

Precision in pQCD

Daniel de Florian
Instituto de Ciencias Físicas
UNSAM - Argentina



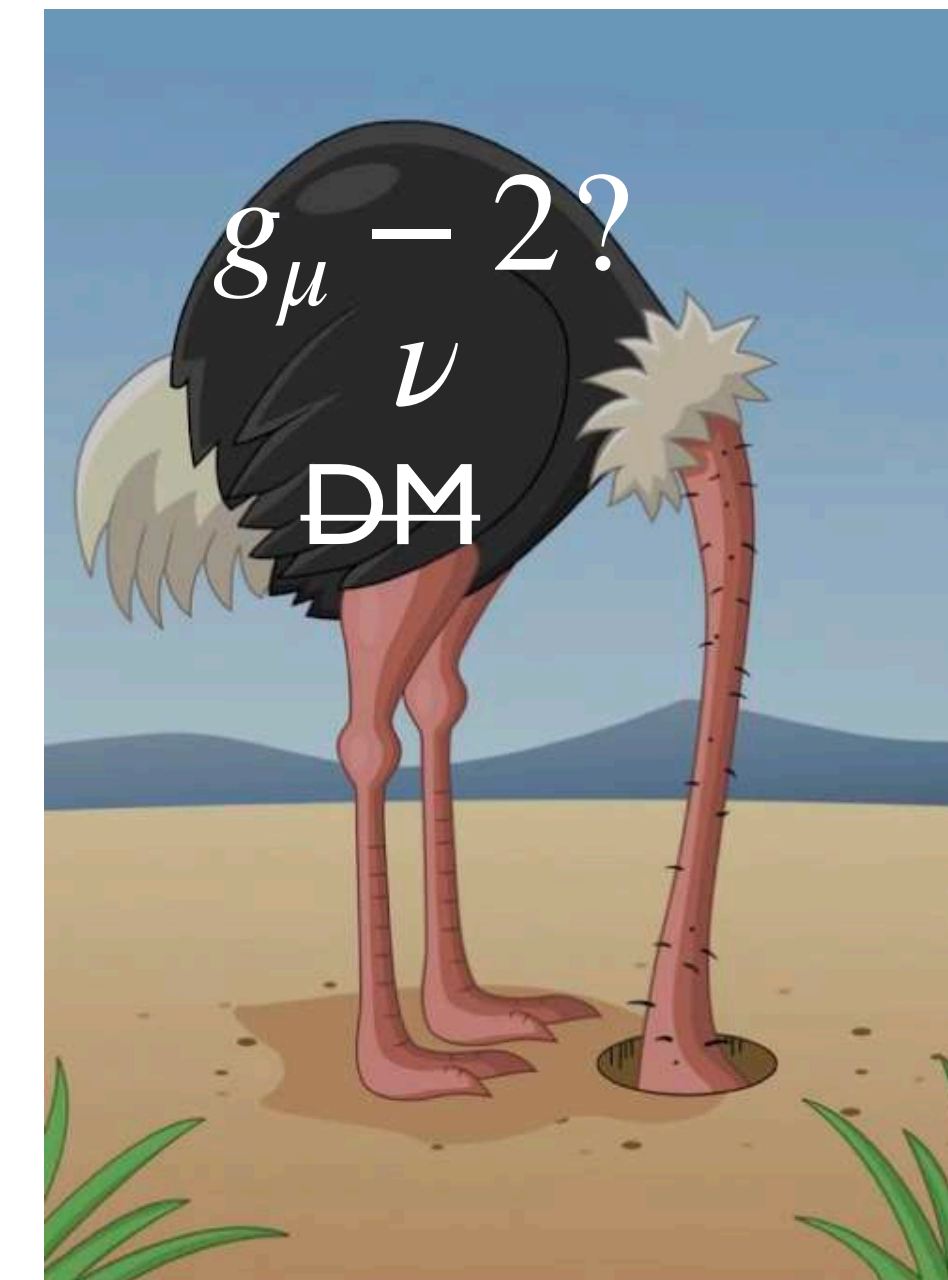
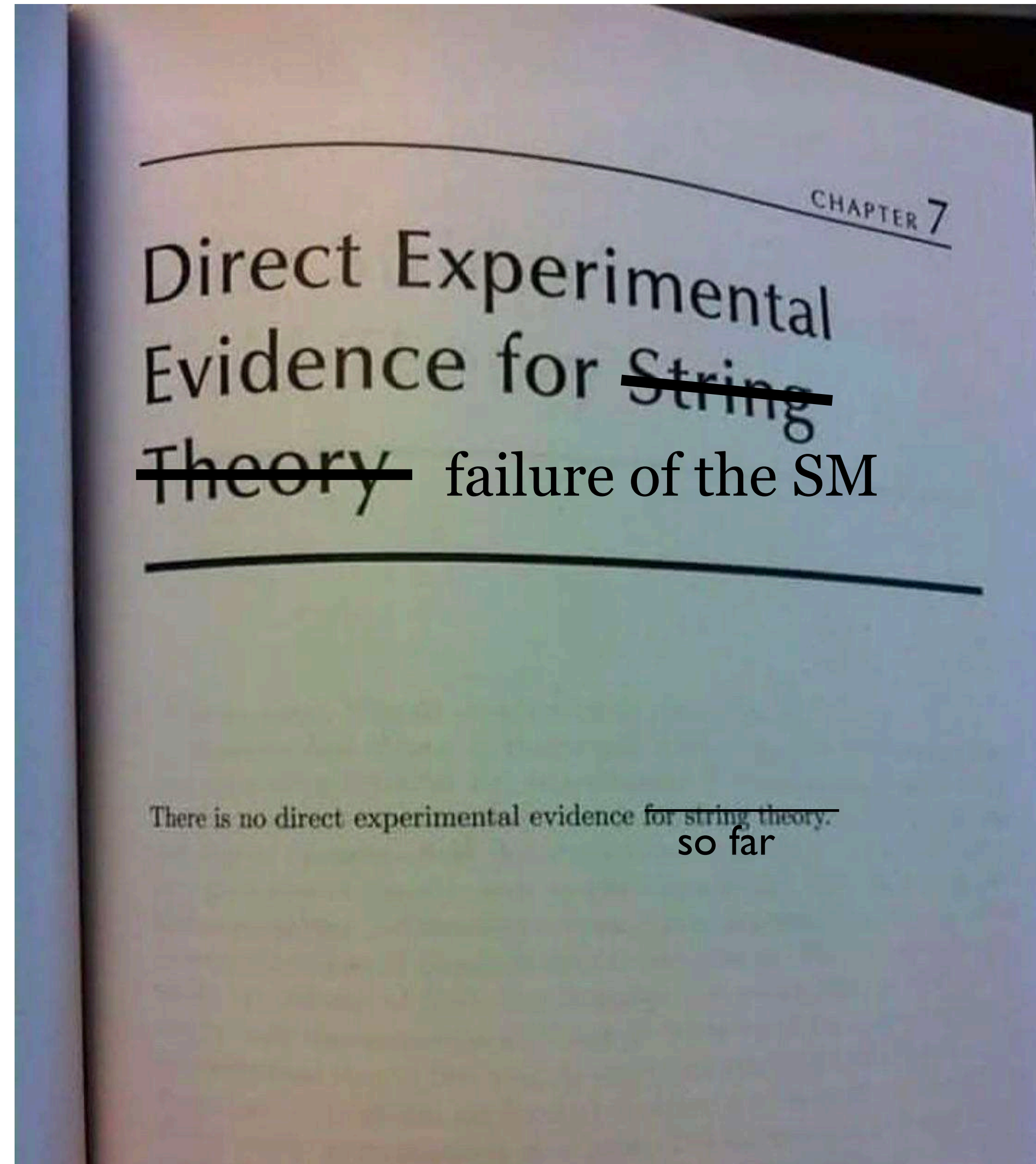
August 10 – 12, 2026
at IFT-UNESP, São Paulo, Brazil

OUTLINE

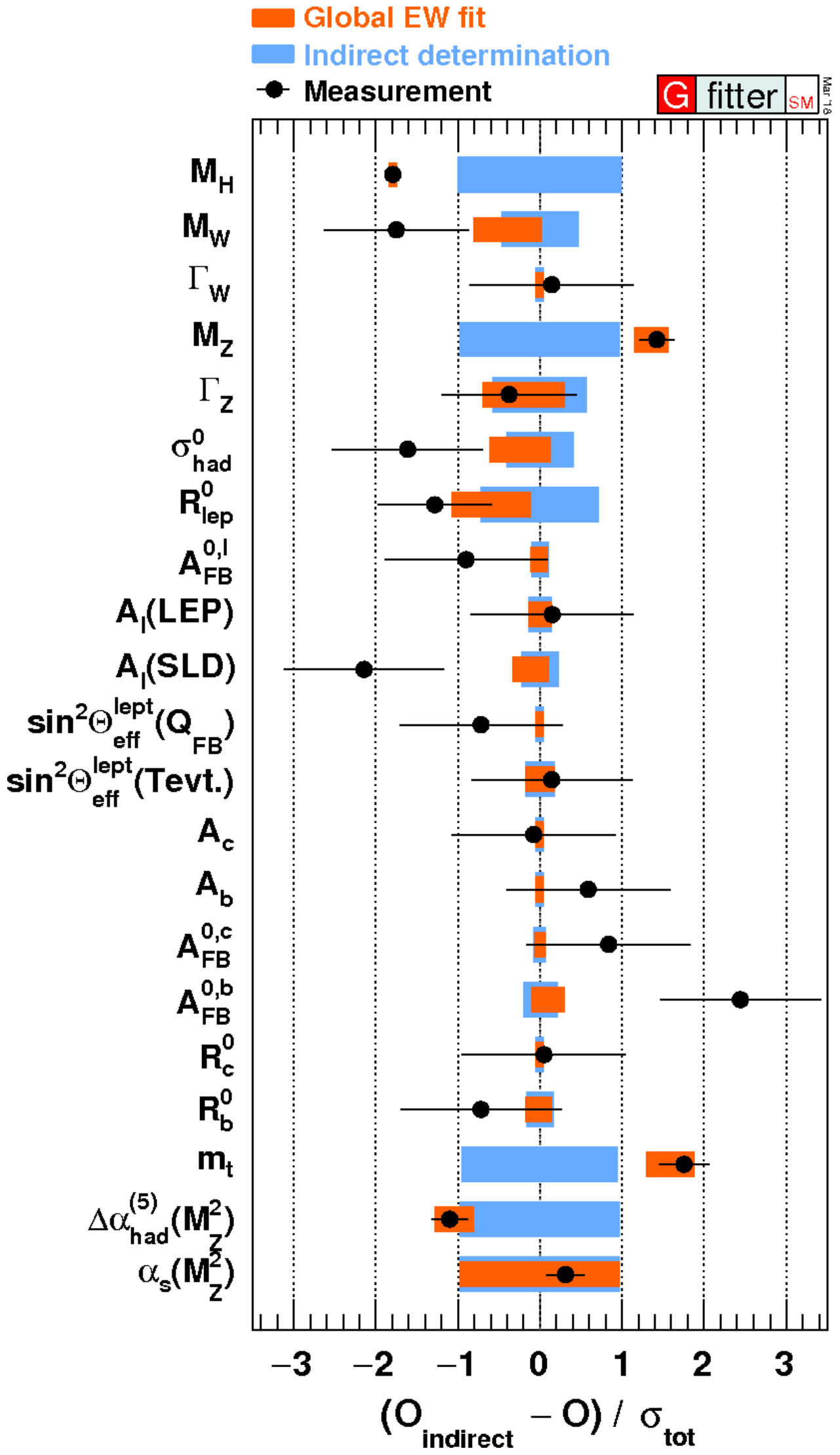
- ▶ 1) QCD and precision at the LHC (the usual stuff)
- ▶ 2) My realistic view about 1)

No soy pesimista, es que el mundo es pésimo

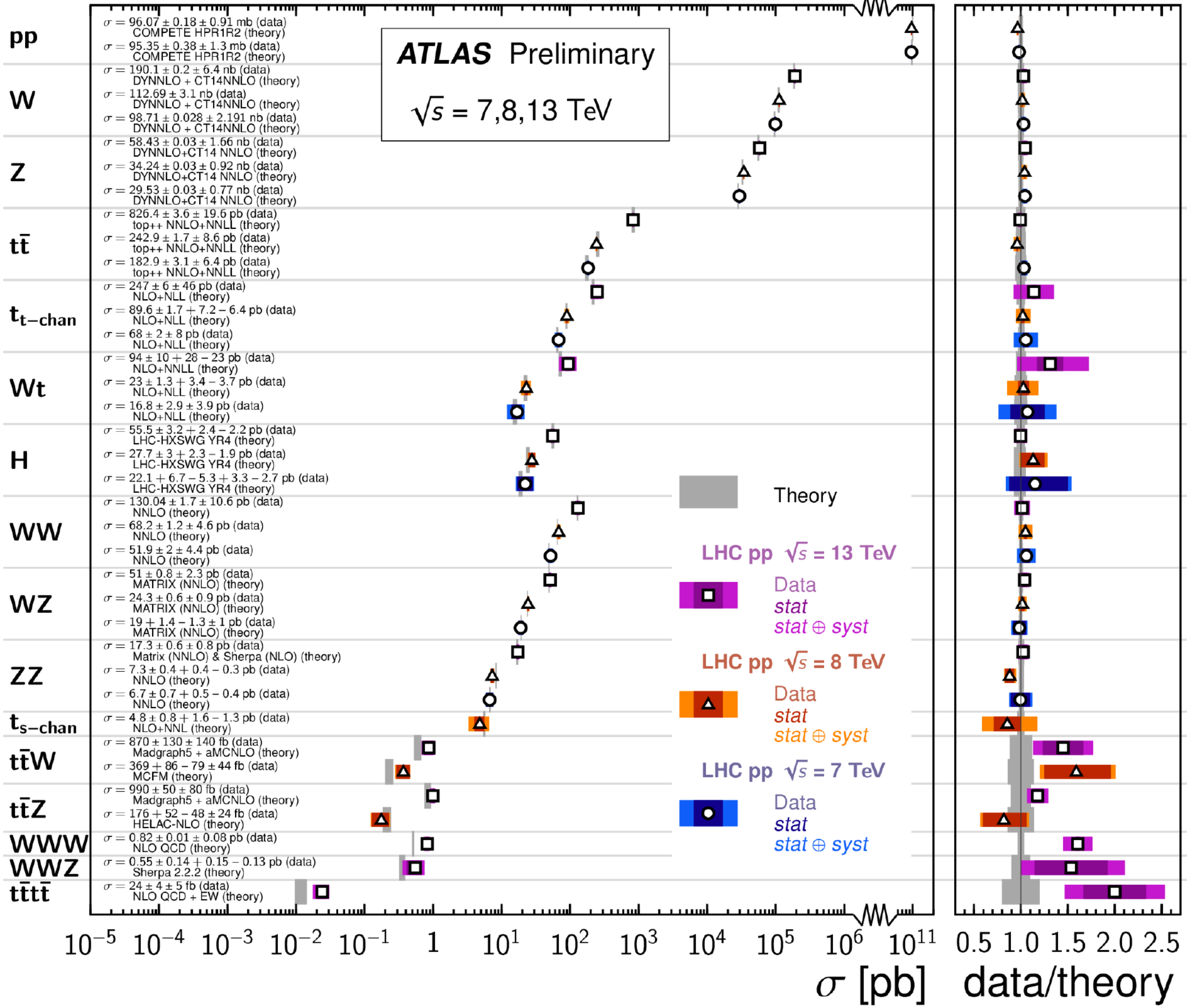
Everything looks SM-like at LHC



Everything looks SM-like at LHC

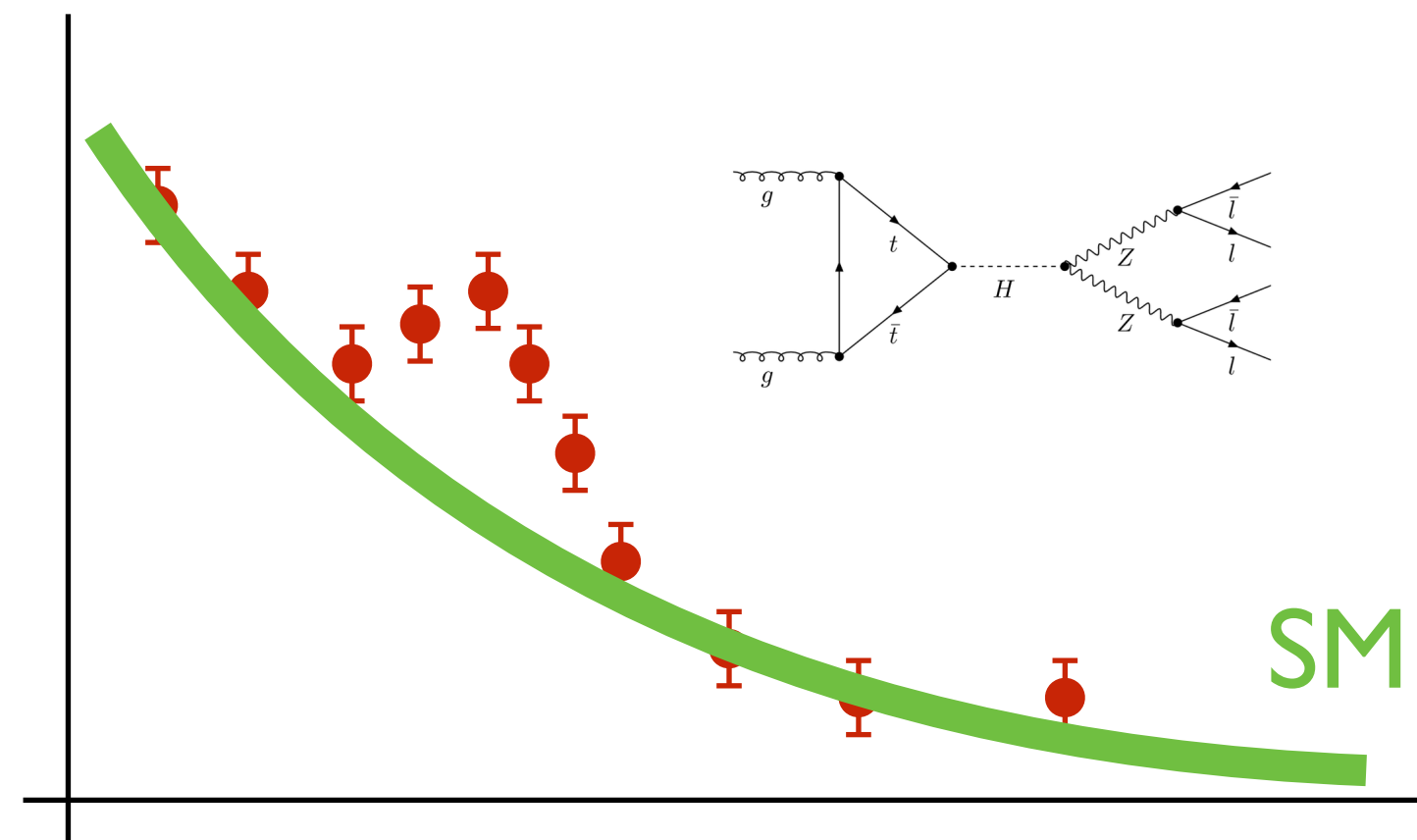


Standard Model Total Production Cross Section Measurements



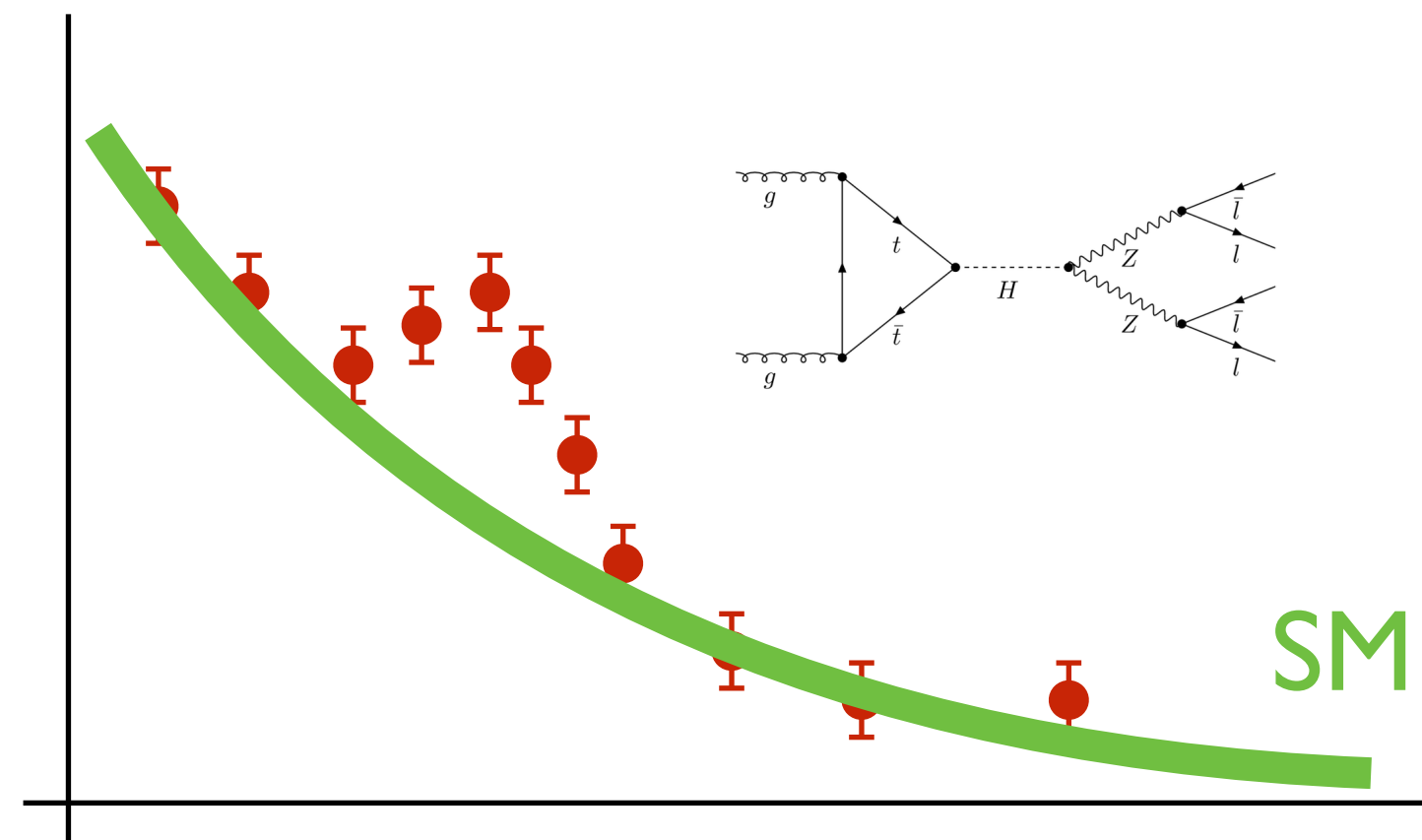
- ▶ Still of room for new discoveries : two main scenarios

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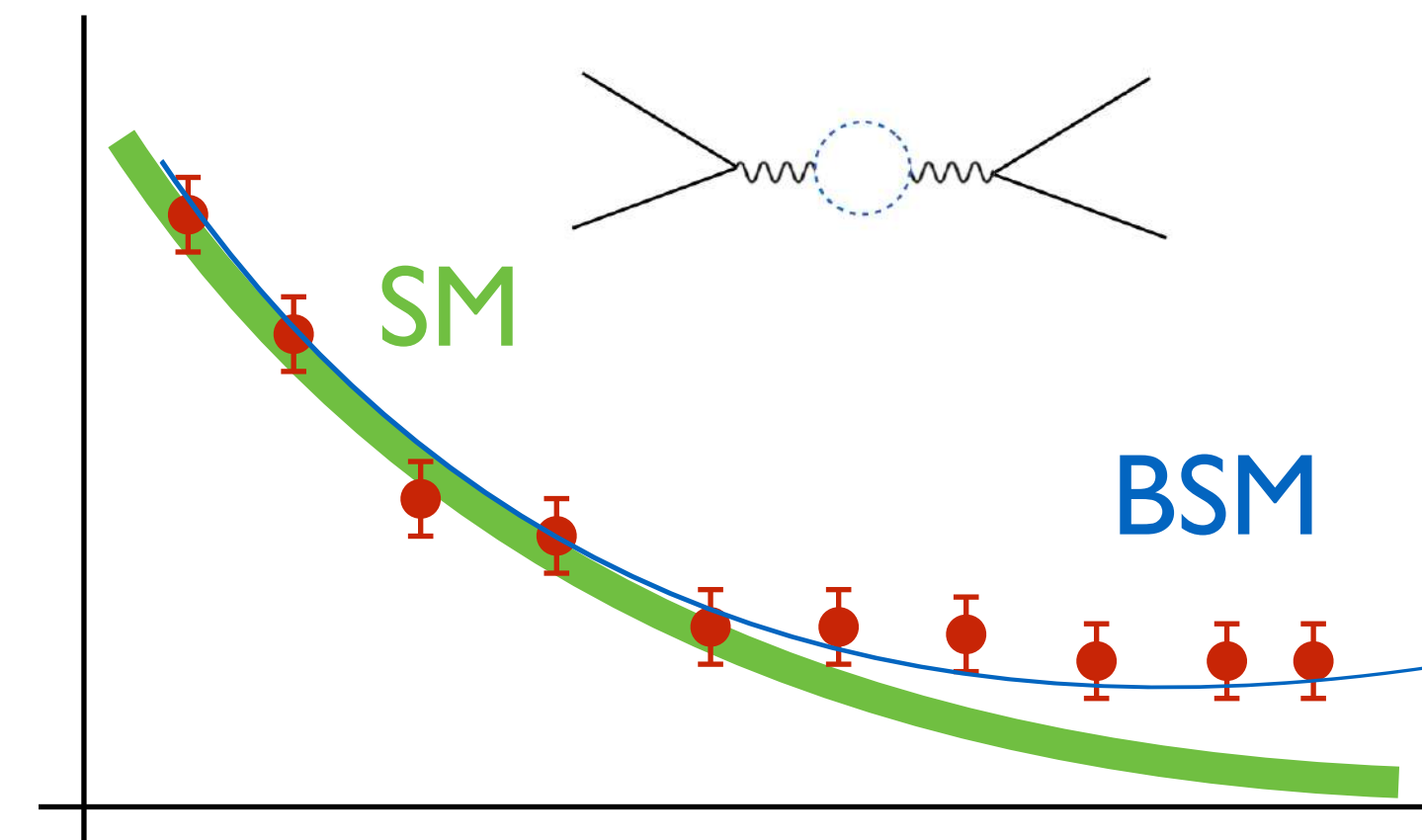
- ▶ Search for (and find) new states
- ▶ Resonance needs “descriptive” TH

