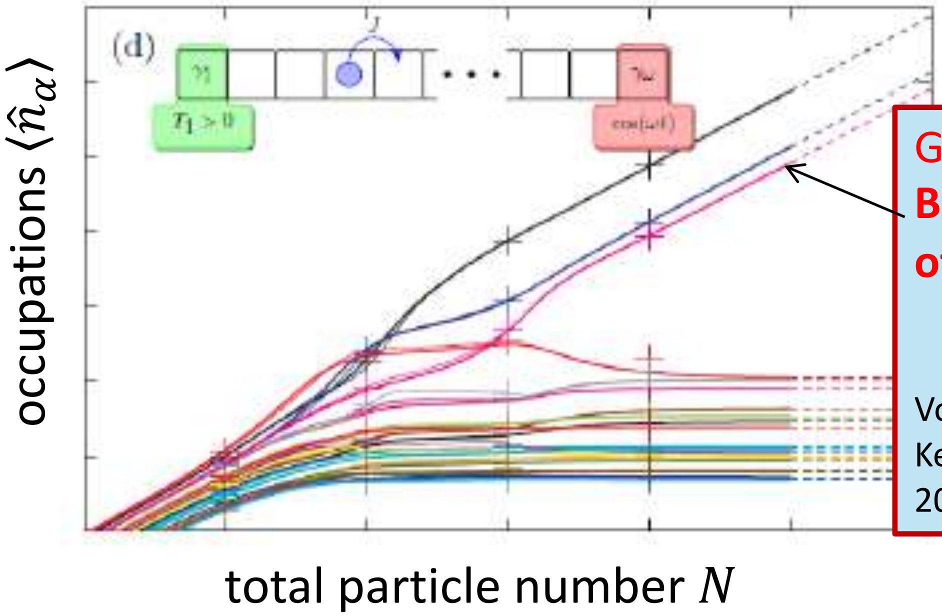


BEC

in ground state

Non-equilibrium steady state

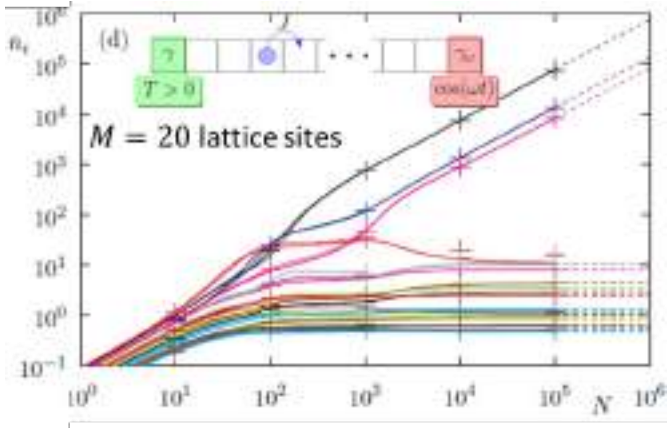




Generic behavior:
Bose selection
of multiple states
(odd number)

Vorberg, Wustmann, Ketzmerick, A.E., PRL 2013

Mean occupations $\bar{n}_i = \langle \hat{n}_i \rangle$



$$\dot{\bar{n}}_i = \sum_j \left[-R_{ji} \langle (\hat{n}_j + 1)\hat{n}_i \rangle + R_{ij} \langle (\hat{n}_i + 1)\hat{n}_j \rangle \right]$$

(1) Non-linear kinetic equations: $\langle \hat{n}_j \hat{n}_i \rangle \approx \bar{n}_j \bar{n}_i$

(2) High-density $(1/n)$-expansion

(2a) Leading order $(\bar{n}_j + 1) \approx \bar{n}_j$

$$\dot{\bar{n}}_i = \underbrace{\bar{n}_i}_{=0} \underbrace{\textstyle\sum_j (R_{ij} - R_{ji})\bar{n}_j}_{=0} = 0$$

$= 0$ or $= 0$

$$\begin{cases} \bar{n}_i = 0 & i \notin S \\ \sum_{j \in S} (R_{ij} - R_{ji})\bar{n}_j = 0 & i \in S \end{cases}$$

$\Rightarrow$ **Odd number of selected states**

(2b) First correction (Bogoliubov-like approx.)

$$\bar{n}_i = \frac{\sum_{j \in S} R_{ij} \bar{n}_j}{-\sum_{j \in S} (R_{ij} - R_{ji})\bar{n}_j} \qquad i \notin S$$

independent of $N \Rightarrow$ **saturation**

(3) Set S uniquely determined by requirement $\bar{n}_i \geq 0 \quad \forall i$

Theory of non-eq. Bose condensation in open Floquet system!

Vorberg, Wustmann, Ketzmerick, A.E. 2013

Beyond ultraweak system-bath coupling:

Lorenz Wanckel
(TU Berlin)

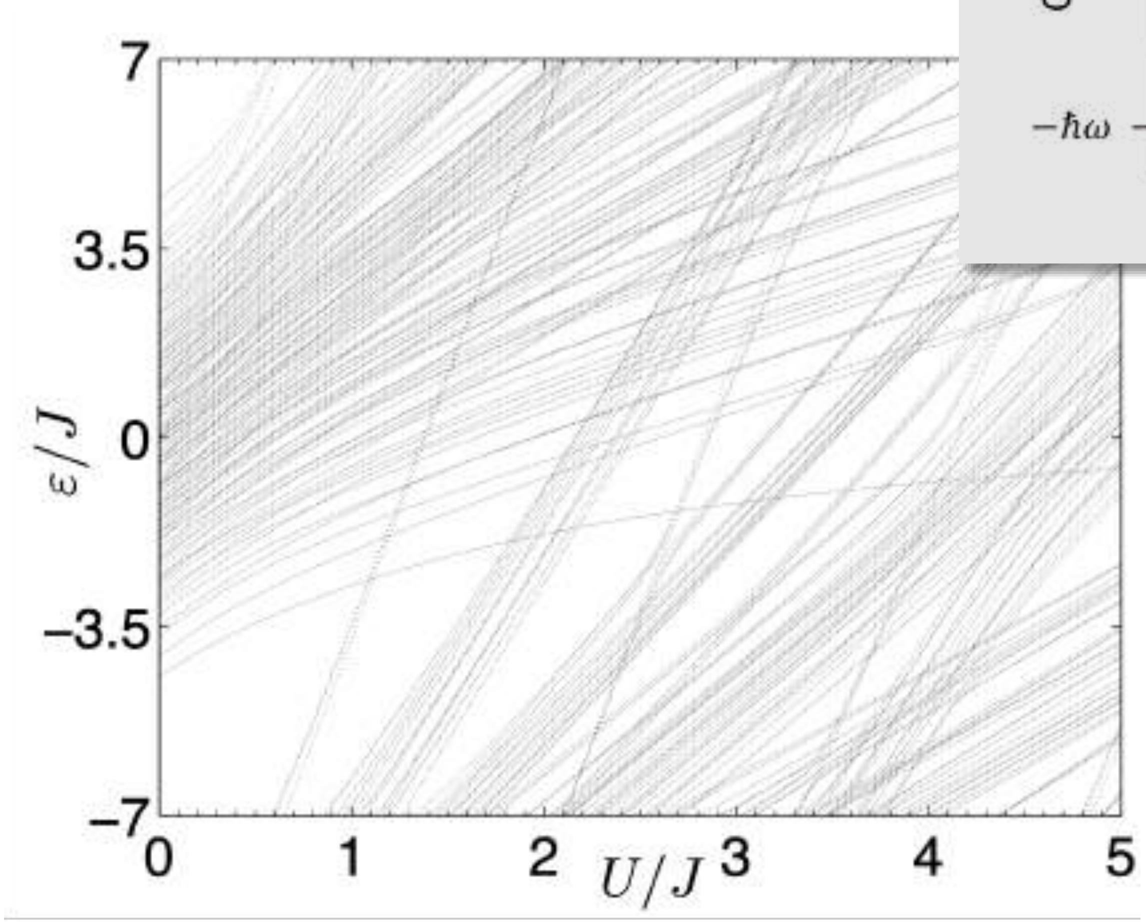
Dissipative Floquet engineering of gapped many-body phases using thermal baths

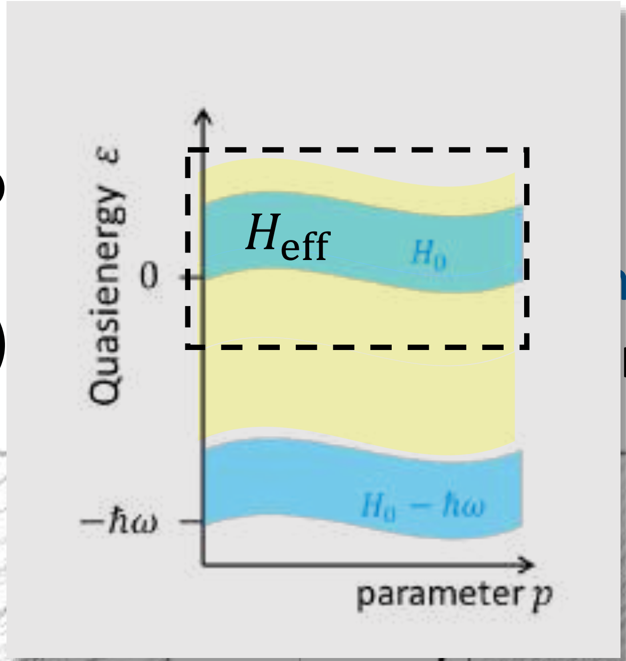
arXiv:2604.01291

Example: Driven B model

Quasienergy spectrum

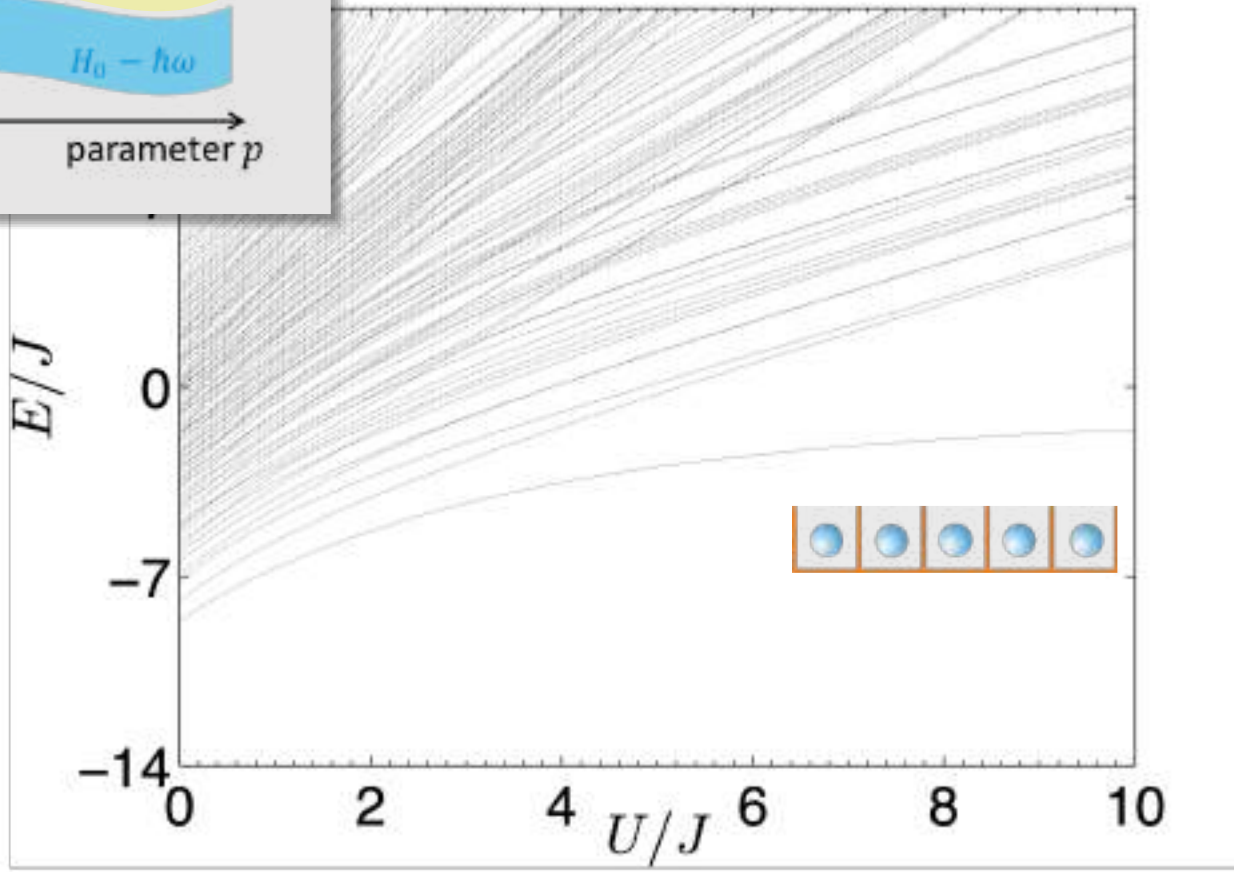
($\hbar\omega = 14\,J$, and $J_{\text{eff}} = \mathrm{J}/2$)





ergy spectrum

ndriven 5 particles on 5 sites)