- ▶ Still of room for new discoveries : two main scenarios

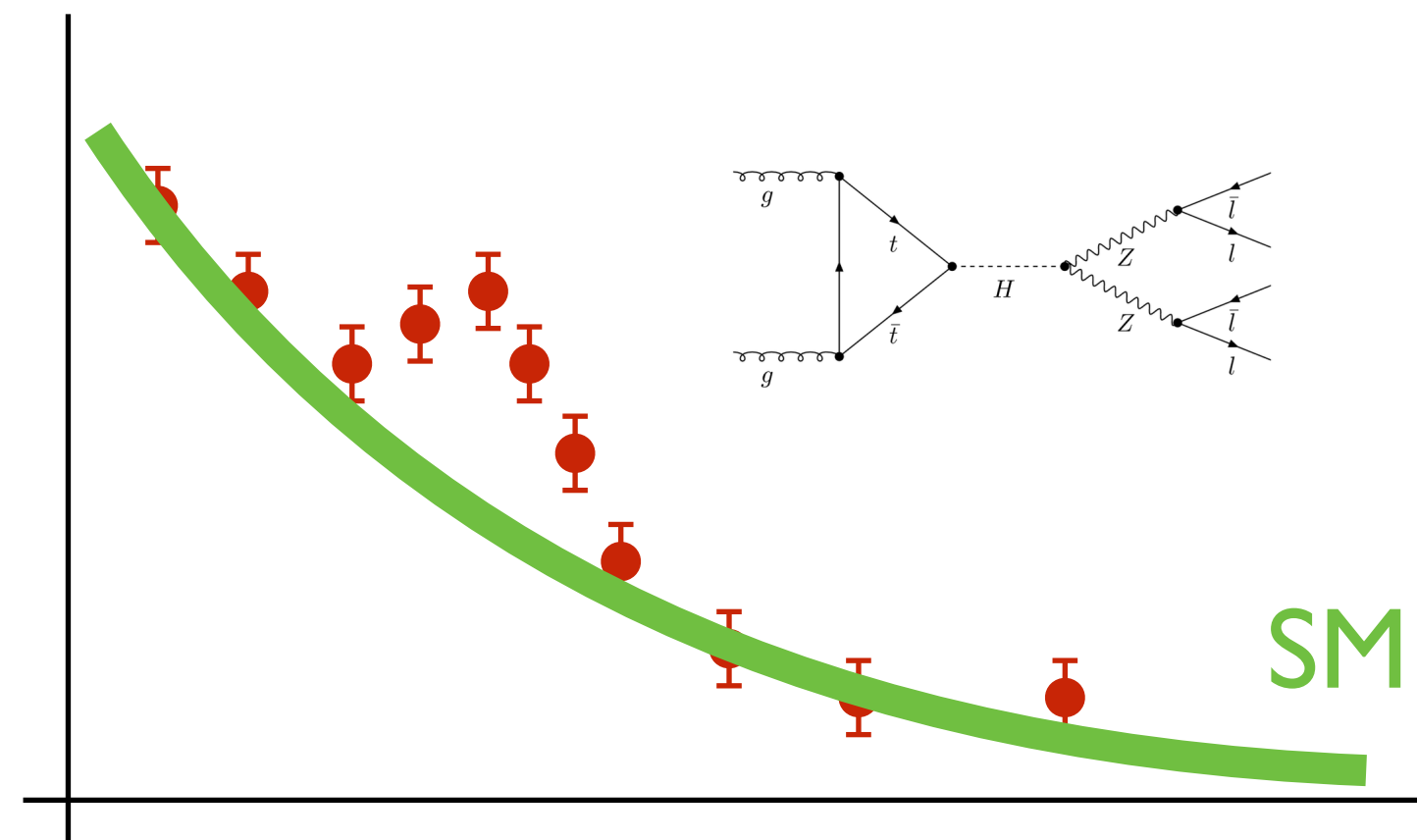


- ▶ Search for (and find) new states
- ▶ Resonance needs “descriptive” TH

- ▶ Most likely look for “new interactions”
- ▶ Small deviations from SM : PRECISION
- ▶ EFT description / BSM model

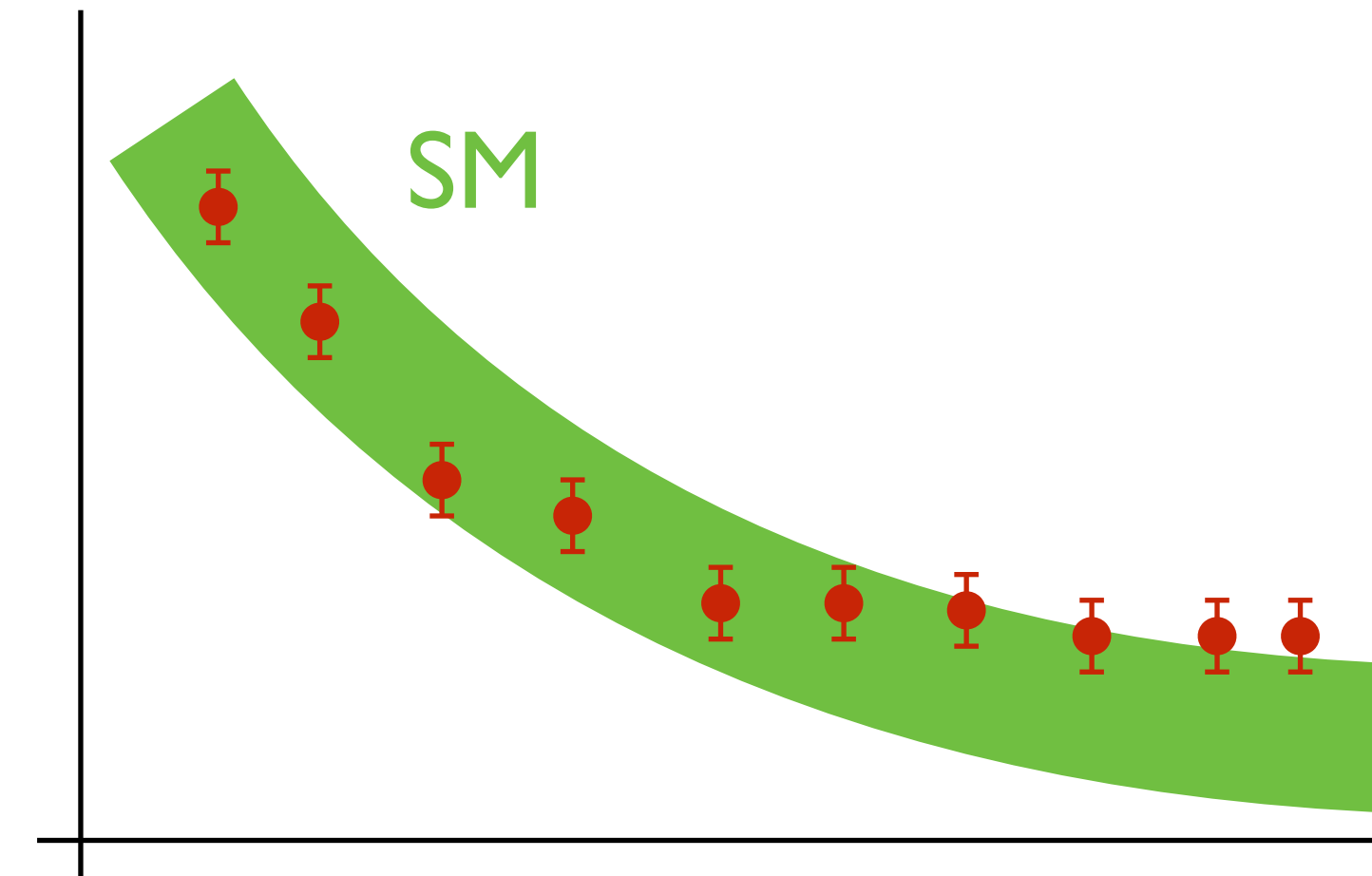


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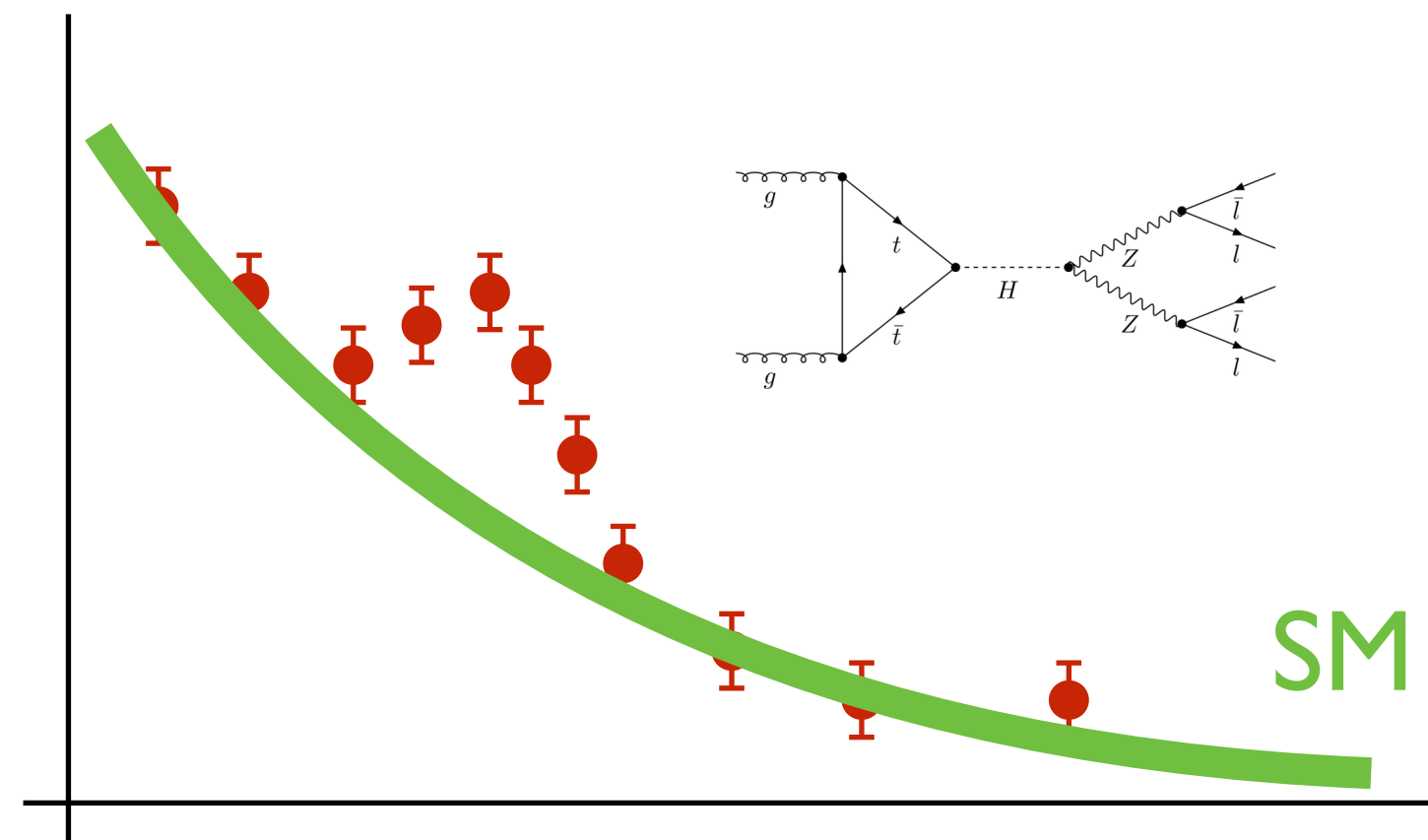


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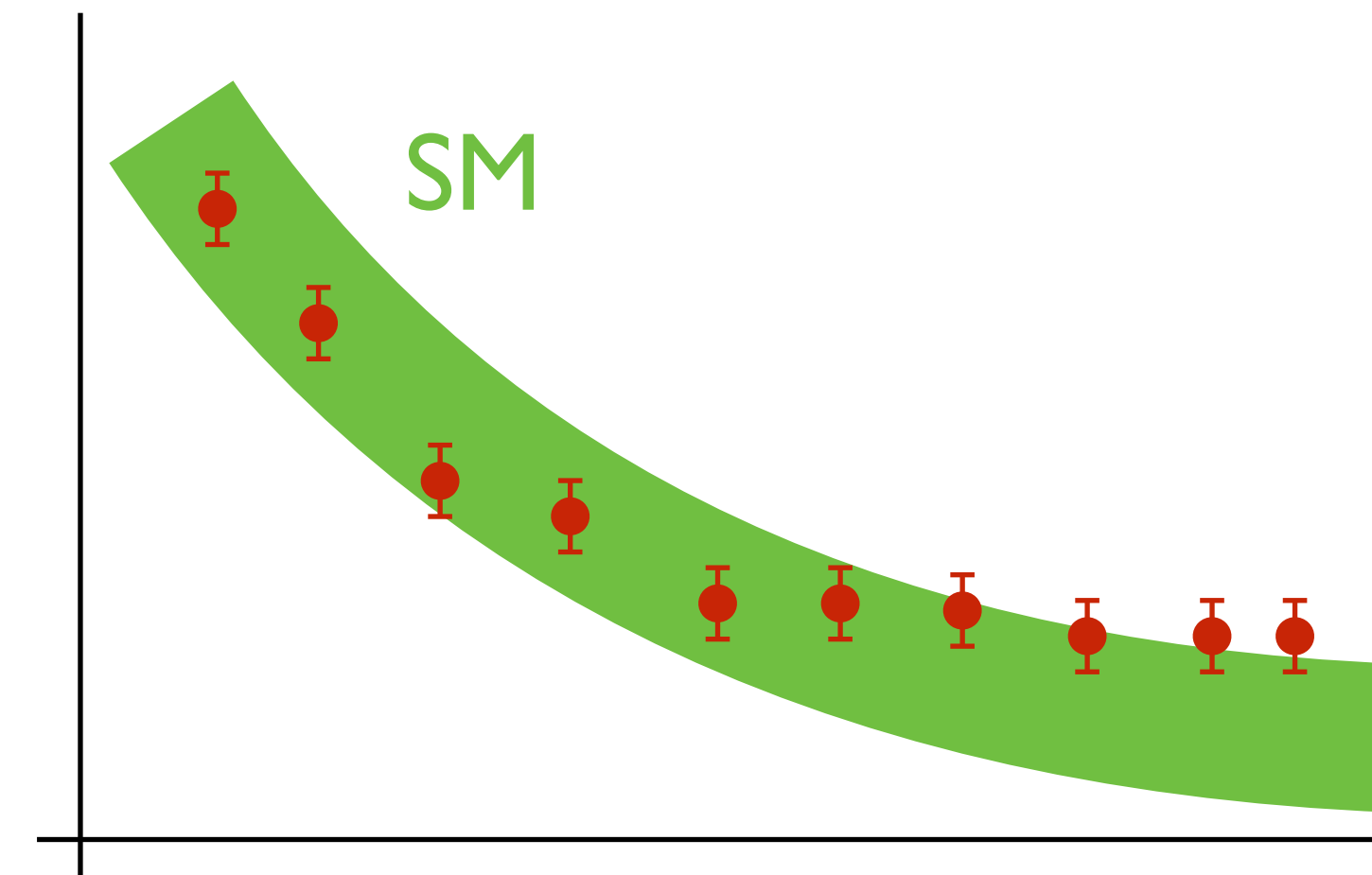


- ▶ Still of room for new discoveries : two main scenarios



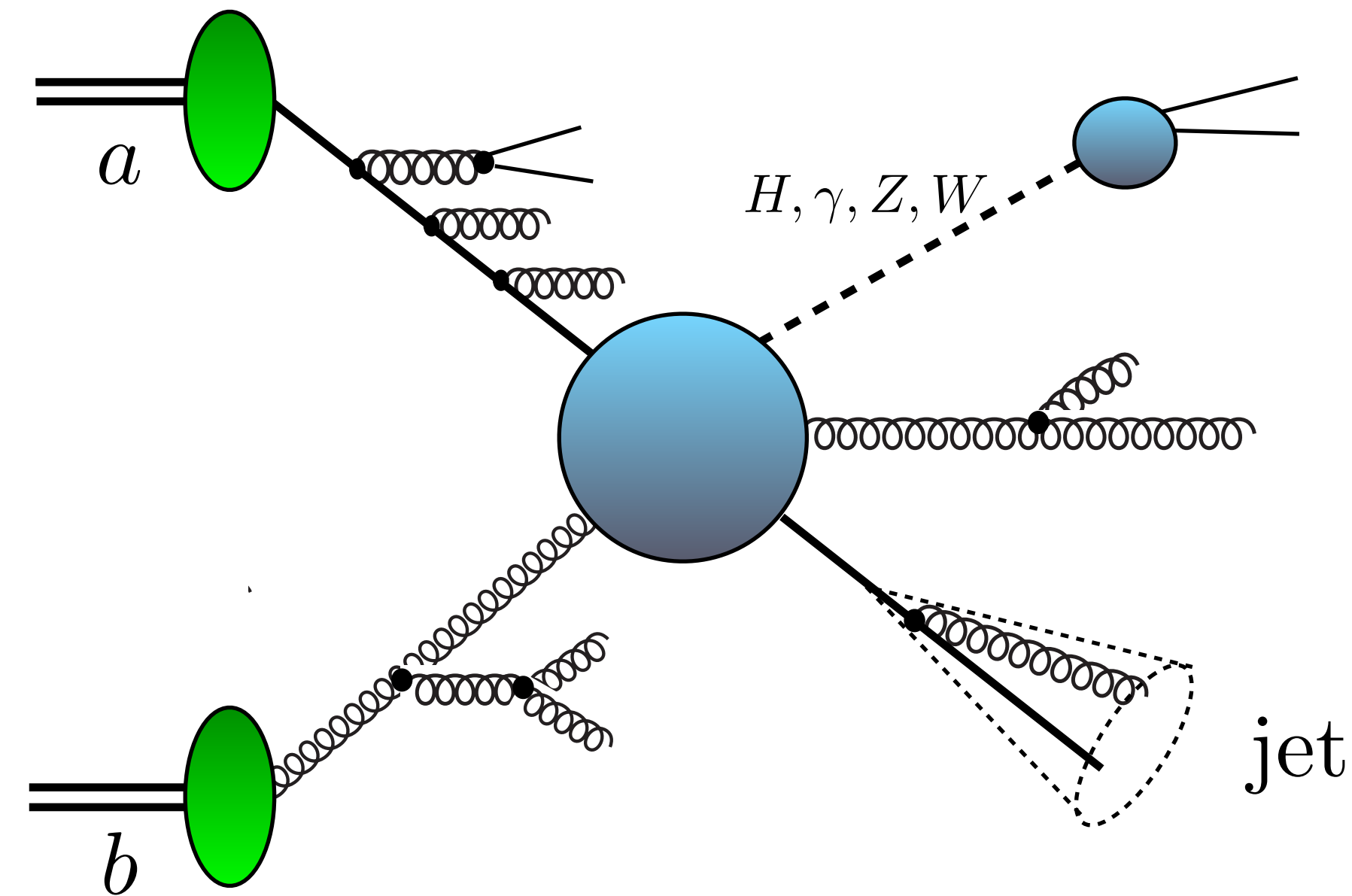
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- ▶ Resonance needs “descriptive” TH

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- ▶ **Precision** is the name of the game for the next decades

- In the LHC era, QCD is everywhere!



non-perturbative parton distributions

$$d\sigma = \sum_{ab} \int dx_a \int dx_b f_a(x_a, \mu_F^2) f_b(x_b, \mu_F^2) \times d\hat{\sigma}_{ab}(x_a, x_b, Q^2, \alpha_s(\mu_R^2)) + \mathcal{O}\left(\left(\frac{\Lambda}{Q}\right)^m\right)$$

perturbative partonic cross-section

Partonic cross-section: expansion in $\alpha_s(\mu_R^2) \ll 1$ $d\hat{\sigma} = \alpha_s^n d\hat{\sigma}^{(0)} + \alpha_s^{n+1} d\hat{\sigma}^{(1)} + \dots$

- Require precision for perturbative and non-perturbative contribution

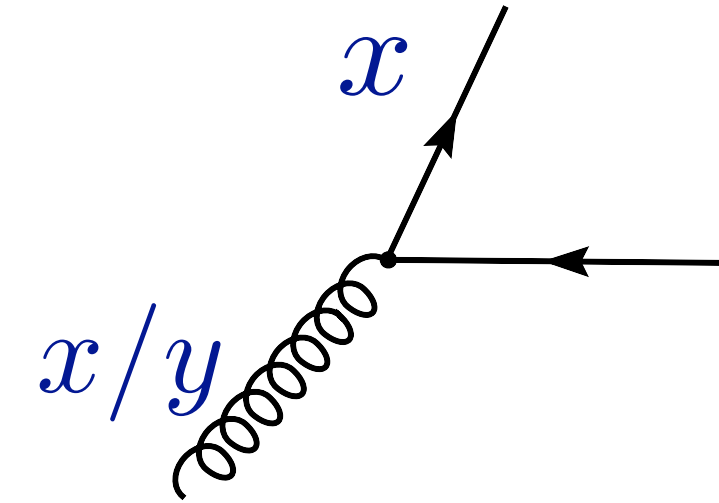
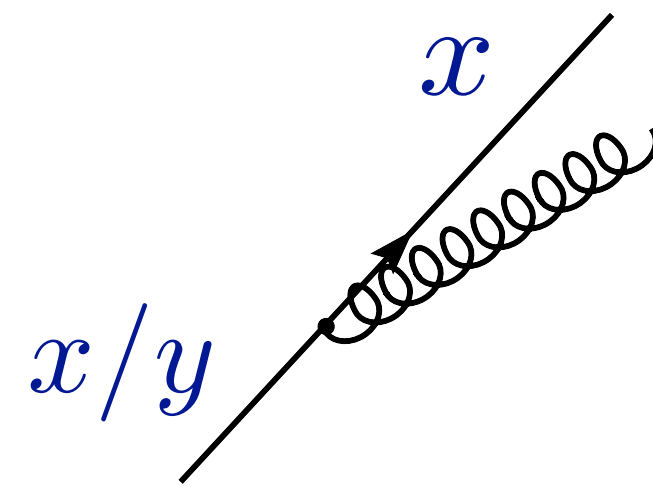
Can not compute the “x” dependence of pdf but we can do the scale one

Altarelli-Parisi equation (RGE like: resummation of collinear logs)

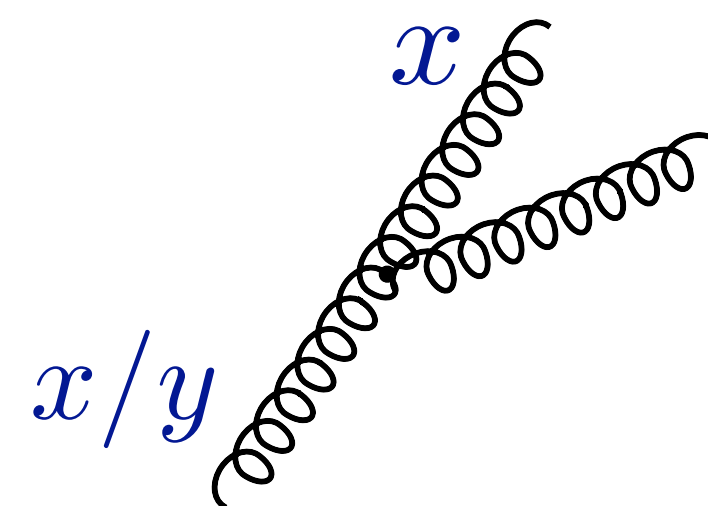
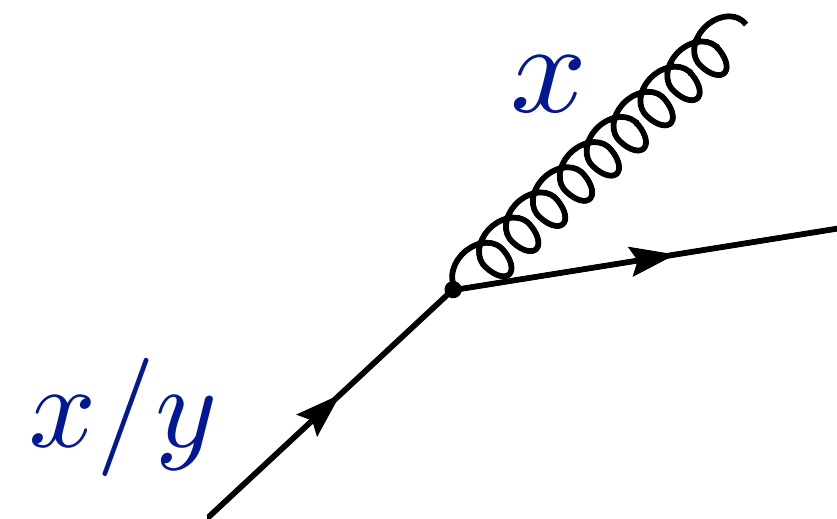
Increase “resolution” scale: resolve more details of “partonic structure”

$$\frac{\partial q(x, \mu_F^2)}{\partial \log(\mu_F^2)} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} P_{qq}(y) q\left(\frac{x}{y}, \mu_F^2\right) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} P_{qg}(y) g\left(\frac{x}{y}, \mu_F^2\right)$$

Probabilistic
interpretation

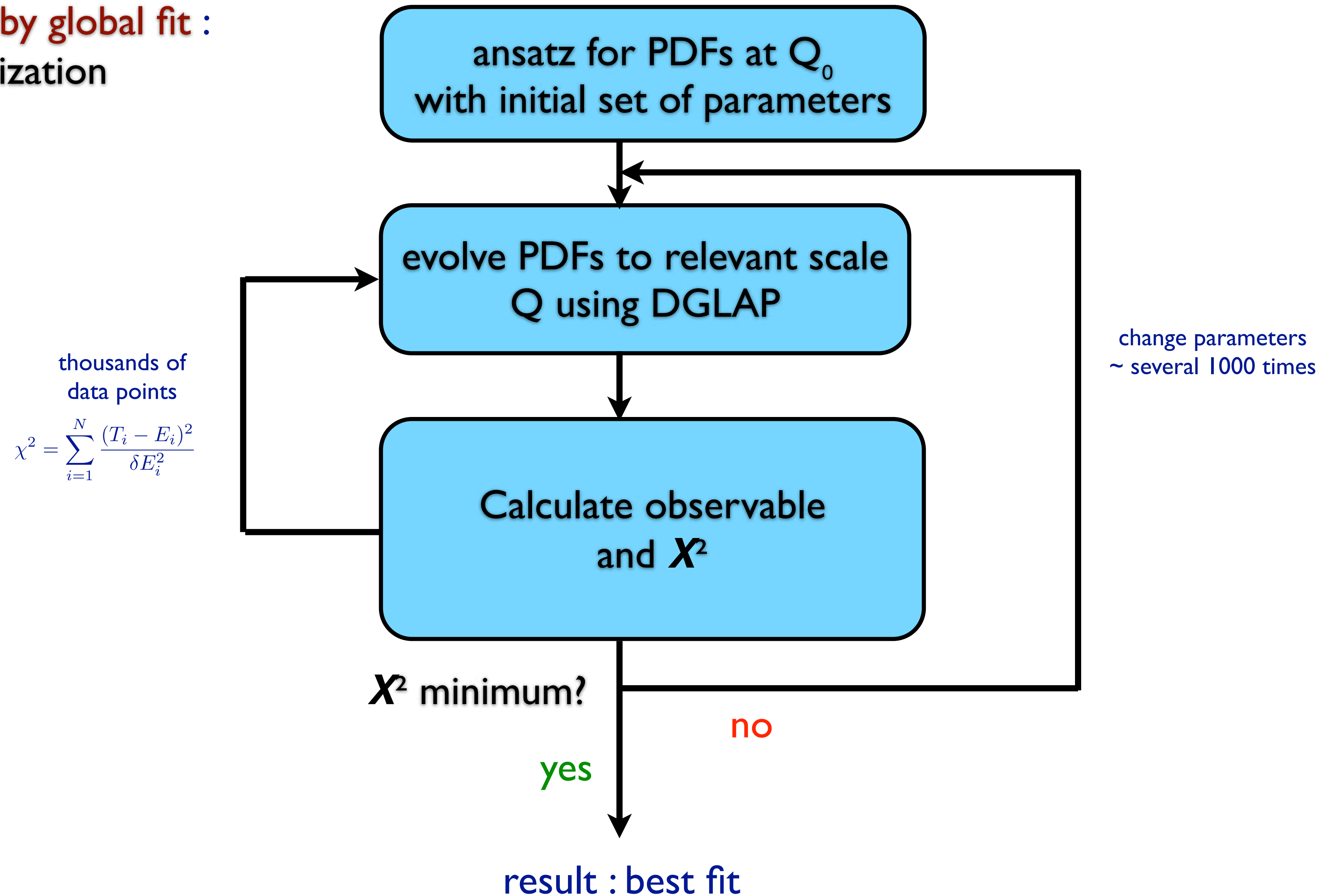


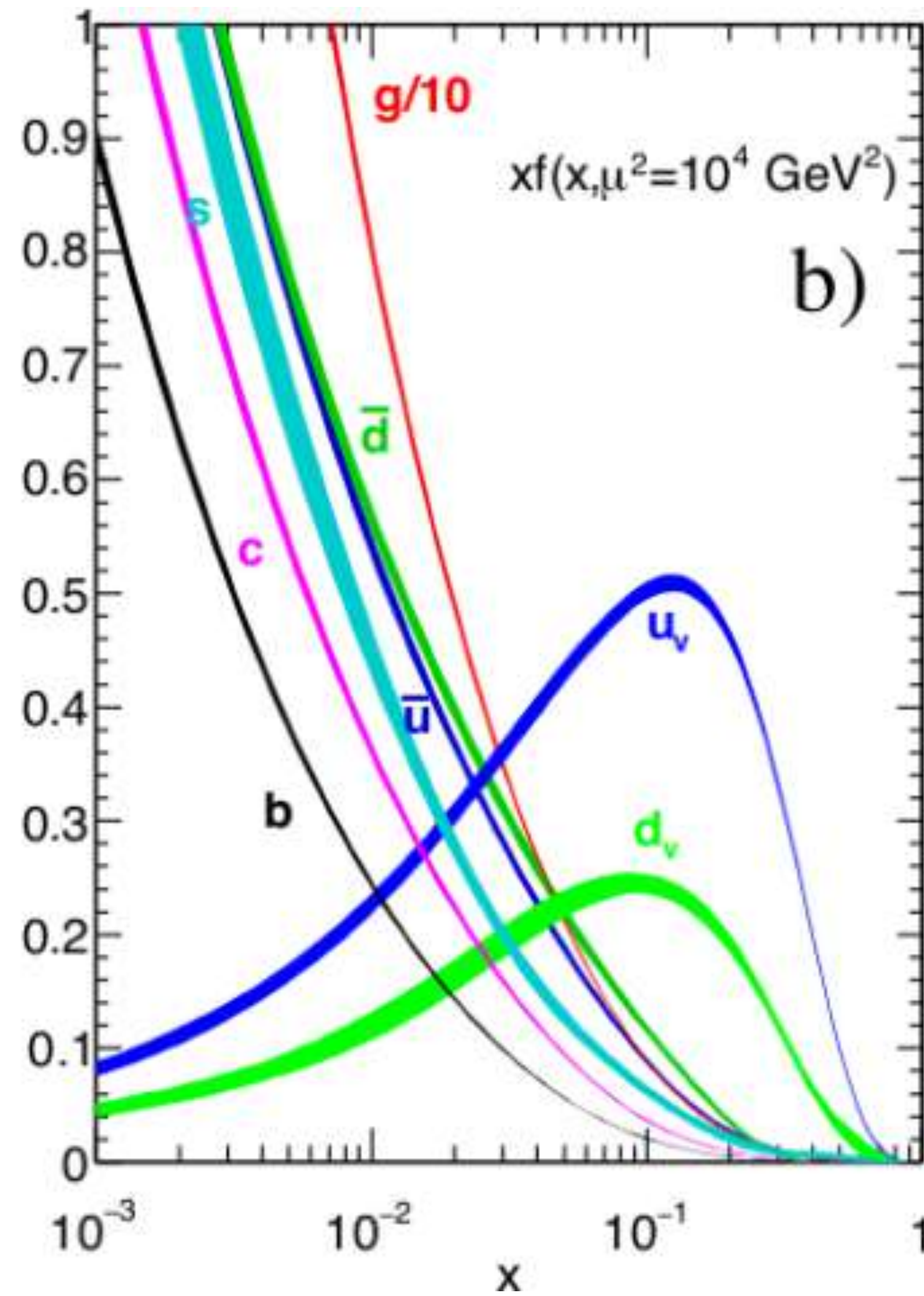
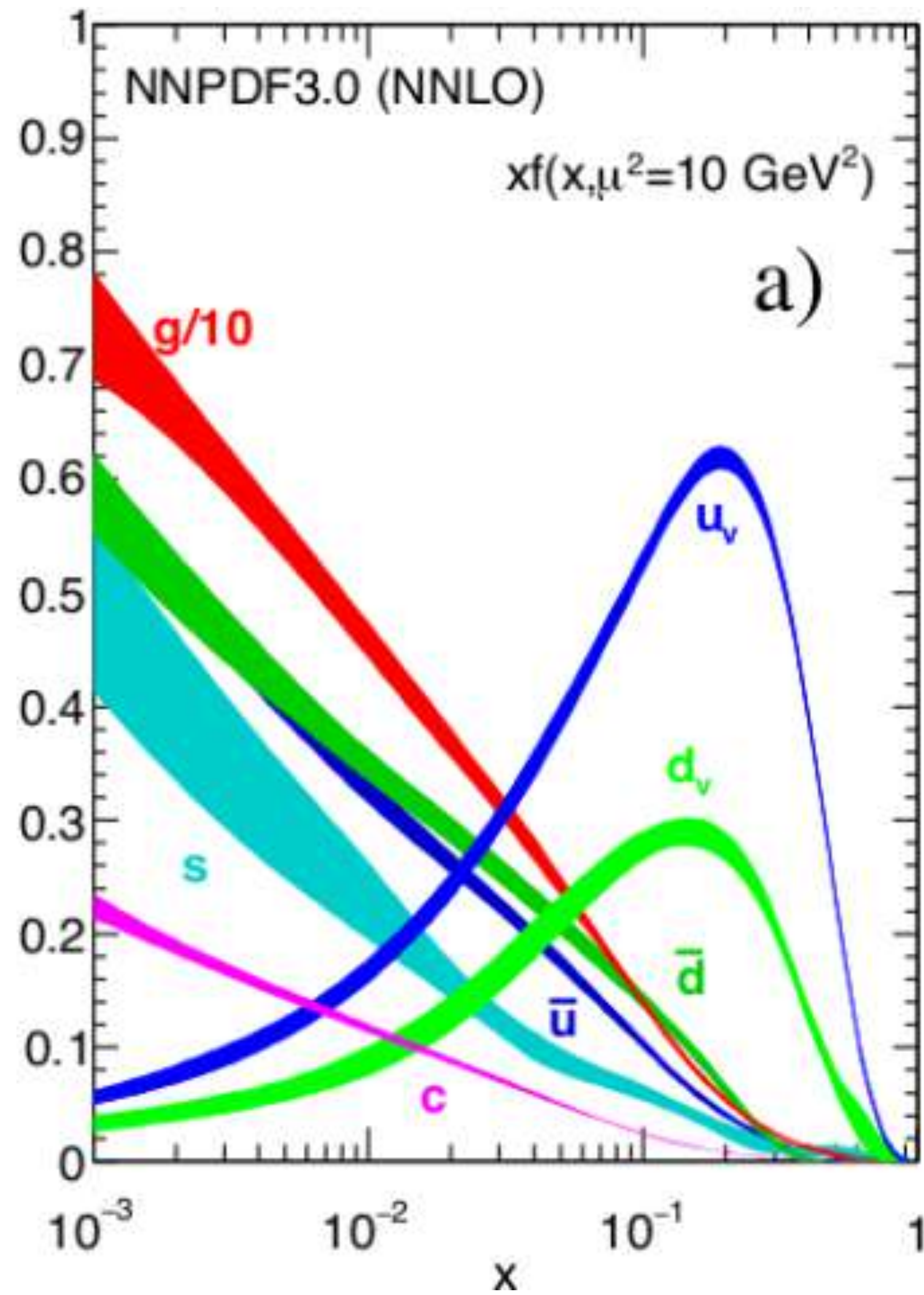
$$\frac{\partial g(x, \mu_F^2)}{\partial \log(\mu_F^2)} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} P_{gq}(y) \sum_q q\left(\frac{x}{y}, \mu_F^2\right) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} P_{gg}(y) g\left(\frac{x}{y}, \mu_F^2\right)$$



$$xf(x, Q_0^2) = Ax^\alpha(1-x)^\beta(1 + \epsilon\sqrt{x} + \gamma x + \dots)$$

PDFs obtained by global fit :
 χ^2 minimization





state of the art: NNLO sets of **pdfs** with variations of (valid) choices

- ▶ fitting methodologies
- ▶ PDF parametrization
- ▶ coupling constant
- ▶ theoretical constraints
- ▶ error estimation techniques
- ▶ dataset selection

CT18
MSHT20
NNPDF4.0
HERA2.0
ATLASpdf21
ABMPI6

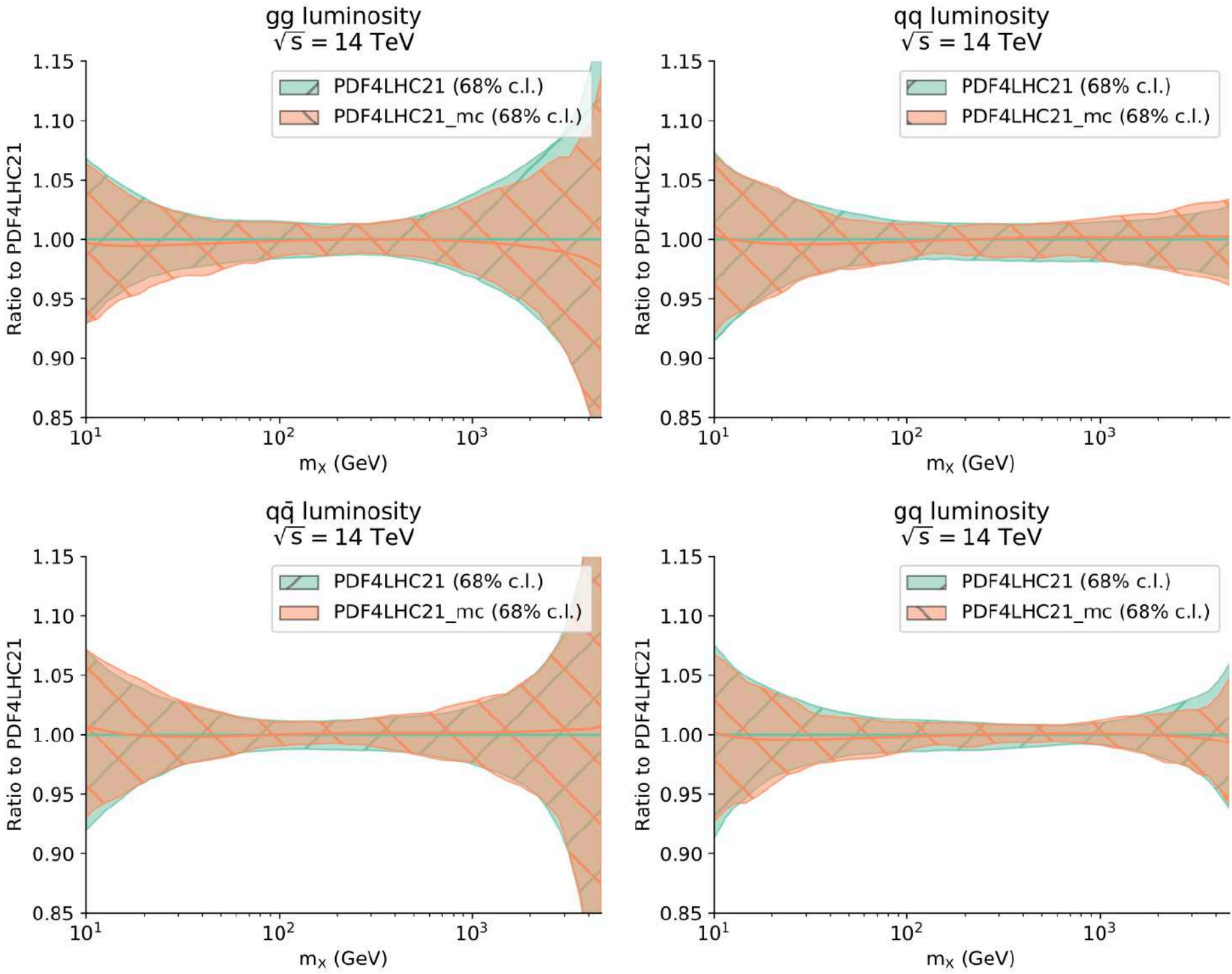
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CT18
MSHT20
NNPDF4.0
HERA2.0
ATLASpdf21
ABMPI6

LHAPDF6 grid name	ErrorType	N_{mem}
PDF4LHC21_MC	replicas	100
PDF4LHC21_40	symmhessian	40
PDF4LHC21_MC_PDFAS	replicas+as	102
PDF4LHC21_40_PDFAS	symmhessian+as	42
PDF4LHC21_MC_NF4	replicas	100
PDF4LHC21_40_NF4	symmhessian	40
PDF4LHC21_MC_PDFAS_NF4	replicas+as	102
PDF4LHC21_40_PDFAS_NF4	symmhessian+as	42

PDF4LHC21 combination



Precision in pQCD

- ▶ MC replicas and Hessian sets
- $\alpha_s(M_Z) = 0.1179 \pm 0.0009$ (PDG)

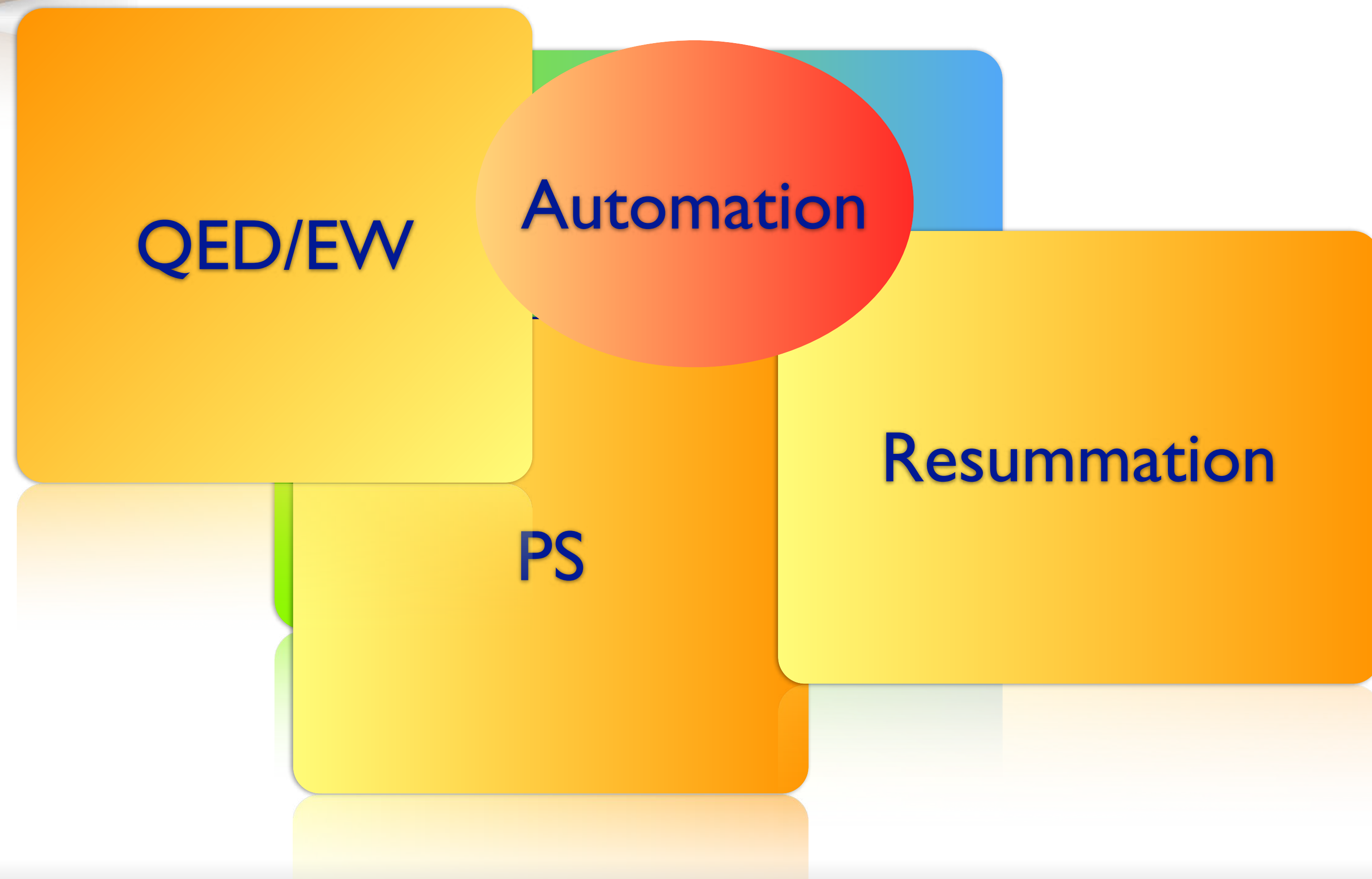
- ▶ **NEW** partial N3LO splitting functions Falcioni, Herzog, Moch, Vogt (2023)
- ▶ **NEW** First attempt to obtain approximated N3LO pdfs MSHT20aN3LO
McGowan, Cridge, Harland-Lang, Thorne (2022)
- ▶ Includes approximations to N3LO kernels, coefficients, K-factors
~20 nuisance parameters
- ▶ Improvement in data description from NNLO
- ▶ aN3LO $\alpha_s(M_Z)$ in agreement with world average
 - **Large impact** in Higgs when going from NNLO to aN3LO pdf

PDF order	σ (pb) + $\Delta\sigma_+$ – $\Delta\sigma_-$ (%)
aN ³ LO (H'_{ij})	45.296 + 1.81% – 1.68%
NNLO	47.817 + 1.17% – 1.22%

- ▶ central value decreases **> 5%**
(beyond expected uncertainties)
- ▶ (TH) uncertainties grow

▶ Many improvements expected in the field during next years towards N3LO, more LHC data in fit and better control of TH uncertainties, but hard to beat the few percent level.... (more uncertainties added)

The perturbative toolkit for precision at colliders



The perturbative toolkit for precision at colliders



Everything starts with a fixed order calculation

- ▶ Partonic cross-section: expansion in $\alpha_s(\mu_R^2) \ll 1$

$$d\hat{\sigma} = \alpha_s^n d\hat{\sigma}^{(0)} + \alpha_s^{n+1} d\hat{\sigma}^{(1)} + \dots$$



Why higher order corrections?

- ▶ Large Corrections : check PT shape and normalization

$\alpha_s \sim 0.1 \longrightarrow$ slow convergence

- ▶ Accurate Theoretical Predictions

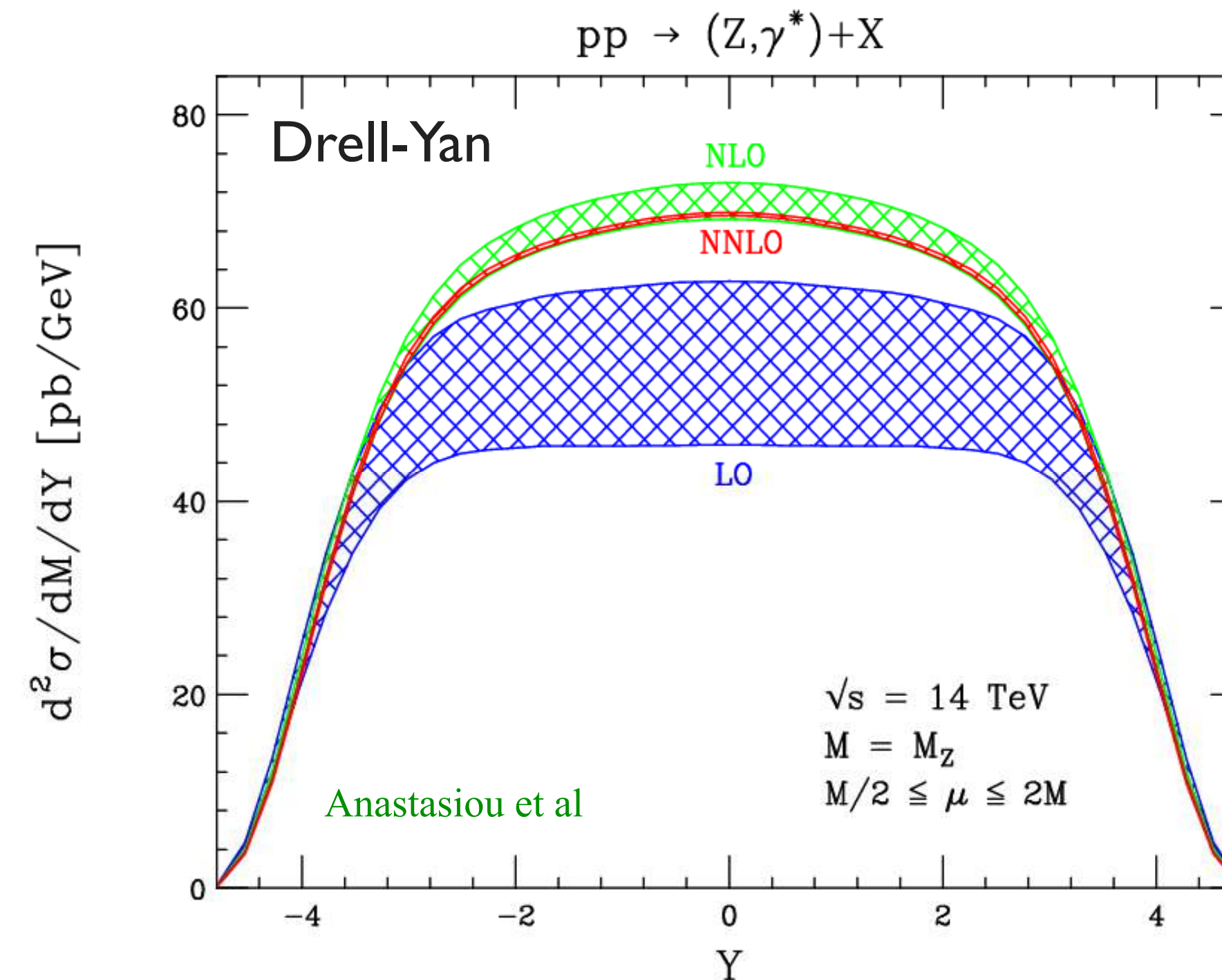
$$\sigma(p_1, p_2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \times \hat{\sigma}_{ab}(x_1 p_1, x_2 p_2, \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$$

μ_R Renormalization scale

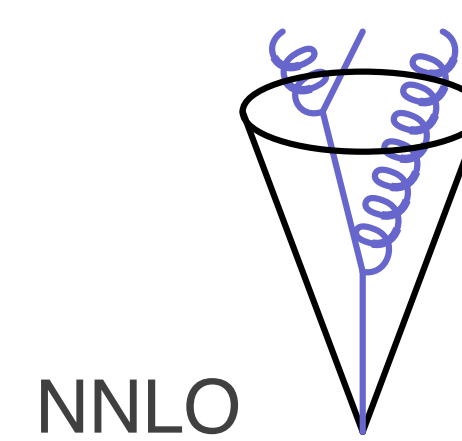
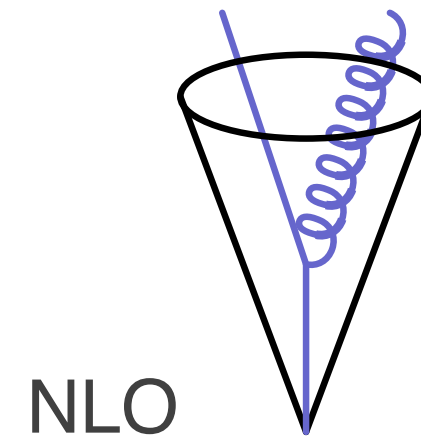
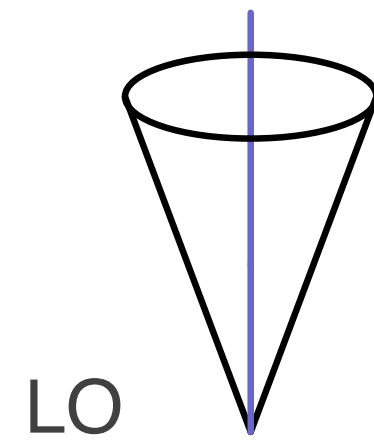
μ_F Factorization scale