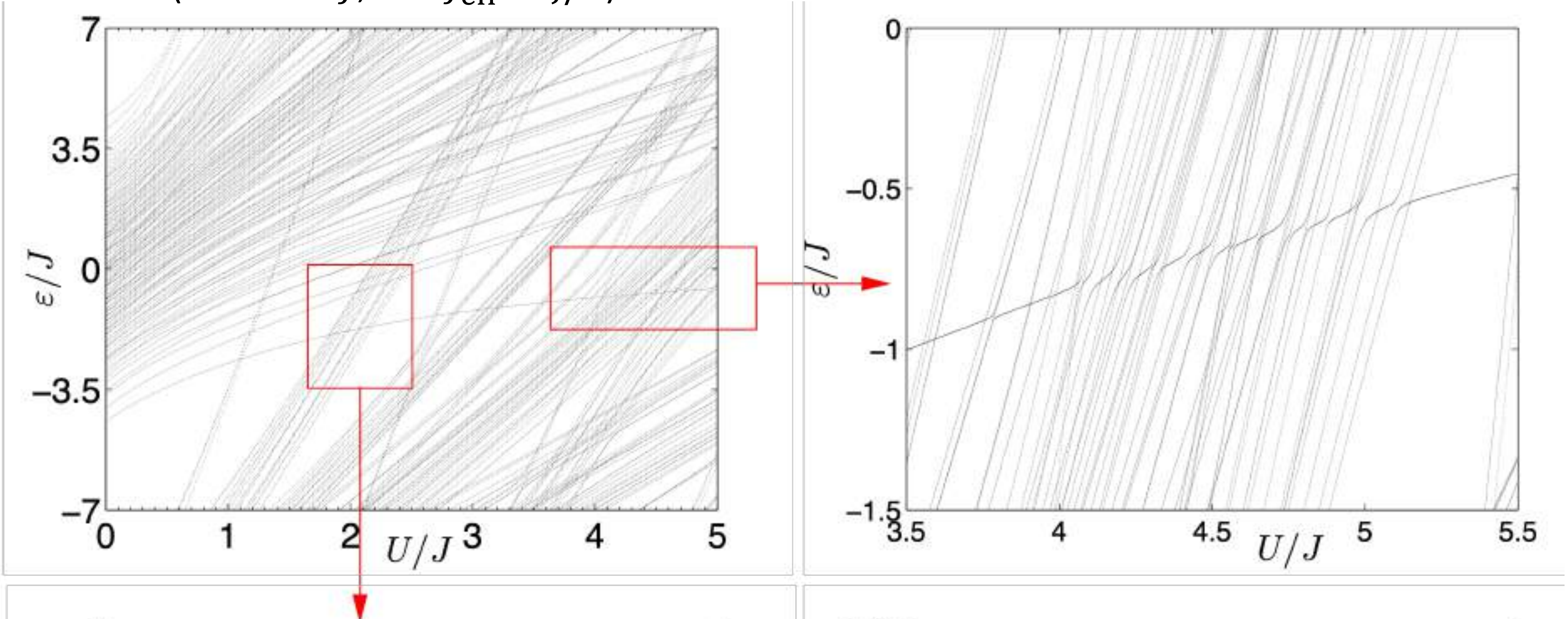


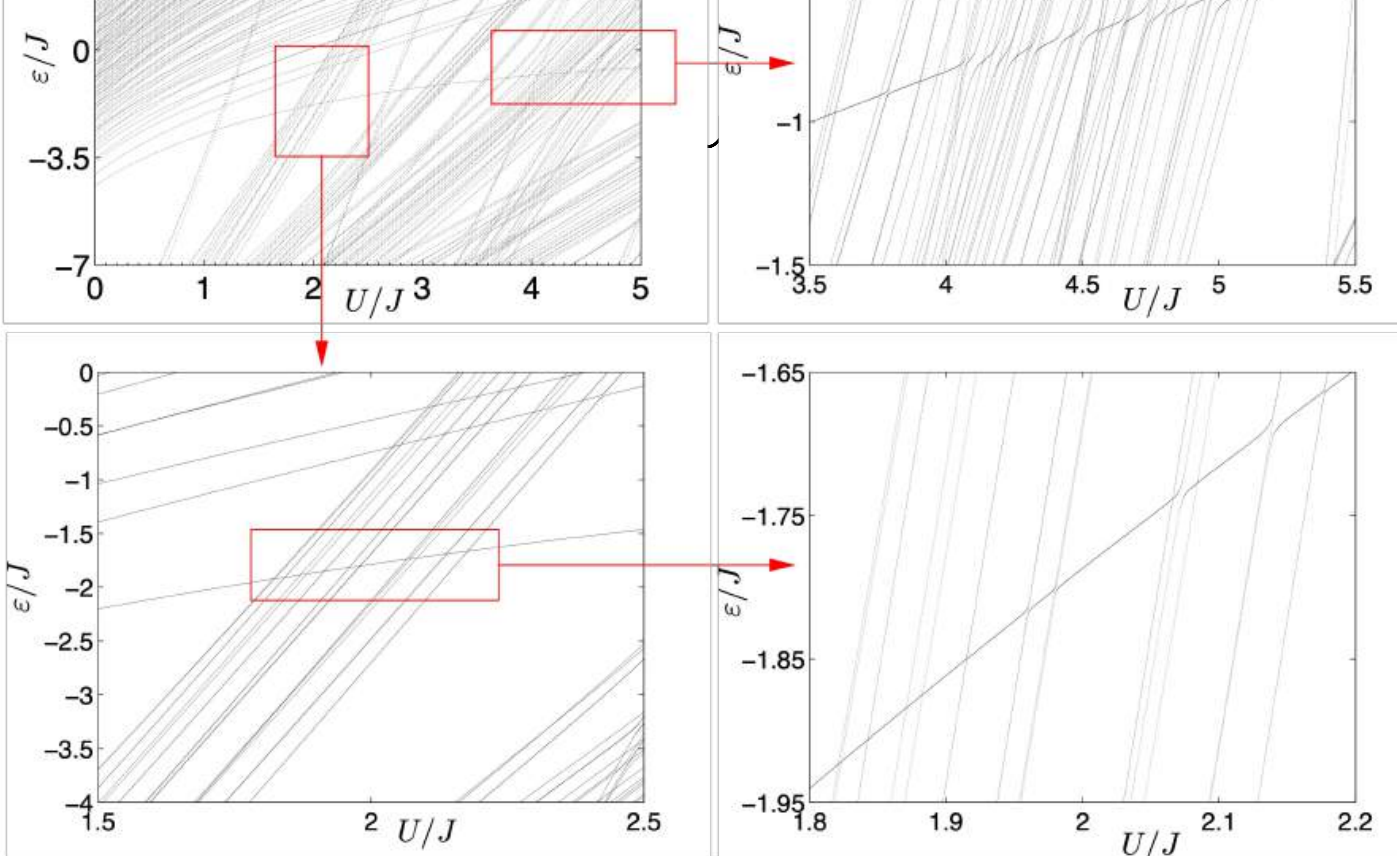
Example: Driven Bose-Hubbard model

Quasienergy spectrum

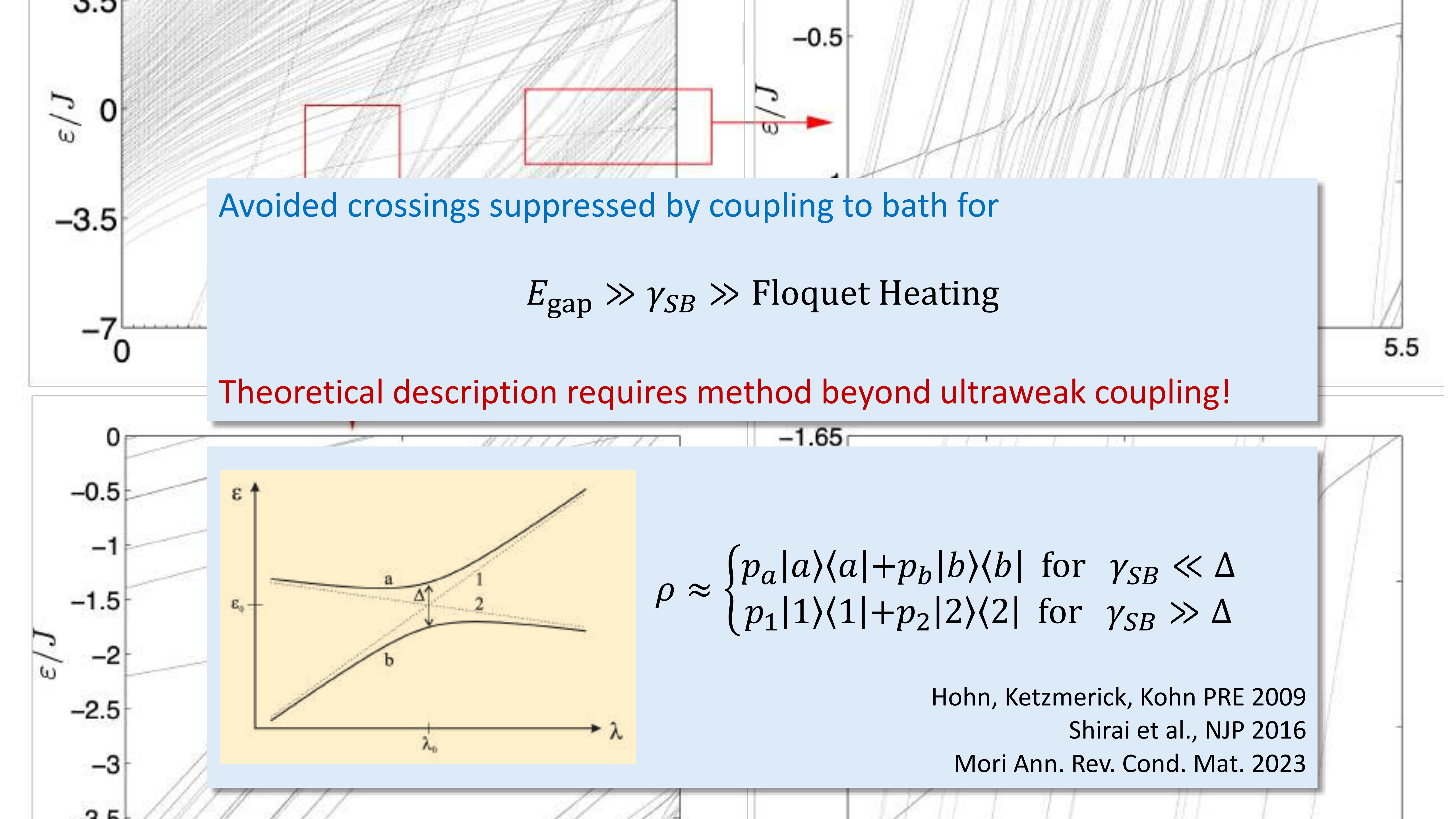
($\hbar\omega = 14\,J$, and $J_{\mathrm{eff}} = \mathrm{J}/2$)