Scale dependence considerably reduced at higher orders

$\longrightarrow$ TH uncertainty



- ▶ Extra radiation : more partons result in better TH/EXP matching

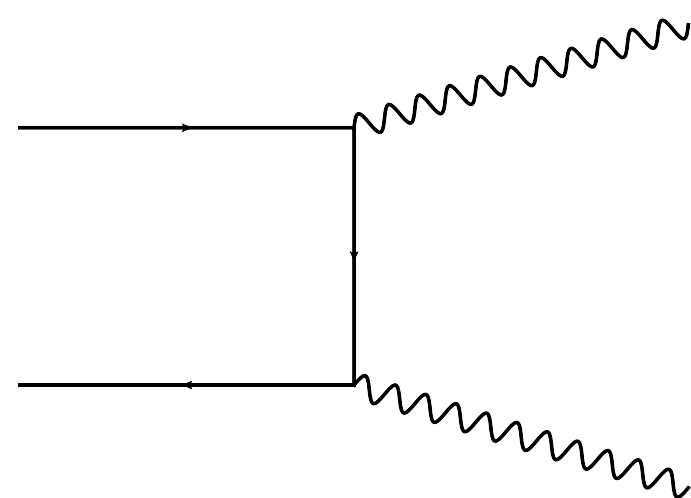


Description of jets, transverse momentum, etc

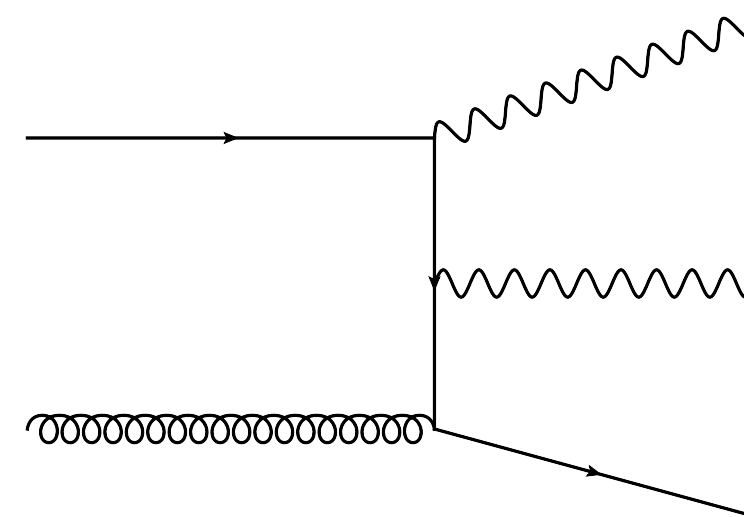
- ▶ Opening of new channels

Sometimes new channels at higher order provide large corrections due to parton luminosity (pdf, non-perturbative-perturbative interplay)

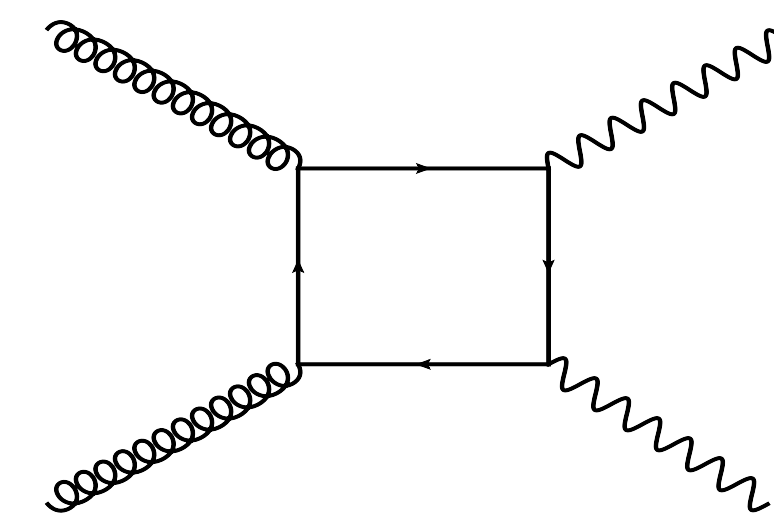
● Diboson production



$\mathcal{O}(\alpha_s^0)$ but $q\bar{q}$ Luminosity

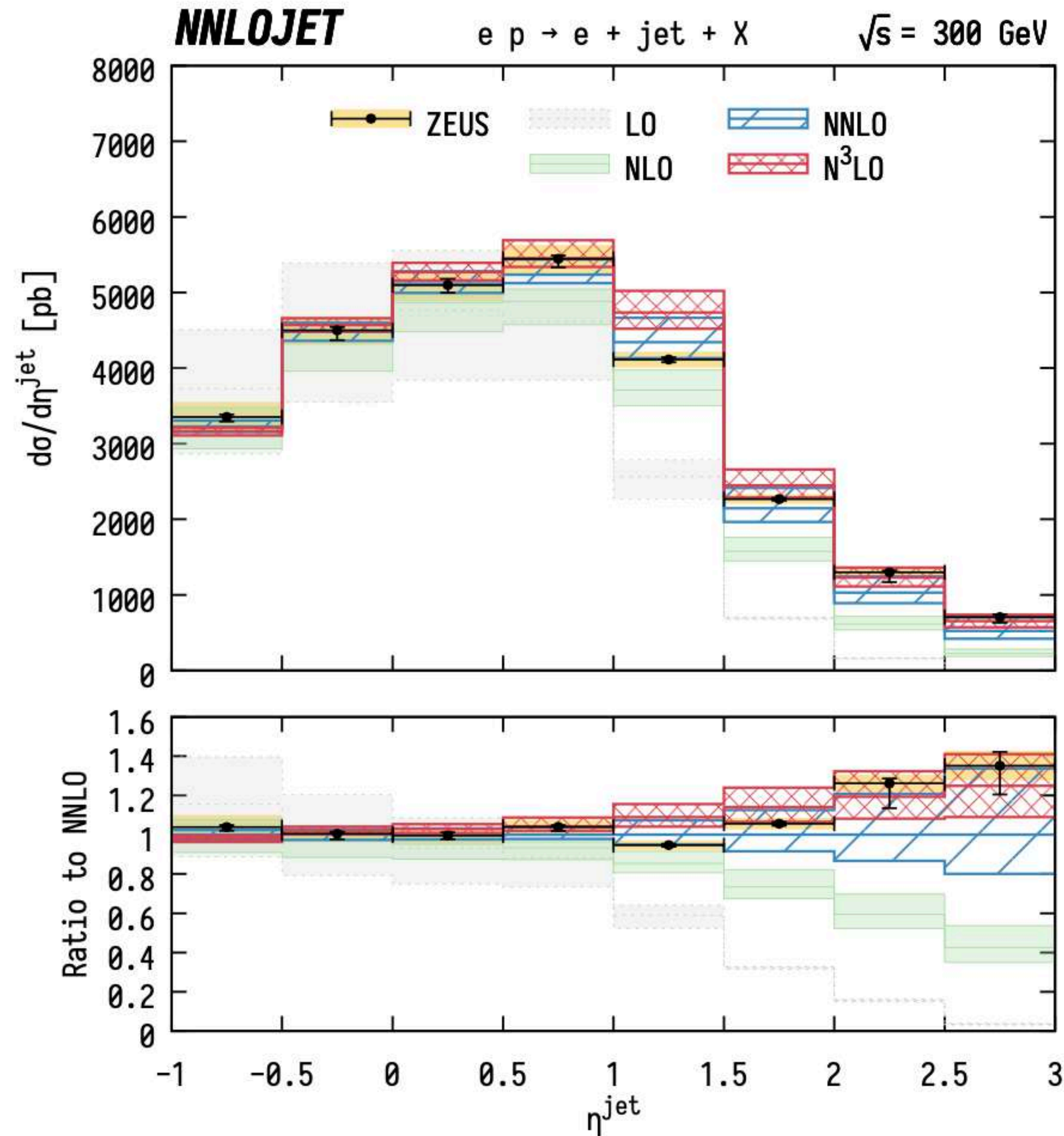


$\mathcal{O}(\alpha_s)$ but Luminosity



$\mathcal{O}(\alpha_s^2)$ but gg Luminosity

► Precision Challenged!



- DIS Single jet production at N3LO

Currie, Gehrmann, Glover, Huss, Niehues, Vogt (2018)

- TH uncertainties still larger than experimental errors (HERA)

Born Cross section

LO : number of tools to compute tree level amplitudes

$$\sigma_{LO} = \int_m |\mathcal{M}^{(0)}(\{p_i\})|^2 \mathbf{S}(\{p_i\}) d\Phi_m$$


Tree level matrix element Measurement function Phase space

- “Brute Force” Feynman Diagrams
- Recursive relations : Berends-Giele, BCFW

Fully automated calculations for very large multiplicities

MADGRAPH, HELAC-PHEGAS, ALPGEN, SHERPA, ComHep, COMIX,...

✓ Born level: simpler to integrate calculation to parton showers

↓ In most cases, LO not enough for precision physics

LO can (almost) be done by chatGPT



Can you write a Fortran 77 code that computes the DIS cross section at LO using LHAPDFs?



```
sql Copy code

Sample Fortran 77 code to compute the DIS cross section at LO using LHAPDFs
in a 5-flavor scheme and neglecting the contribution from the photon PDF

implicit none

Declare the input variables
real*8 x, q2, y, s, tau, pdf, sigma
integer flavor, nset, ievtnum
character*255 pdfname

Declare the LHAPDF functions
external initpdfset, evolvepdf, alphapdf, endpdfset

Set the input kinematics
x = 0.1d0
q2 = 10.d0**6
y = 0.5d0
s = q2 / x / y
tau = q2 / s

Set the LHAPDF set and initialize it
pdfname = 'cteq66'
call initpdfset(nset,pdfname,0)

Compute the LO partonic cross section for each flavor
sigma = 0.d0
do flavor = 1, 5
    pdf = evolvepdf(x, q2, flavor) / x ! divide by x to get the PDF for the parton
    sigma = sigma + alpha_em**2 / (2.d0 * s * q2) * pdf * &
        ((4.d0 / 9.d0 + 16.d0 / 27.d0 * delta(flavor,2)) * (1.d0 + (1.d0 - y)**2)) &
        + (4.d0 / 27.d0 * delta(flavor,2)) * (1.d0 + y**2)
end do

Compute the DIS cross section by convolving with the PDF of the proton
sigma = sigma * 3.d0 / 2.d0 / x / q2 / (1.d0 - y)
```

Why higher orders so complicated?

Blame Feynman!

- Real and virtual contributions : separately divergent

$$\begin{array}{ccc}
 \int_0^\infty dk & \text{1 loop} & + \int dk \\
 \text{IR} & \text{UV} & \\
 \text{IR} & \text{IR} & \\
 \text{1 loop} & & \text{1 extra parton} \\
 \text{IR in soft/collinear configurations} & &
 \end{array}$$

dimensional regularization $\epsilon = d - 4$ $\rightarrow \frac{1}{\epsilon^2}$

1 extra parton
IR in soft/collinear configurations

Why higher orders so complicated?

Blame Feynman!

- Real and virtual contributions : separately divergent

$$\begin{array}{c} \text{UV} \\ \int_0^\infty dk \\ \text{IR} \end{array} \quad \begin{array}{c} \text{1 loop} \end{array} \quad + \quad \int dk \quad \begin{array}{c} \text{1 extra parton} \\ \text{IR in soft/collinear configurations} \end{array}$$

$$\frac{-1}{(p-k)^2} = \frac{1}{2p \cdot k} = \frac{1}{2E_q E_g (1 - \cos \theta)}$$

dimensional regularization $\epsilon = d - 4$ $\rightarrow \frac{1}{\epsilon^2}$

Subtraction Method : need **local** subtraction counter-term

$$\int_0^1 (|M_R|^2 - \mathcal{S}) dPS + \int \mathcal{S} dPS + \int |M_V|^2 dPS'$$

Finite

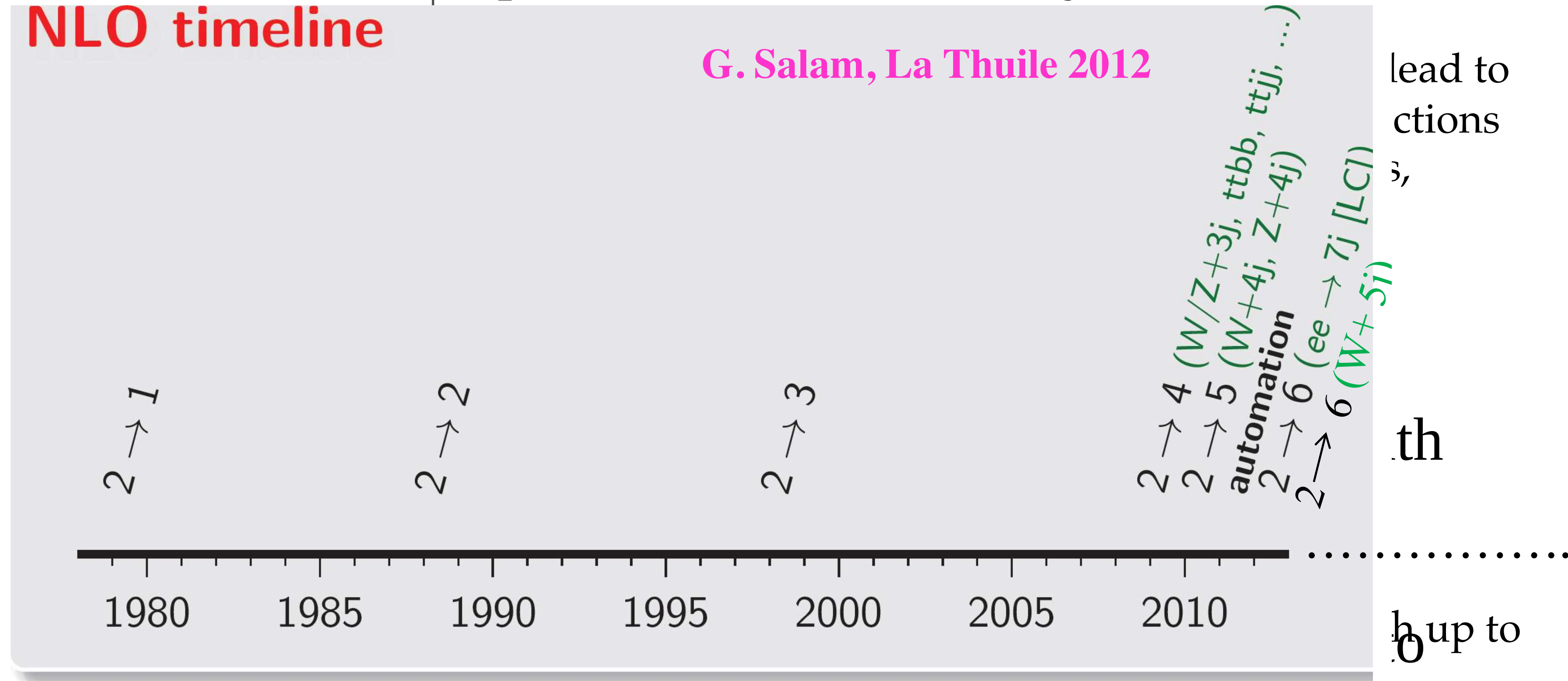
Computed “analytically”
cancel divergences

The NLO revolution standard automation

NLO predictions are currently available for

NLO timeline

G. Salam, La Thuile 2012



- Final goal: Really automatic NLO calculations, **zero cost for humans**

5) Higgs and jets

- Apart from **very large multiplicities and loop induced processes** automatic NLO “conceptually” solved

- Uncertainties, EW corrections, masses, matching to PS, etc
- Bern, Dixon, Kosower, Berger, Forde, Maitre, Febres-Cordero, Gleisberg, Papadopoulos, Ossola, Pittau, Czakon, Worek, Bevilacqua, Ellis, Kunszt, Giele, Zanderighi, Melia, Rountsh, Denner, Dittmaier, Pozzorini, Kallweit

How easy is NLO these days?

- Specify the process (input card)
- Input parameters
- Define final cuts
- in a few years a number of codes

HELAC-NLO, Rocket, BlackHat+SHERPA, GoSam+SHERPA/MADGRAPH, NJet+SHERPA, Madgraph5-aMC@NLO, RECOLA, OpenLoops+SHERPA...

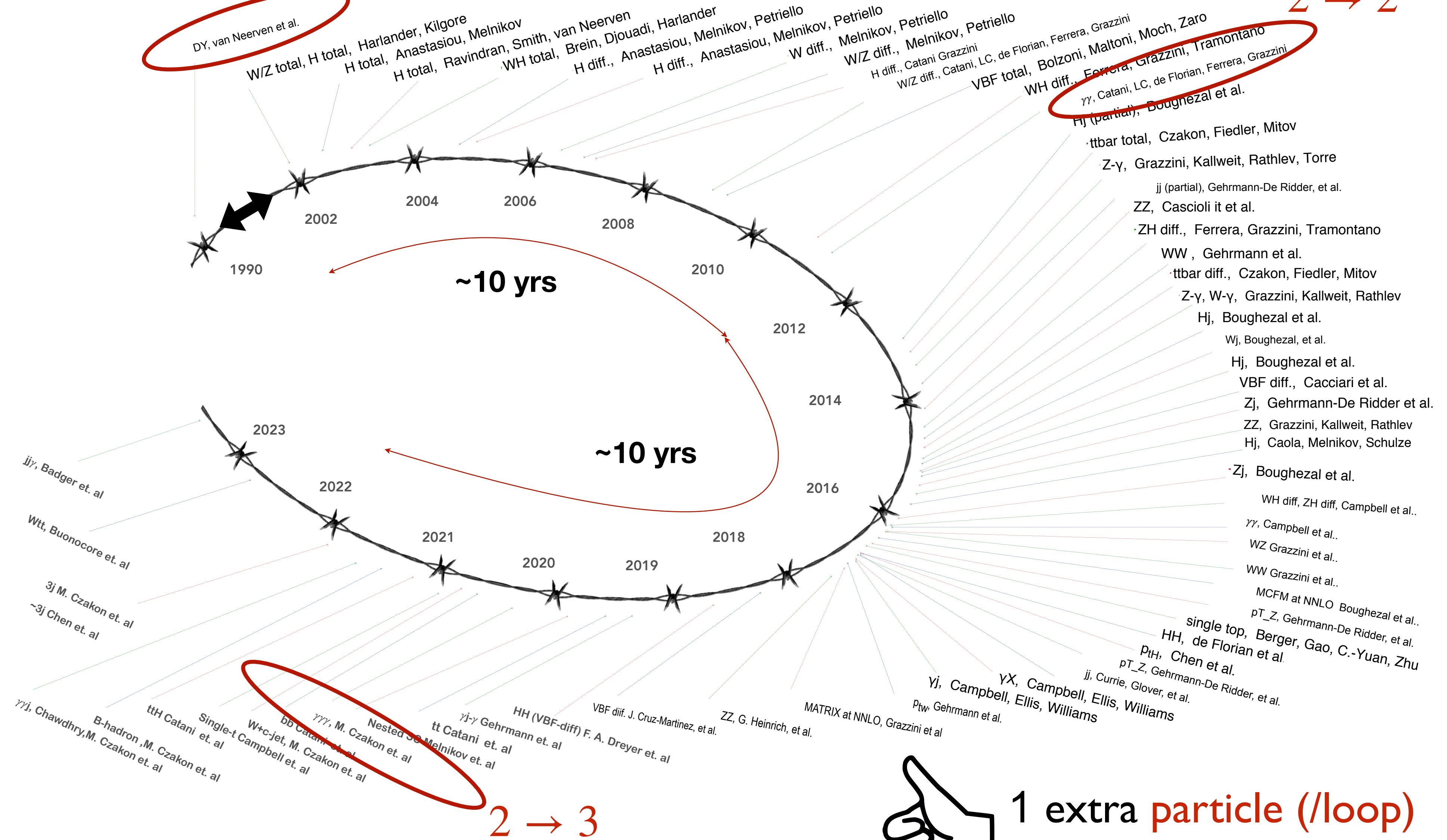
- compete on precision, flexibility, speed, stability, ...
- many features : uncertainties, Parton shower, ...

[illegible]

The NNLO revolution standard

2 → 1

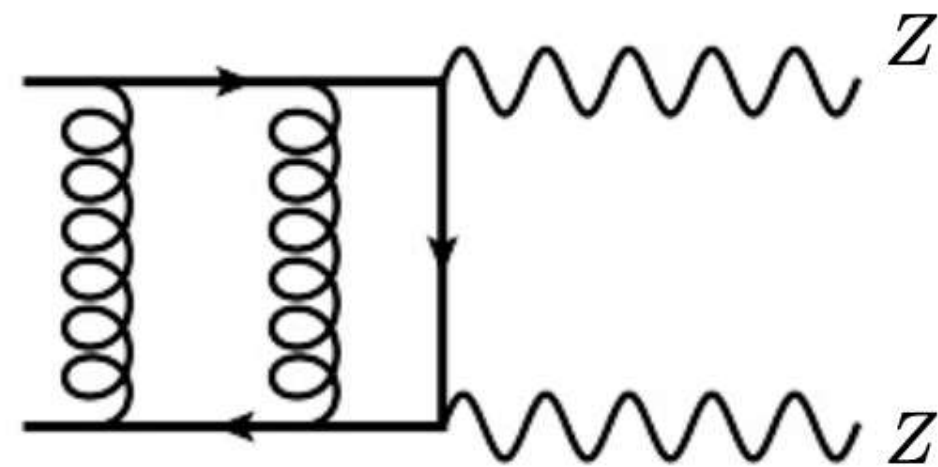
2 → 2



1 extra particle (/loop)
every 10 years

Degree of complexity at NNLO

► 2 loop



loop integrals

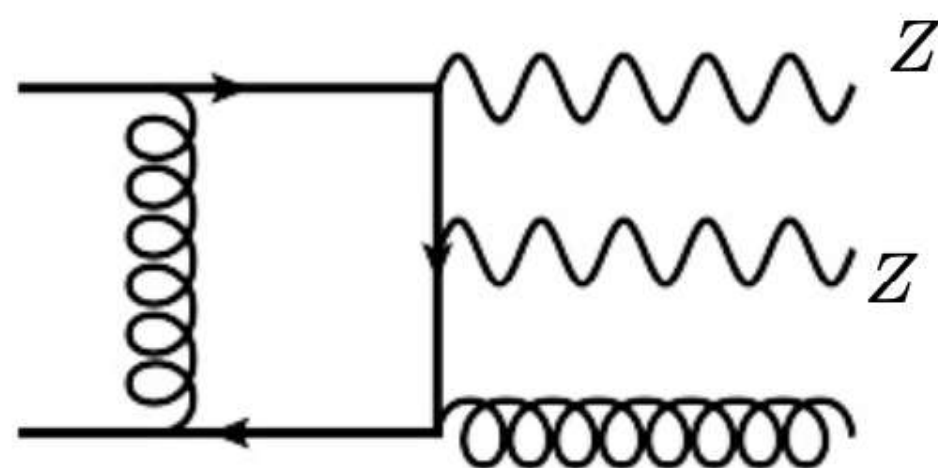


explicit infrared poles

$$\frac{1}{\epsilon^4}$$

- **Bottleneck** for larger multiplicities
- Many becoming available

► 1 loop + single emission



“NLO complexity” : loop



$$\frac{1}{\epsilon^2}$$

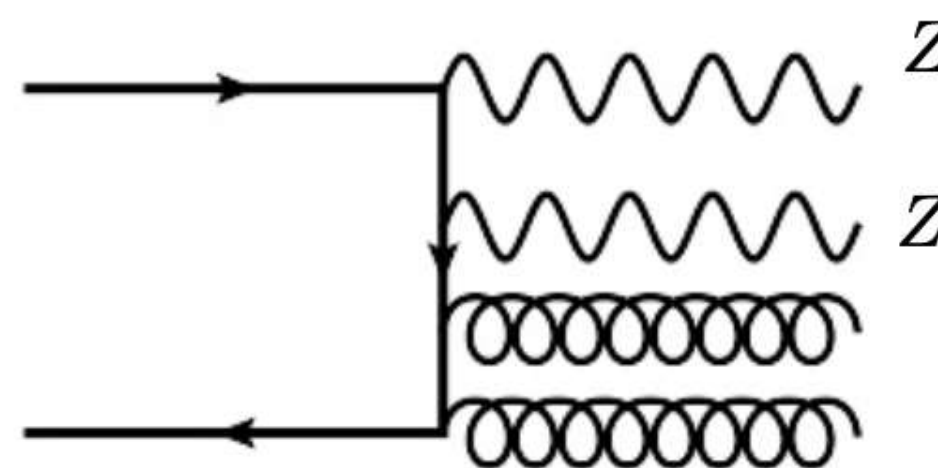
singular emission (extra)



$$\frac{1}{\epsilon^2}$$

► Double real emission

Tree level a Hell of infrared singularities



- **Bottleneck** for larger multiplicities: implementation

after integration over
unresolved partons

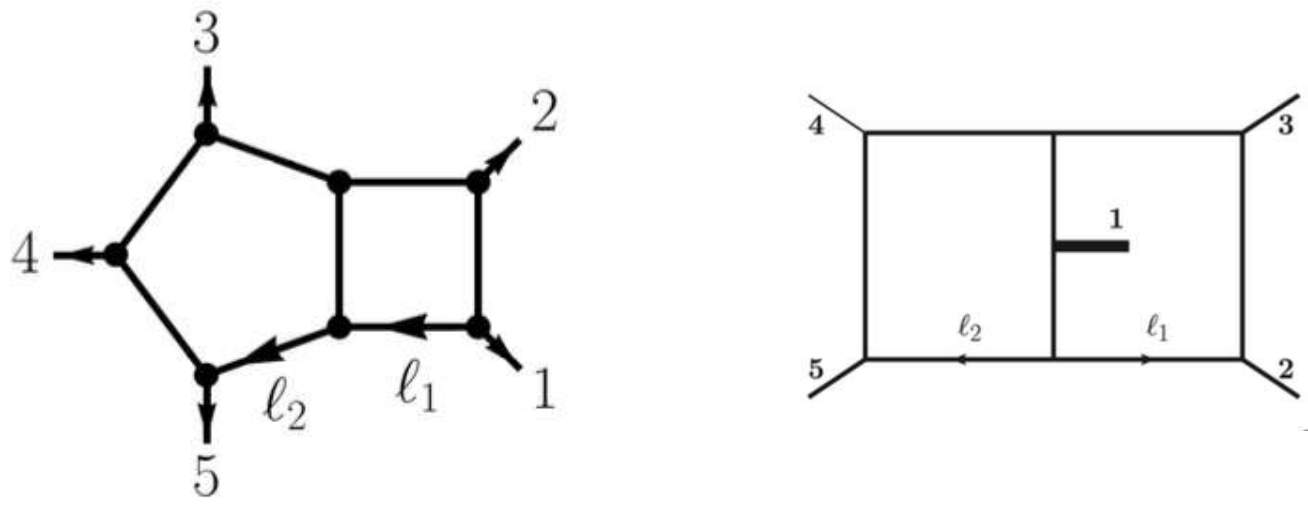


$$\frac{1}{\epsilon^4} \text{ poles}$$

State of the art for 2 loop (five point) amplitudes

$$\mathcal{A} = \sum_{\text{Coefficients}} c_i(\vec{p}; \epsilon) m_i(\vec{p}; \epsilon)$$