Floquet heating indicated by avoided crossings





ε/J
0
−3.5
−7
0
1
2
3
4
5
U/J
−1
−1.5
3.5
4
4.5
5
5.5
0
−0.5
−1
−1.5
−2
−2.5
−3
−3.5
−4
1.5
2
2.5
−1.65
−1.75
−1.85
−1.95
1.8
1.9
2
2.1
2.2



Avoided crossings suppressed by coupling to bath for

$$E_{\text{gap}} \gg \gamma_{SB} \gg \text{Floquet Heating}$$

Theoretical description requires method beyond ultraweak coupling!

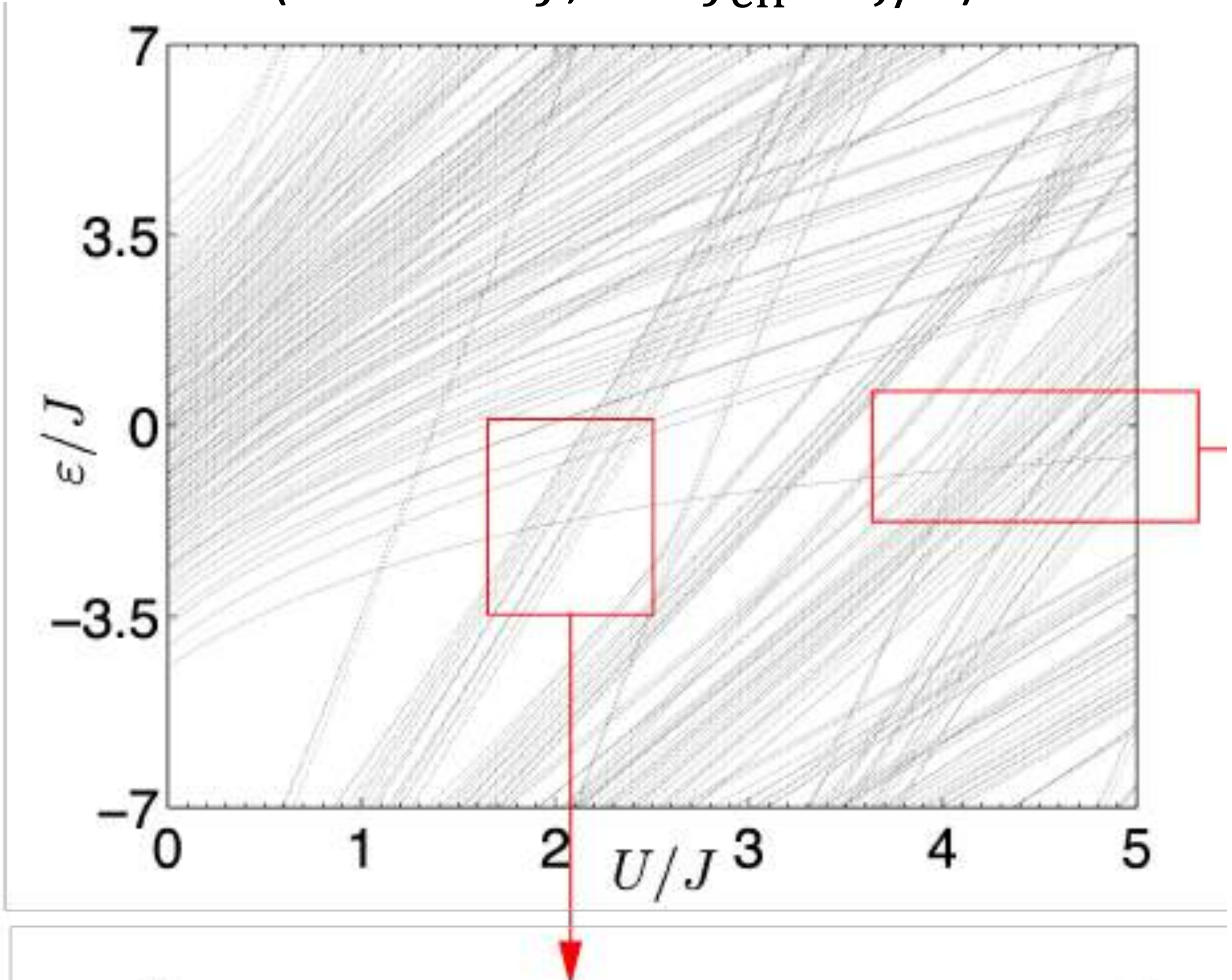
$$\rho \approx \begin{cases} p_a|a\rangle\langle a| + p_b|b\rangle\langle b| & \text{for } \gamma_{SB} \ll \Delta \\ p_1|1\rangle\langle 1| + p_2|2\rangle\langle 2| & \text{for } \gamma_{SB} \gg \Delta \end{cases}$$

Hohn, Ketzmerick, Kohn PRE 2009
Shirai et al., NJP 2016
Mori Ann. Rev. Cond. Mat. 2023

Example: Driven Bose-Hubbard model

Quasienergy spectrum

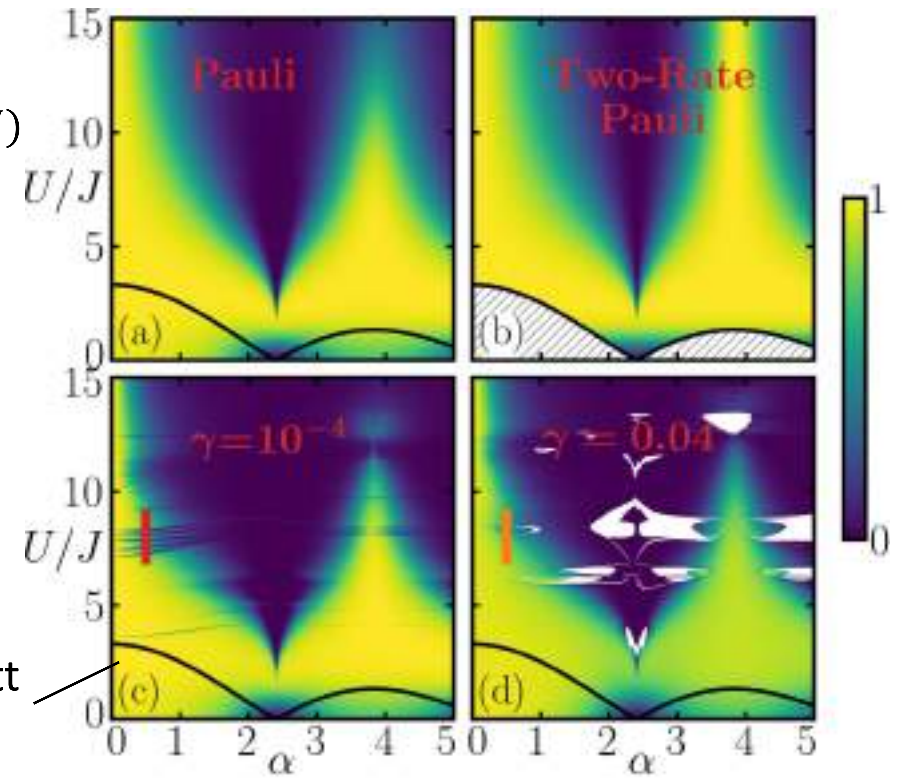
($\hbar\omega = 14\,J$, and $J_{\mathrm{eff}} = \mathrm{J}/2$)



Overlap with effective ground state

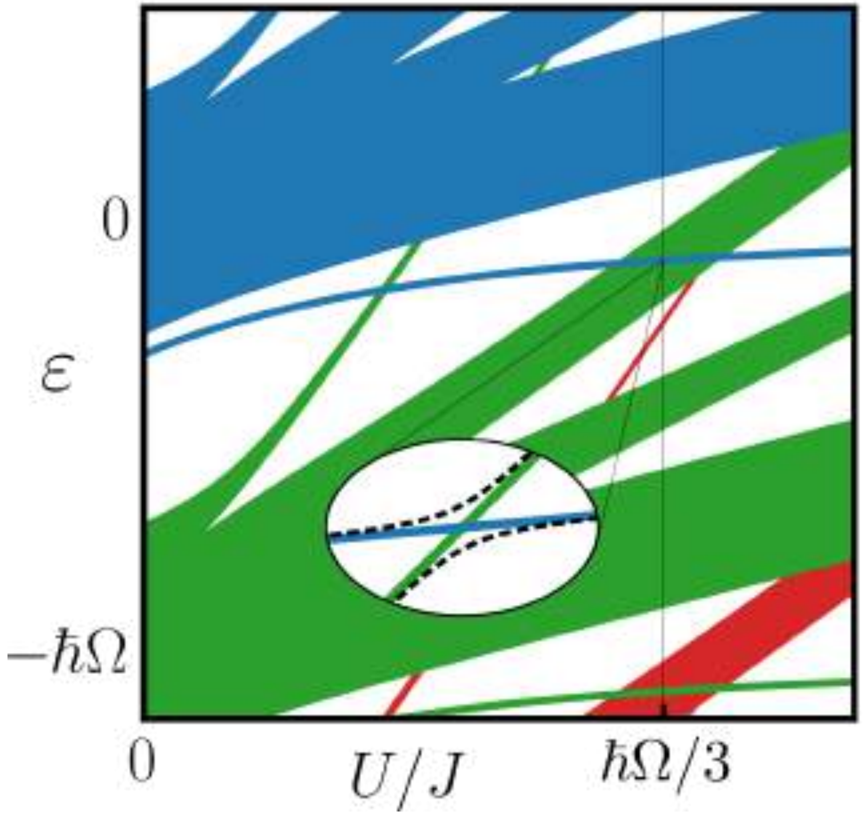
Driven system coupled to thermal bath

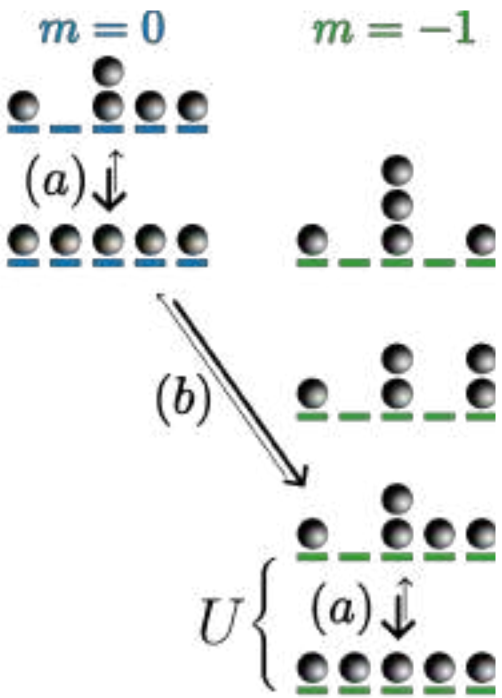
$T = 0.1J$
$E_{\mathrm{cut}} = \max(U,J)$
$\hbar\omega = 20\,J$
$N = M = 4$