Master Integrals



- ▶ **Master integrals** decomposed in pentagon functions (efficient evaluation)
- ▶ **Coefficients** are process dependent (Feynman diagrams, generalized unitarity)
- ▶ IBP, differential eqns, numerical, partial-fractioning, rational reconstruction, etc

- **5 massless / 4 massless + 1 massive**
- **Two-loop amplitudes in pheno-ready form**
- **NEW: planar and non-planar**

Abreu, Chicherin, Ita, Page, Sotnikov, Tschernow, Zoia (2023)

- **NEW: Two-loop 6 massless (planar)!!**

Henn, Matijašić, Miczajka, Peraro, Xu, Zhang (2024)

Abreu, Monni, Page, Usovitsch (2024)

Liu, Matijašić, Peraro, Xu, Yang, Zhang (2026)

$pp \rightarrow jjj$	l.c.
$pp \rightarrow \gamma\gamma\gamma$	l.c.*
$pp \rightarrow \gamma\gamma j$	
$pp \rightarrow \gamma jj$	$\bar{\text{N}}\bar{\text{L}}\text{O}$ loop
$gg \rightarrow \gamma\gamma g$	induced
$pp \rightarrow Wb\bar{b}$	l.c.*, on-shell W
$pp \rightarrow W(l\nu)b\bar{b}$	l.c.
$pp \rightarrow W(l\nu)jj$	l.c.
$pp \rightarrow Z(l\bar{l})jj$	l.c.*
$pp \rightarrow W(l\nu)\gamma j$	l.c.*
$pp \rightarrow Hb\bar{b}$	l.c., b -quark Yukawa

V. Sotnikov 2207.12295

l.c. : leading color
l.c. *: l.c. only planar

State of the art for real/subtraction

$$\int |M_R|^2 dPS$$

- **Subtraction Method** : need **local** subtraction counter-term

$$\int_0^1 (|M_R|^2 - \mathcal{S}) dPS + \int \mathcal{S} dPS$$

Finite

Computed
“analytically”

Sector-improved residue **Czakon et al**
 Nested Soft-Collinear subtraction **Caola et al**
 Local Analytical subtraction **Magnea et al**
 Geometric **Herzog**
 Antenna Subtraction **Gehrmann et al**
 Colorful subtraction **Troscanyi**

- **Phase space slicing** : split phase space according to singular configurations

$$\int_\delta^1 |M_R|^2 dPS + \int_0^\delta |M_R|^2 dPS$$

singular

q_T **Catani, Grazzini, deF, et al**
 Jettiness **Boughezal et al, Gaunt et al**

Regularized by cut-off from resummation

Several methods resulting in many NNLO calculations

Most efficient implementation more important than refinements?

- Not automatic NNLO yet, but several processes available in public codes

MATRIX v2.1.0

Grazzini, Kallweit, Wieseemann + et al

$\text{\$}\{\text{process_id}\}$	process	description
pph21	$pp/p\bar{p} \rightarrow H$	on-shell Higgs-boson production
ppz01	$pp/p\bar{p} \rightarrow Z$	on-shell Z production (EW)
ppw01	$pp/p\bar{p} \rightarrow W^-$	on-shell W^- production with CKM
ppwx01	$pp/p\bar{p} \rightarrow W^+$	on-shell W^+ production with CKM
ppeex02	$pp/p\bar{p} \rightarrow e^-e^+$	Z production with decay (EW)
ppnenex02	$pp/p\bar{p} \rightarrow \nu_e\bar{\nu}_e$	Z production with decay (EW)
ppnenex02	$pp/p\bar{p} \rightarrow e^-\bar{\nu}_e$	W^- production with decay and CKM (EW)
ppexne02	$pp/p\bar{p} \rightarrow e^+\nu_e$	W^+ production with decay and CKM (EW)
ppaa02	$pp/p\bar{p} \rightarrow \gamma\gamma$	$\gamma\gamma$ production
ppeexa03	$pp/p\bar{p} \rightarrow e^-e^+\gamma$	$Z\gamma$ production with decay
ppnenexa03	$pp/p\bar{p} \rightarrow \nu_e\bar{\nu}_e\gamma$	$Z\gamma$ production with decay
ppnenexa03	$pp/p\bar{p} \rightarrow e^-\bar{\nu}_e\gamma$	$W^-\gamma$ with decay
ppexnea03	$pp/p\bar{p} \rightarrow e^+\nu_e\gamma$	$W^+\gamma$ with decay
ppzz02	$pp/p\bar{p} \rightarrow ZZ$	on-shell ZZ production
ppwxw02	$pp/p\bar{p} \rightarrow W^+W^-$	on-shell W^+W^- production
ppemexmx04	$pp/p\bar{p} \rightarrow e^-\mu^-e^+\mu^+$	ZZ production with decay (gg NLO, EW)
pppeexex04	$pp/p\bar{p} \rightarrow e^-e^-e^+e^+$	ZZ production with decay (gg NLO, EW)
ppexxnmnm04	$pp/p\bar{p} \rightarrow e^-e^+\nu_\mu\bar{\nu}_\mu$	ZZ production with decay (gg NLO, EW)
ppemxnmnex04	$pp/p\bar{p} \rightarrow e^-\mu^+\nu_\mu\bar{\nu}_e$	W^+W^- production with decay (gg NLO, EW)
ppexxnenex04	$pp/p\bar{p} \rightarrow e^-e^+\nu_e\bar{\nu}_e$	ZZ / W^+W^- production with decay (gg NLO, EW)
ppemxnmx04	$pp/p\bar{p} \rightarrow e^-\mu^-e^+\bar{\nu}_\mu$	W^-Z production with decay (EW)
pppeexnex04	$pp/p\bar{p} \rightarrow e^-e^-e^+\bar{\nu}_e$	W^-Z production with decay (EW)
ppexmxnm04	$pp/p\bar{p} \rightarrow e^-e^+\mu^+\nu_\mu$	W^+Z production with decay (EW)
pppeexxne04	$pp/p\bar{p} \rightarrow e^-e^+e^+\nu_e$	W^+Z production with decay (EW)
ppttx20	$pp/p\bar{p} \rightarrow t\bar{t}$	on-shell top-pair production
ppaaa03	$pp/p\bar{p} \rightarrow \gamma\gamma\gamma$	$\gamma\gamma\gamma$ production

NEW $pp \rightarrow t\bar{t}H$

2 $\rightarrow$ 3 process with massive colored particles (missing exact 2 loop)

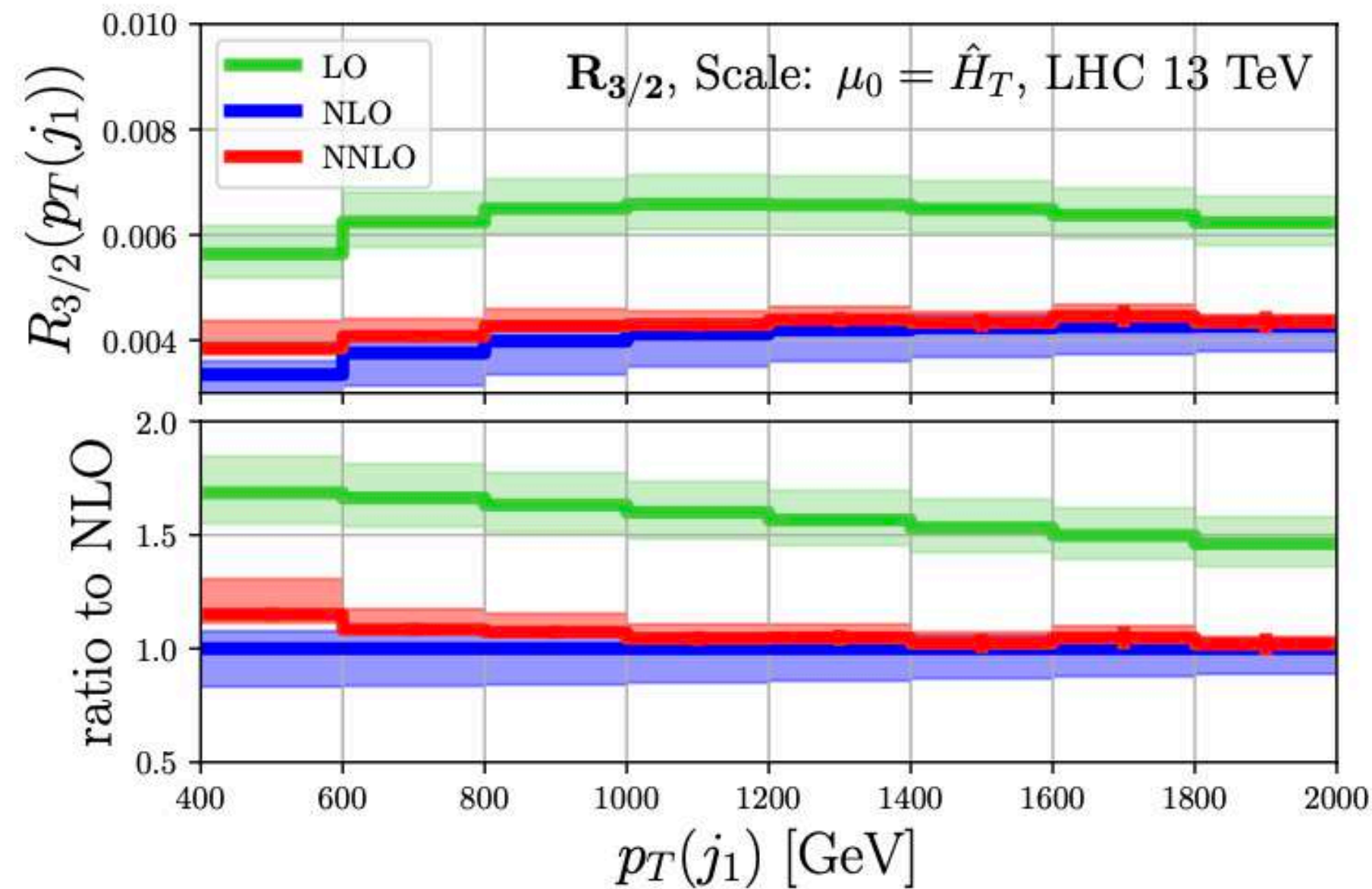
MC2FM

Campbell, Ellis, Neumann, Williams

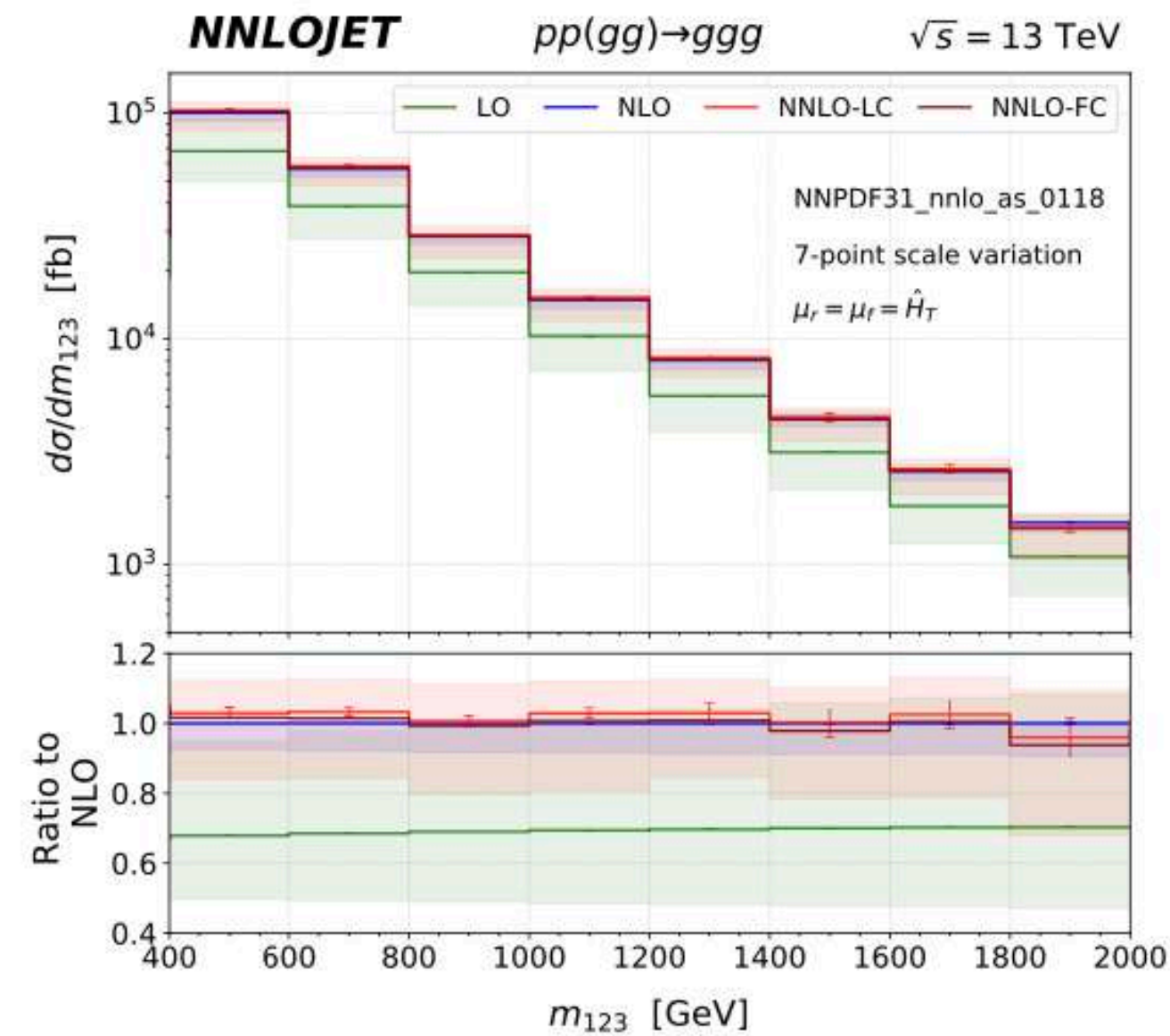
Final state	input file
H	input_H_qt.ini input_H_scet.ini
Z	input_Z_qt.ini input_Z_scet.ini
W^-	input_W-_qt.ini input_W-_scet.ini
$W^\pm$	input_W+_qt.ini input_W+_scet.ini
ZH	input_ZH_qt.ini input_ZH_scet.ini
$W^-H \pm W^\pm H$	input_WH_qt.ini input_WH_scet.ini
$e^-e^\pm\gamma$	input_eexa_qt.ini input_eexa_scet.ini
$e^-\nu_e\gamma$	input_enexa_qt.ini input_enexa_scet.ini
$\nu_e e^\pm\gamma$	input_neexa_qt.ini input_neexa_scet.ini
$\gamma\gamma$	input_aa_qt.ini input_aa_scet.ini
$e^-\mu^\pm\nu_e\nu_\mu$	input_emxnexnm_qt.ini input_emxnexnm_scet.ini
$e^-\nu_e\mu^-\mu^\pm$	input_enexmmx_qt.ini input_enexmmx_scet.ini
$\nu_e e^\pm\mu^-\mu^\pm$	input_neexmmx_qt.ini input_neexmmx_scet.ini
$e^-e^\pm\mu^-\mu^\pm$	input_eexmmx_qt.ini input_eexmmx_scet.ini

3-jet production at LHC @NNLO (LC)

Czakon, van Hameren, Mitov, Poncelet (2019)



Chen, Gehrmann, Glover, Huss, Mo (2022)



$$\hat{H}_T = \sum_{i \in \text{partons}} p_{T,i}$$

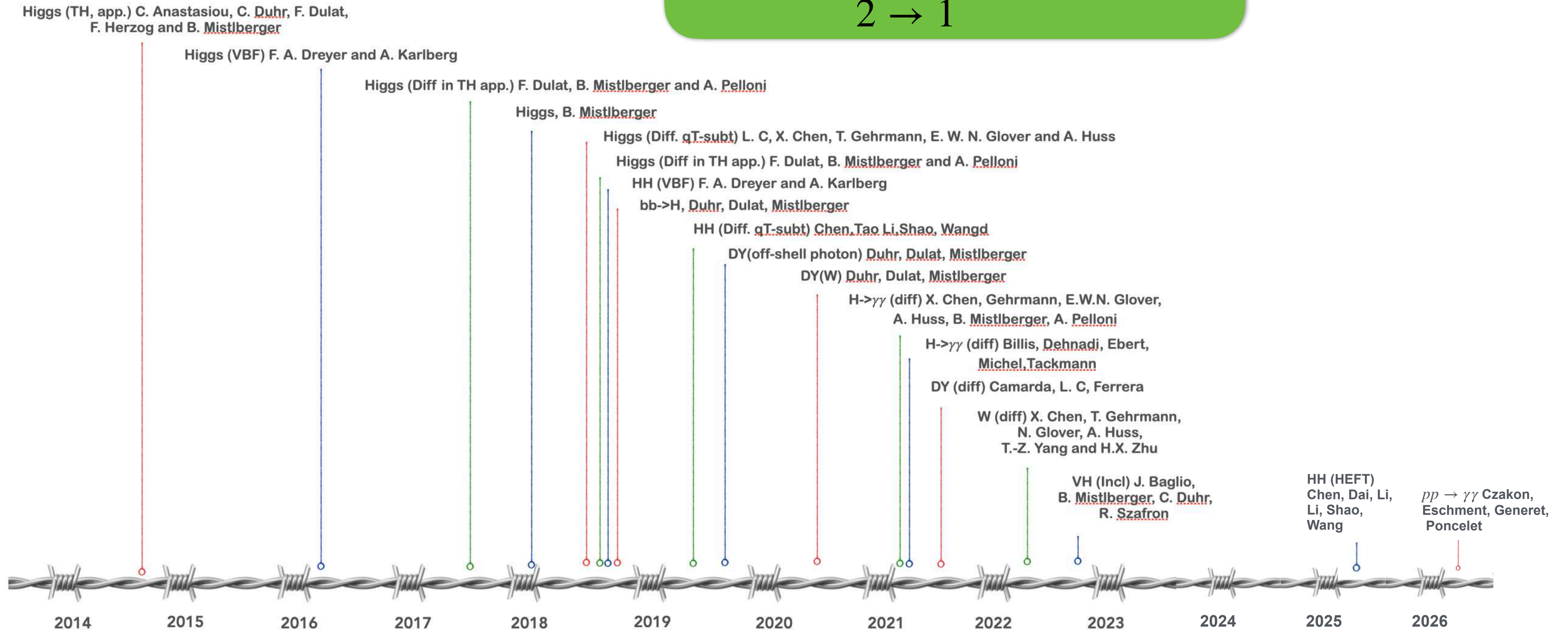
- ▶ Achieved using two subtraction methods: sector-improve residue vs antenna
- ▶ $R_{3/2}$ and event shapes important to extract α_s . Big NLO K-factor
NNLO stabilizes and smaller uncertainties
- ▶ Limitation in 2-loop Leading Color amplitudes and cancellation between double-real and subtraction terms (technical)  10^6 CPU hours
- ▶ **NEW** : $pp \rightarrow \gamma jj$ at N3LO (full color)

Badger, Czakon, Hartanto, Moodie, Peraro, Poncelet, Zoia (2023)

The N3LO revolution

DY-like (no color in final state)

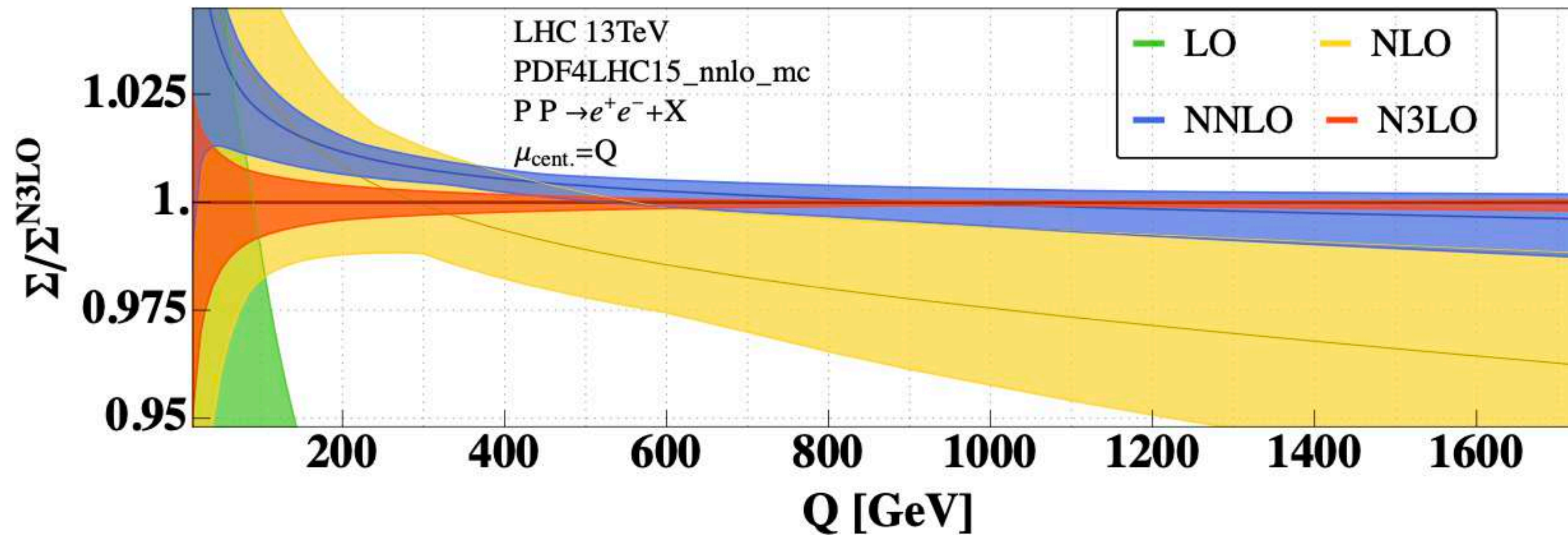
$$2 \rightarrow 1$$



Lepton-pair production at N3LO

Duhr, Mistelberger (2021)

Neutral current Z/γ exchange (axial-vector coupling)

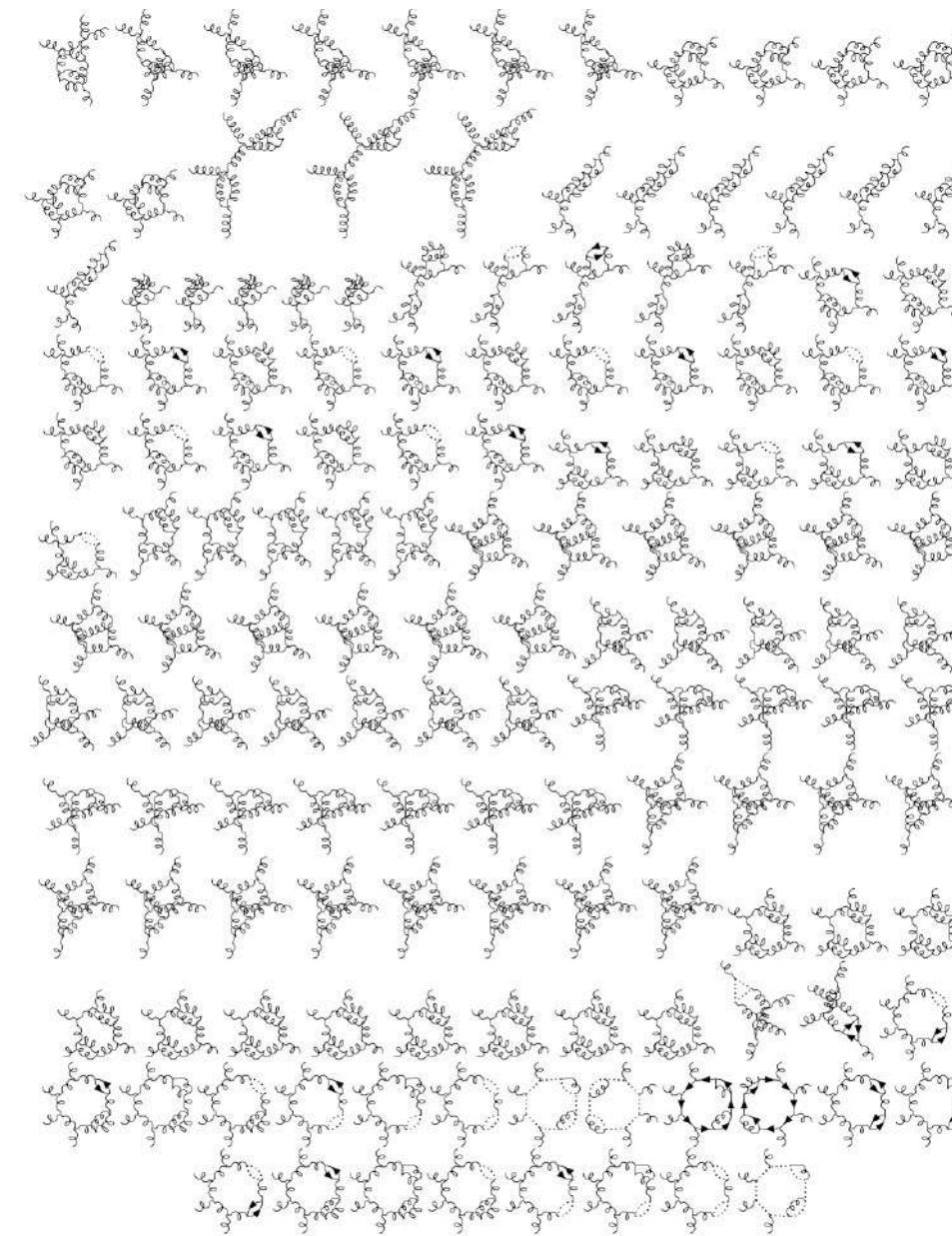


- ▶ bands obtained varying scales by 2 with $\frac{1}{2} \leq \frac{\mu_R}{\mu_F} \leq 2$
- ▶ small corrections but NNLO and N3LO do not overlap
- ▶ conventional scale variation underestimates size of corrections
- *missing N3LO pdfs

N3LO for processes with “2 final state particles”?