Superfulid-Mott transition line

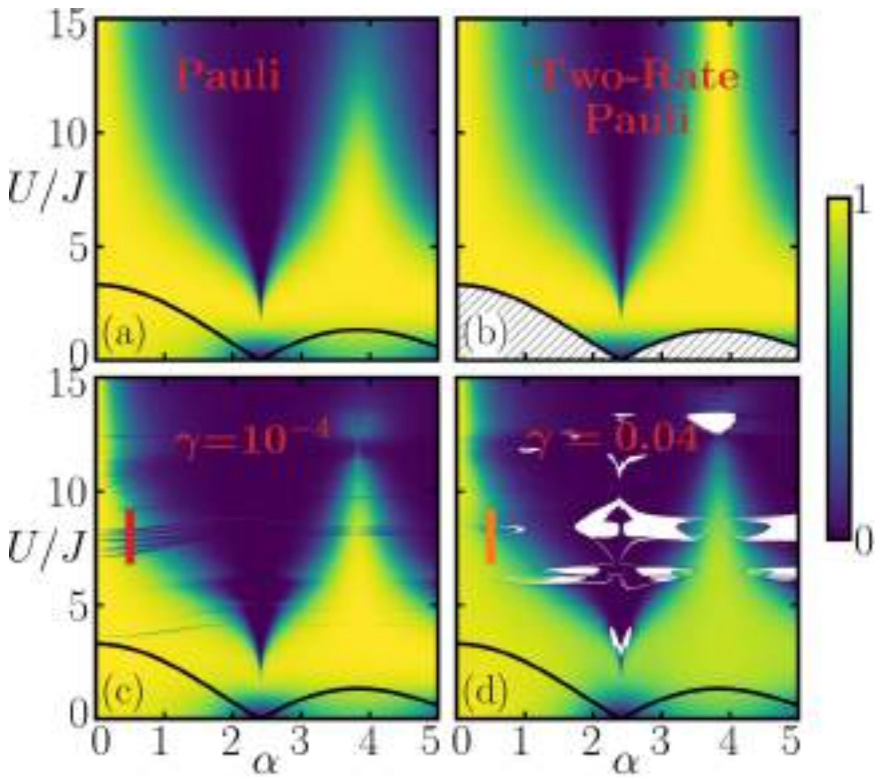
Example: Driven Bose-Hubbard model



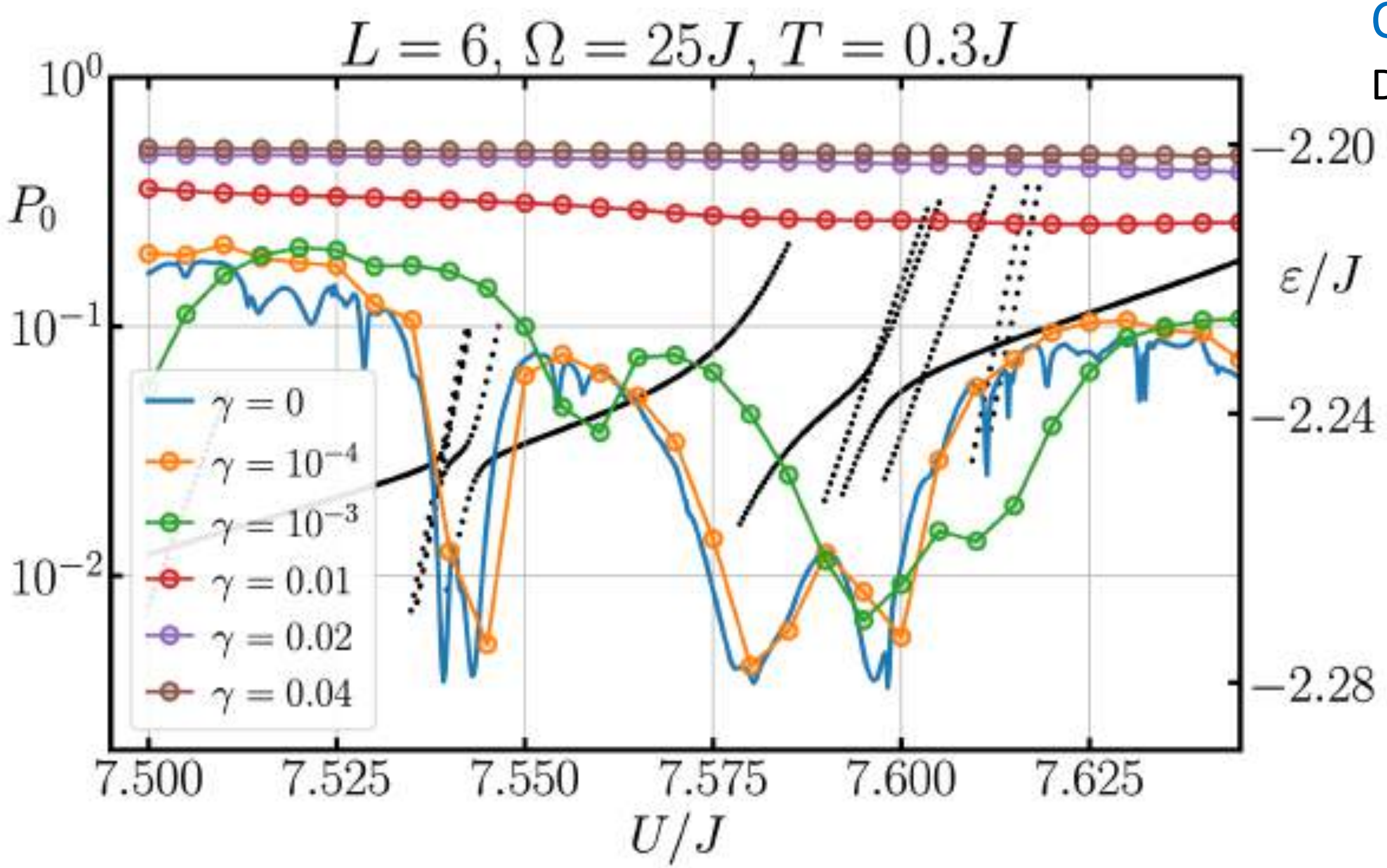


Overlap with effective ground state

Driven system coupled to thermal bath

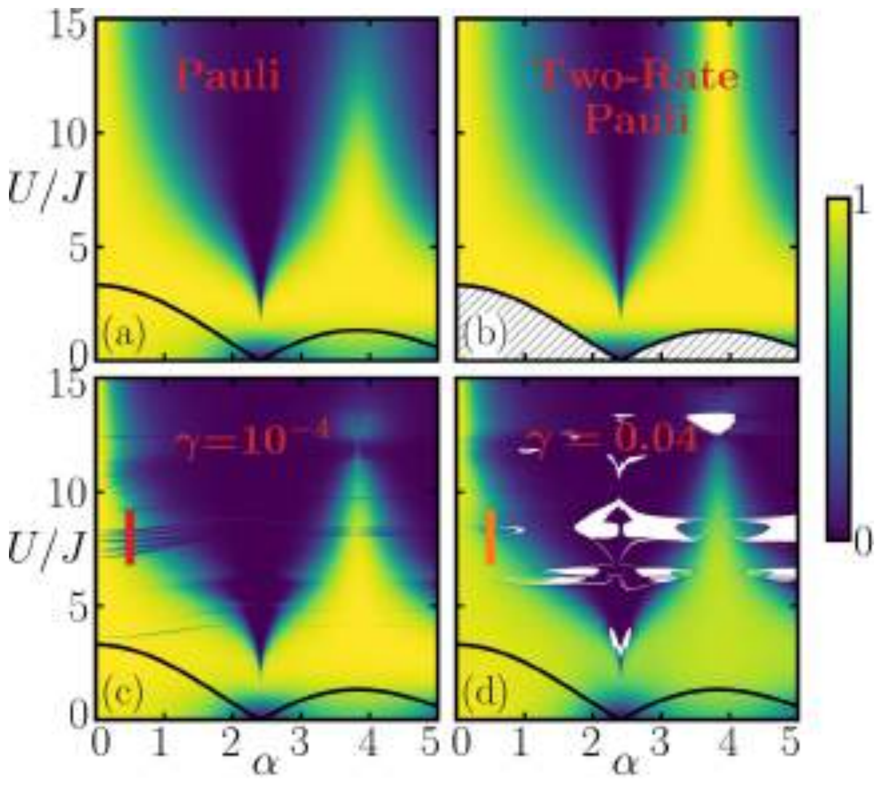


Example: Driven Bose-Hubbard model



Overlap with effective ground state

Driven system coupled to thermal bath



General scheme

ε_X: energy change induced by applying operator X to the ground state $|\tilde{u}_0\rangle$ of H_0.

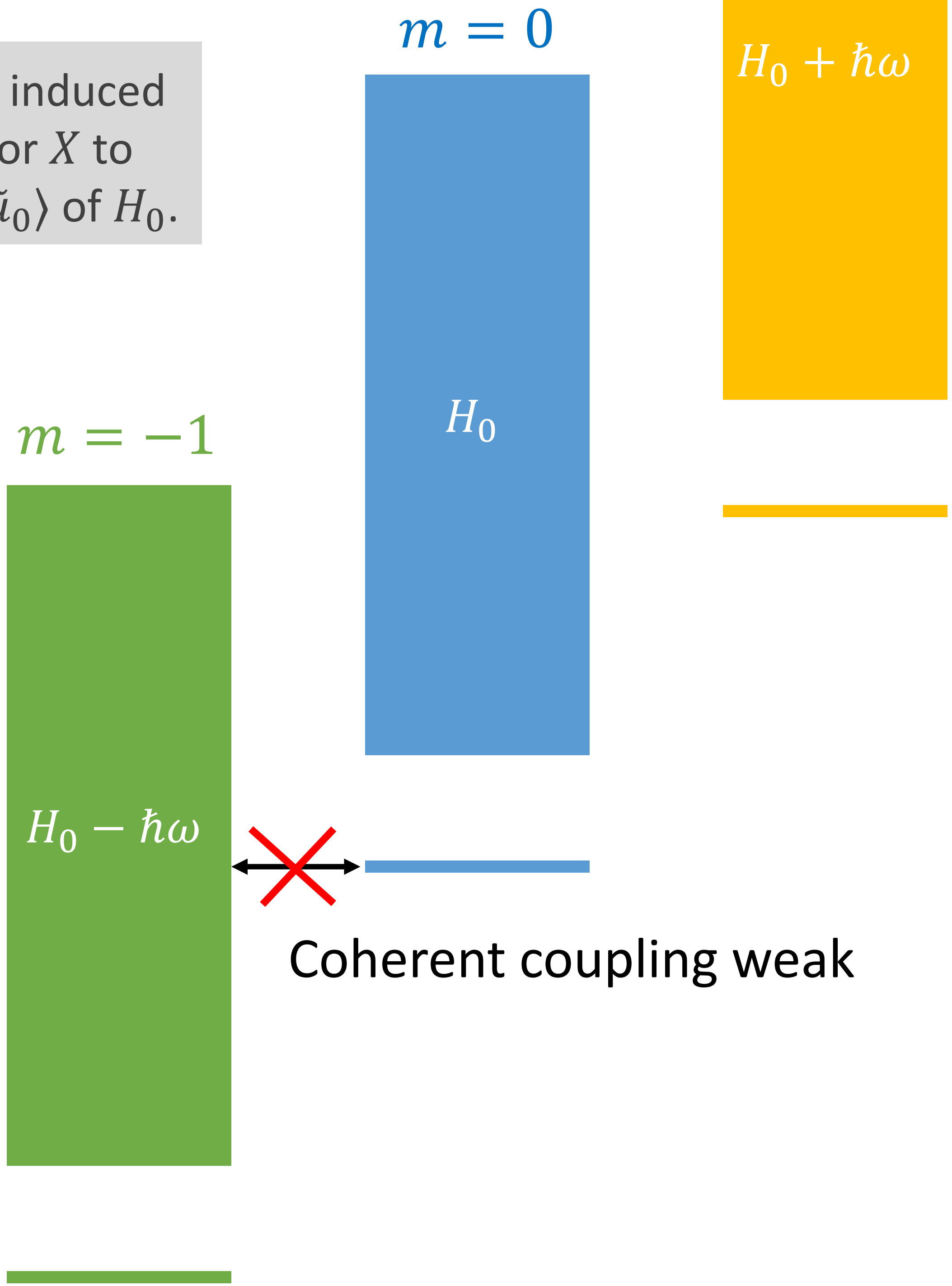
Floquet heating slow: $\mathcal{E}_{H_\mu} \ll \hbar\omega$

General condition for Floquet engineering

Suppress Floquet heating: $\tau_{\mathrm{relax}} \ll \tau_{\mathrm{heating}}$

Coherent resonant excitation suppressed by system-bath coupling

[Hohn, Ketzmerick, Kohn PRE2009; Shirai et al., NJP 2016]



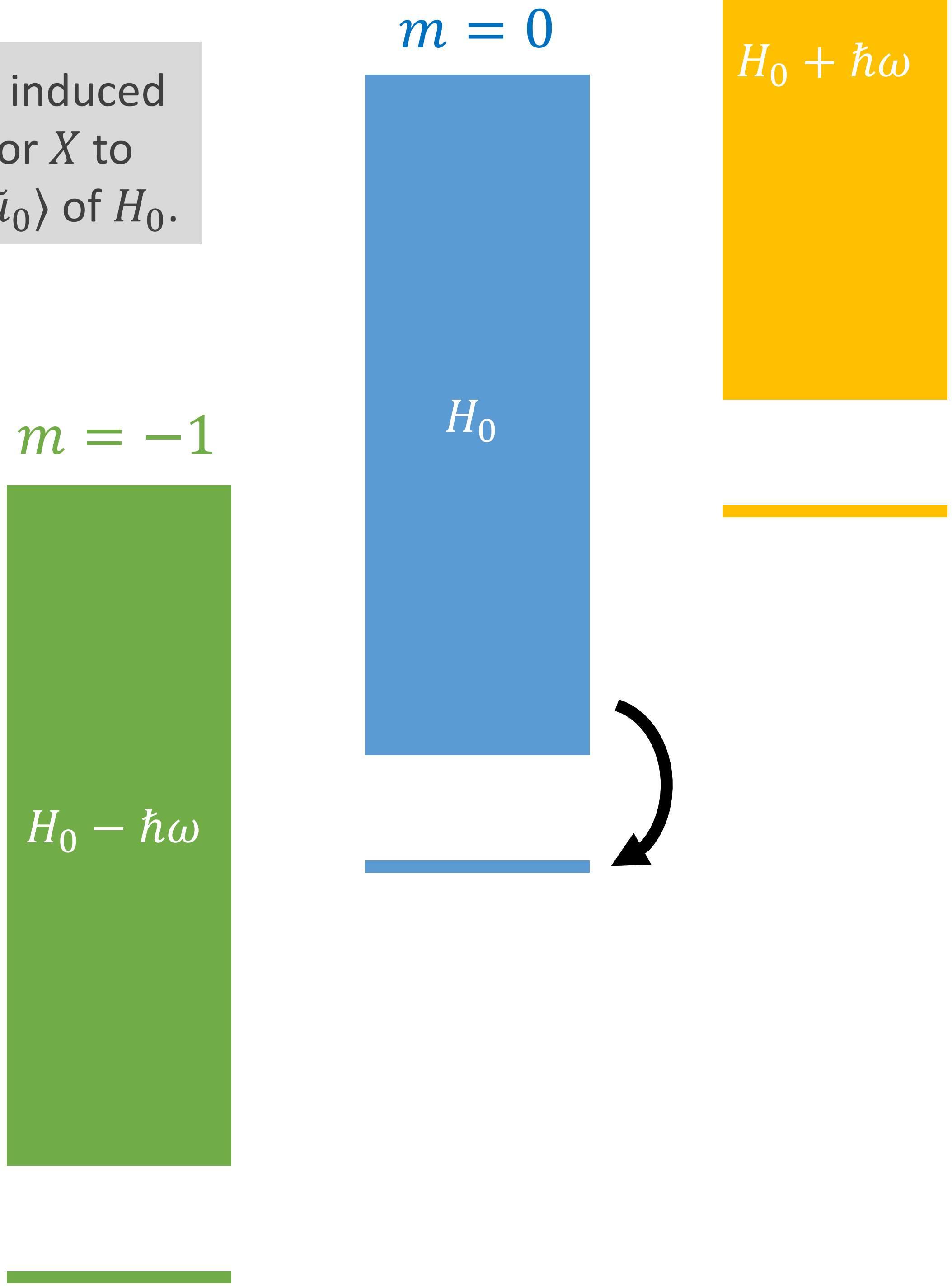
General scheme

$\mathcal{E}_X$: energy change induced by applying operator X to the ground state $|\tilde{u}_0\rangle$ of H_0.