3 loop amplitudes for $pp \rightarrow jj$

Caola, Chakraborty, Gambuti,
von Manteuffel, Tancredi (2022)



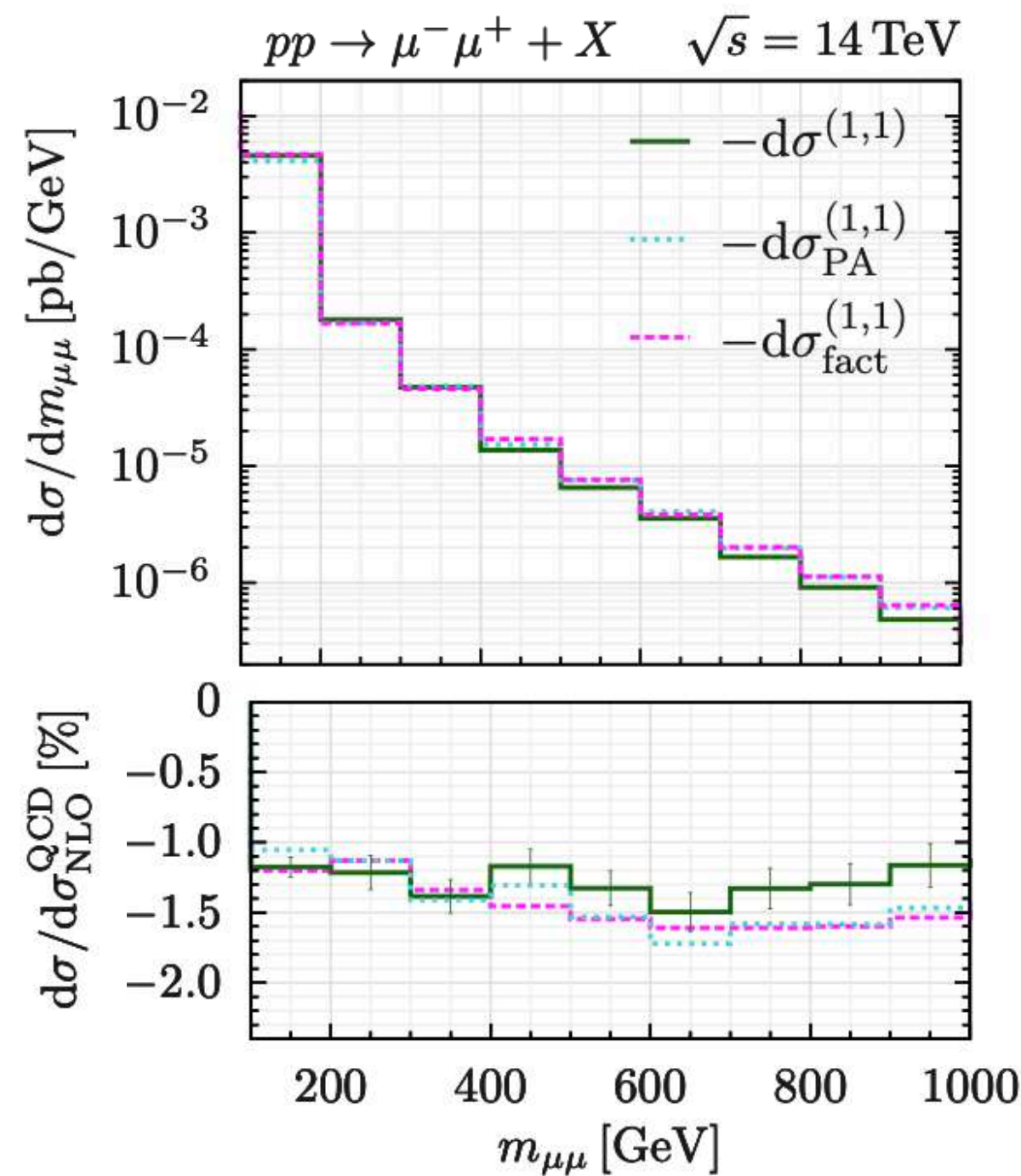
- ▶ first step towards N3LO for dijet production (all 3 loop available)
- ▶ performing N3LO subtraction for process “beyond DY-like” **difficult**
- ▶ $pp \rightarrow \gamma\gamma$ is a rather simple case (no color in final state)
- ▶ Beware! violation of strict collinear factorization at N3LO (IS/SL)

Catani, D.deF, Rodrigo (2012)

QCD+QED/EW effects

Notice that $\alpha_s^2 \sim \alpha \sim 0.01$ and EW effects can be enhanced..

$$\sigma(\alpha_s, \alpha) = \underbrace{\sigma^{(0,0)}}_{\text{LO}} + \underbrace{\alpha_s \sigma^{(1,0)}}_{\text{NLO QCD}} + \underbrace{\alpha_s^2 \sigma^{(2,0)}}_{\text{NNLO QCD}} + \dots + \underbrace{\alpha \sigma^{(0,1)}}_{\text{NLO EW}} + \dots + \underbrace{\alpha_s \alpha \sigma^{(1,1)}}_{\text{NNLO mixed QCD-EW}} + \dots$$



Bonciani et al (2021)

recent effort in mixed QCD-EW/QED for dileptons

Delto, Jaquier, Melnikov, Röntsch (2020)

Cieri, deF., Der, Mazzitelli (2020)

Buonocore, Grazzini, Kallweir, Savoini, Tramontano (2021)

Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch (2020)

Dittmaier, Schmidt, Schwarz (2020)

Buccioni et al (2022)

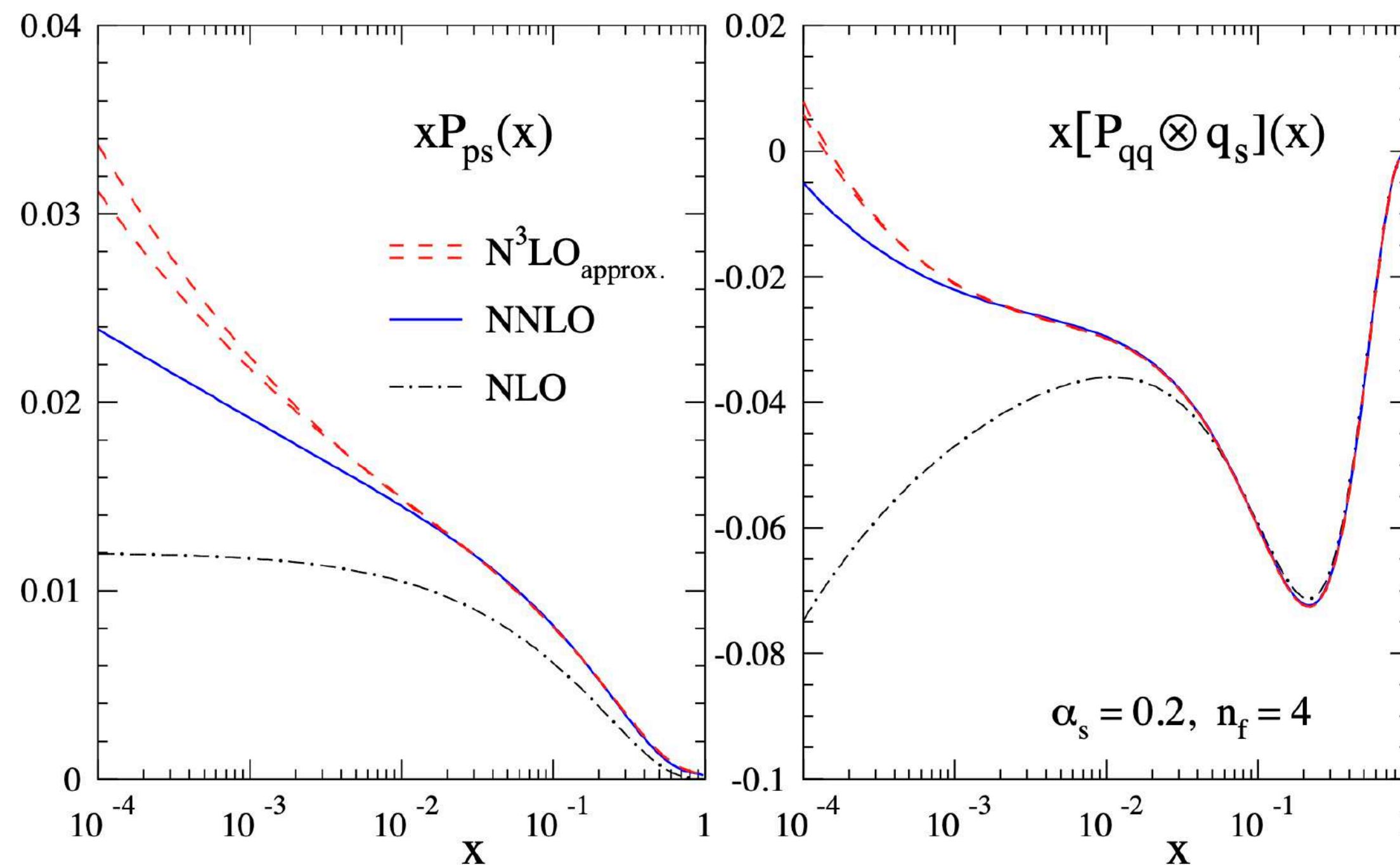
- mixed QCD-QED splitting functions known

deF. Rodrigo, Sborlini (2016)

- Corrections typically $\sim 1\%$ but can be enhanced in certain kinematical regions

N³LO Splitting functions

- ▶ All 4 loop splitting functions Falcioni, Herzog, Moch, Pelloni, Vogt (2023, 2024)
- ▶ N=22 Mellin moments
- ▶ +endpoint constraints provide a reconstruction of quark and gluon kernels
- ▶ Enough for N3LO evolution to achieve % level precision

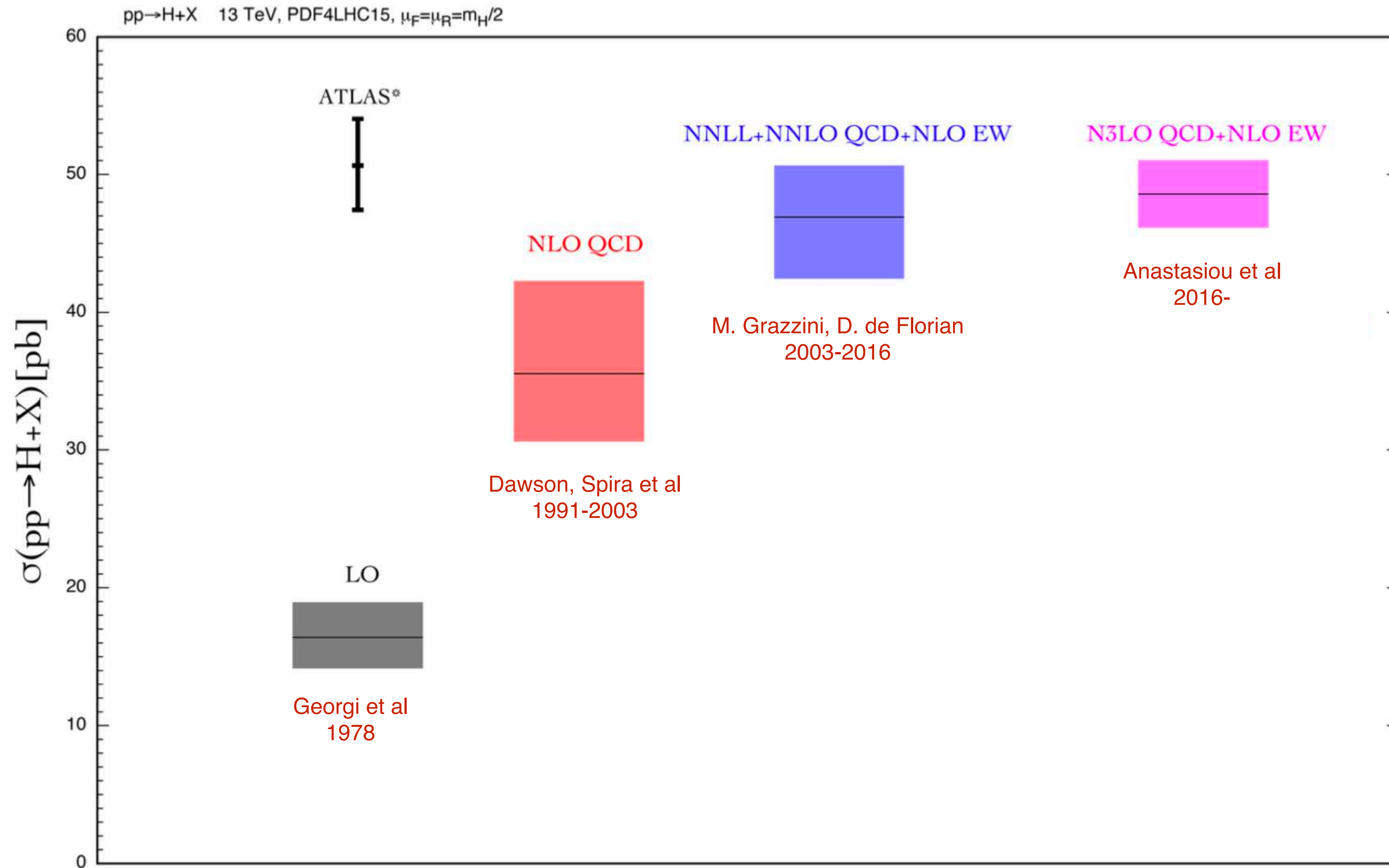


$$xq_s(x, \mu_0^2) = 0.6x^{-0.3}(1-x)^{3.5}(1+5.0x^{0.8})$$

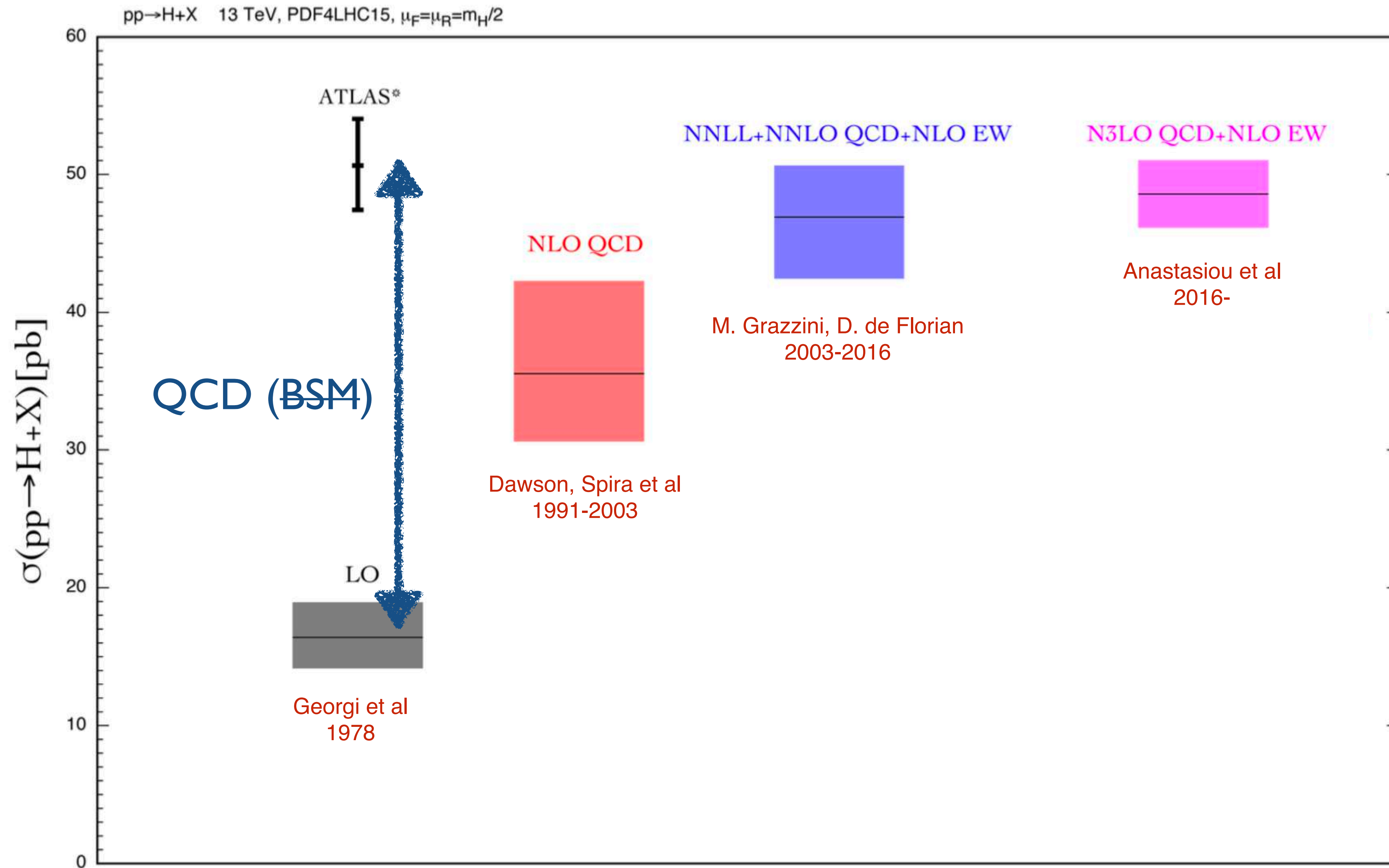
Non-Singlet available fully analytically Gehrmann, von Manteuffel, Sotnikov, Yang, (2026)

- ▶ Even some moments at 5 loops... Herzog et al (2019)

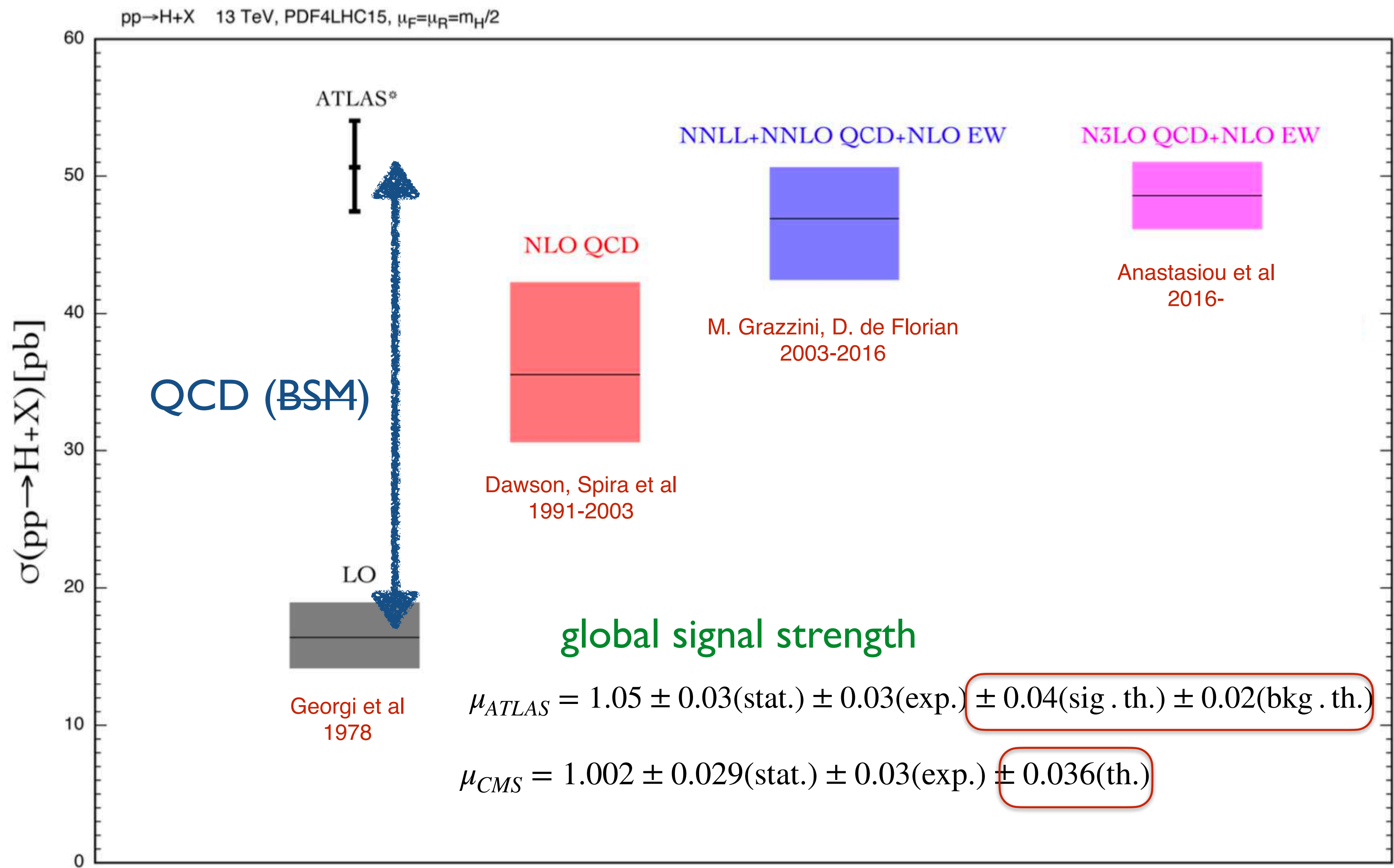
Inclusive Higgs : an example of precision



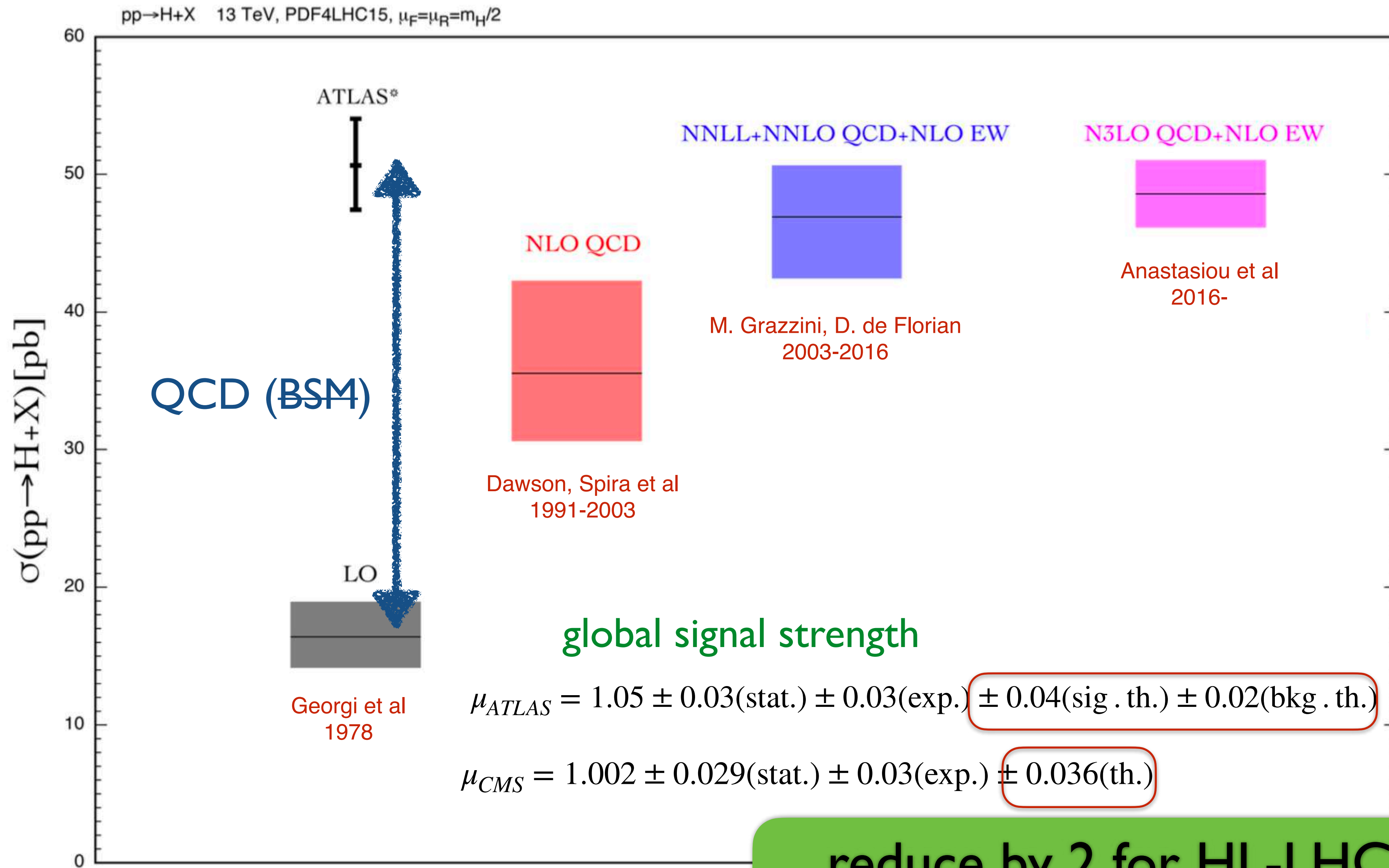
Inclusive Higgs : an example of precision