Floquet heating slow: $\mathcal{E}_{H_\mu} \ll \hbar\omega$

Suppress Floquet heating: $\tau_{\mathrm{relax}} \ll \tau_{\mathrm{heating}}$

Allow bath-induced cooling: $E_{\mathrm{gap}} \lesssim \Delta_{\mathrm{bath}}$



General scheme

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Floquet heating slow: $\mathcal{E}_{H_\mu} \ll \hbar\omega$

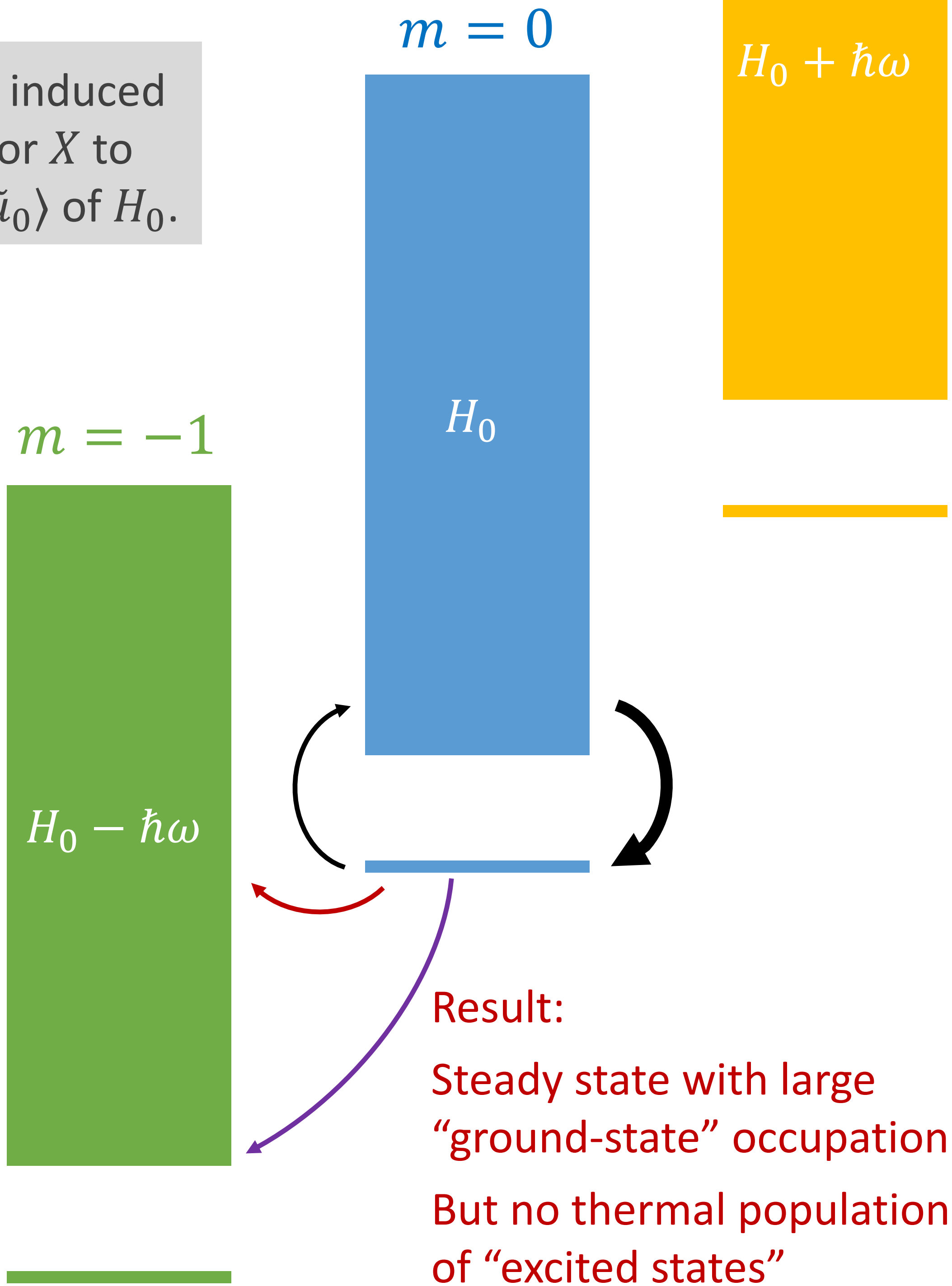
Suppress Floquet heating: $\tau_{\mathrm{relax}} \ll \tau_{\mathrm{heating}}$

Allow bath-induced cooling: $E_{\mathrm{gap}} \lesssim \Delta_{\mathrm{bath}}$

Suppress bath-induced heating:

$$\mathcal{E}_{H_{SB}} \ll E_{\mathrm{gap}}\,, \quad \beta^{-1} \ll E_{\mathrm{gap}}$$

$$\mathcal{E}_{H_{SB}} \ll \hbar\omega\,, \qquad \Delta_{\mathrm{bath}} \ll \hbar\omega$$



Result:

Steady state with large "ground-state" occupation

But no thermal population of "excited states"

Challenges beyond ultraweak coupling

$$\dot{\rho} = -(i/\hbar)\,[H,\rho] + \mathcal{D}[\rho]$$

1) Determine dissipator!

2) Solve equation!

Both challen
many-body s
with expone
large state s

Local approx. to Redfield equation
A. Schnell, PRL 2025

Truncated Redfield equation
T. Becker, L.-N. Wu, A.E., PRE 2021

Efficient quantum trajectories for Redfield equation
T. Becker, C. Netzer. A.E., PRL 2023; Becker & A.E. PRE 2025

Hard to derive
Unperturbed problem
is full system Hamiltonian

Non-local equation
of motion

Not of Lindblad Form
$\Rightarrow$ Not straightforward to unravel in stochastic eq.

Born-Markov-
Redfield

Not consistent with thermal state $\rho_{\mathrm{thermal}} \propto \mathrm{tr}_{\mathrm{Bath}}\{e^{-\beta H_{\mathrm{tot}}}\}$

Canonically consistent quantum master equation
T. Becker, A. Schnell, J. Thingna, PRL 2022

$0 + \epsilon$

Ulatraweak
coupling

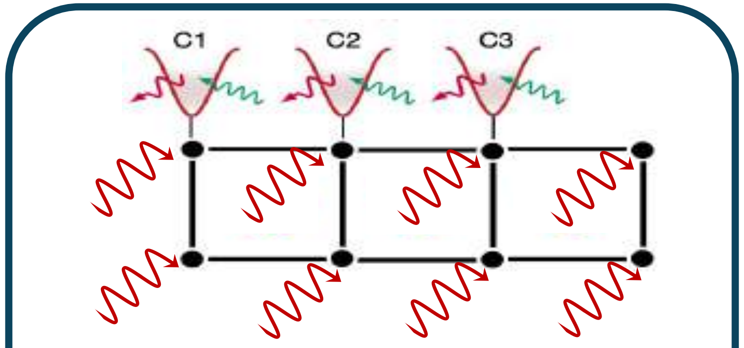
Quantum optical
master equation
of GKSL-type

Methods extending the state space

- Reaction-coordinate/start-to-chain mapping
- Hierarchie of eqs. of motion (HEOM)/pure states (HOPS)
- Influence-Matrix / Process-Tensor approach

Coupling
strength

Conclusions

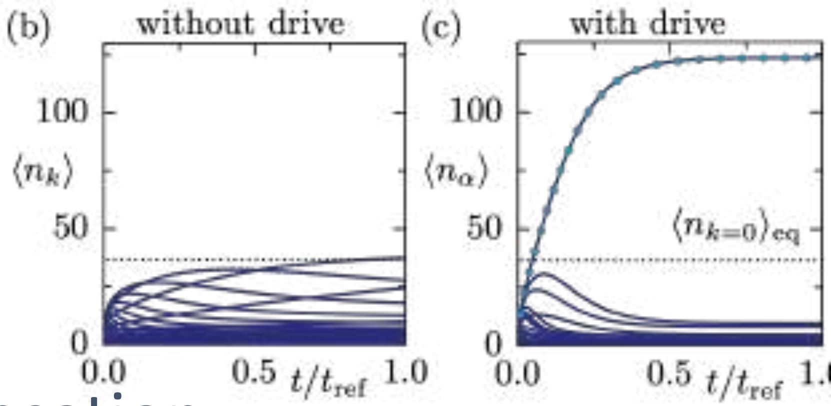


Ground-state cooling of H_{eff} ?

Combining driving and dissipation is powerful tool for state preparation.

Prepare effective ground-states of interacting systems

- Suppression of Floquet heating
- No adiabatic state preparation required



Prepare novel non-equilibrium states

- Non-equilibrium high-temperature Bose condensation

Need for efficient theoretical methods beyond ultraweak coupling!