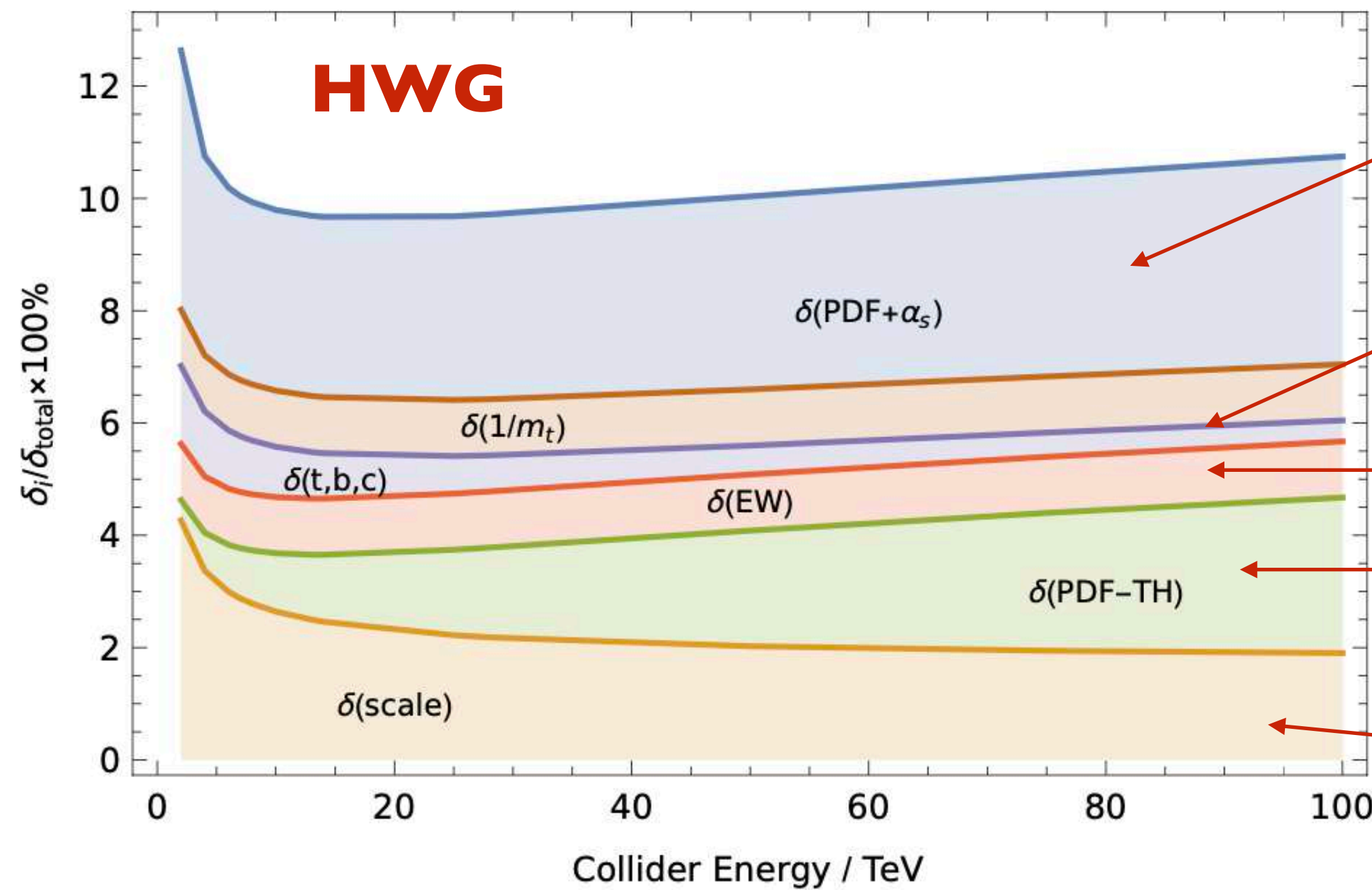
Inclusive Higgs : an example of precision



Inclusive Higgs : an example of precision



Inclusive Higgs : anatomy of precision



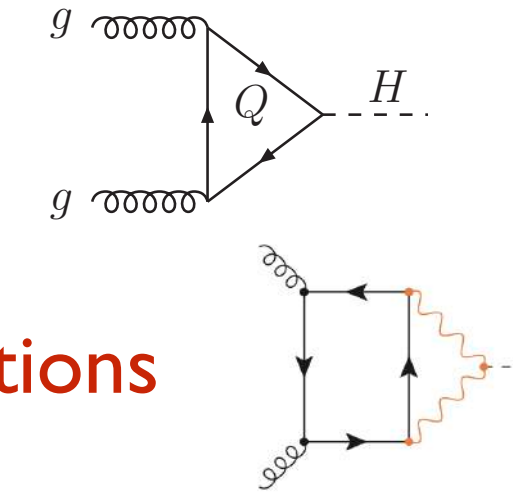
requires more precise pdf/coupling
HL-LHC + TH developments

Quark mass dependence

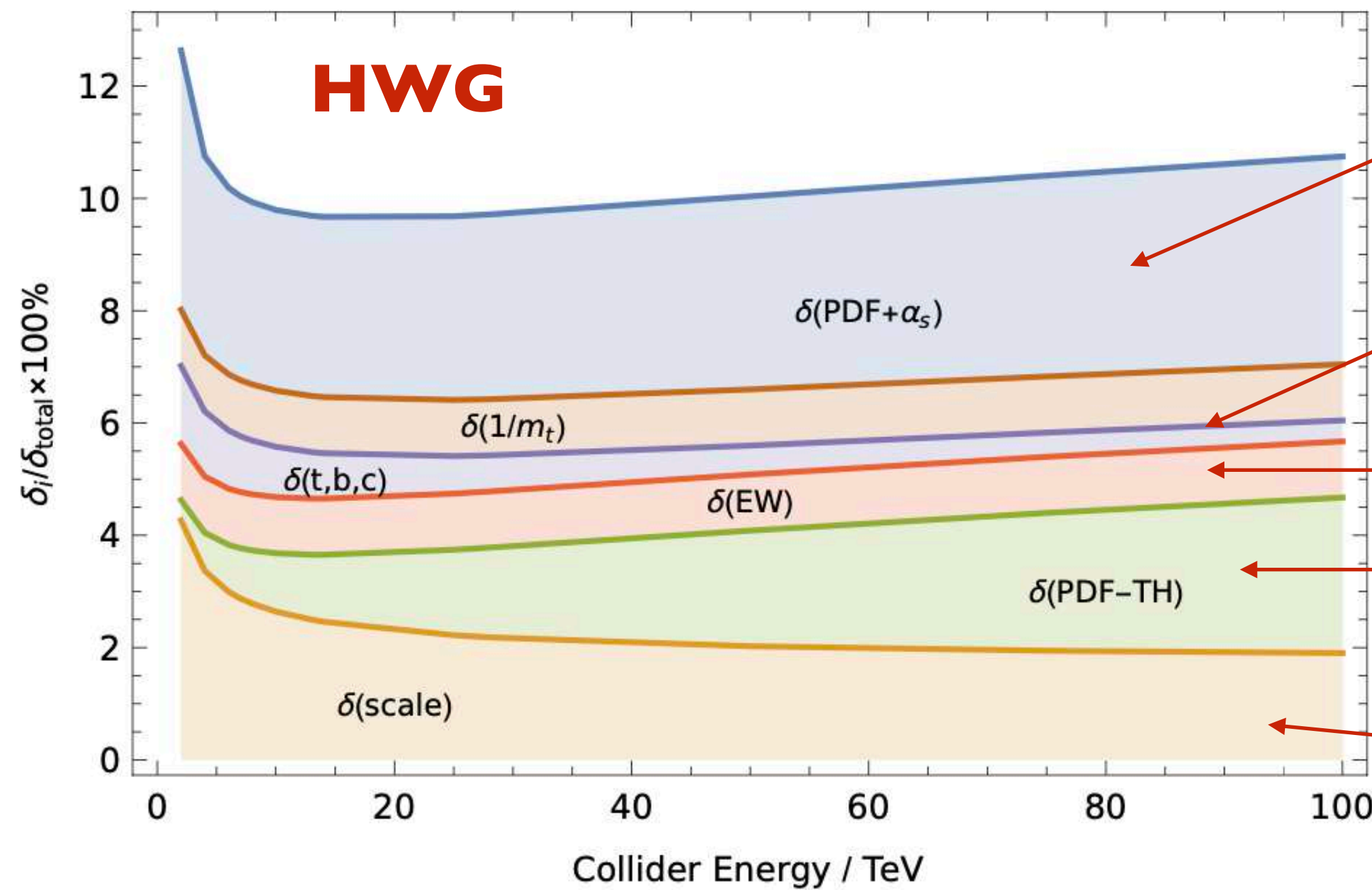
mixed EWxQCD corrections

requires N3LO pdfs
on the way, splitting functions

requires beyond N3LO calculation
Tough (ten years rule?)



Inclusive Higgs : anatomy of precision



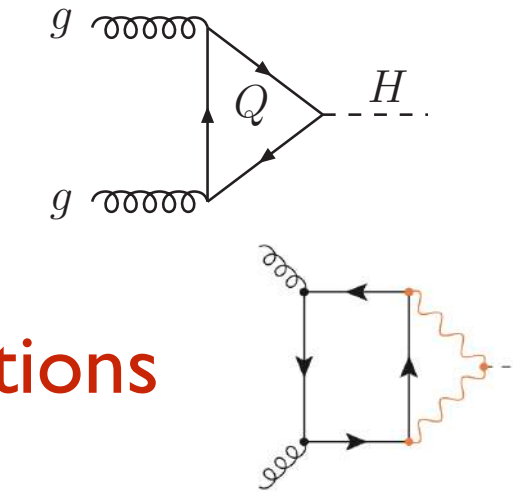
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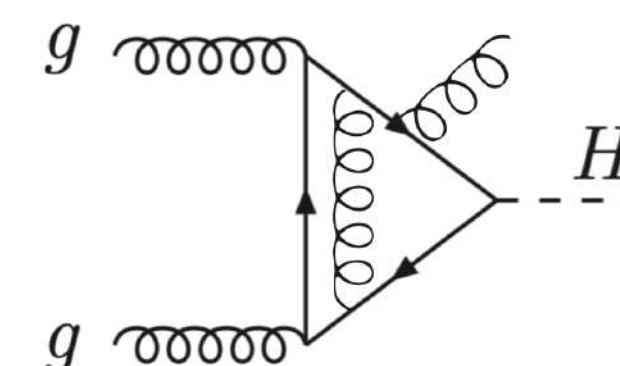
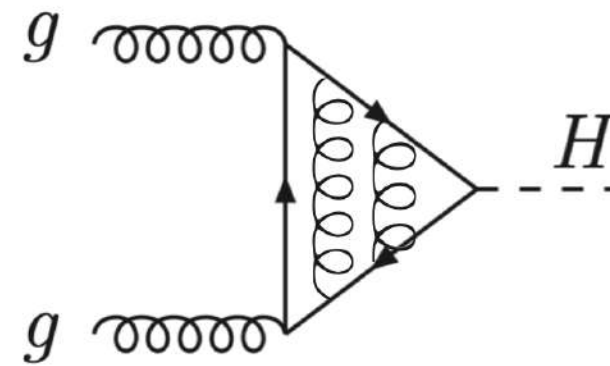
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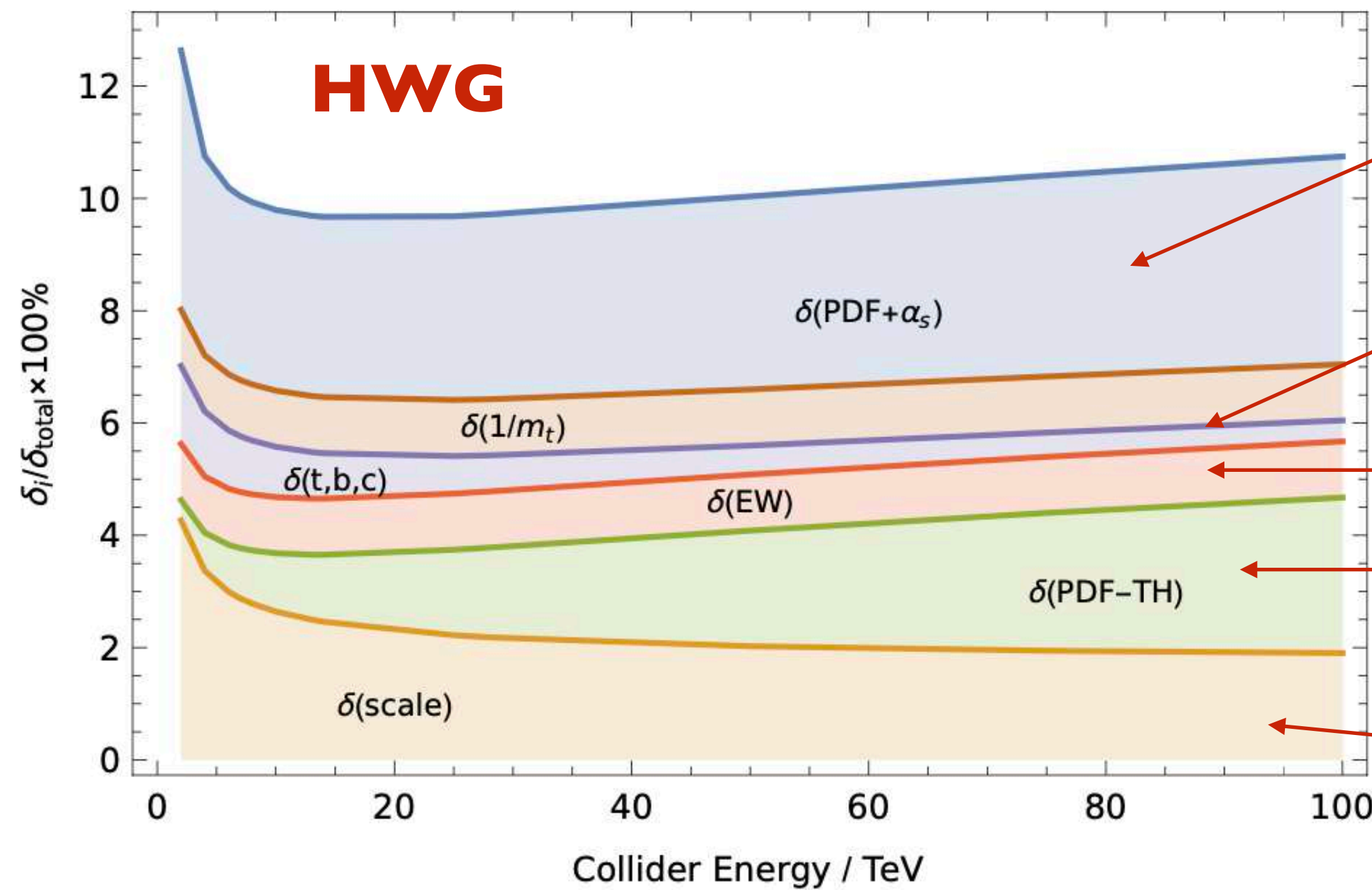


NNLO exact top mass dependence
-0.26% effect at 13 TeV
b and c on the way



Czakon, Harlander, Klapperty, Niggetiedt (2021)

Inclusive Higgs : anatomy of precision



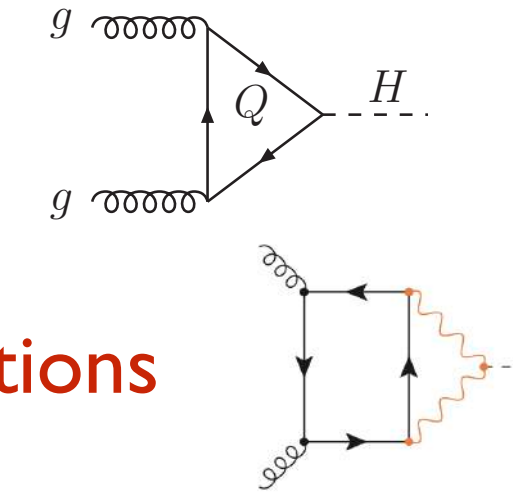
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Quark mass dependence

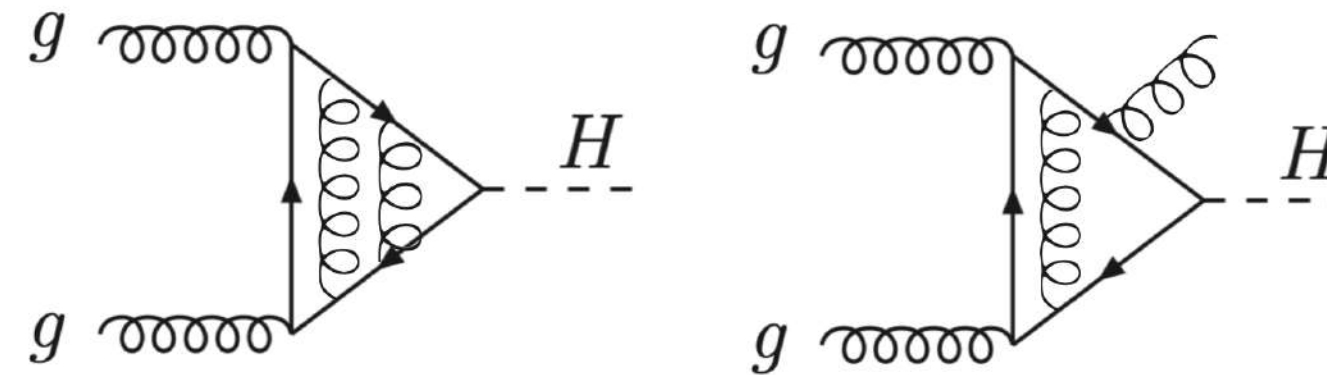
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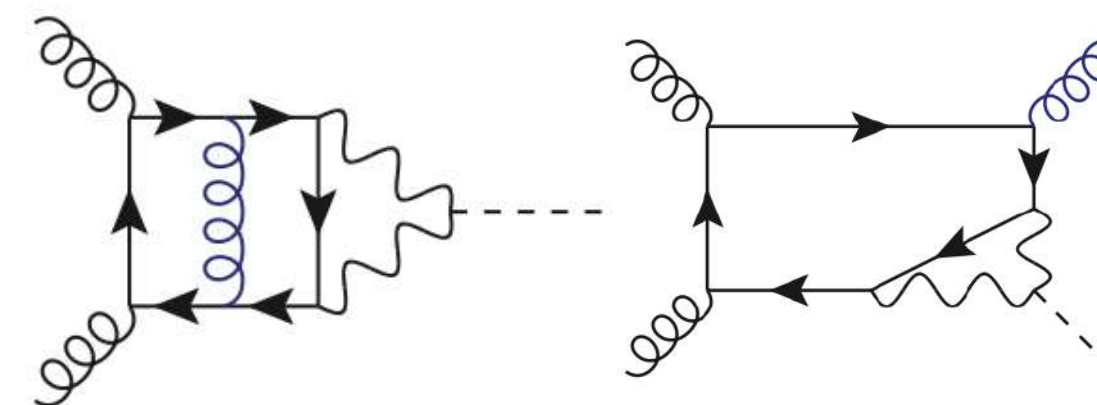


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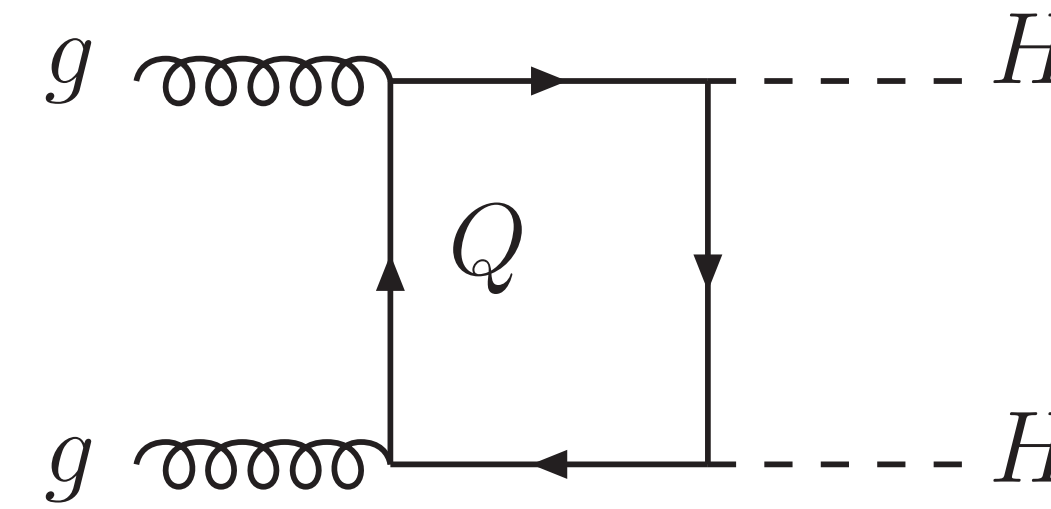
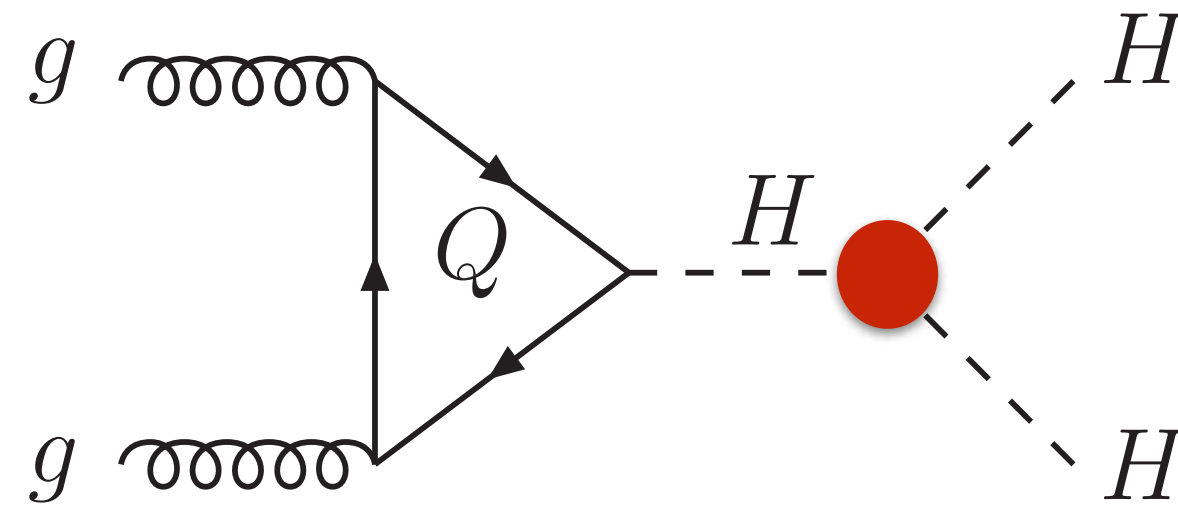
NLO QCDxEW (light quarks)
 $\alpha_s^3 \alpha^2$ 50% improvement
top on the way



$2.19 \pm 0.26 \text{ pb}$ vs $2.37 \pm 0.49 \text{ pb}$
New Old

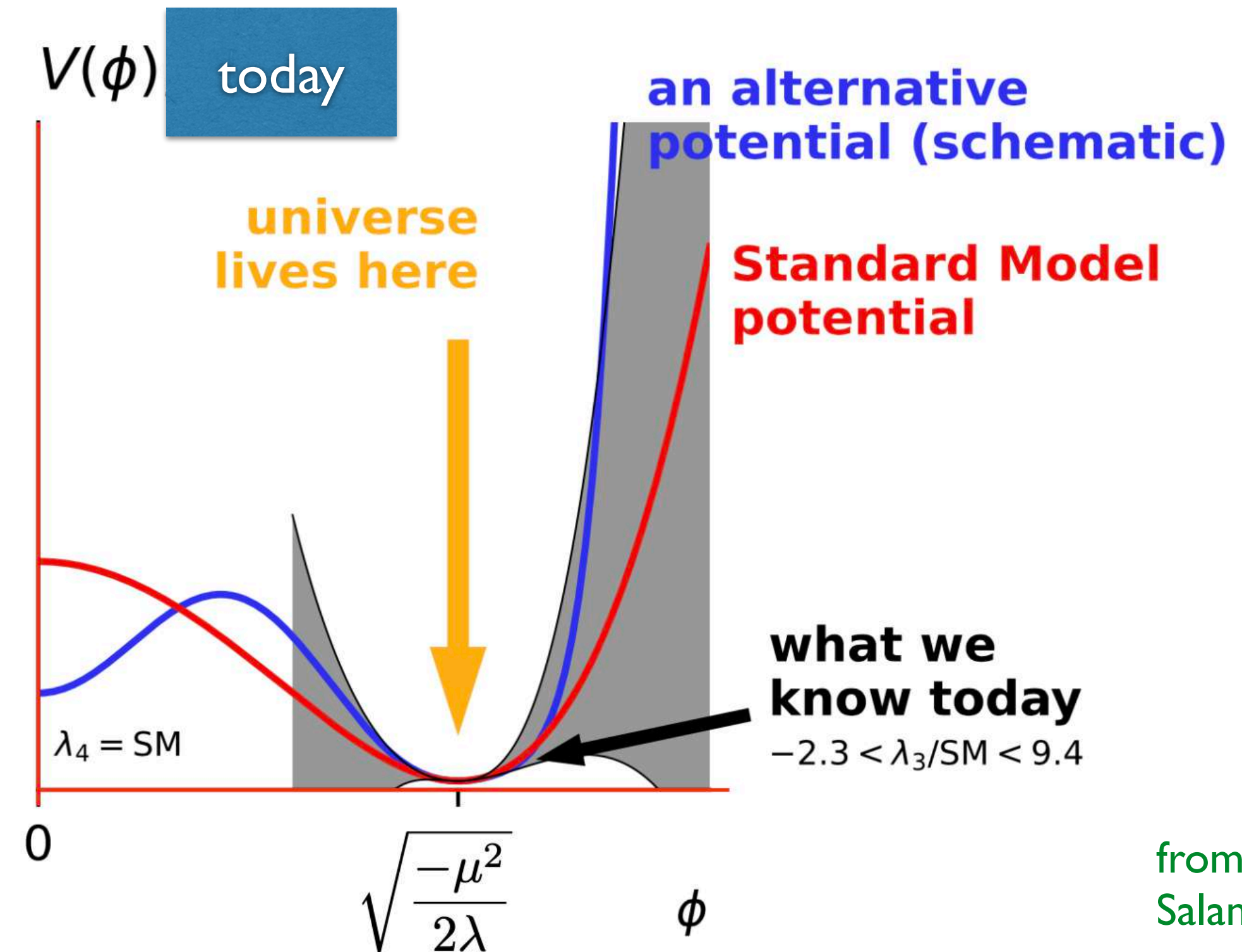
Becchetti, Bonciani, Del Duca, Hirschi, Moriello, Schweitzer (2020)

HH : The next discovery?



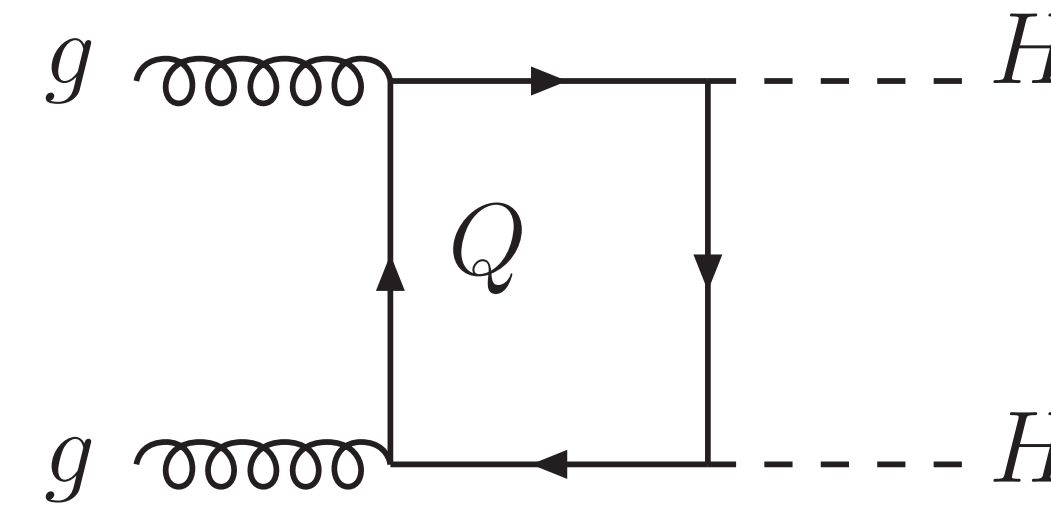
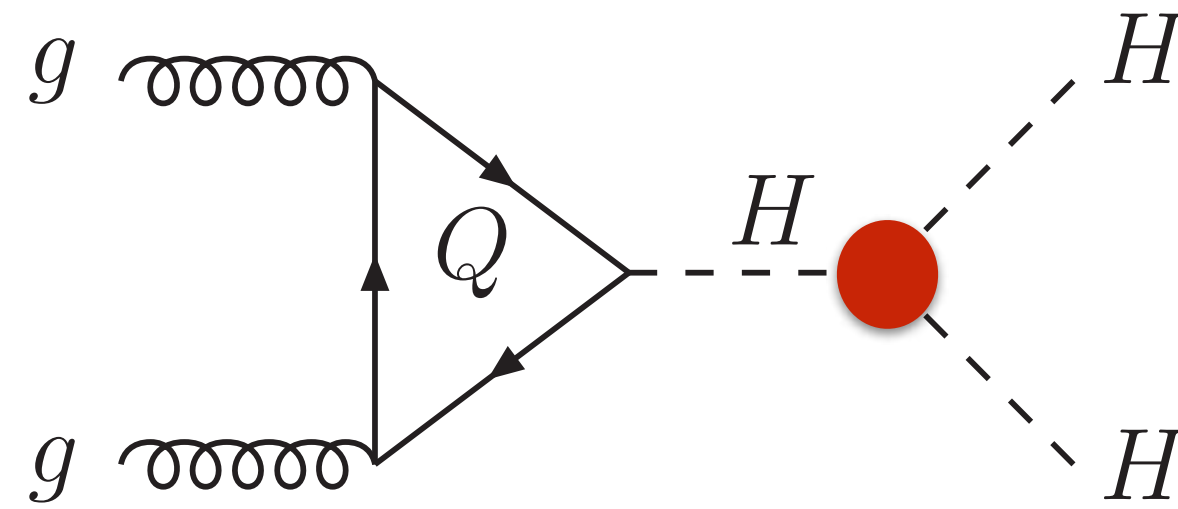
- Possibility to measure HHH coupling (and single-Higgs & no-Higgs)
- Directly check SSB mechanism

$$V = \frac{1}{2}(2\lambda v^2)H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4$$



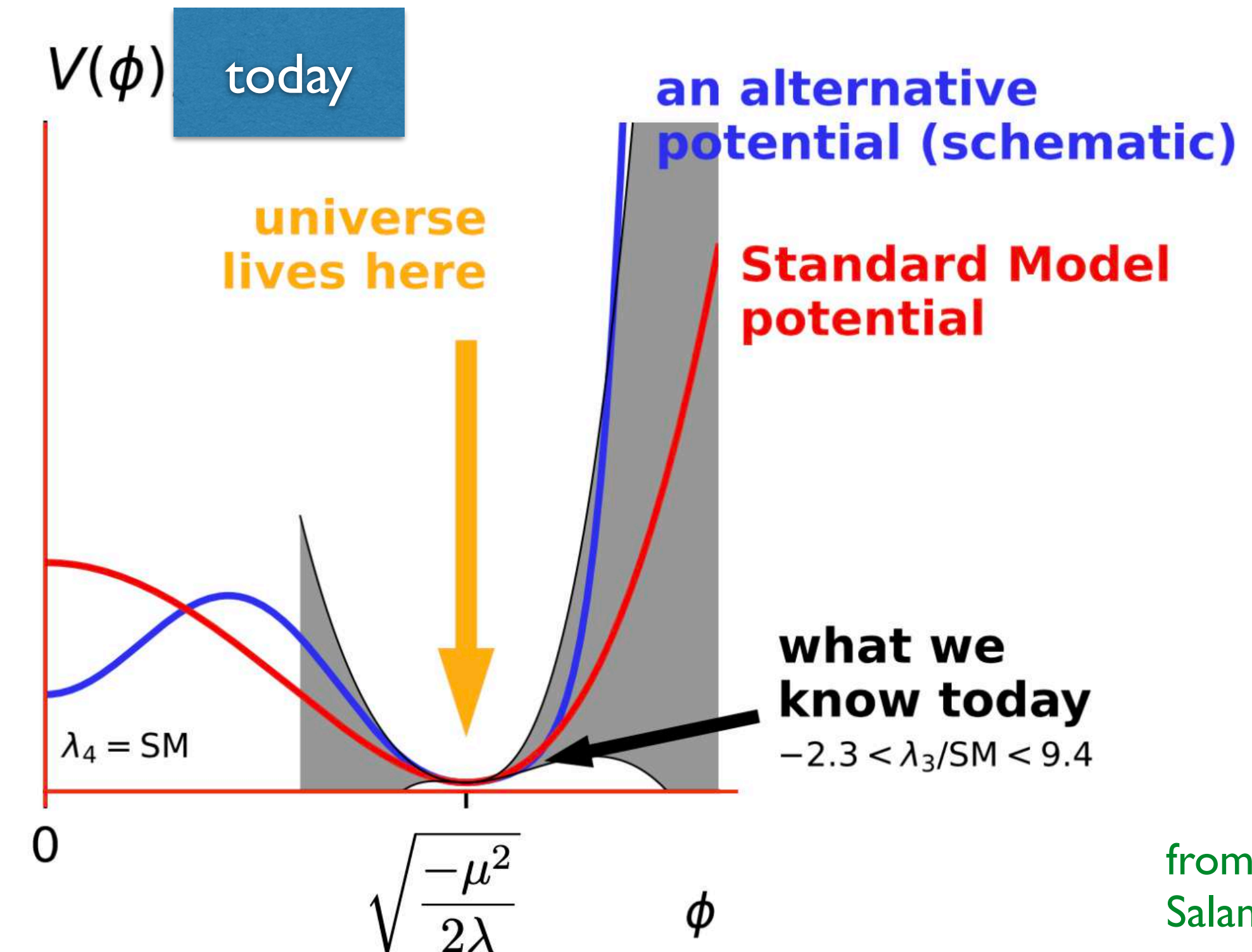
from G. Zanderighi,
Salam, Wang

HH : The next discovery?



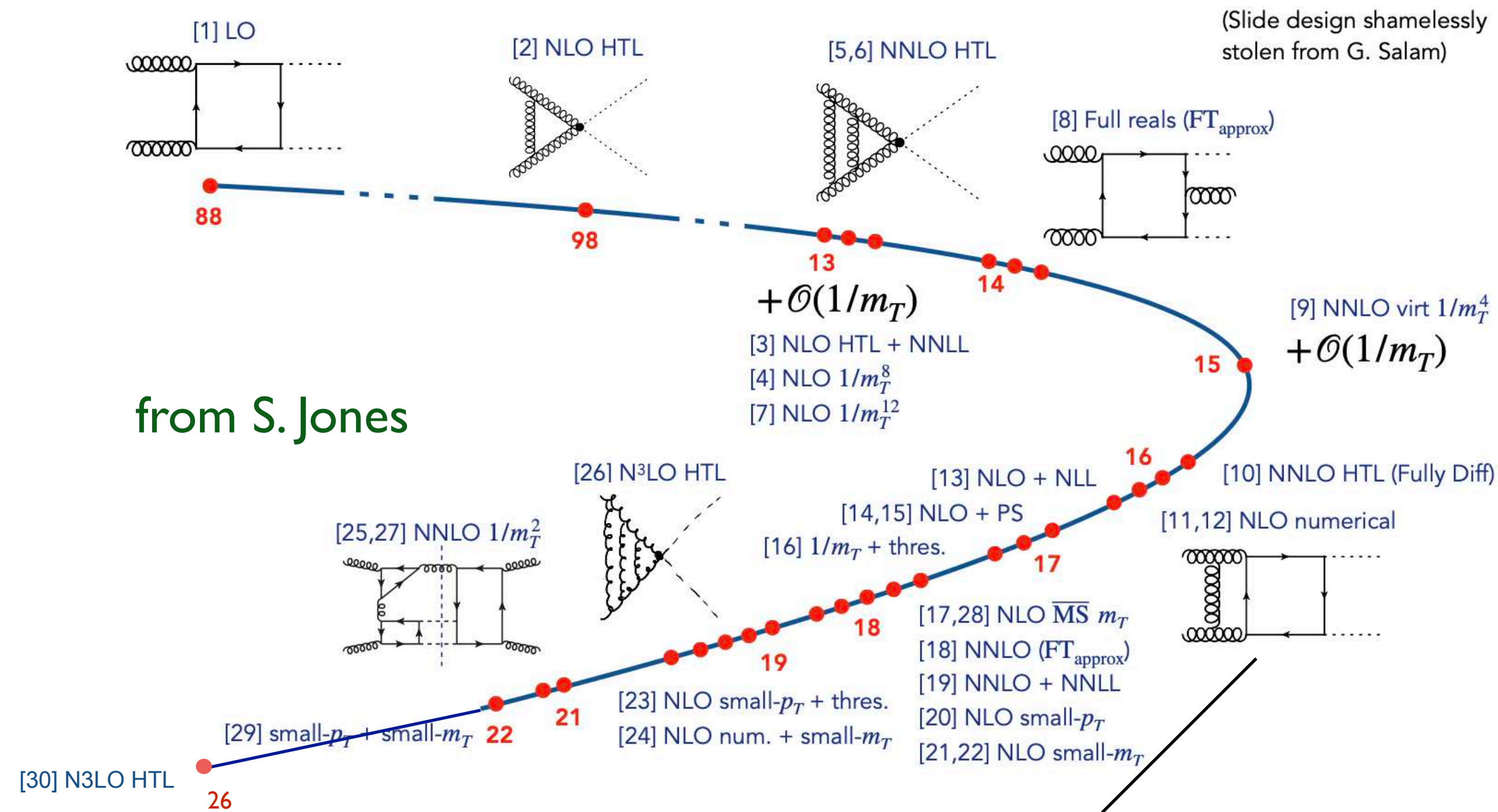
- Possibility to measure HHH coupling (and single-Higgs & no-Higgs)
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$$V = \frac{1}{2}(2\lambda v^2)H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4$$



from G. Zanderighi,
Salam, Wang

HH : precision (or not)

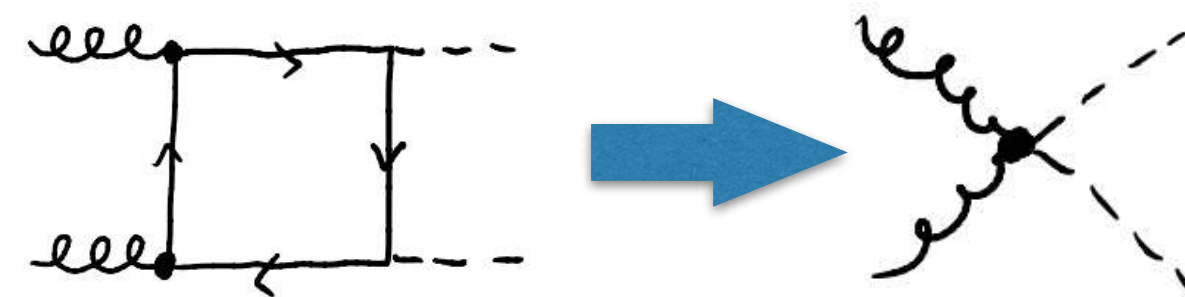


HWG

$\sqrt{s}$	14 TeV
$\sigma_{\text{NNLO FTapprox}} [\text{fb}]$	36.69
PDF unc.	$\pm 2.1\%$
α_S unc.	$\pm 2.1\%$
PDF+ α_S unc.	$\pm 3.0\%$
Scale unc.	-4.9% $+2.1\%$
m_{top} unc.	-18% $+4\%$
Scale+ m_{top} unc.	-23% $+6\%$
m_{top} approx. unc.	$\pm 2.7\%$

- High order calculations: $< 5\%$ uncertainty from scale variations... BUT

- Many based on HTL (only NLO exact)



HTL valid for $\sqrt{\hat{s}} \ll 2m_T$
 HH production $2m_H < \sqrt{\hat{s}}$

$\sim 3\%$ and much worse for distributions

- Large uncertainties from mass scheme and mass scale up to 18% !
- Important reduction by resumming logs using SCET (not full mass at NNLO yet)

Jaskiewicz, Jones, Szafron, Ulrich(2025)

Conclusions

- ▶ Amazing progress in calculations during the last two decades 
Driven by LHC
 - More precise/combined analysis of pdfs
 - Automation of NLO
 - Several NNLO processes $2 \rightarrow 2$ and already a few $2 \rightarrow 3$
 - Even N³LO for simpler kinematics and first set of splitting functions
 - Account for QED/EW effects
 - Resummation reaching N3LL
 - Parton showers with improved logarithmic accuracy
- ▶ But... **Reaching new bottlenecks for large multiplicities**
 - ▶ 2-loop amplitudes beyond $2 \rightarrow 3$
 - ▶ Real radiation far from trivial (**numerical infrared treatment**)
 - ▶ N³LO beyond Drell-Yan like processes require significant developments
- ▶ Higgs Pair Production will be fundamental : more work needed

Need a more rigorous treatment of TH uncertainties

Paremunpóc*

- ▶ So far, that was the traditional talk about precision in QCD
- ▶ But what kind of precision we are talking about in practice?
- ▶ And more important, is that feasible at the HL-LHC?

* ~catalán version of Paremos un poco

What is the SM today?

Effective Field Theory approach



$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & + i\bar{\psi}\not{D}\psi + h.c. \\ & + \psi_i y_{ij} \psi_j \phi + h.c. \\ & + |D_\mu \phi|^2 - V(\phi)\end{aligned}$$

} gauge

} Higgs: EWSB, vacuum stability
flavour problem

What is the SM today?

Effective Field Theory approach

Λ = scale of new physics

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & + i\bar{\psi}\not{D}\psi + h.c. \\ & + \psi_i y_{ij} \psi_j \phi + h.c. \\ & + |D_\mu \phi|^2 - V(\phi) \\ & + \frac{1}{\Lambda^2} \sum_i C_i^{(6)} \mathcal{Q}_i^{(6)} \\ & + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)\end{aligned}$$

} gauge

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} “Heavy” New Physics (large gap)

What is the SM today?

Effective Field Theory approach



$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & + i\bar{\psi}\not{D}\psi + h.c. \\ & + \psi_i y_{ij} \psi_j \phi + h.c. \\ & + |D_\mu \phi|^2 - V(\phi) \\ & + \frac{1}{\Lambda^2} \sum_i C_i^{(6)} \mathcal{Q}_i^{(6)} \\ & + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)\end{aligned}$$

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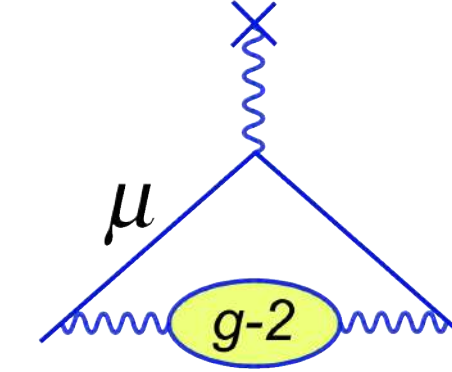
} “Heavy” New Physics (large gap)

Λ = scale of new physics

m = typical scale of the process

$$\mathcal{O} = \mathcal{O}_{SM} \left(1 + \mathcal{C} \left(\frac{m}{\Lambda} \right)^2 \right)$$

very high precision, very low mass $(g - 2)_\mu$



$$\Lambda = 2 \text{ TeV}$$

$$m = m_\mu = 105 \text{ MeV}$$

$$C \sim \mathcal{O}(1)$$

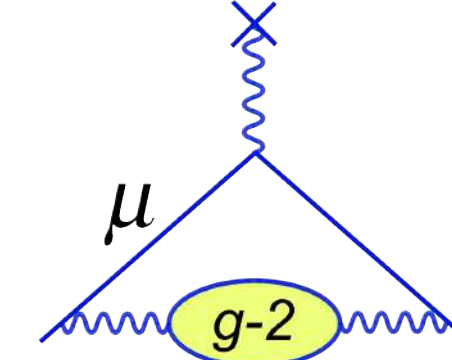
$$O = O_{SM} \left(1 + \mathcal{C} \left(\frac{m}{\Lambda} \right)^2 \right)$$

$$\Lambda = 2 \text{ TeV} \rightarrow \text{BSM} \sim 3 \times 10^{-9}$$

$$\Delta = 2 \times 10^{-10}$$

$$\Lambda \sim 7 \text{ TeV}$$

very high precision, very low mass $(g-2)_\mu$



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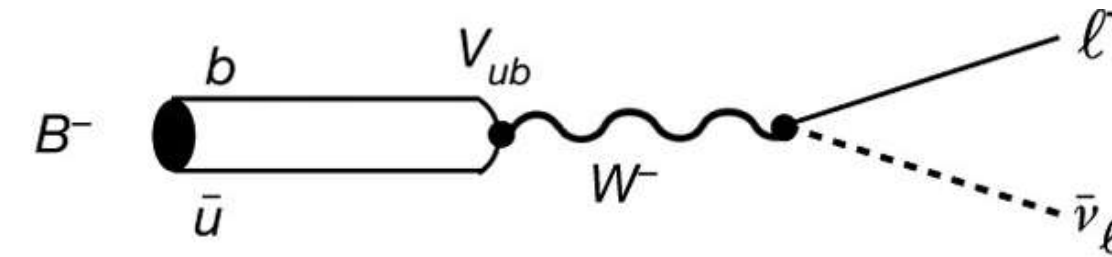
$$\Delta = 2 \times 10^{-10}$$

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high precision, low mass B meson

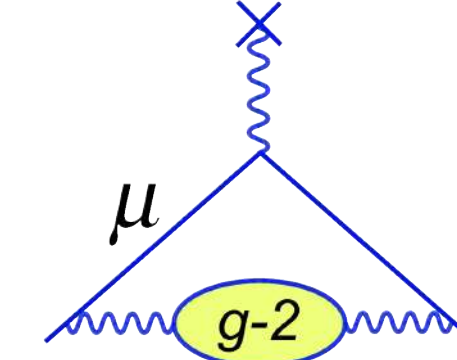
$$m = m_b = 4.2 \text{ GeV}$$

$$\text{BSM} \sim 4 \times 10^{-6}$$



very different
situation if $\text{SM}=0$!
but there is background

very high precision, very low mass $(g-2)_\mu$



$$\Lambda = 2 \text{ TeV}$$

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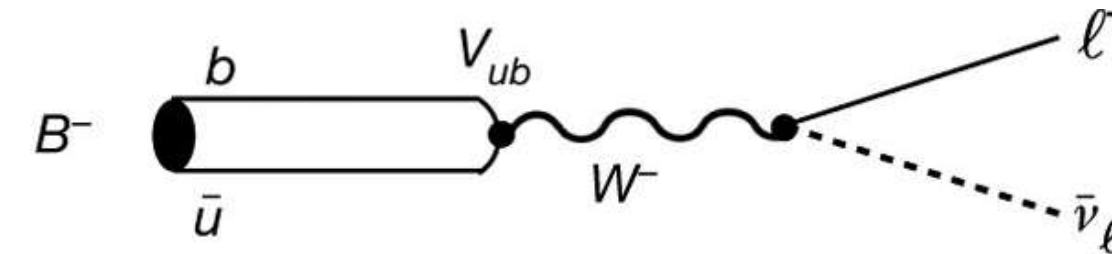
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precision, high mass

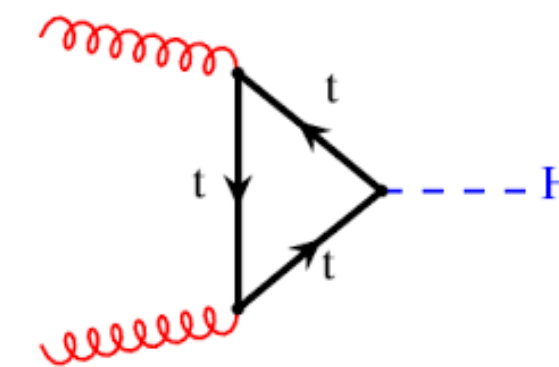
$$m = m_H = 125 \text{ GeV}$$

$$m = v = 246 \text{ GeV}$$

Single Higgs

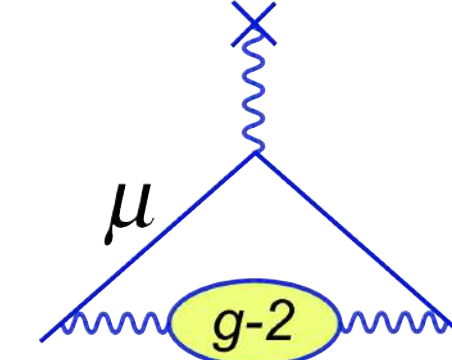
$$\text{BSM} \sim 0.5 \%$$

$$\text{BSM} \sim 1.5 \%$$



TH and EXP
can attempt it?

very high precision, very low mass $(g-2)_\mu$



$$\Lambda = 2 \text{ TeV}$$

$$m = m_\mu = 105 \text{ MeV}$$

$$C \sim \mathcal{O}(1)$$

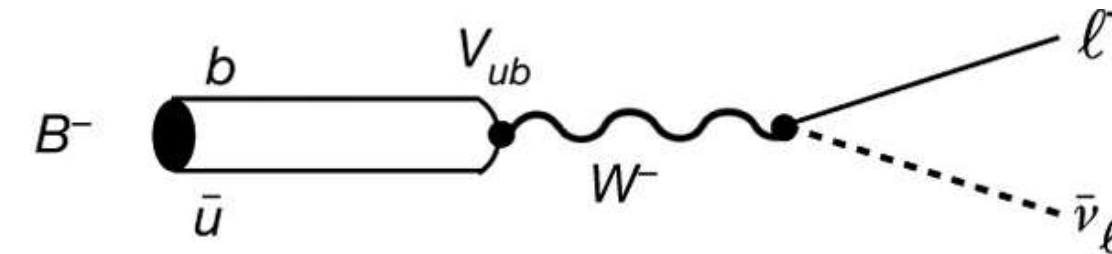
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high precision, low mass B meson



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precision, high mass

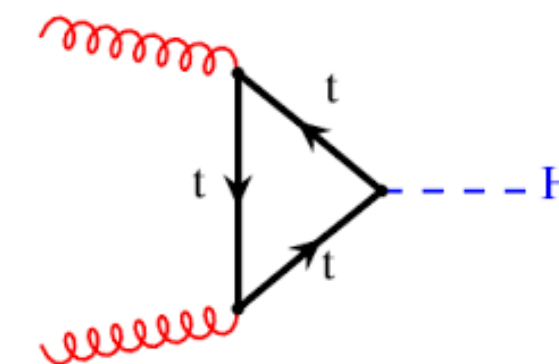
Single Higgs

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$$m = v = 246 \text{ GeV}$$

$$\text{BSM} \sim 1.5 \%$$

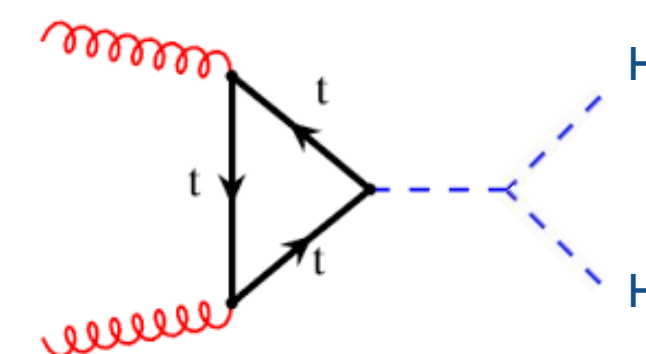


TH and EXP
can attempt it?

lower precision, higher mass Double Higgs

$$m > 2m_H = 250 \text{ GeV}$$

$$\text{BSM} \sim 2\%$$



TH can do it?
EXP impossible

higher scales (p_T) vs lower statistics

Only if $\mathcal{C}$ is very large and Λ rather small one can expect to see “some deviation”

$$O = O_{SM} \left(1 + \mathcal{C} \left(\frac{m}{\Lambda} \right)^2 \right)$$

Only if $\mathcal{C}$ is very large and Λ rather small one can expect to see “some deviation”

$$O = O_{SM} \left(1 + \mathcal{C} \left(\frac{m}{\Lambda} \right)^2 \right)$$

I hope I'm wrong but...

It is very likely that the HL-LHC will not find ANYTHING ELSE

(double Higgs production could be the “new discovery”)

Therefore, I think that...

